SGWB Generated by Superconducting Cosmic Strings

Based on: I. Yu. Rybak & L. Sousa, JCAP 11 (2022) 024 (arXiv: 2209.01068)
I. Yu. Rybak & L. Sousa, Phys.Rev.D 111 (2025) 8, 083502 (arXiv: 2412.17154)

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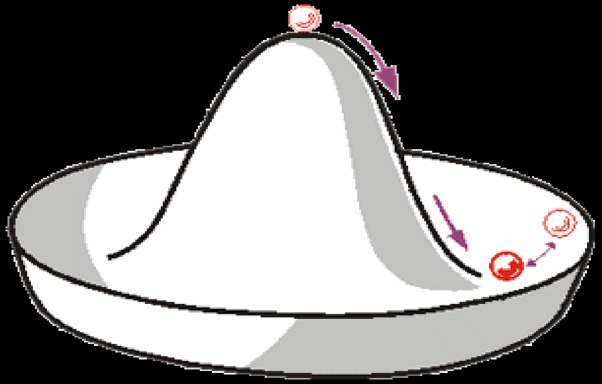
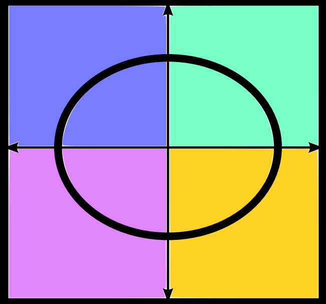
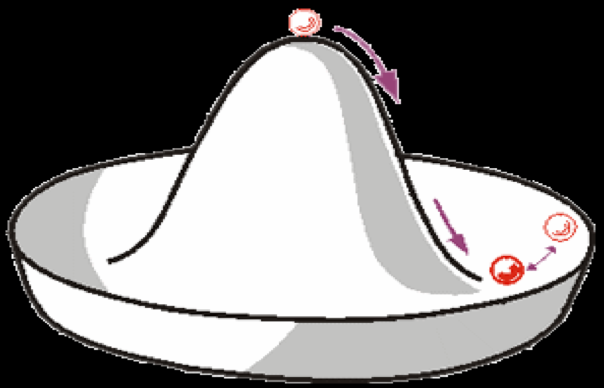
Cosmic String Formation

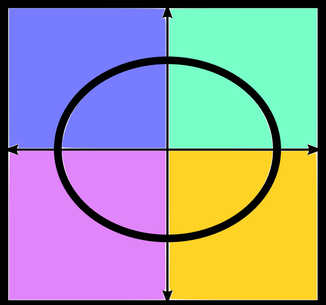


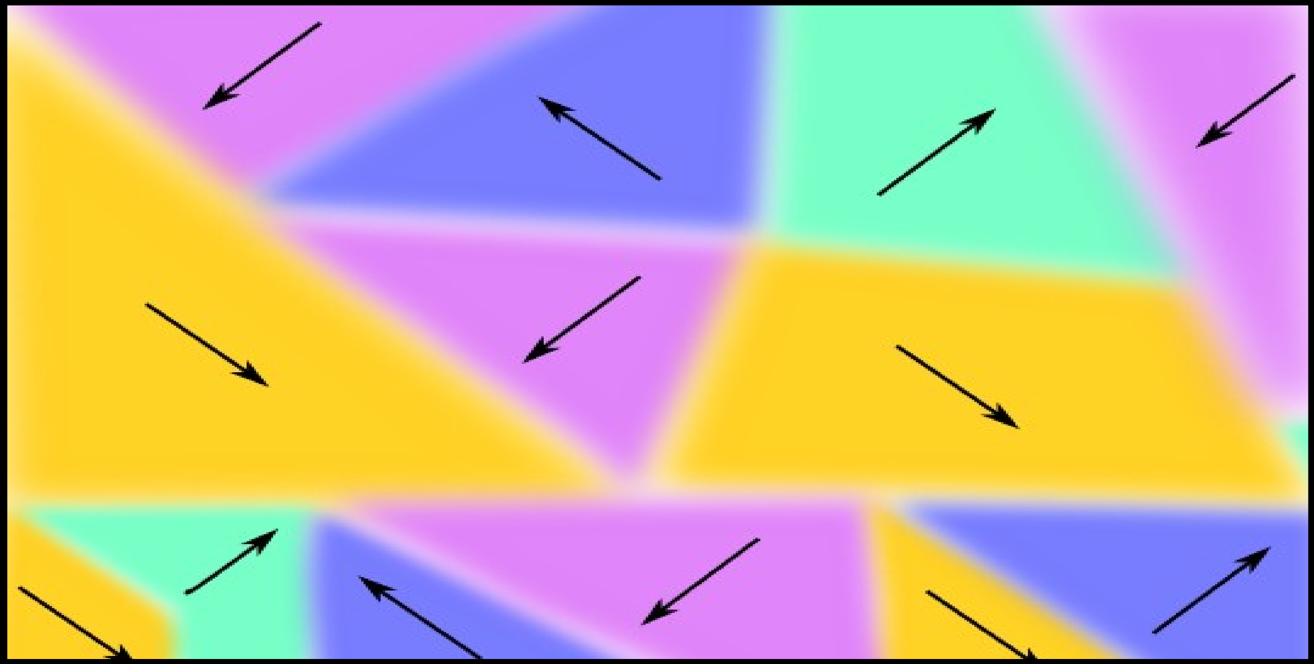


Cosmic String Formation

Different patches of the universe fall into different minima!

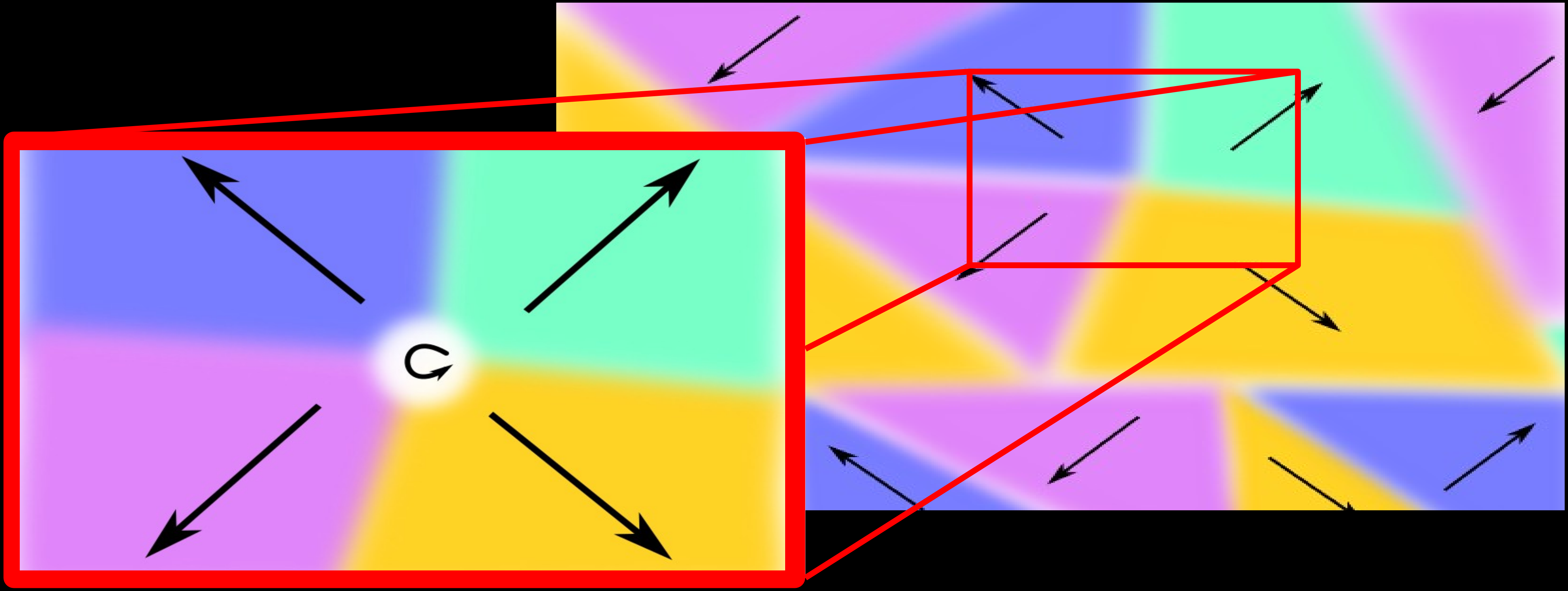






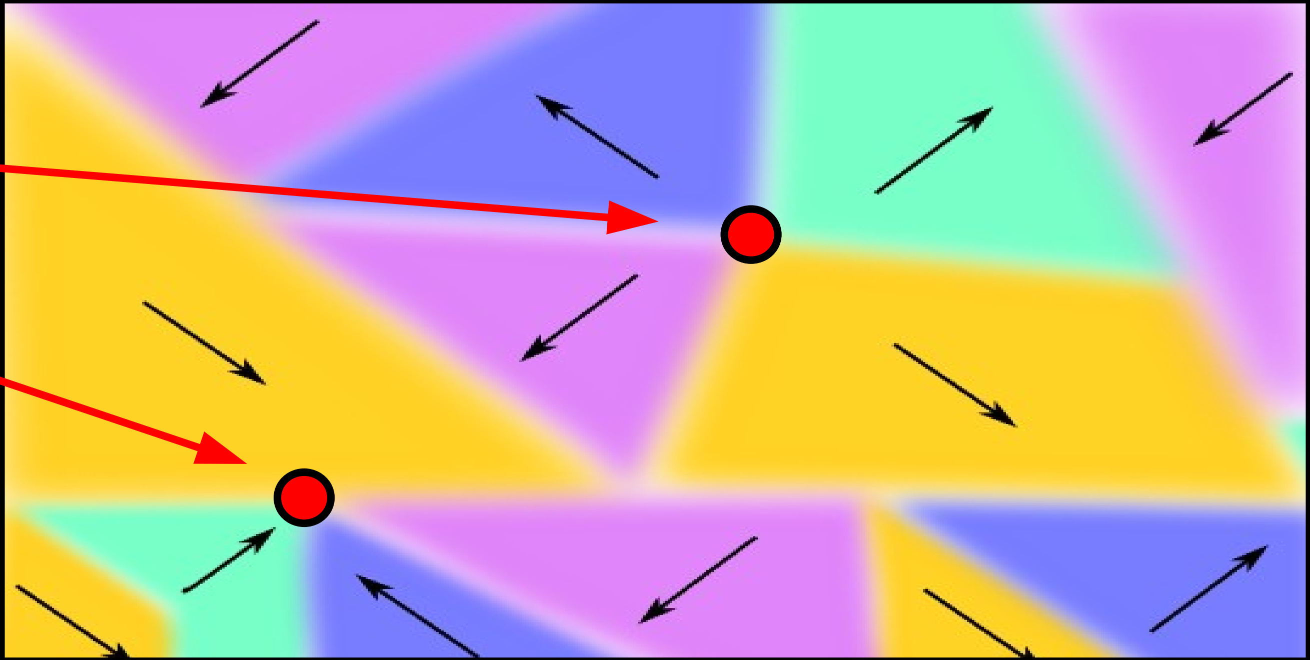
Cosmic String Formation

What lurks in the edges?



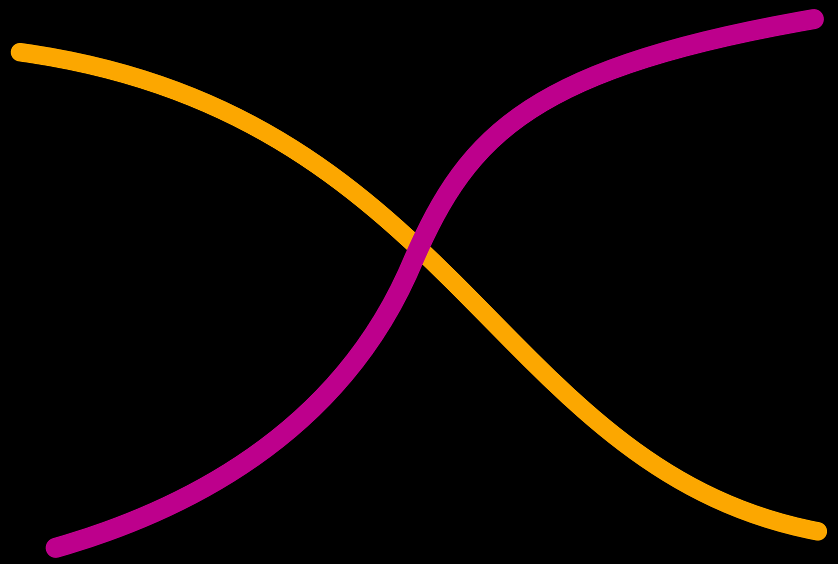
Cosmic String Formation

COSMIC
STRINGS!



Cosmic String Loops & GWs

Upon collision



Cosmic String Loops & GWs

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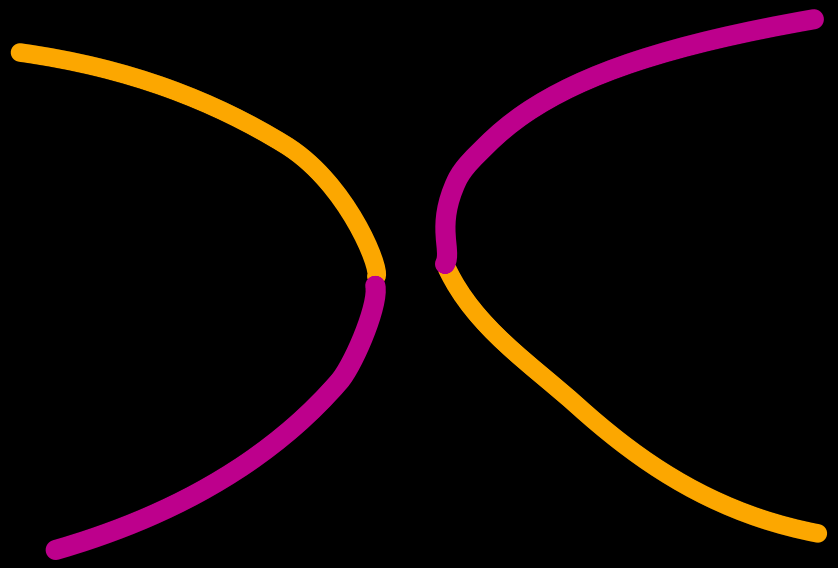
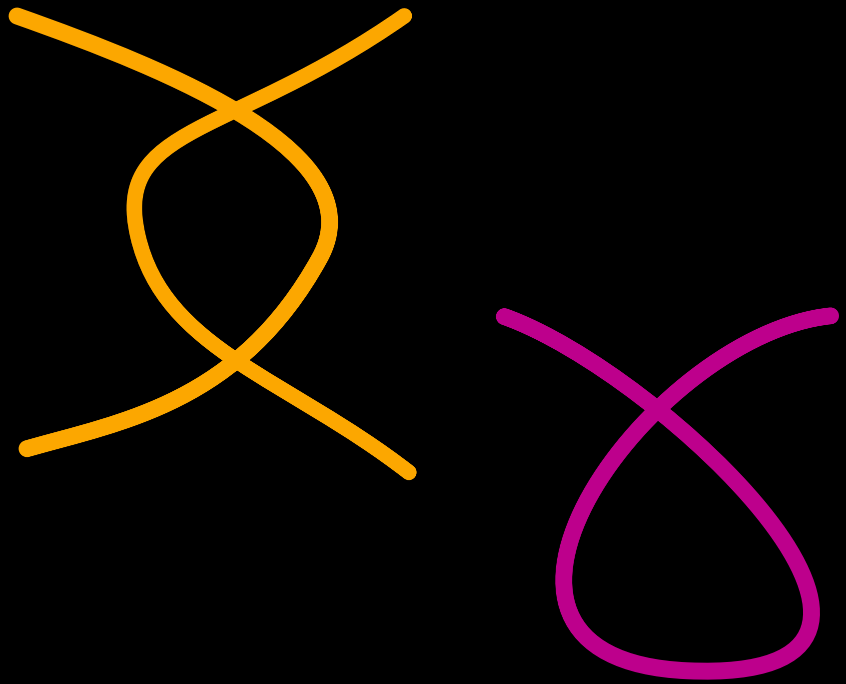


Strings exchange partners and intercommute.

Cosmic String Loops & GWs

Some situations…



Cosmic String Loops & GWs

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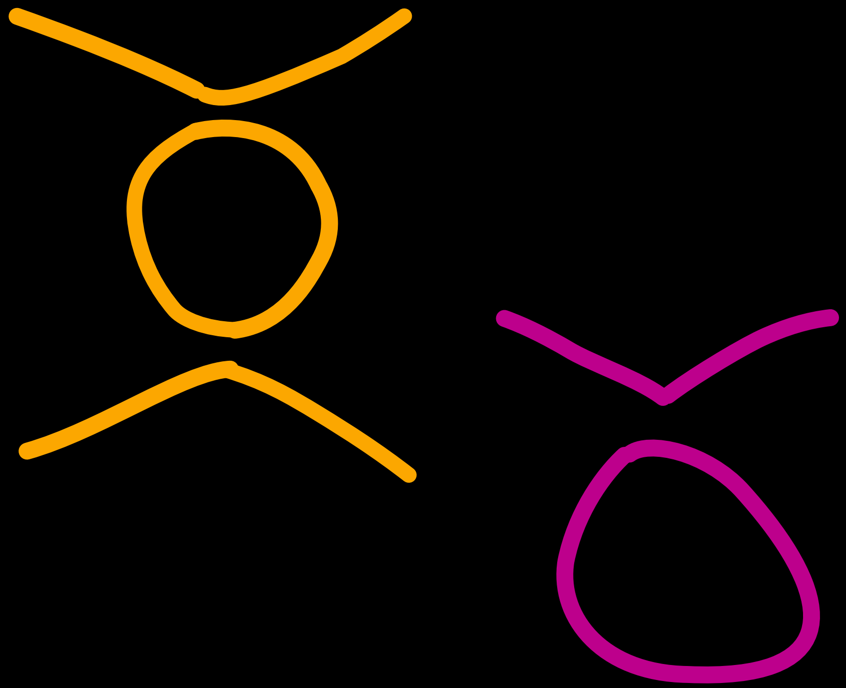
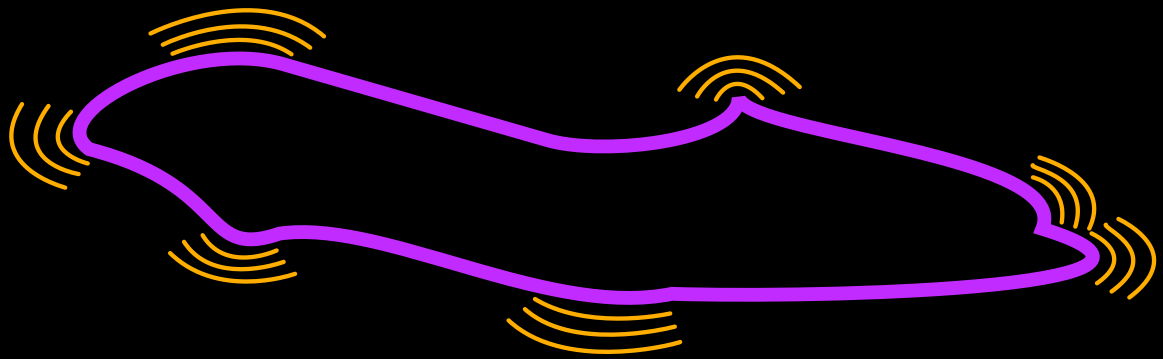


lead to the copious production of cosmic string loops!

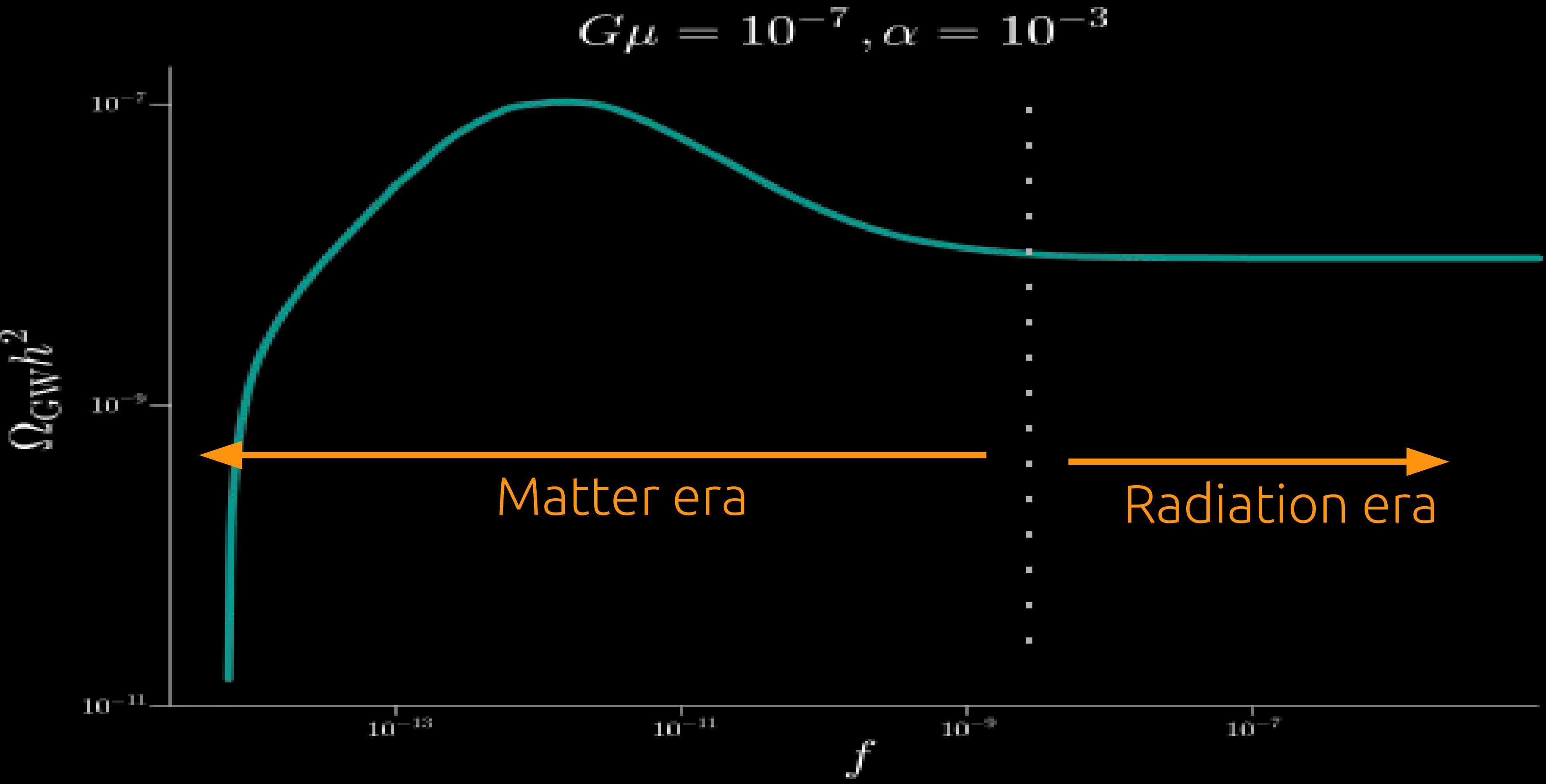
Cosmic String Loops & GWs

Loops detach form the network and start to oscilate



and generate transient gravitational wave signals.

The Typical Cosmic String Sgwb



Loops and Gravitational Waves

TO COMPUTE THE SGWB GENERATED BY COSMIC STRINGS WE NEED TO KNOW THE NUMBER DENSITY OF LOOPS $n(\ell(t), t)$

How many loops are created?

How much of the energy of the loops goes into GWs?

What is the emission spectrum of loops?

SUPERCONDUCTING STRINGS

In many GUTs, cosmic strings may carry currents and behave as thin superconductor wires (Witten 1985).

Spontaneous current generation may happen even in the standard GUT paradigm (Davis and Peter, 1995).

Superconducting Strings

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REALISTIC COSMIC STRINGS MAY
BE CURRENT-CURRYING AND THUS HAVE
INTERNAL DEGREES OF FREEDOM

SUPERCONDUCTING STRINGS

SUPERCONDUCTING STRINGS BEHAVE AS ELASTIC STRINGS which have a distinct tension and energy per unit length

There are two types of perturbations:

* Wiggles: wordsheet displacements
* Woggles: longitudinal displacements

Superconducting Strings

SUPERCONDUCTING STRINGS BEHAVE AS ELASTIC STRINGS which have a distinct tension and energy per unit length

There are two types of perturbations:

* Wiggles: wordsheet displacements
* Woggles: longitudinal displacements

We use TRANSONIC ELASTIC STRINGS, for which the speed of propagation of wiggles and woggles are equal, as a proxy to study the gravitational wave background generated by superconducting strings

Superconducting Strings

TRANSONIC STRING LOOPS

$$X^\mu = \frac{T_\ell}{2\pi}\left[X_+^\mu\left(\sigma_+\right) + X_-^\mu\left(\sigma_-\right)\right],$$

$$\phi = \frac{T_\ell}{2\pi}\left[F_+\left(\sigma_+\right) + F_-\left(\sigma_-\right)\right],$$

with

$$\mathbf{X}_+^{\prime\,2}(\sigma_+) = 1 - F_+^{\prime\,2}(\sigma_+), \quad \mathbf{X}_-^{\prime\,2}(\sigma_-) = 1 - F_-^{\prime\,2}(\sigma_-)$$

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GW Emission Emission Efficiency

Power emitted in GWs:

$$P = G\mu_0^2\Gamma, \quad \Gamma \equiv \sum_{j=0}^{\infty}\Gamma_j = \sum_{j=0}^{\infty}\int \omega_j^2 \frac{\left|I_j^{22} - I_j^{33}\right|^2 + \left|I_j^{23} + I_j^{32}\right|^2}{2^3\pi T_\ell^2} d\Omega,$$

GRAVITATIONAL WAVE
EMISSION EFFICIENCY

GW Emission Efficiency

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GRAVITATIONAL WAVE EMISSION EFFICIENCY

SPECTRUM OF EMISSION:

Tells us how the power is split into the different harmonic modes

$$\omega_j = 2\pi j / T_\ell$$

GW Emission Efficiency

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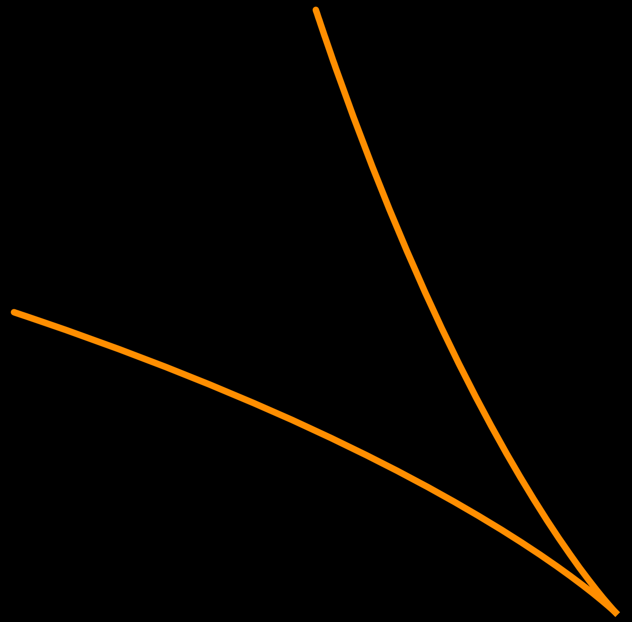
with

$$I_\pm^\mu = \frac{T_\ell}{\pi}\int_0^{2\pi} X_\pm'^\mu \mathrm{e}^{-\frac{iT_\ell}{2\pi}k_\nu X_\pm^\nu} d\sigma_\pm$$

$$\tilde{T}^{\mu\nu}(\omega, \mathbf{k}) = \frac{1}{T_\ell}\int T^{\mu\nu}\mathrm{e}^{-ik_\nu X^\nu}\, d^4x = \frac{\mu_0}{2T_\ell} I_+^{(\mu} I_-^{\nu)}$$

Emission from a Cusp-like Point

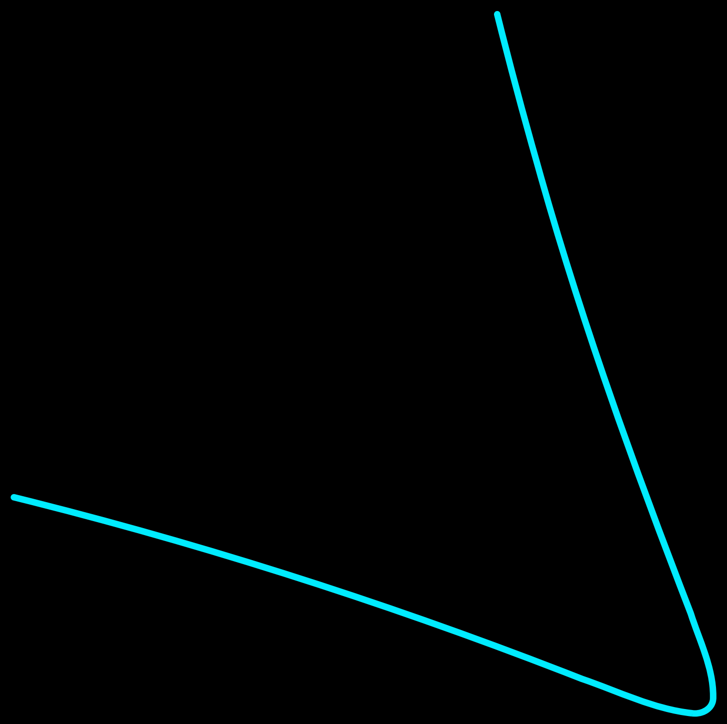
At some points on string loops, known as CUSPS, string velocity can equal the speed of light.



CUSPS EMIT
HIGHLY BEAMED
GRAVITATIONAL WAVE
BURSTS

EMISSION FROM A CUSP-LIKE POINT

For current-carrying strings, CUSPS CAN NO LONGER FORM.



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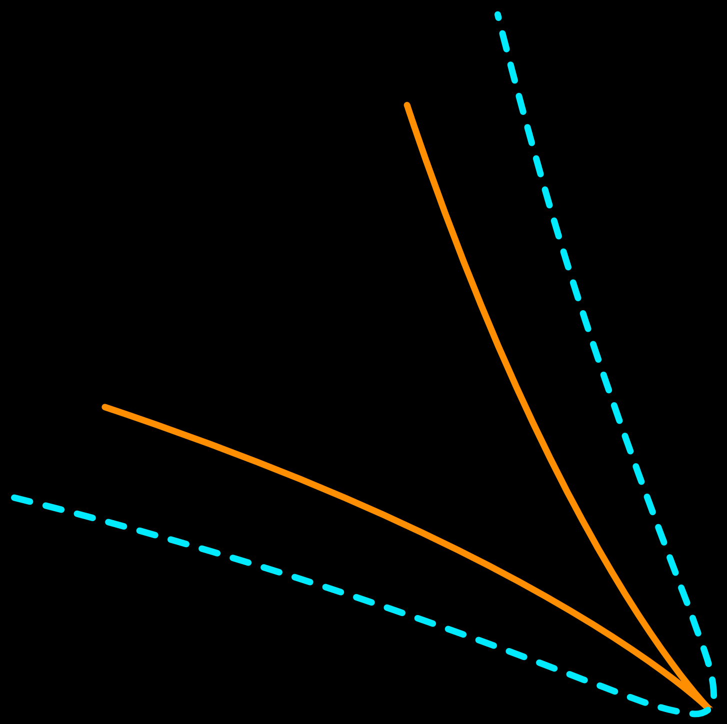
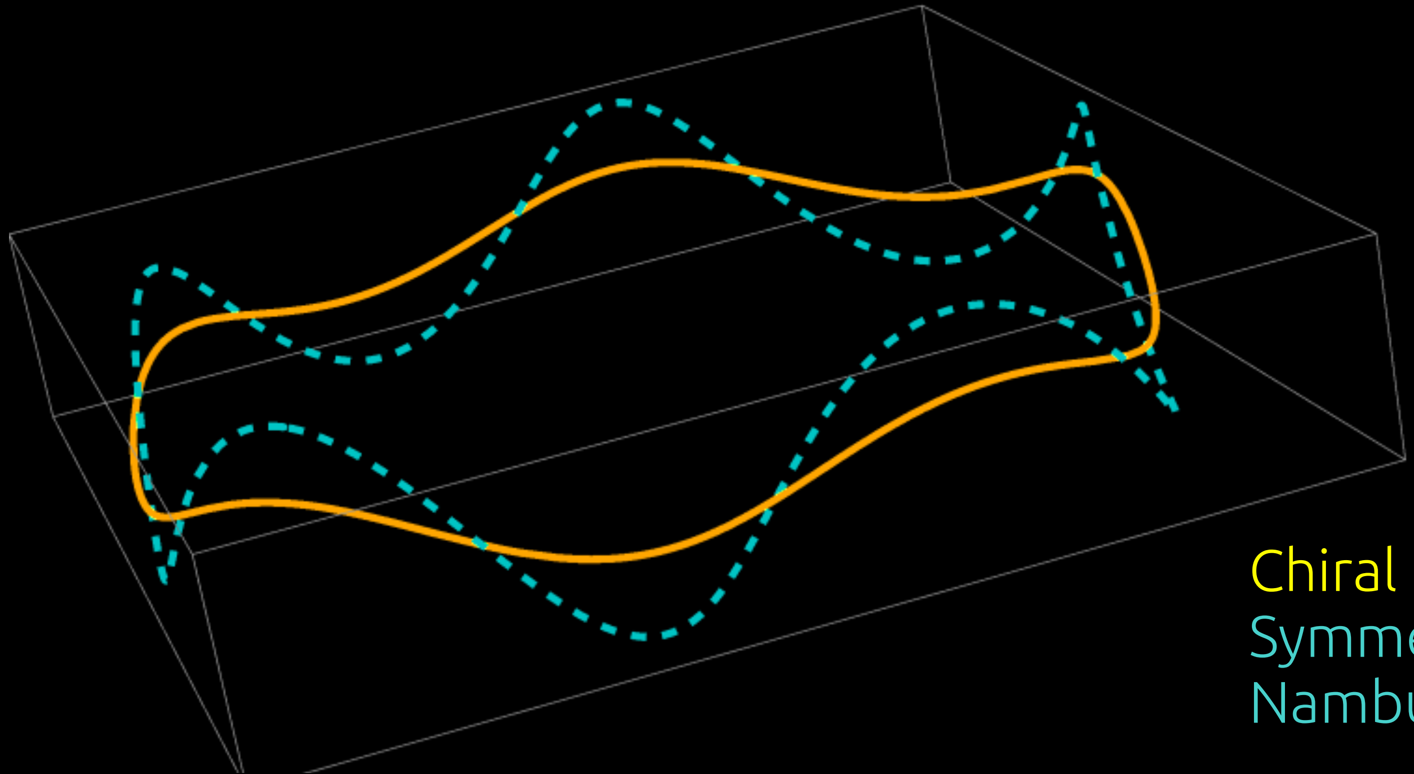


IF CURRENT
IS SYMMETRICAL,
HOWEVER,
CUSP-LIKE POINTS MAY
FORM, BUT NOW
THEIR VELOCITY IS
SUBLUMINAL

Emission from Burden LOOPS

Burden Loops create cusps (or cusp-like points) at discrete instants of time

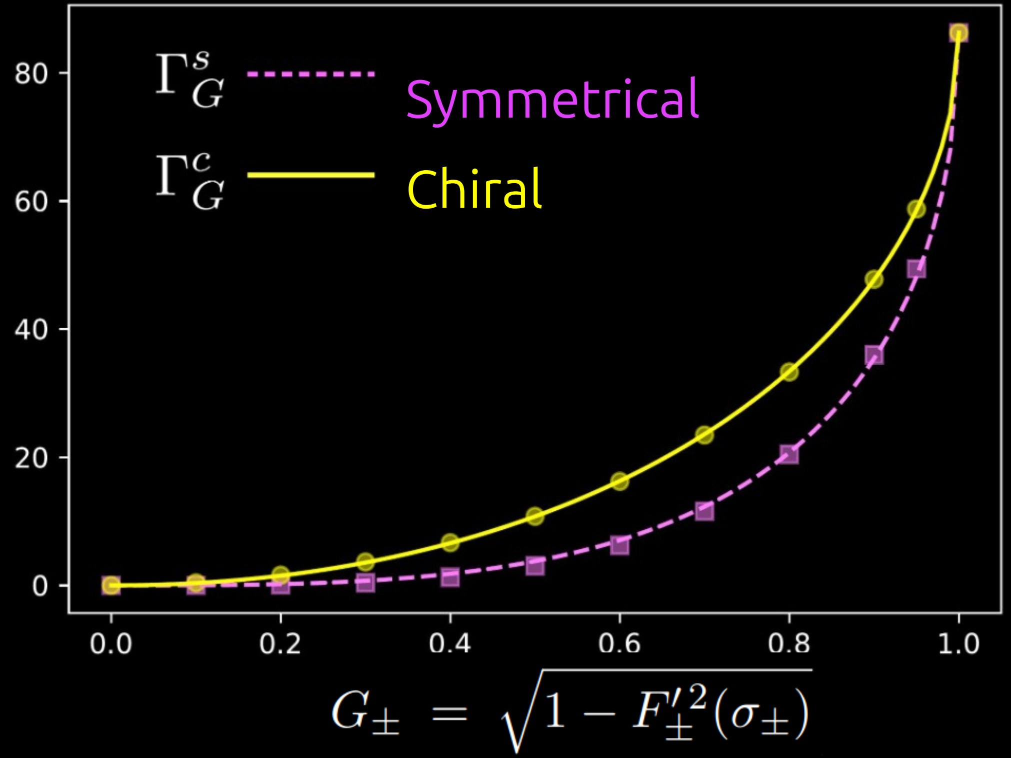


Chiral

Symmetrical and Nambu-Goto

Spectrum of Emission of Burden Loops

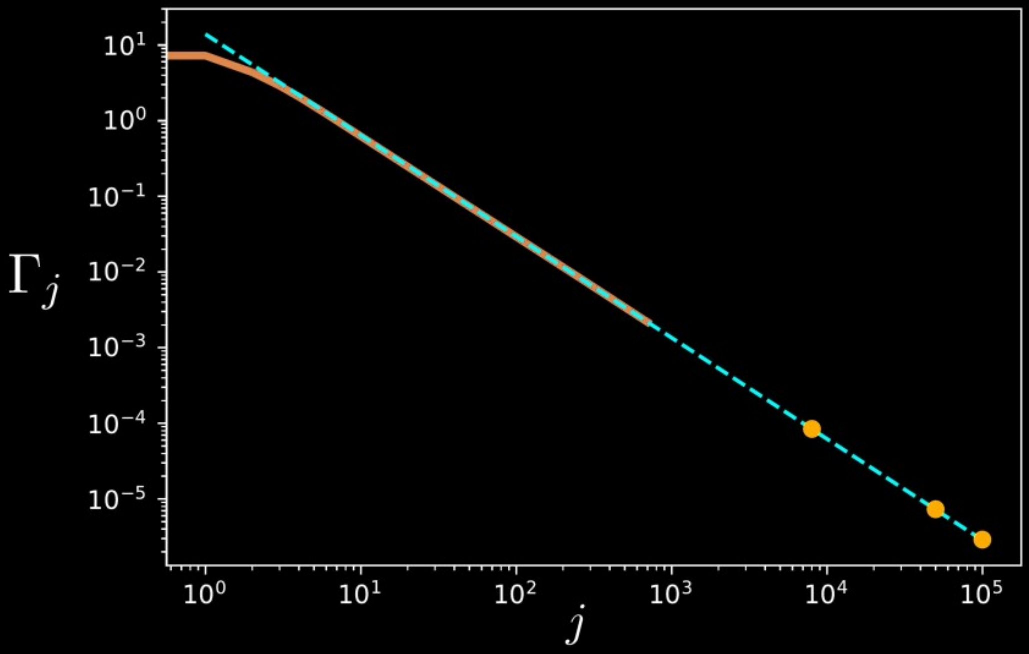
Gravitational wave emission efficiency decreases with increasing current



Spectrum of Emission of Burden Loops

THE PRESENCE OF CURRENT LEADS TO A SUPPRESSION OF THE EMISSION OF GRAVITATIONAL RADIATION

Spectrum of Emission of Burden Loops



Without current, we have a power law spectrum

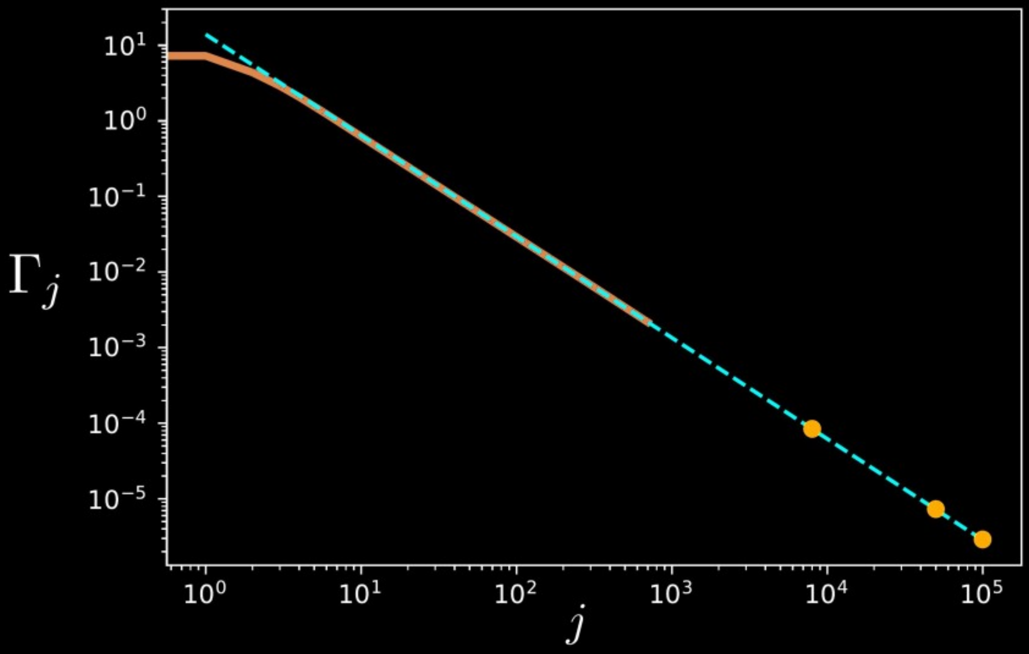
$$\Gamma_j \sim j^{-q}$$

For large enough j.

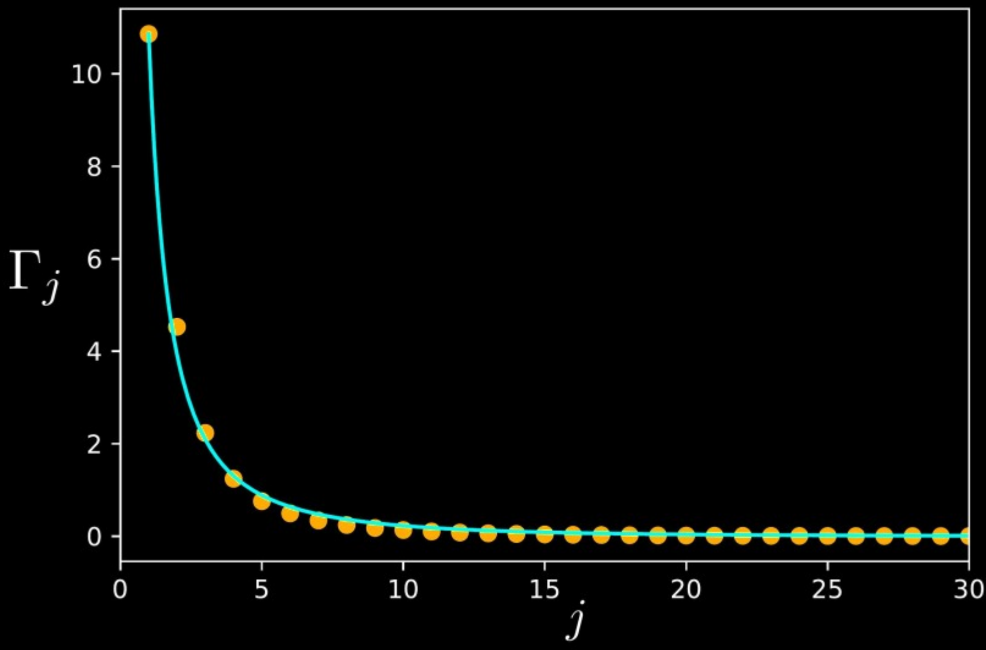
(Vaschaspati & Vilenkin, 1985)

Spectrum of Emission of Burden Loops

Without current



With current



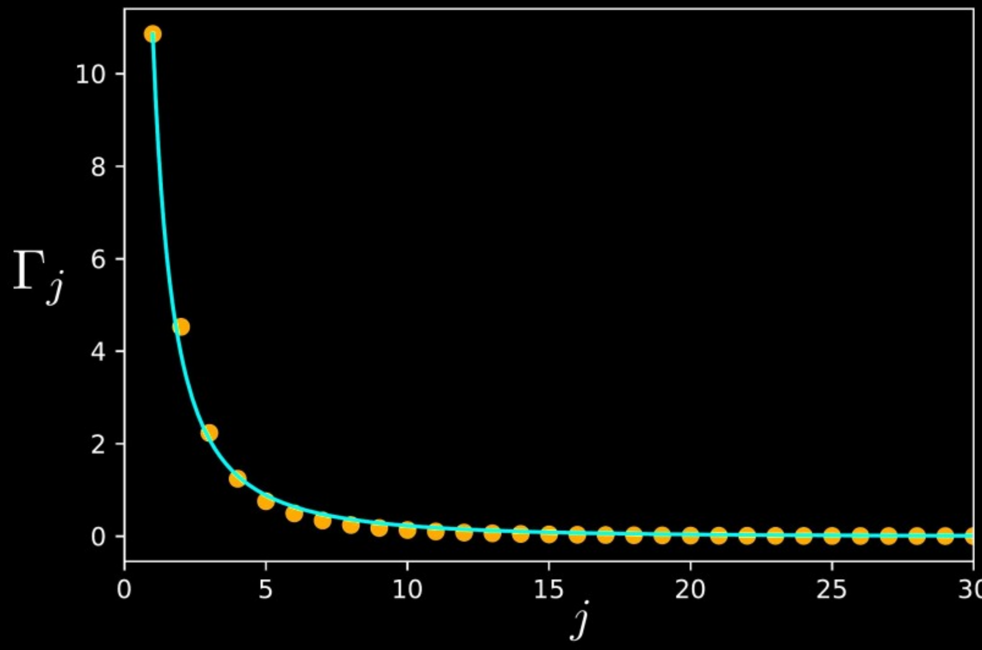
Spectrum of Emission of Burden Loops

With current, we have an
exponential decay

$$\Gamma_j \sim j^{-q} \mathrm{e}^{-j f_m(G_\pm)}$$

with

$$f_m(G_\pm) = a_m(1 - \sqrt{G_\pm})^{b_m}$$



Spectrum of Emission of Burden Loops

THE POWER EMITTED IN GWS BY
A LOOP WITH QUASI-CUSPS DECAYS
EXPONENTIALLY WITH INCREASING
HARMONIC MODE FOR
NULL AND SYMMETRICAL CURRENTS

Emission from a Kink

String collisions and intercommutation give rise to discontinuities in the string tangent, known as KINKS.

For Nambu-Goto strings, kinks travel along the string at the speed of light.

KINKS EMIT "FAN-LIKE" GRAVITATIONAL WAVE BURSTS

Emission from a Kink

String collisions and intercommutation give rise to discontinuities in the string tangent, known as KINKS.

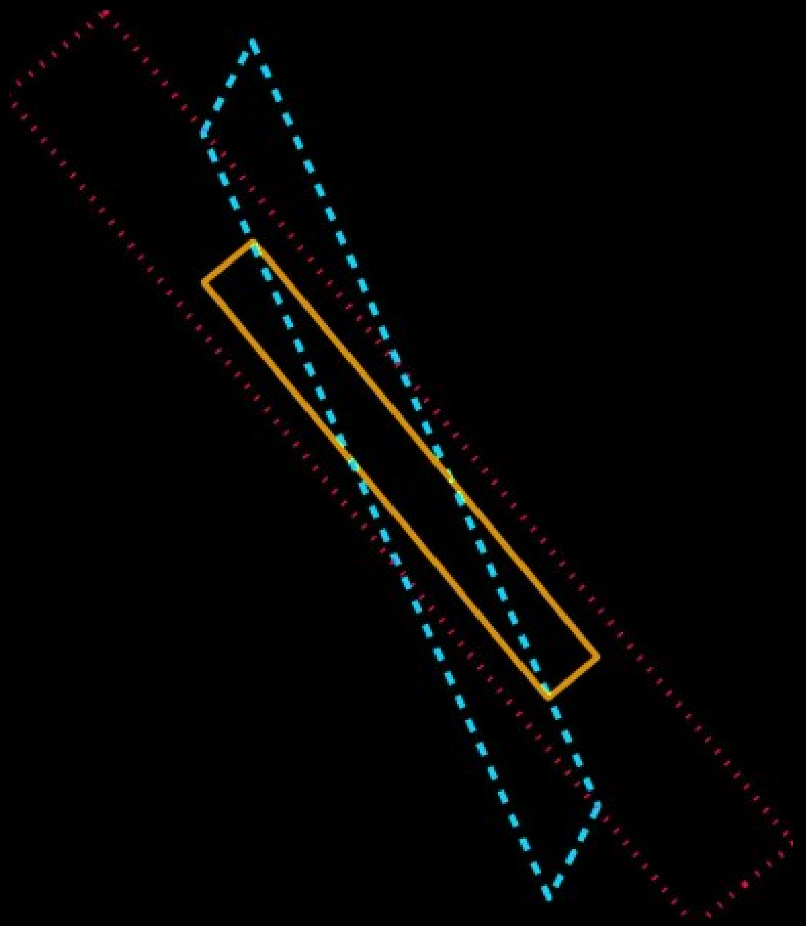
If there is current traveling along the string, kinks are necessarily slower

GRAVITATIONAL WAVE BURST FROM KINKS ON SUPERCONDUCTING STRINGS ARE WEAKER

Emission from Cuspless Loops

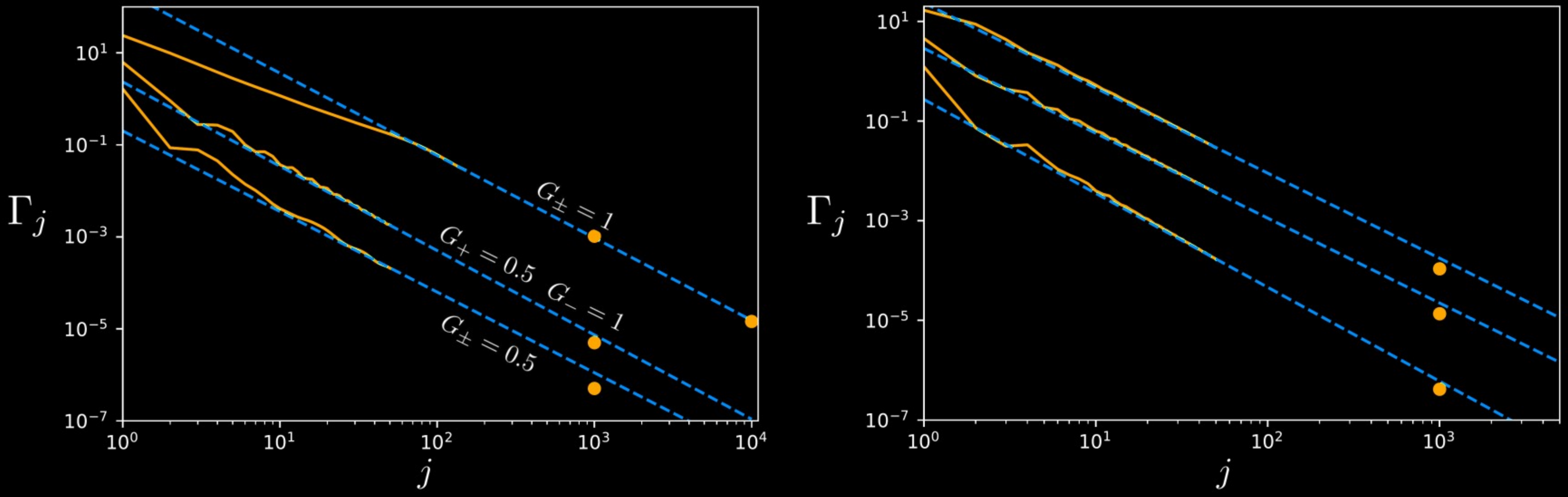
Loops that are
Piece-wise straight
And have no cusps

Chiral
Symmetrical
Nambu-Goto



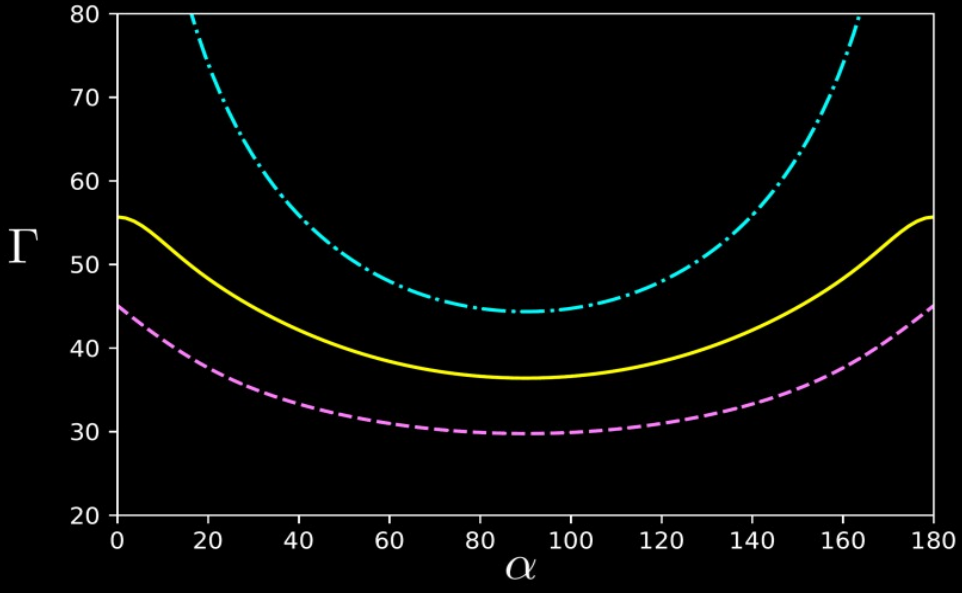
EMISSION FROM CUSPLESS LOOPS

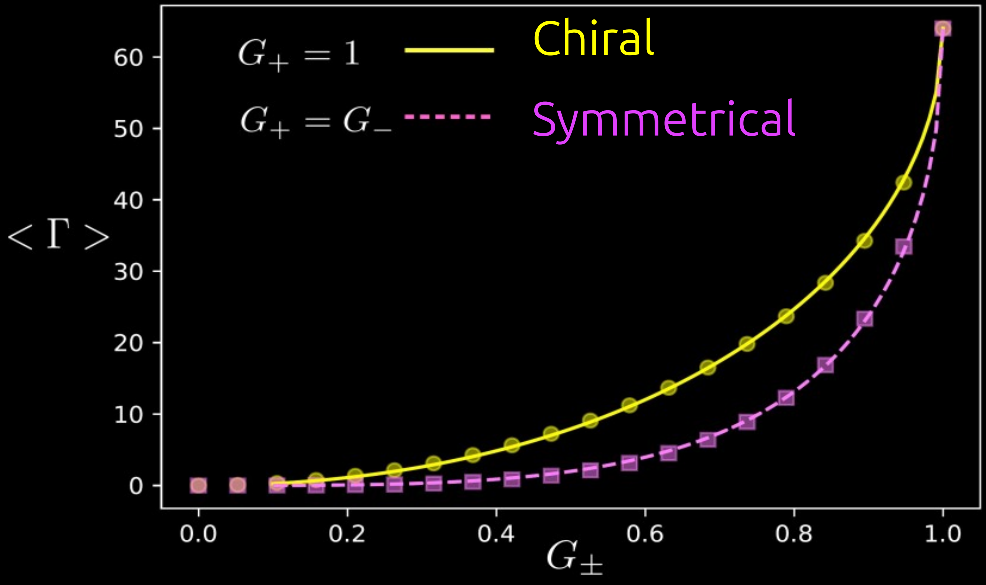
In this case, the spectrum of emission is not affected by the inclusion of current: it is still a power law



EMISSION FROM CUSPLESS LOOPS

The gravitational wave efficiency, however, is affected precisely in the same way as before





Impact of Current on GW Efficiency

Gravitational wave emission efficiency for current-carrying strings is given by

$$\Gamma_G^m = \Gamma_0 (1 - |F'_\pm|)^{B_\Gamma^m} ,$$

Where, for symmetrical currents,

$$B_\Gamma^s \approx 2$$ For loops with cusps

$$B_\Gamma^s \approx 1.5$$ For loops with kinks

And $B_\Gamma^s \approx \sqrt{2} B_\Gamma^c$ for chiral superconducting loops.

Loops and Gravitational Waves

TO COMPUTE THE SGWB GENERATED BY COSMIC STRINGS WE NEED TO KNOW THE NUMBER DENSITY OF LOOPS $n(\ell(t), t)$

How many loops are created?

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LOOPS AND VECTOR RADIATION

Power emitted in Vector radiation:

$$P = \sum_j P_j = e^2 \Gamma^{\text{em}},$$

VECTOR RADIATION EMISSION EFFICIENCY

$$\frac{dP_j}{d\Omega} = -\frac{\omega_j^2}{2\pi} j^\mu j_\mu^* = e^2 \frac{d\Gamma^{\text{em}}}{d\Omega}$$

SPECTRUM OF EMISSION:

Tells us how the power is split into the different harmonic modes

Loops and Vector Radiation

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with

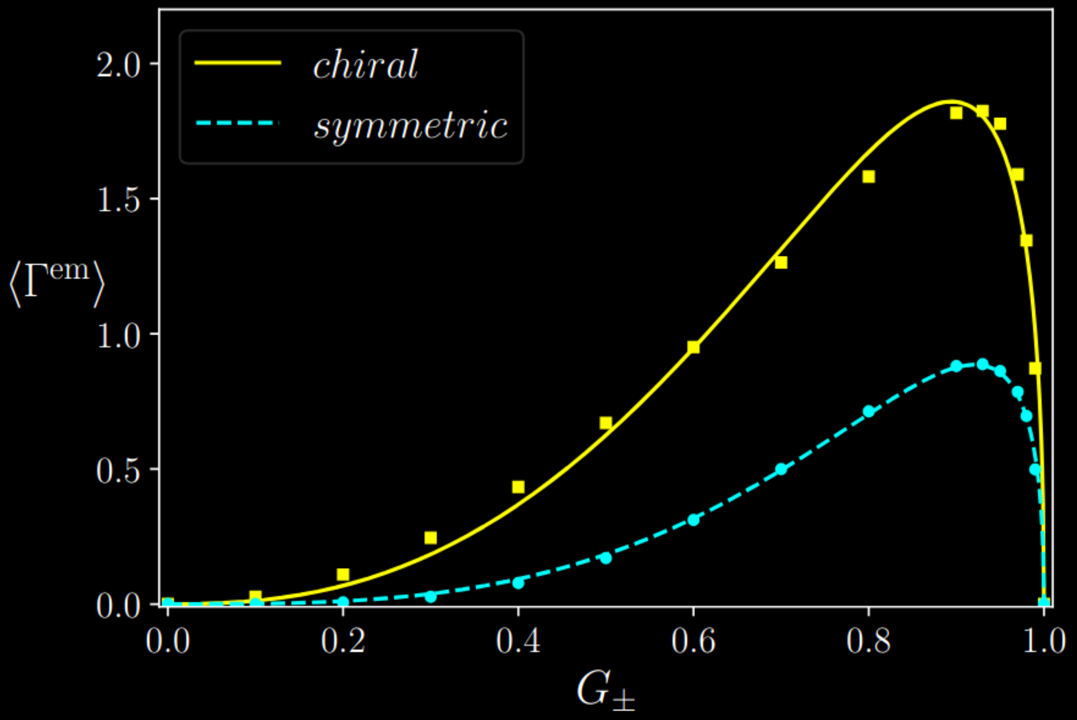
$$j^\mu = e \frac{L}{8\pi^2} \left(I_- J_+^\mu - I_+ J_-^\mu \right),$$

$$I_\pm = \int_0^{2\pi} F'_\pm \mathrm{e}^{ij\left(\sigma_\pm - \frac{2\pi}{L} \mathbf{n}\cdot\mathbf{X}_\pm\right)} d\sigma_\pm,$$

$$J_\pm^\mu = \int_0^{2\pi} X'^\mu_\pm \mathrm{e}^{ij\left(\sigma_\pm - \frac{2\pi}{L} \mathbf{n}\cdot\mathbf{X}_\pm\right)} d\sigma_\pm .$$

Loops and Vector Radiation

Vector radiation emission efficiency also decreases with increasing current, but is also suppressed for very low currents



Loops and Vector Radiation

Vector radiation emission efficiency for current-carrying strings is given by

$$\langle \Gamma^{\mathrm{em}} \rangle = \Gamma_0^{\mathrm{em}} \left| F'_{\pm} \right| \left(1 - \left| F'_{\pm} \right| \right)^D$$

With

	Γ_0^{em}	D
Symmetric loop with quasi-cusps	4.9	1.6
Symmetric loop with kinks	10.5	1.8
Chiral loop with quasi-cusps	8.6	1.1
Chiral loop with kinks	8.6	1.2

Superconducting Loops Evolution

Loop length:

$$\dot{\ell} = -G\mu_0\Gamma^{\rm gr}(\mathcal{Y}) - \tilde{e}^2\Gamma^{\rm em}(\mathcal{Y}),$$

Superconducting Loops Evolution

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$$\dot{\ell} = -G\mu_0 \Gamma^{\rm gr}(\mathcal{Y}) - \tilde{e}^2 \Gamma^{\rm em}(\mathcal{Y}),$$

Current:

$$\dot{\mathcal{Y}}\Big|_{\rm rad} = -\frac{\mathcal{Y}}{\ell}\,\dot{\ell}\Big|_{\rm rad}$$

SUPERCONDUCTING LOOPS EVOLUTION

Loop length:

$$\dot{\ell} = -G\mu_0\Gamma^{\rm gr}(\mathcal{Y}) - \tilde{e}^2\Gamma^{\rm em}(\mathcal{Y}),$$

Current:

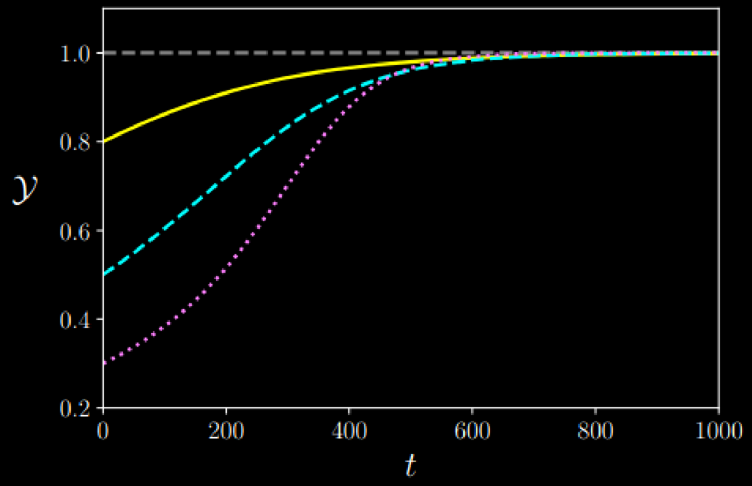
$$\dot{\mathcal{Y}} = \frac{\mathcal{Y}}{\ell}\left[G\mu_0\Gamma^{\rm gr}(\mathcal{Y}) + \tilde{e}^2\Gamma^{\rm em}(\mathcal{Y}) - A(\mathcal{Y})\right]$$

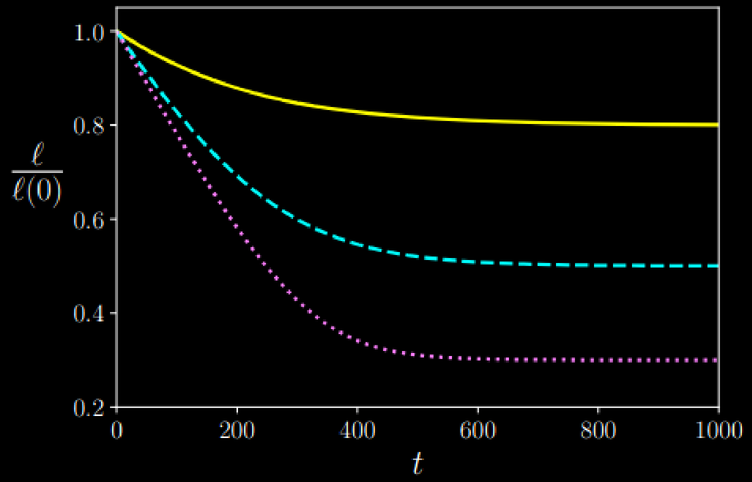
Charge leakage

SUPERCONDUCTING LOOPS EVOLUTION

In the absence of charge leakage,
VORTONS FORM

NO SIGNIFICANT
GW EMISSION
IS
EXPECTED



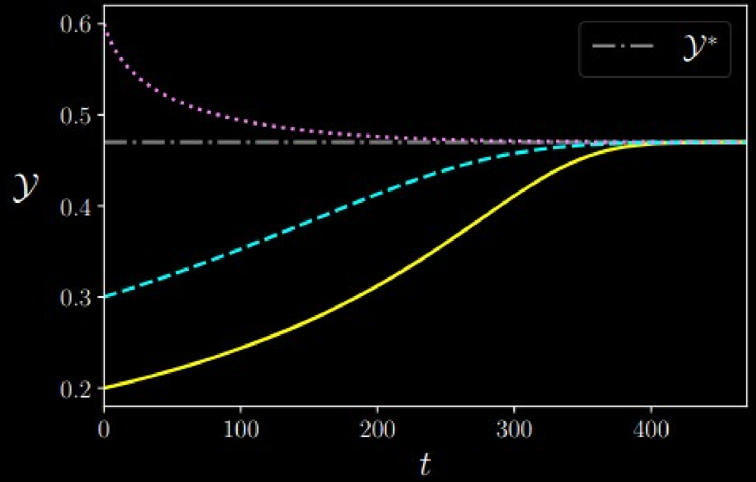


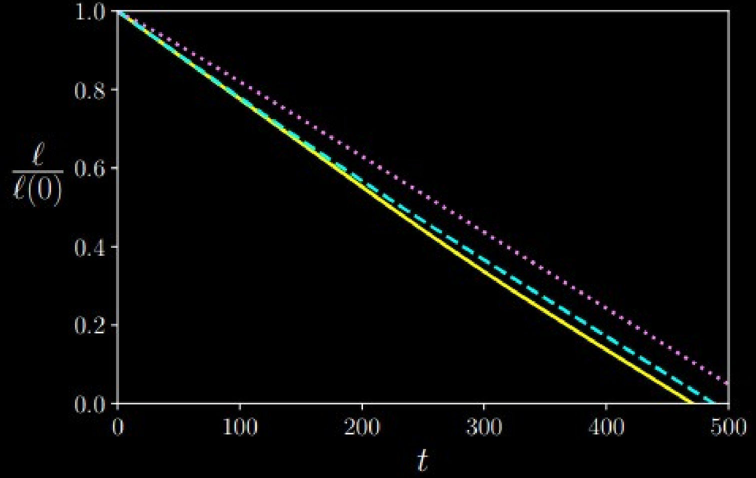
SUPERCONDUCTING LOOPS EVOLUTION

But in the presence of charge leakage, A SCALING SOLUTION EMERGES:

$$\dot{\ell} = \text{constant} \quad \text{and} \quad \dot{\mathcal{Y}} = 0$$

$$G\mu_0 \Gamma^{\text{gr}}(\mathcal{Y}^*) + \tilde{e}^2 \Gamma^{\text{em}}(\mathcal{Y}^*) = A(\mathcal{Y}^*)$$





Superconducting Loops Evolution

WE MAY HAVE A SIGNIFICANT SGWB FROM SUPERCONDUCTING LOOPS WITH CONSTANT CURRENT

Loops and Gravitational Waves

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How many loops are created?

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(SEMI-)ANALYTICAL APPROACH

Sousa & Avelino 2013 (arXiv: 1304.2445); Sousa, Avelino & Guedes 2020 (arXiv: 2002.01709)

PREMISE: the large-scale evolution of the string network determines how much energy goes into loops:

$$\dot{\rho}_\ell = \left.\frac{d\rho}{dt}\right|_{loops} = \tilde{c}v\frac{\rho}{L} \sim \frac{v}{L^3}$$

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ASSUMPTION: Loops are created with a length that is a fixed fraction of the characteristic length $\ell = \alpha L$

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NUMBER DENSITY OF LOOPS PRODUCED:

$$\dot{n}_l = \frac{\dot{\rho}_\ell}{\alpha L}$$

Evolution of Strings with Current

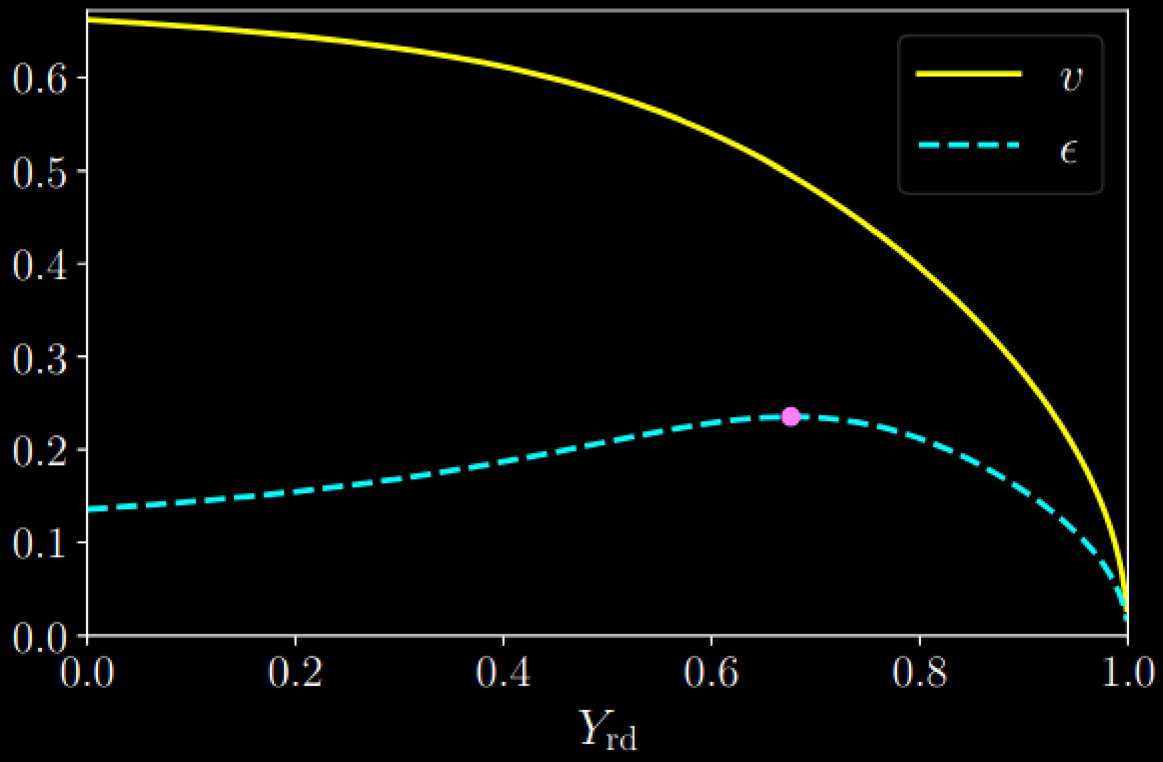
Martins, Peter, Rybak & Shellard 2021 (arXiv: 2011.09700 and arXiv: 2108.03147)

IN THE RADIATION ERA:

$$L_{\rm ph} = \xi_r t \quad \text{and} \quad v = v_r \,,$$

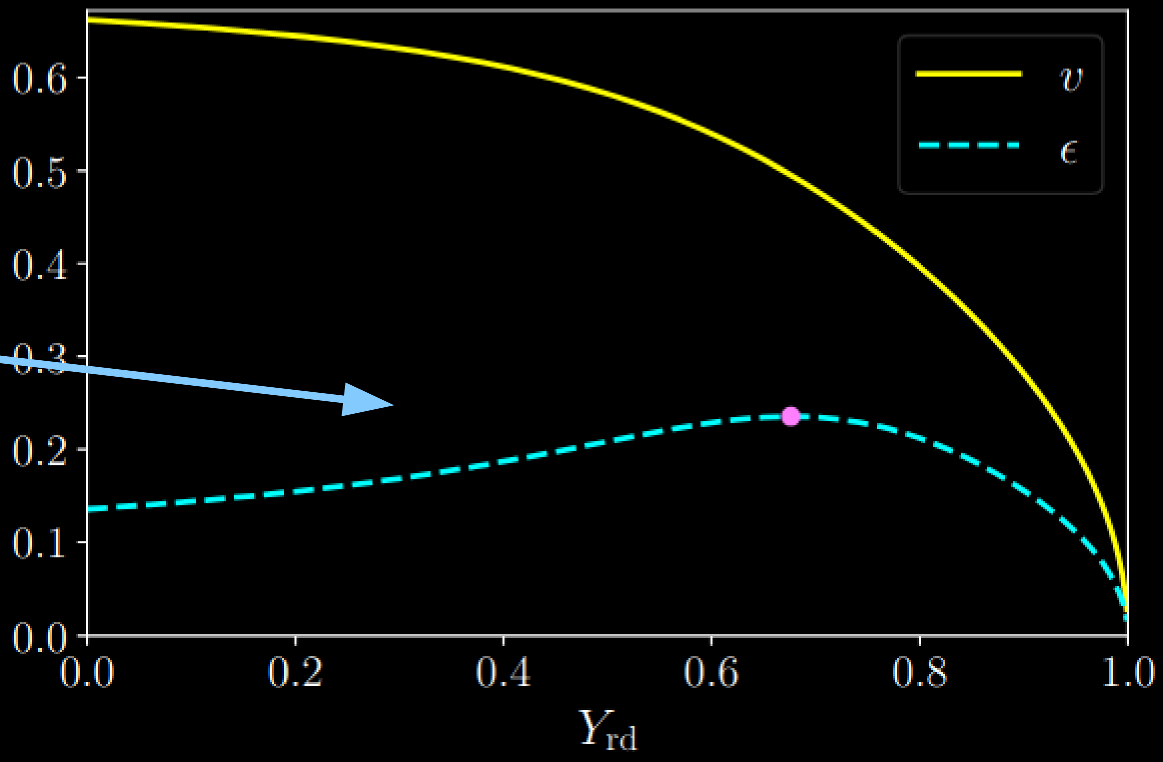
$$\xi_r^2 = (1-Y)k(k+\tilde{c}) \quad \text{and} \quad v_r^2 = (1-Y)\frac{k}{k+\tilde{c}}$$

Evolution of Strings with Current



Evolution of Strings with Current

Number of loops produced is suppressed for small currents



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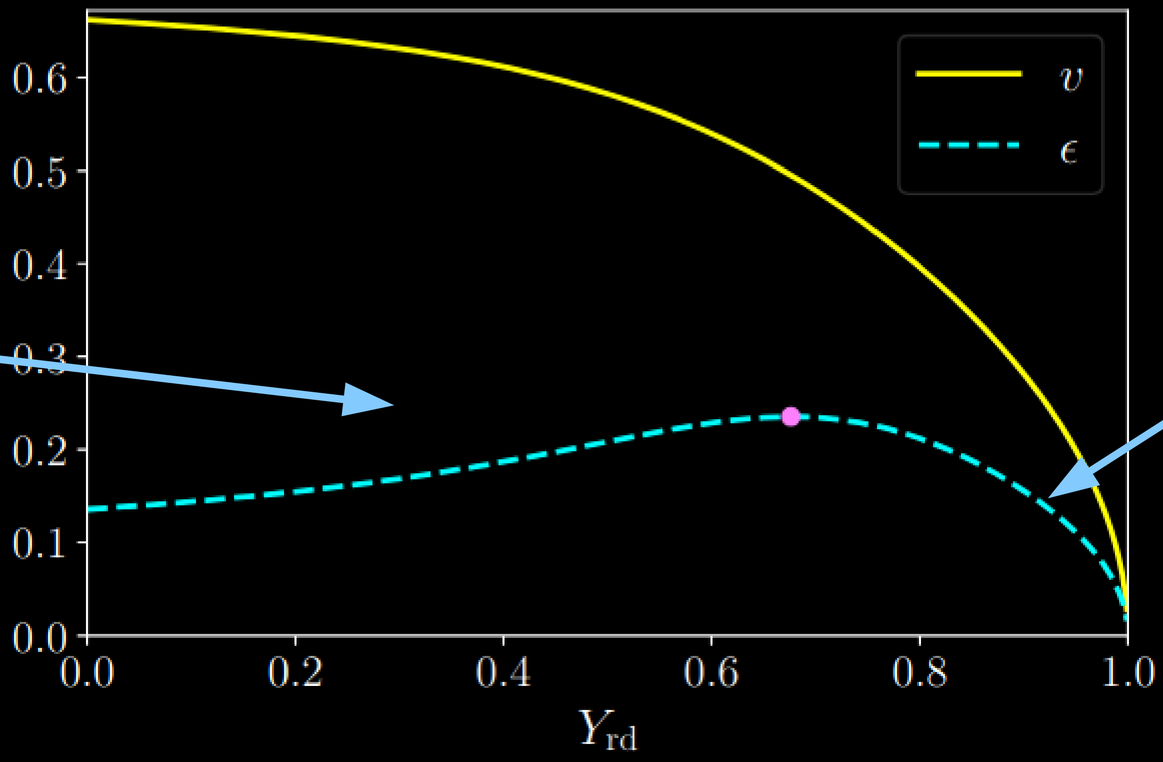
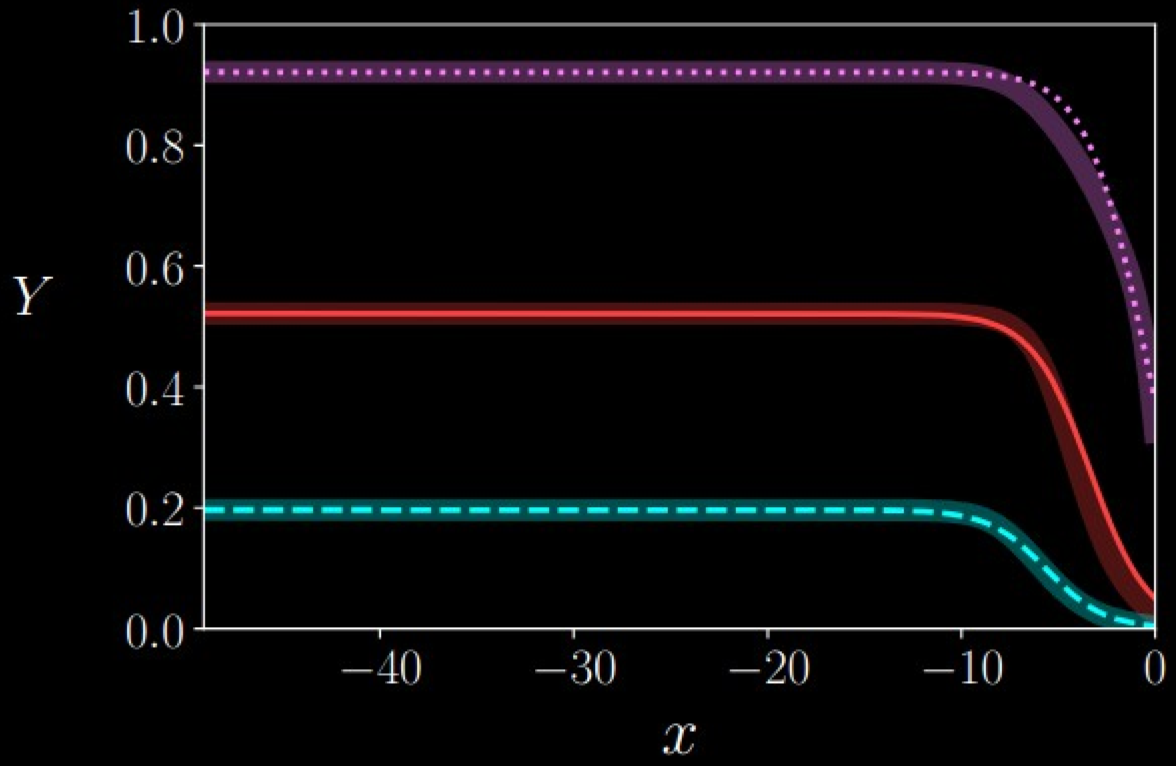


Number of loops produced is enhanced for large currents

Evolution of Strings with Current

Current is quickly dissipated in the matter era



Impact of Current on the SGWB

IN THE RADIATION ERA:

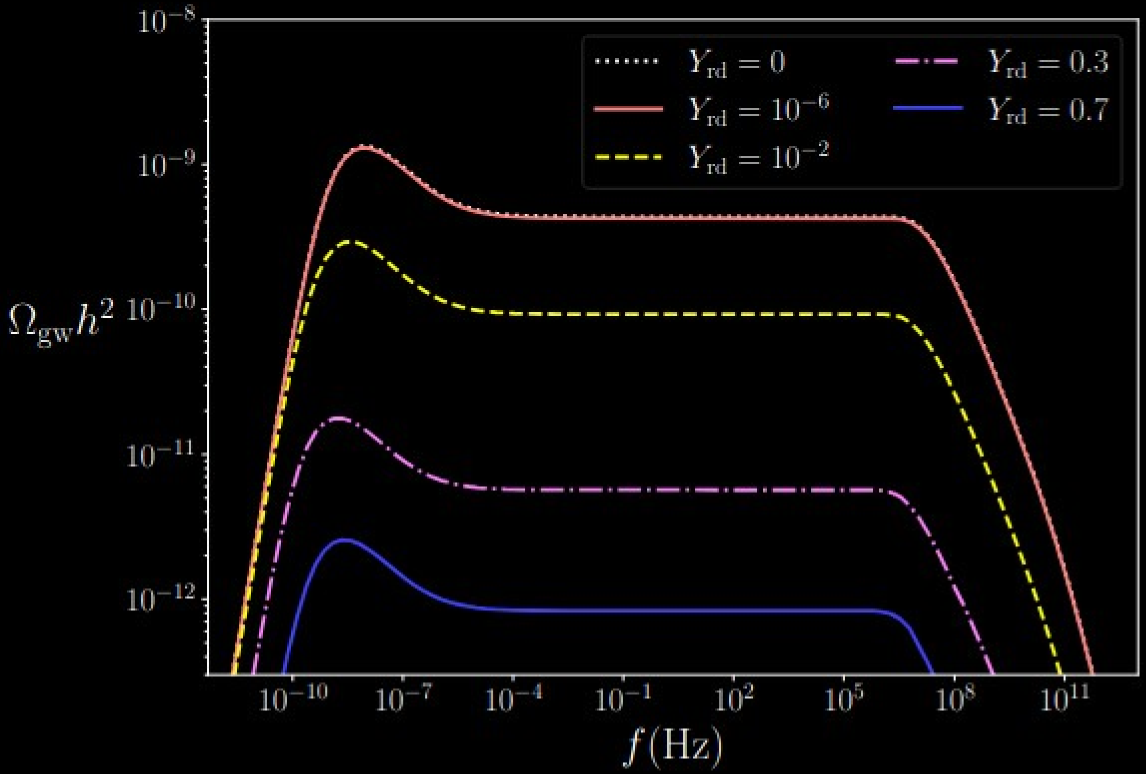
THE AMPLITUDE OF THE PLATEAU
MAY BE SIGNIFICANTLY AFFECTED AND
DEPENDS ON THE VALUE OF CURRENT

IN THE MATTER ERA:

THE CONTRIBUTION OF MATTER-ERA
LOOPS IS (MOSTLY) UNAFFECTED

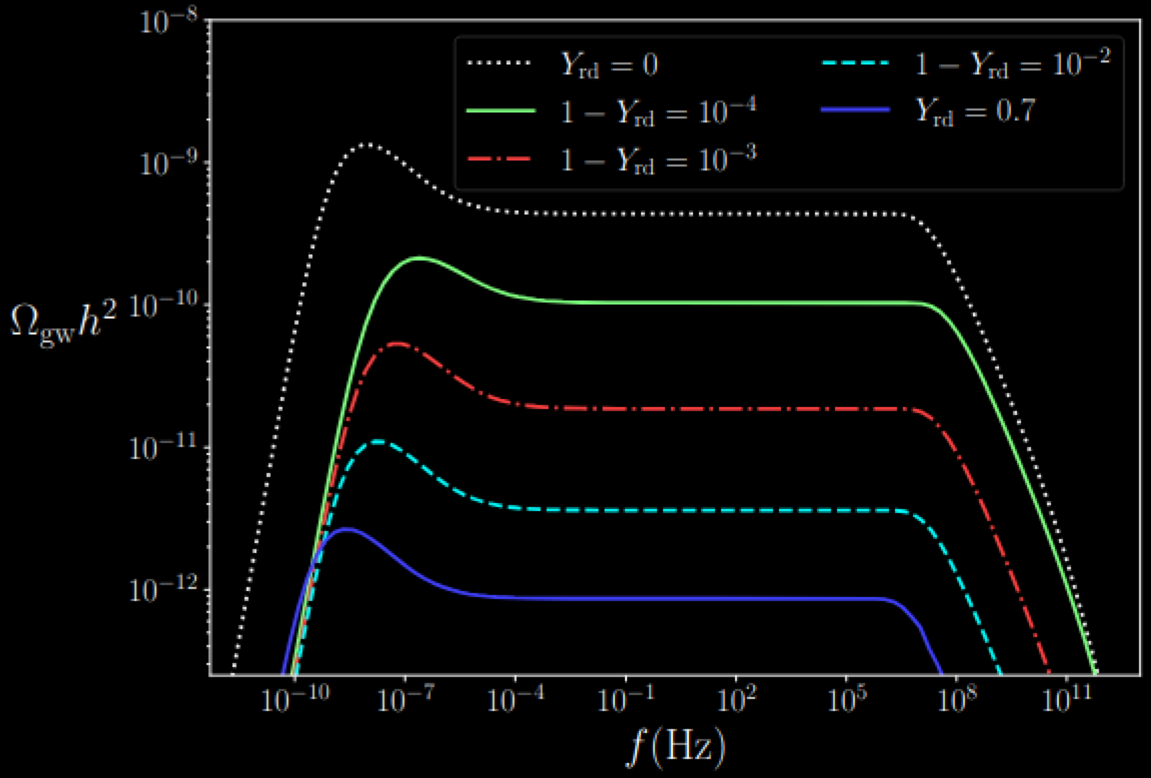
IMPACT OF CURRENT ON THE SGWB

For small enough currents:



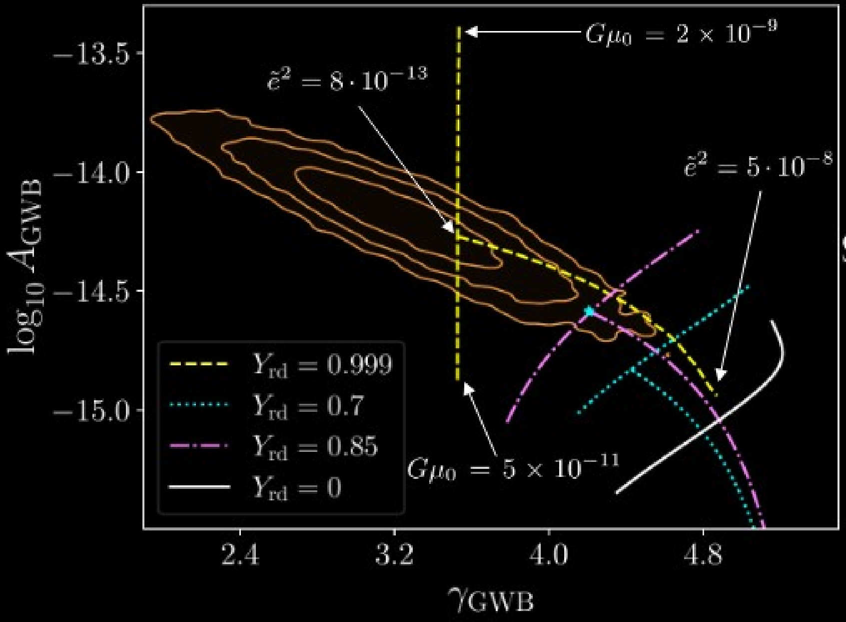
Impact of Current on the SGWB

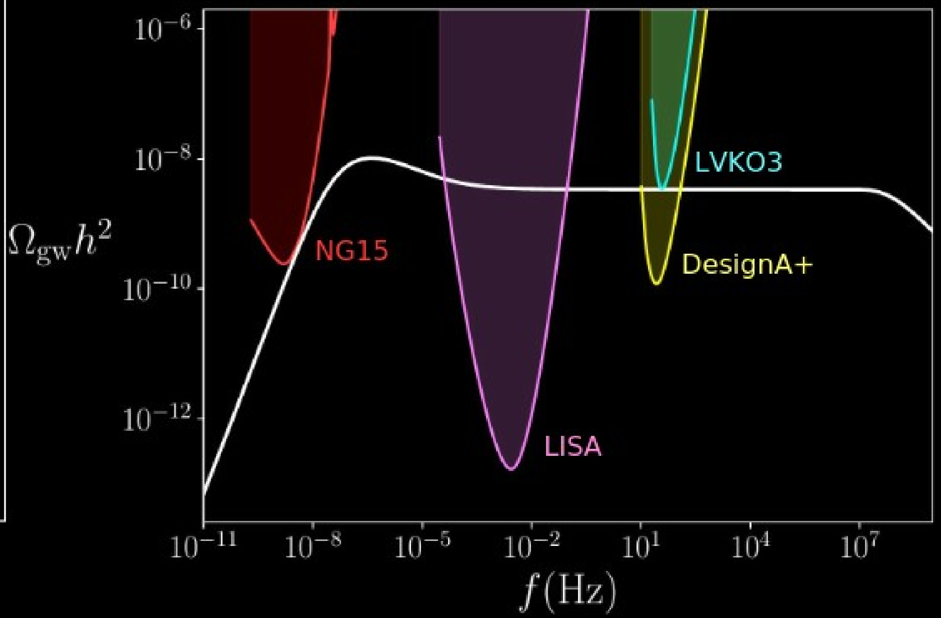
For large enough currents:



Impact of Current on the SGWB

Superconducting strings with high current fit better NANOGrav data than standard cosmic strings



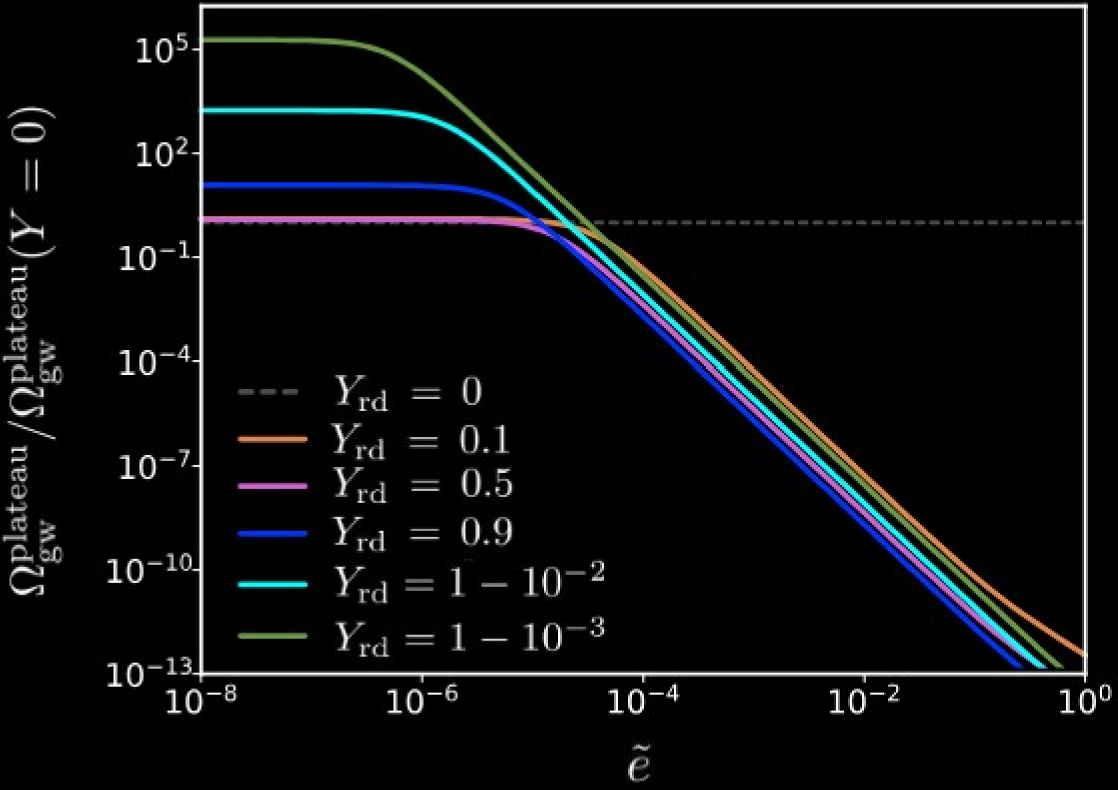


Impact of Current on the SGWB

CURRENT MAY ACTUALLY HELP RECONCILE COSMIC STRINGS WITH NANOGRAV DATA!

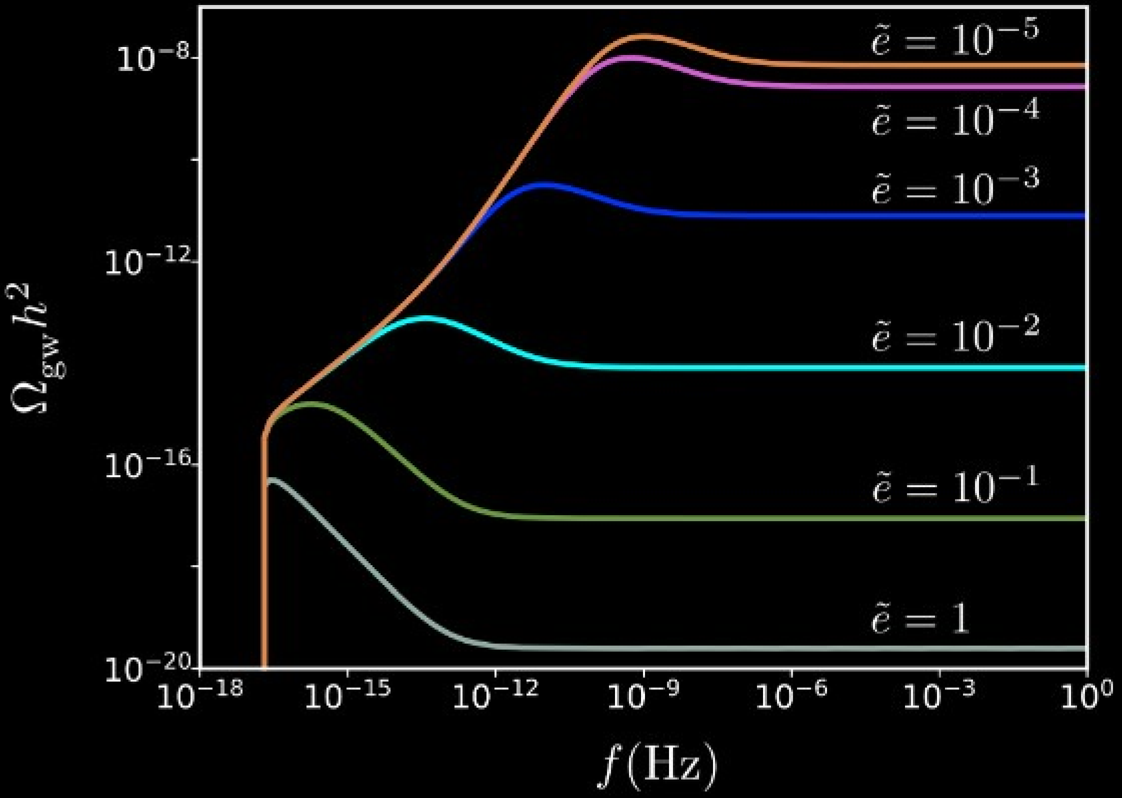
Impact of Charge on the SGWB

As the charge of the current carriers increases, the amplitude is suppressed:



Impact of Charge on the SGWB

But the shape of the spectrum is also affected



Impact of Current on the SGWB

THERE MAY BE DISTINCTIVE SIGNATURES
ON THE SGWB SPECTRUM GENERATED
BY STRINGS, WHICH MAY ALLOW US
TO PROBE THE UNDERLYING
PARTICLE PHYSICS SCENARIO

To Sum Up

*Current may significantly decrease the efficiency of emission of gravitational radiation by cosmic strings and affect the spectrum of emission;

*The amplitude of the SGWB for chiral superconducting strings is highly dependent on the value of current and on the charge of current carriers;

*There may also be significant changes to the shape of the spectrum if the charge of current carriers is large enough;

THANKS!