Non-topological solitons and the effects of an external field on them

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August/13/2025@Numerical Simulations of Early Universe Sources of Gravitational Waves

Siyao Li, MY, Ying-li Zhang, 2507.13276 [hep-ph]

(Many slides and figures are given by Siyao's courtesy !!)

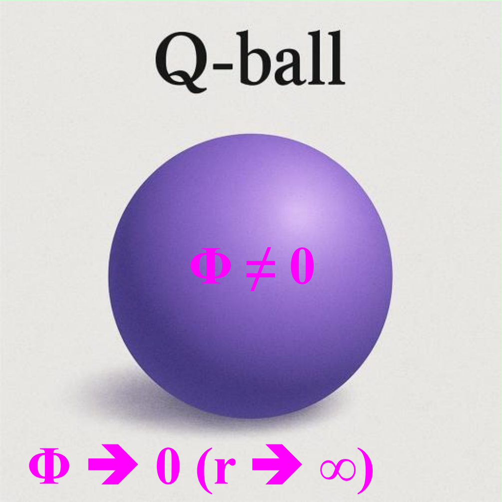
$c = \hbar = 1, \quad M_G = 1/\sqrt{8\pi G} \sim 2.4 \times 10^{18}\mathrm{GeV}.$

Contents

Q ball

What's Q ball ? (Coleman 1985)

Q-ball : stable, spherical symmetric, localized field configuration that can store the charge Q.



By ChatGPT

- **It usually forms in a complex scalar field with a global U(1) symmetry.**
- **Given fixed charge Q0, it is energetically stable and has a lower energy state than a collection of free particles.**
- **Charge conservation guarantees its stability.**

Why is Q ball so interesting ???

$\Rightarrow$

Affleck-Dine baryogenesis

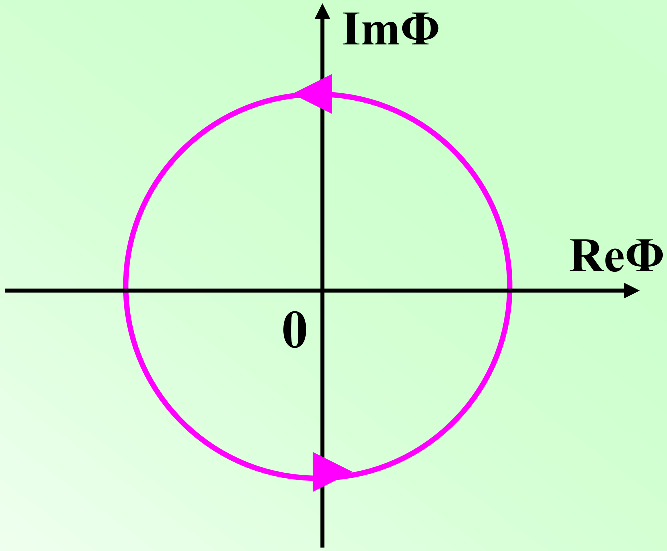
Scalar field condensation

(Affleck and Dine 1985)

Affleck-Dine field Φ:

A complex scalar field with baryon or lepton number q

$$\Rightarrow \quad n_B = iq\left(\dot{\Phi}^*\Phi - \Phi^*\dot{\Phi}\right) = 2q\,\omega\Phi_0^2 \quad \text{when} \quad \Phi = \Phi_0 e^{i\omega t}$$



Affleck-Dine field acquires angular momentum in field space ⇔ **B or L number is generated.**

Flat directions in a SUSY standard model

Some combinations of squark and/or slepton fields (AD field):

- **No classical potential in the SUSY exact limit (called flat directions).**
- **Often have baryon and/or lepton charge**
- **Lifted by SUSY breaking effects and the non-renormalizable terms**
- **Acquire a large amount of VEV during inflation**

$$V(\Phi) = (m_\Phi^2 - cH^2)|\Phi|^2 + \frac{|\Phi|^{2n-2}}{M^{2n-6}} + \frac{m_{3/2}}{nM^{n-3}}(a_m\Phi^n + a_m^*\Phi^{*n}) + \frac{H}{nM^{n-3}}(a_H\Phi^n + a_H^*\Phi^{*n})$$

soft mass **non-renormalizable terms** **A-term**

B or L is conserved **violate**

B or L is generated dynamically thanks to A-term when a flat direction has baryon and/or lepton charge.

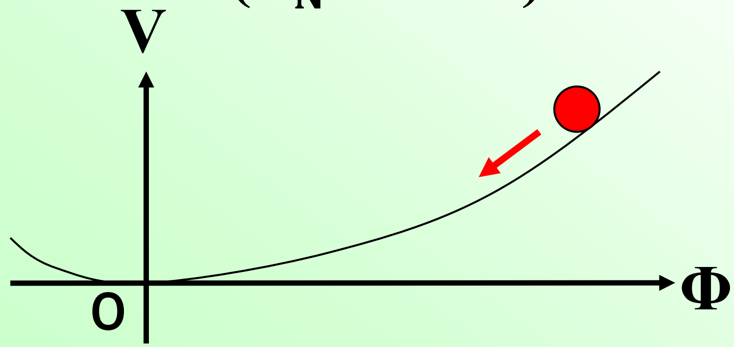
Cosmological scenario

(Dine, Randall, and Thomas 1995)

Φ acquires a large VEV during inflation thanks to large negative mass squared (- c H² |Φ|²). $(G_N V \sim H^2)$

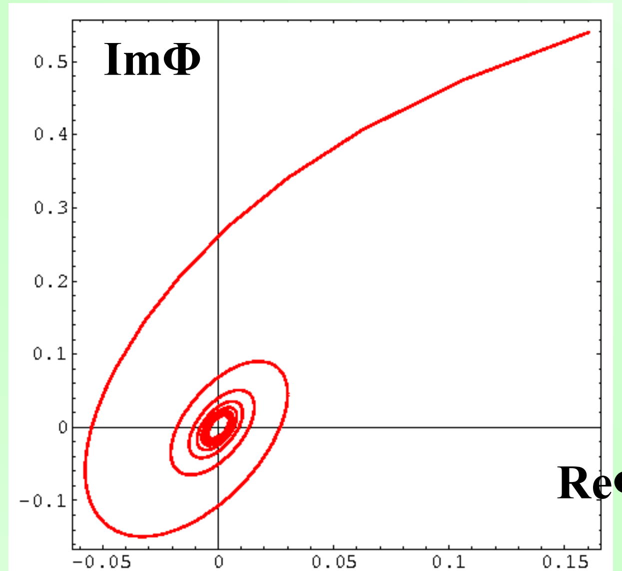
After inflation

It starts oscillation around the origin around $H \lesssim m_\Phi$.



At the same time, it starts rotation due to A-term and B and/or L number is produced.

Φ decays into quarks and/or leptons, the stored B or L number is transferred to them.



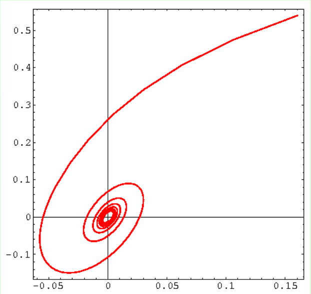
Taken from Fujii's master thesis

Q ball formation in cosmology

(Dvali et al. 1998, Kusenko & Shaposhnikov 1998, etc)

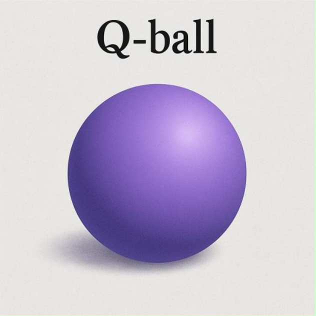
Which is energetically favored, given Q inside horizon?

- **Keeping a homogeneously rotating solution**



OR

- **A stable, spherical symmetric, localized field configuration called Q ball is formed**



What's the condition to form Q balls ???

Q-ball solution and existing conditions

Φ : a complex scalar field

$$\mathcal{L} = \eta^{\mu\nu}\partial_\mu\Phi^*\partial_\nu\Phi - V(|\Phi|)$$

- **What's the lowest energy configuration E, given fixed charge Q0 ???**

$$E = \int d^3x\,\mathcal{H} = \int d^3x\left(|\dot{\Phi}|^2 + |\nabla\Phi|^2 + V(|\Phi|)\right)$$

$$E_\omega = E + \omega\left[Q_0 - iq\int d^3x(\Phi^*\dot{\Phi} - \dot{\Phi}^*\Phi)\right]$$ with ω : Lagrange multiplier

$\frac{\delta E_\omega}{\delta\Phi} = 0$

$$= \int d^3x\left[\underline{|\dot{\Phi} - i\omega q\Phi|^2} + \underline{|\nabla\Phi|^2} + V(|\Phi|) - \omega^2q^2|\Phi|^2\right] + \omega Q_0$$

= 0 **(Phase of ϕ must be const => ϕ : real)**

$$\Phi(\boldsymbol{x}, t) = \frac{1}{\sqrt{2}}e^{iq\omega t}\phi(\boldsymbol{x})$$

$$E_\omega = \int d^3x\left[\frac{1}{2}(\nabla\phi(\boldsymbol{x}))^2 + \underbrace{V(\phi) - \frac{1}{2}\omega^2q^2\phi(\boldsymbol{x})}_{\equiv V_\omega(\phi)}\right] + \omega Q_0$$

Bounce solution

$$E_\omega = \int d^3x \left[\frac{1}{2}(\nabla\phi(\boldsymbol{x}))^2 + \underbrace{V(\phi) - \frac{1}{2}\omega^2 q^2 \phi^2(\boldsymbol{x})}_{\equiv V_\omega(\phi)}\right] + \omega Q_0$$

To find the minimum of Eω

$$\Rightarrow \quad \frac{\delta E_\omega}{\delta \phi(\boldsymbol{x})} = 0 \quad \Leftrightarrow \quad \textbf{3D bounce solution}$$

(Coleman et al. 1978)

$$\Rightarrow \quad \phi(\boldsymbol{x}) = \phi(r), \quad r = |\boldsymbol{x}|$$

$$\frac{\delta E_\omega}{\delta \phi(r)} = 0$$

$$\Rightarrow \quad \phi''(r) + \frac{2}{r}\phi'(r) - \frac{\mathrm{d}V_\omega}{\mathrm{d}\phi} = 0$$

$$\Leftrightarrow \quad \frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{2}\phi'^2(r) - V_\omega(\phi(r))\right] = -\frac{2}{r}\phi'^2(r)$$

with 2 B.C. $\phi(r \to \infty) = 0$ **to avoid the divergence of Q and Eω**

$\phi'(r = 0) = 0$ **to guarantee the regularity of ϕ at r = 0**

Shooting problem by regarding r as time !!

Q-ball existing conditions

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{2}\phi'^2(r) - V_\omega(\phi(r))\right] = -\frac{2}{r}\phi'^2(r) \ \ (< 0)$$

with 2 B.C. $\phi(r \to \infty) = 0$ **to avoid the divergence of Q and Eω**

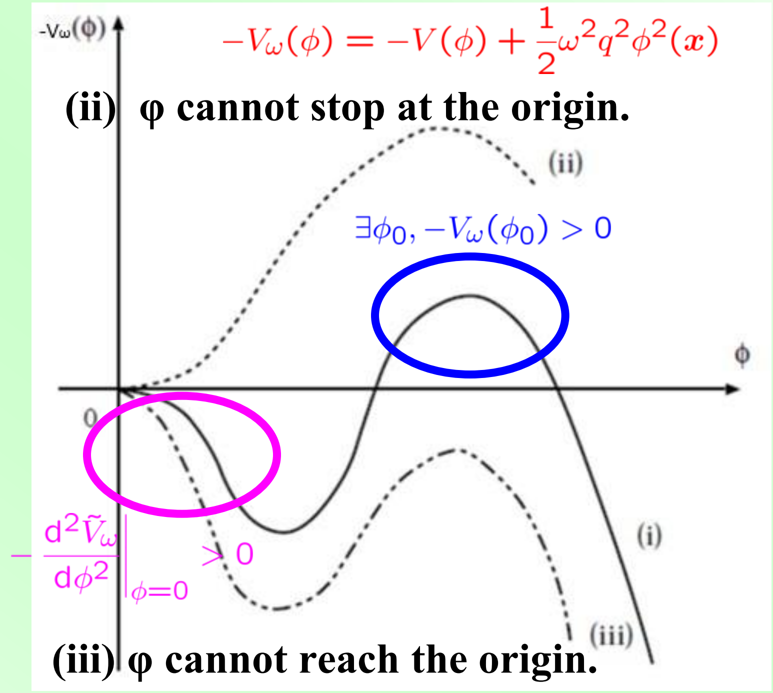
$\phi'(r = 0) = 0$ **to guarantee the regularity of ϕ at r = 0**

Observation:

For only case (i) : there exists a solution satisfying the boundary conditions.

⇕

$$\omega_0^2 \equiv \min_\phi \left(\frac{2V(\phi)}{\phi^2}\right) < \omega^2 q^2 < m^2 = \left.\frac{\mathrm{d}^2 V}{\mathrm{d}\phi^2}\right|_{\phi=0}$$



(Simply saying, the potential V(ϕ) should be flatter than quadratic one !!)

Generic properties

- **Topological defects :**

Gradient energy ~ Potential energy

$$\left(\frac{v}{R}\right)^2 \quad \sim \quad V(0) \ \left(\sim \lambda v^4\right) \quad \Rightarrow \quad R \sim \frac{1}{\sqrt{\lambda} v}$$

- **Q balls :**

Gradient energy ~ Potential energy ~ Kinetic (rotation) energy

$$\left(\frac{\phi}{R}\right)^2 \quad \sim \quad V(\phi) \ \left(\sim m^2\phi^2 \text{ or } M^4\right) \ \sim \ \omega^2\phi^2$$

$$\Rightarrow \quad Q \sim n_Q R^3 \sim \omega\phi^2 R^3 \sim \left(\frac{\phi}{m}\right)^2 \text{ or } \left(\frac{\phi}{M}\right)^4$$

Revised scenario

After inflation, around $H \lesssim m_\phi$, it starts oscillation around the origin.

At the same time, it starts rotation due to A-term and B and/or L number is produced.

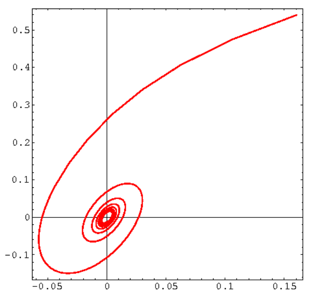


Figure taken from Kasuya & Kawasaki 2000

A non-topological soliton called Q ball is formed.

Q ball serves as DM !!

A part of baryon/lepton charge is striped off from Q ball.
=> Baryogenesis !!

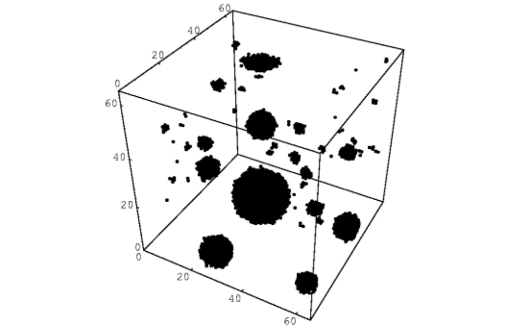


FIG. 1. Configuration of Q balls on three dimensional lattice. More than 30 Q balls are formed, and the largest one has the charge with $Q \simeq 1.96 \times 10^{16}$.

PLANCK results were released and are interesting

Constraints on scalar and tensor perturbations from the PLANCK satellite

Observational constraints :

$$
\begin{cases}
\Delta_\zeta(k_0) = 2.099^{+0.030}_{-0.029} \times 10^{-9}, \\
n_s = 0.9649 \pm 0.0042, \\
r < 0.10, \quad \text{(95\% CL TT,TE,EE+lowE+lensing)} \\
\text{at} \quad k_0 = 0.002\text{Mpc}^{-1}.
\end{cases}
$$

Theoretical predictions :

$$
\begin{cases}
\Delta_\zeta(k) \simeq \dfrac{1}{8\pi^2\epsilon}\left(\dfrac{H}{M_G}\right)^2, \\
n_s - 1 = \dfrac{d\ln\Delta_\zeta(k)}{d\ln k} \simeq -2\epsilon - 2\eta, \quad \left(\epsilon = -\dfrac{\dot{H}}{H^2}, \quad \eta = \dfrac{\dot{\epsilon}}{\epsilon H}\right) \\
\Delta_h(k) \simeq \dfrac{2}{\pi^2}\left(\dfrac{H}{M_G}\right)^2, \quad n_T = \dfrac{d\ln\Delta_h(k)}{d\ln k} \simeq -2\epsilon, \\
r \equiv \dfrac{\Delta_h(k)}{\Delta_\zeta(k)} \simeq 16\epsilon\,(= -8n_T)\,.
\end{cases}
$$

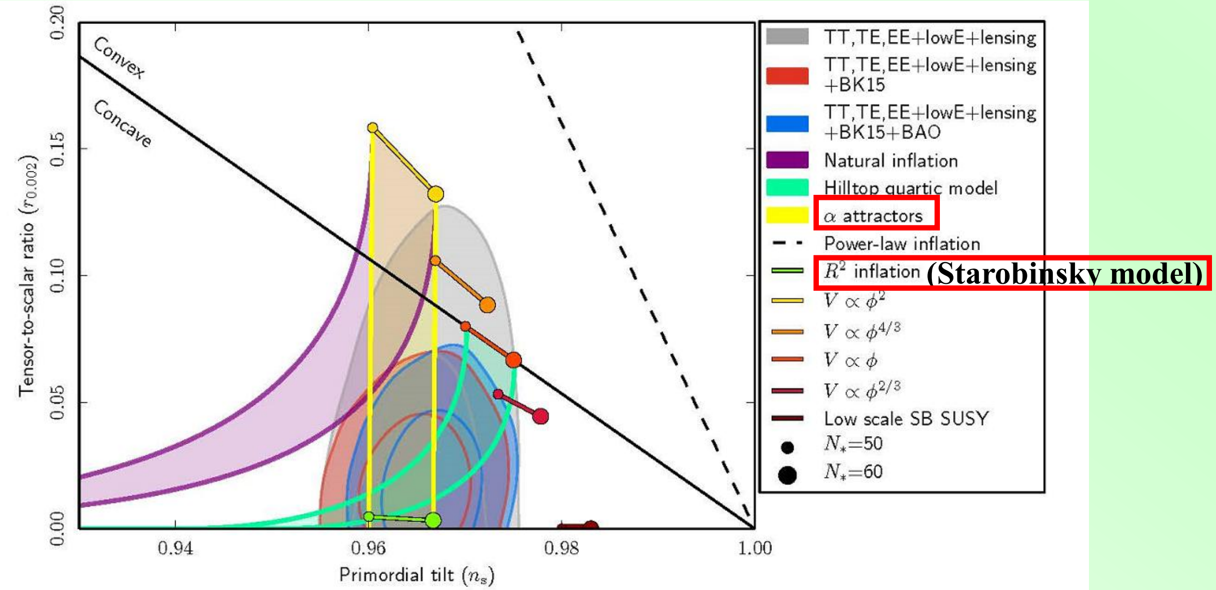


Fig. 8. Marginalized joint 68 % and 95 % CL regions for n_s and r at $k = 0.002\,\text{Mpc}^{-1}$ from *Planck* alone and in combination with BK15 or BK15+BAO data, compared to the theoretical predictions of selected inflationary models. Note that the marginalized joint 68 % and 95 % CL regions assume $dn_s/d\ln k = 0$.

Attractor models like Starobinsky model fit the data well.

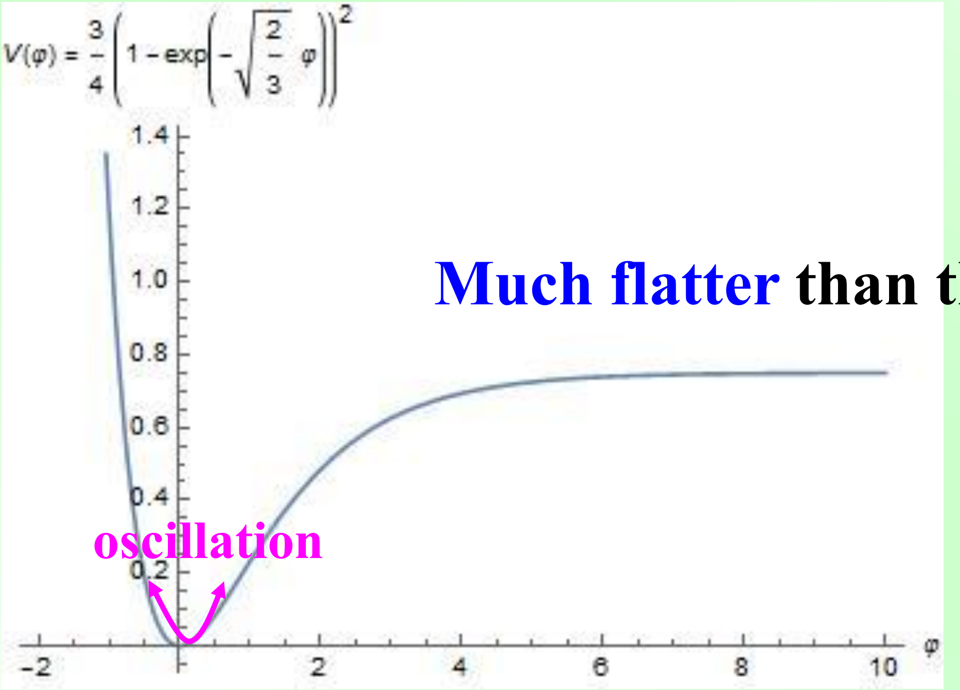
Planck 2018 results. X 1807.06211

R^2 (Starobinsky) model

(Starobinsky 1980)

($M_G = 1$)

$$\begin{cases} S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R + \frac{R^2}{12M^2}\right) \\ S_{\rm eq} = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{3M^2}{4}\left(1 - \exp^{-\sqrt{2/3}\,\varphi}\right)^2\right] \end{cases}$$



Much flatter than the mass term, φ^2

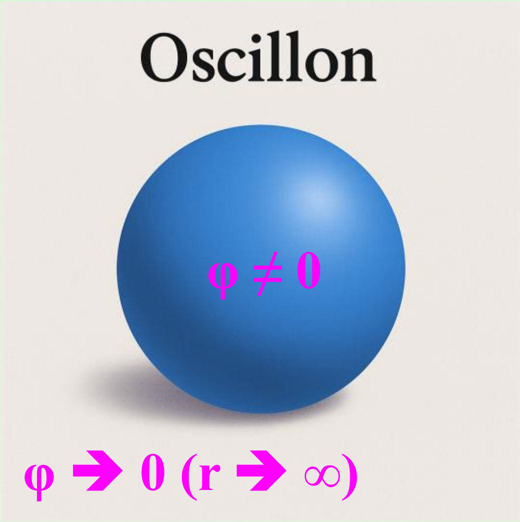
- **During the oscillation period, can a non-topological soliton like Q ball be formed in attractor models ???**
- **But, apparently, there is no conserved charge for a real scalar field !!**

Oscillon/I ball

What's Oscillon/I ball ?

(Gleiser 1994, Copeland et al, 1995, Kasuya et al. 2002, etc)

Oscillon/I ball : ***long-lived*** **($\tau \sim 10^{\lesssim 11} m^{-1}$), spherical symmetric, localized field configuration**



By ChatGPT

- **Oscillons/I balls could be formed after inflation.**
- **It usually forms in a real scalar field with (approximately) adiabatic conserved quantity.**
- **Given fixed adiabatic quantity I0, it is energetically favored.**

Oscillon/I ball solution

Oscillon/I ball as analogy of Q ball :

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - V_{\mathrm{nl}}(\phi), \qquad \left(V_{\mathrm{nl}}(\phi) = \sum_{n\geq 3} \frac{g_n}{n}\phi^n \right)$$

ϕ : a real scalar field,

ϕ is regarded as real part of a complex scalar Φ : $\phi = \mathrm{Re}(\Phi)$

(Mukaida & Takimoto 2014, etc)

$$\Rightarrow \quad V_{\mathrm{nl}}(\phi) = V_{\mathrm{nl}}(\mathrm{Re}(\Phi)) = V_{U(1)}(|\Phi|) + V_B(\Phi, \Phi^*)$$

e.g.

$$V_{\mathrm{nl}}(\phi) = -\frac{g_4}{4}\phi^4 + \frac{g_6}{6}\phi^6$$
$$V_{U(1)}(|\Phi|) = -\frac{3g_4}{8}|\Phi|^4 + \frac{5g_6}{24}|\Phi|^6$$
$$V_B(\Phi, \Phi^*) = -\frac{g_4}{16}\left(\Phi^4 + \Phi^{*4}\right) + \frac{g_6}{16}|\Phi|^2\left(\Phi^4 + \Phi^{*4}\right)$$

Equation of motion:

$$\begin{aligned} 0 &= (\Box + m^2)\,\mathrm{Re}(\Phi) + V'_{\mathrm{nl}}(\mathrm{Re}(\Phi)) \\ &= \mathrm{Re}\left[(\Box + m^2)\Phi + \frac{\partial V_{U(1)}(|\Phi|)}{\partial \Phi^*} + \frac{\partial V_B(\Phi, \Phi^*)}{\partial \Phi^*}\right] \end{aligned}$$

If we can neglect the violating term, the EOM approximately respects $U(1)$ symmetry.

Non-relativistic limit

Expand Φ(t,x) into rapidly oscillating parts e^{-imt} & δΦ and a slowly varying envelope Ψ:

$$\Phi(t,\mathrm{x}) = e^{-imt}\,\Psi(t,\mathrm{x}) + \delta\Phi(t,\mathrm{x})$$

$|\delta\Phi| \ll |\Psi|, \quad |\ddot{\Psi}| \ll m|\dot{\Psi}| \ll m^2|\Psi|, \quad |\nabla\Psi| \ll m|\Psi|, \quad$ (slow variation) **Adiabatic invariance**

$$0 = (\Box + m^2)\Phi + \frac{\partial V_{U(1)}(|\Phi|)}{\partial\Phi^*} + \frac{\partial V_B(\Phi,\Phi^*)}{\partial\Phi^*}$$
$$\simeq e^{-imt}\left[-2im\frac{\partial\Psi}{\partial t} - \nabla^2\Psi + \frac{\partial V_{U(1)}(|\Psi|)}{\partial\Psi^*}\right] + \cancel{\frac{\partial V_B(e^{-imt}\Psi, e^{imt}\psi)}{\partial\Phi^*}}$$

Rapid oscillating parts (more than e^{imt}) averaged to be zero over a period

$$\mathcal{L}_{\mathrm{NR}} = \left|\dot{\Psi} - im\Psi\right|^2 - |\nabla\Psi|^2 - m^2|\Psi|^2 - V_{U(1)}(|\Psi|) \qquad \mathcal{L}$$

$$\left(\mathcal{E} = |\dot{\Psi} - im\Psi|^2 + |\nabla\Psi|^2 + m^2|\Psi|^2 + V_{U(1)}(|\Psi|)\right) \qquad \mathcal{H}$$

LNR respects U(1) symmetry : $\Psi \to \Psi' = e^{i\alpha}\Psi$

(Approximate) Charge Conservation

$$\mathcal{L}_{\rm NR} = \left|\dot\Psi - im\Psi\right|^2 - |\nabla\Psi|^2 - m^2|\Psi|^2 - V_{U(1)}(|\Psi|)$$

$\mathcal{L}_{\rm NR}$ respects U(1) symmetry : $\Psi \to \Psi' = e^{i\alpha}\Psi$

$\Rightarrow$ $Q = -i\int d^3x \left[\Psi(\dot\Psi^\dagger + im\Psi^\dagger) - \Psi^\dagger(\dot\Psi - im\Psi)\right]$ **is conserved**

$\Rightarrow$ **Oscillon : "Q ball" solution minimizing E for given Q_0**

$$\left(\mathcal{E} = |\dot\Psi - im\Psi|^2 + |\nabla\Psi|^2 + m^2|\Psi|^2 + V_{U(1)}(|\Psi|)\right)$$

$\Leftarrow$ $I_{Q_0} = \int d^3x\,\mathcal{E} + \omega(Q_0 - Q)$ with ω : Lagrange multiplier

$$\frac{\delta I_{Q_0}}{\delta\Psi} = 0 \qquad = \int d^3x \left[\underline{|\dot\Psi - i(m-\omega)\Psi|^2} + \underline{|\nabla\Psi|^2} + (m^2-\omega^2)|\Psi|^2 + V_{U(1)}(|\Psi|)\right] + \omega Q_0$$

$\Rightarrow$ $\Psi(t,\boldsymbol{x}) = e^{i\mu t}\psi(\boldsymbol{x}),\;\; \mu \equiv m - \omega \ll m$

$\Rightarrow$ **Bounce solution :** $\psi(\boldsymbol{x}) = \psi(r)$, $\quad \left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right)\psi + (\omega^2 - m^2)\psi + \frac{1}{2}\frac{\mathrm{d}V_{U(1)}}{\mathrm{d}\psi} = 0$

$\Leftarrow$ **Existing conditions :** $\min_\psi\left(\frac{V_{U(1)}(\psi)}{\psi^2}\right) < \omega^2 - m^2 < 0$

Oscillon from Adiabatic Invariance

(Kasuya et al. 2002, etc)

What's the physical meaning of Q $\left(= 8\pi\omega \int dr\, r^2\, \psi^2(r)\right)$ **?**

- **Periodic system characterized by some parameter λ(t) satisfying**

$$\left|\frac{\dot{\lambda}}{\lambda}\right| \ll T^{-1}$$

(T : period)

⇒ **Adiabatic invariant :** $I \equiv \frac{1}{2\pi} \sum_i \oint p_i\, dq_i$

- **Scalar field system :** $\mathcal{L} = \frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi - V(\phi, \lambda), \quad \lambda = \lambda(t, \mathbf{x})$: external parameter

⇒ $$I = \frac{1}{2\pi} \int \mathrm{d}^3x \oint \dot{\phi}\, d\phi = \frac{1}{2\pi} \int \mathrm{d}^3x \int_0^T \mathrm{d}t\, \dot{\phi}^2 = \frac{1}{\omega} \int \mathrm{d}^3x\, \overline{\dot{\phi}^2}$$

$$\left(\overline{\dot{\phi}^2} \equiv \frac{1}{T} \int_0^T \mathrm{d}t\, \dot{\phi}^2 \quad \text{where} \quad \omega = \frac{2\pi}{T}\right)$$

$\phi = \mathrm{Re}(\Phi) \simeq 2\psi(r)\cos(\omega t)$

⇒ $$I = 8\pi\omega \int dr\, r^2\, \psi^2(r) = Q$$

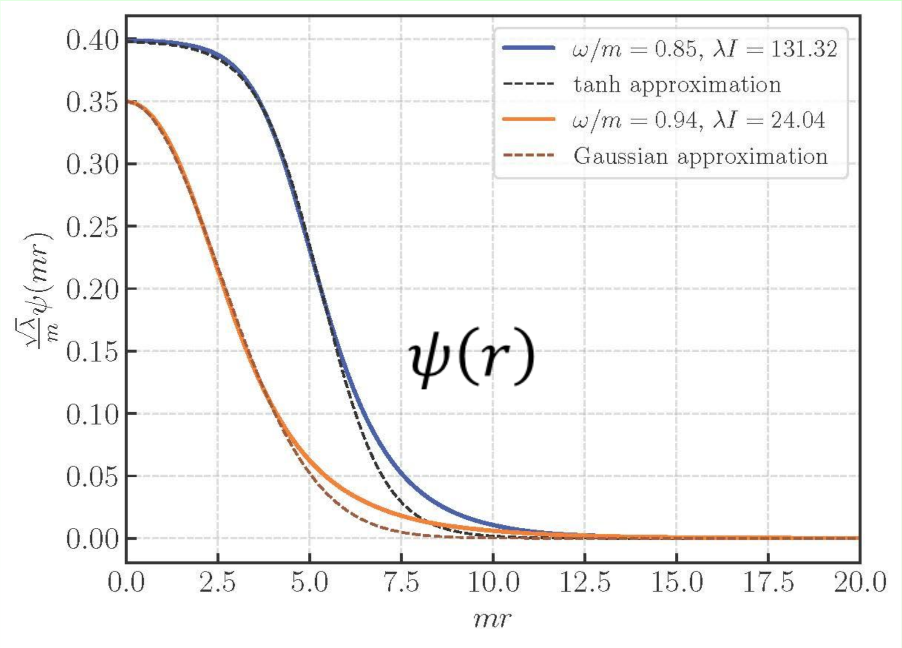
Oscillon/I ball is *long-lived* thanks to this adiabatic invariance.

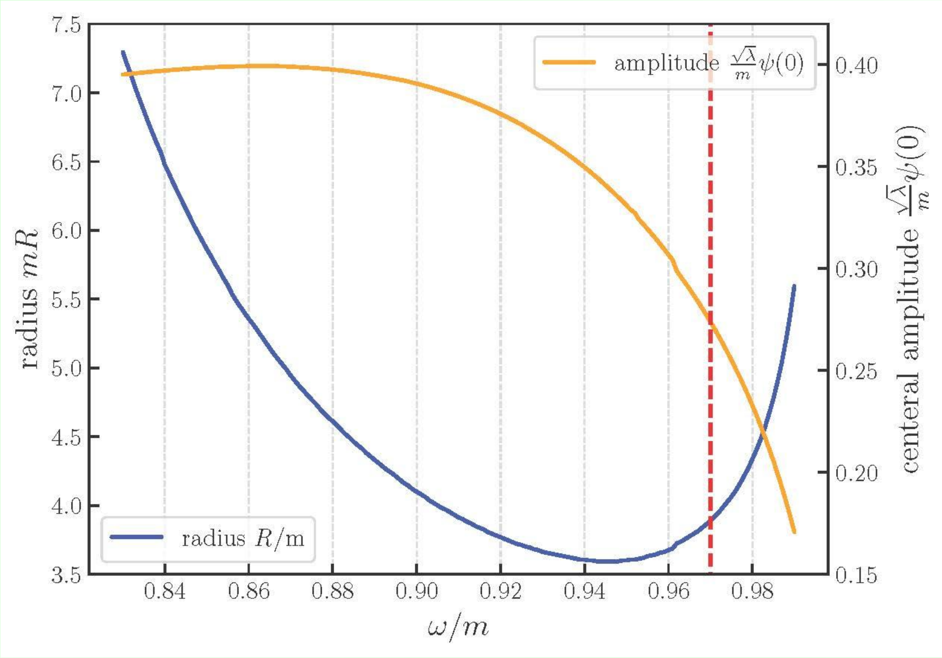
Oscillon configurations

$$\phi(t,\mathbf{x}) \simeq 2\psi(r)\cos(\omega t)$$

EOM : $\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right)\psi+(\omega^2-m^2)\psi+\frac{1}{2}\frac{\mathrm{d}V_{\mathrm{eff}}}{\mathrm{d}\psi}=0,\qquad V_{\mathrm{eff}}(\psi)=\overline{V_{\mathrm{nl}}(\phi)}$

$$V_{\mathrm{nl}}(\phi)=-\lambda\phi^4+g_6\phi^6,\quad g_6 m^2/\lambda=\frac{16}{15}$$





Radius shrinks first and amplitude decreases when oscillon becomes small.

End of Oscillon

- **Adiabatic charge :**

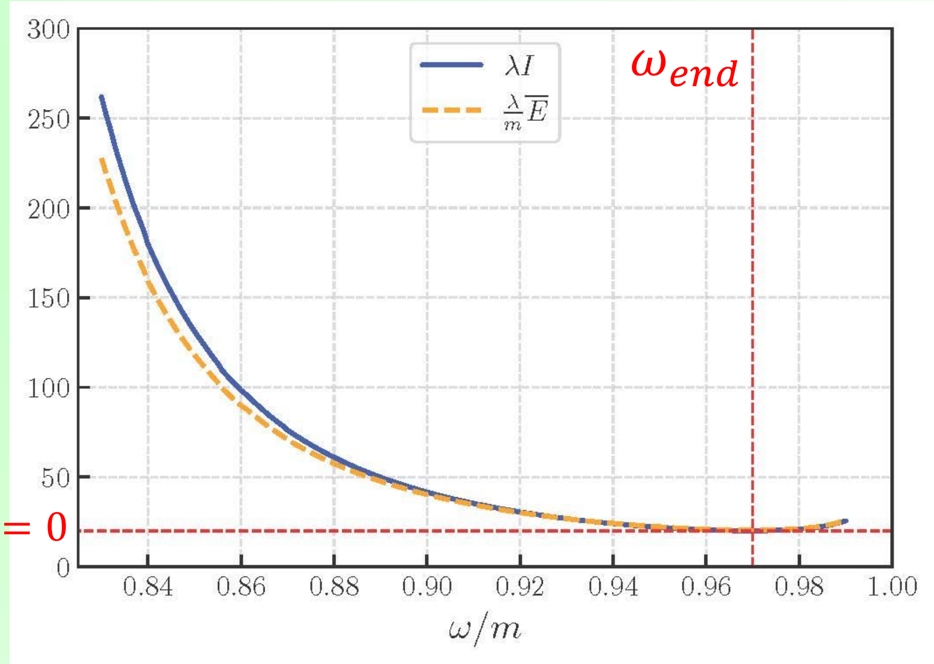
$$I \equiv \frac{1}{\omega}\int d^3x\,\overline{\dot{\phi}^2} = 8\pi\omega\int dr\,r^2\psi(r)^2$$

- **Time-averaged energy :**

$$\overline{E} = \int d^3x\left(\frac{1}{2}\overline{\dot{\phi}^2} + \frac{1}{2}\overline{(\nabla\phi)^2} + \frac{1}{2}m^2\overline{\phi^2} + \overline{V_{\rm nl}(\phi)}\right)$$
$$= 4\pi\int dr\,r^2\left[\omega^2\psi^2 + \left(\frac{\partial\psi}{\partial r}\right)^2 + m^2\psi^2 + V_{\rm eff}(\psi)\right]$$
$$V_{\rm eff}(\psi) \equiv \overline{V_{\rm nl}(\phi)}$$

"Energetic end" at $\bar{E}_{end}(\omega_{end})$ where $\frac{\partial\bar{E}}{\partial\omega} = 0$

decay direction



Oscillons cannot be stable against small perturbations for $\omega > \omega_{end}$, which we define as the end of oscillons and use it to estimate its lifetime.

Semi-analytic estimate of decay and lifetime of oscillons

(Ibe et al. 2019, Zhang et al. 2020)

$$\phi(t,\mathbf{x}) = \mathrm{Re}(\Phi) = 2\psi(r)\cos(\omega t) + \xi(t,\boldsymbol{x}).$$

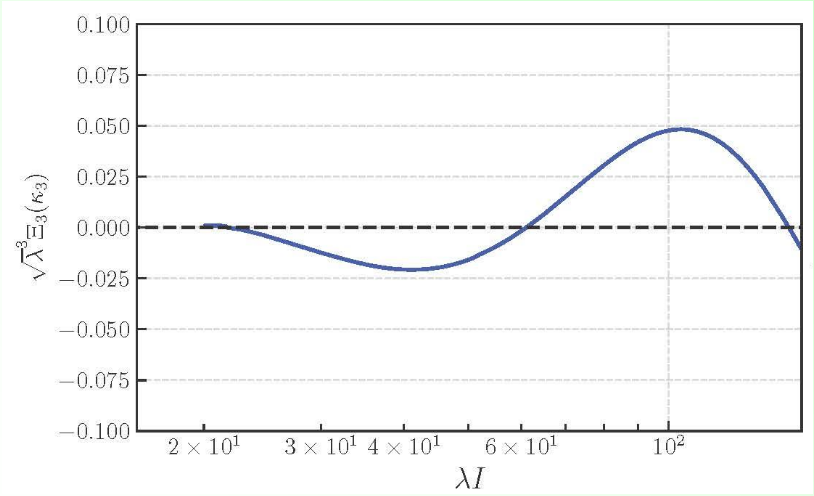
With Green's function technique, $\xi(t,r)$ can be solved in terms of $\psi(r)$ and ω.

For $V_{\mathrm{nl}}(\phi) = -\lambda\phi^4 + g_6\phi^6$

outgoing spherical waves

$$\xi(t,r) = -\frac{1}{2\pi}\left([4\lambda\Xi_3(\kappa_3) + 30g_6\Xi_5(\kappa_3)]\frac{\cos(3\omega t - \kappa_3 r)}{r} + 6g_6\Xi_5(\kappa_5)\frac{\cos(5\omega t - \kappa_5 r)}{r}\right)$$

$\Xi_3(\kappa_3)$: dominant mode



$$\Xi_n(\kappa_j) = 4\pi\int dr\,\psi^n(r)\frac{r\sin(\kappa_j r)}{\kappa_j}, \qquad \kappa_j = \sqrt{(j\omega)^2 - m^2} \quad (j>1)$$

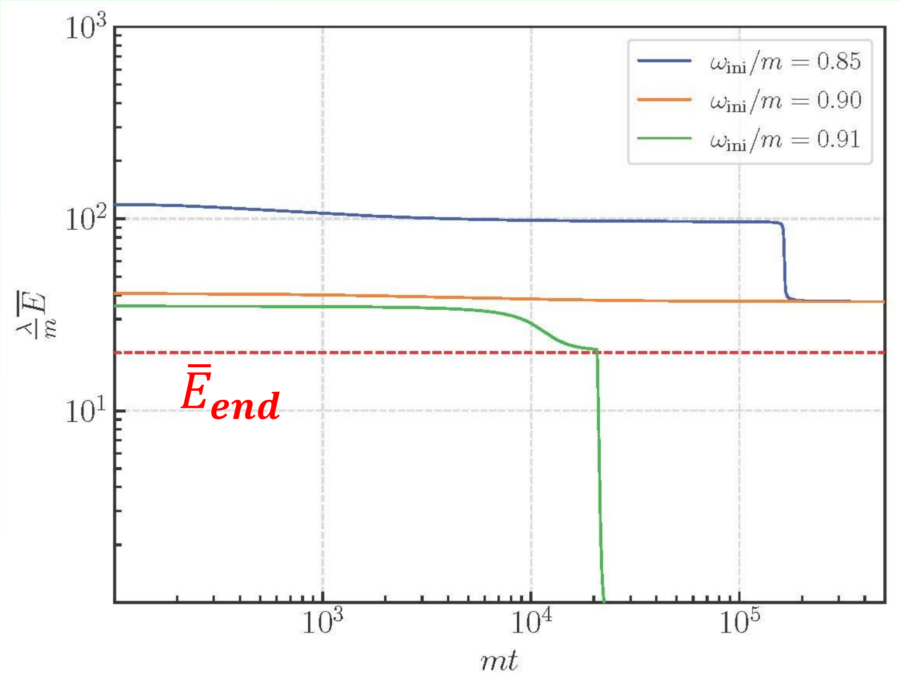
Energy decay rate and lifetime:

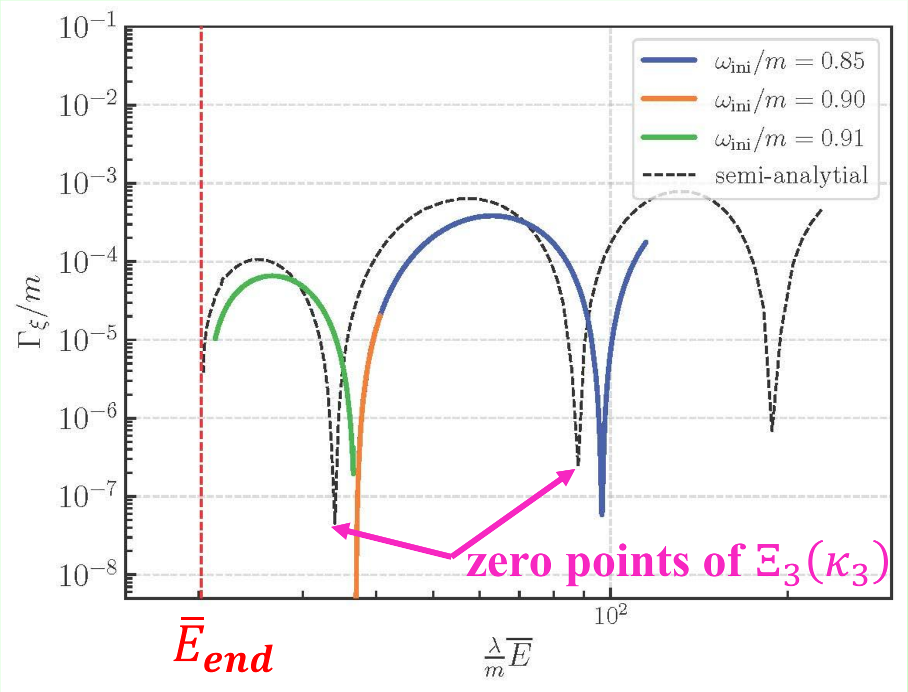
$$\Gamma_\xi \equiv \frac{1}{\overline{E}}\left|\frac{d\overline{E}}{dt}\right| = 4\pi r^2\frac{|\overline{T_{0r}}|}{\overline{E}} = 4\pi r^2\frac{|\overline{\partial_t\xi\,\partial_r\xi}|}{\overline{E}}$$

$$\tau(\overline{E}_{\mathrm{ini}}) = \int_{\overline{E}_{\mathrm{end}}}^{\overline{E}_{\mathrm{ini}}}\frac{dE}{\Gamma_\xi(E)}$$

Numerical estimate of decay and lifetime of oscillons

We evolve the EoM of $\phi(t,r)$ numerically.





decay direction

$$E = \frac{1}{T}\int_{t-T}^{t} dt \int d^3x \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2 + V_{\rm nl}(\phi)\right]$$

Inflaton must couple to another field to reheat the universe.

What's the effect of an external field on oscillons ?

Let's consider an interaction $g\phi^2\chi^2$ as an example.

Siyao Li, MY, Ying-li Zhang, 2507.13276 [hep-ph]
(c.f. Shafi et al. 2024)

Effects of external field χ on oscillons

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - V_{\rm nl}(\phi) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\chi^2\chi^2 - g\phi^2\chi^2$$

Taking an oscillon solution $\phi(t,x) \simeq 2\psi(r)\cos\omega t$ **as background,**

EOM of χ

$$\ddot{\chi}_k + (k^2 + m_\chi^2)\chi_k + 8g\cos^2(\omega t)\underline{\int \frac{d^3k'}{(2\pi)^3}\Psi(k-k')\chi_{k'}} = 0$$

Convolution because of the inhomogeneity of ϕ

(χ_k : Fourier mode of $\chi(r)$, $\Psi(k)$: **Fourier mode of** $\psi^2(r)$)

Mathieu-like equation

Floquet theorem: $\chi_k(t) \propto \mathcal{P}_k(t)e^{\mu_k t}$

For $\mathrm{Re}[\mu_k] > 0$, χ_k **grows as** $e^{\mathrm{Re}[\mu_k]t}$.

Neglecting inhomogeneous oscillon profile

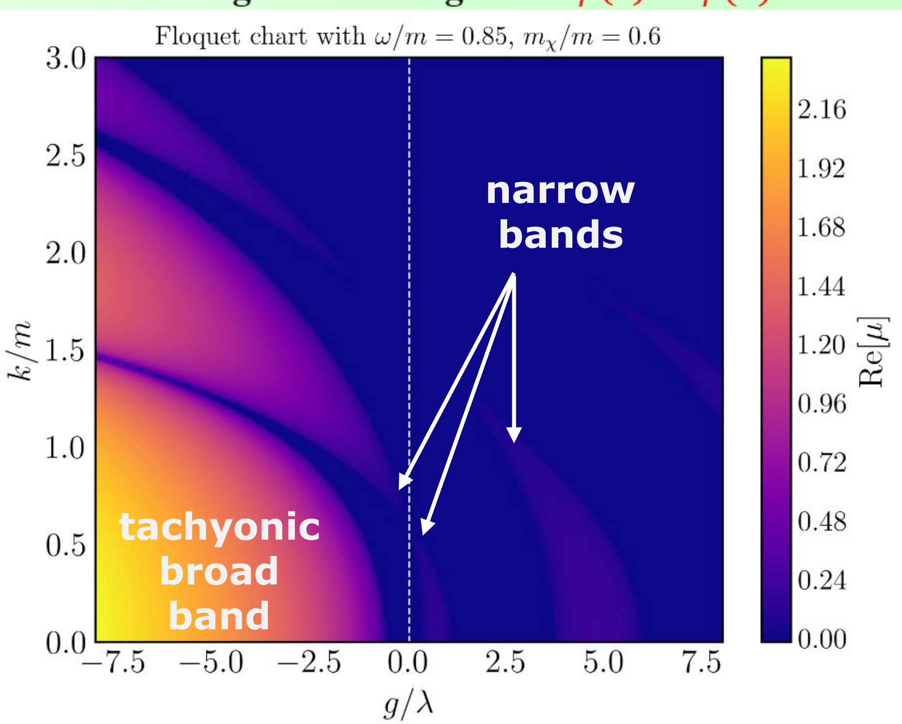
$\psi^2(r) \Rightarrow \psi^2(0) = \psi_0^2$: **homogeneous profile !!**

EOM of χ

$$\chi_k'' + (A_k + 2q\cos(2z))\,\chi_k = 0, \qquad \left(A_k = \frac{k^2 + m_\chi^2}{\omega^2} + 2q,\ q = \frac{2g\psi_0^2}{\omega^2},\ z = \omega t \right)$$

Standard Mathieu equation

Floquet chart with
homogeneous background $\psi(r) \approx \psi(0)$



For $g > 0$ (attractive interaction), in the first narrow band,

$$\mu_{\max} \approx \frac{|q|}{2} = \frac{\psi_0^2}{\omega^2} g$$

$$(\mu_{\max} \equiv \max(\mathrm{Re}(\mu)))$$

For $g < 0$ (repulsive interaction),

$$\mu_{\max} \approx \begin{cases} \dfrac{|q|}{2} = \dfrac{\psi_0^2}{\omega^2}|g|, & \text{if } -\dfrac{m_\chi^2}{4\psi_0^2} \lesssim g \\[2ex] \dfrac{1}{\omega}\sqrt{m_\chi^2 + 4\psi_0^2 g}, & \text{if } g \lesssim -\dfrac{m_\chi^2}{4\psi_0^2} \end{cases}$$

(Tachyonic instability, $A_k < 0$)

Inhomogeneous oscillon profile

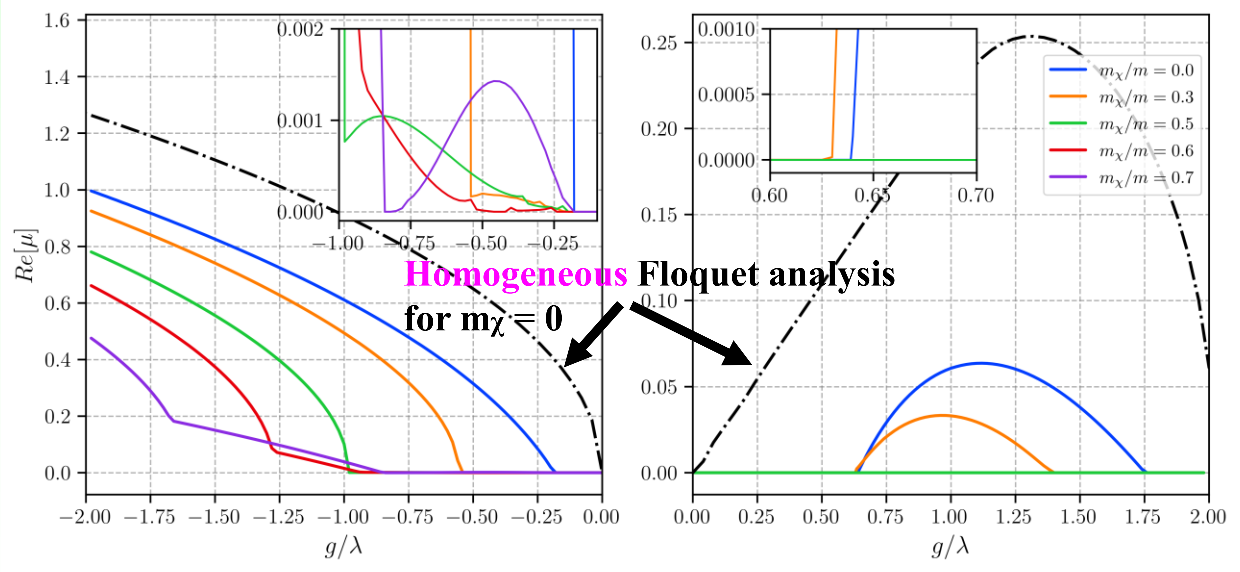
Numerical calculation : $\ddot{\chi}_k + (k^2 + m_\chi^2)\chi_k + 8g\cos^2(\omega t)\int \frac{d^3k'}{(2\pi)^3}\Psi(k-k')\chi_{k'} = 0$

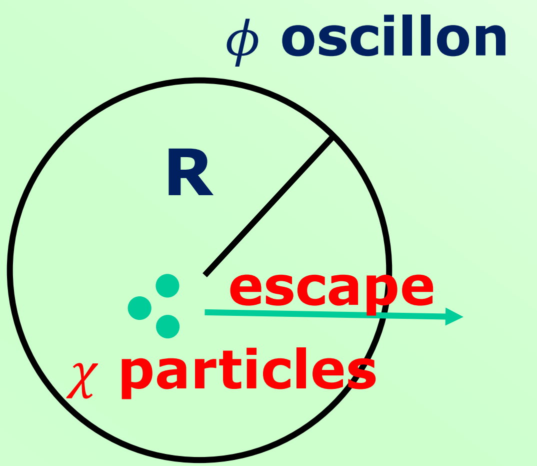
ϕ: **fixed as oscillon profile,** $\phi = 2\psi(r)\cos(\omega t)$

χ: **initially Gaussian function**

Energy density of χ sector: $E_\chi(t) = \frac{1}{T_{\rm ave}}\int_t^{t+T_{\rm ave}} dt \int_0^{r_{\rm max}} dr\, 4\pi r^2 \left(\frac{1}{2}\dot{\chi}^2 + \frac{1}{2}(\partial_r\chi)^2 + \frac{1}{2}m_\chi^2\chi^2\right)$

We fit the Floquet exponent, Re(μ), by $E_\chi(t) \propto e^{2\mathrm{Re}(\mu)\omega t}$





Inhomogeneous oscillon profile prevents resonance because of Particle escaping from the resonance region, i.e. oscillon radius.

You may wonder if

the lifetime of oscillons with external coupling would be $\tau \sim \frac{1}{\mathrm{Re}[\mu]}$

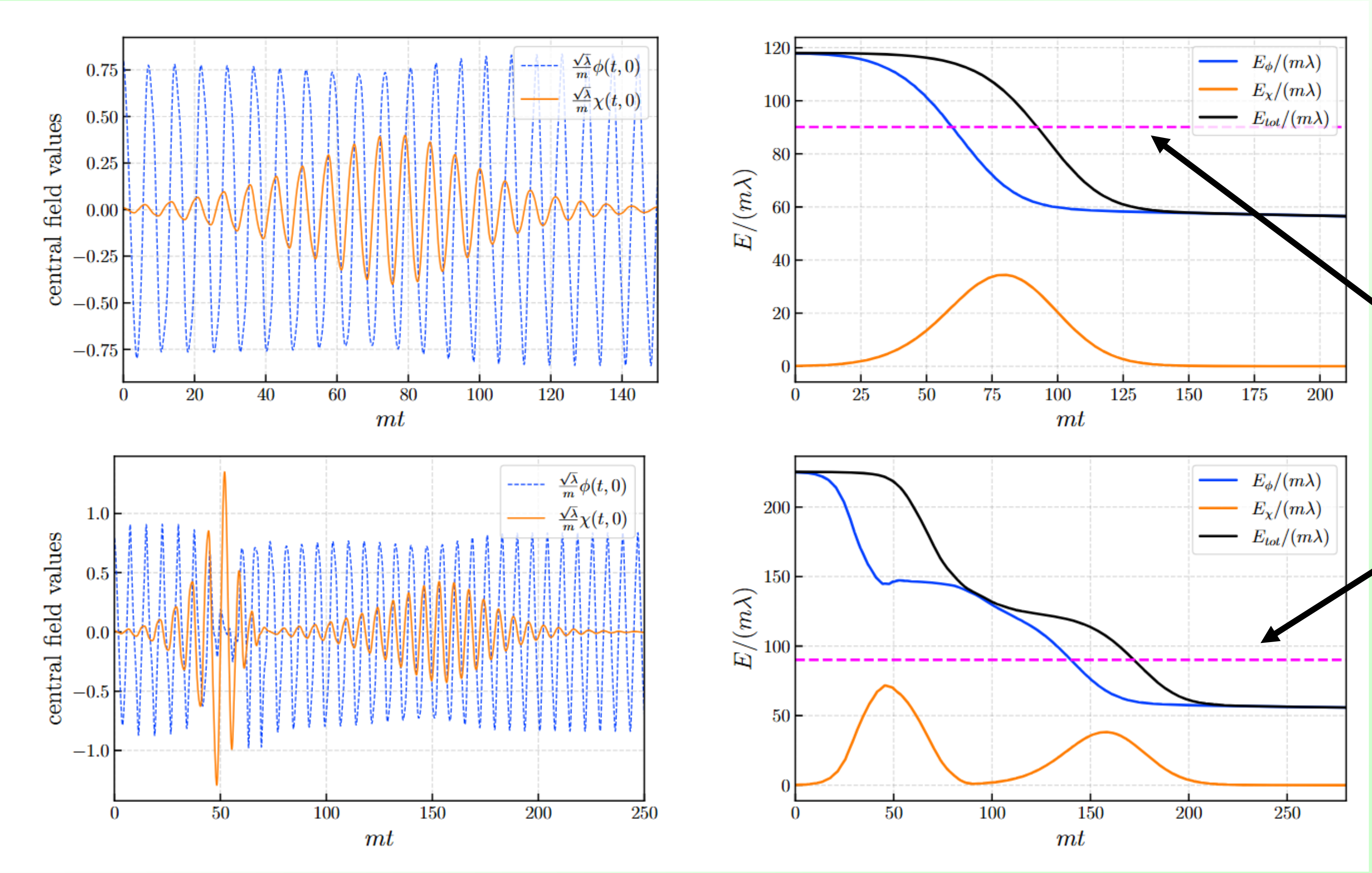
because, once the resonance starts, oscillons would be destroyed immediately ?

Two-field simulation

We evolve the EoM of two fields numerically, starting from initial conditions:

- ϕ: **oscillon profile as** $\phi = 2\psi(r)\cos(\omega t)$
- χ: **time-averaged profile from previous simulation**

Two-field simulation (g > 0)

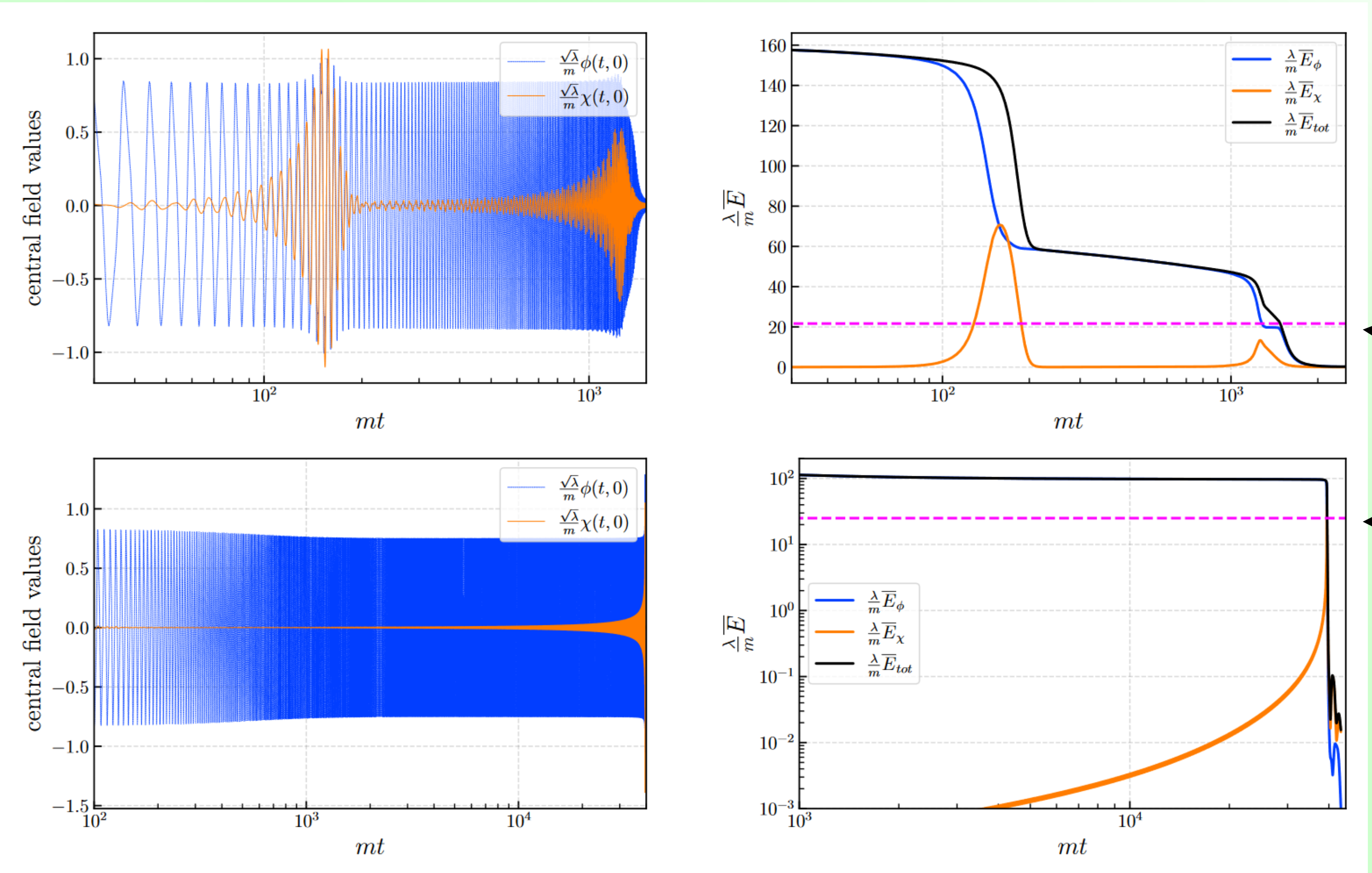


Critical oscillon energy, $\bar{E}_0^{\mathrm{osc}}$, below which Re[μ] = 0

Top: $\omega/m = 0.85,\ m_\chi/m = 0.3,\ g/\lambda = 1.0$ **Bottom:** $\omega/m = 0.83,\ m_\chi/m = 0.3,\ \ g/\lambda = 1.0$

Even though the resonant growth of χ happens,
the oscillon is not necessarily destroyed immediately => different profile !!

Two-field simulation (g < 0)



Critical oscillon energy, $\bar{E}_0^{\rm osc}$, below which Re[μ] = 0

Top :$\omega/m = 0.84,\ m_\chi/m = 0.6,\ g/\lambda\ = -1.0,$ **Bottom:**$\omega/m = 0.84,\ m_\chi/m = 0.6,\ g/\lambda = -0.8$

χ causes E_ϕ to drop below E_{end},
thereby driving the oscillon to immediate destruction.

Oscillon lifetime with external coupling

$$\begin{cases} \text{For } \bar{E}_0^{\text{osc}} \gtrsim \bar{E}_{\text{end}}, & \Gamma(E) \sim \begin{cases} \mathrm{Re}(\mu), & \bar{E} \gtrsim \bar{E}_0^{\text{osc}} \\ \Gamma_{\text{single}}, & \bar{E}_{\text{end}} < \bar{E} \lesssim \bar{E}_0^{\text{osc}} \end{cases} \\ \text{For } \bar{E}_0^{\text{osc}} \lesssim \bar{E}_{\text{end}}, & \Gamma(E) \sim \mathrm{Re}(\mu), \quad \bar{E} \gtrsim \bar{E}_{\text{end}} \end{cases}$$

Total lifetime : $\tau(\bar{E}_{\text{ini}}) \sim \sum_i \frac{1}{\mathrm{Re}(\mu_i)} + \tau_{\text{single}}\left(\min\left(\bar{E}_0^{\text{osc}}, \bar{E}_{\text{ini}}\right)\right)$

- **i runs different stage from initial $\bar{E}_{\text{ini}}$ to $\bar{E}_0^{\text{osc}}$, and the first term becomes zero if $\bar{E}_{\text{ini}} \leq \bar{E}_0^{\text{osc}}$.**
- $\boldsymbol{\tau_{single}(\bar{E}_0^{\text{osc}}) = 0}$ **if** $\boldsymbol{\bar{E}_0^{\text{osc}} \leq \bar{E}_{\text{end}}}$

Summary

- Q balls are stable, spherical symmetric, localized field configuration, whose stability is guaranteed by U(1) symmetry.
- They are paid attention to in the context of baryogenesis.
- Oscillons/I balls are long-lived, spherical symmetric, localized field configuration, whose long lifetime is guaranteed by adiabatic invariance.
- They are paid attention to in the context of attractor models of inflation like Starobinsky inflation.
- We have discussed the effects of an external field on oscillons.
- Even though the resonant growth of χ happens, the oscillon is not necessarily destroyed immediately.
- We gave the rough estimate of lifetime of oscillons.