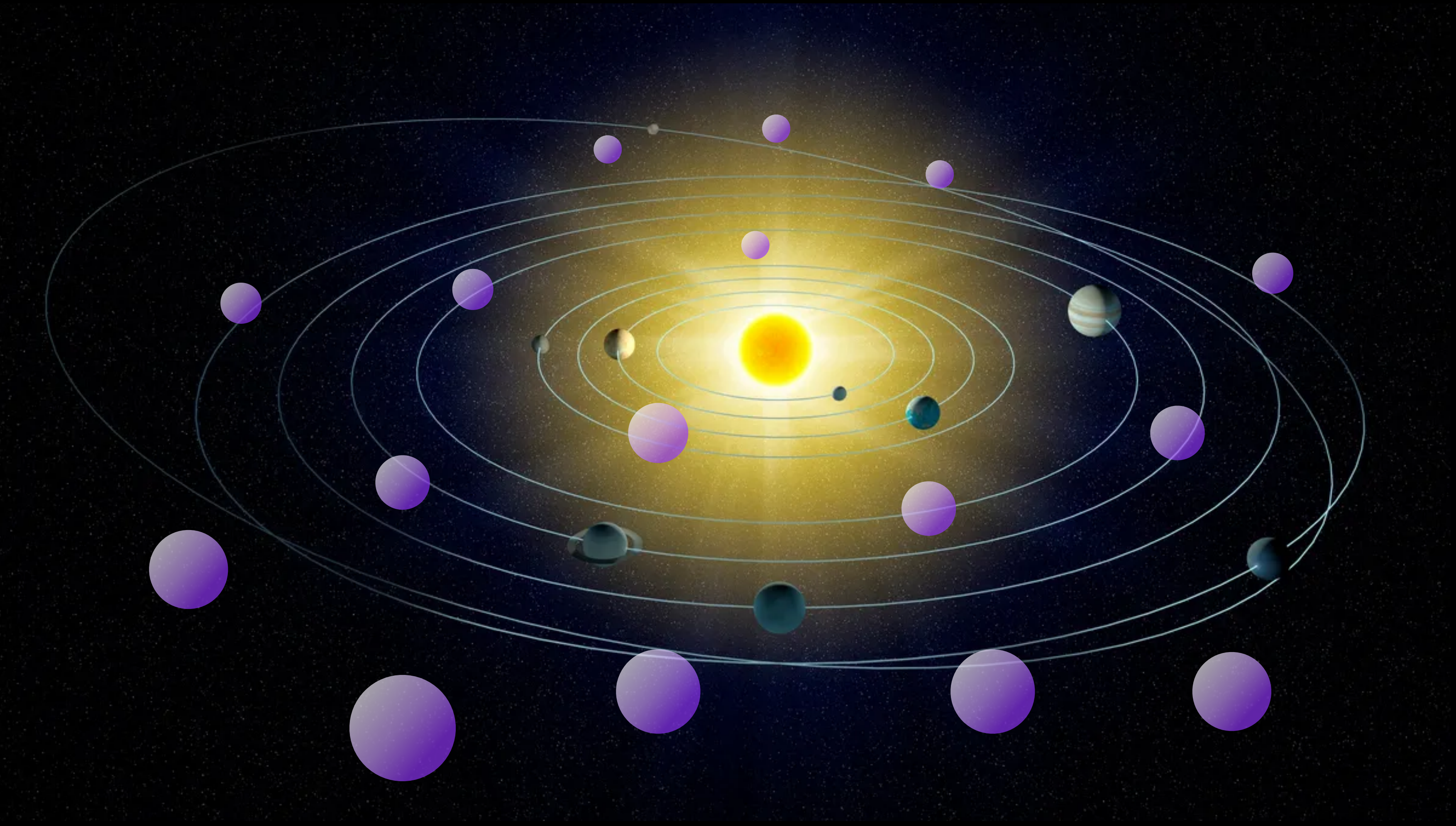
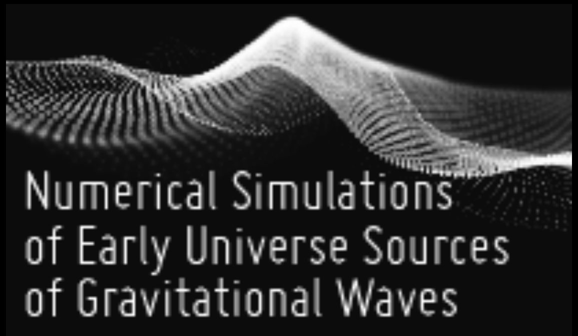
More axion stars and GWs from strings

Marco Gorghetto



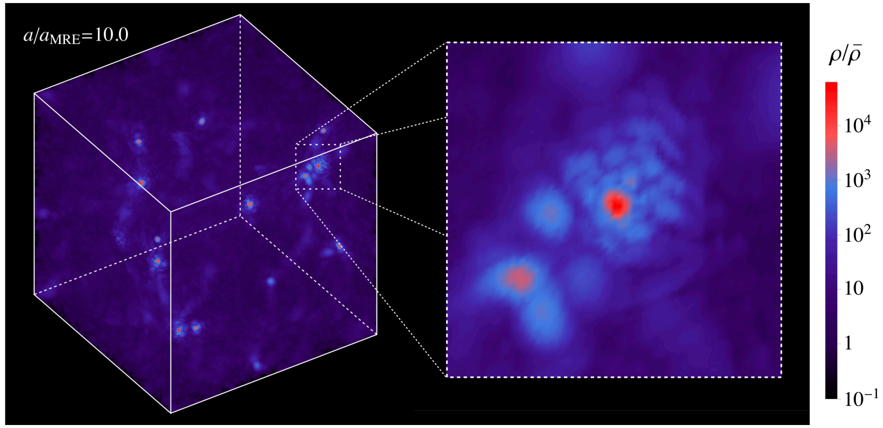
- MG, E.Hardy, G.Villadoro
  [2405.19389]

- MG, E.Hardy, H.Nicolaescu
  [2101.11007]

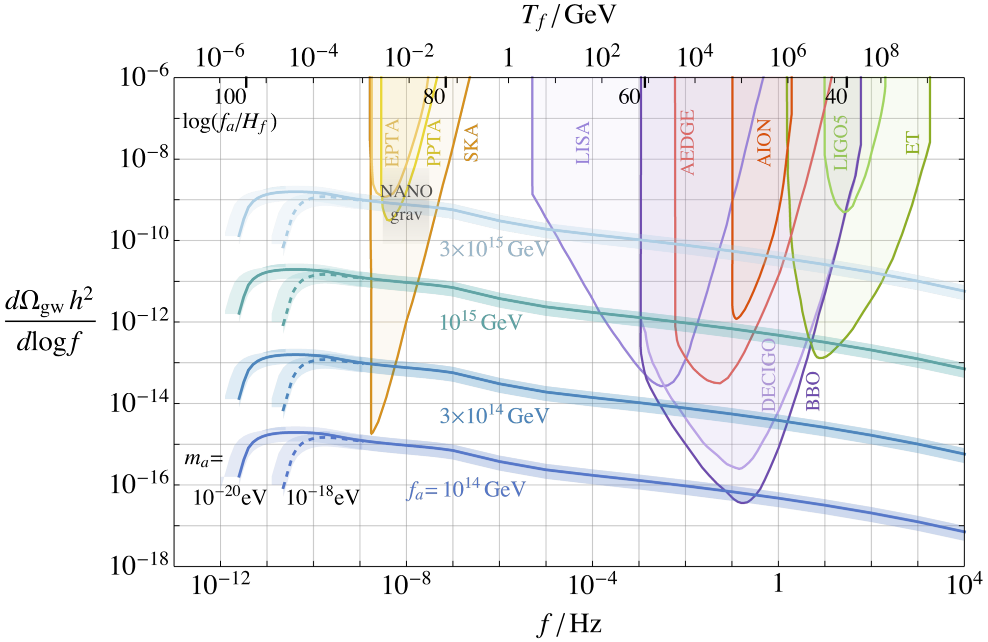


Outline

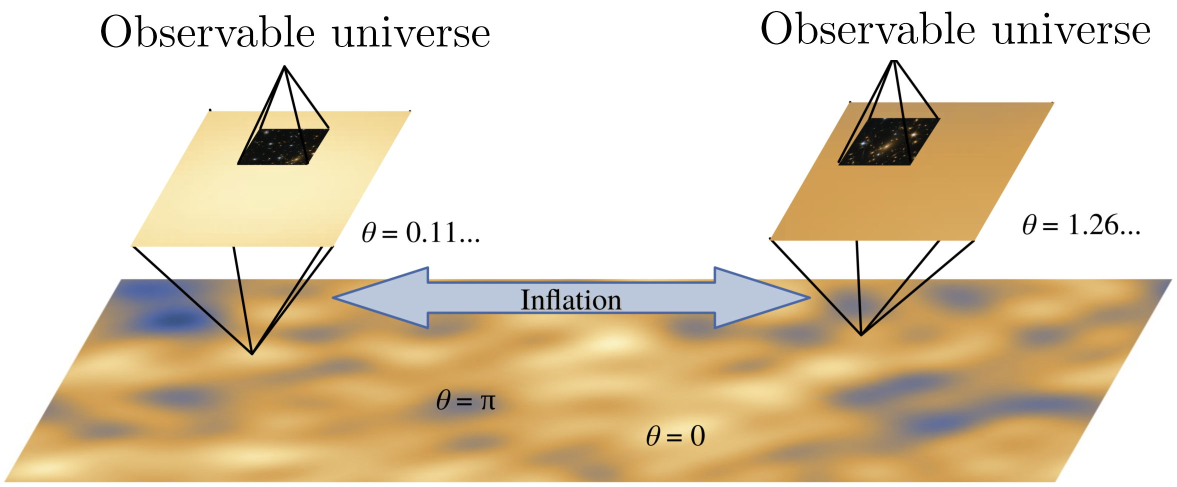
1. Axion dark matter abundance and stars



2. Gravitational waves from ALP strings



Pre-inflationary

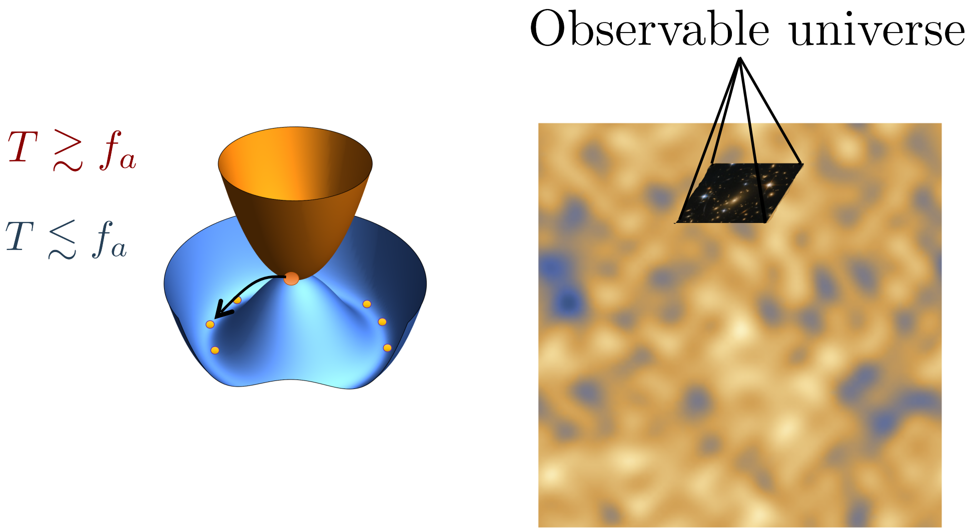


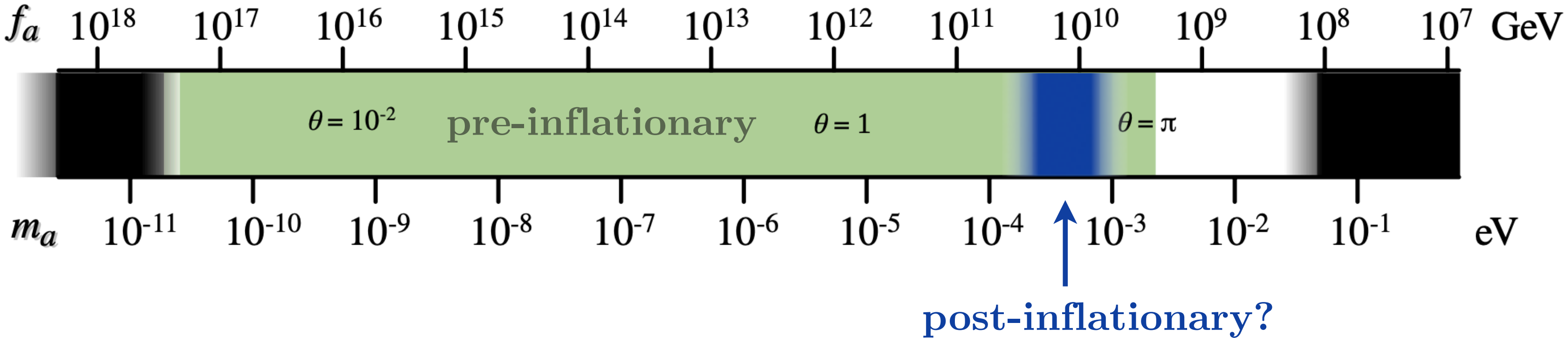
$$\theta \equiv \frac{a}{f_a} \in [-\pi, \pi]$$

$$\Omega_a \simeq \theta_0^2 \left(\frac{f_a}{10^{12}\,\mathrm{GeV}} \right)^{1.2} \Omega_{\mathrm{DM}}$$

misalignment

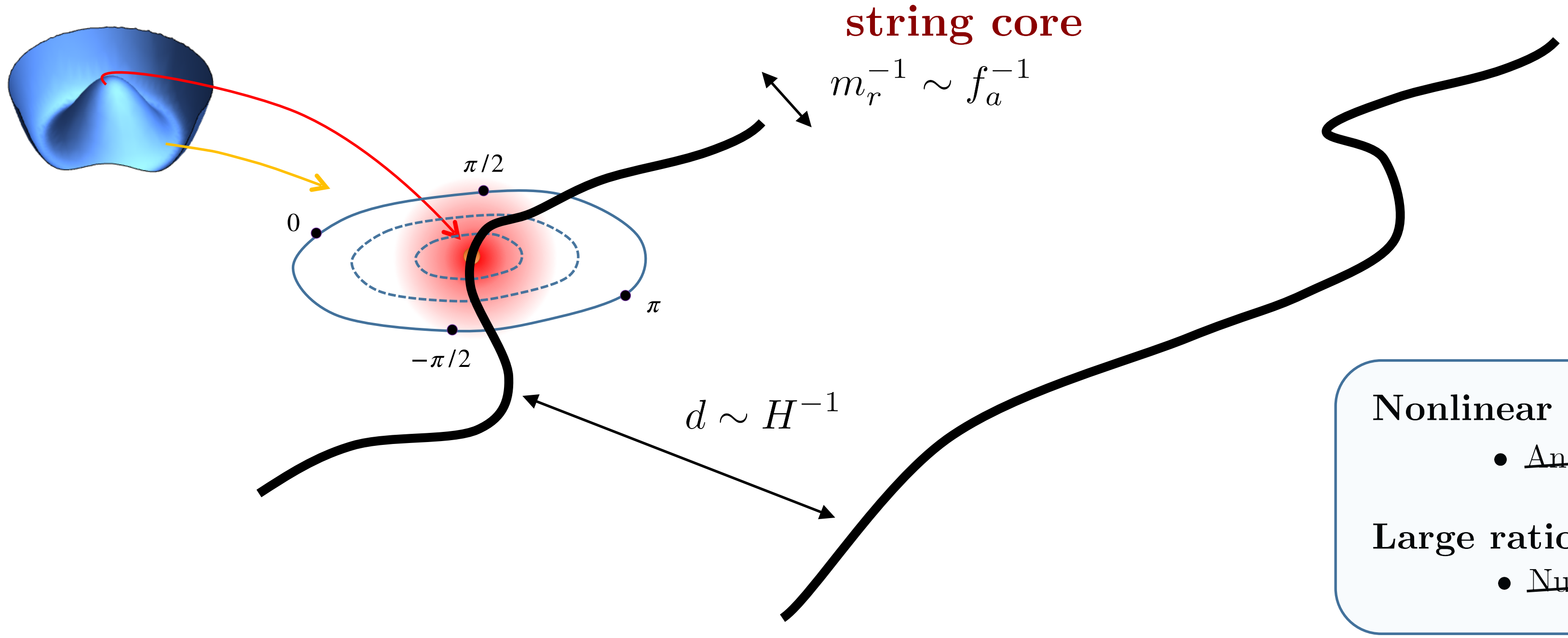
Post-inflationary





@ $T \simeq f_a$ (or $H \simeq f_a$)

Kibble mechanism $\Longrightarrow$ **Axion strings**



Nonlinear dynamics:

- ~~Analytical approach~~ ☹

Large ratio of scales:

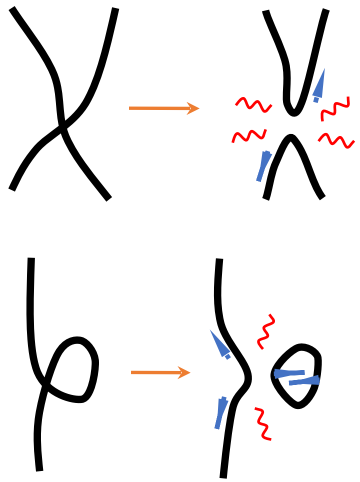
- ~~Numerical approach~~ ☹

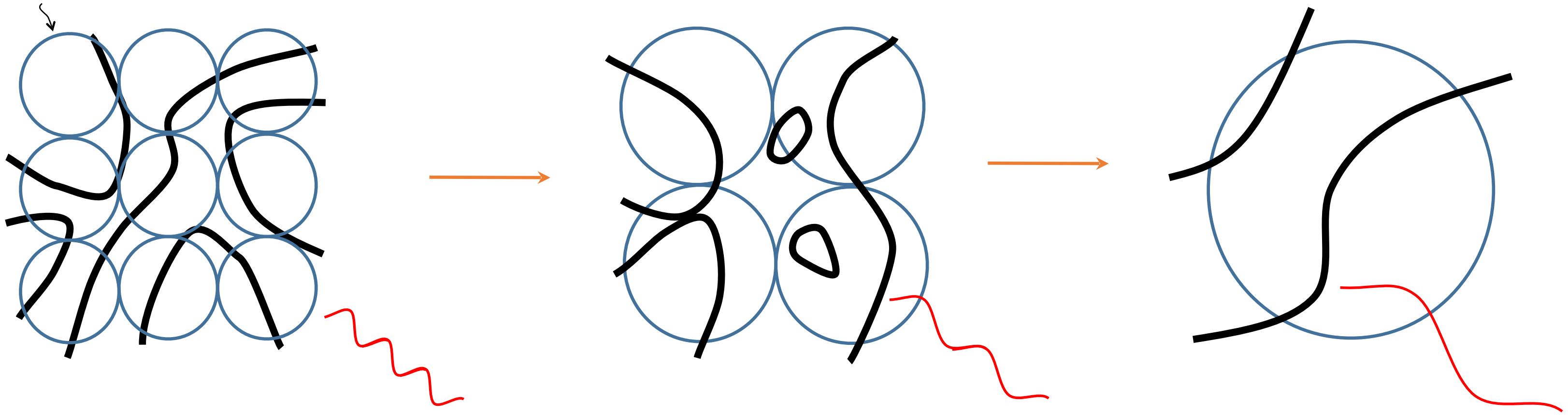
string tension

$$\mu = \frac{E}{L} \sim \underbrace{\boxed{\pi f_a^2}}_{\text{core}} \underbrace{\boxed{\log \frac{d}{m_r^{-1}}}}_{\text{axion gradient}} \sim \pi f_a^2 \log \frac{m_r}{\underbrace{H}_{T^2/M_p}}$$

grows logarithmically in time

The Scaling Regime

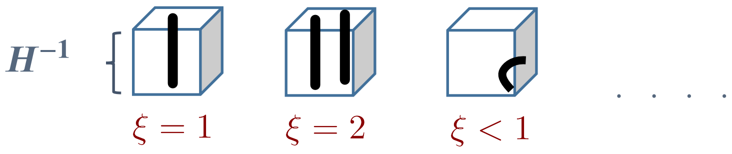


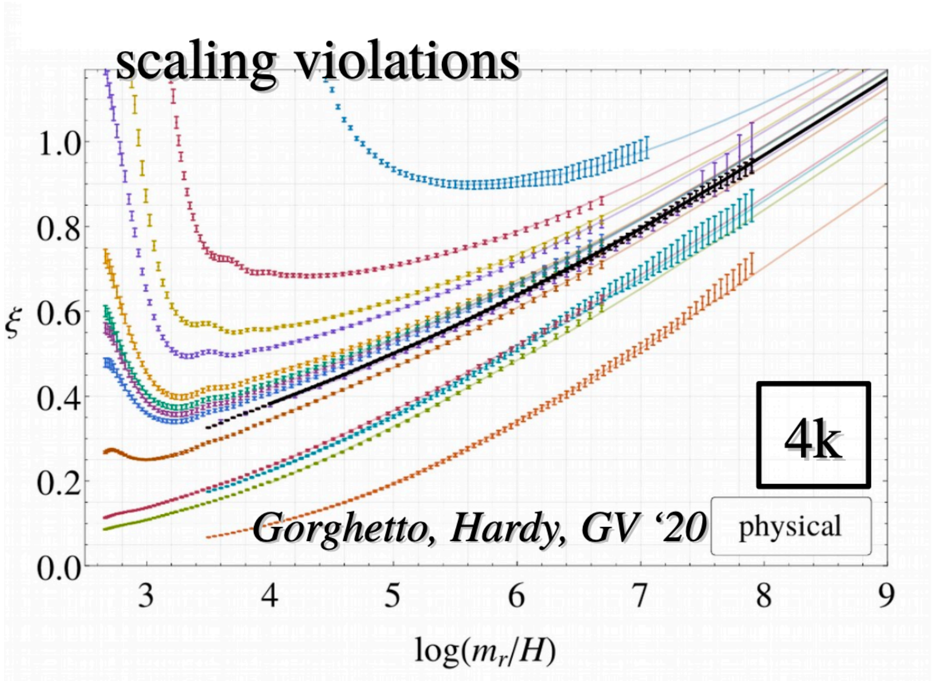


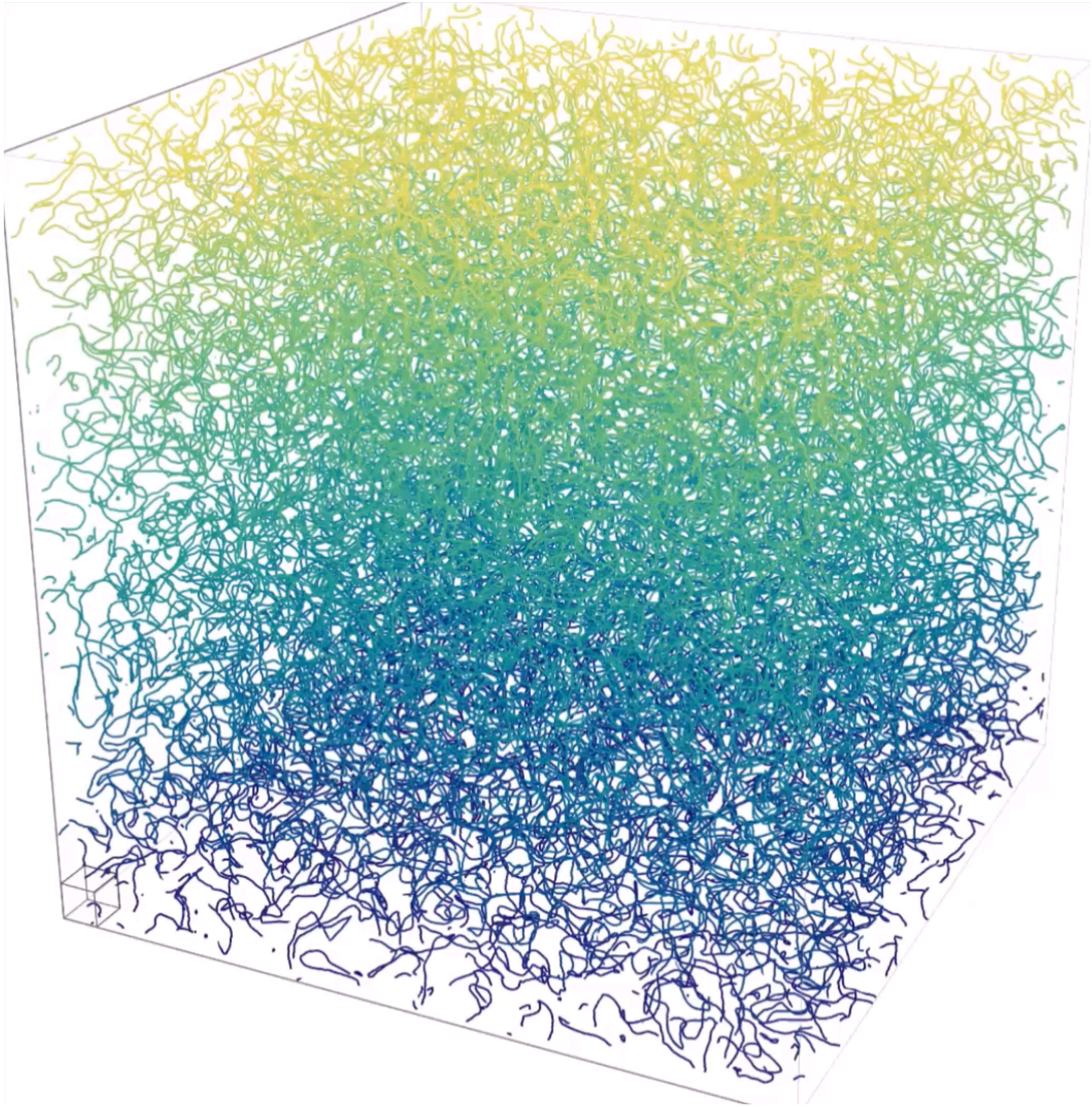
rate of energy loss:

$$\Gamma \simeq \frac{\boxed{\xi}\mu}{t^3}$$

number of strings per Hubble patch



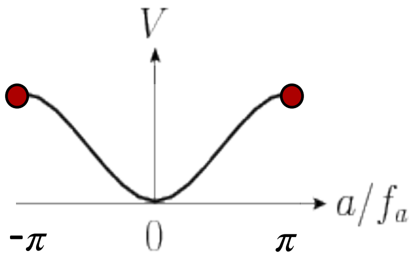


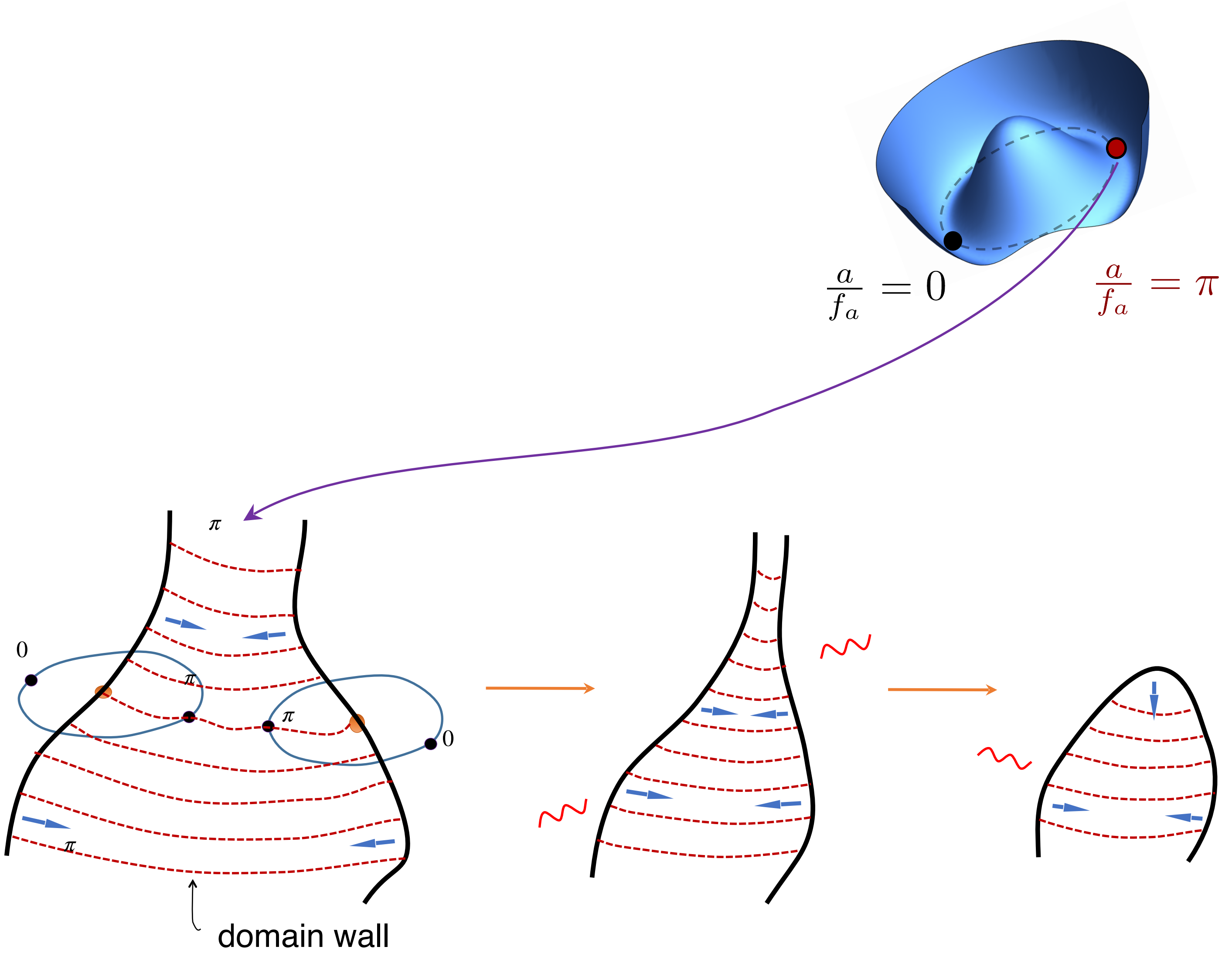


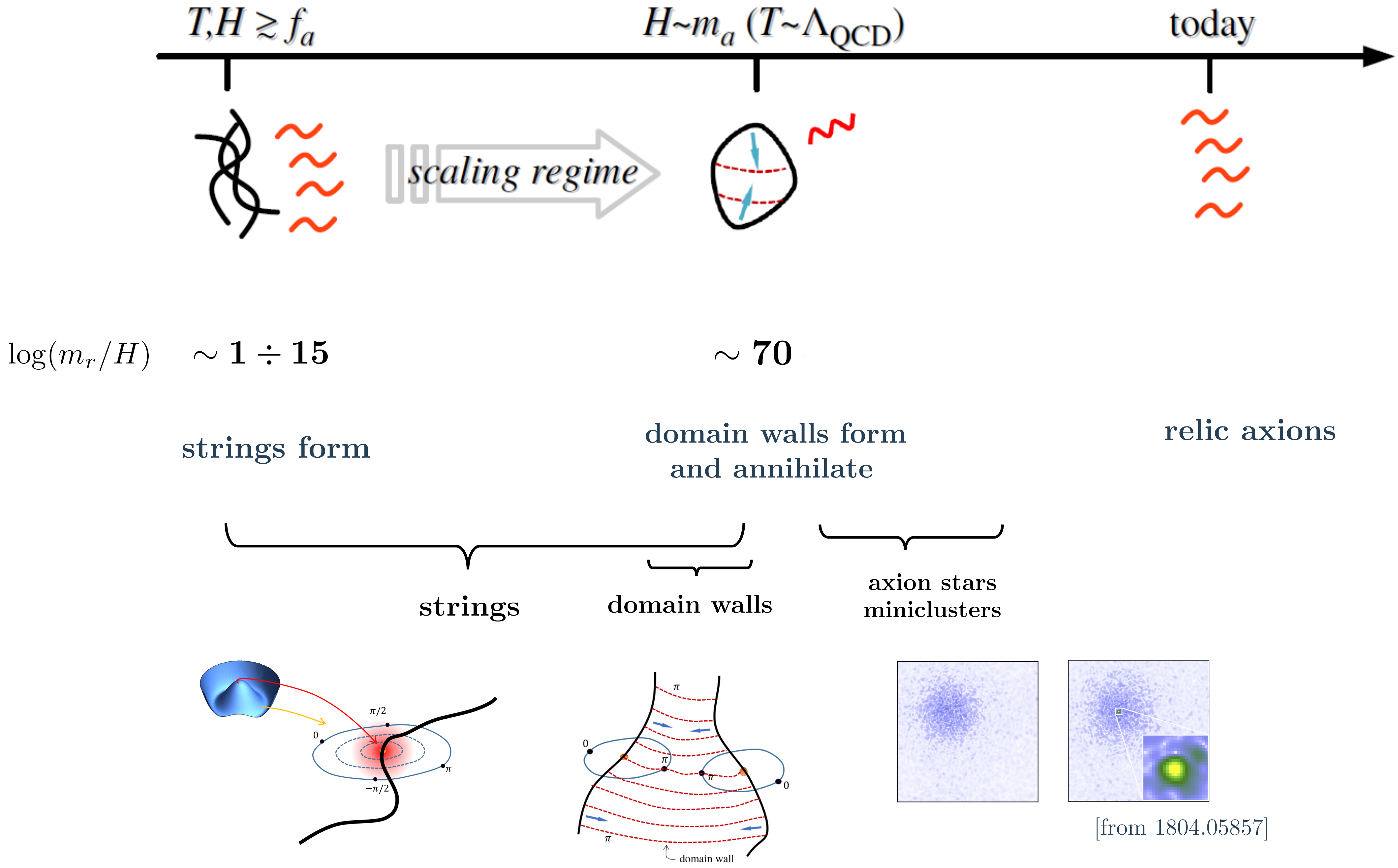
Domain Walls

@ $T \simeq 1\,\mathrm{GeV}$ $\quad (\; m = H \equiv H_\star \;)$

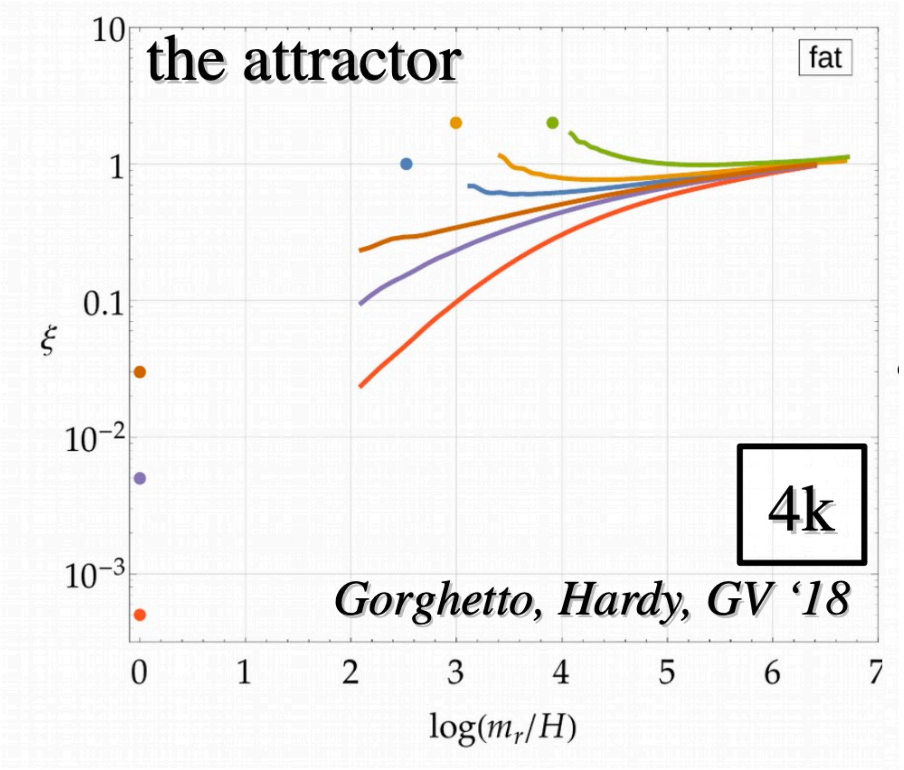
Axion potential from QCD:

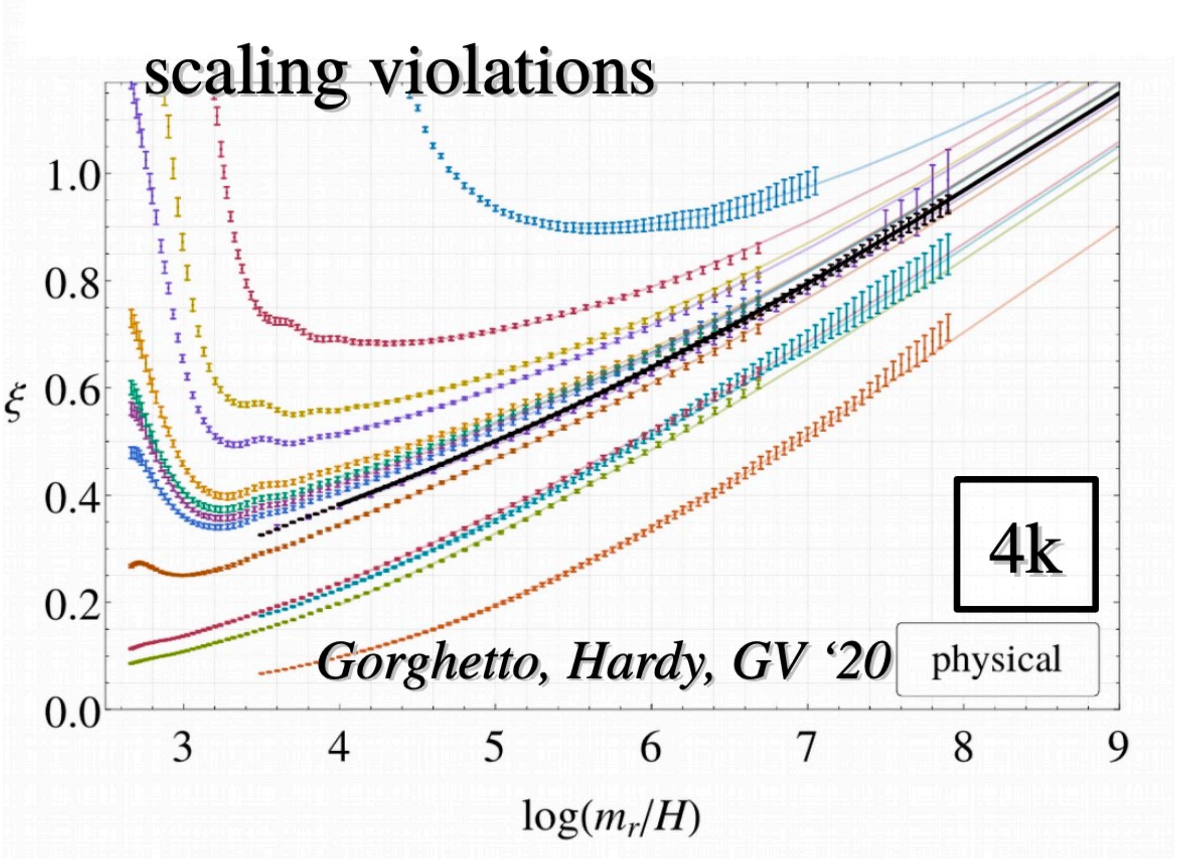






$T,H \gtrsim f_a$
$H \sim m_a \ (T \sim \Lambda_{QCD})$
today
scaling regime
$\log(m_r/H)$
$\sim \mathbf{1 \div 15}$
$\sim \mathbf{70}$
strings form
domain walls form
and annihilate
relic axions
strings
domain walls
axion stars
miniclusters
$\pi/2$
0
π
$-\pi/2$
domain wall
[from 1804.05857]

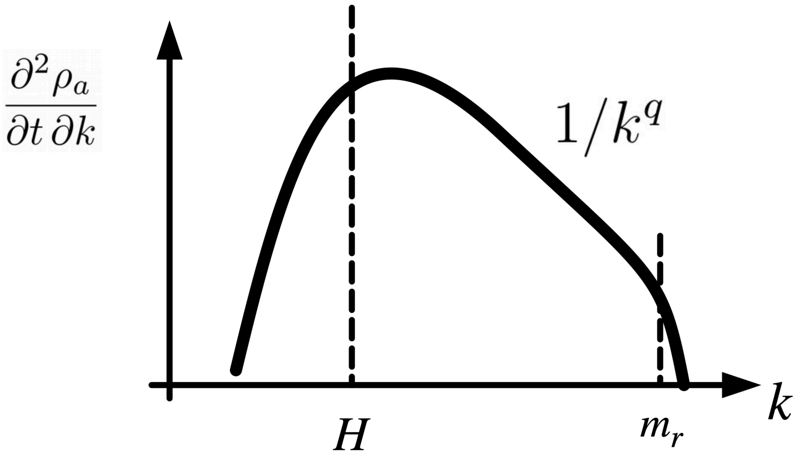




$$\xi \to \frac{\log(m_r/H)}{4 \div 5}$$

$$\boxed{\xrightarrow{\log \to 70} 15(2)}$$

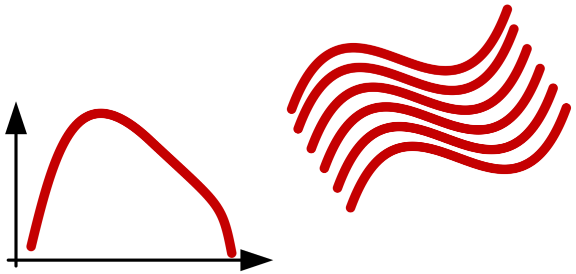
The Spectrum



$$n \sim \frac{\rho}{\langle k \rangle}$$

IR

$q > 1$

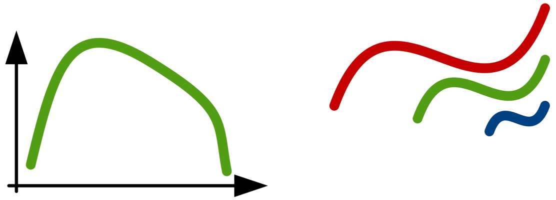


Davies, Shellard, ...

$$n \sim \frac{\rho}{H} \sim \xi \log f^2 H \sim \boxed{\xi \log} n^{mis}$$

$\sim 10^3$

$q = 1$

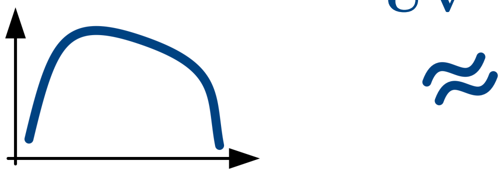


Sikivie, ...

$$n \sim \frac{\rho}{H \log} \sim \xi f^2 H \sim \boxed{\xi} n^{mis}$$

UV

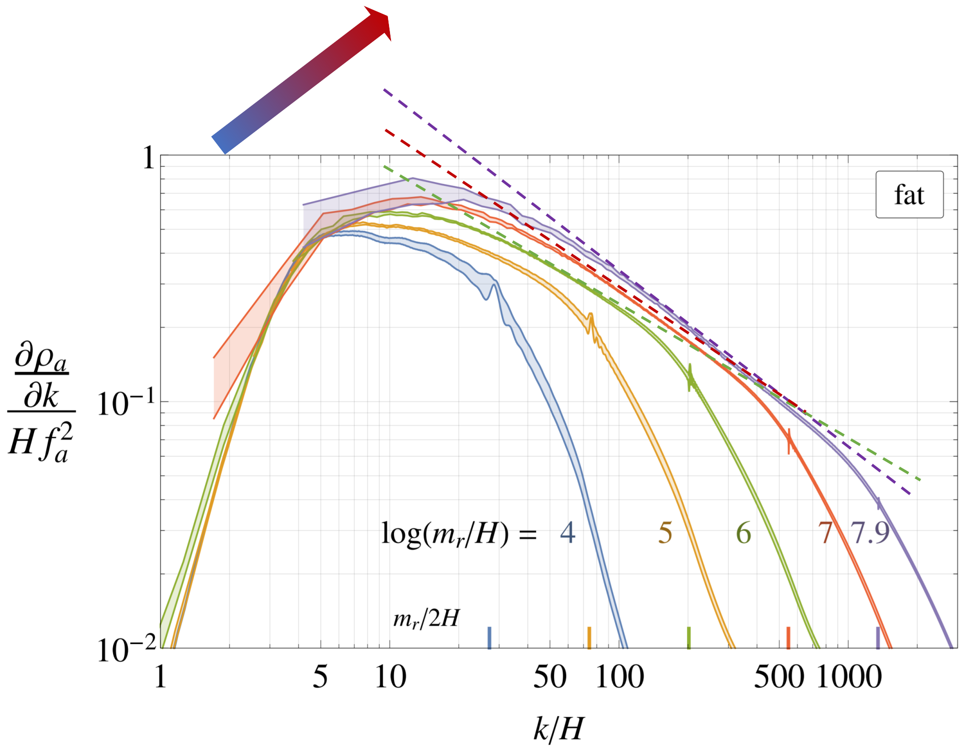
$q < 1$

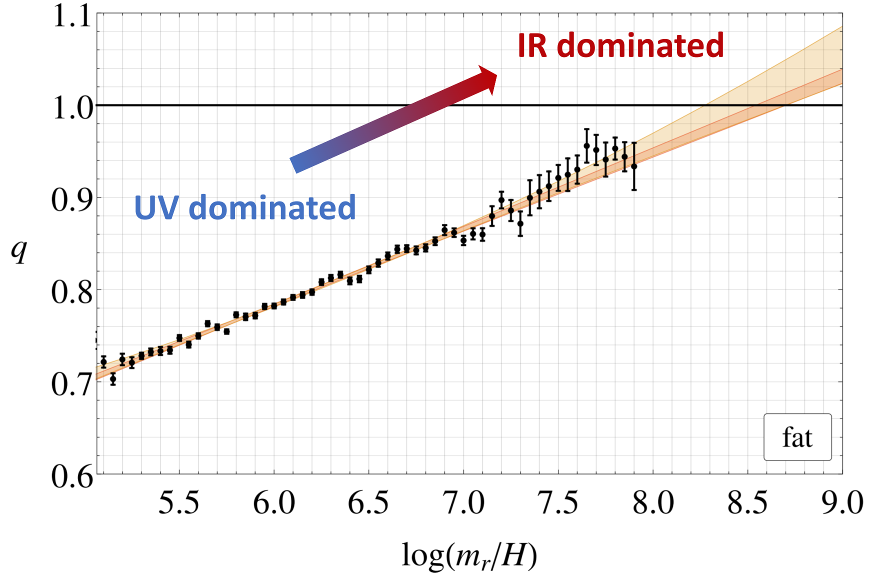


$$n \sim \frac{\rho}{H} \left(\frac{H}{m_r}\right)^{1-q} \sim n^{mis} \boxed{\left(\frac{H}{m_r}\right)^{1-q}}$$

$\lll 1$

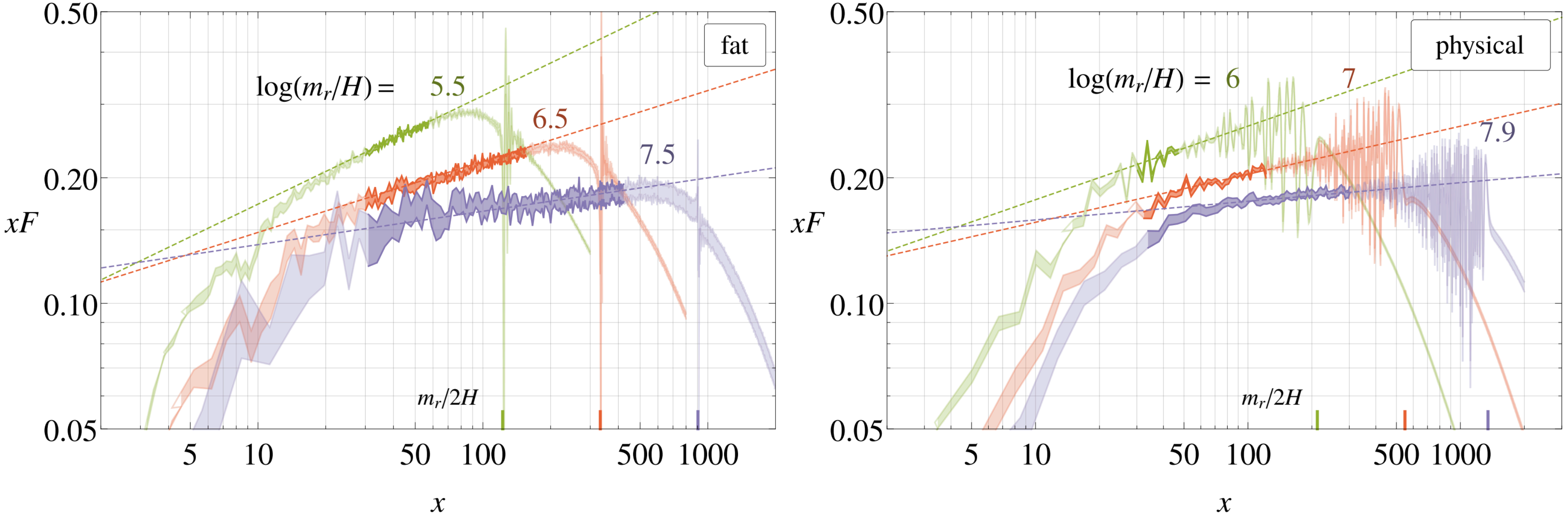
The Spectral Index





Running of q $\Rightarrow$ $\underset{\log \to 70}{q > 1}$

$$f_a \simeq (\overset{q>1}{1} \div \overset{q=1}{6}) \cdot 10^{10}\ \text{GeV} \quad + \text{DW?}$$



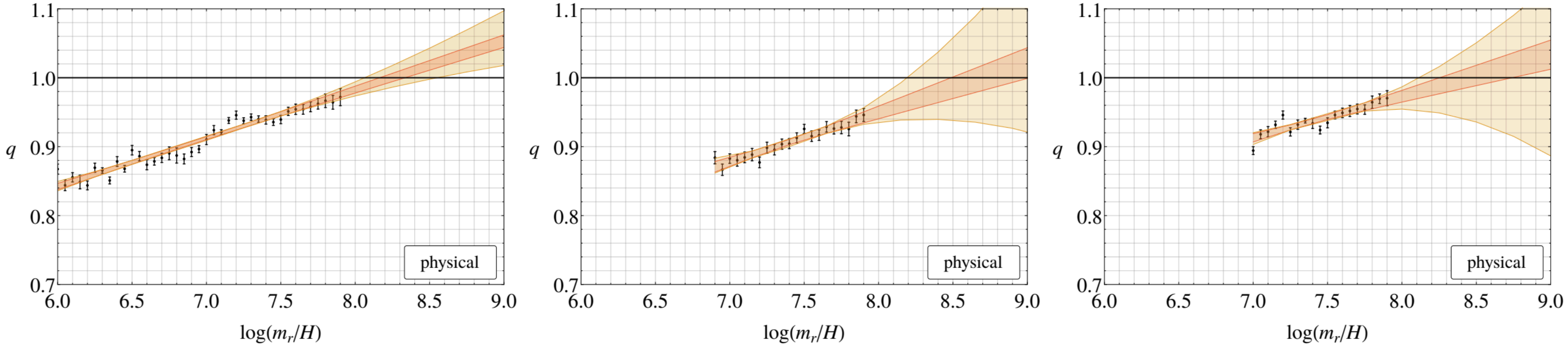
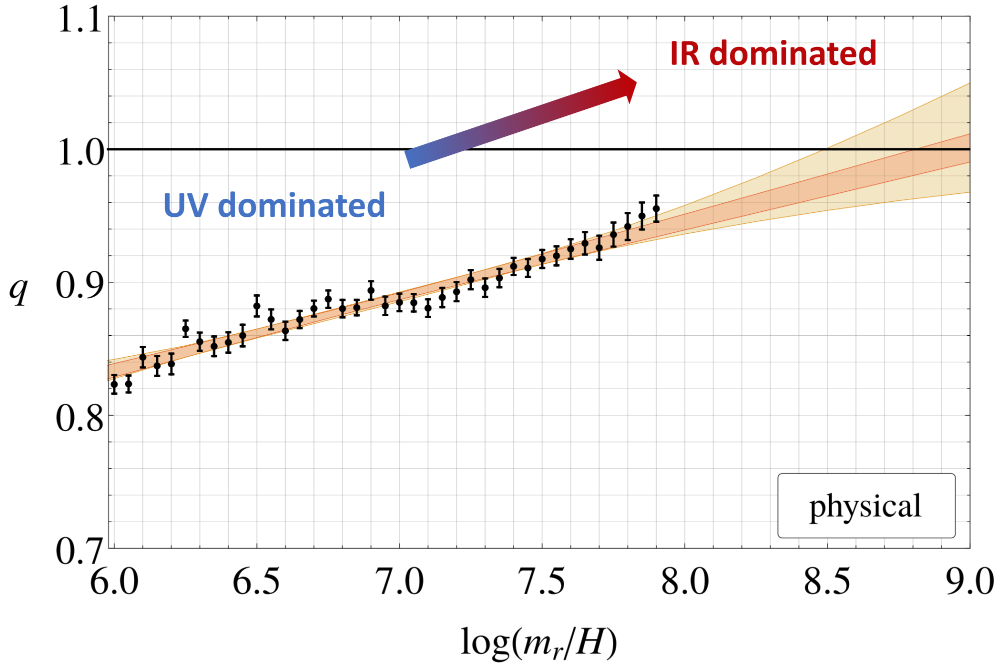
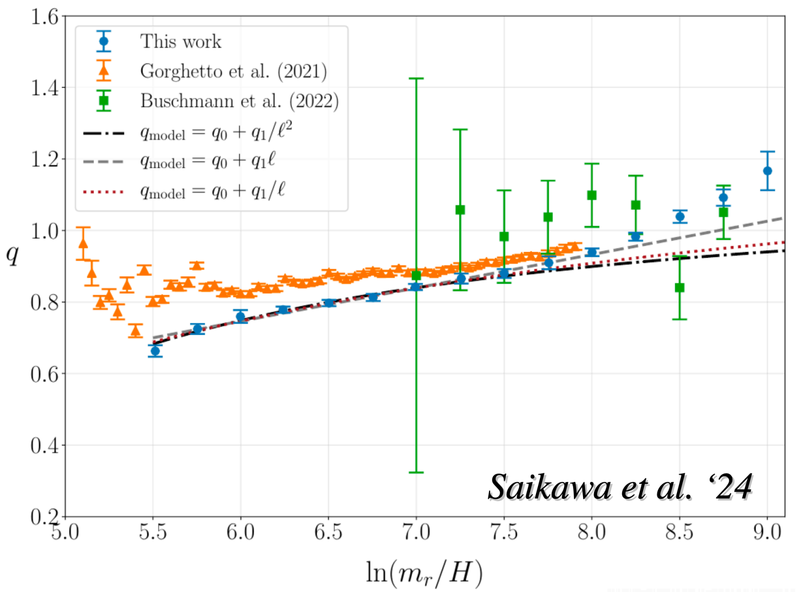
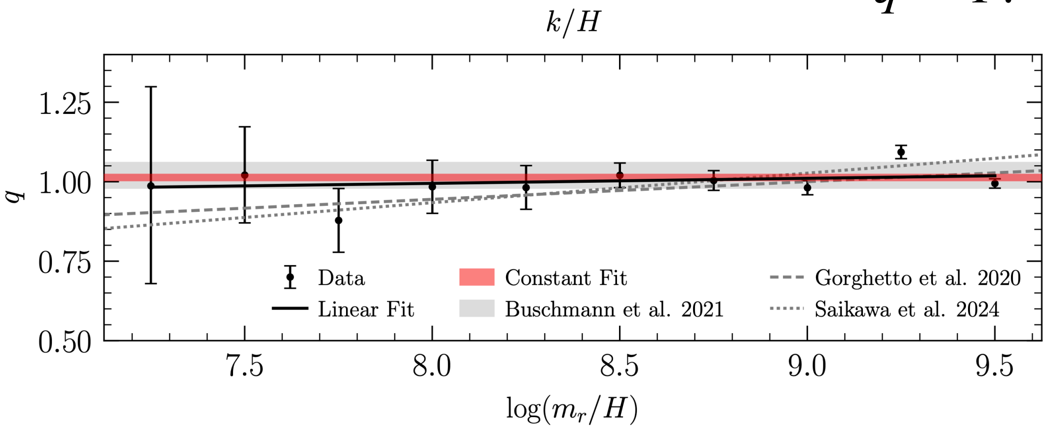


Figure 18: The best fit values of q as a function of log for physical strings, with statistical error bars. Left: results with q fit over the momentum range $[30H, m_r/4]$ for data with lattice spacing at the final time $m_r\Delta_f = 1.5$. Center: q fit in the range $[50H, m_r/6]$ for $m_r\Delta_a = 1$. Right: q fit in the range $[50H, m_r/6]$ for $m_r\Delta_f = 1.5$.

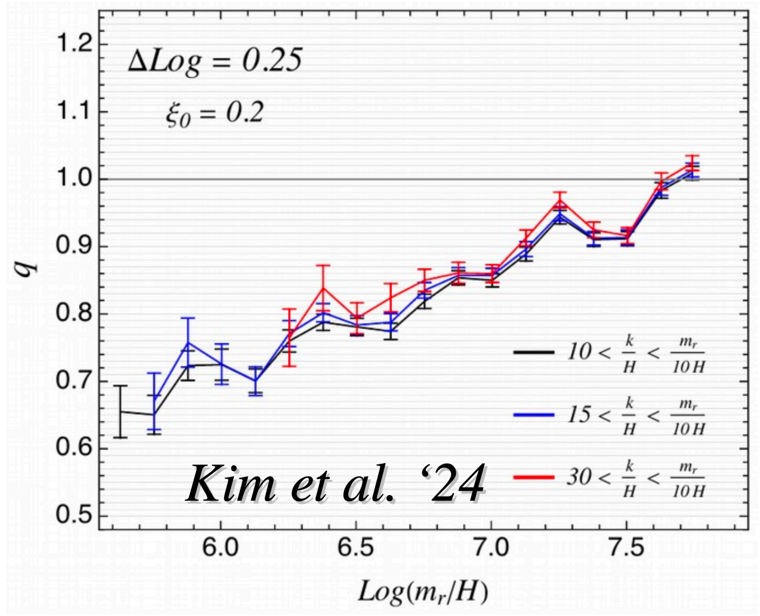


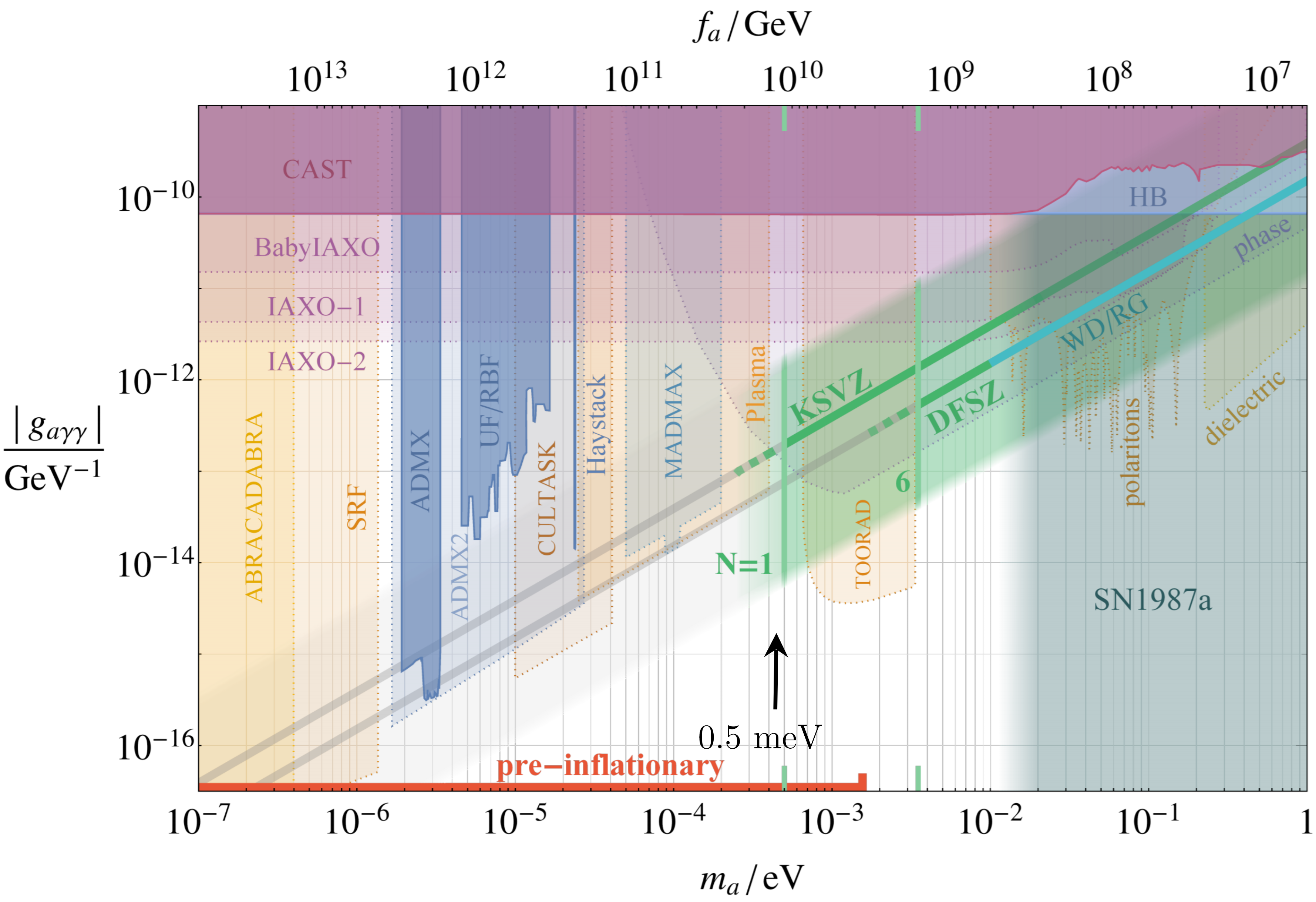


$q \simeq 1?$



Benabou+, 25'



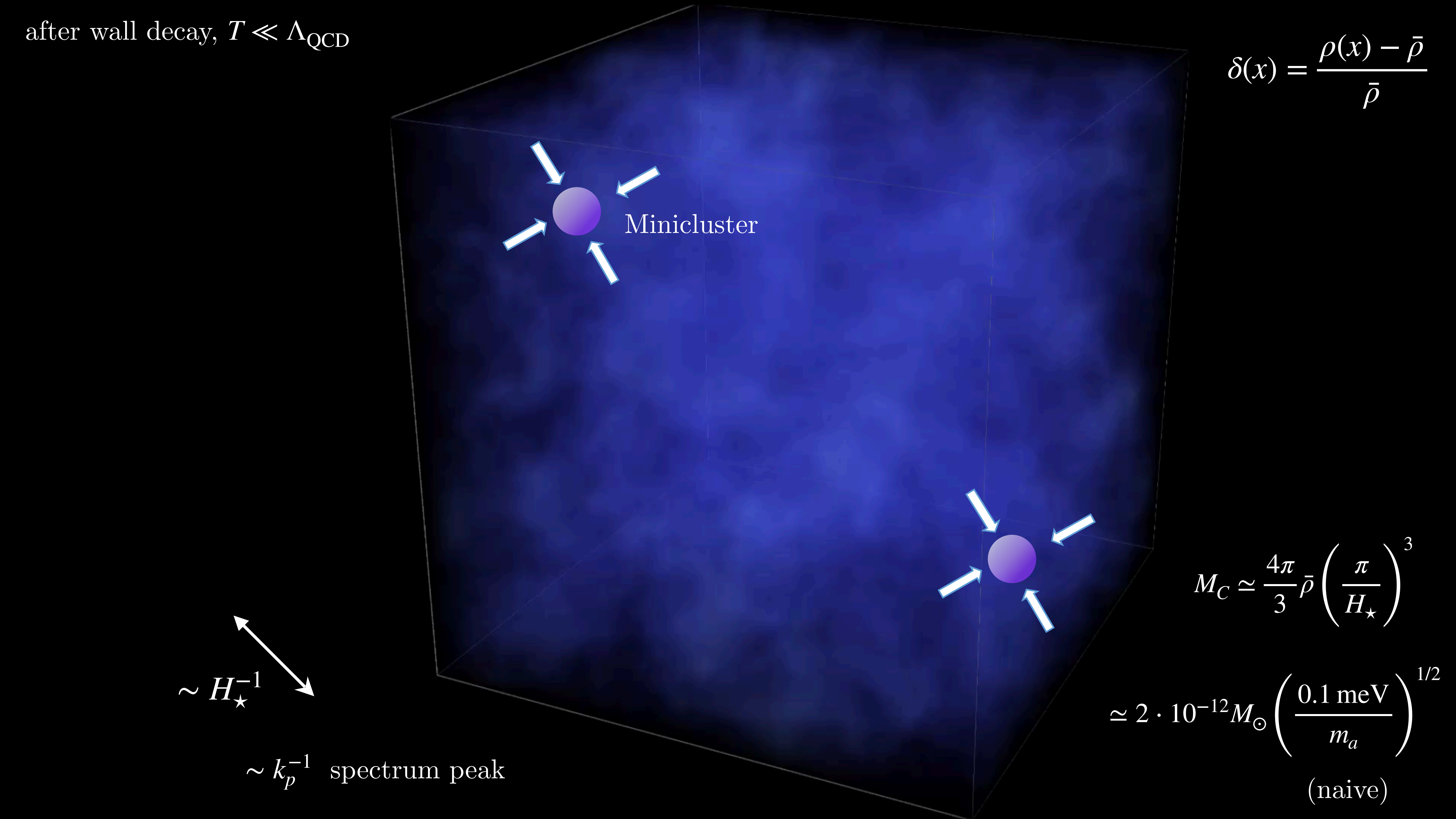


$q \simeq 1$ $\qquad$ $q > 1$

$f_a \lesssim 10^{10}$ GeV $\quad$ from DM overproduction

$\simeq$ $\quad$ if domain walls negligible

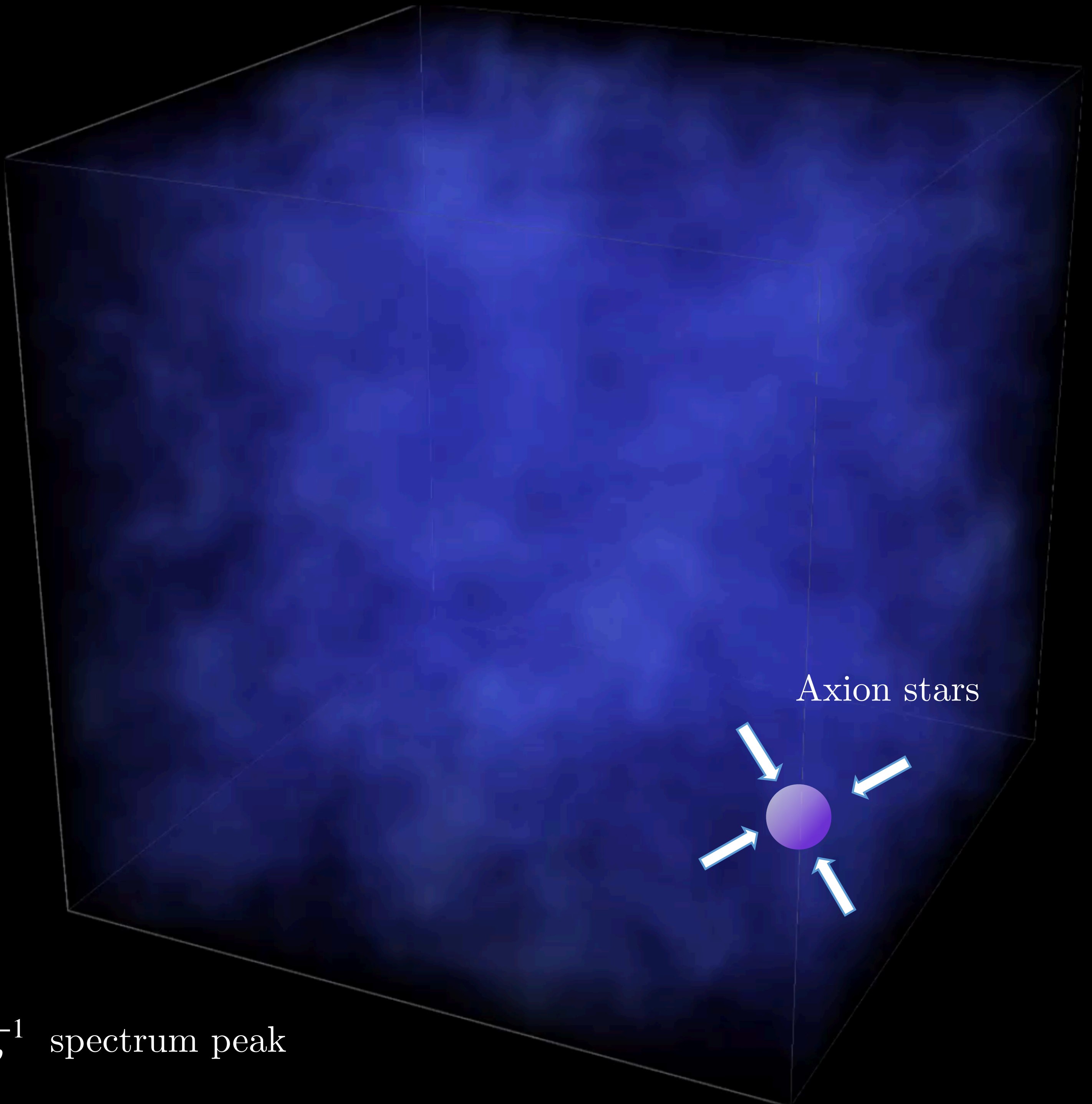
Formation of structures



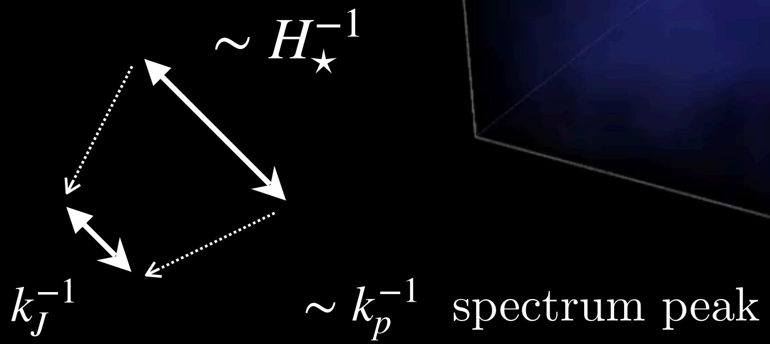
after wall decay, $T \ll \Lambda_{\rm QCD}$
$\delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$
Minicluster
$\sim H_\star^{-1}$
$\sim k_p^{-1}$ spectrum peak
$M_C \simeq \frac{4\pi}{3} \bar{\rho} \left(\frac{\pi}{H_\star} \right)^3$
$\simeq 2 \cdot 10^{-12} M_\odot \left(\frac{0.1\,\mathrm{meV}}{m_a} \right)^{1/2}$
(naive)

at $T \simeq \Lambda_{\rm QCD}$:

$$\delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$$

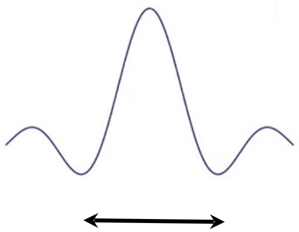


M_C is smaller



Gravitational collapse *vs* quantum Jeans scale

overdensity ρ



R

spatial size of the overdensity $\longleftrightarrow$ de Broglie wavelength of the particles in the resulting clump

$$R \qquad \qquad \frac{1}{mv}$$

$$\simeq \frac{1}{m\left(\frac{GM}{R}\right)^{1/2}} \simeq \frac{1}{R(4\pi G\rho m^2)^{1/2}}$$

$$4\pi\rho R^3/3$$

$\downarrow$

$$R_{\rm crit} \simeq \lambda_J \simeq (16\pi G\rho m^2)^{-1/4}$$

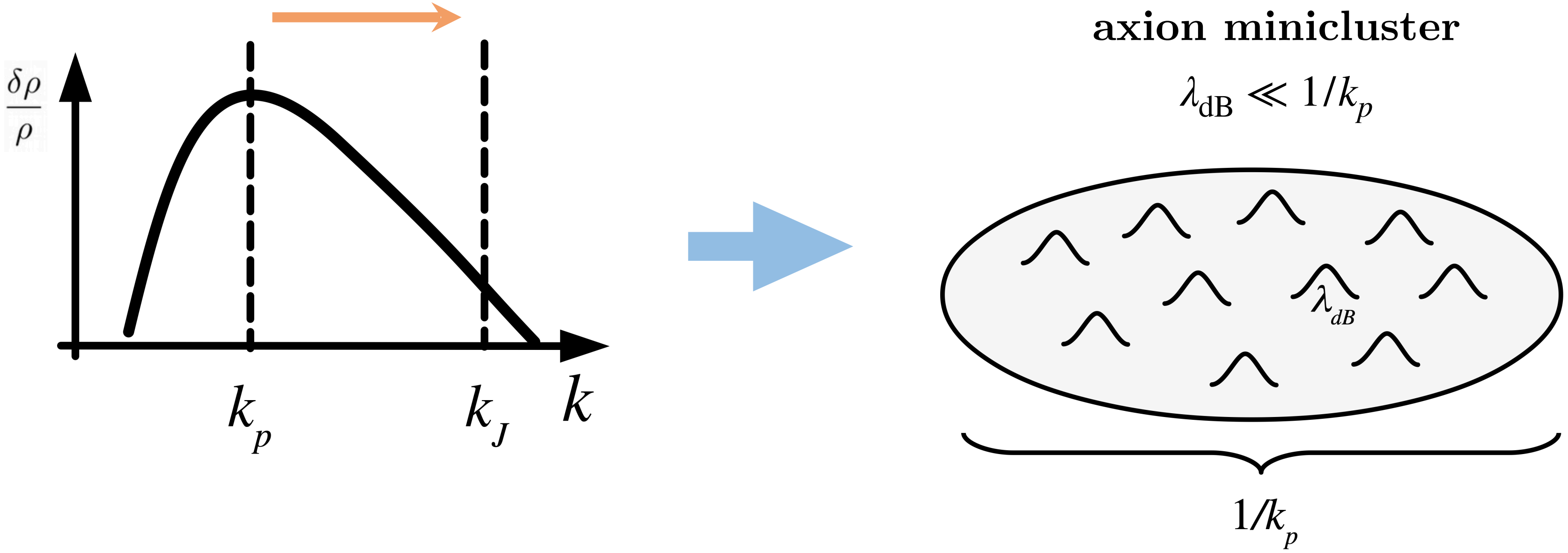
quantum Jeans length $\lambda_J = 2\pi/k_J$ $\quad\equiv\quad$ smallest scale an overdensity can have before wave effects (quantum pressure) have to be considered

The standard lore after DW decay

quantum Jeans scale

$k_J \equiv (16\pi G\rho m^2)^{\frac{1}{4}}$

@MRE

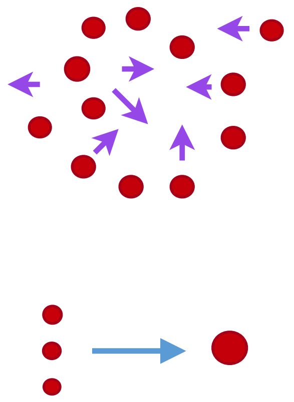


$$\left.\frac{k_p}{k_J}\right|_{\rm MRE} \simeq \left(\frac{f_a}{M_p}\right)^{1/3} \frac{k_{p\star}}{H_\star} \sim 10^{-3}\,\frac{k_{p\star}}{H_\star}$$

Naive because k_p increases due to the self-interactions and becomes of order k_J

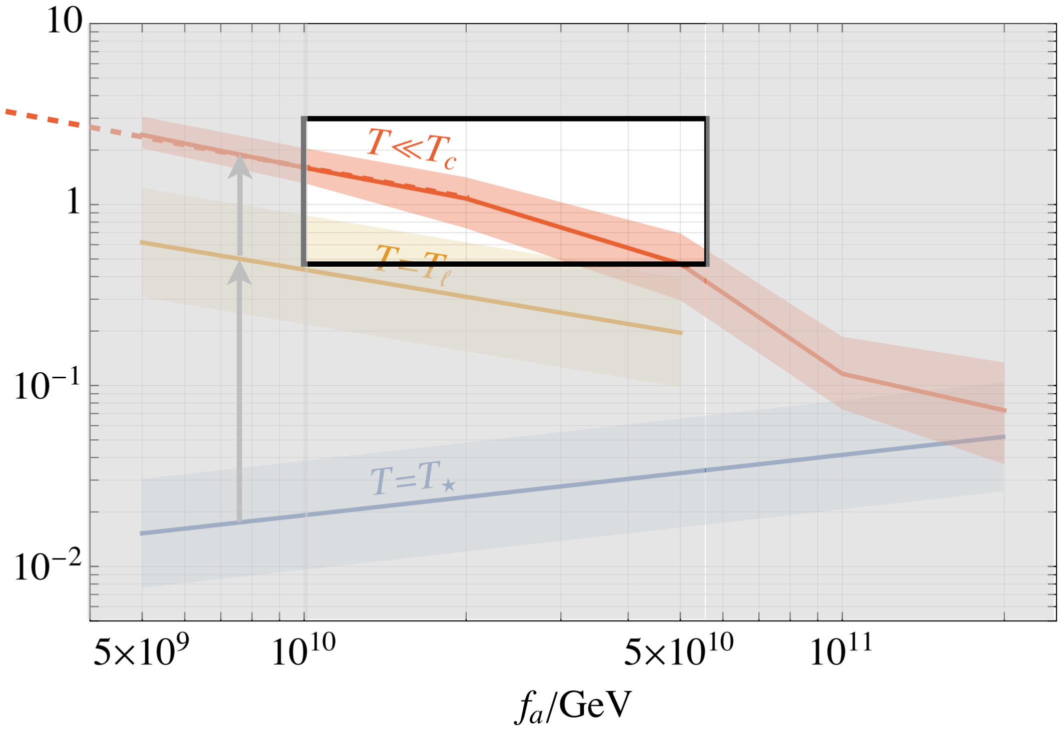
The remarkable coincidence

$$\left.\frac{k_p}{k_J}\right|_{\mathrm{MRE}} \simeq 0.4\left(\frac{10^{10}\ \mathrm{GeV}}{f_a}\right)^{1/2}$$



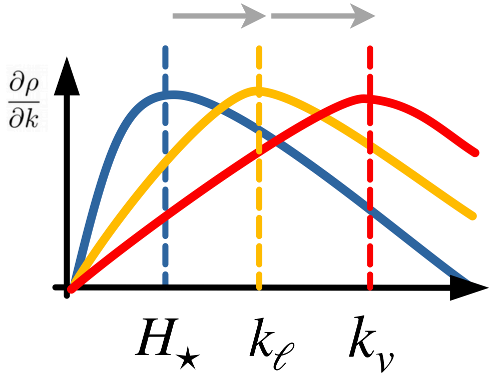
$3 \to 1$

$$\left.\frac{k_p}{k_J}\right|_{\mathrm{MRE}}$$



grows $\propto T^{-8}$

until $T \simeq 150\,\mathrm{MeV}$

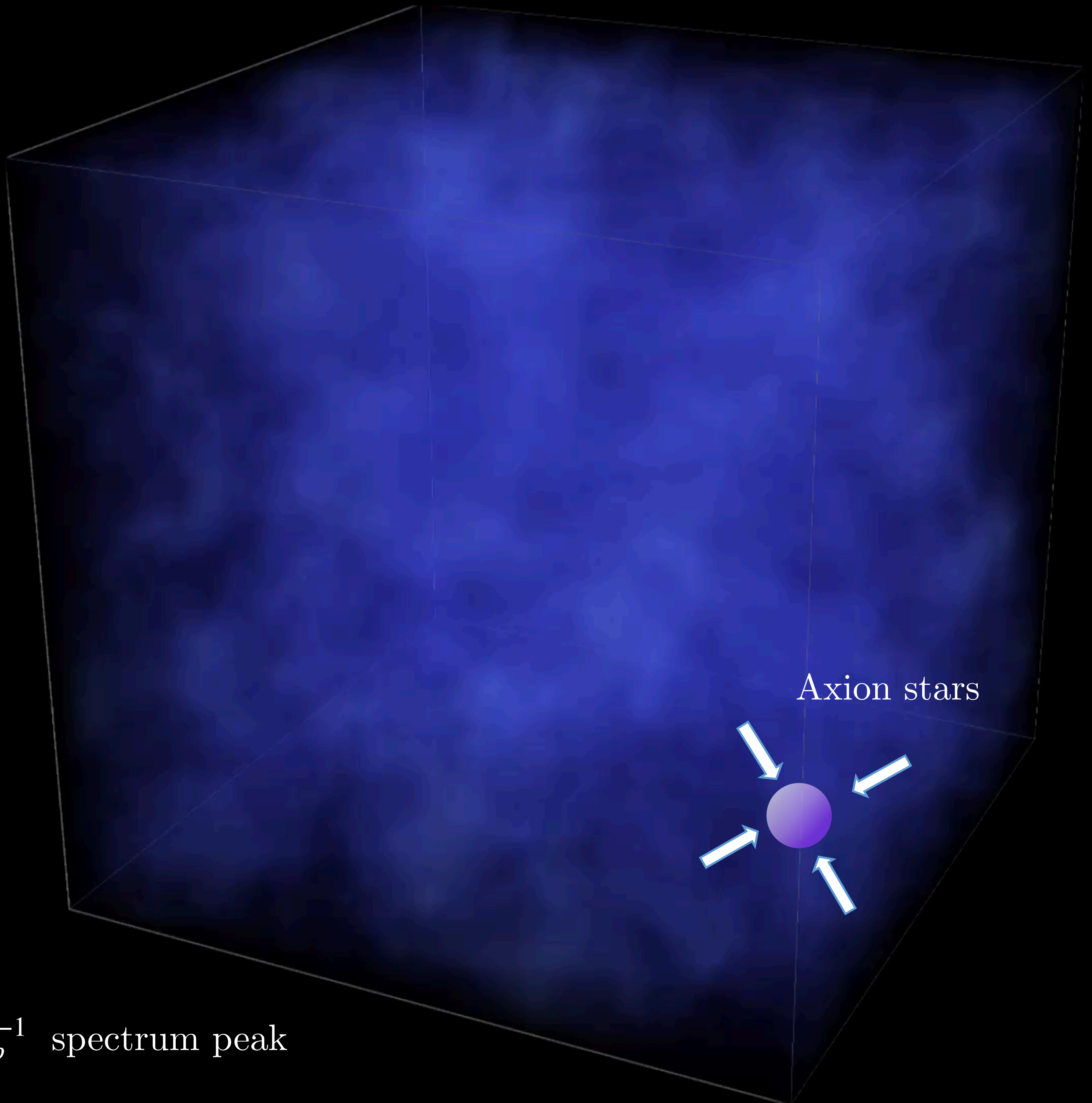


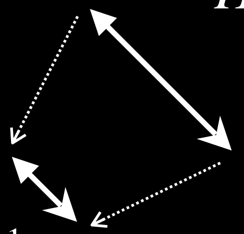
$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\boxed{\lambda}|\psi|^2}{8a^3 m_0 m^2}\right)\psi = 0$$

$$k_p \to k_v = \sqrt{\lambda\langle\phi^2\rangle} \simeq \sqrt{\rho}/f_a$$

at $T \simeq \Lambda_{\rm QCD}$:

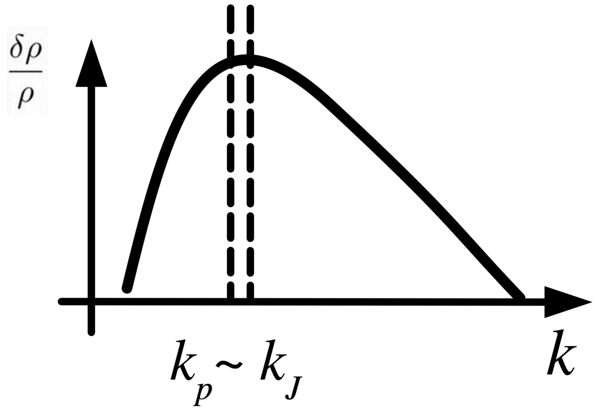
$$\delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$$

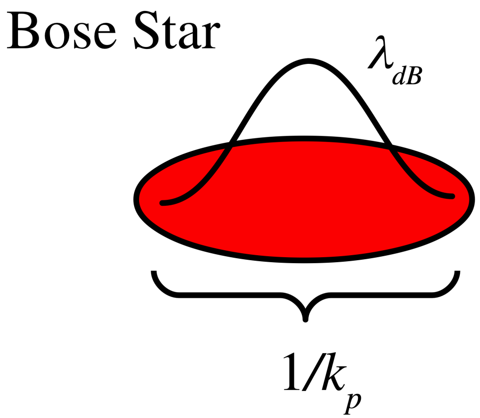




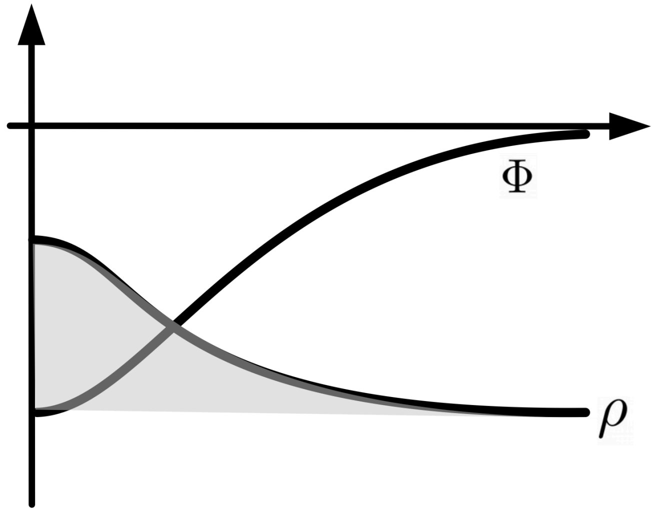
M_C is smaller

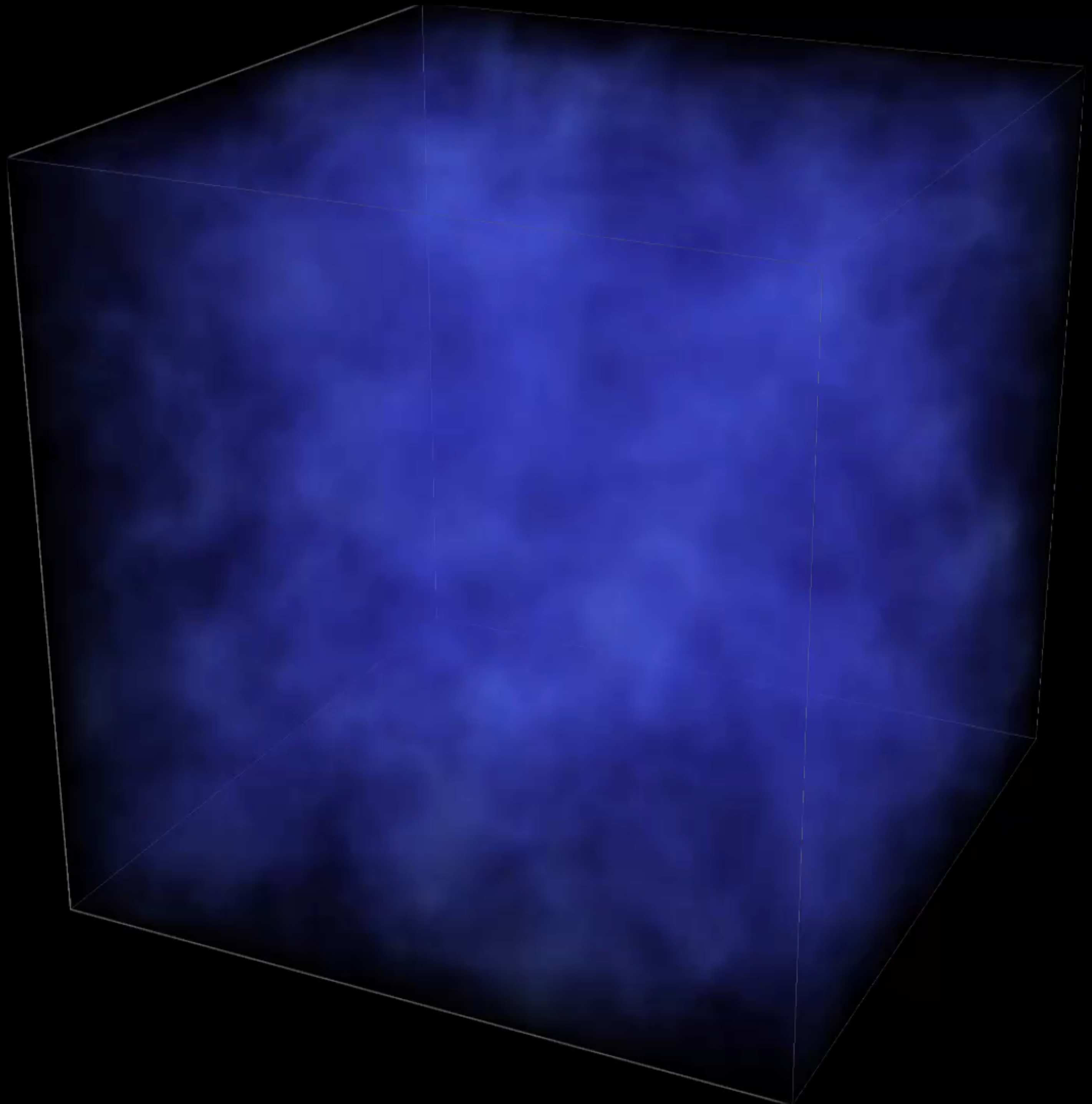
Axion stars:



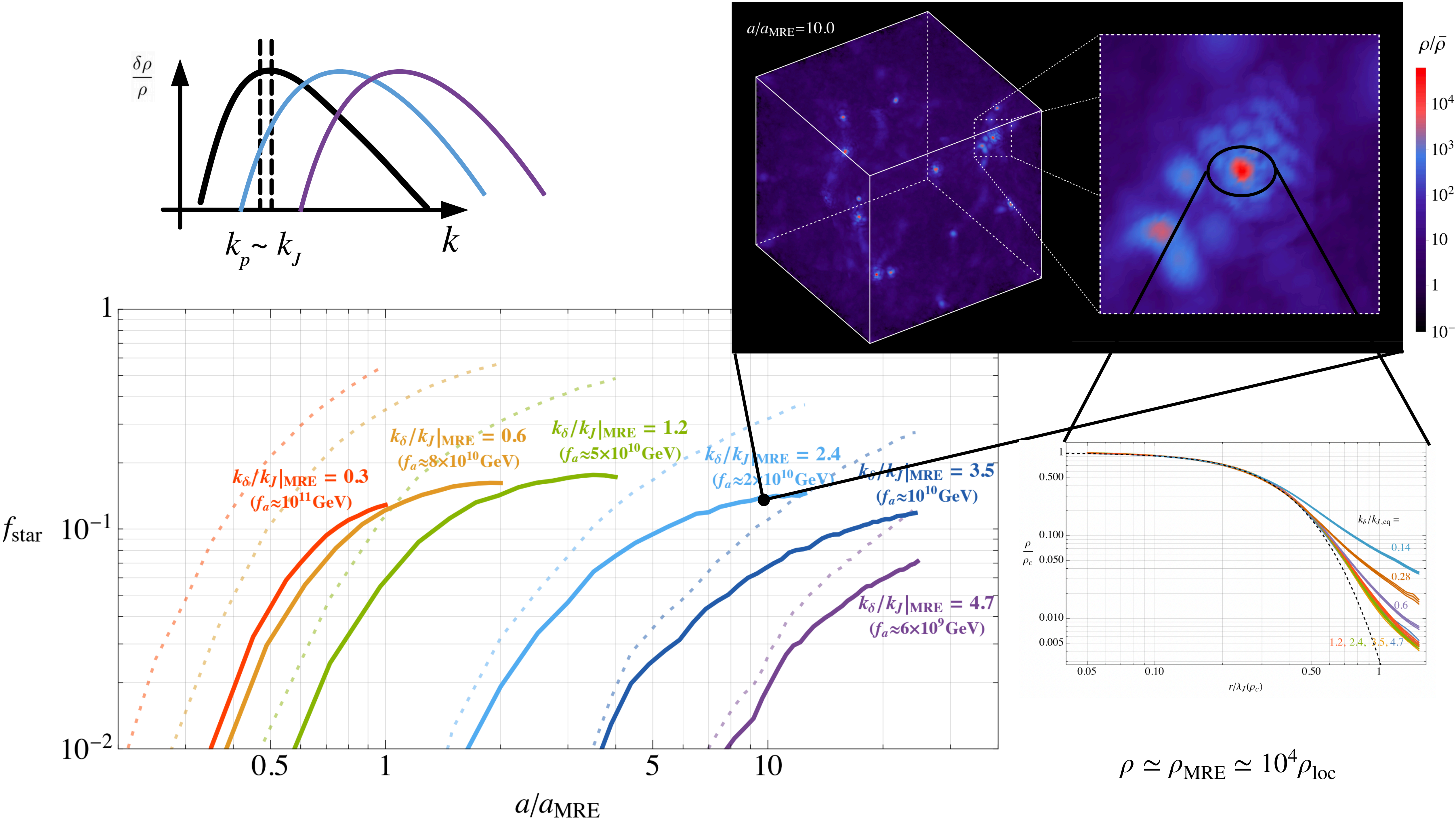


$$\begin{cases} \dot{\psi} + \frac{\nabla^2}{2m}\psi + m\Phi\psi = 0 \\ \nabla^2\Phi = 4\pi G|\psi|^2 \end{cases} \rightarrow \begin{cases} \nabla^2\sqrt{\rho} = 2m^2\Phi\sqrt{\rho} \\ \nabla^2\Phi = 4\pi G\rho \end{cases} \qquad \rho = |\psi|^2$$



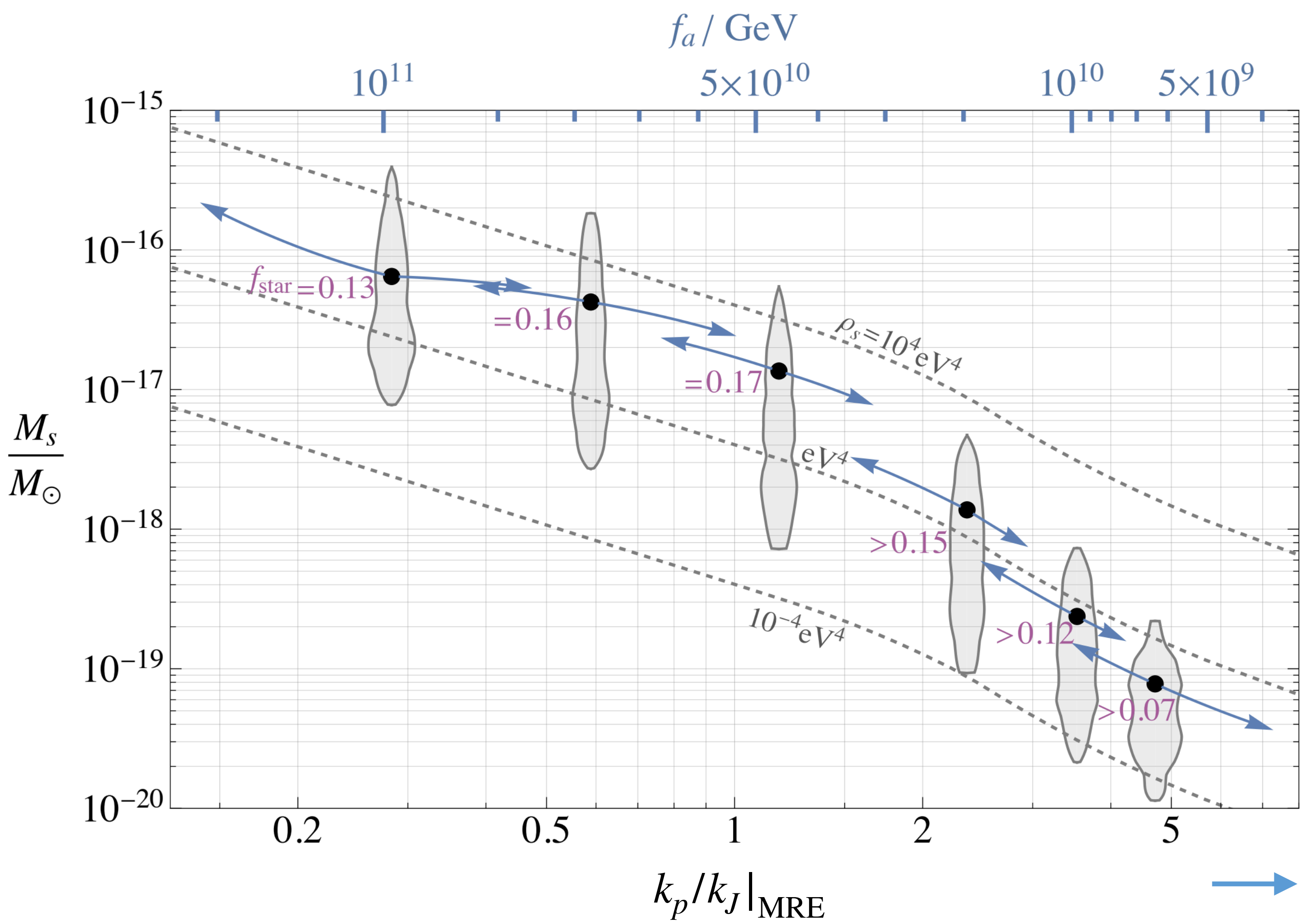


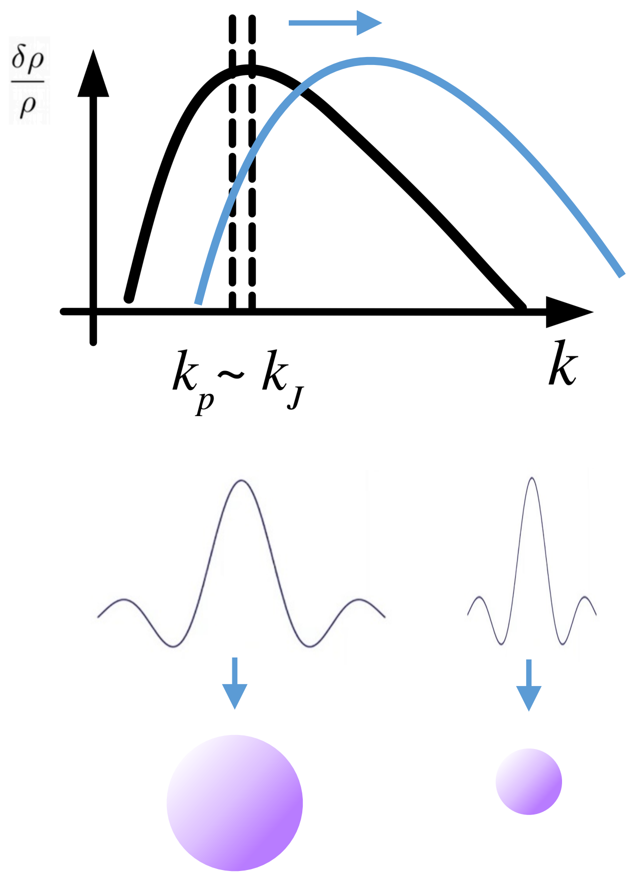
$$0.5 < \frac{a}{a_{\mathrm{eq}}} < 7$$



$\frac{\delta\rho}{\rho}$
$k_p \sim k_J$
k
$a/a_{\rm MRE}$=10.0
$\rho/\bar{\rho}$
$k_\delta/k_J|_{\rm MRE}$ = 0.3 ($f_a \approx 10^{11}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 0.6 ($f_a \approx 8\times10^{10}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 1.2 ($f_a \approx 5\times10^{10}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 2.4 ($f_a \approx 2\times10^{10}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 3.5 ($f_a \approx 10^{10}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 4.7 ($f_a \approx 6\times10^{9}$GeV)
$f_{\rm star}$
$a/a_{\rm MRE}$
$\frac{\rho}{\rho_c}$
$k_\delta/k_{J,\rm eq}$ =
0.14
0.28
0.6
1.2, 2.4, 3.5, 4.7
$r/\lambda_J(\rho_c)$
$\rho \simeq \rho_{\rm MRE} \simeq 10^4 \rho_{\rm loc}$

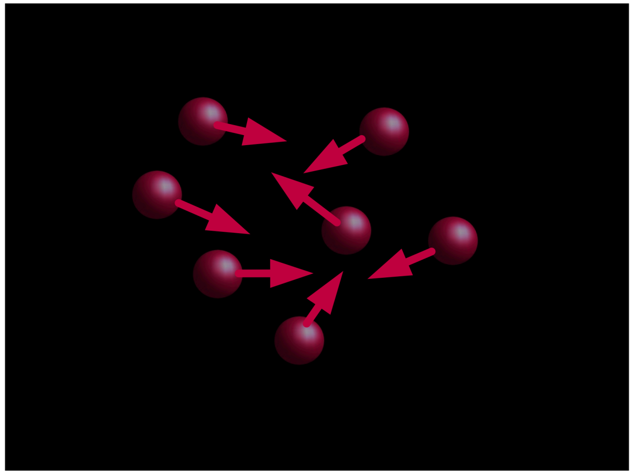
Axion stars properties:

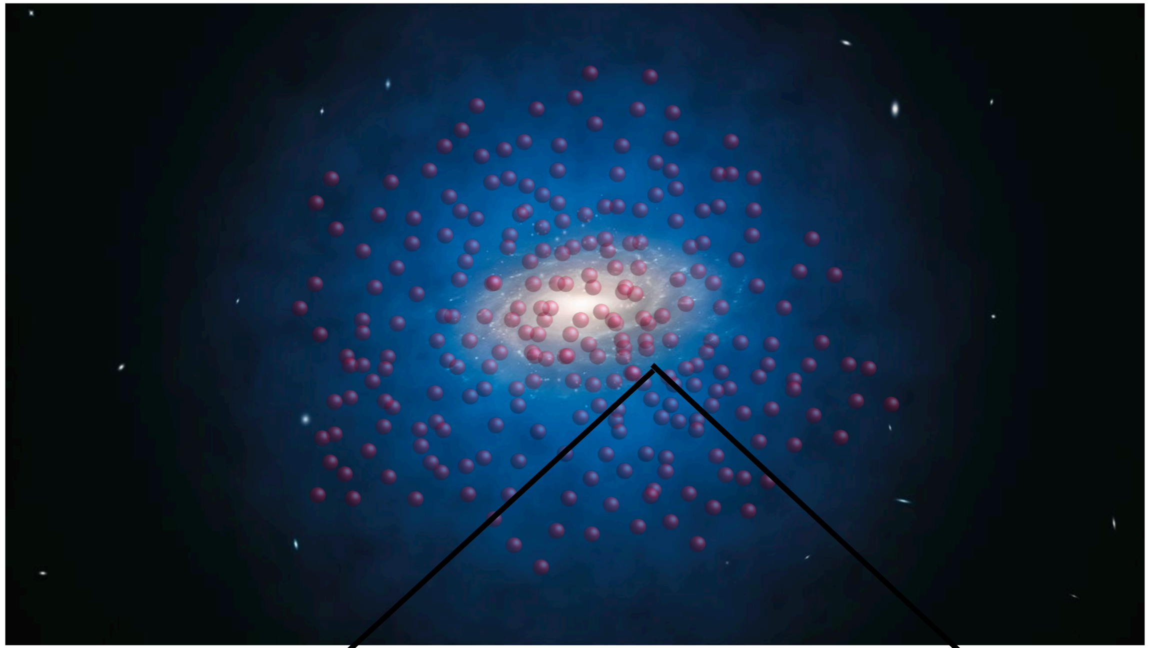




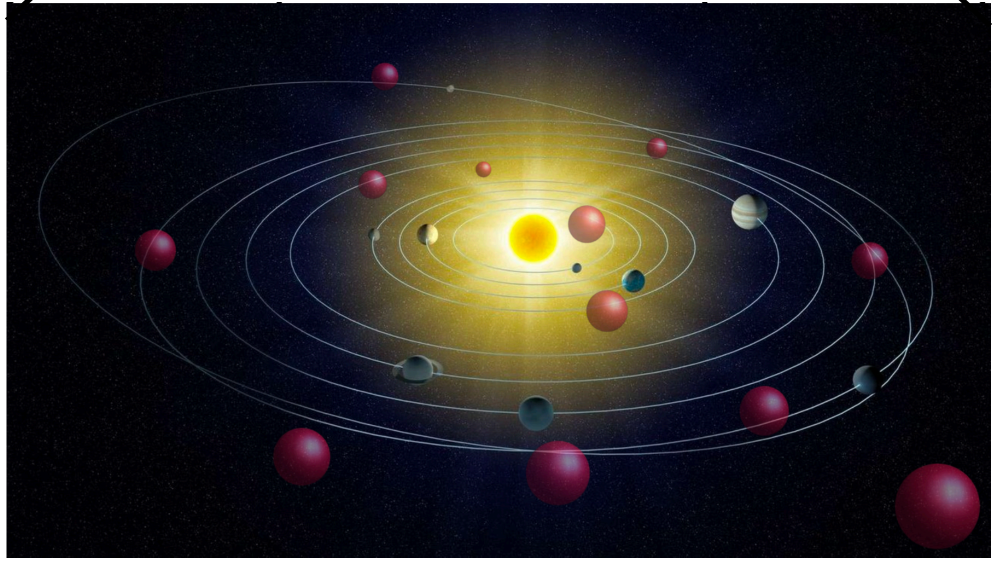
$$\bar{R}_{0.1} \approx 2.1\cdot 10^6\ \mathrm{km}\ \left(\frac{10^{10}\ \mathrm{GeV}}{f_a}\right)^{\frac{1}{2}} \qquad v_a \approx \mathrm{mm/s}$$

Axion stars (after MRE):



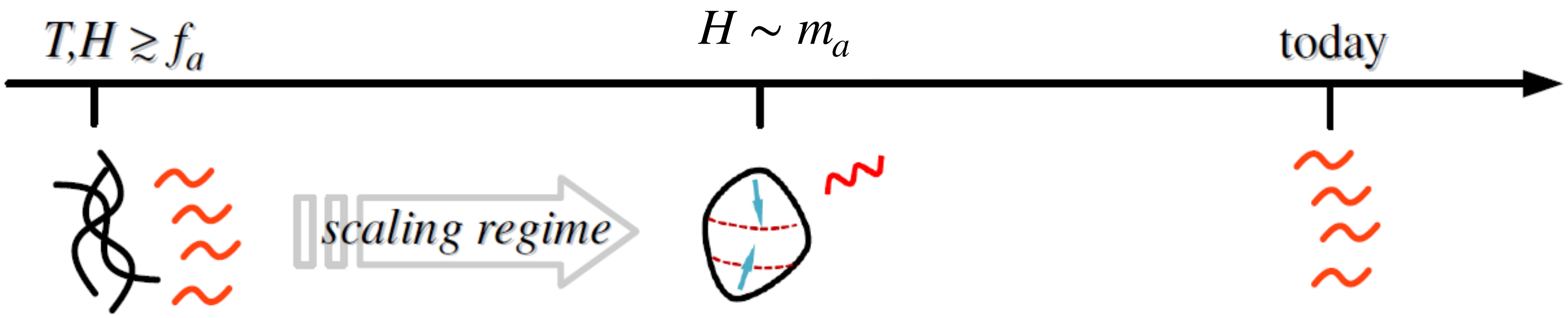


e.g. for $\begin{cases} M_s = 10^{-19} M_\odot \\ f_a = 10^{10}\ \text{GeV} \\ f_s = 0.1 \end{cases} \longrightarrow \begin{cases} n_s^{-1/3} = 1.4 \cdot 10^8\,\text{km} \\ \tau_\oplus = 5\ \text{yrs} \\ \Delta t \simeq 8\ \text{hrs} \end{cases}$



Gravitational waves from ALPs

Gravitational Waves



relic axions

$\log(m_r/H)$ $\quad \sim \mathbf{1 \div 15}$ $\quad \sim \mathbf{70 \div 100}$

GWs from the scaling regime

1) GW energy emission rate: $\Gamma_g(t)$

2) momentum distribution: $\frac{\partial \Gamma_g}{\partial k}$

Nambu–Goto EFT

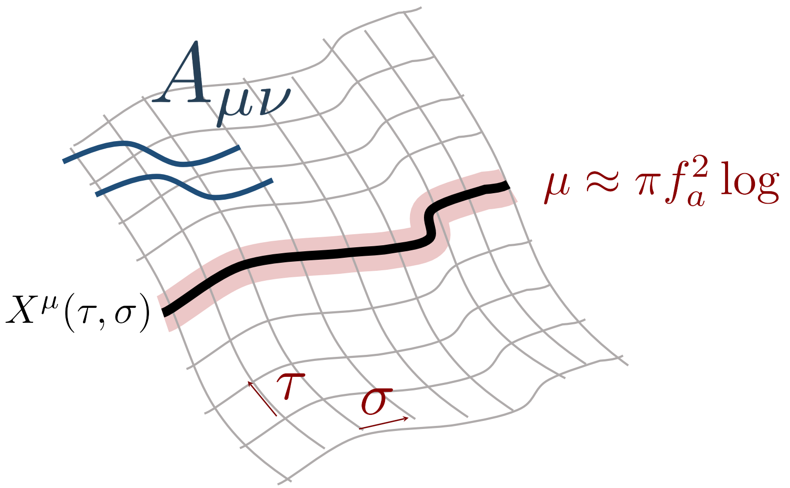
$\Gamma_g \longleftrightarrow \Gamma_a$

String Effective Theory

Degrees of freedom:

- $a \quad \longleftrightarrow \quad A_{\mu\nu}$
- $X^\mu(\tau, \sigma)$

$$\partial A \sim F^{\mu\nu\rho} = \epsilon^{\mu\nu\rho\sigma}\partial_\sigma a$$



$S_\phi[\phi]$

$\Downarrow$

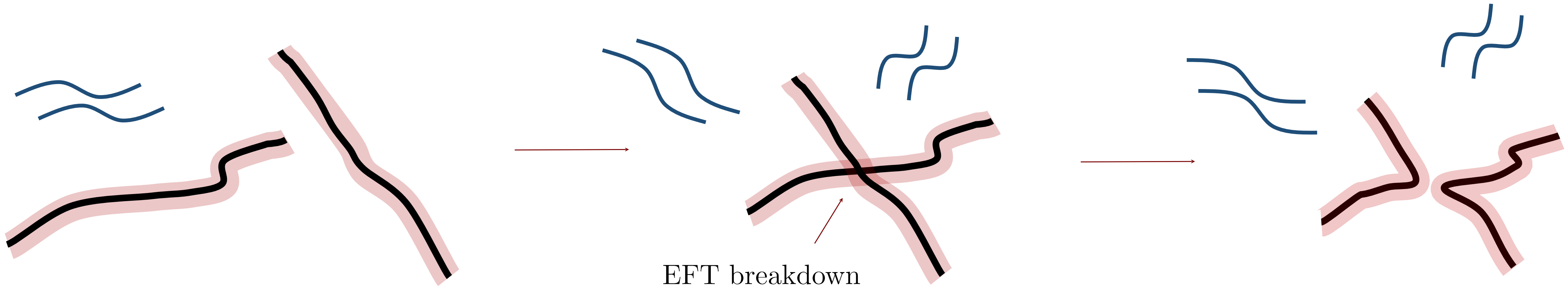
axion-to-string coupling

$$S_{\rm EFT}[X, A] = \underbrace{-\mu \int d\tau d\sigma \, \sqrt{-\gamma}}_{\text{Nambu–Goto action}} \; \underbrace{-\frac{1}{6}\int d^4x \, (\partial A)^2}_{\text{Axion kinetic term}} \; \underbrace{+\boxed{2\pi f_a}\int d\tau d\sigma \, \partial_\tau X^\mu \partial_\sigma X^\nu A_{\mu\nu}}_{\text{Axion-string interaction (Kalb–Ramond action)}}$$

$$\gamma_{ab} = \partial_a X^\mu \partial_b X_\mu$$

1) GW energy emission rate: $\Gamma_g(t)$

2) momentum distribution: $\frac{\partial \Gamma_g}{\partial k}$



Gravitational Wave Emission

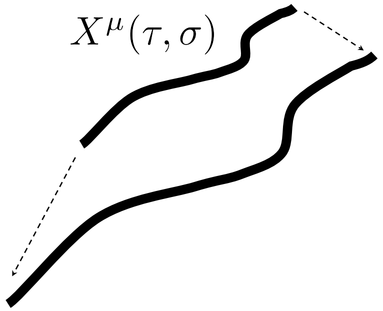
EoM: $\Box_x A^{\mu\nu} = 2\pi \boxed{f_a} \int d\sigma \dot{X}^{[\mu} X'^{\nu]} \delta^3(\vec{x} - \vec{X})$

Einstein Eq: $\Box_x h^{\mu\nu} = 16\pi G \left(T_s^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} T_{s\,\lambda}^{\lambda}\right)$

$$T_s^{\mu\nu} = \boxed{\mu} \int d\sigma \left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu\right) \delta^3(\vec{x} - \vec{X})$$

$$\frac{dE_a}{dt} = \underbrace{r_a[X]} \boxed{f_a^2} \qquad \frac{dE_g}{dt} = \underbrace{r_g[X]} G\boxed{\mu^2}$$

dimensionless functionals of the shape of the string trajectory X^μ



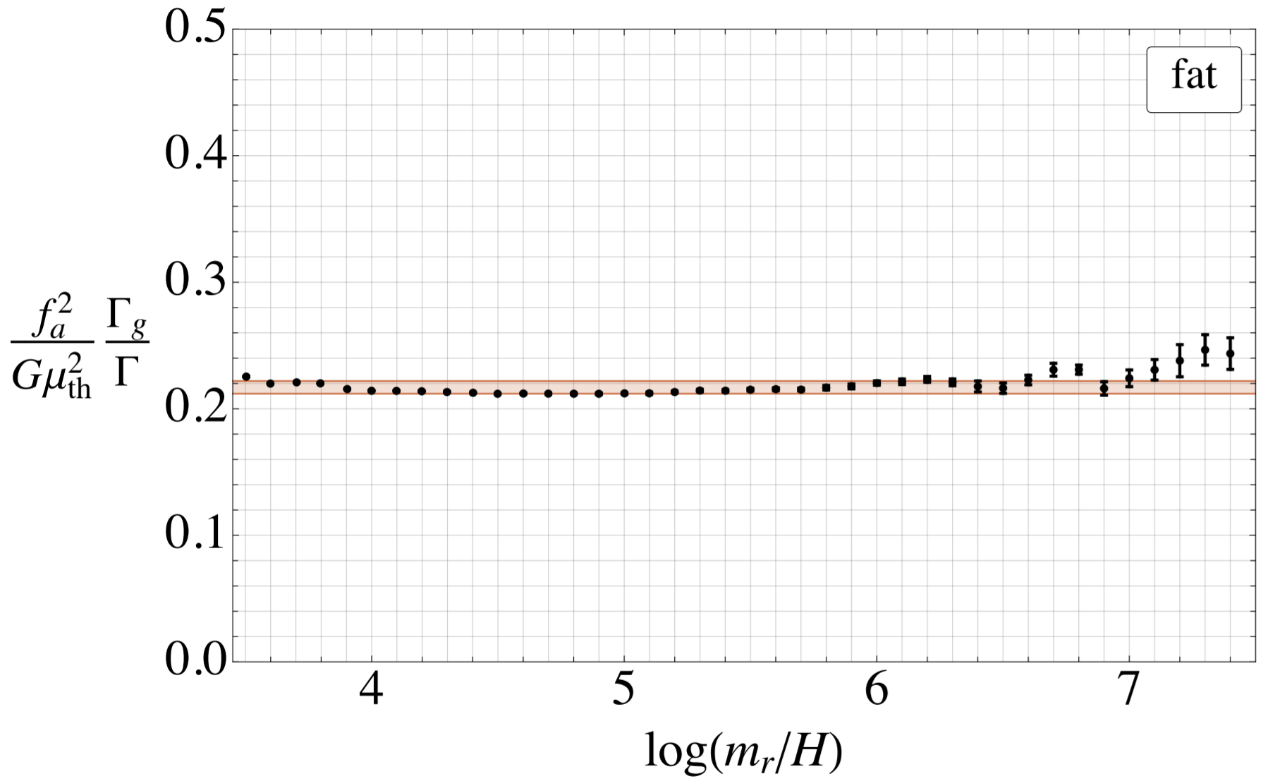
$$\boxed{\Gamma_g = r \frac{G\mu^2}{f_a^2} \Gamma_a} \quad \propto \frac{\log^4}{t^3}$$

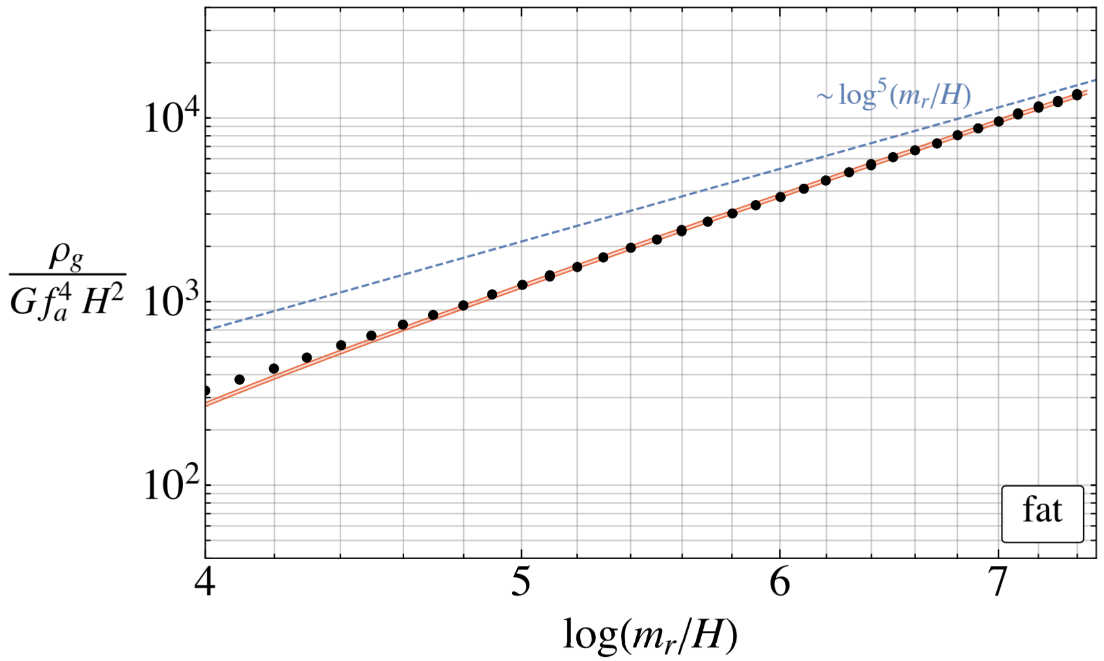
$$\frac{r_g[X]}{r_a[X]} = \text{const} = \mathcal{O}(1) \qquad \frac{\xi\mu}{t^3}$$

Comparison with field theory simulations

$$\boxed{\Gamma_g = r\frac{G\mu^2}{f_a^2}\Gamma_a}$$

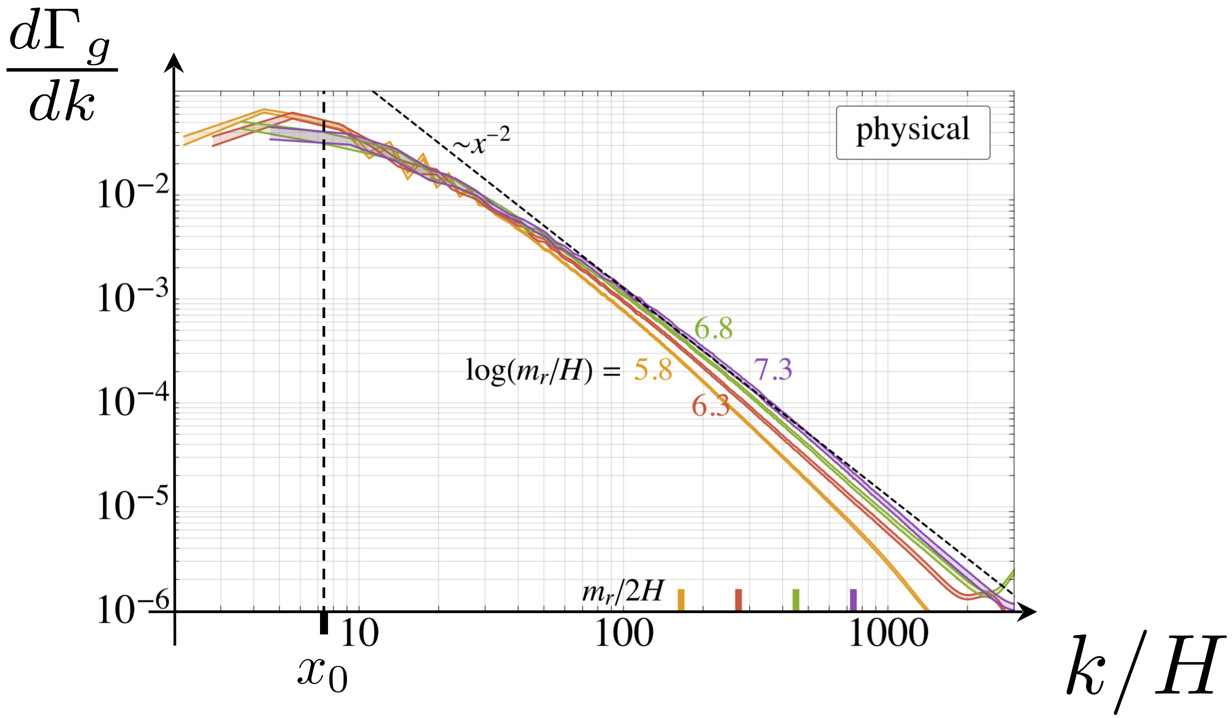
$\frac{\xi\mu}{t^3}$





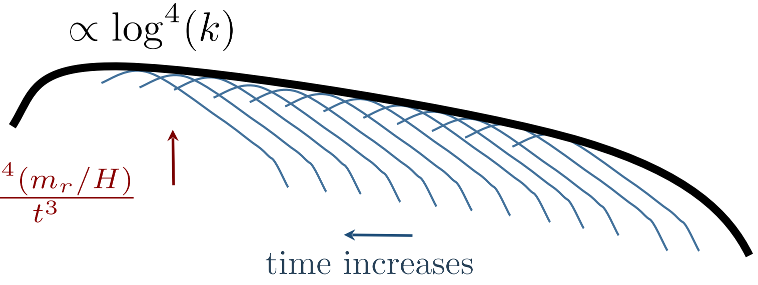
The Gravitational Wave Spectrum

1) GW energy emission rate: $\Gamma_g(t)$
2) momentum distribution: $\frac{\partial \Gamma_g}{\partial k}$



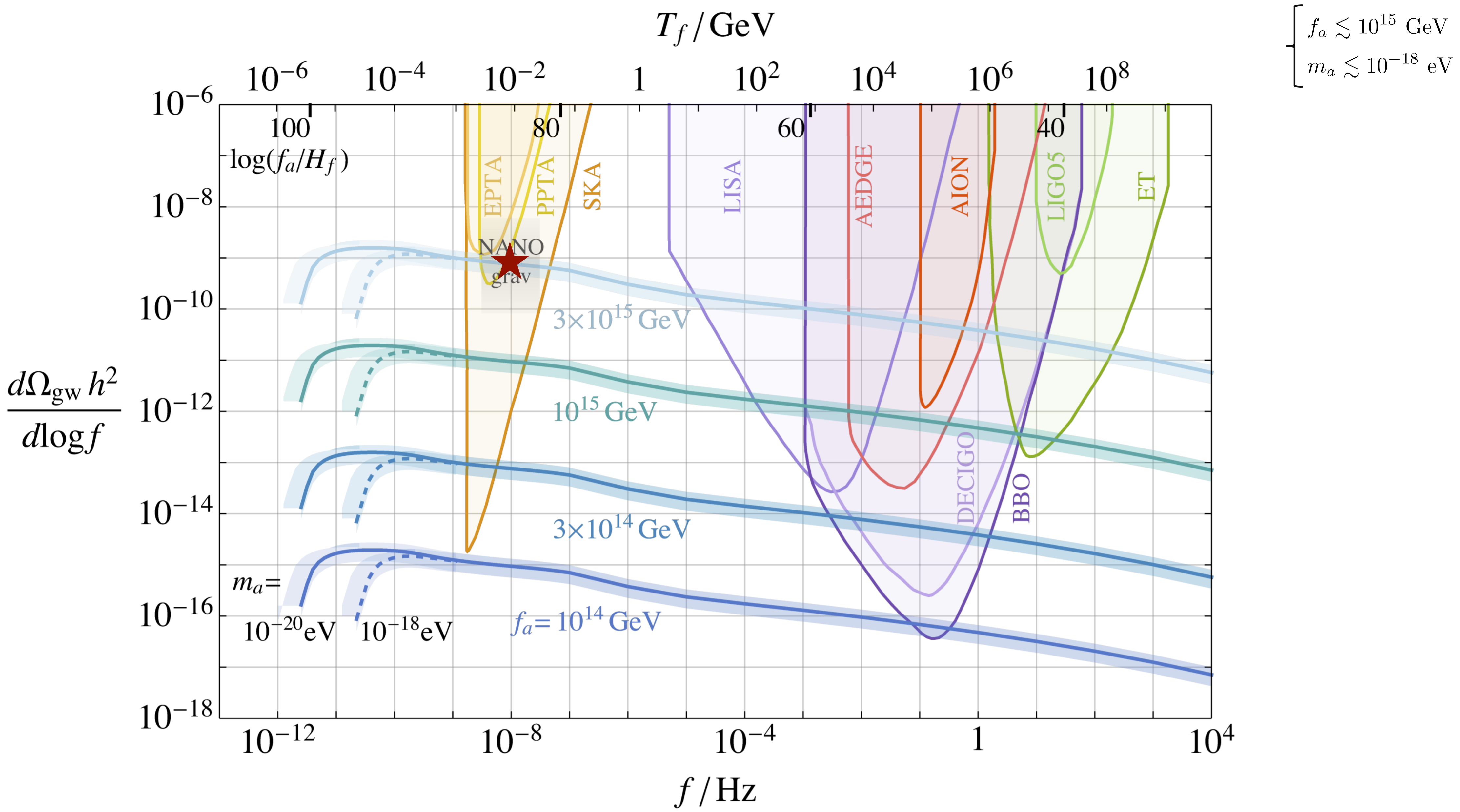
$$\propto \frac{\log^4}{t^3}$$

$$\frac{\partial \rho_g}{\partial \log k} \equiv \int dt' \frac{d\boxed{\Gamma'_g}}{d\log k}\left(\frac{R'}{R}\right)^4 \quad =$$



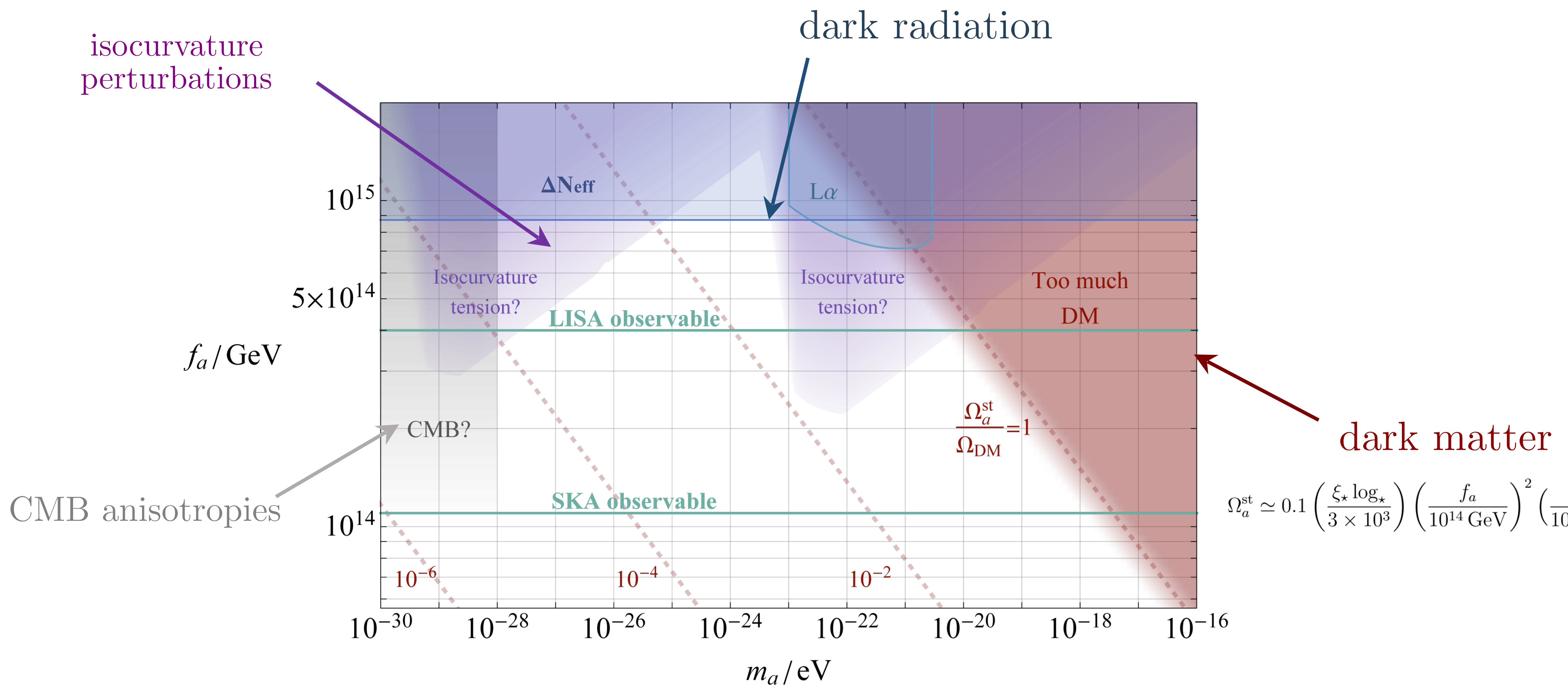
$$\Gamma_g \propto \frac{\log^4(m_r/H)}{t^3}$$

- approximately scale invariant
- $\log^4$ enhancement



See also: C. Chang and Y. Cui, [1910.04781]
Y. Gouttenoire, G. Servant, P. Simakachorn [1912.02569]
D. Figueroa + [2007.03337]; Baeza-Ballesteros + [2308.08456]

Bounds on the Post-Inflationary Scenario



$$\Omega_a^{st} \simeq 0.1 \left(\frac{\xi_\star \log_\star}{3\times 10^3}\right)\left(\frac{f_a}{10^{14}\,\mathrm{GeV}}\right)^2\left(\frac{m_a}{10^{-18}\,\mathrm{eV}}\right)^{\frac{1}{2}}$$

Cf: G. Servant and P. Simakachorn [2307.03121] for best fit on new NANOgrav

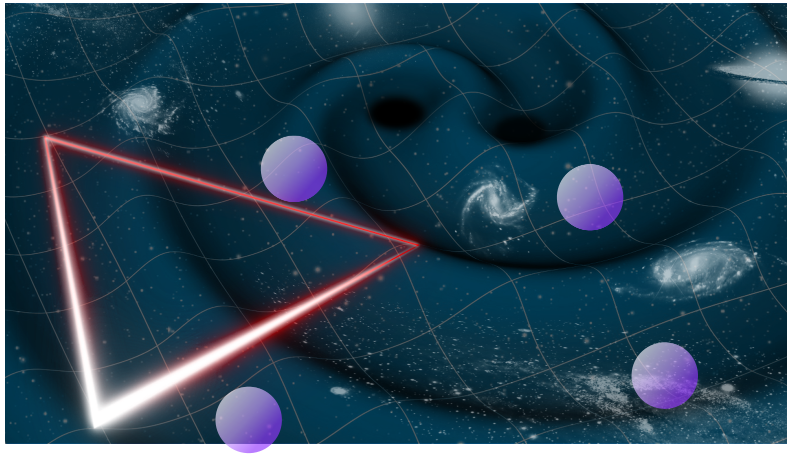
Conclusions

- Post-inflationary abundance **uncertain**, despite progress

$$f_a \lesssim 10^{10}\,\mathrm{GeV} \quad \text{or} \quad m_a \gtrsim 0.5\,\mathrm{meV} \quad \text{from dark matter over-production}$$

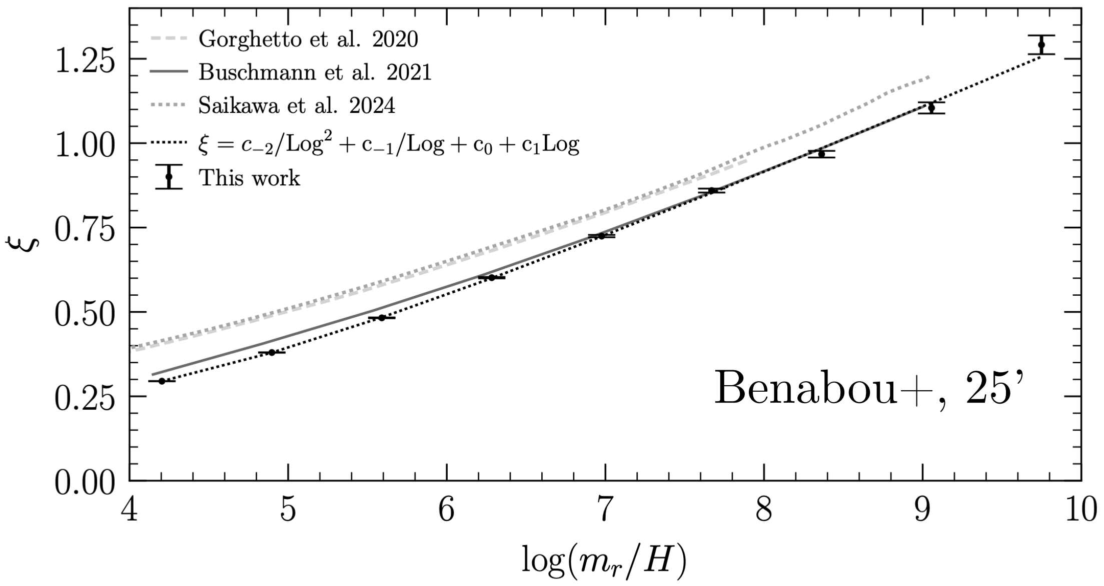
- **Axion star** formation enhanced at MRE

  ➡ Potential for new observational opportunities

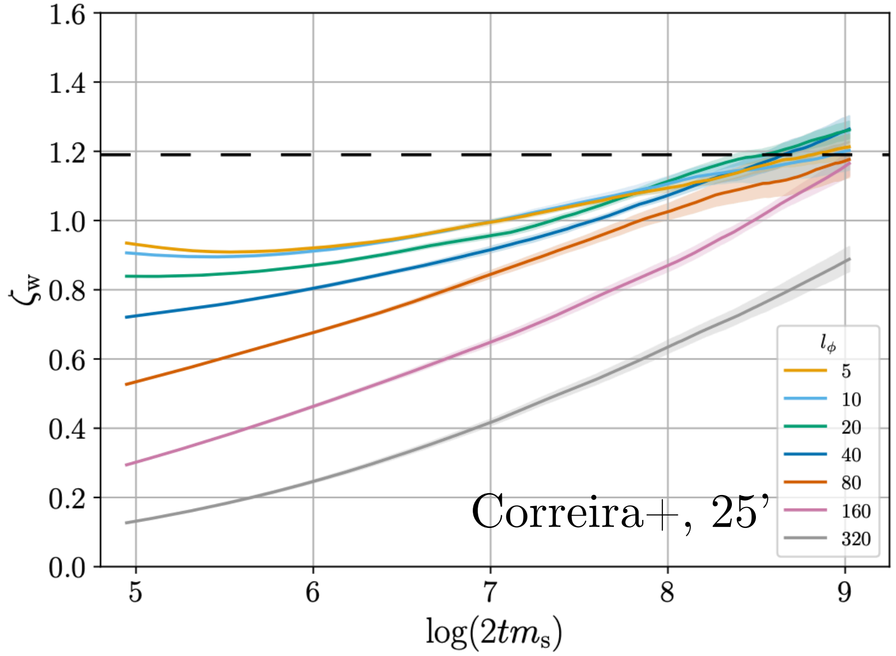


- **GWs** from ALP strings observable for $f_a \gtrsim 10^{14}\,\mathrm{eV}$

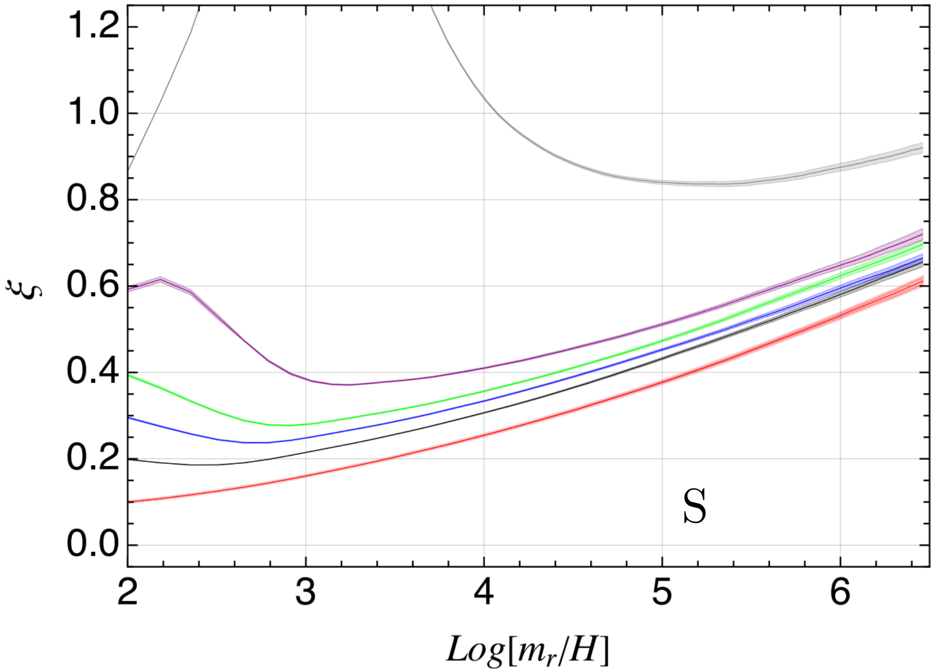
Backup



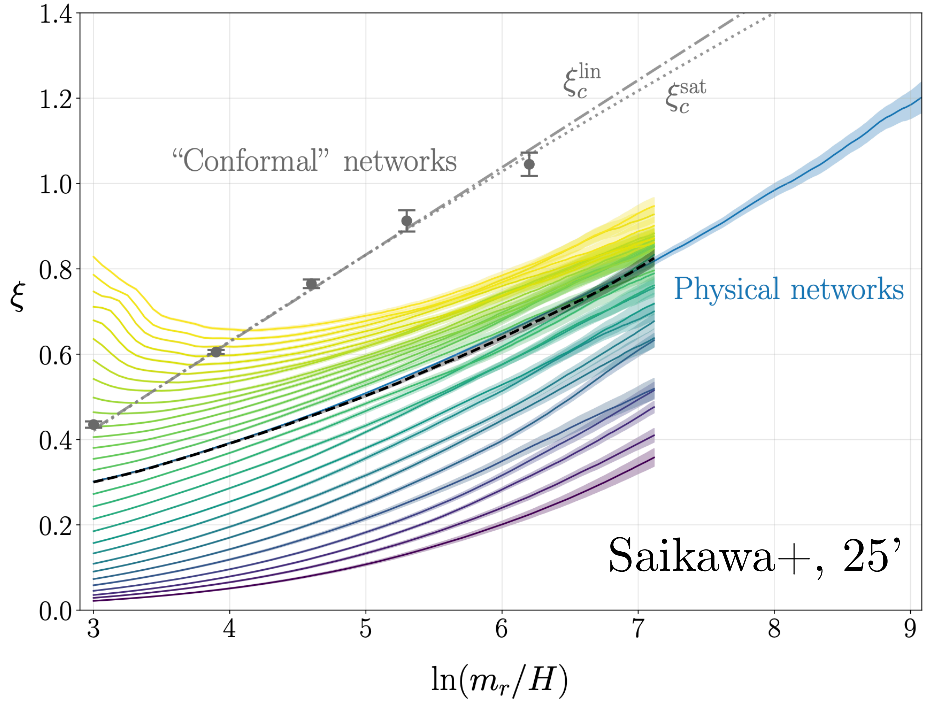
Gorghetto et al. 2020
Buschmann et al. 2021
Saikawa et al. 2024
$\xi = c_{-2}/\mathrm{Log}^2 + c_{-1}/\mathrm{Log} + c_0 + c_1\mathrm{Log}$
This work
Benabou+, 25'
ξ
$\log(m_r/H)$



l_ϕ
5
10
20
40
80
160
320
Correira+, 25'
ζ_{w}
$\log(2tm_{\mathrm{s}})$



S
ξ
$Log[m_r/H]$

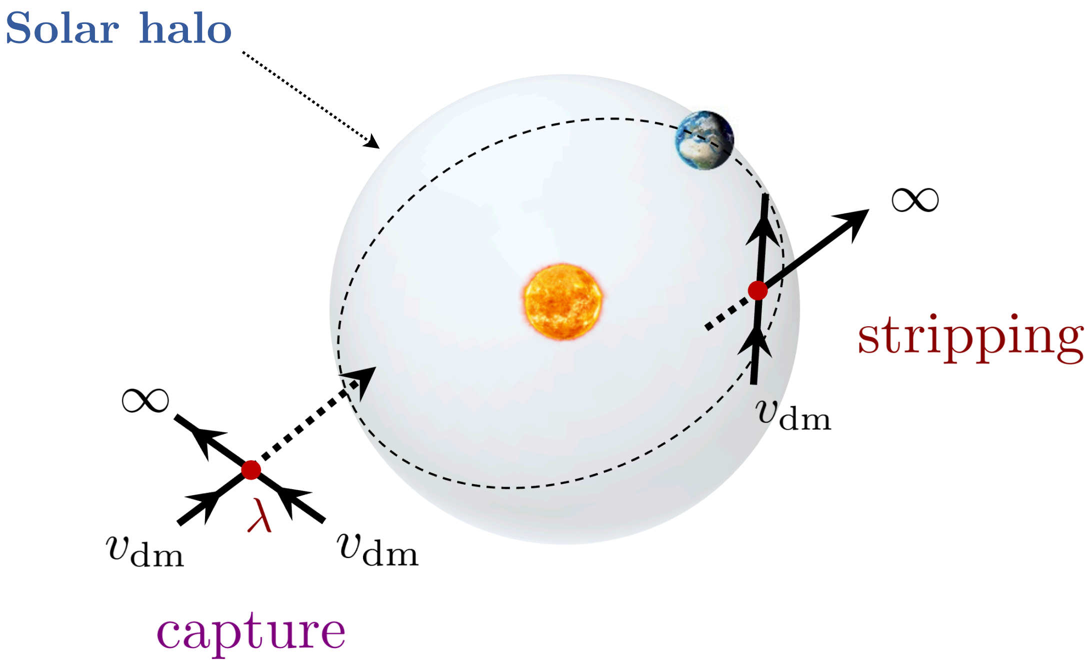


ξ_c^{lin}
ξ_c^{sat}
"Conformal" networks
Physical networks
Saikawa+, 25'
ξ
$\ln(m_r/H)$

Advert:

Solar halos of ultra-light dark matter

- DM is ϕ with $V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\underbrace{\frac{m^2}{f_a^2}}_{-\lambda}\phi^4 + \ldots$



Capture dominates over stripping when:

$$v_{\rm dm} \simeq 10^{-3} \lesssim 2\pi\,\alpha$$

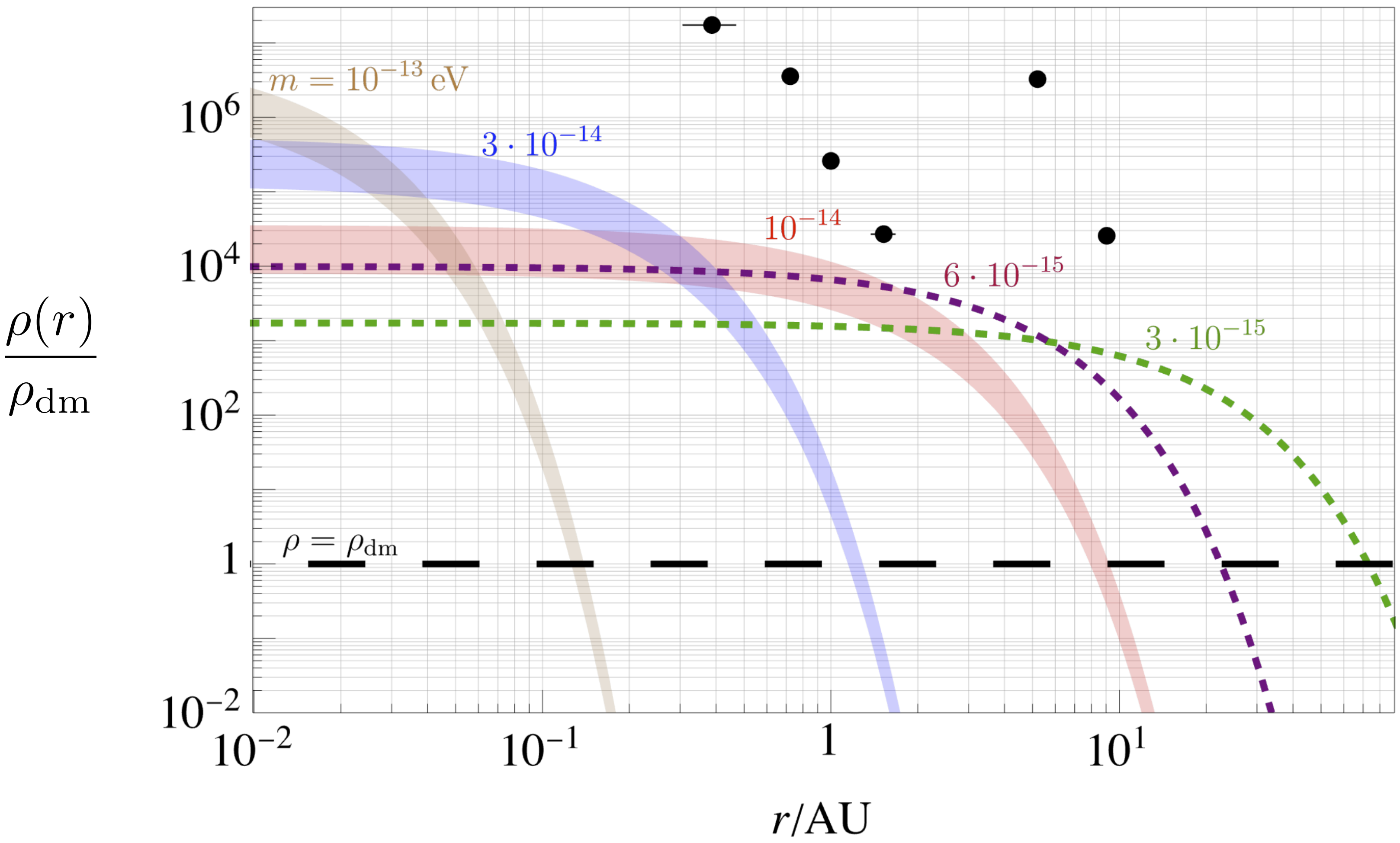
i.e. if $\quad m \gtrsim 1.5 \cdot 10^{-14}\,\text{eV}$

$$R_\star = \frac{1}{m\underbrace{\alpha}_{GM_\odot m}} = 1\,\text{AU}\left[\frac{1.3\cdot 10^{-14}\,\text{eV}}{m}\right]^2$$

Bohr radius

$$\dot{N}_{\rm bound} \sim \Gamma(m, f_a)\;\cdot\; N_{\rm bound} \quad \rightarrow \quad N_{\rm bound} \propto e^{\Gamma t}$$

density profile after 5 Gyr



- bands have $v_{\rm dm} = 50 \div 240\,\mathrm{km/s}$
- f_a (or λ) fixed in $10^7 \div 10^8\,\mathrm{GeV}$