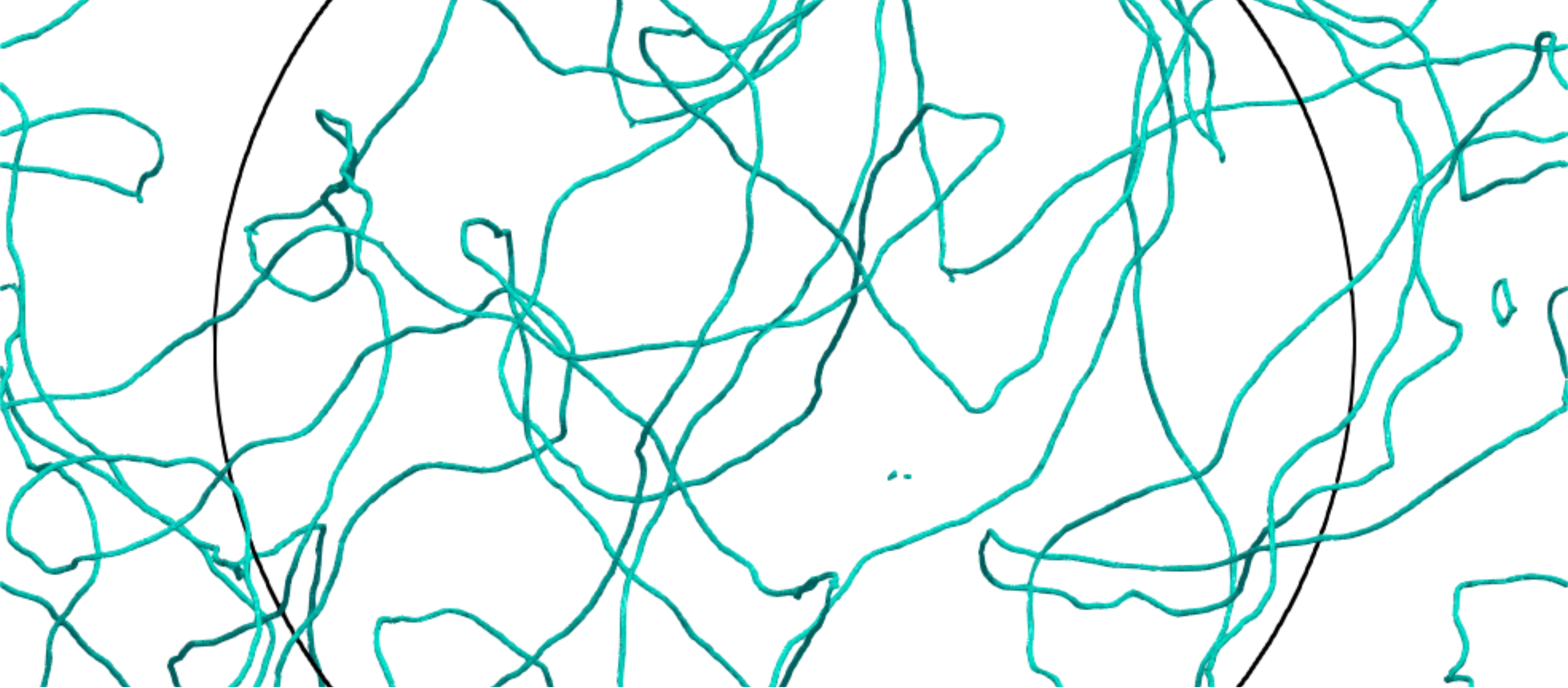




Terasite Simulations of Axion String Networks

Based on [arXiv:2410.18064](#) / **PhysRevD 111 063532**, **Preliminary, Preliminary**
Computational resources from CSC Finland: 462000084, 462000672

J. Correia, M. Hindmarsh, J.
Lizarraga, A. Lopez-Eiguren,
K. Rummukainen, J. Urrestilla



CSC



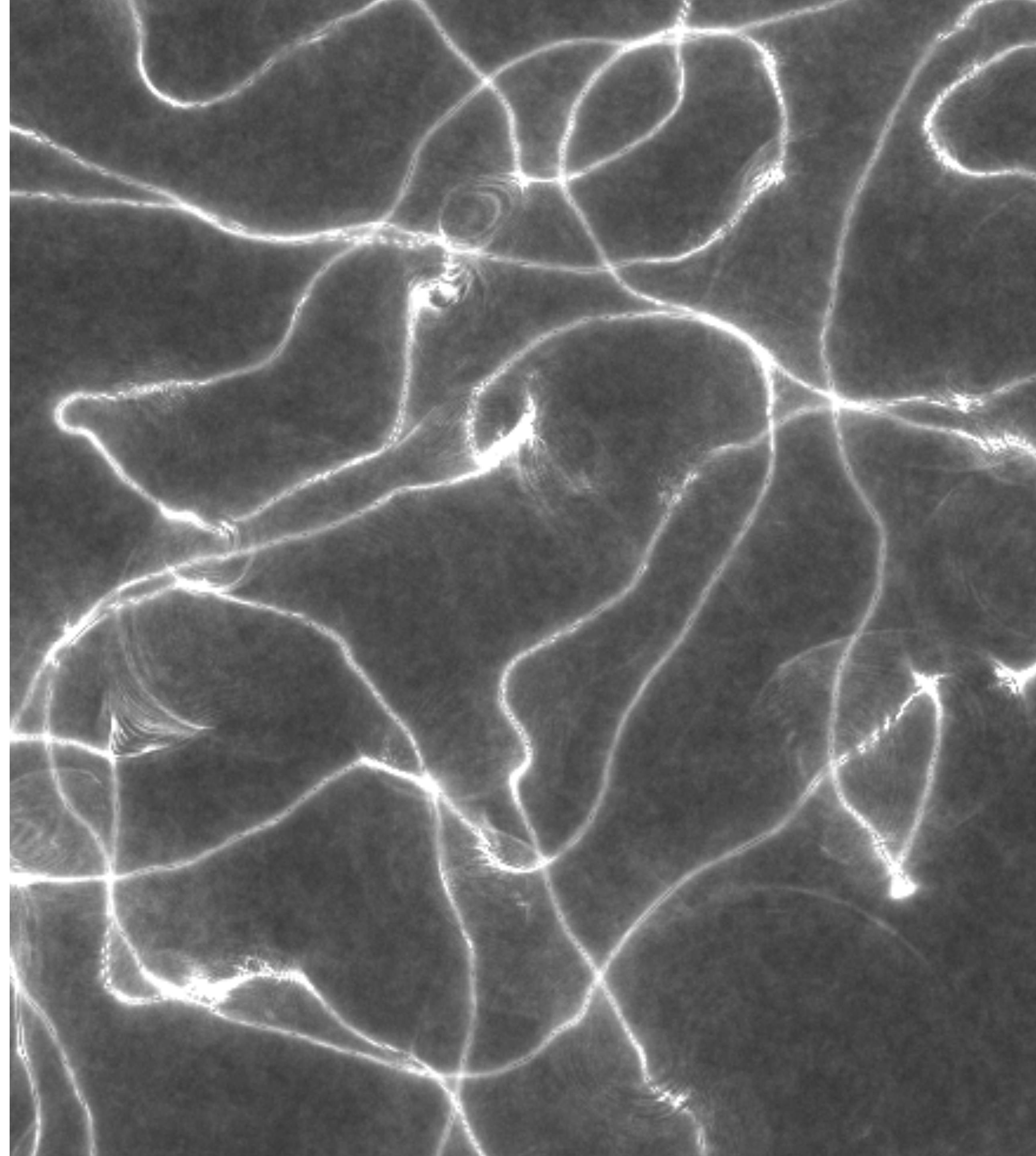
HELSINKI
UNIVERSITY OF
TECHNOLOGY



HELSINKI
INSTITUTE OF
PHYSICS

Introduction

- Introduction
- Quantitative String Evolution
- Scaling density of Axion Strings in Terasite Simulations
- Conclusions & Next Steps

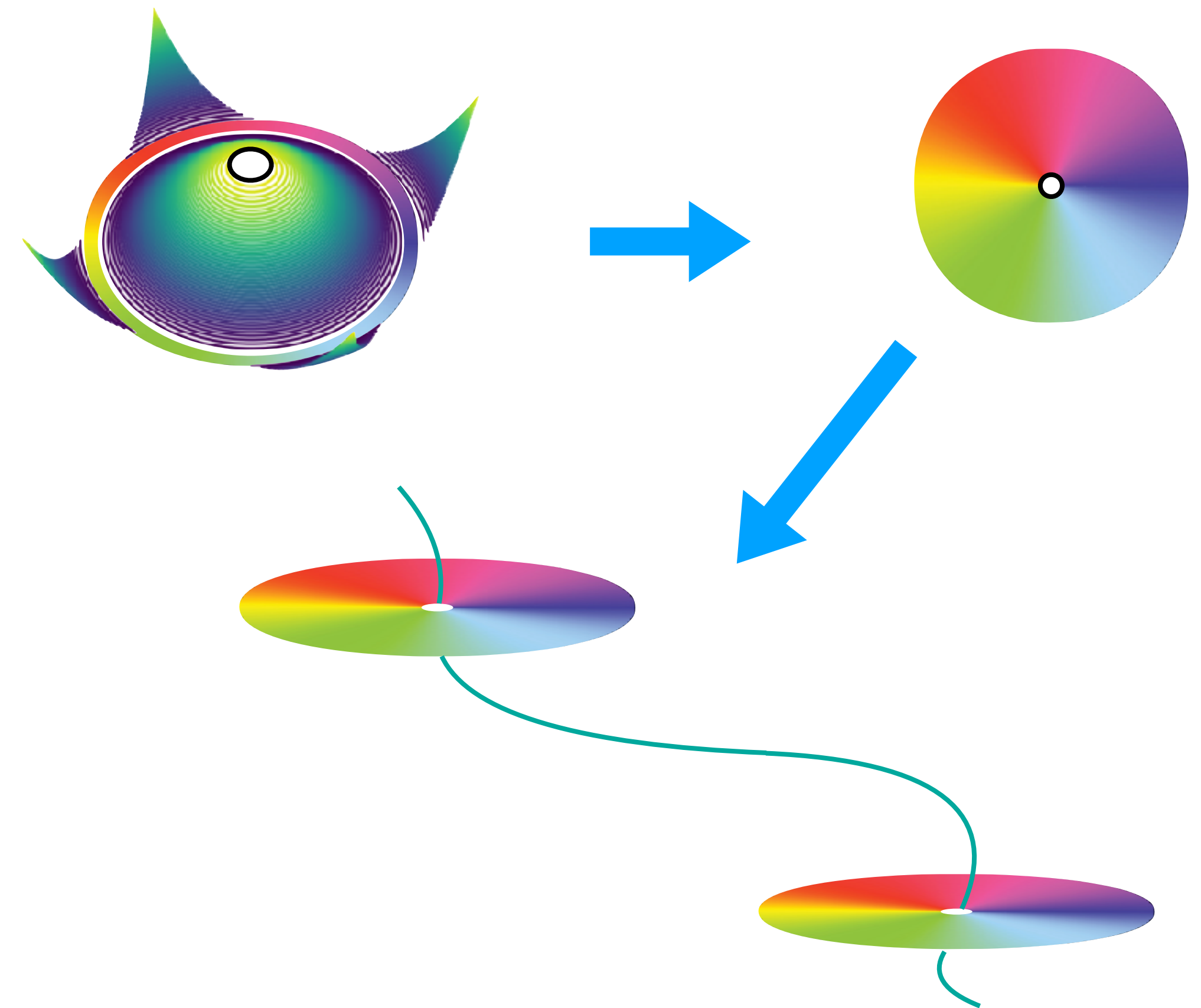


Global strings

- Consider a singlet **scalar field** invariant under **U(1) Global** symmetry, (ϕ is a complex scalar field), which spontaneously breaks

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi^* \partial^\mu \phi - \frac{1}{4} \lambda (\phi^* \phi - \eta^2)^2 \right)$$

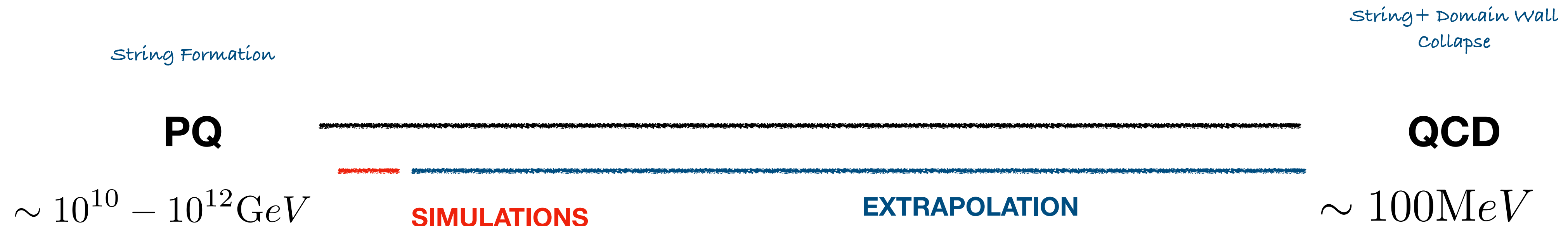
- String solutions** can then exist $\rightarrow$
- Global strings** have different properties from “conventional” gauged/local strings:
 - Energy not** entirely **localised** to the string world sheet
 - Existence of **long-range interactions**
 - Massless radiation** expected to be emitted



[Kibble, 1976]

Axions in the post-inflationary scenario I

- Peccei-Quinn mechanism [Peccei & Quinn, 1977] was proposed to solve the Strong CP problem...
- Extra field is added to Standard Model, and the angular excitations of this field are known as **Axions**
 - Solid dishwashing product, but also happens to be a dark matter candidate.
- If the **PQ phase transition** is **after inflation**, one will form **global strings** [Davis & Shellard, 1986]
 - They survive until the QCD phase transition (for QCD axions), roughly 70 e-folds later
 - And the **string network** will emit **axion waves** continuously throughout this time
- At QCD PT, a system of domain walls connecting strings will form and the whole system will collapse ($N_{\text{DW}}=1$)



Axions in the post-inflationary scenario II

- Multiple factors which can greatly impact the predicted dark matter abundance, we highlight two:

- The **number of strings per Hubble** volume at the QCD phase transition ζ

$\zeta \approx \mathcal{O}(1) \approx \text{const.}$ or $\zeta \propto \log(m_s t)$

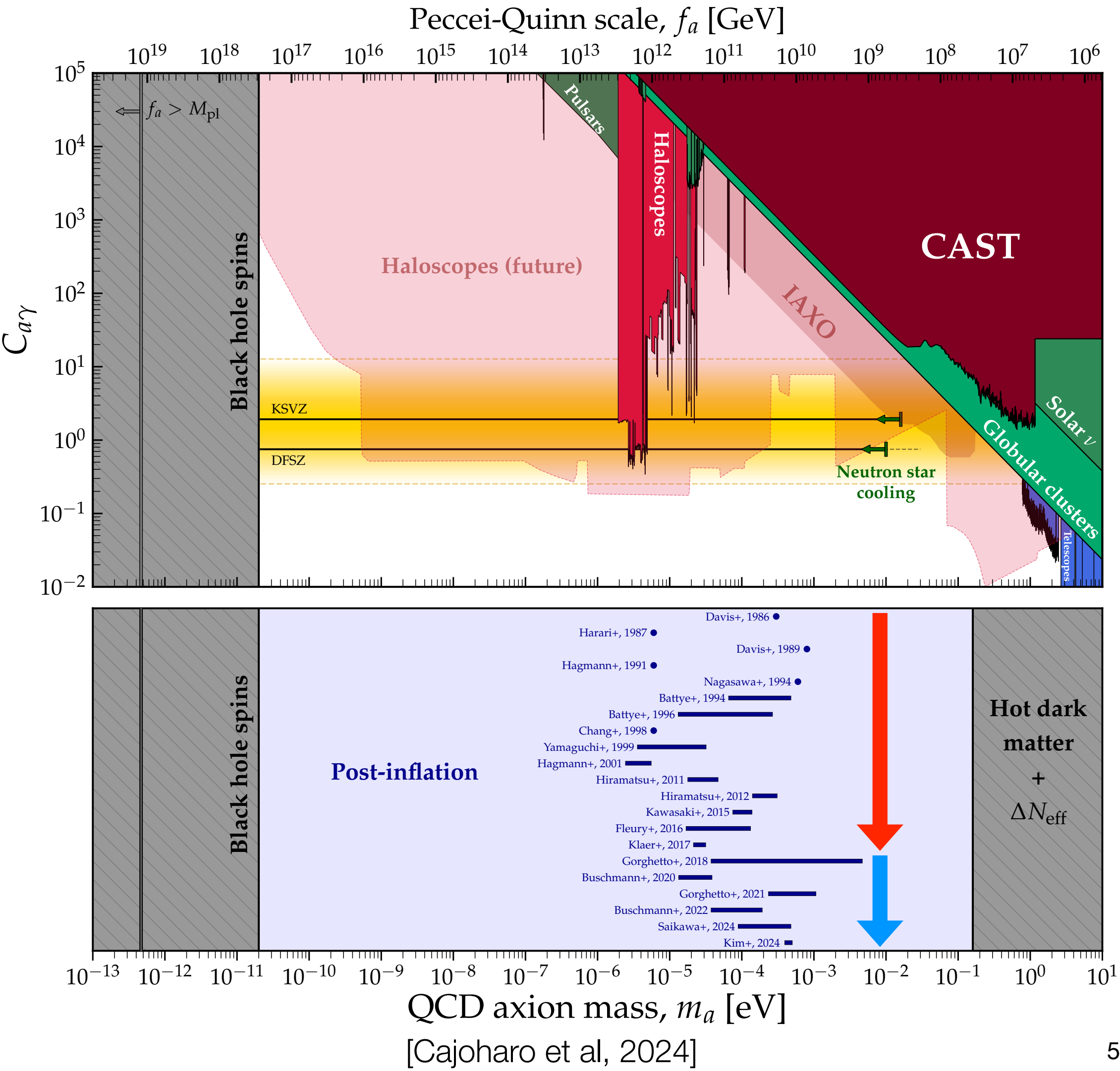
- The shape of the axion emission spectra. **Spectral index:**

$$q \approx 1 \quad \text{or} \quad q > 1 ?$$

[Hariri & Sikivie, 1987]

[Davis & Shellard, 1986]

Summarised on the right are mass predictions of multiple simulation groups: rather active research area! Important for axion detection.



Standard Scaling

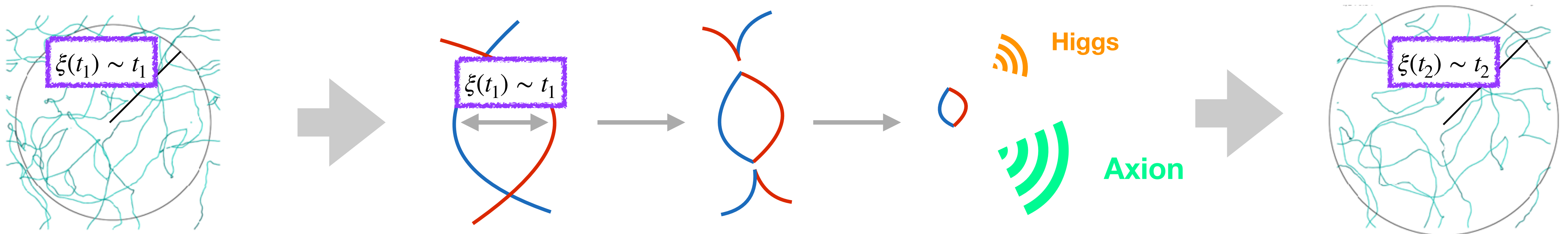
- Defect networks are expected (deep in eras where $a \propto t^\lambda$) to evolve according to the **scaling regime / scale-invariant regime** [Kibble, 1976], [Zeldovich, 1980], [Villenkin, 1981 & 1982] where,

- Mean String **separation**, $\xi \propto t$ and the string **velocity** $v \propto \text{const}$ and therefore $\zeta = \text{const}$
- Scaling given $t \gg t_c$; $\xi, \bar{\xi} \gg w_s$ and results in:

$$\xi \sim \bar{\xi} \sim d_H$$

(w_s - string width, $\bar{\xi}$ - mean string curvature)

- Self-similar evolution sustained via energy loss mechanisms (loop chopping, expansion, emission of radiation)



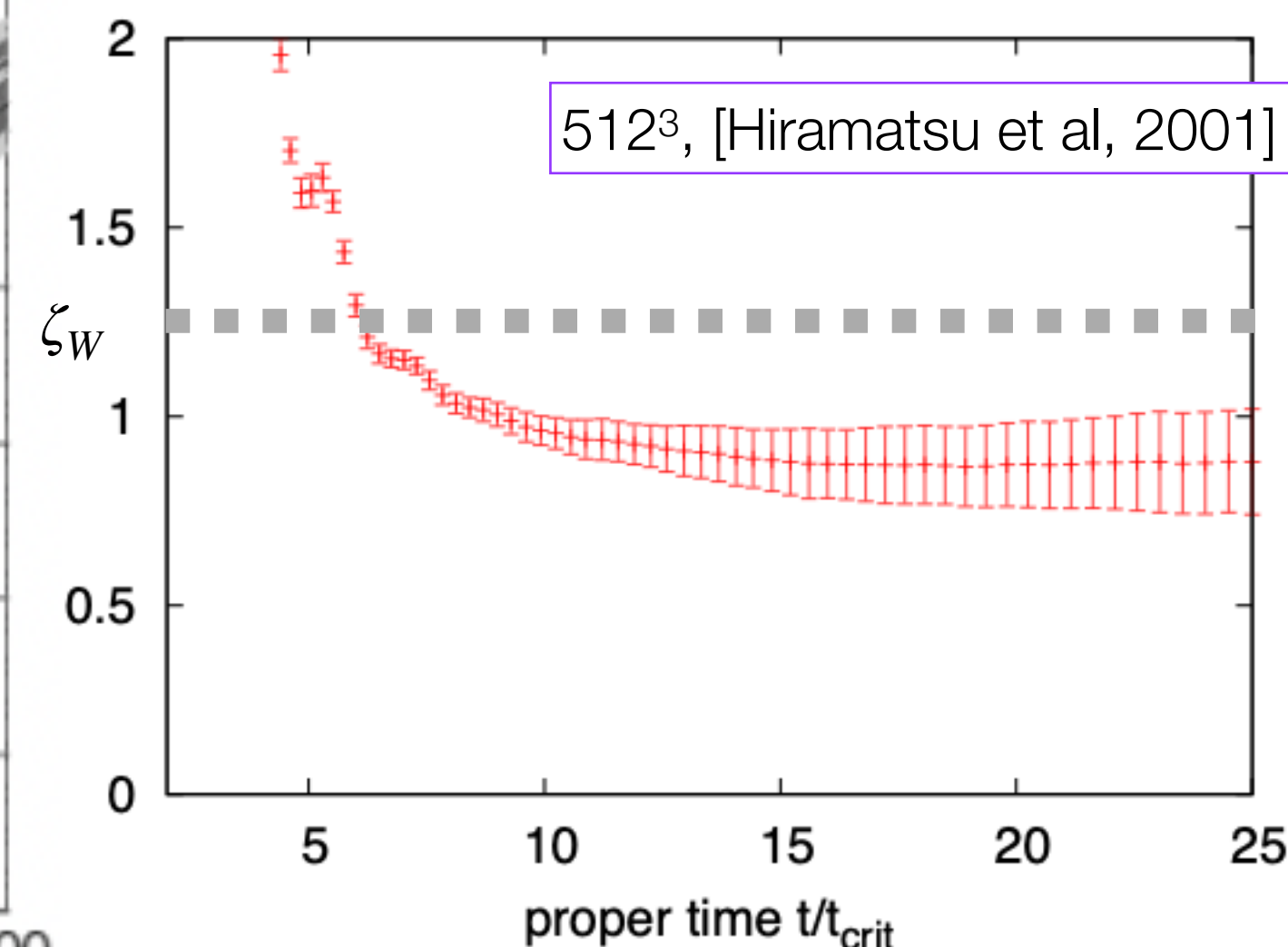
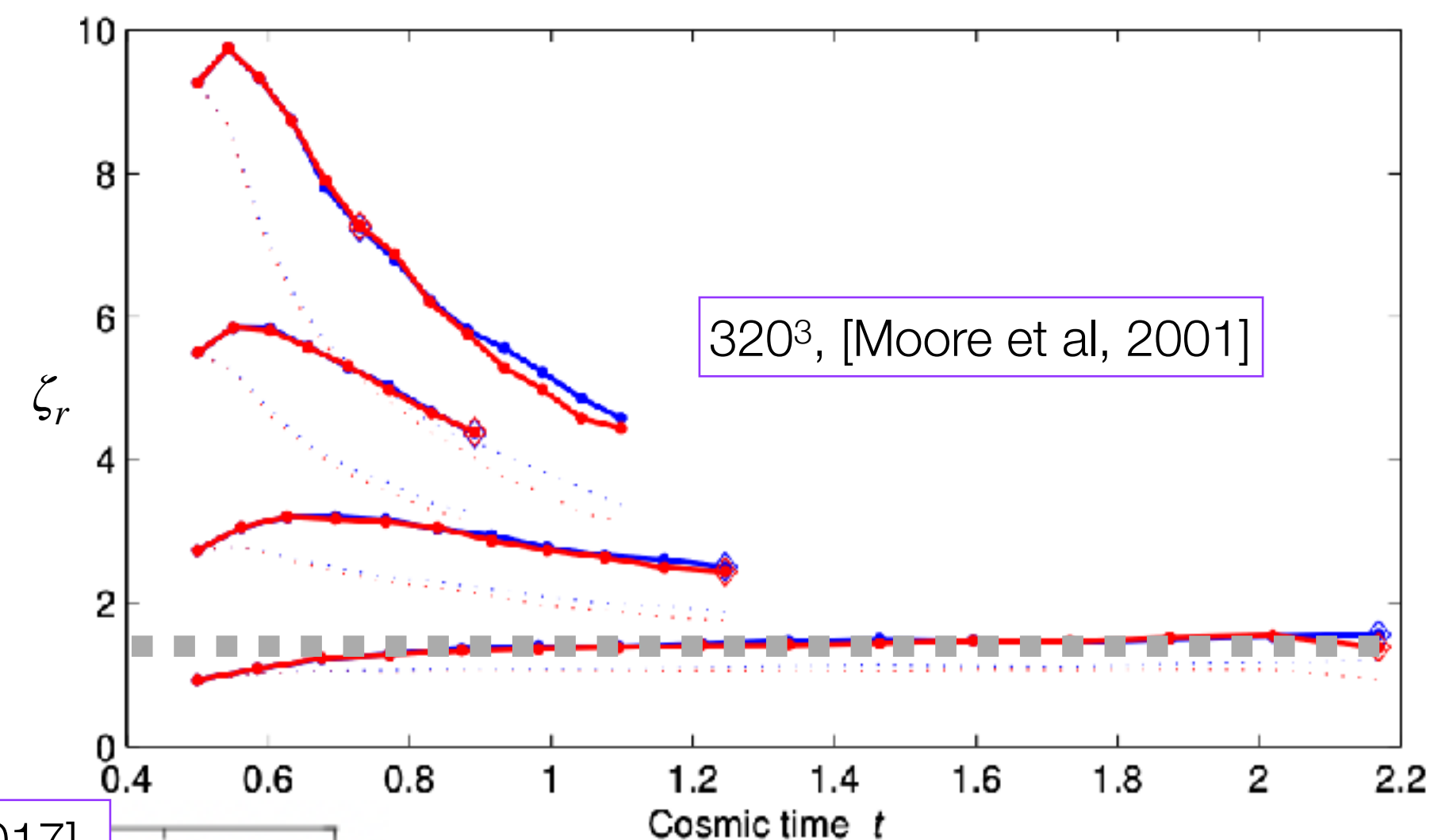
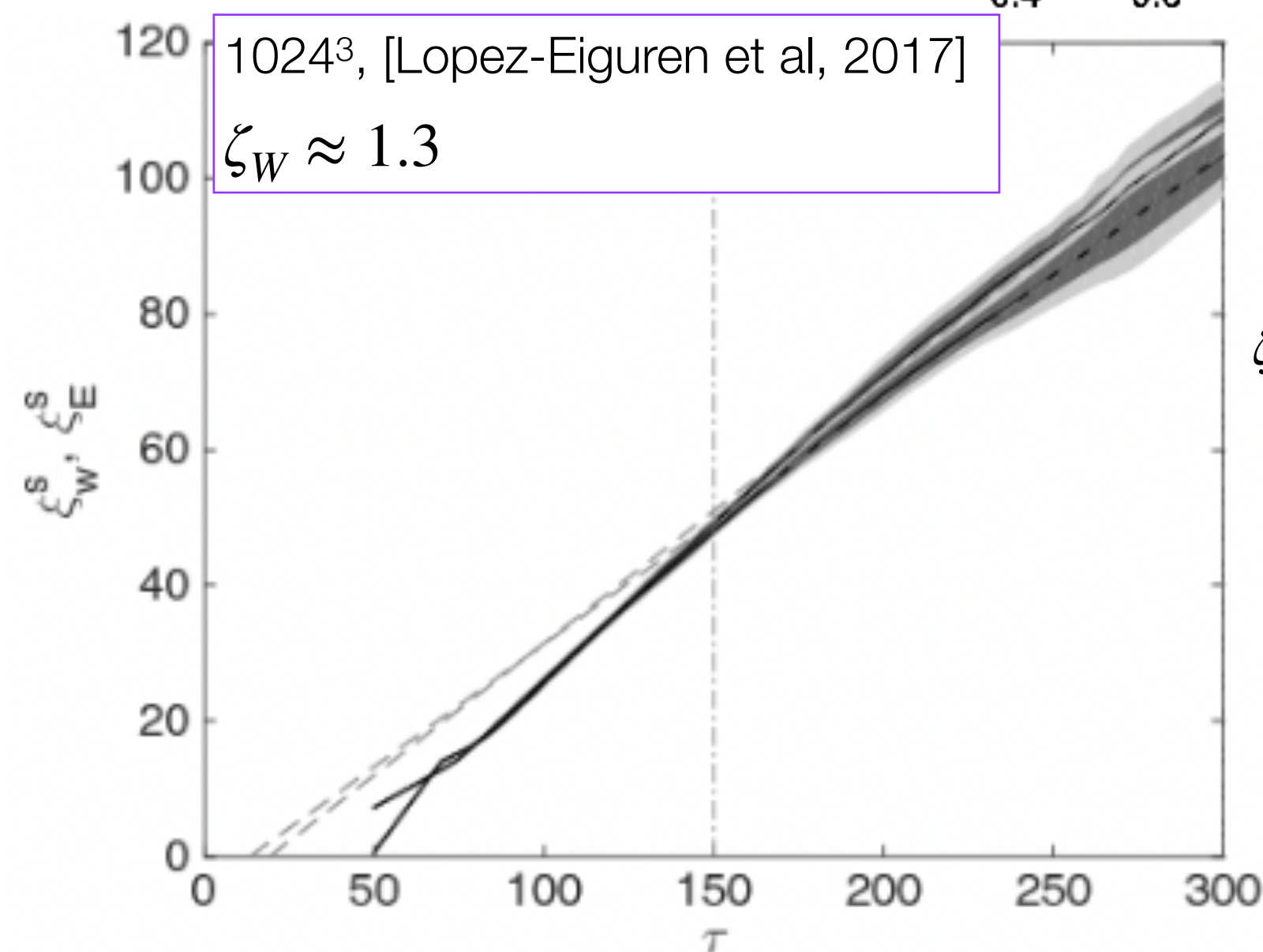
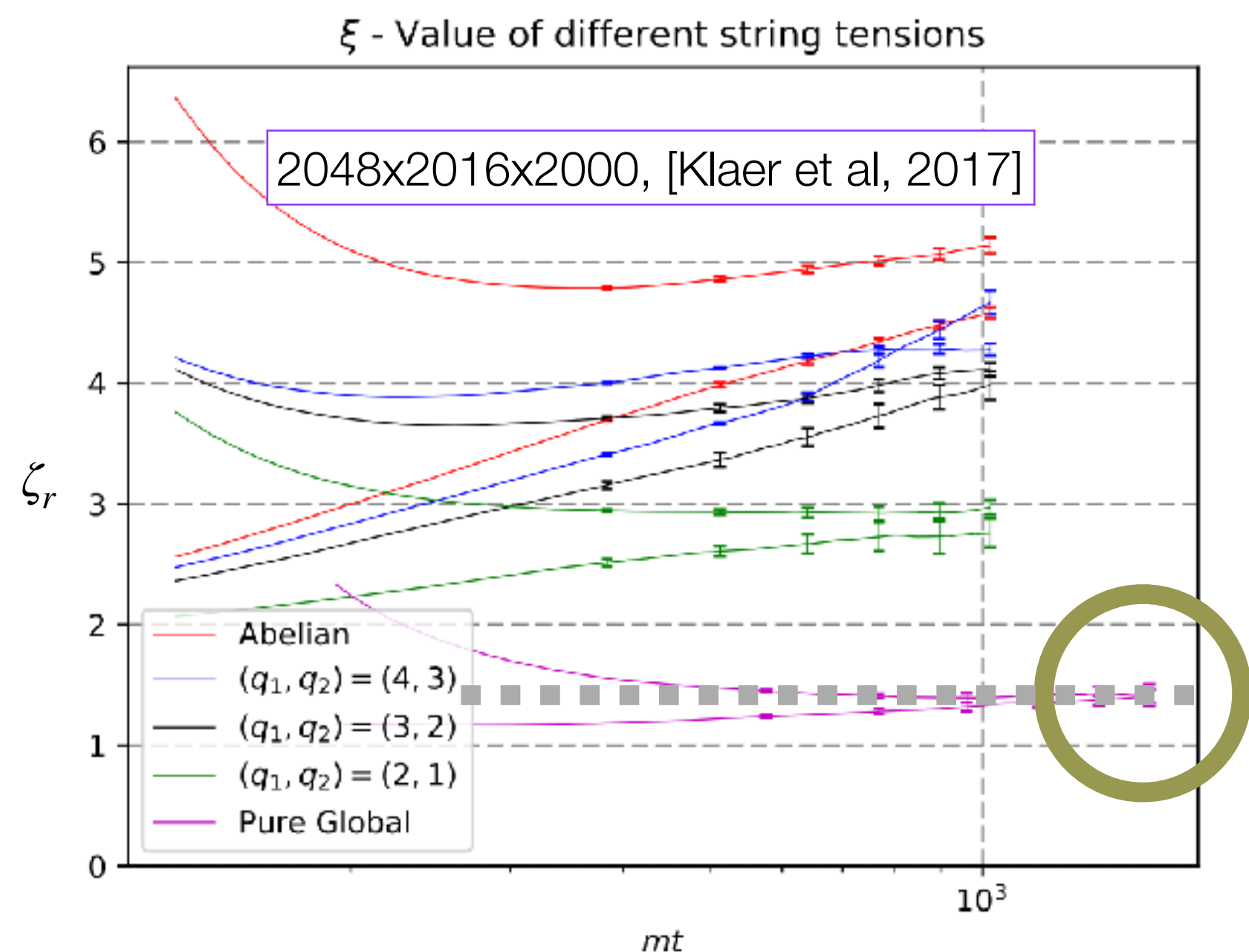
Axion strings - Standard Scaling (pre-2018)

- So far, all consistent with string density parameter / number of strings per Hubble patch $\zeta \approx 1$ (grey dashed line, foreshadows our latest result)

- Some definitions for **rest-frame** and **Universe-frame**:

$$\zeta_r = \gamma \zeta_W$$

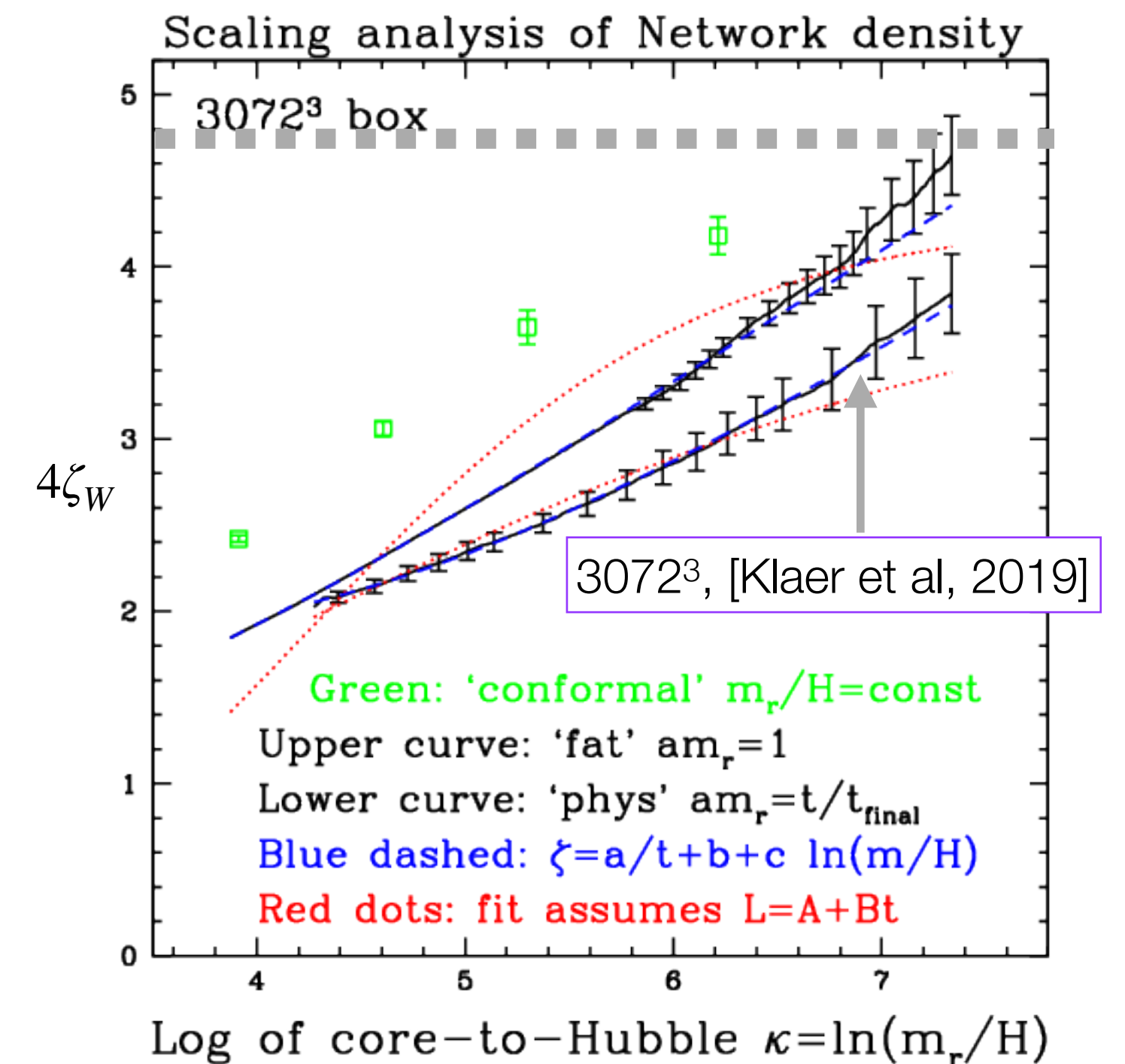
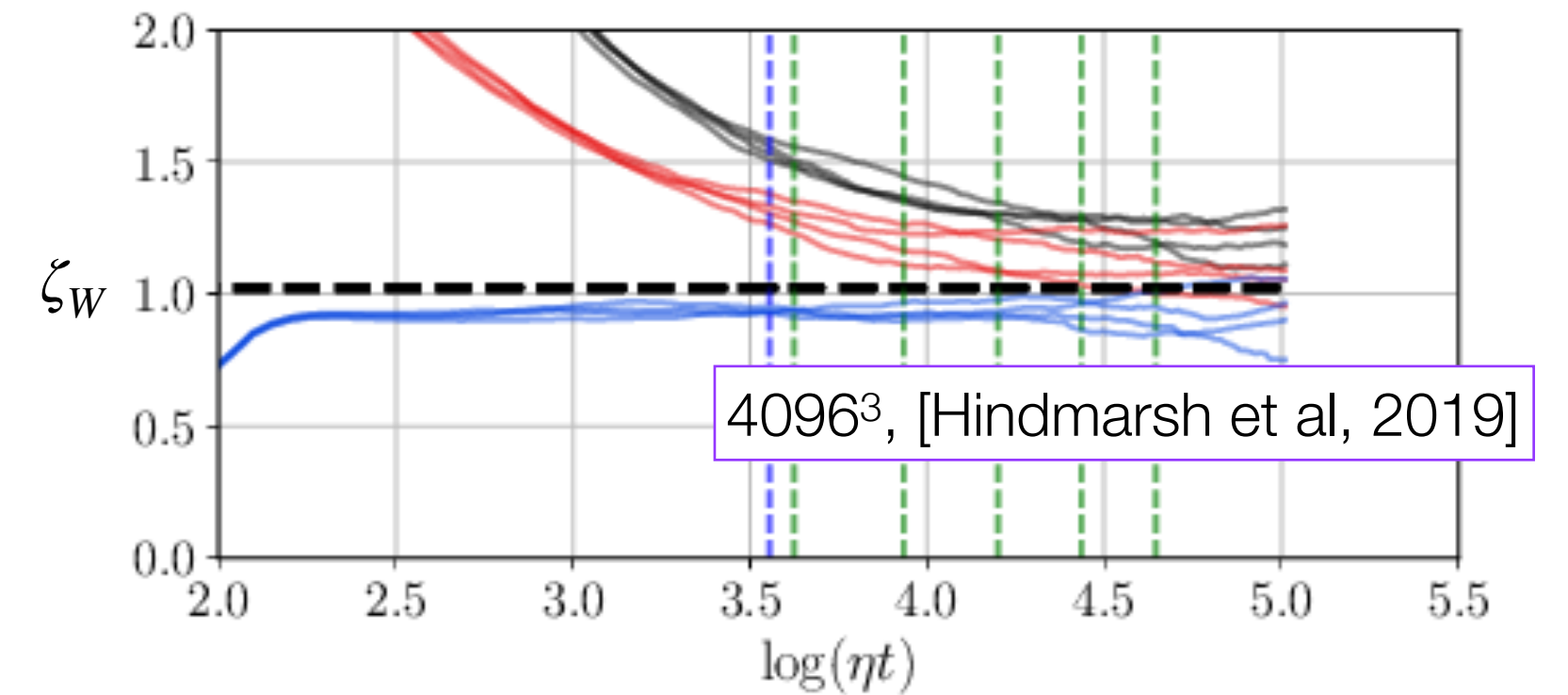
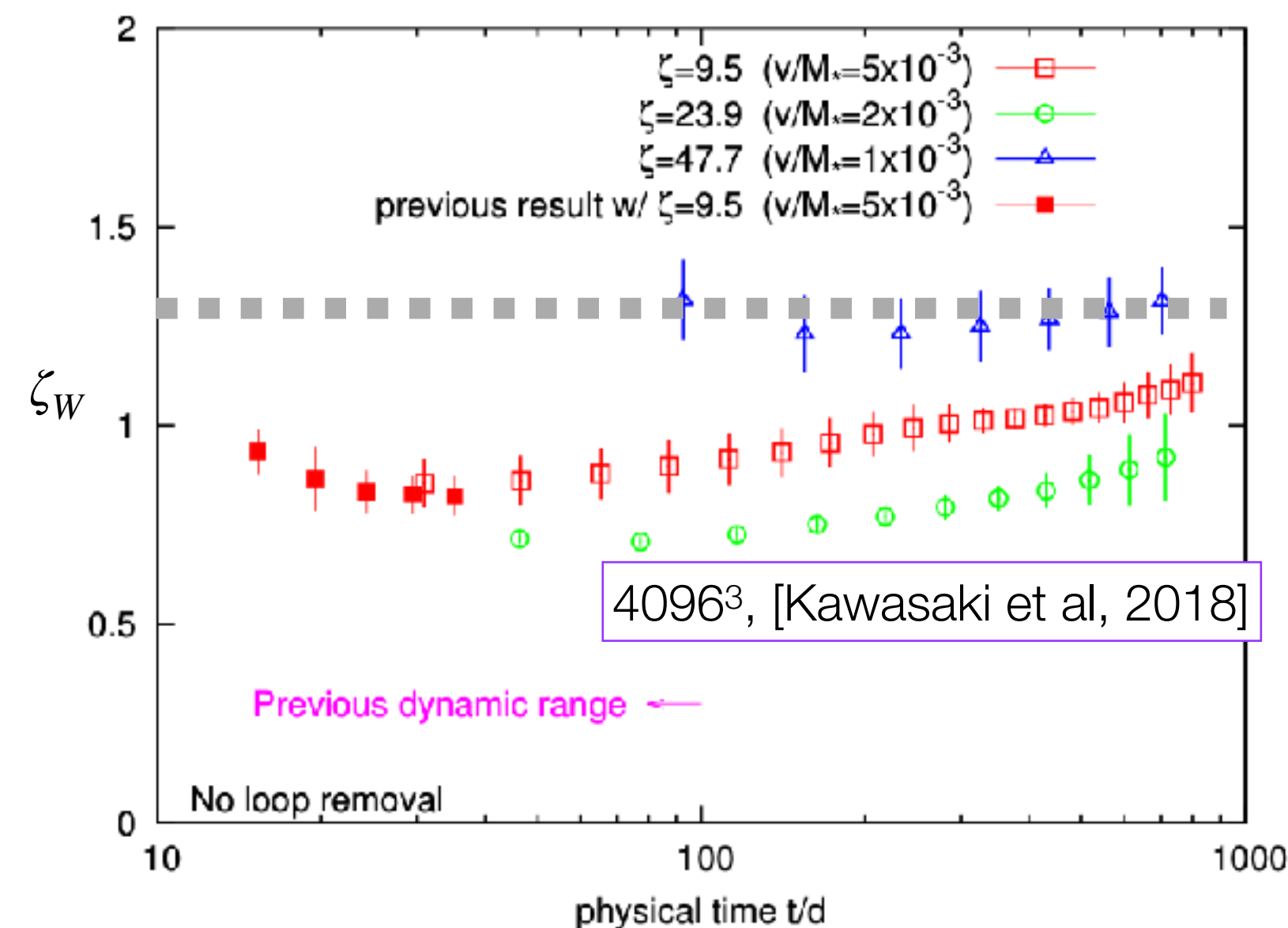
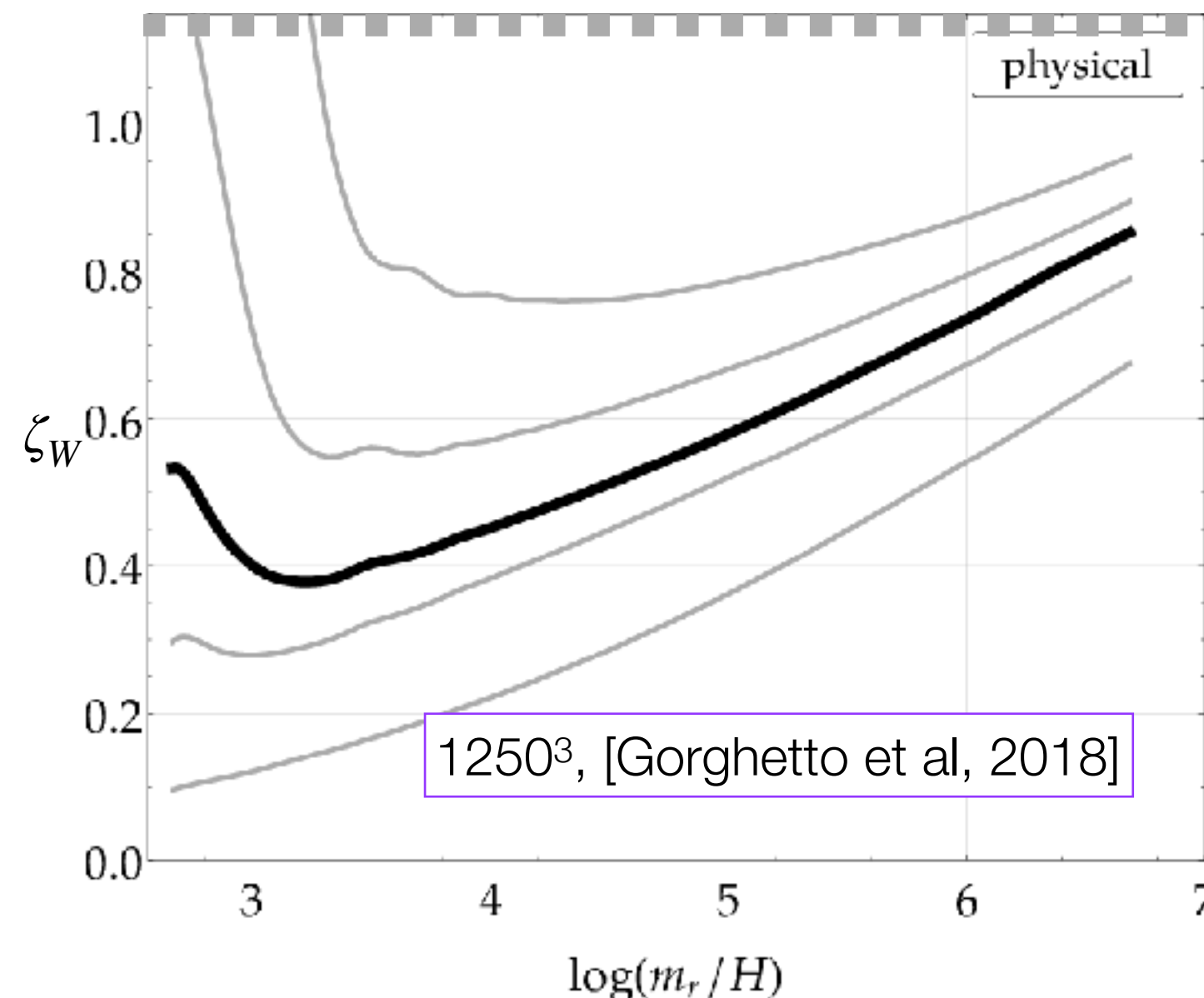
- Root-mean-square velocity $v \approx 0.6$; $\gamma \approx 1.3$ [Klaer et al, 2017]



Axion strings - Scaling Violation (2018/2019)

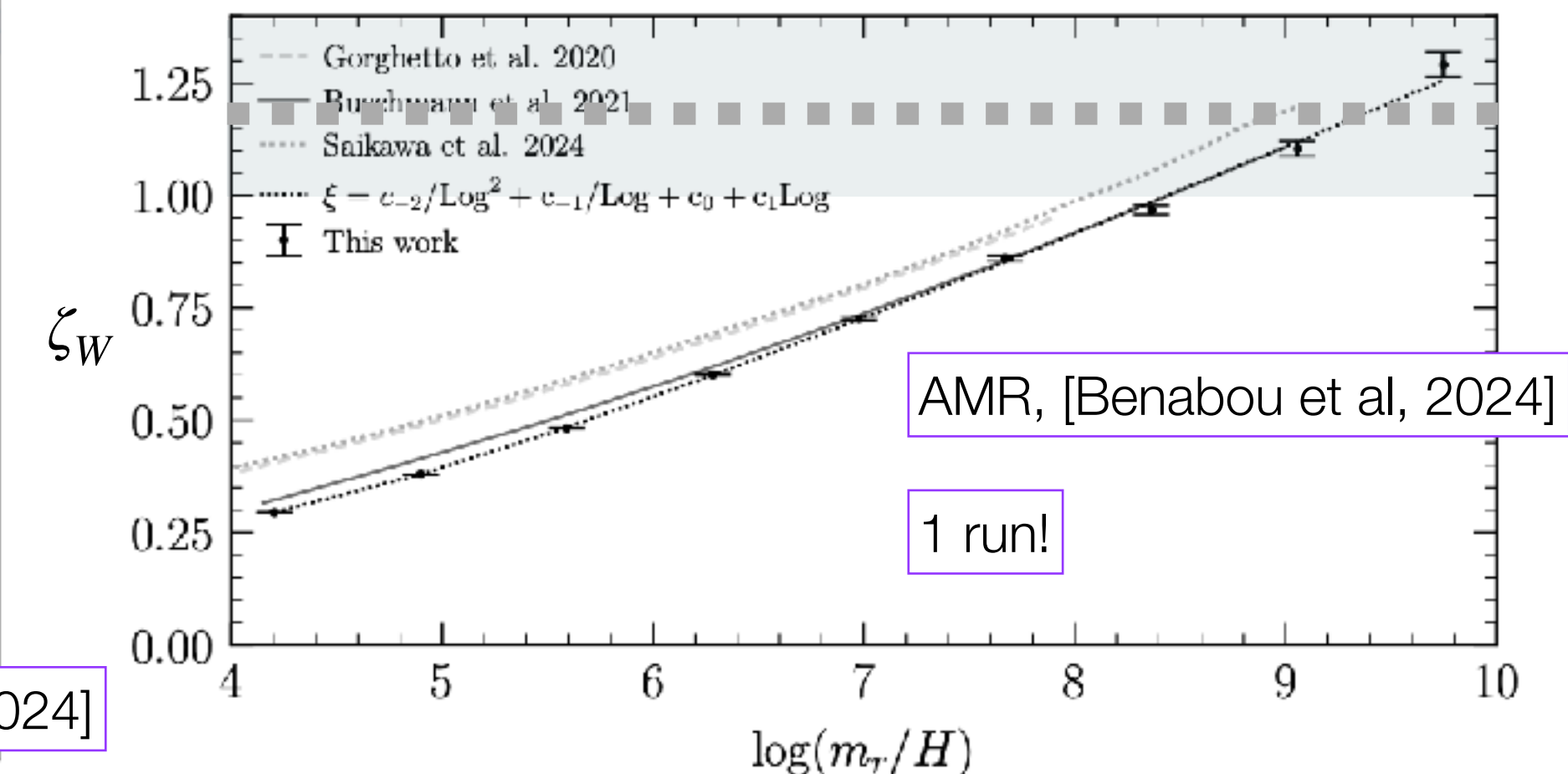
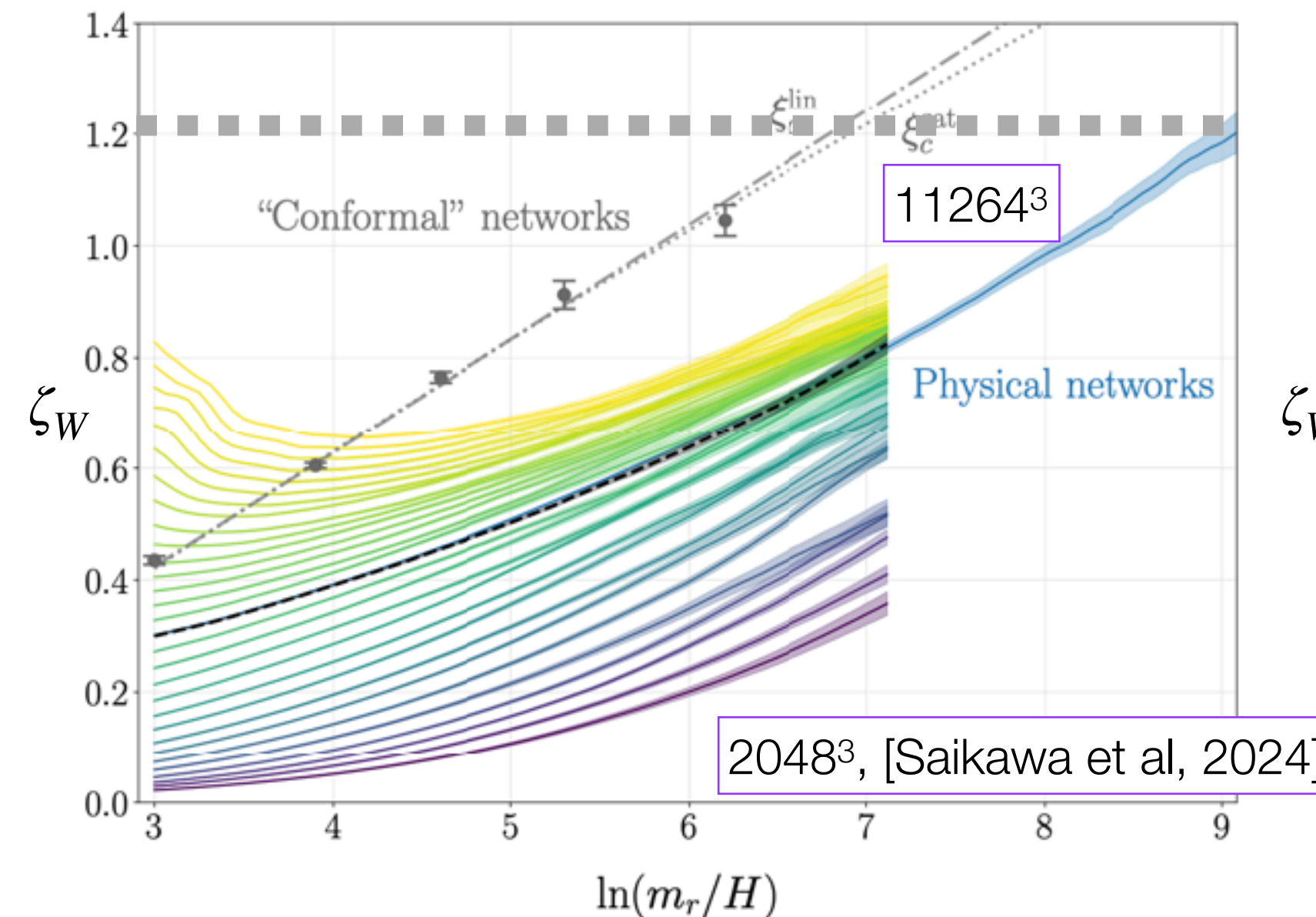
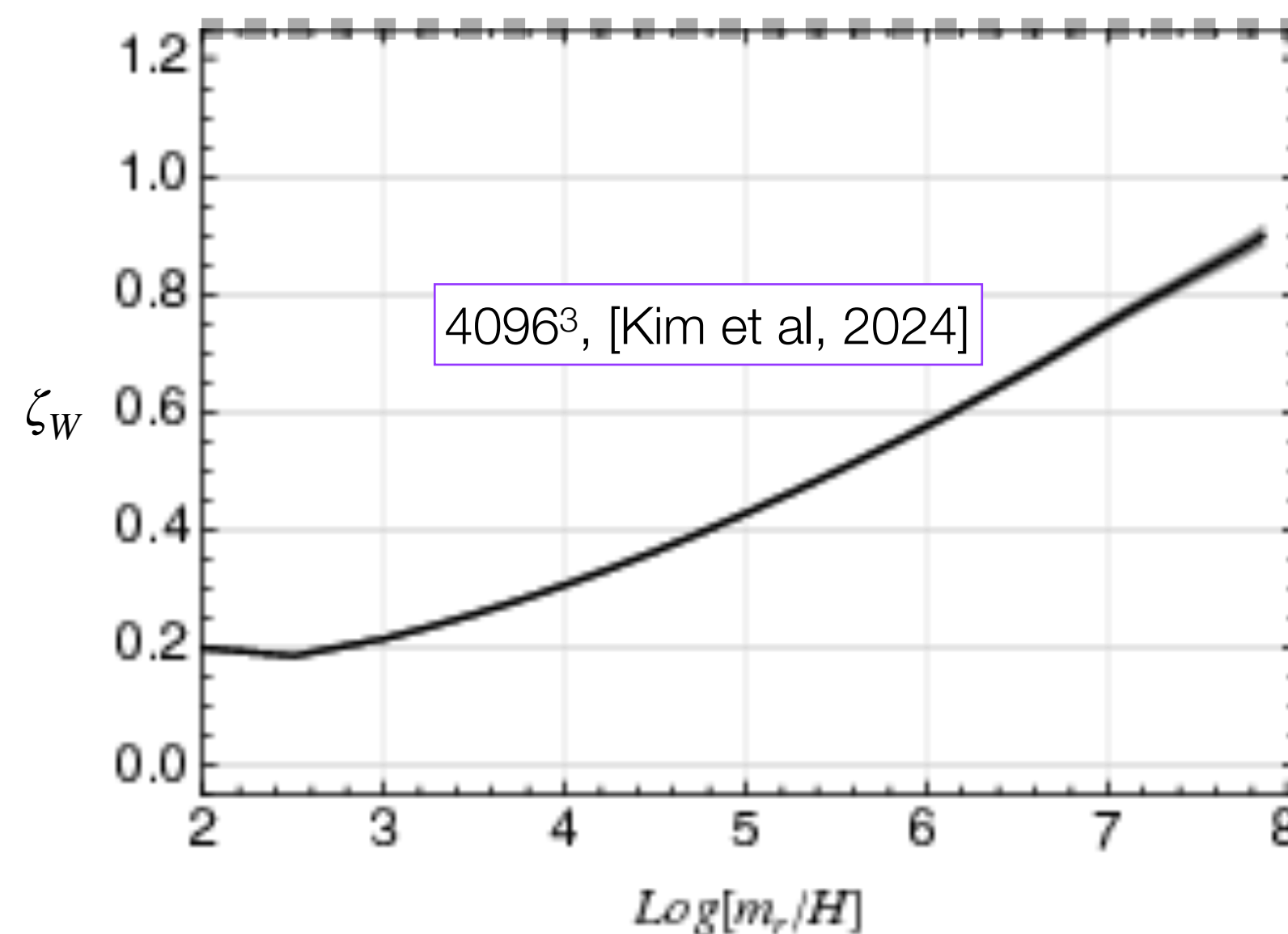
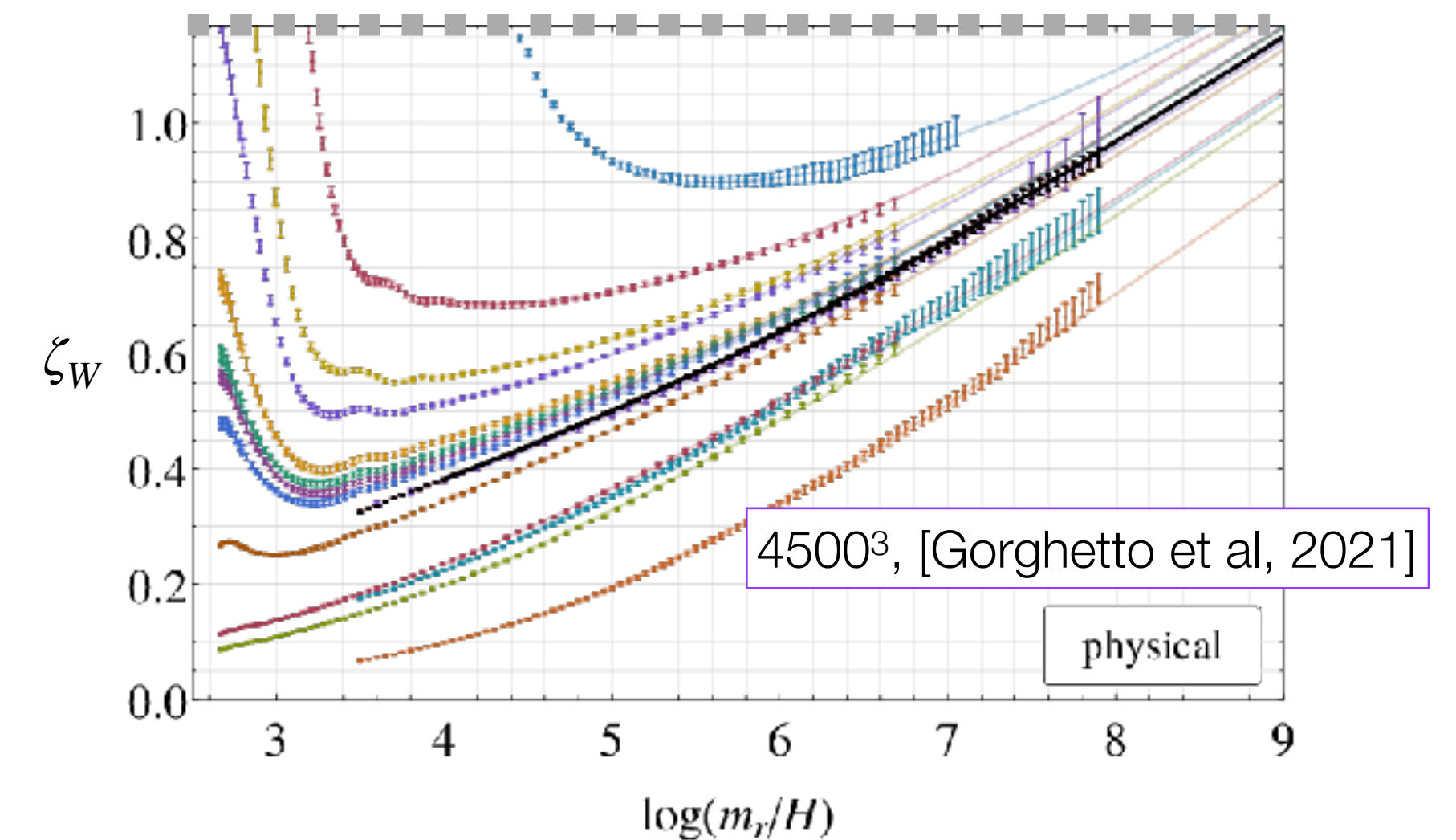
- In [Gorghetto et al, 2018] it is proposed that the density parameter should grow logarithmically with time due to the time-dependent string tension $\mu(t) \propto \log(t)$. **NOTE:** this is **not scaling** in the original sense. Could this be a low-density transient? (See upper right figure)

- Here fits of the form $c_1 \log(t) + c_0$ to $\zeta_W = \frac{t^2 l}{a^2 V}$ are used to argue that this is the true long-term behaviour of the network. However, final values $\zeta_W \sim 1$.



Axion strings - Scaling Violation (2020s)

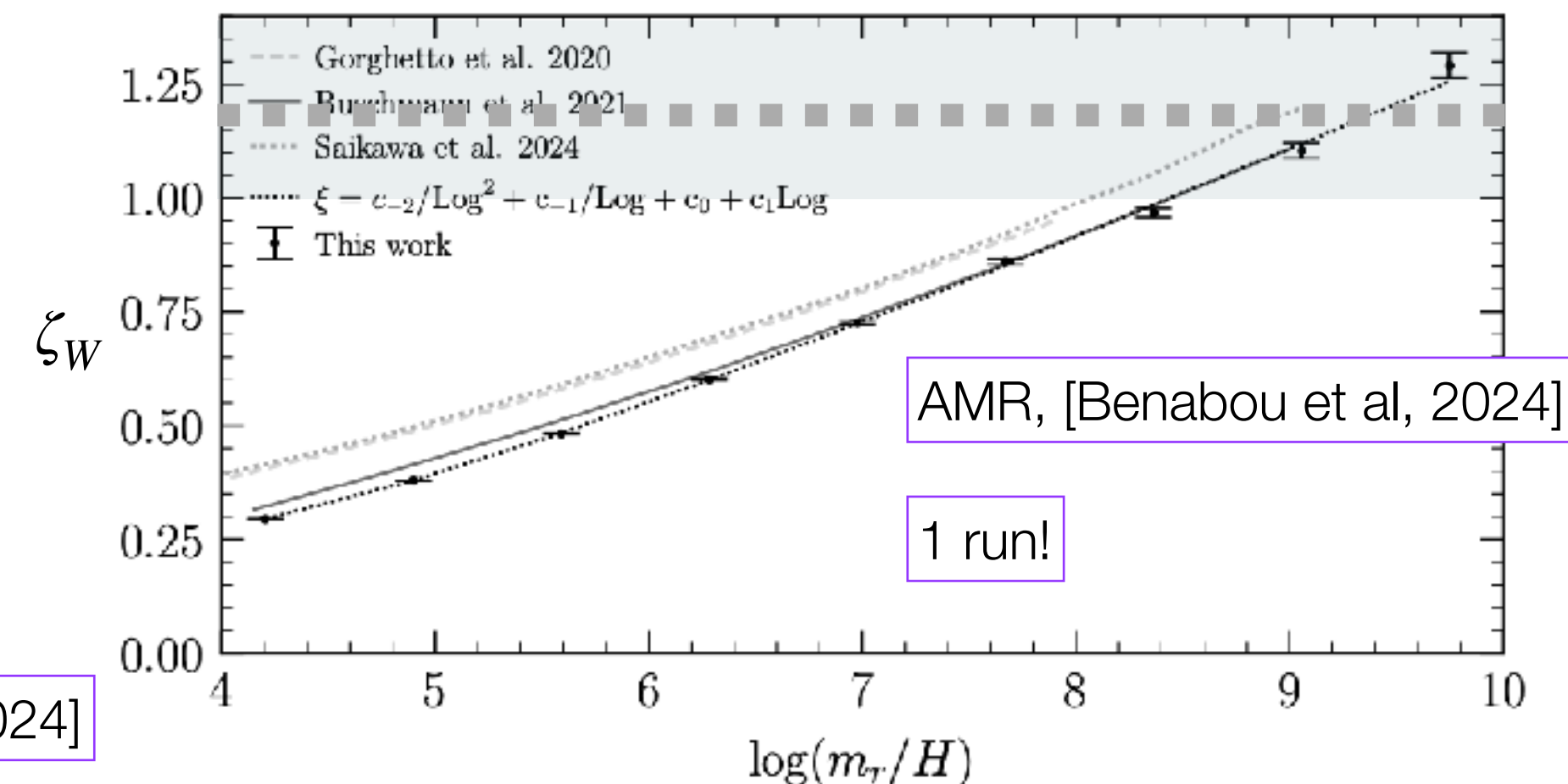
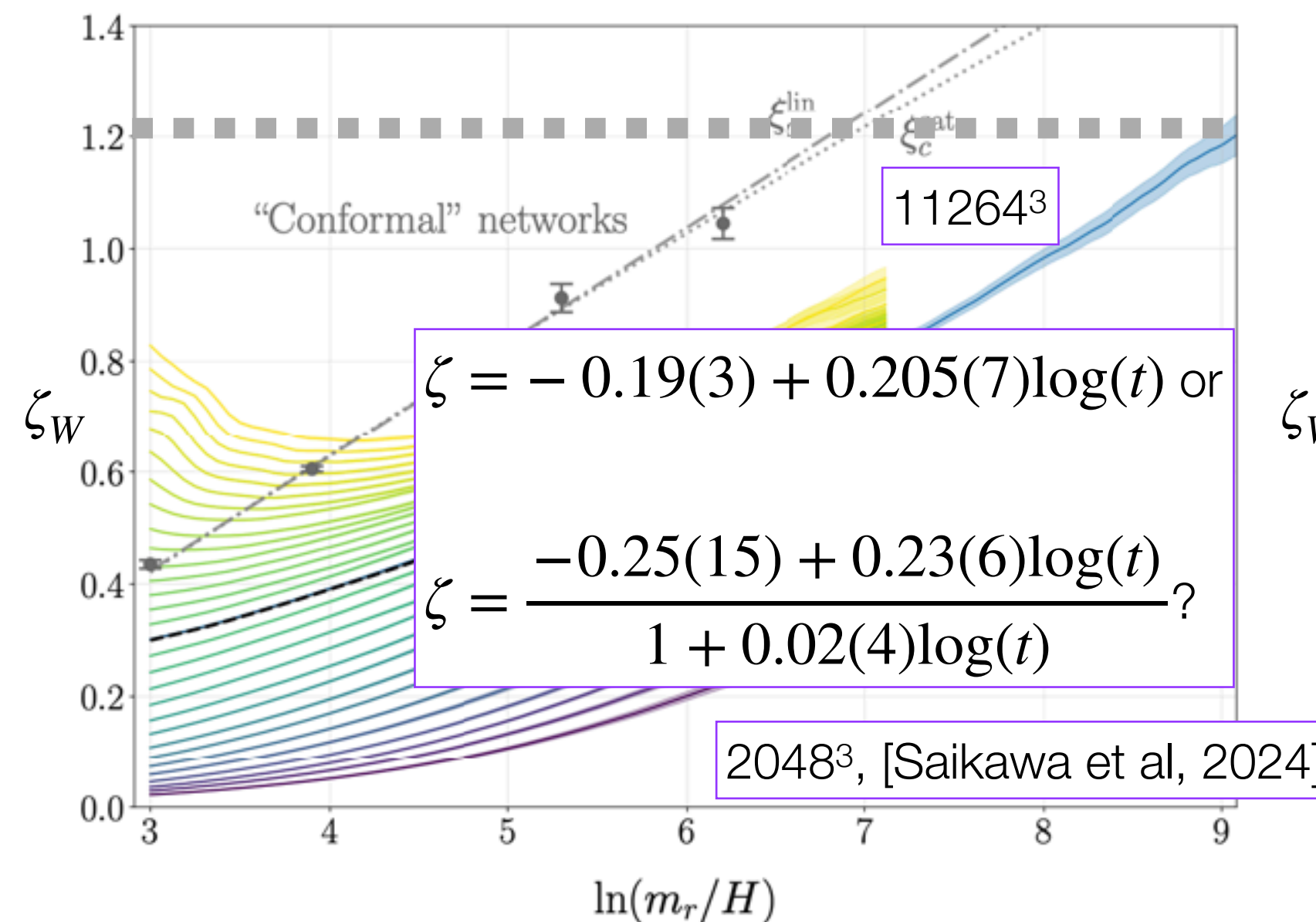
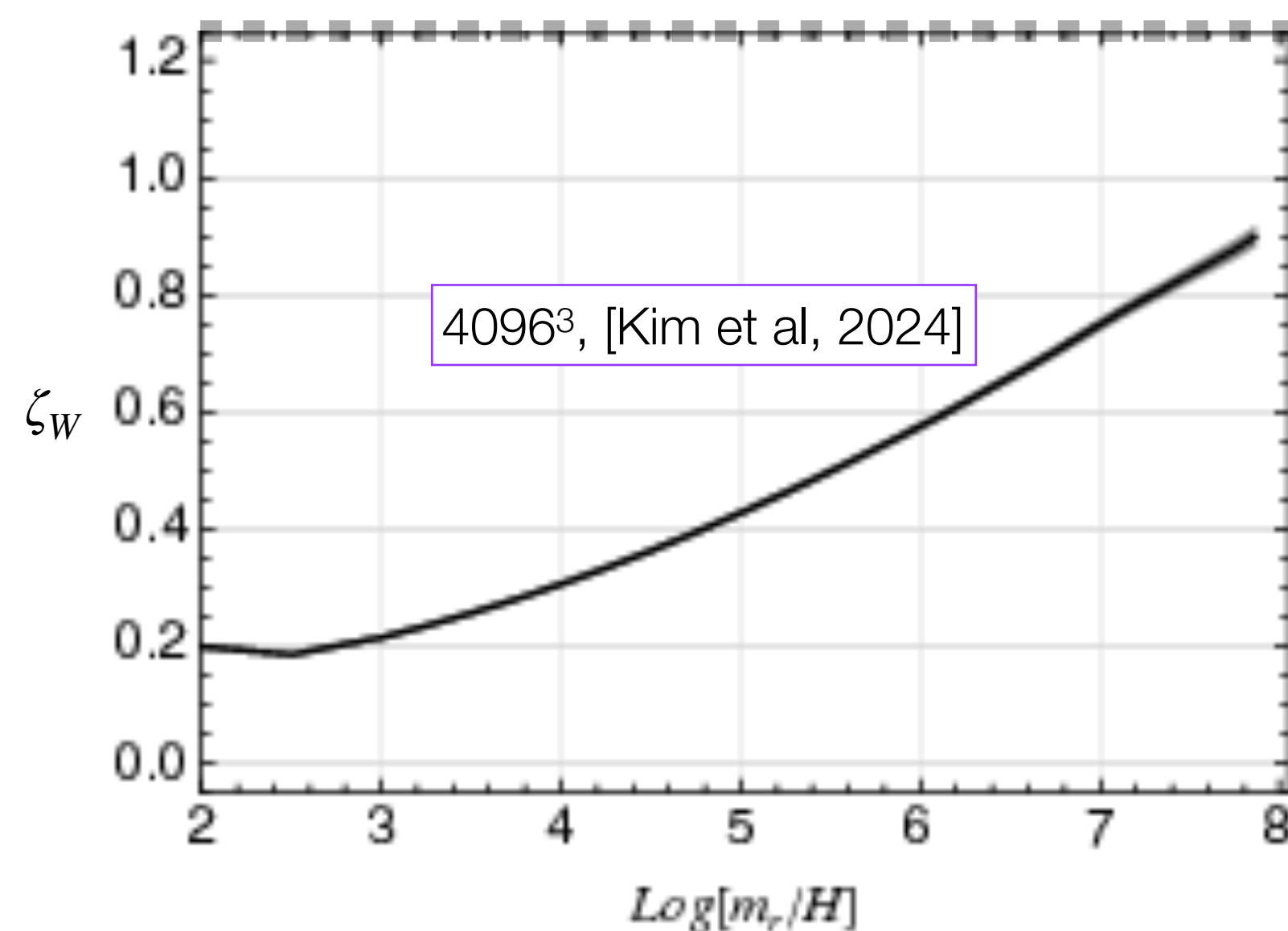
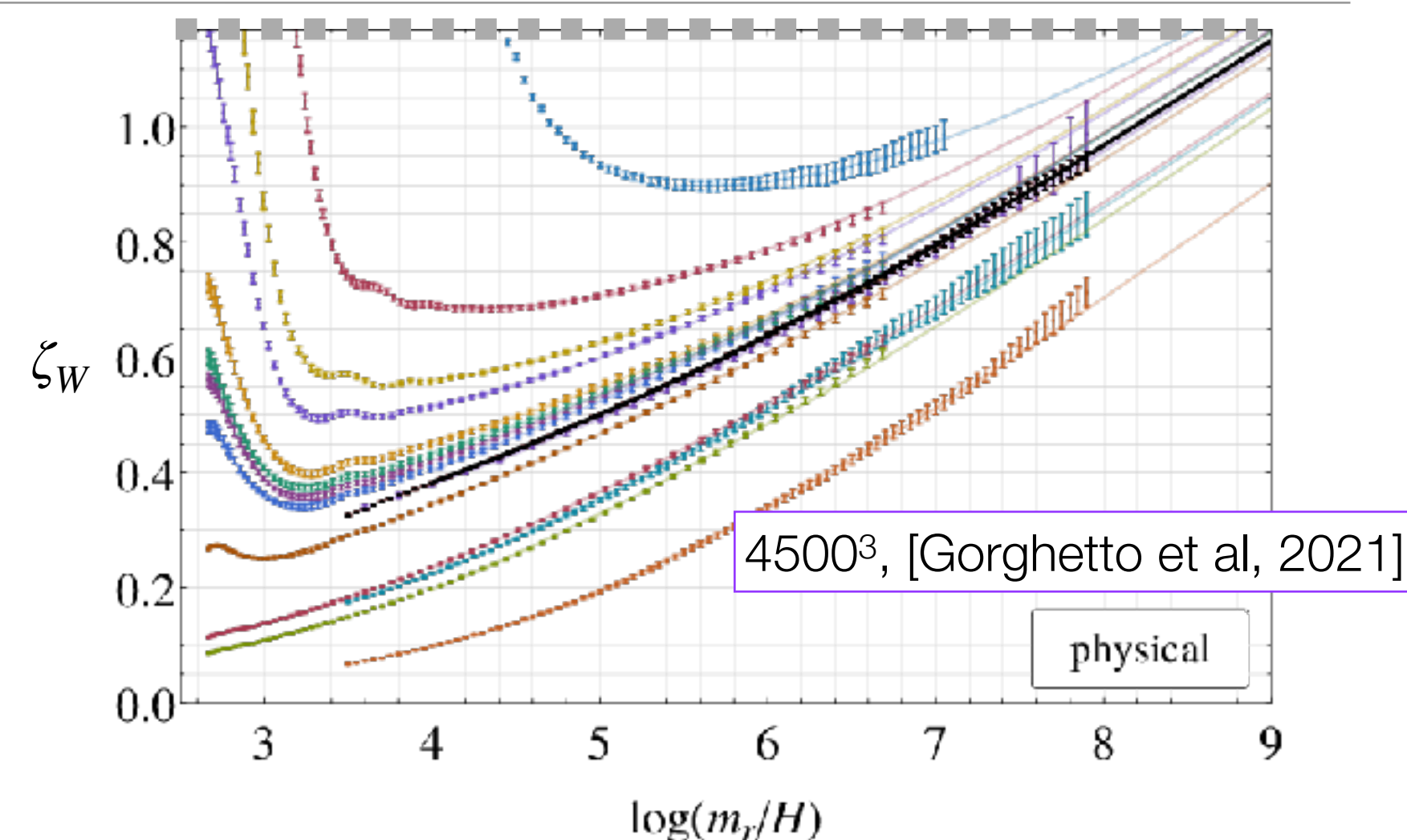
- In [Gorghetto et al, 2018] it is proposed that the density parameter should grow logarithmically with time due to the time-dependent string tension $\mu(t) \propto \log(t)$. **NOTE:** this is **not scaling** in the original sense. Could this be a low-density transient?
- Here fits of the form $c_1 \log(t) + c_0$ to $\zeta_W = \frac{t^2 l}{a^2 V}$ are used to argue that this is the true long-term behaviour of the network. However, final values $\zeta_W \sim 1$.



Axion strings - Scaling Violation (2020s)

- In [Gorghetto et al, 2018] it is proposed that the density parameter should grow logarithmically with time due to the time-dependent string tension $\mu(t) \propto \log(t)$. **NOTE:** this is **not scaling** in the original sense. Could this be a low-density transient?

- Here fits of the form $c_1 \log(t) + c_0$ to $\zeta_W = \frac{t^2 l}{a^2 V}$ are used to argue that this is the true long-term behaviour of the network. However, final values $\zeta_W \sim 1$.



Quantitative String Evolution

[C. J. A. P. Martins & E. P. S. Shellard, 1996]

Quantitative String Evolution

C. J. A. P. Martins* *and* **E. P. S. Shellard**

*Department of Applied Mathematics and Theoretical Physics
University of Cambridge
Silver Street, Cambridge CB3 9EW, UK[†]*

Abstract

An analytic model of long string network evolution, recently developed by the authors, is presented in detail, and modified to describe string loop evolution. By treating the average string velocity, as well as the characteristic lengthscale, as dynamical variables, one can include the effects of frictional forces on the evolution of the network. This generalized ‘one-scale’ model provides a quantitative picture of the complete evolution of a string network, including the prediction of previously unknown transient scaling regimes and a detailed analysis of the evolution of the loop population. The evolution of all cosmologically interesting string networks is studied in detail, and possible consequences of our results are discussed.

String Evolution as an Autonomous System

- Take the action for an ideal axion string [Kalb et al, 1974; Lund et al. 1976; Dabholkar et al, 1990]. Here we will work with world sheet coordinates $X^\mu = X^\mu(\sigma^0, \sigma^1)$, which represent 1+1 world sheet in space-time:

$$S = -\mu \int \sqrt{-\gamma} d^2\sigma - \frac{1}{6} \int H^2 d^4x - 2\pi f_a \int B_{\mu\nu} d\sigma^{\mu\nu}$$

$\gamma = \det(\gamma_{ab})$ is the determinant of the induced metric, $\gamma_{ab} = g_{\mu,\nu} x^\mu_{,a} x^\nu_{,b}$. Let us neglect **interactions (& axion)** for now.

- Assume transverse-temporal gauge: $\sigma_0 = \tau$ (or, $X^\mu = (\tau, x^i)$) and $\dot{X} \cdot X' = 0$ and the equations of motion in an FLRW background are:

$$\ddot{x} + 2\frac{\dot{a}}{a}\left(1 - \dot{x}\right)\dot{x} = \epsilon^{-1}\left(\epsilon^{-1}x'\right) \quad \text{and} \quad \dot{\epsilon} = -2\frac{\dot{a}}{a}\dot{x}^2\epsilon$$

- Where $\epsilon = \sqrt{\frac{x'^2}{1 - \dot{x}^2}}$. Apply an averaging procedure on **rest-frame length** and **r.m.s velocity**:

$$l_r = \int \epsilon d\sigma = \frac{a^2 V}{\xi_r^2} \quad \text{and} \quad v^2 = \int \dot{x}^2 \epsilon d\sigma / l_r$$

Assume One-Scale approximation:
 $\xi \sim \bar{\xi} \sim L$ (correlation length)

Add Loop chopping:
 $\frac{d\rho}{dt_{\text{loops}}} = cv \frac{\rho}{L}$

String Evolution as an Autonomous System

Result: **Velocity-dependant One-Scale** (VOS) [Martins & Shellard, 1996]. For ideal Nambu-Goto strings:

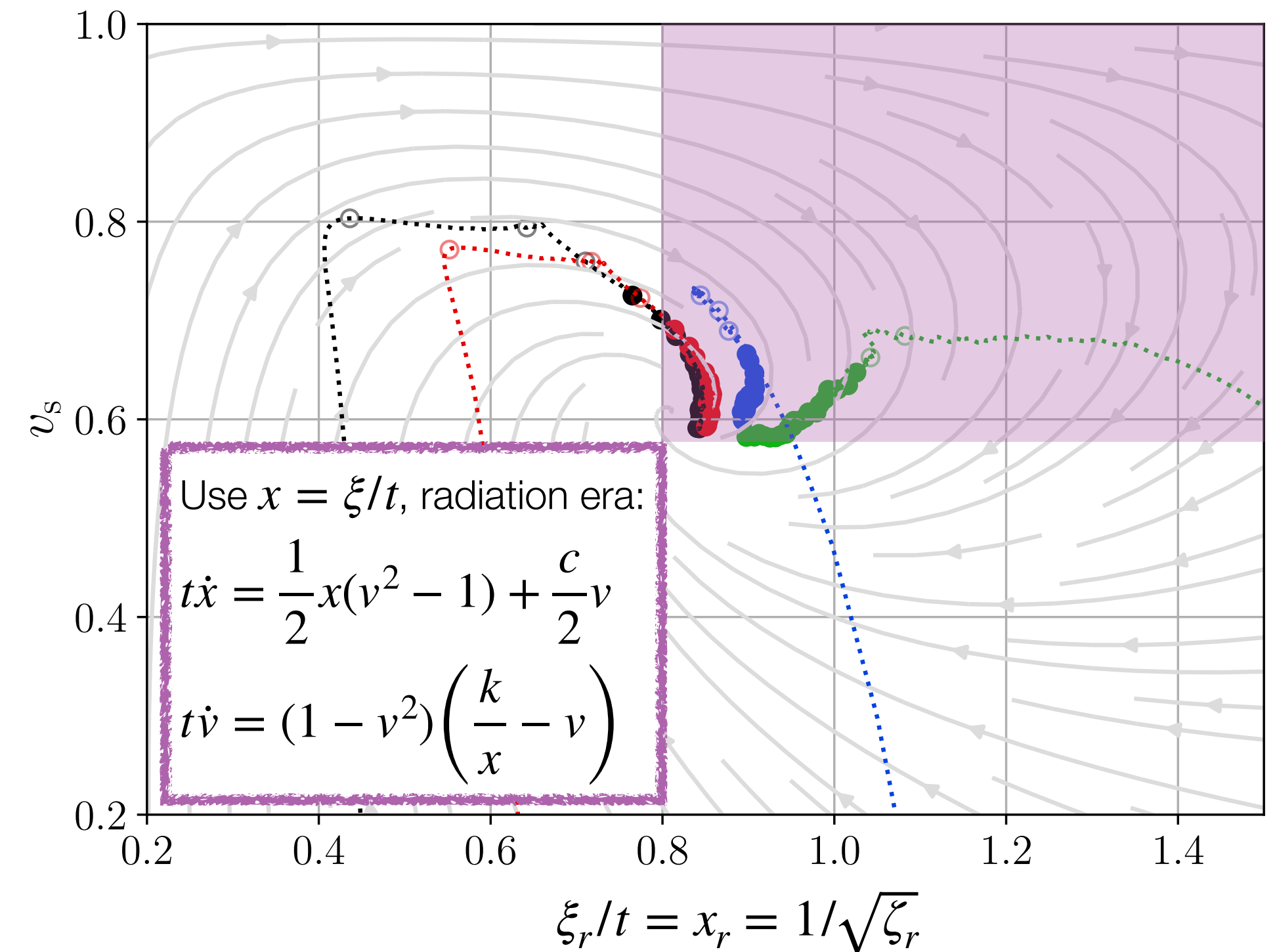
$$2\frac{d\xi}{dt} = 2H\xi(1 + v^2) + cv$$

c = loop chopping efficiency
k = curvature parameter

$$\frac{dv}{dt} = \left(1 - v^2\right) \left(\frac{k(v)}{\xi} - 2Hv\right)$$

Scaling appears naturally as an **attractor fixed point**.

- Energy conservation + Newton's second law (relates acceleration with curvature)
- In [Hindmarsh et al, 2021] a simplified version of this model is used for Global Strings...
 - ...where the curvature parameter is taken as constant (string interaction force absorbed)



[Hindmarsh et al, 2021] used this simple model to estimate a axion strings FIXED POINT AT:

$$x_* = 0.814 \pm 0.037 \quad v_* = 0.609 \pm 0.014$$

$$\zeta_* = 1.20 \pm 0.09$$

String Evolution as an Autonomous System

Result: **Velocity-dependant One-Scale** (VOS) [Martins & Shellard, 1996]. For ideal Nambu-Goto strings:

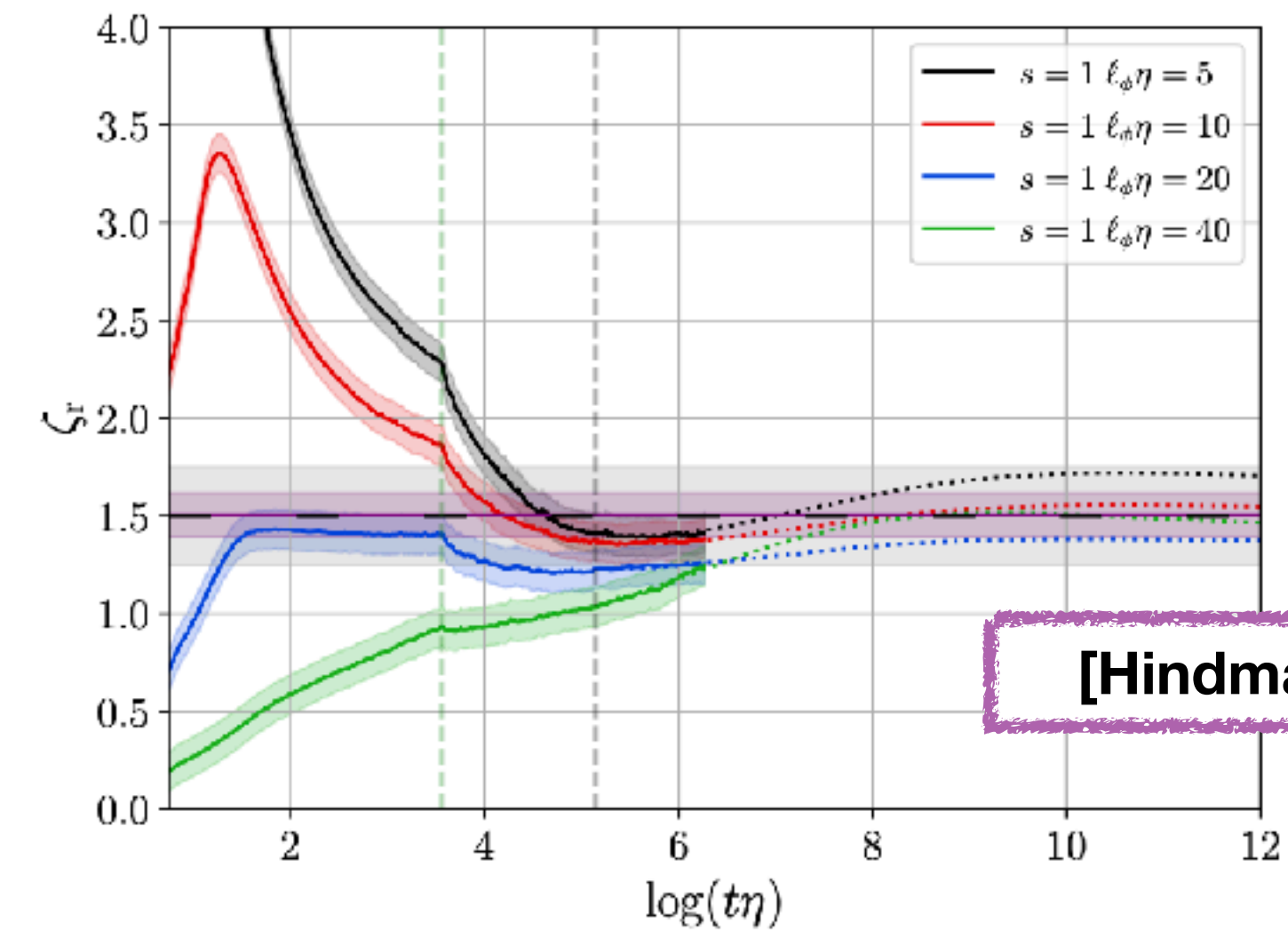
$$2\frac{d\xi}{dt} = 2H\xi(1 + v^2) + cv$$

c = loop chopping efficiency
k = curvature parameter

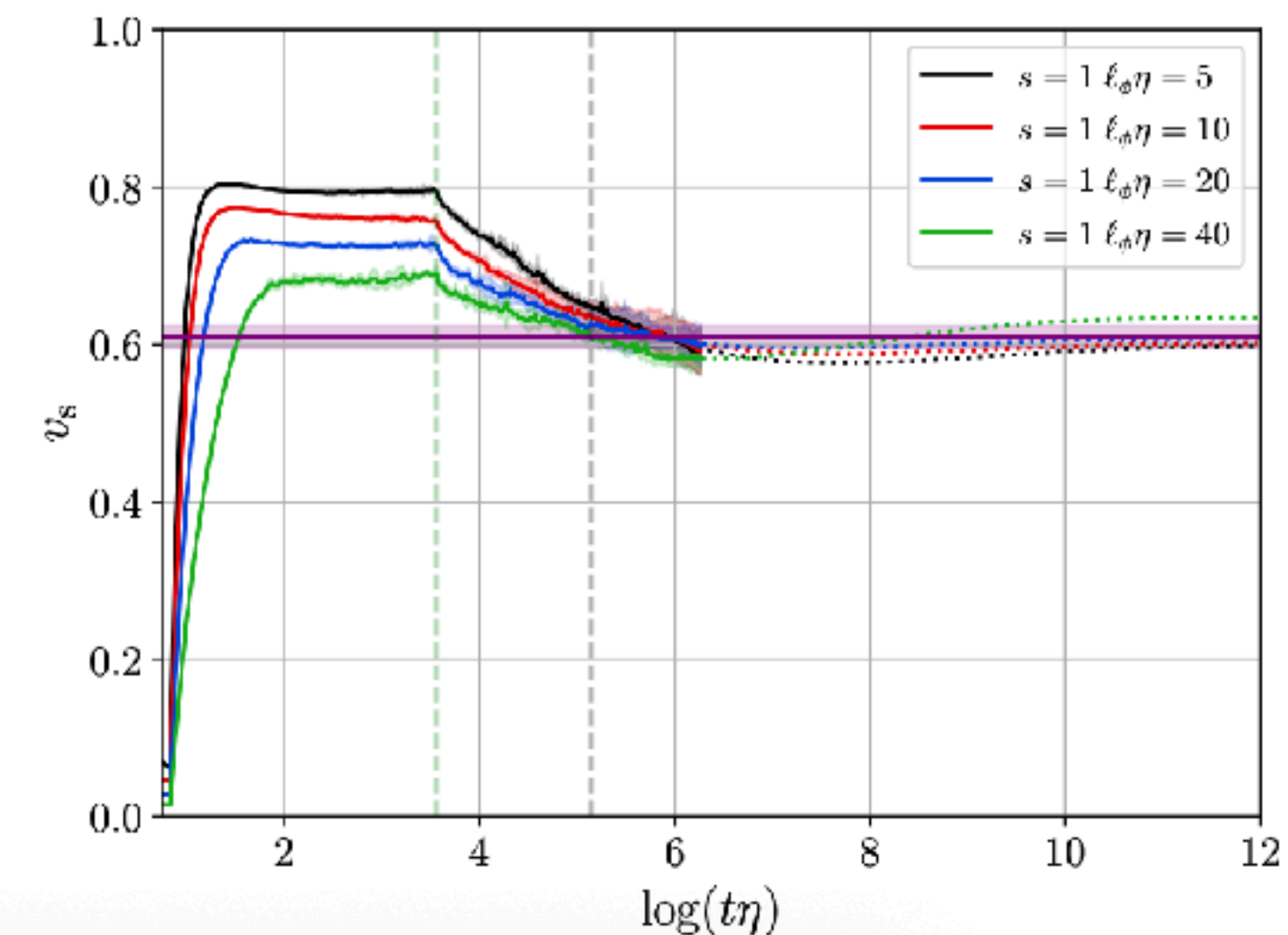
$$\frac{dv}{dt} = \left(1 - v^2\right) \left(\frac{k(v)}{\xi} - 2Hv\right)$$

Scaling appears naturally as an **attractor fixed point**.

- Energy conservation + Newton's second law (relates acceleration with curvature)
- In [Hindmarsh et al, 2021] a simplified version of this model is used for Global Strings...
 - ...where the curvature parameter is taken as constant (string interaction force absorbed)



[Hindmarsh et al, 2021]



String Evolution as an Autonomous System

Result: **Velocity-dependant One-Scale** (VOS) [Martins & Shellard, 1996]. For ideal Nambu-Goto strings:

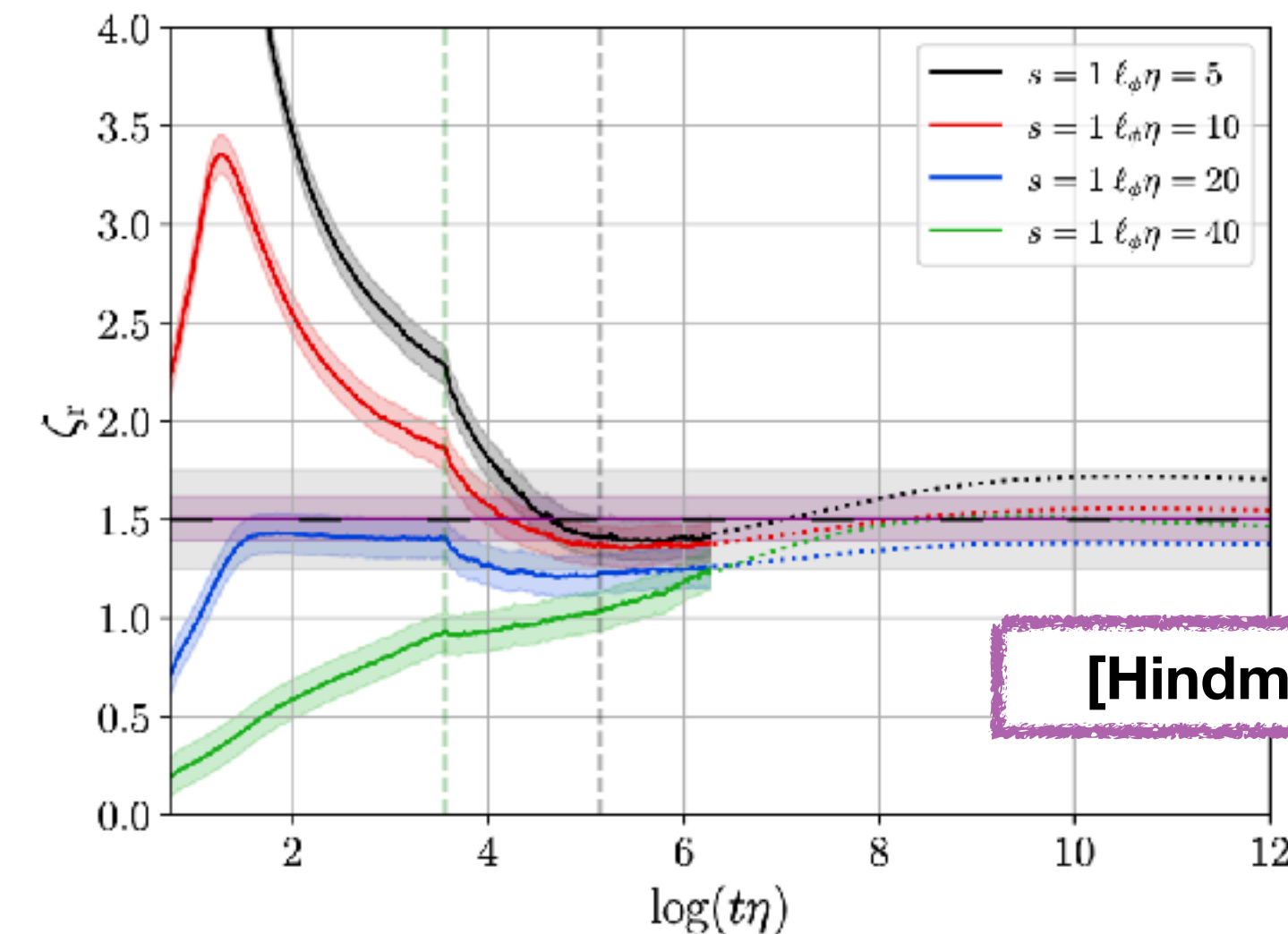
$$2\frac{d\xi}{dt} = 2H\xi(1 + v^2) + cv$$

c = loop chopping efficiency
k = curvature parameter

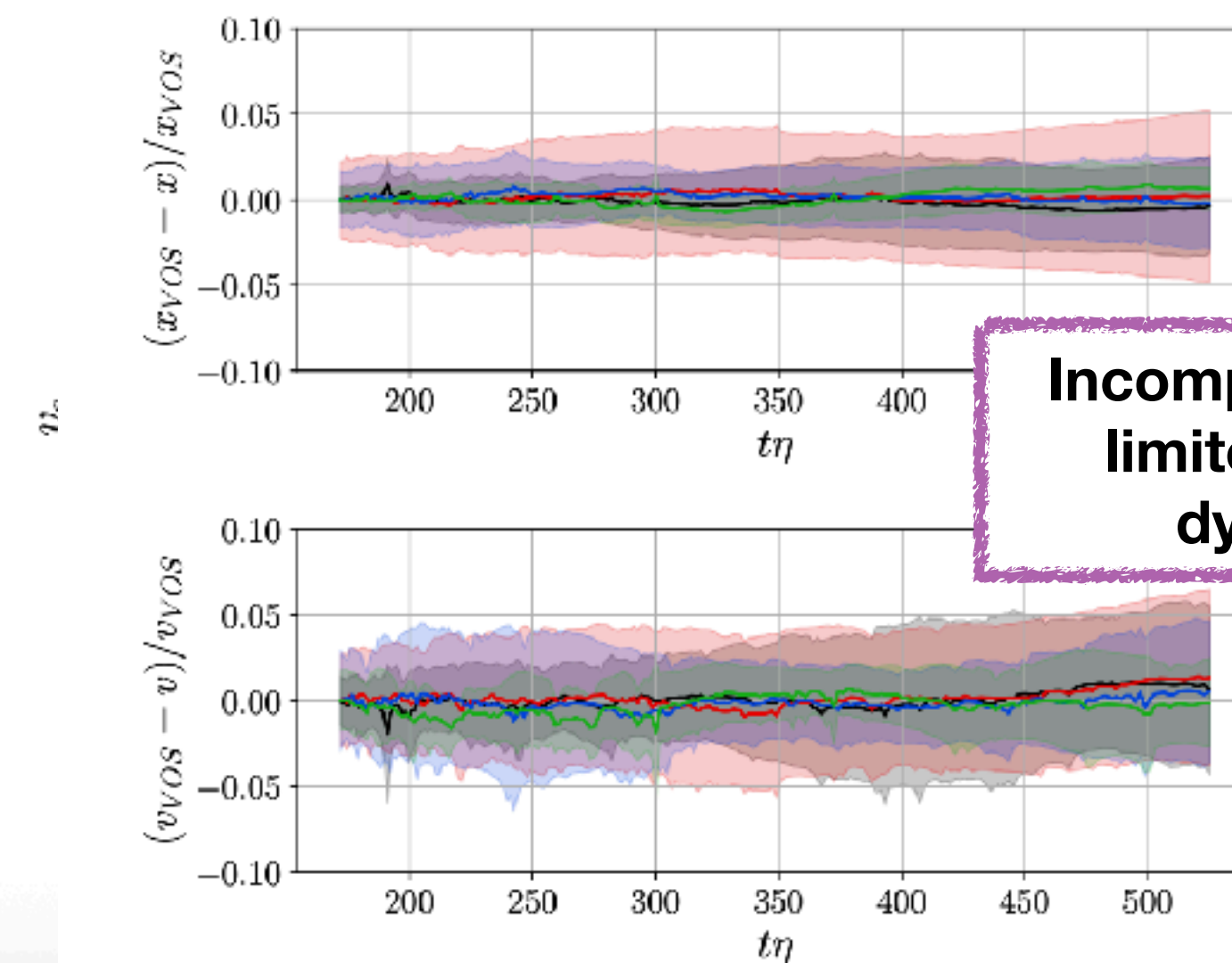
$$\frac{dv}{dt} = \left(1 - v^2\right) \left(\frac{k(v)}{\xi} - 2Hv\right)$$

Scaling appears naturally as an **attractor fixed point**.

- Energy conservation + Newton's second law (relates acceleration with curvature)
- In [Hindmarsh et al, 2021] a simplified version of this model is used for Global Strings...
- ...where the curvature parameter is taken as constant (string interaction force absorbed)



[Hindmarsh et al, 2021]



Incomplete model ok for
limited curves, short
dynamic range.

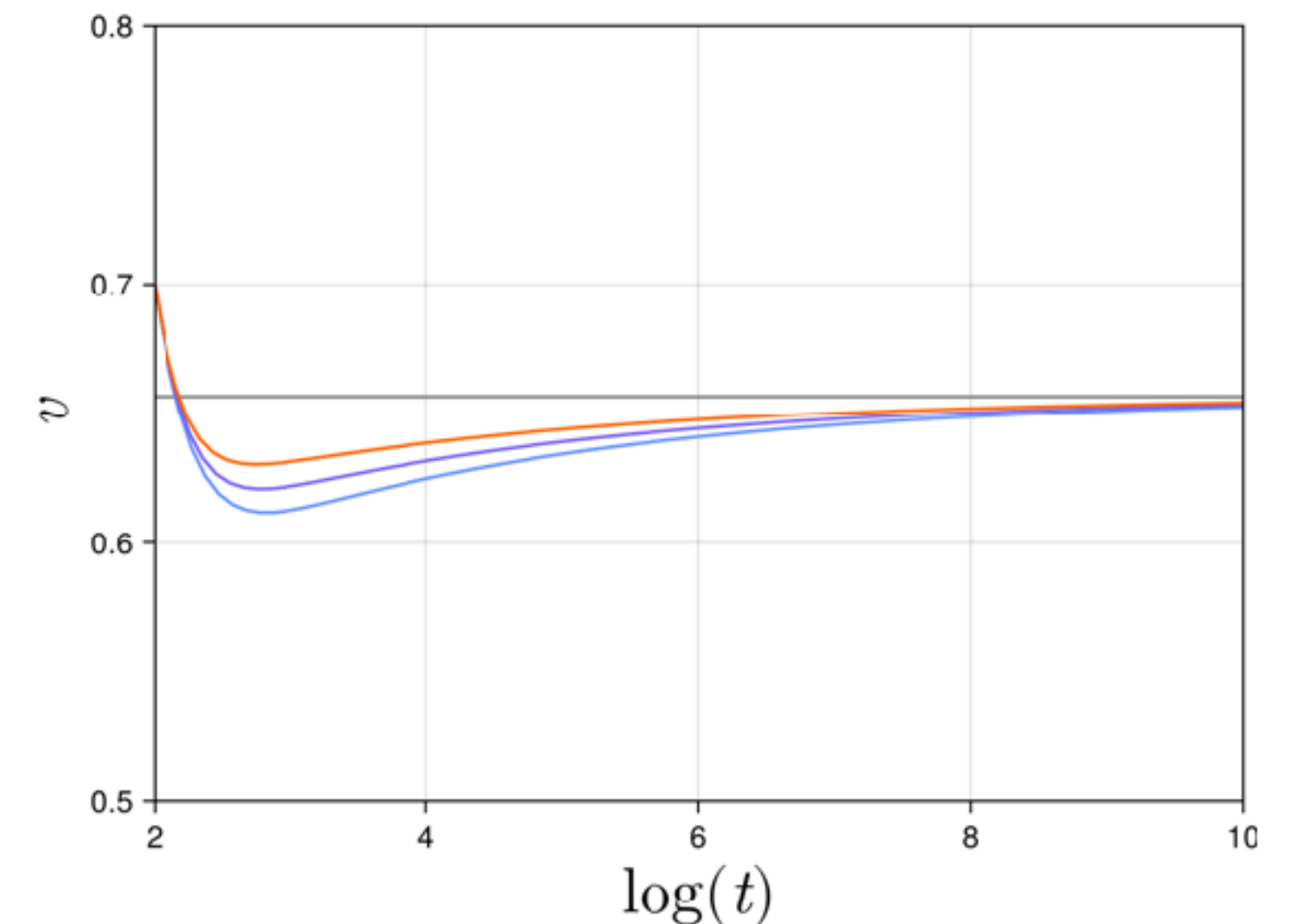
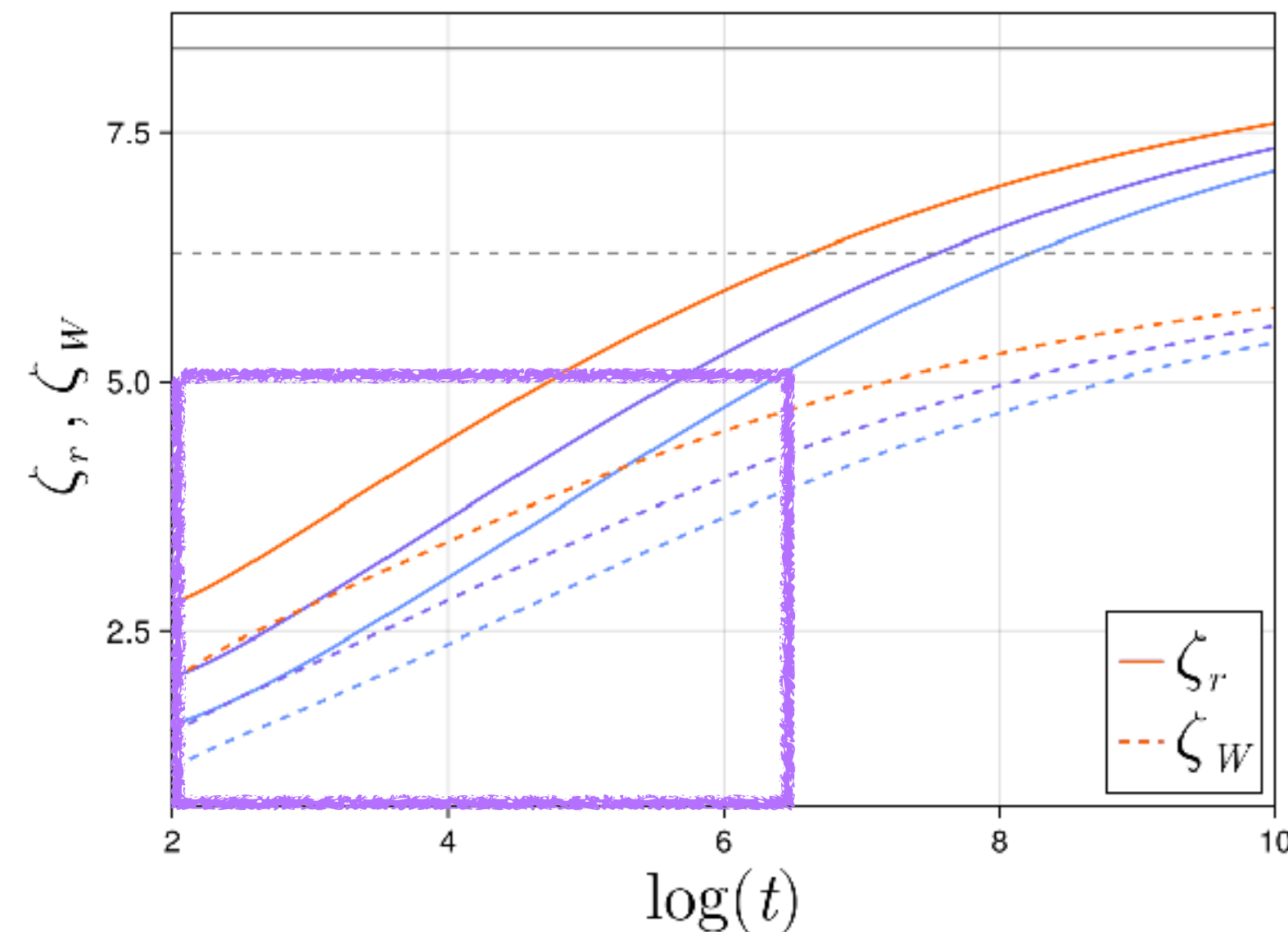
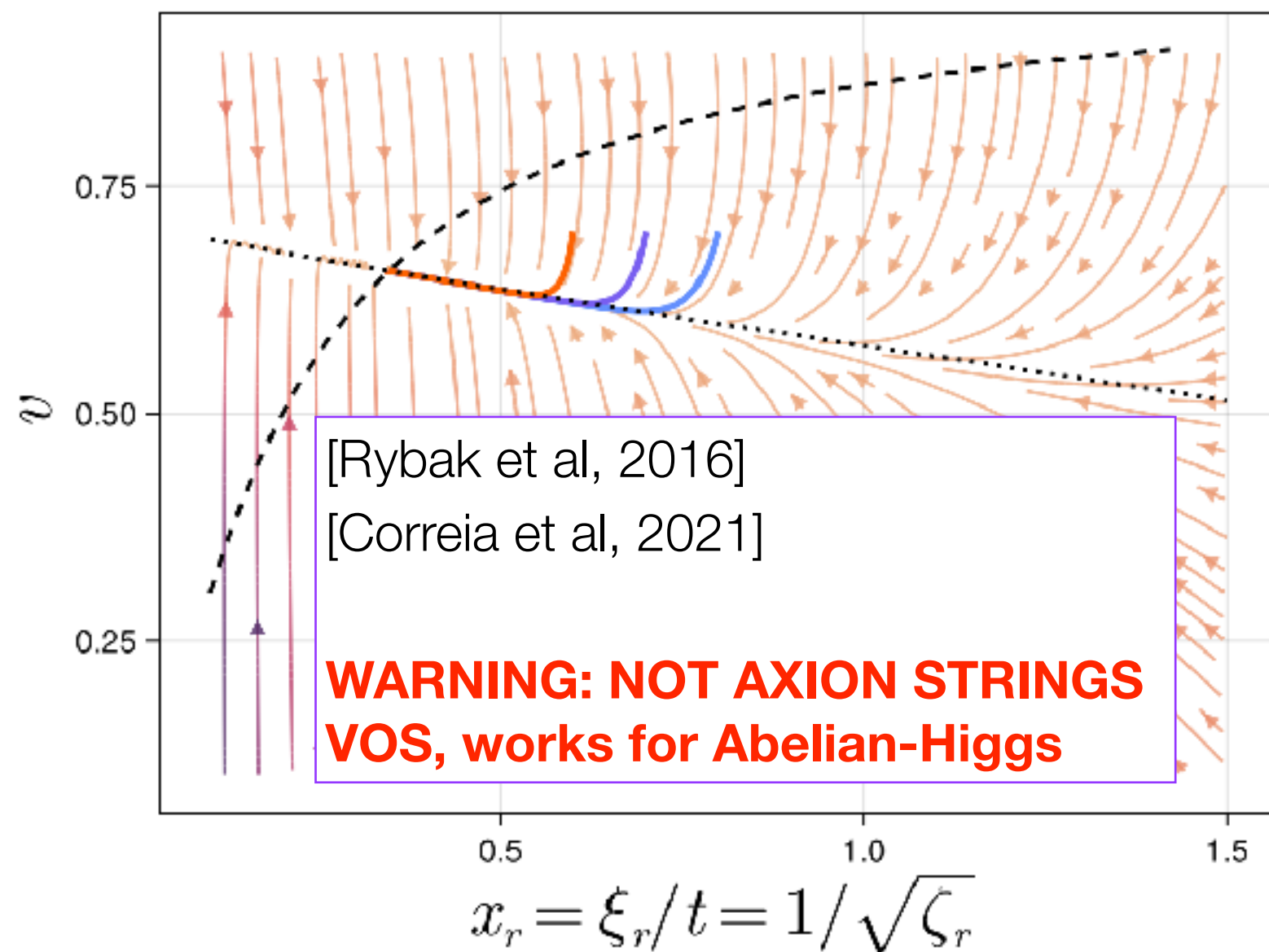
Scaling Violation or Slow approach to fixed point?

- The previous VOS dynamical system can always be recast into the following form (autonomous system, no dependence on $t \rightarrow$ follows from conservation laws),

$$\frac{dx}{d\Lambda} = f_x(x, v) \qquad \frac{dv}{d\Lambda} = f_v(x, v)$$

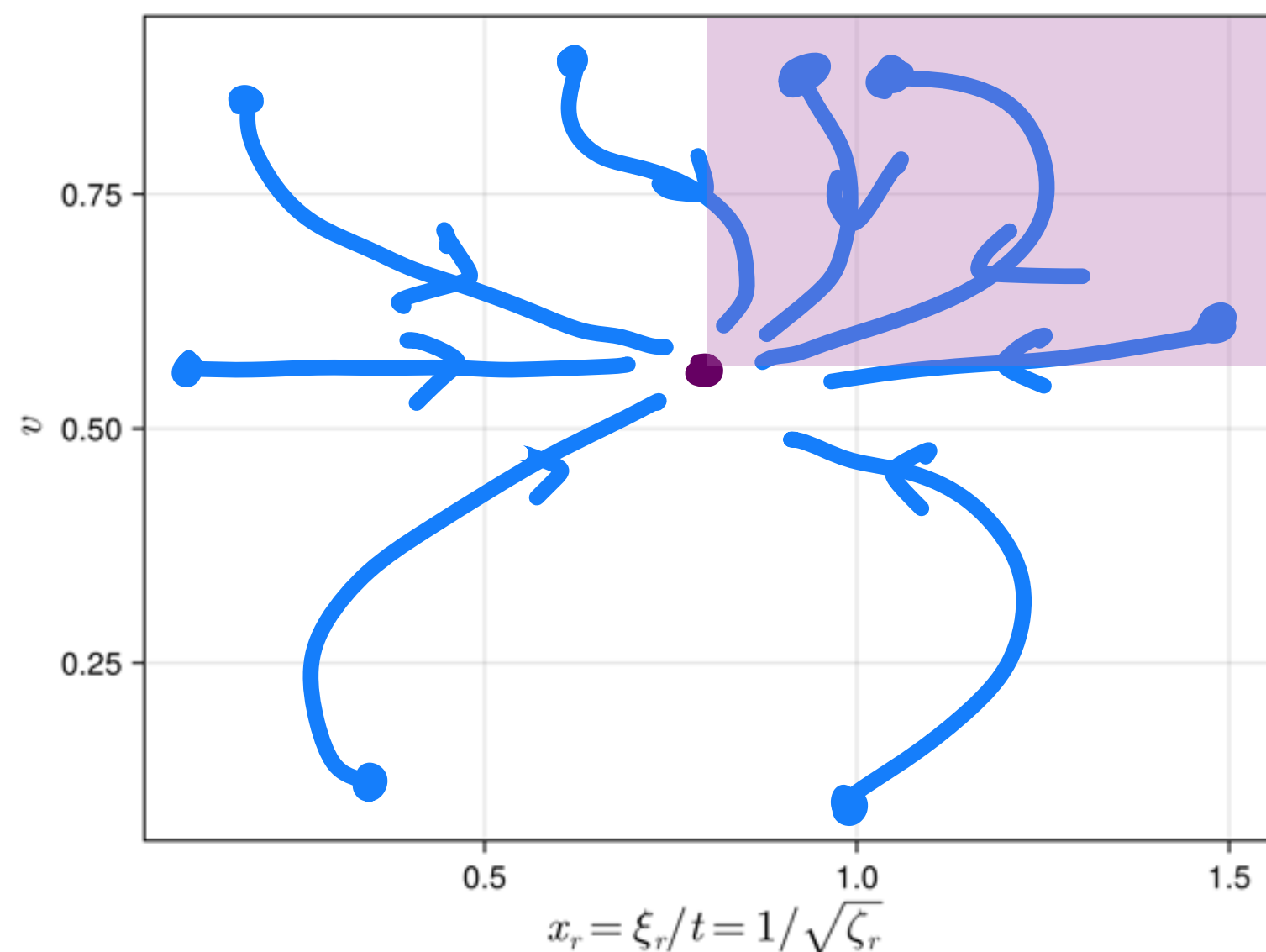
Where we have introduced $\Lambda = \log(t/t_0)$. This implies **no drift** of the fixed point.

- Non-spiral approach to a fixed point at **low** ζ near $dv/dt = 0$ null-cline can lead to similar behaviour to scaling violation.

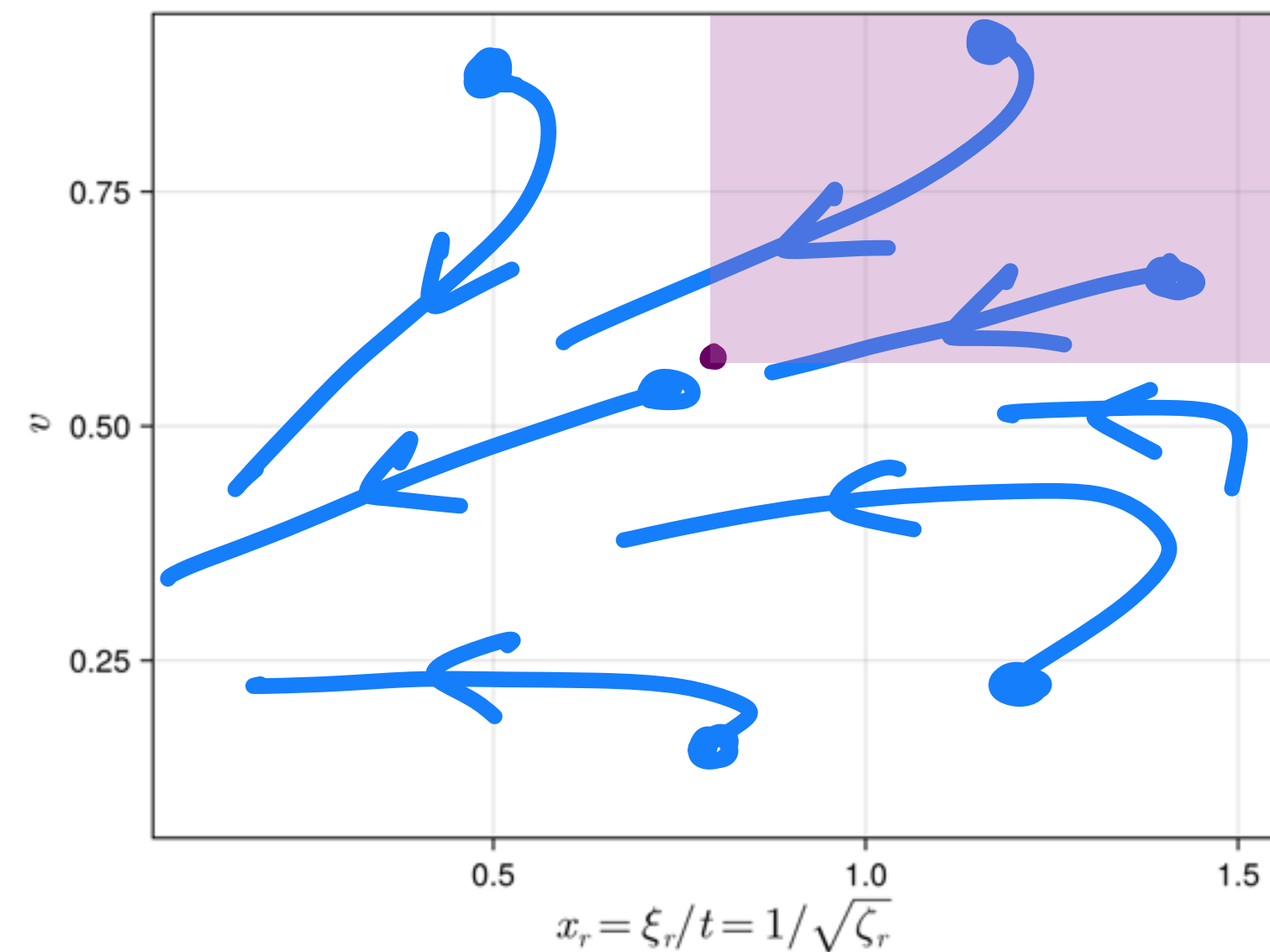


Scaling Violation or Slow approach to fixed point?

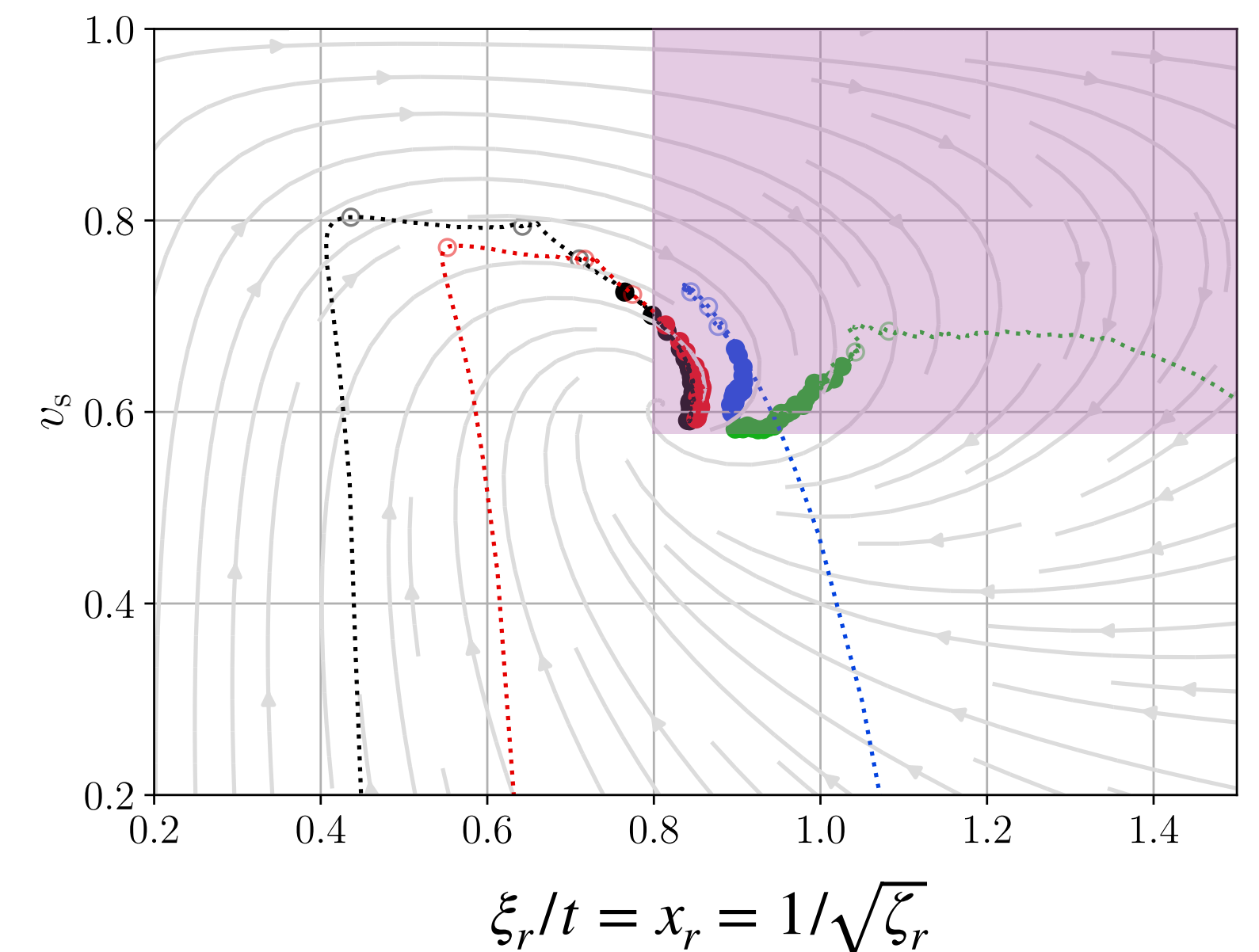
- Can we somehow fill the phase space to **distinguish** between a **fixed point** and a **scaling violation**?
- We should start with initial conditions in all four “quadrants” of the phase space (**high/low density** and **velocity**) and not only the purple part (high velocity, low density). Some work is ongoing on this.
- On the other hand a properly calibrated VOS model will shed on this issue too. Some parts of the phase space are harder to access than others. Calibration is a computationally expensive task [Martins et al, 2016; Correia & Martins, 2018...].



Non-spiral approach to fixed point, sketched



Scaling Violation, sketched



Simulation data [Hindmarsh, 2021]

Scaling Violation or Slow approach to fixed point?

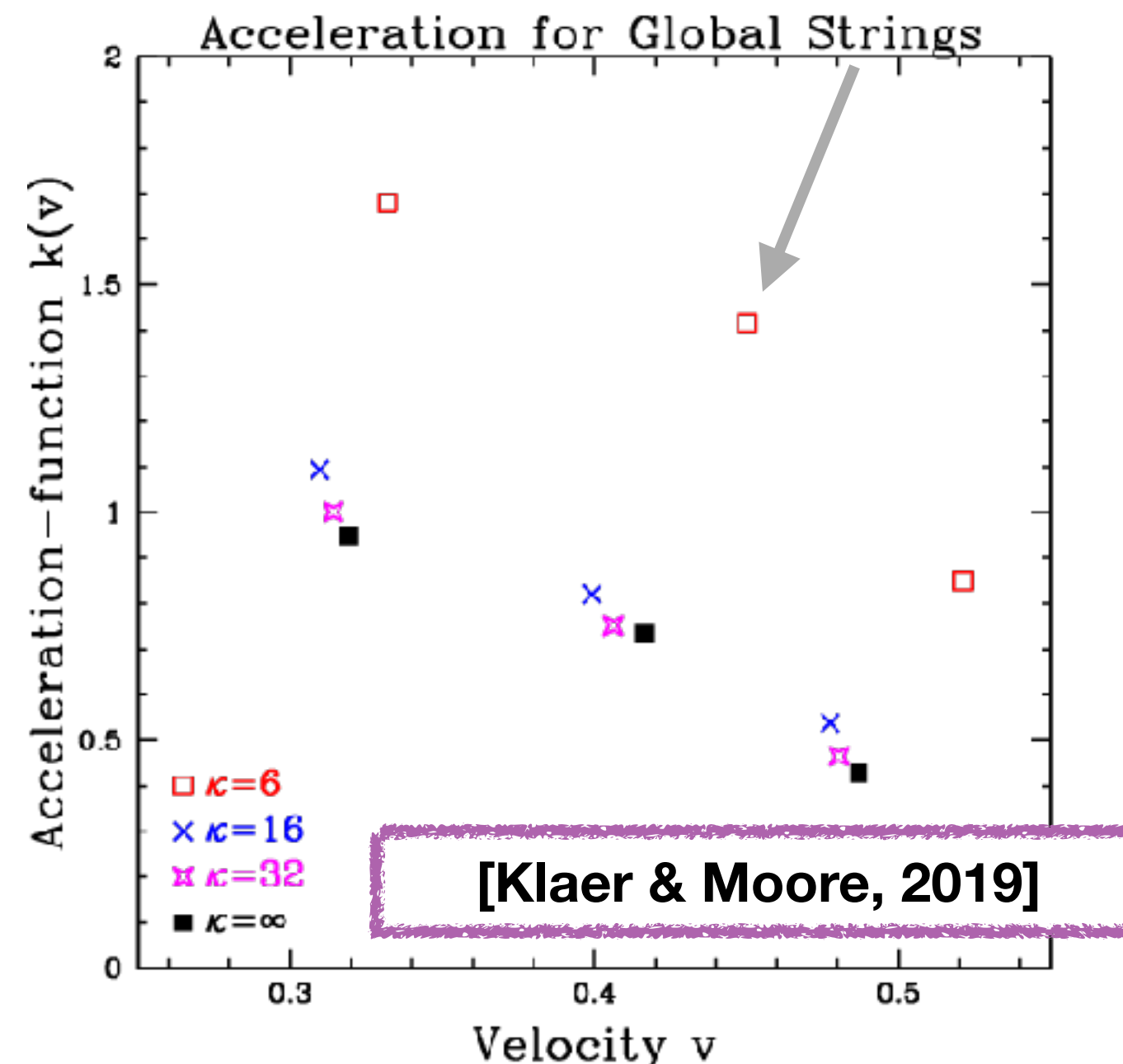
- Can we somehow fill the phase space to **distinguish** between a **fixed point** and a **scaling violation**?
- We should start with initial conditions in all four “quadrants” of the phase space (**high/low density** and **velocity**) and not only the purple part (high velocity, low density). Some work is ongoing on this.
- On the other hand a properly calibrated VOS model will shed on this issue too. Some parts of the phase space are harder to access than others. Calibration is a computationally expensive task [Martins et al, 2016; Correia & Martins, 2018...].

Radiative model of global strings [C. J. A. P. Martins et al, 2000]

Long strings

$$2\frac{d\xi}{dt} = 2H\xi(1 + v^2) + cv + \frac{sv^6}{\ln(L/\delta)}$$

$$\frac{dv}{dt} = \left(1 - v^2\right) \left(\frac{k(v)}{\xi} - 2Hv\right)$$



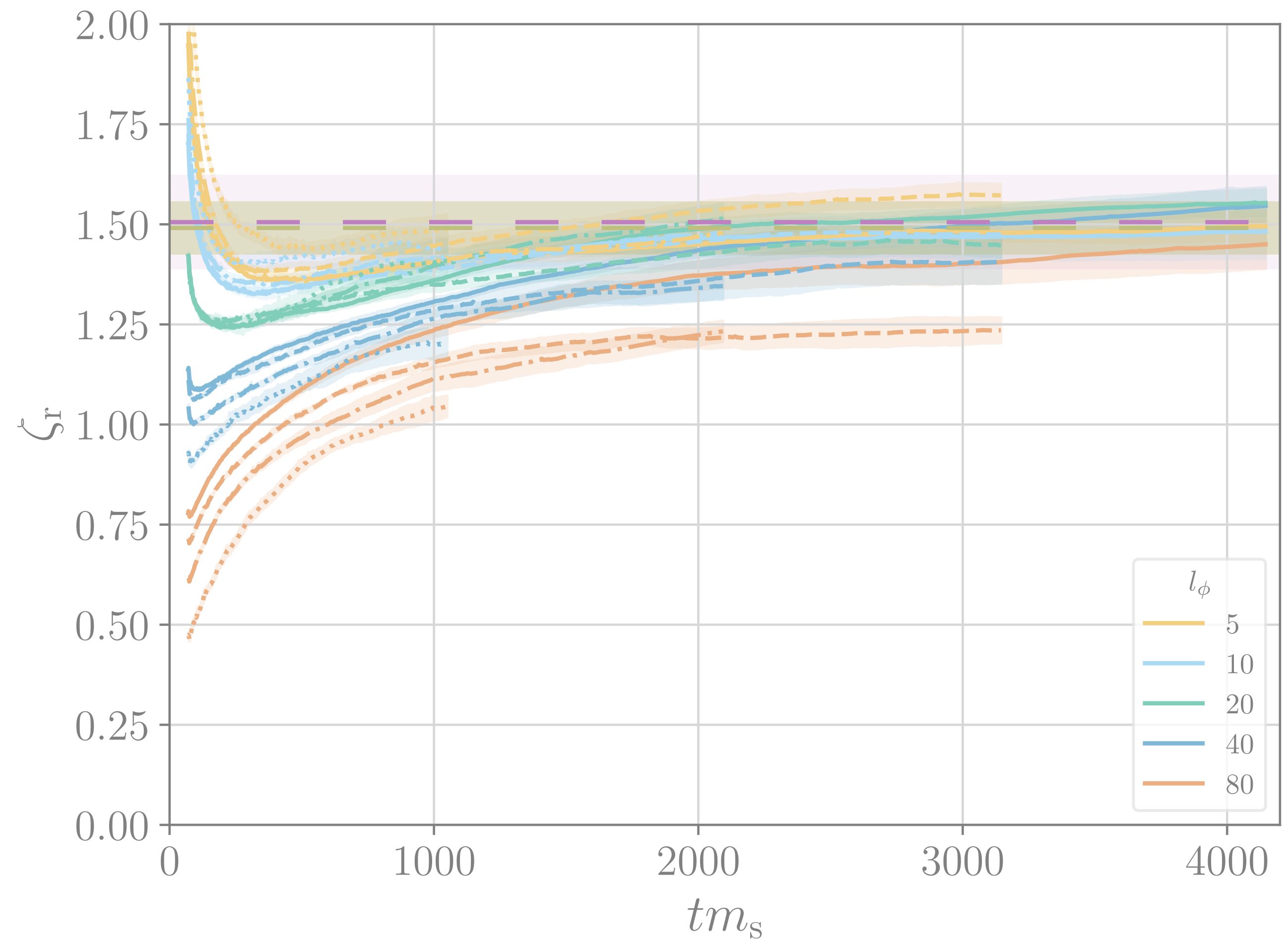
Tension model of [Yamaguchi, 2005] [C. J. A. P. Martins, 2018]

$$2\frac{d\xi}{dt} = 2H\xi(1 + v^2) + cv + \frac{L}{\mu} \frac{d\mu}{dt}$$

$$\frac{dv}{dt} = \left(1 - v^2\right) \left(\frac{k(v)}{\xi} - 2Hv\right)$$

The Scaling Density of Axion Strings in Terasite Simulations

arXiv:2410.18064, PhysRevD 111 063532
CSC project: 462000084



Simulations setup: initial conditions

- We report on **16384³** fixed grid simulations performed using HILA (<https://github.com/CFT-HY/HILA>), ran at **LUMI** (8th at TOP500),

$$\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} - \nabla^2\phi = -a^2\lambda(\phi^T\phi - \eta^2)\phi$$

- “Initial conditions don’t matter** - string networks will be driven towards an attractor”. Yes, but **dynamic range is not infinite**. It is always possible one never quite reaches this attractor.

- Gaussian random field with different initial correlation l_ϕ lengths at conformal time $\tau = 50$, diffusion until $\tau_{diff} = 70$
- “core growth” (also known as conformal strings) until τ_{cg} , after that radiation epoch evolution until half-a-light crossing time

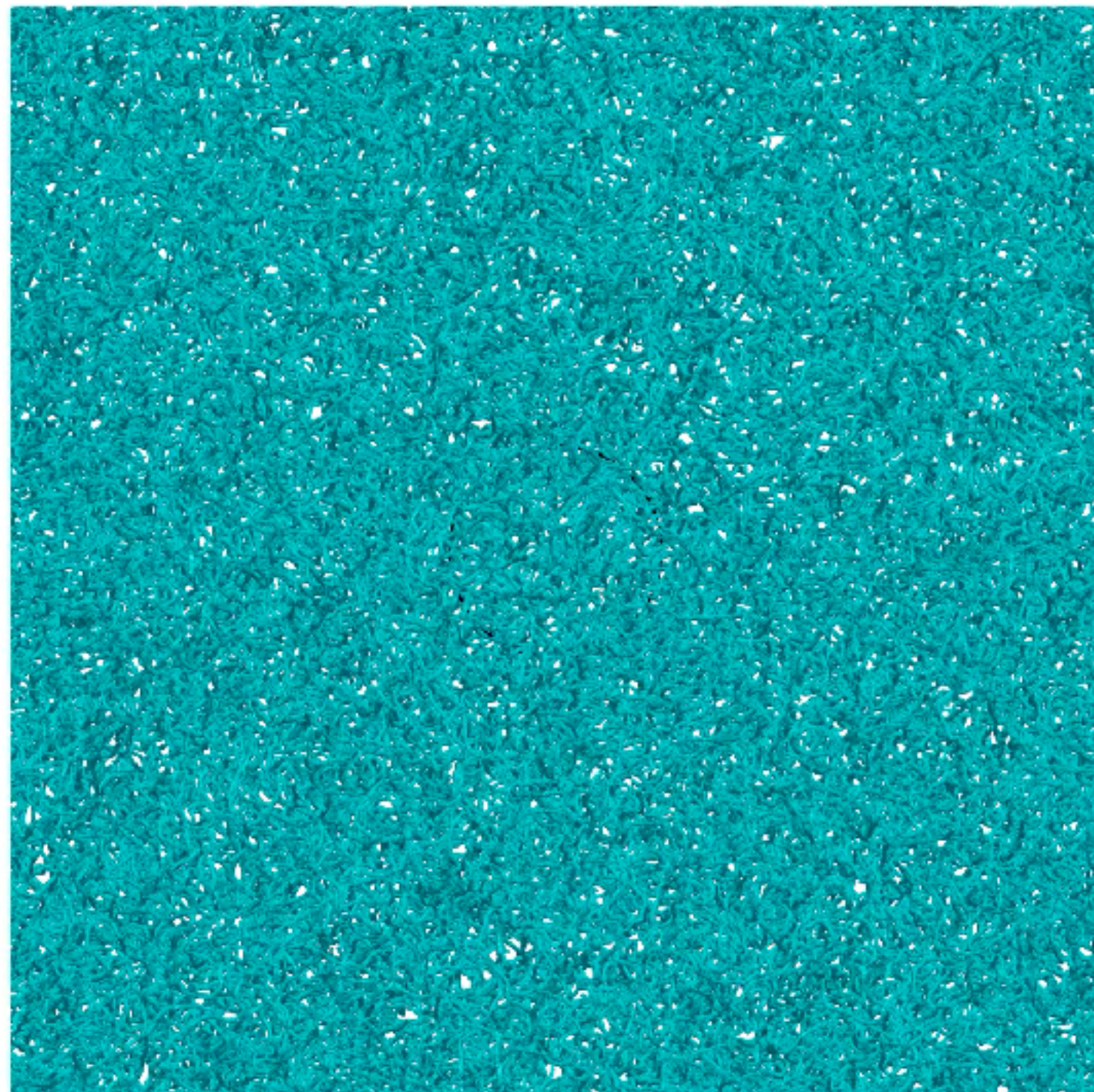


Box size	Seeds	Runs	l_ϕ
4k	9	45	5,10,20,40,80
8k	9	45	5,10,20,40,80
12k	6	30	5,10,20,40,80
16k	8	56	5,10,20,40,80,160,320

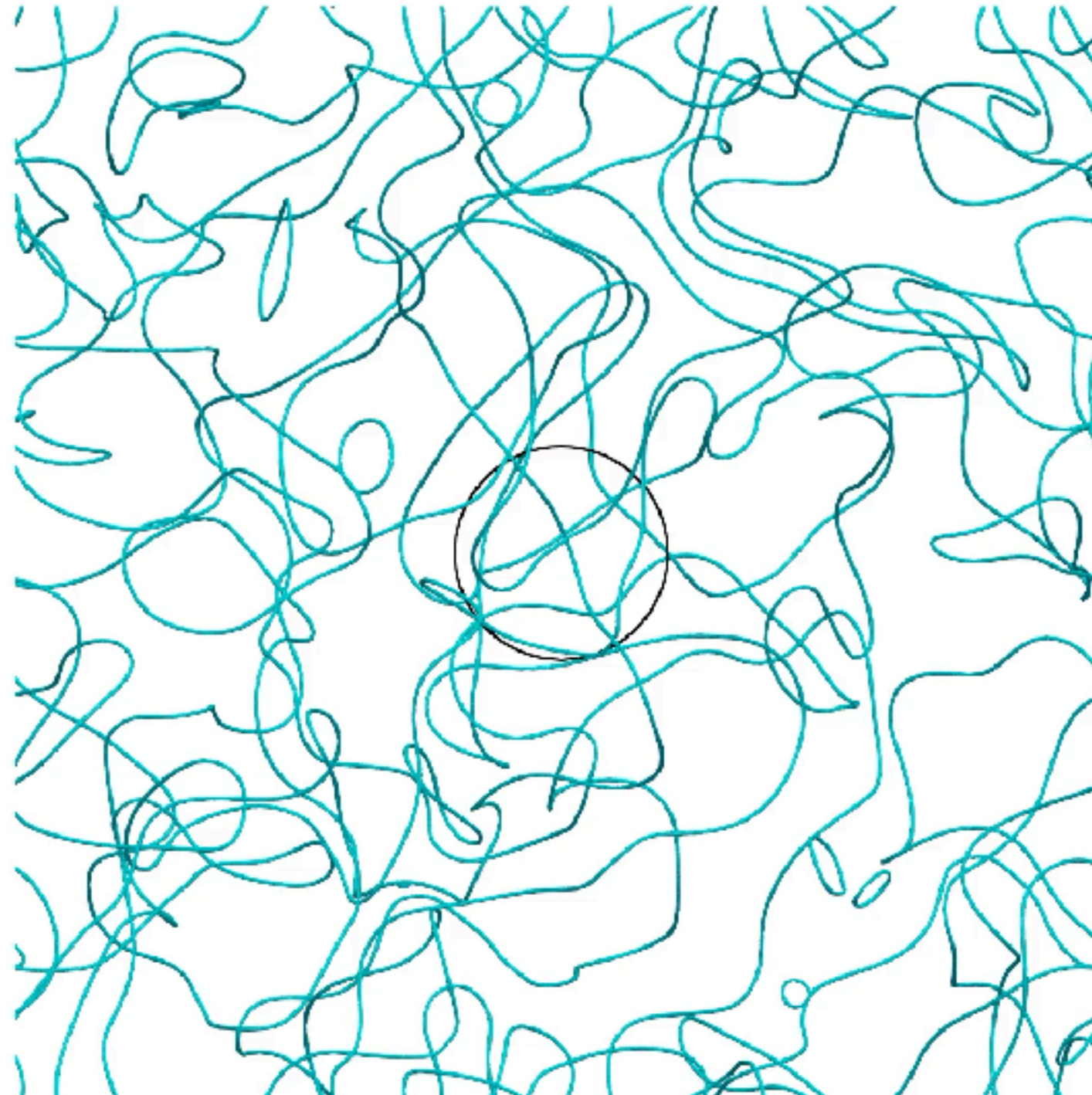
Simulations setup: initial conditions

- Visually it is obvious that low initial density can lead to a state where the number of strings per horizon is lower than the fixed point value $\zeta_W \leq 1.2$ (comoving version $\zeta_c \approx 4$).
- Compare $l_\phi = 5$ on the left-hand-side and $l_\phi = 80$ on the right-hand-side

$\tau/L : 0.10$ $tm_s : 18$



$\tau/L : 0.10$ $tm_s : 18$



- $|\phi| = 0.6$ isosurfaces
- Circle proportional to conformal time τ
- Order four strings per sphere of comoving Hubble radius ($\zeta_c \approx 4$)

Simulations setup: observables

- Understanding and interpreting the evolution of this dynamical system, requires measuring both **string length** and **velocity**
- To explain a bit more our estimators. Take then straight static string boosted with some rms velocity $v^2 = \langle \dot{X}^2 \rangle \dots$

$$\Pi(x, t) = \gamma \dot{X} \cdot \partial \phi(x_s)$$

$$\partial \phi(x, t) = \gamma \hat{v} (\hat{v} \cdot \partial \phi_s) + \partial^\perp \phi(x, t)$$

where $\gamma = 1/\sqrt{1 - \dot{X}^2}$, $\hat{v}$ is a velocity unit vector and $D^\perp \phi(x, t) = (\delta_{ij} - \hat{v}_i \hat{v}_j) \partial_j \phi(x, t)$

- The field energy will be proportional to rest frame string length

$\mu_{\mathcal{W}} \rightarrow$ wighted energy per unit length

$f_{V,\mathcal{W}} \rightarrow$ fraction contributed by the weighted potential energy density

$$E = E_\pi + E_D + E_V = \mu_{\mathcal{W}}(1 - f_{V,\mathcal{W}}v^2)l_r.$$

$$L = E_\pi - E_D - E_V = -\mu_{\mathcal{W}}(1 - v^2)l_r$$

- Then for length measures and velocity estimator,

$$\ell_w = \frac{2}{3}n_p \Delta x \quad \ell_r = \frac{E + f_{V,\mathcal{W}}L}{\mu_{\mathcal{W}}(1 - f_{V,\mathcal{W}})} \quad v_L^2 = \frac{E - L}{E + f_{V,\mathcal{W}}L}$$

Simulations setup: observables

- Understanding and interpreting the evolution of this dynamical system, requires measuring both **string length** and **velocity**
- Measure string length, in both **Universe frame** (left, other groups do this only) and **string rest-frame** (right, related to **energy density, what VOS predicts**):

$$\ell_w = \frac{2}{3} n_p \Delta x$$

$n_p \rightarrow$ number of plaquettes pierced by strings

$$\ell_w = \ell_r \langle \gamma^{-1} \rangle$$



$$\ell_r = \frac{E + f_{V,W} L}{\mu_W (1 - f_{V,W})}$$

$\mu_W \rightarrow$ wighted energy per unit length

$f_{V,W} \rightarrow$ fraction contributed by the weighted potential energy density

$E \rightarrow$ Energy

$L \rightarrow$ Lagrangian

- String separation $\xi = \sqrt{\frac{V}{l}}$ and the related $x = \xi/t$

String density parameter $\zeta = \frac{lt^2}{a^2 V} = \frac{1}{x^2}$.

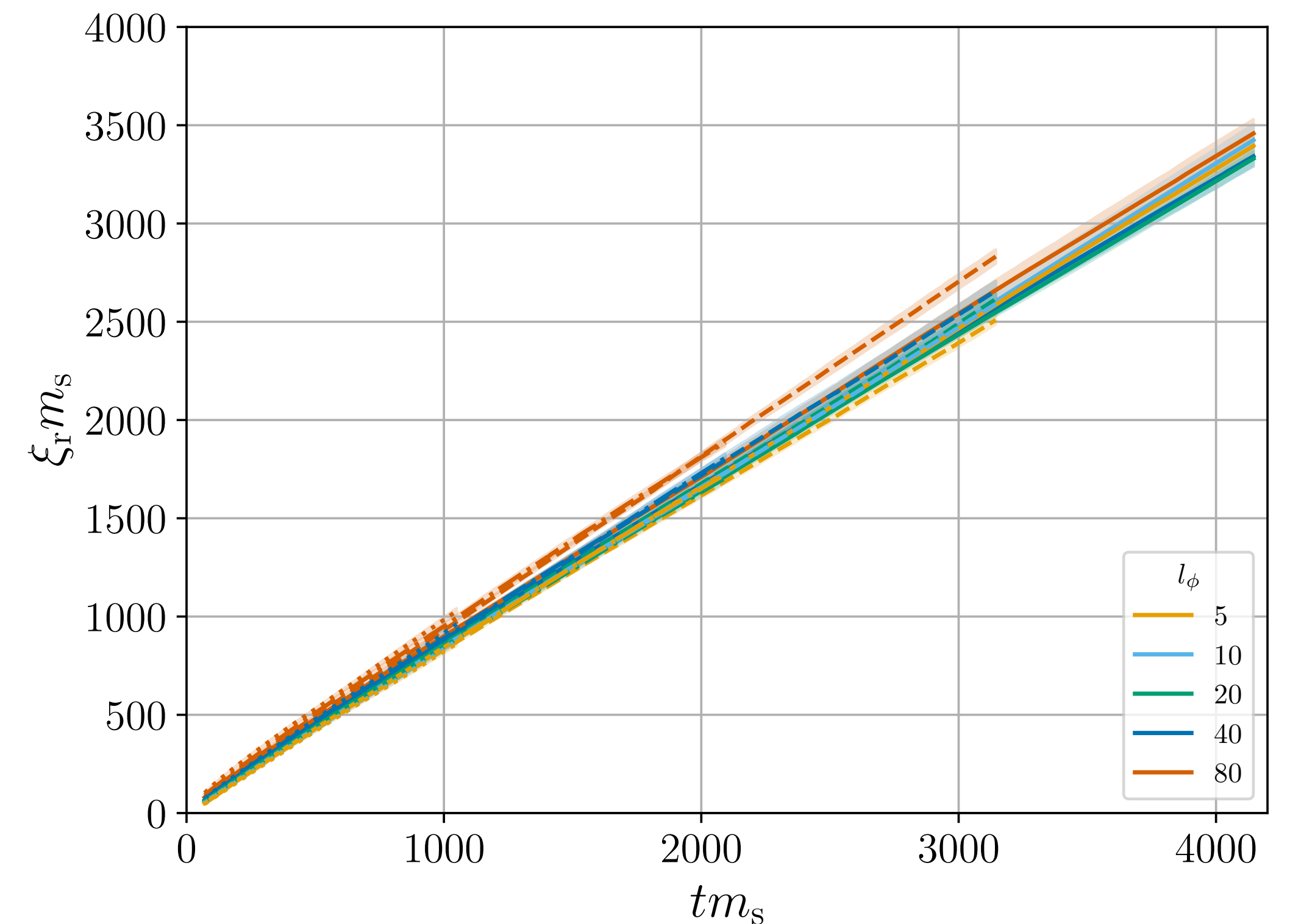
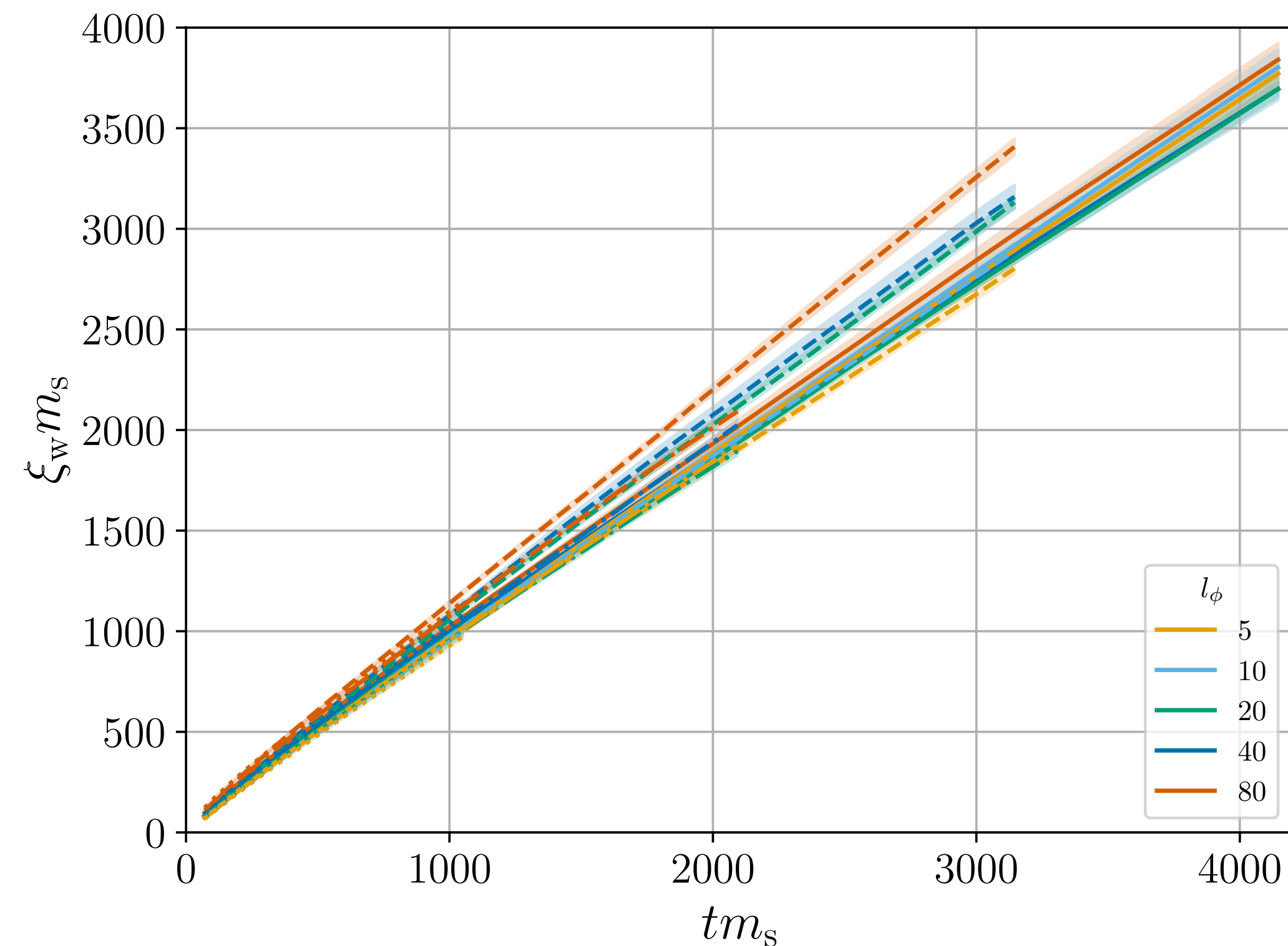
- Some groups measure **velocity**, although not all:

$$v_L^2 = \frac{E - L}{E + f_{V,V} L}$$

Important **consistency check**: $\zeta_r \gamma^{-1} \approx \zeta_W$

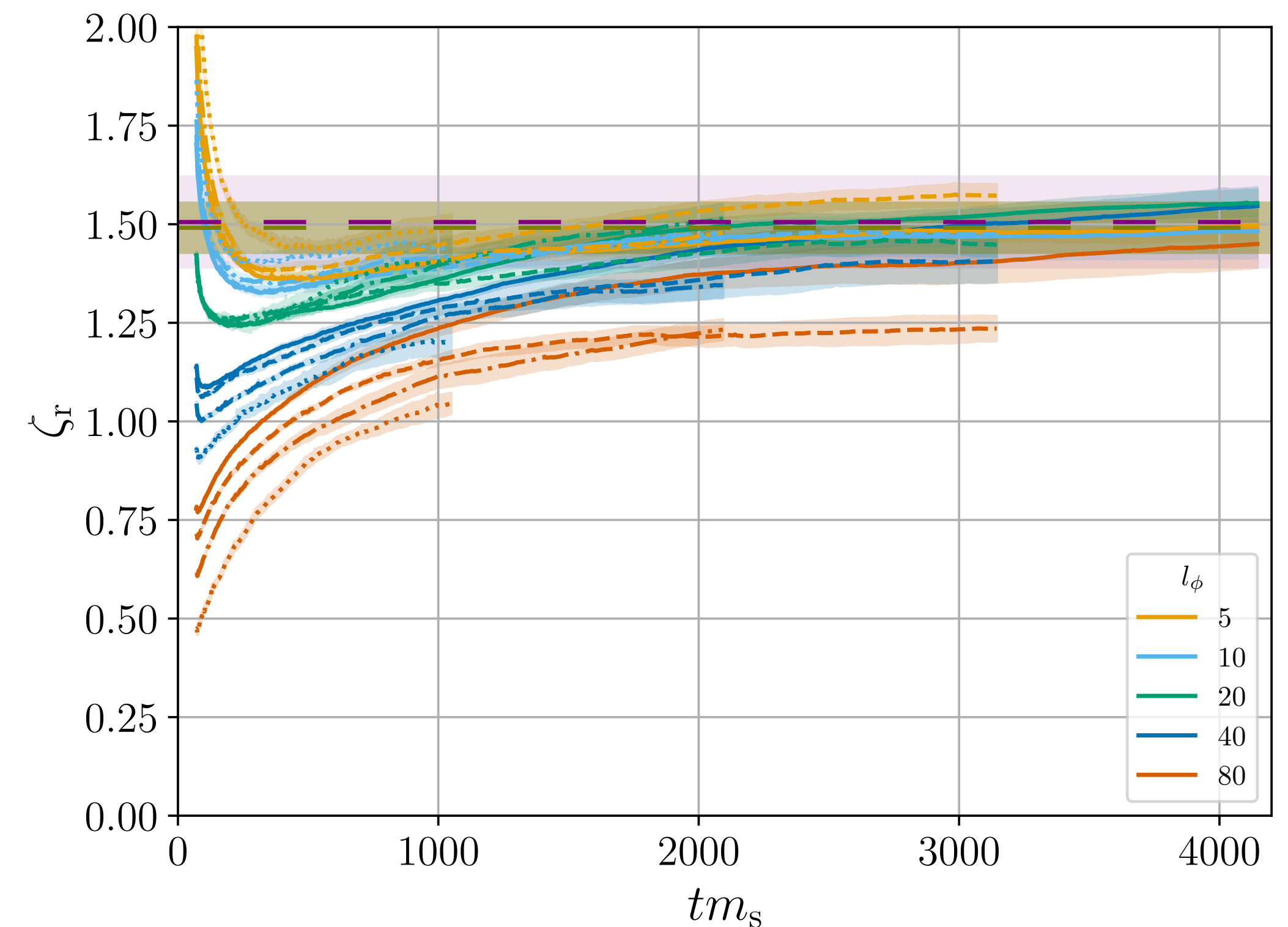
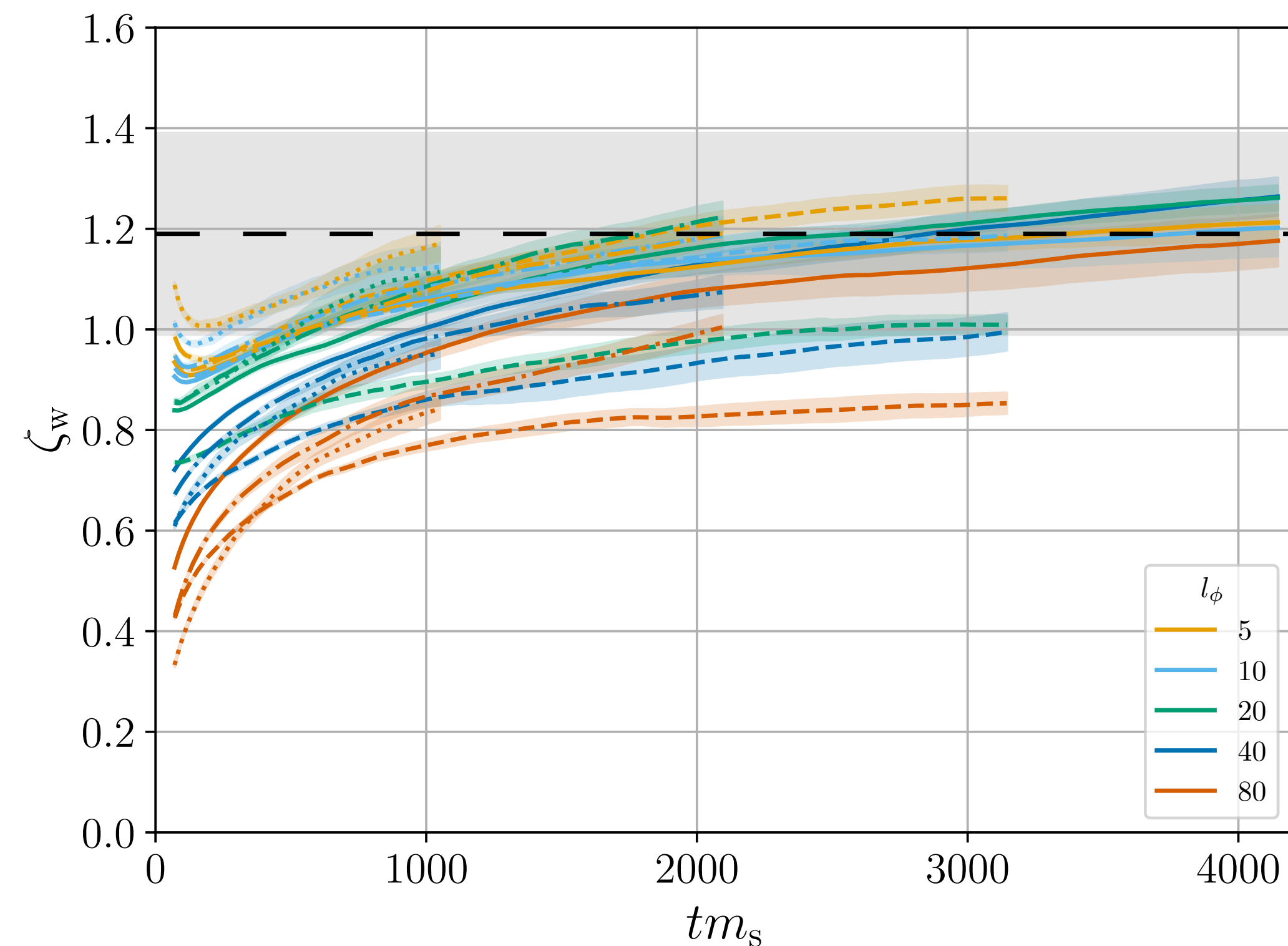
Standard Scaling Analysis: Time Series Data

- From looking at string separation, one does **not** see any **evidence** of **deviations** from **standard scaling**
- Here it looks like using either **Universe frame** or **rest-frame** measures of length is 100% **inconsequential**...



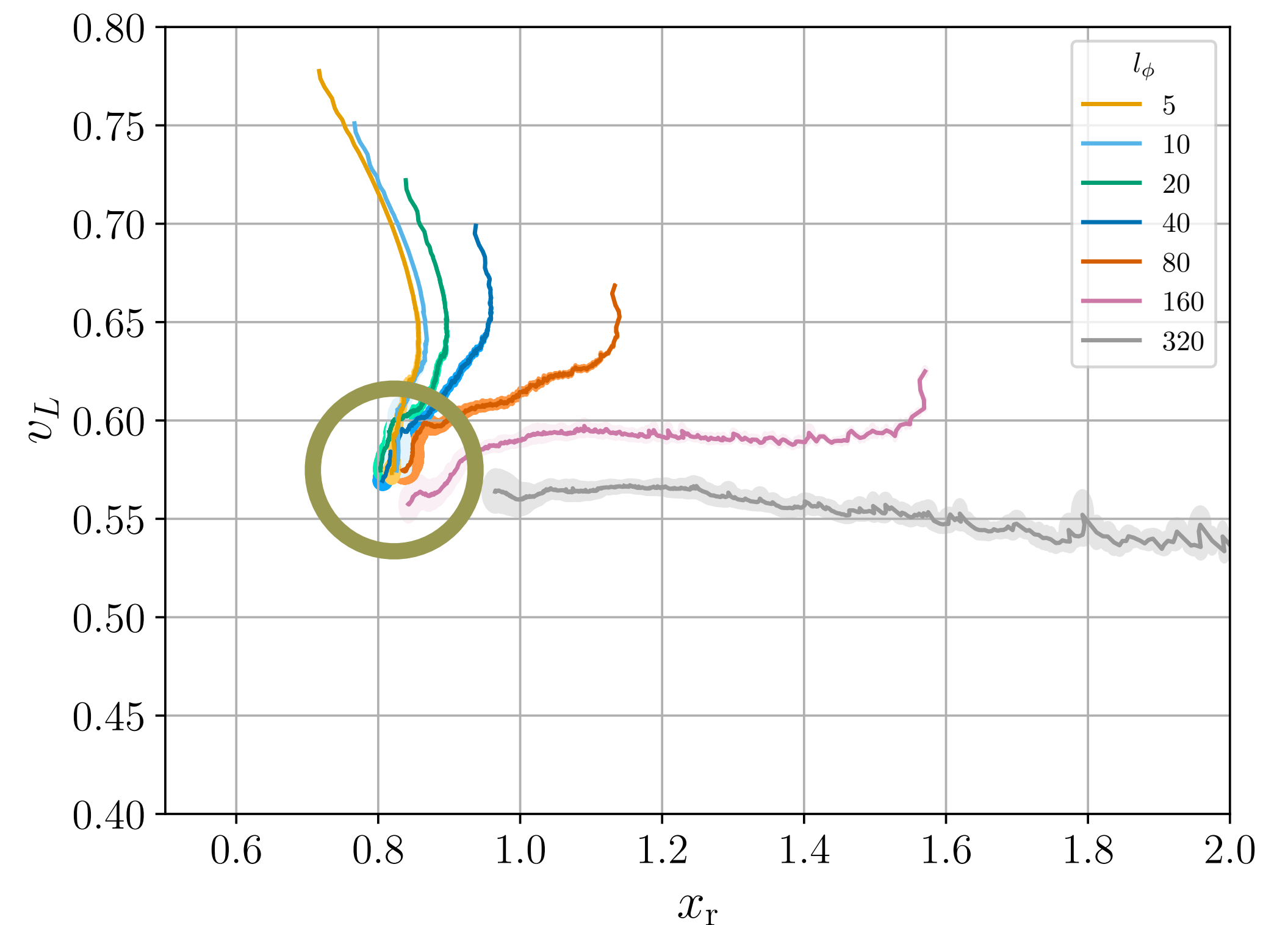
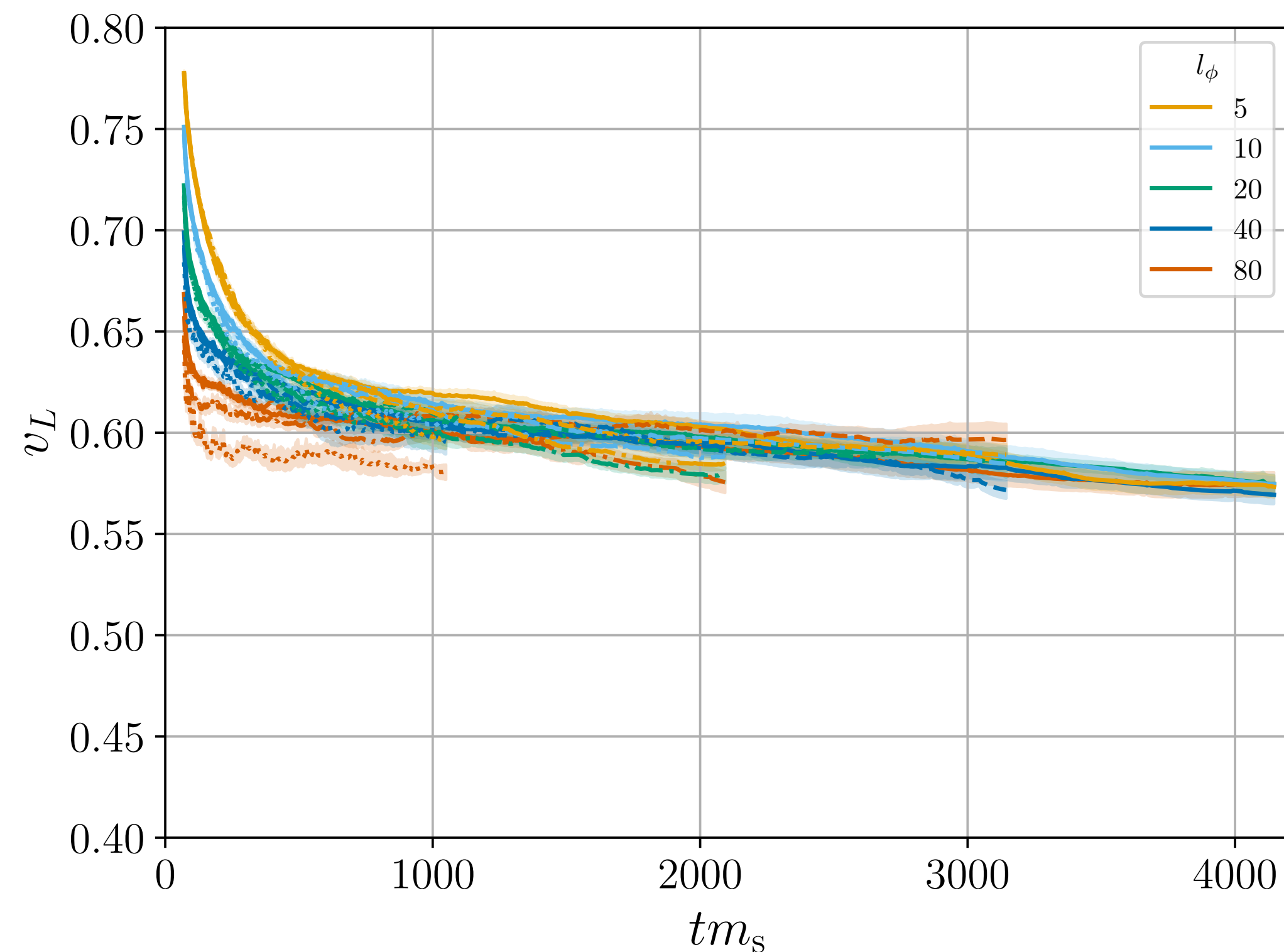
Standard Scaling Analysis: Time Series Data

- We remind the audience that other groups only measure ζ_W , but not ζ_r (and most of the times not v)
- It seems there is a difference between **Universe frame** and **rest-frame** string density parameters. Now we must look at **velocities**. **Rest-frame converges** to constant - depending on initial condition and dynamic range.



Standard Scaling Analysis: Time Series Data

- **Velocity** is still **evolving**. Although **timid**, it is sufficient to take make the length contracted $\zeta_W = \gamma^{-1}\zeta_r$ evolve as well
- The phase space plot (showing only 16k) is quite informative too:
 - **Null-clines** at $dv/dt = 0$ and $dx_r/dt = 0$ explain how to interpret evolution and their crossing leads to a fixed point

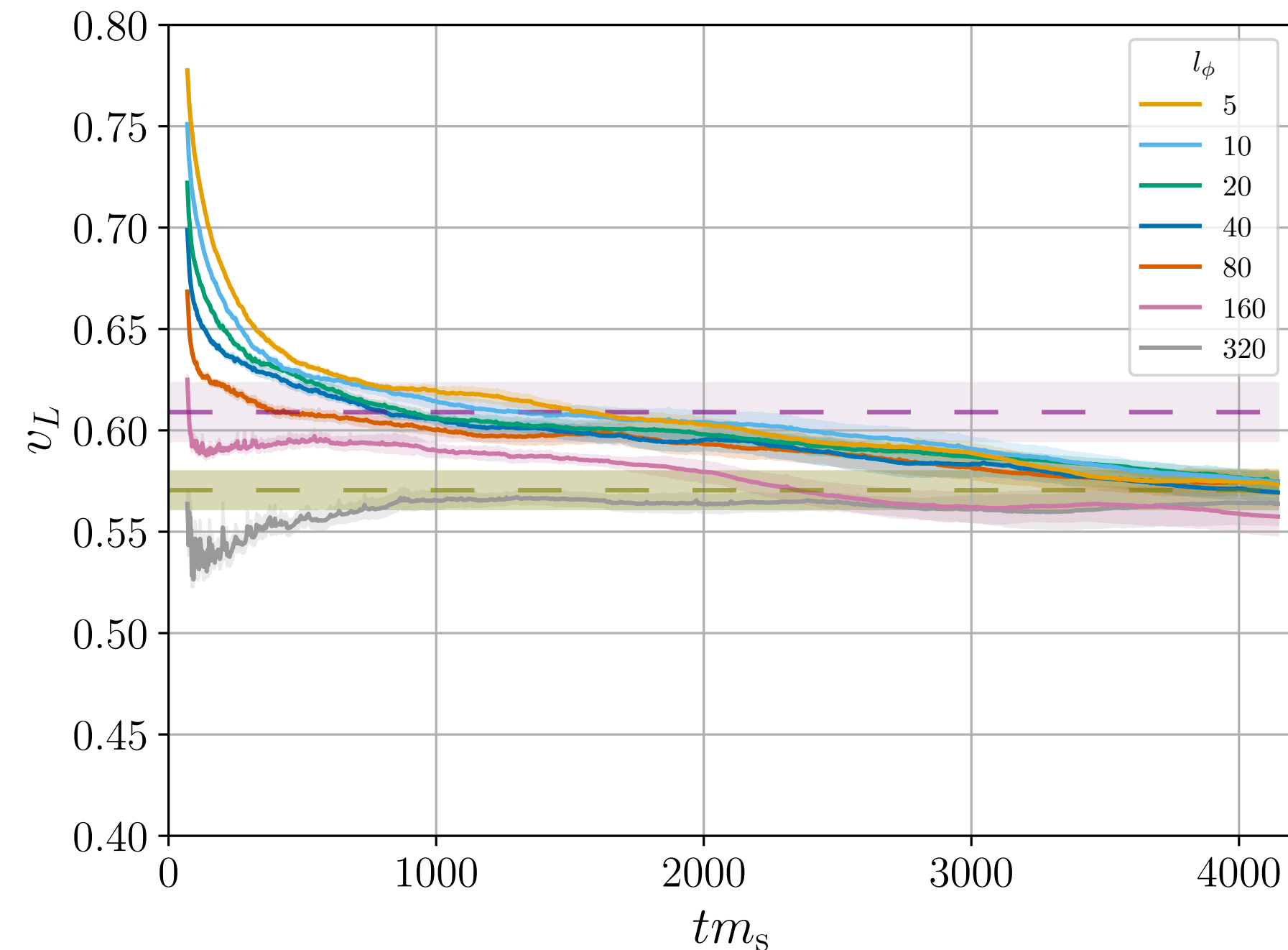
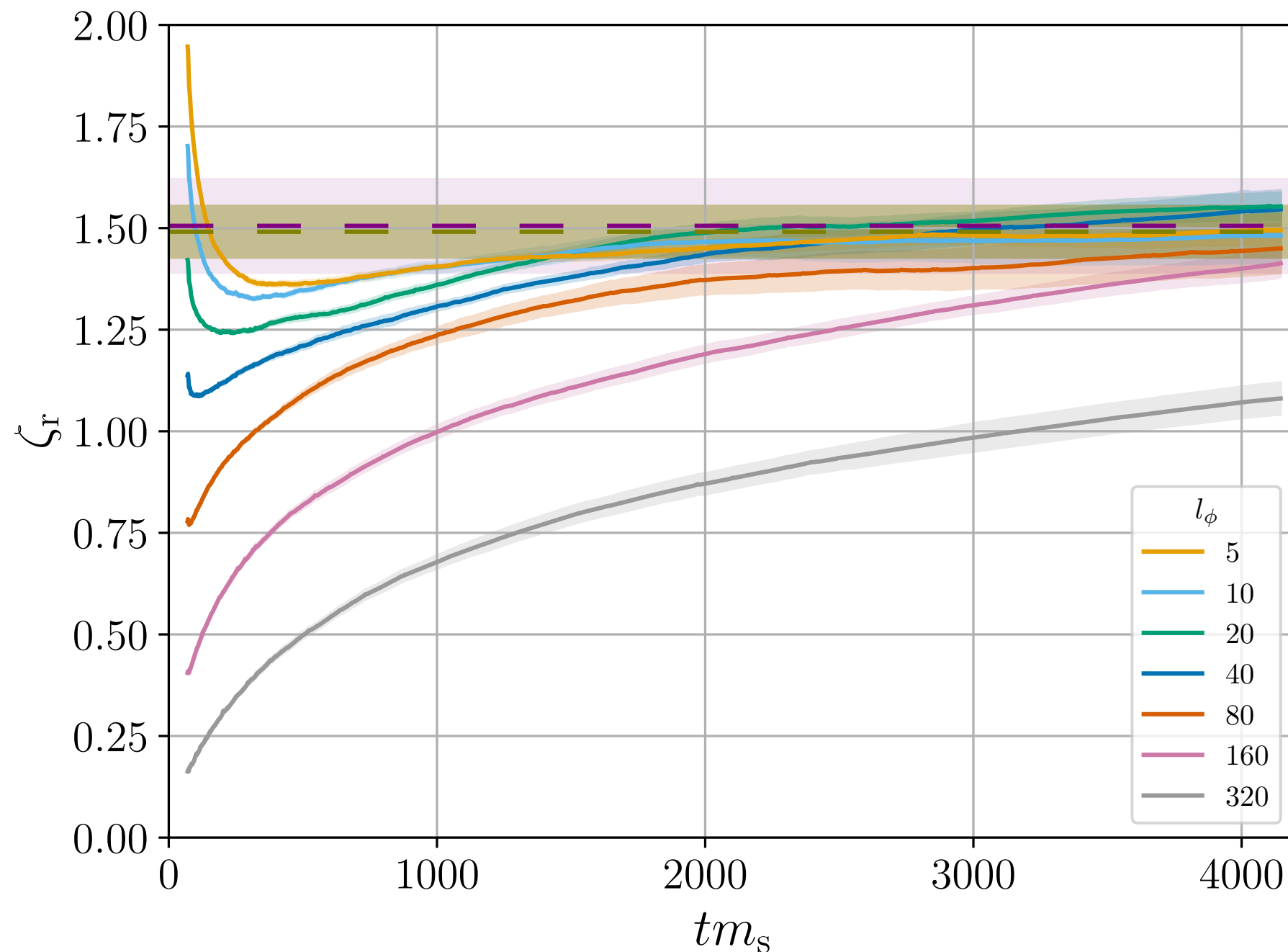


Standard Scaling Analysis: Main result

- We observe **convergence** of **rest-frame** string density parameter, ζ_r . **Well-predicted** by the naive (c, k) VOS!
- Velocity** is **under predicted** by the simple VOS roughly by 5% $\rightarrow$ we need a more **complete model**.
 - We therefore find it justified to extrapolate from standard scaling:

$$\hat{\zeta}_{r,*} = 1.491(93) \quad \text{and} \quad \hat{v}_{L,*} = 0.5705(93)$$

$$\hat{\zeta}_{w,*} = 1.220(57) \quad \quad \quad \max(\zeta_w) = 1.46$$



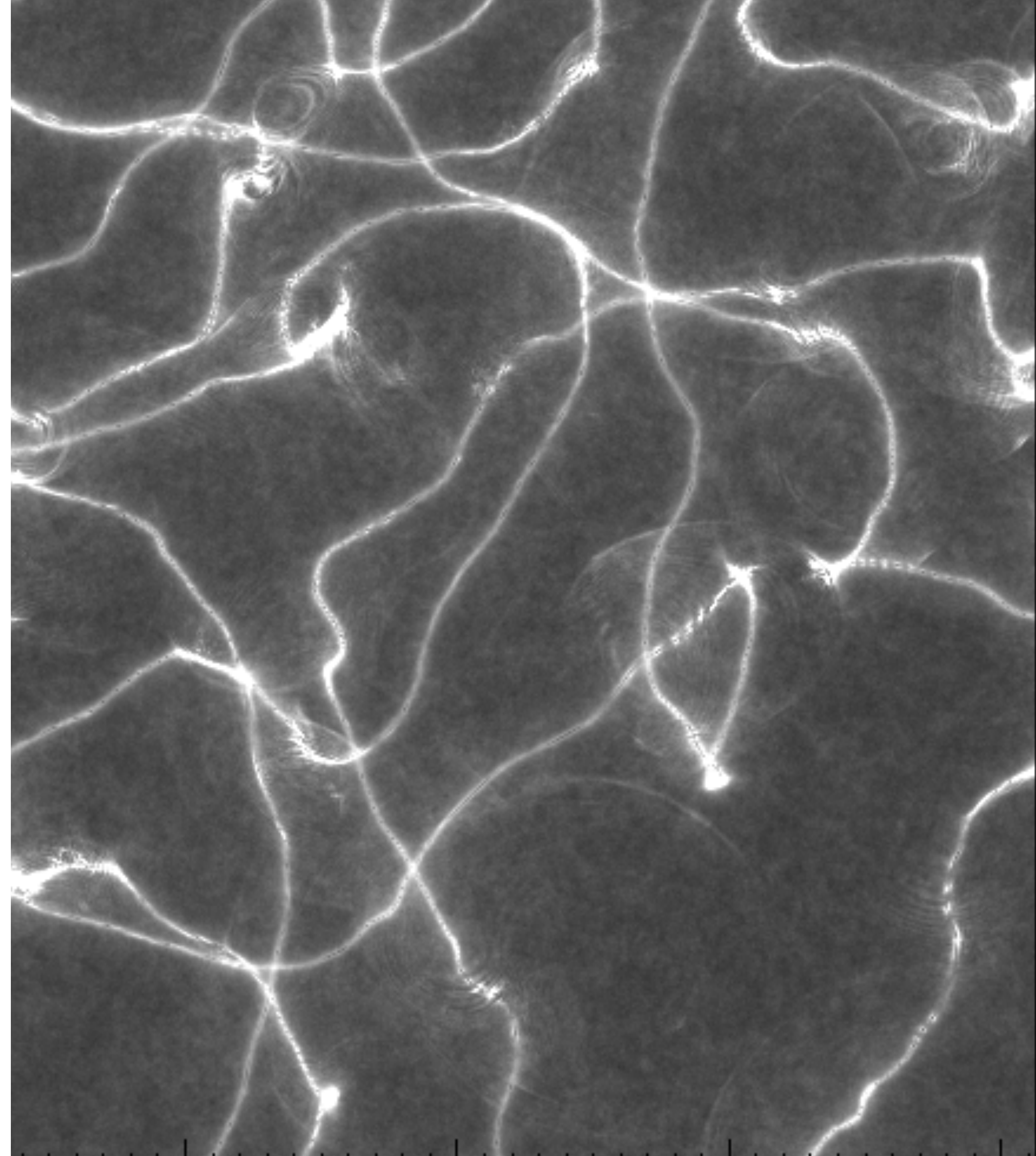
NOTE:

$\hat{\xi}_w \gamma(\hat{v}_L)$ **coincides** with $\hat{\xi}_r$.
Convergence after boost!

l_ϕ	$\hat{\zeta}_w$	$\hat{\zeta}_r \gamma^{-1}(\hat{v}_L)$
5	1.211(34)	1.214(24)
10	1.200(56)	1.202(46)
20	1.258(24)	1.259(21)
40	1.259(37)	1.257(30)
80	1.172(51)	1.174(42)
160	1.156(34)	1.156(24)
320	0.882(32)	0.880(34)

Conclusions & Next Steps

"The report of my death was an exaggeration"
- Mark Twain / Standard scaling



Conclusions

- We have, for the first time, observed **convergence of** ζ_r for simulations with $5 \leq l_\phi \leq 20$
- **RMS velocity convergence** observed in $l_\phi = 320$

- Our results support **standard scaling** (5% accuracy) with,

$$\zeta_{r,*} = 1.491(93) \qquad \hat{\zeta}_w = 1.220(57) \qquad v_{L,*} = 0.5705(93)$$

- Evolution in ζ_w due to two factors (**two null-clines**):

x_r at low initial string densities

v_L at higher initial densities

- ζ_r and $\zeta_w \gamma$ values **agree** at the end of the simulations:

suggesting that the **universe length** is **close** to **converging** too

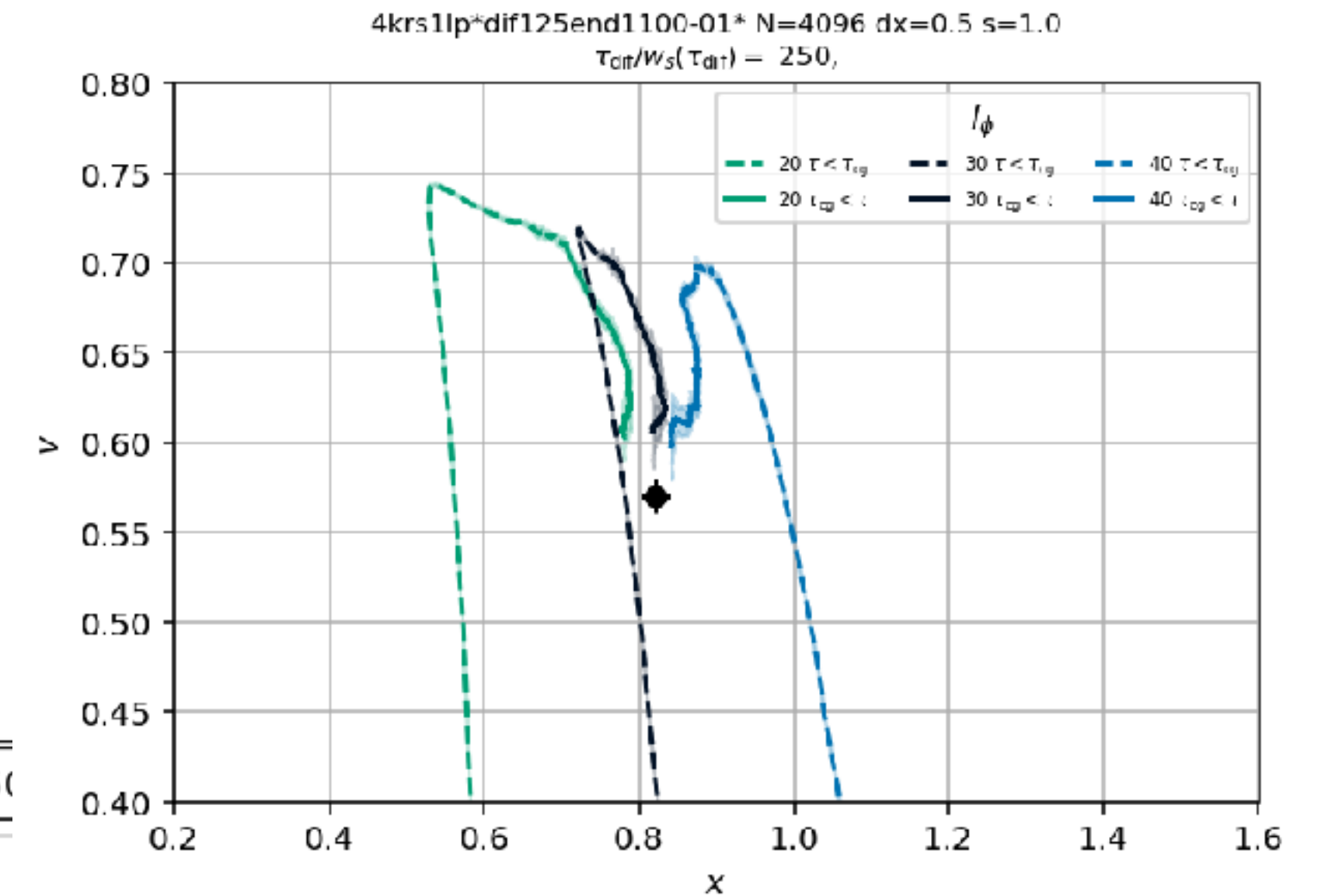
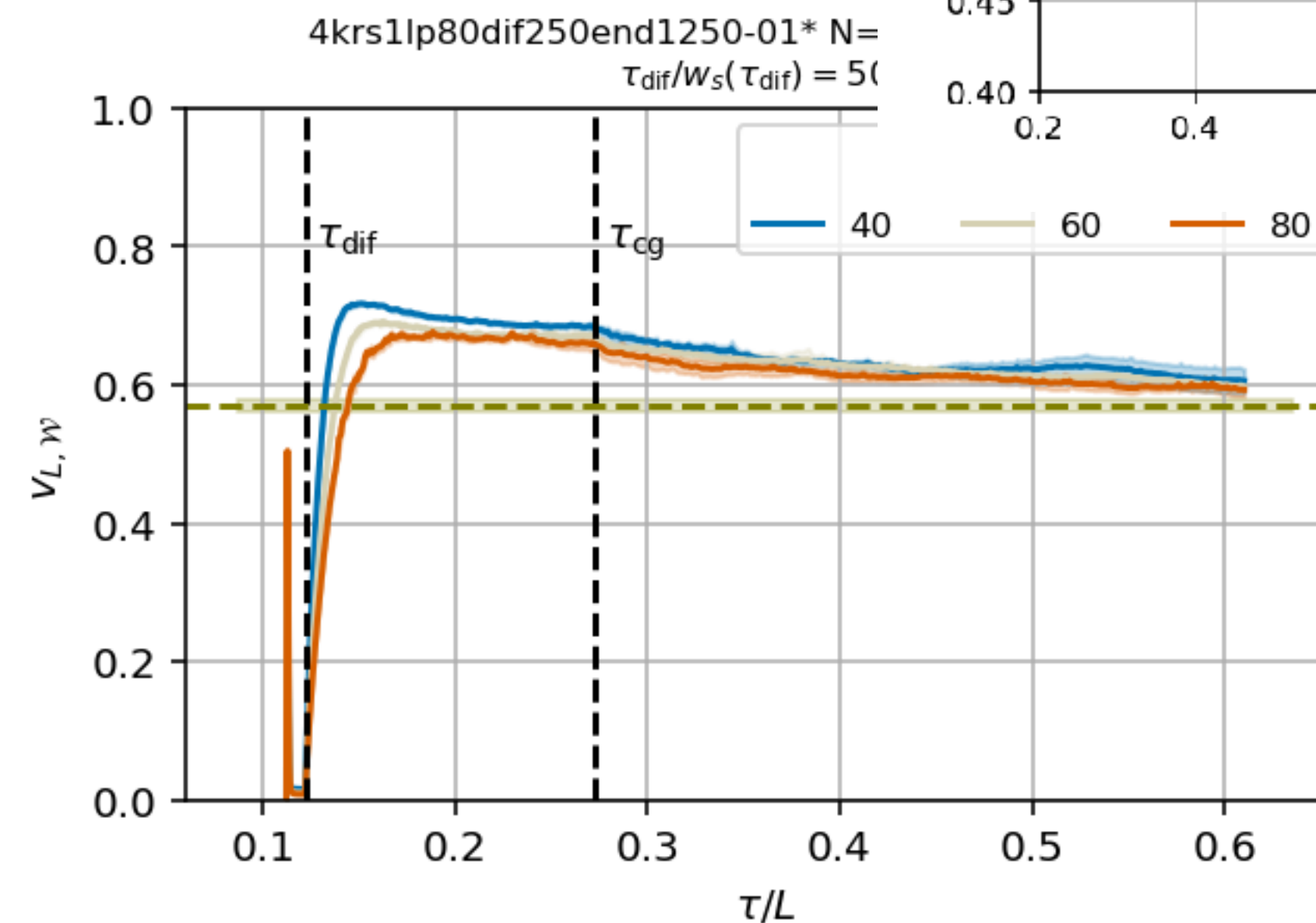
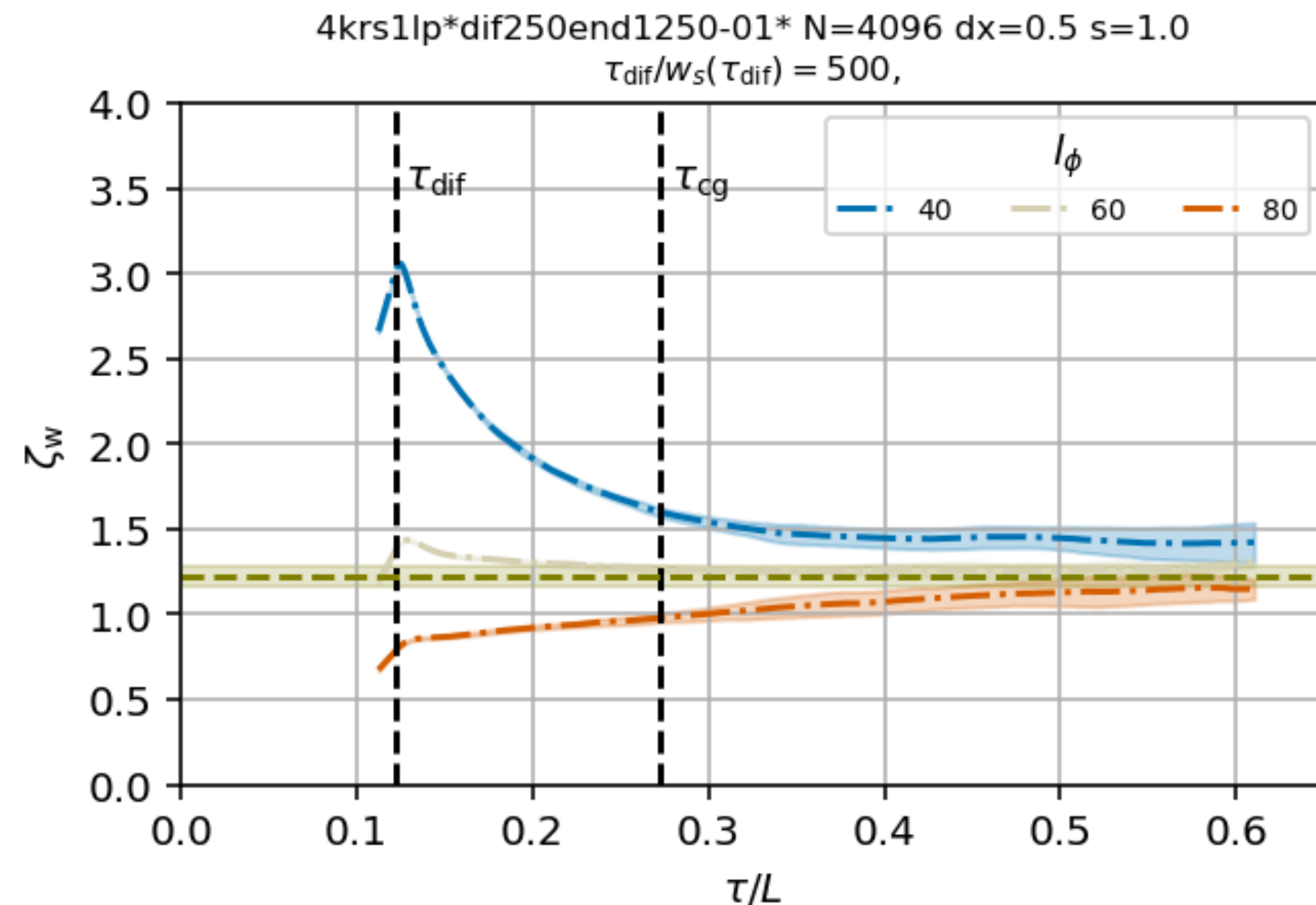
Implied **scaling violation** of $\zeta_w \approx 0.24 \log(m_s t)$ can be linked to **low initial density** and a symptom of an approach to a fixed point.

Conclusions

- Extrapolation from 4k simulations, **agree** with 16k simulations for ζ_r . 16k **velocity** is roughly 5% **lower**.
- A **VOS** model for global strings **needs** to be **developed and calibrated**. Attempts are scattered throughout the literature:
 - [Martins & Shellard, 2000], [Klaer & Moore, 2018], [Martins, 2018], [Revello & Villa, 2024] are some examples either with axion emission terms or time-varying tensions (the latter reparametrization invariant).
 - Testing corrections to the fixed point must be done via **full model calibration**, taking approach into account.
- Our methods of generating initial conditions either end in a constant rest-frame string density parameter, **or** a constant velocity $\rightarrow$ refining methods of generating ICs would benefit all groups.
 - Some ongoing work on modifying ICs to start physical evolution in different parts of phase space;
 - Will eventually allow us to start as close as possible to the fixed point = maximises useful dynamic range.
- **Axion mass estimate** should be closer to $\sim 28 \pm 2, 26.2 \pm 3.4 \mu eV$ [Borsanyi et al, 2016][Klaer et al, 2019] assuming of course no extra gauge fields [Klaer et al, 2019]. Spectra in backup slides!

Next Steps: Filling in the Phase Space

- Some ongoing work on modifying ICs to start physical evolution in different parts of phase space;
- Will eventually allow us to start as close as possible to the fixed point = maximises useful dynamic range.
- VOS model calibration must come next.



PRELIMINARY

Next Steps: Gravitational Waves from Axion Strings

- Multiple possibilities are being reviewed for LISA cosWG project
“Cosmic Strings beyond minimally-coupled models”
- VOS models** for Global strings (weak ζ logarithmic drift, plateaus)
 [Gouttenoire et al, 2020][Chang et al, 2019 & 2021]:

- Spectrum $\Omega_{gw}(f) \propto \log^3(f)$

- Field theory simulations + energy-momentum tensor:

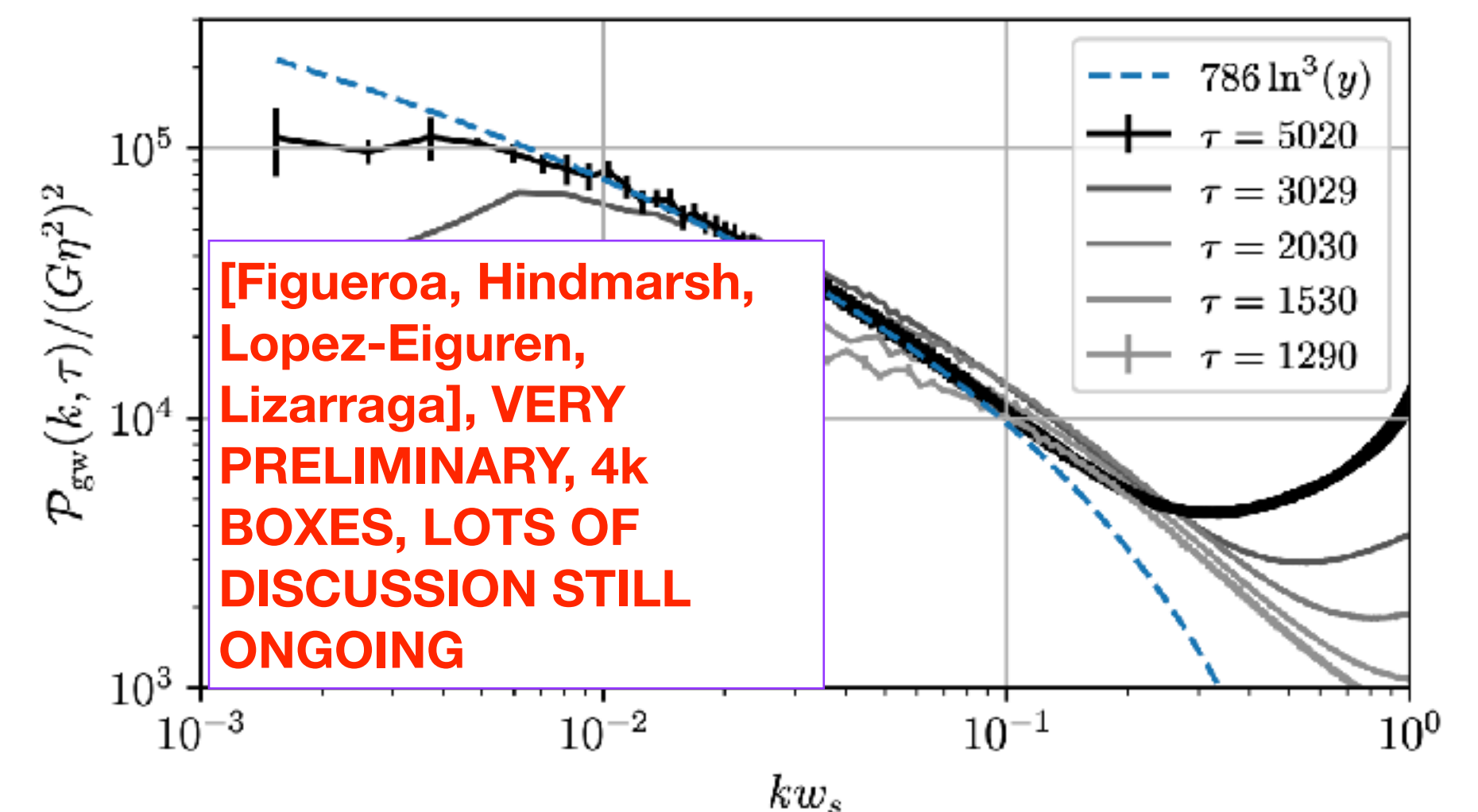
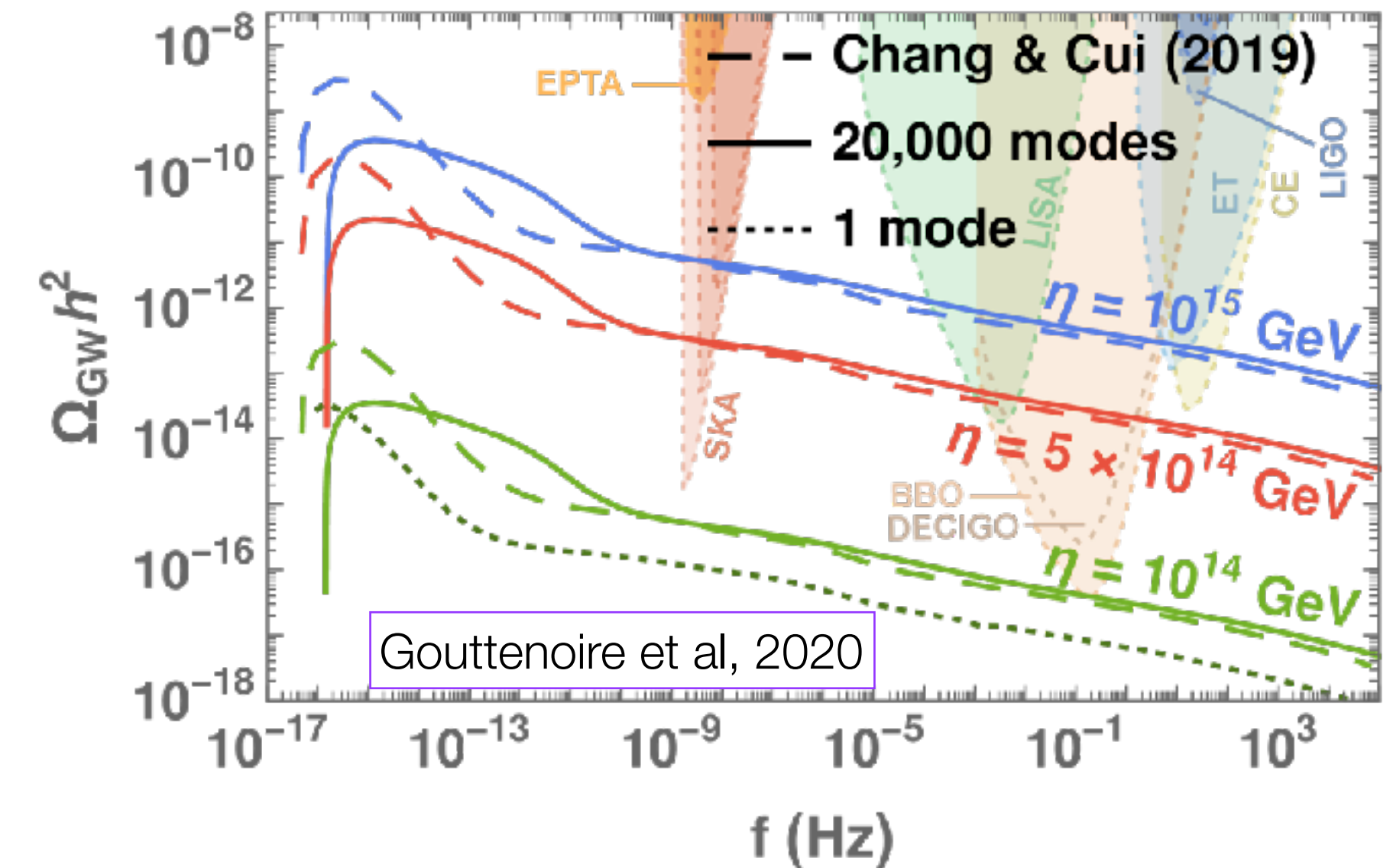
- Marco’s talk yesterday (and [Gorghetto et al, 2021])

Spectrum $\Omega_{gw}(f) \propto \log^4(f)$ Extra $\log(f)$ from $\zeta \propto \log(t)$

- See O(N) paper [Figueroa et. al, 2020] for scaling textures

Exact scale-invariant spectra claimed, to be updated

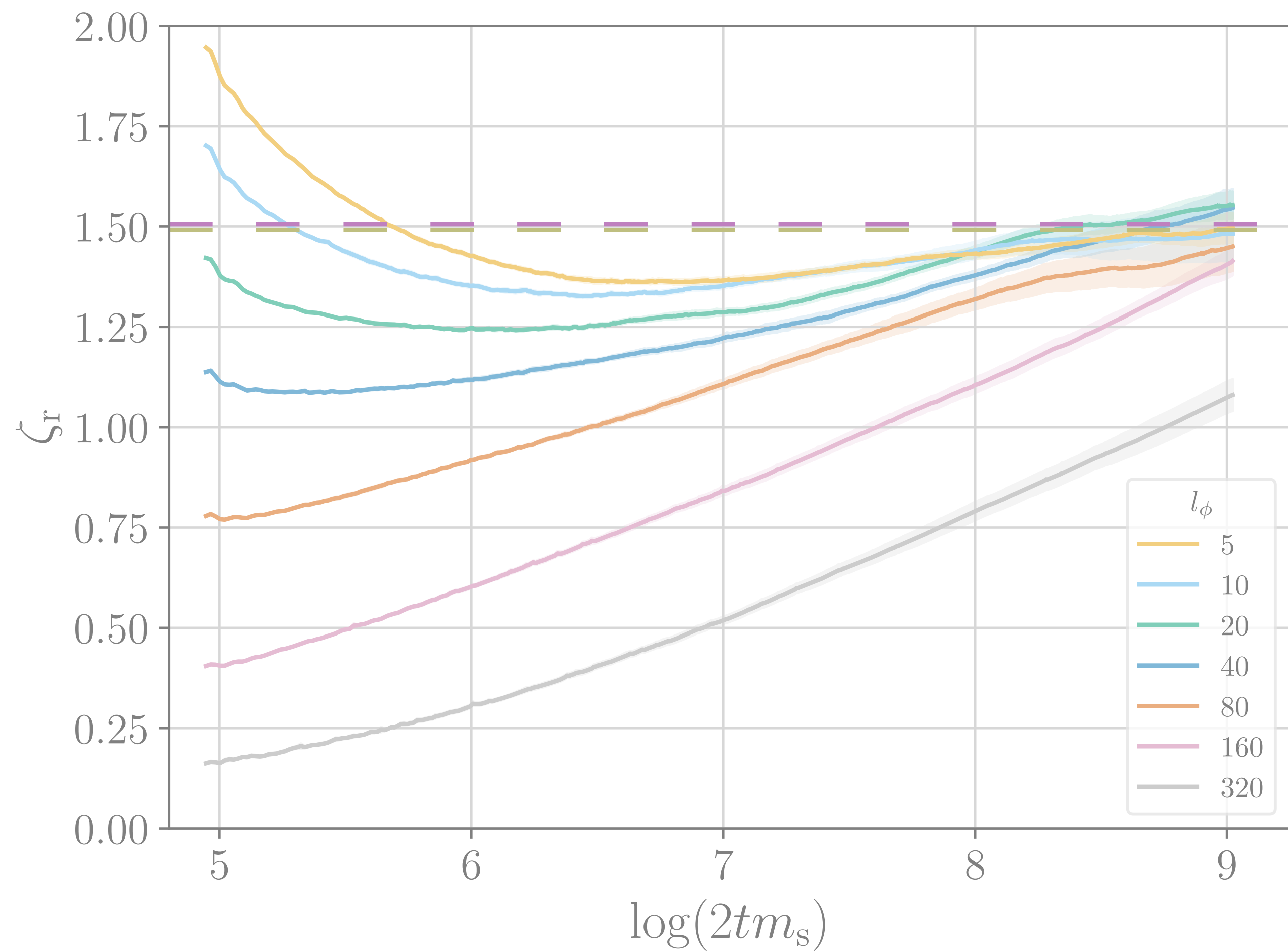
Work is being done on O(2), we need larger simulations →



Tack så mycket! /
Thank you! /
Obrigado!

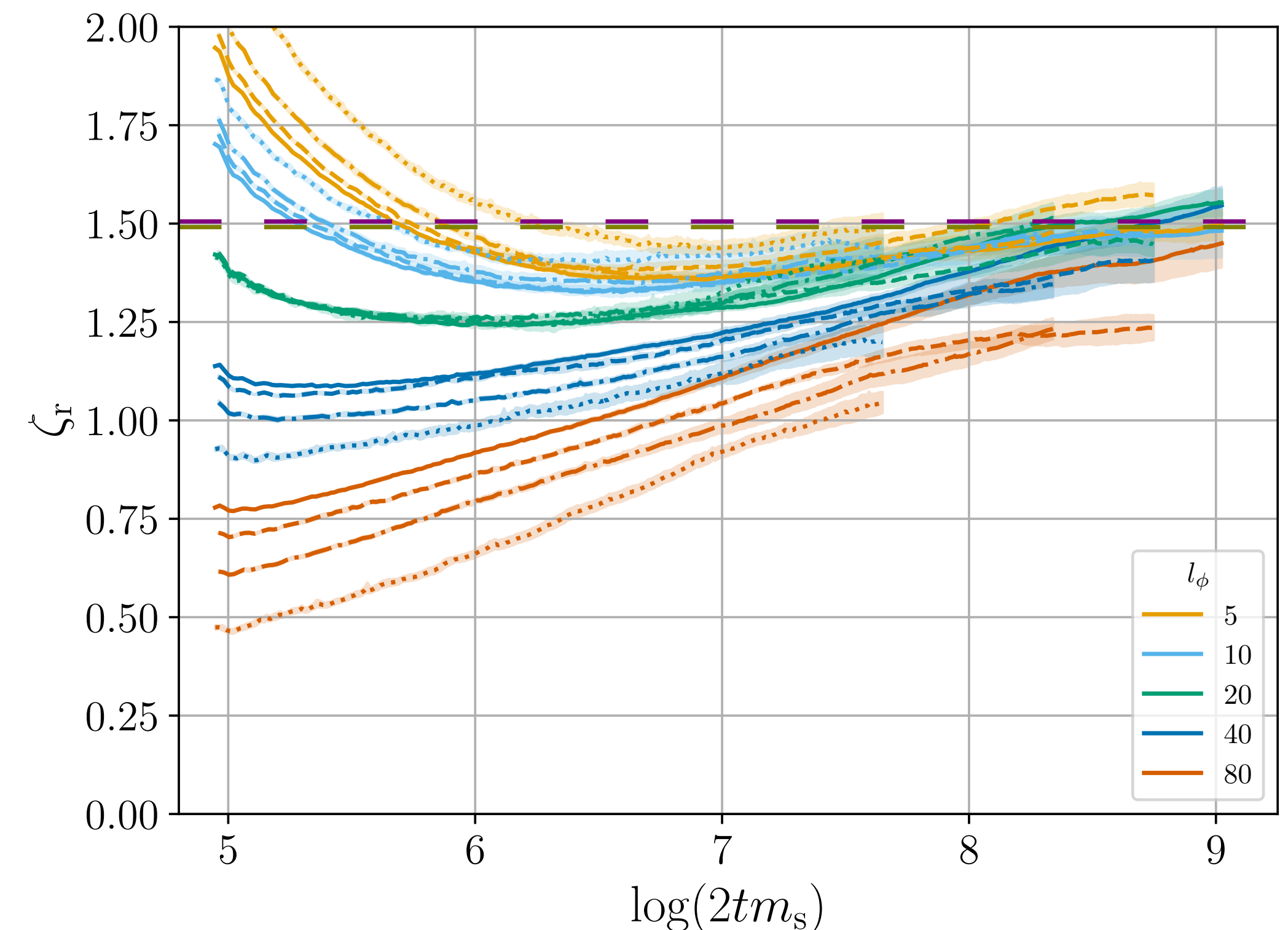
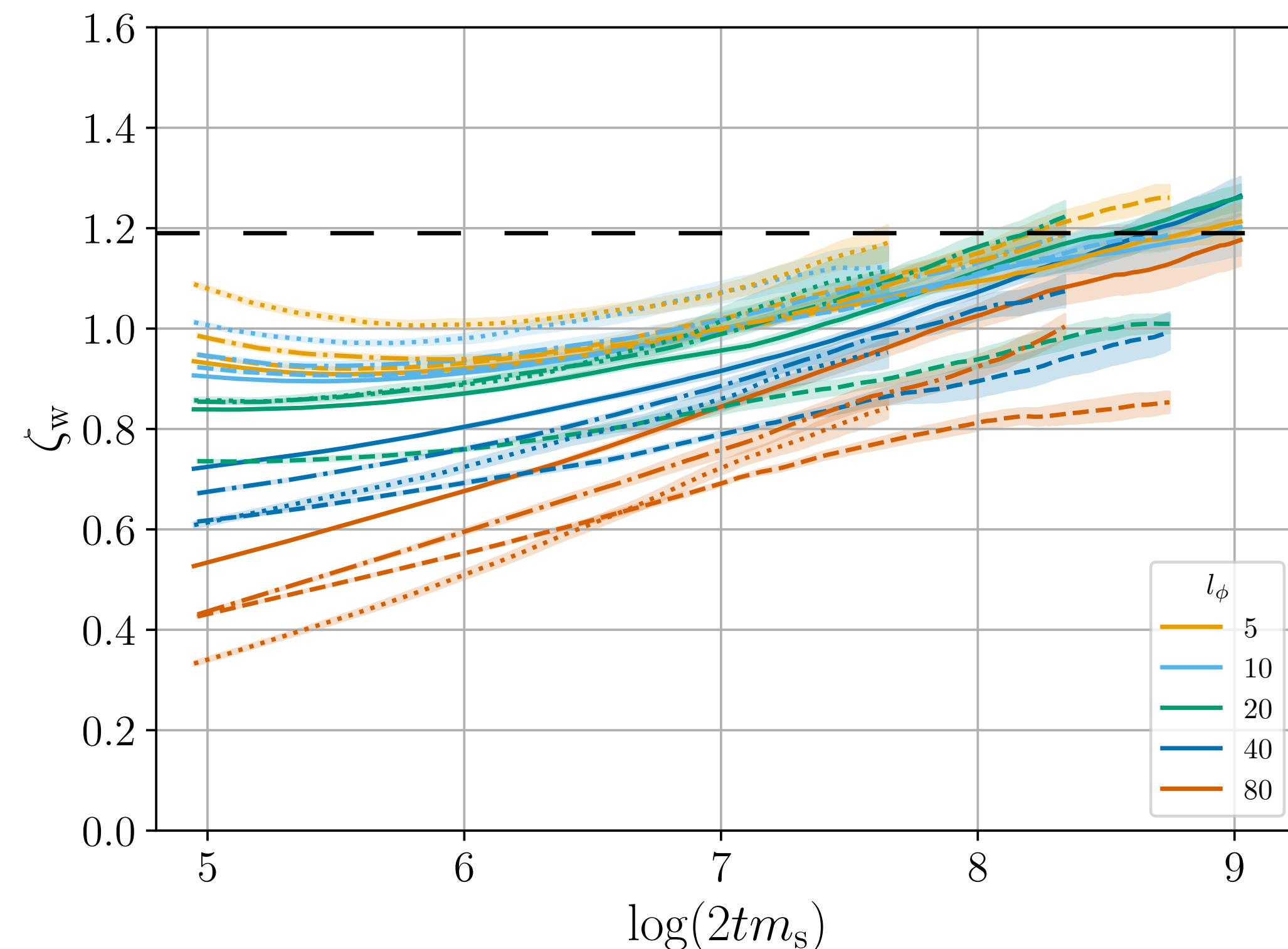
Long Term Growth Analysis

arXiv:2410.18064, PhysRevD 111 063532
CSC project: 462000084



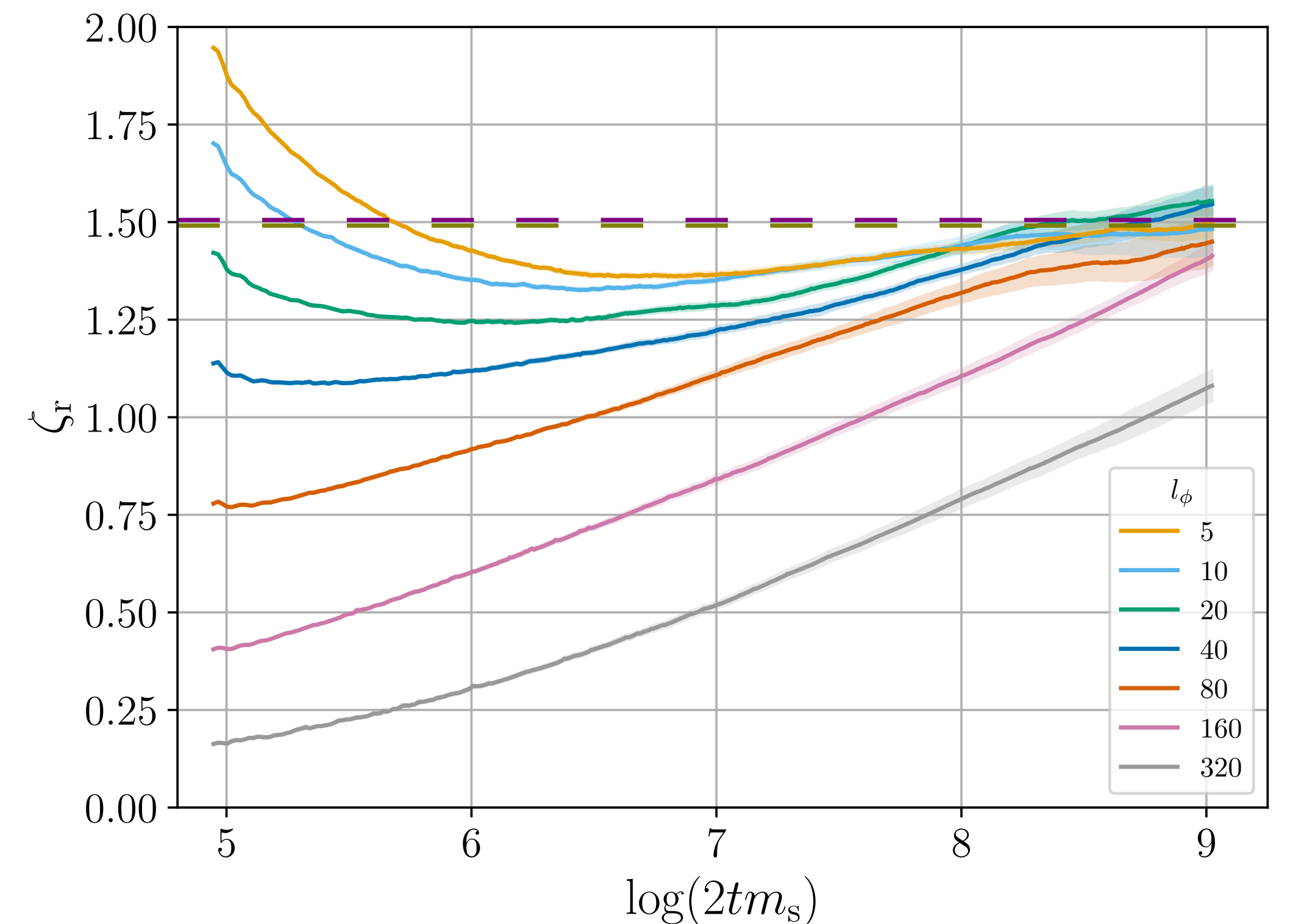
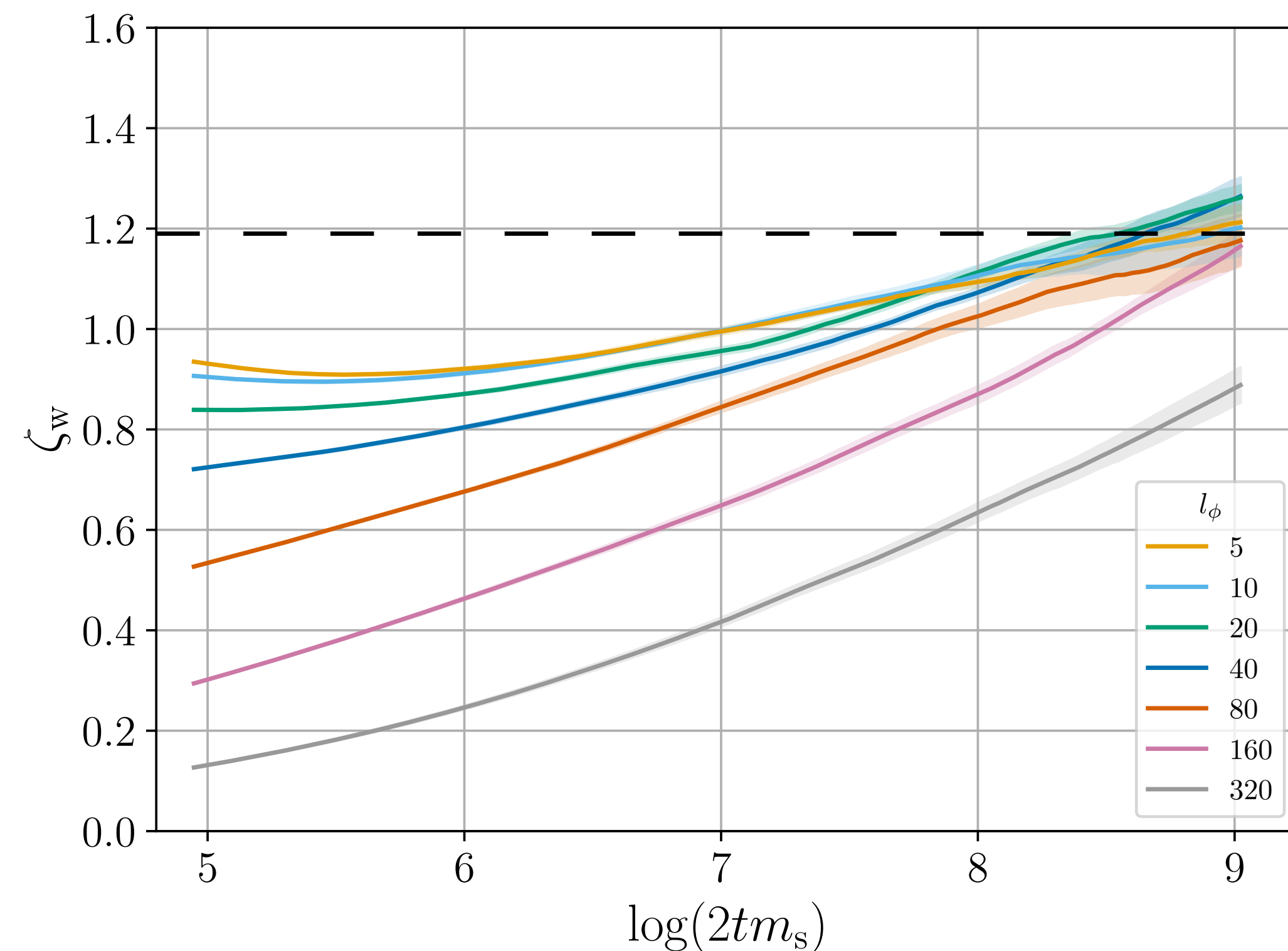
Long Term Growth Analysis

- Although one can seemingly presume that these ζ_W curves imply long-term growth (at least from a cursory look at all resolutions / dynamics ranges altogether)...
- Finding the scaling violation at low resolution and **extrapolating forwards** would **significantly exceed the fixed point values**. **This does not occur**. Again we note most groups choose ICs with $\zeta_W \approx 0.5$ at $\log(2tm_s) = 5$



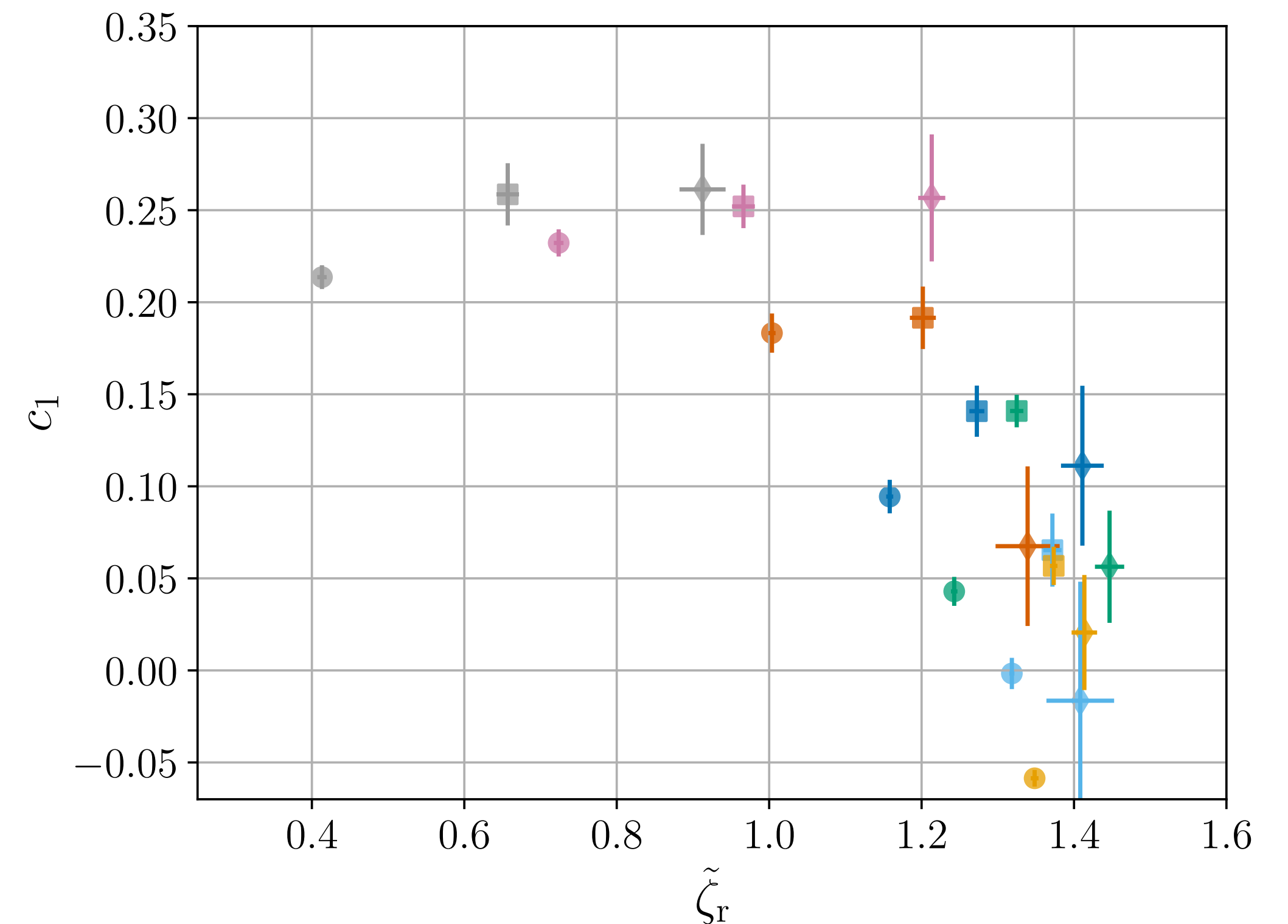
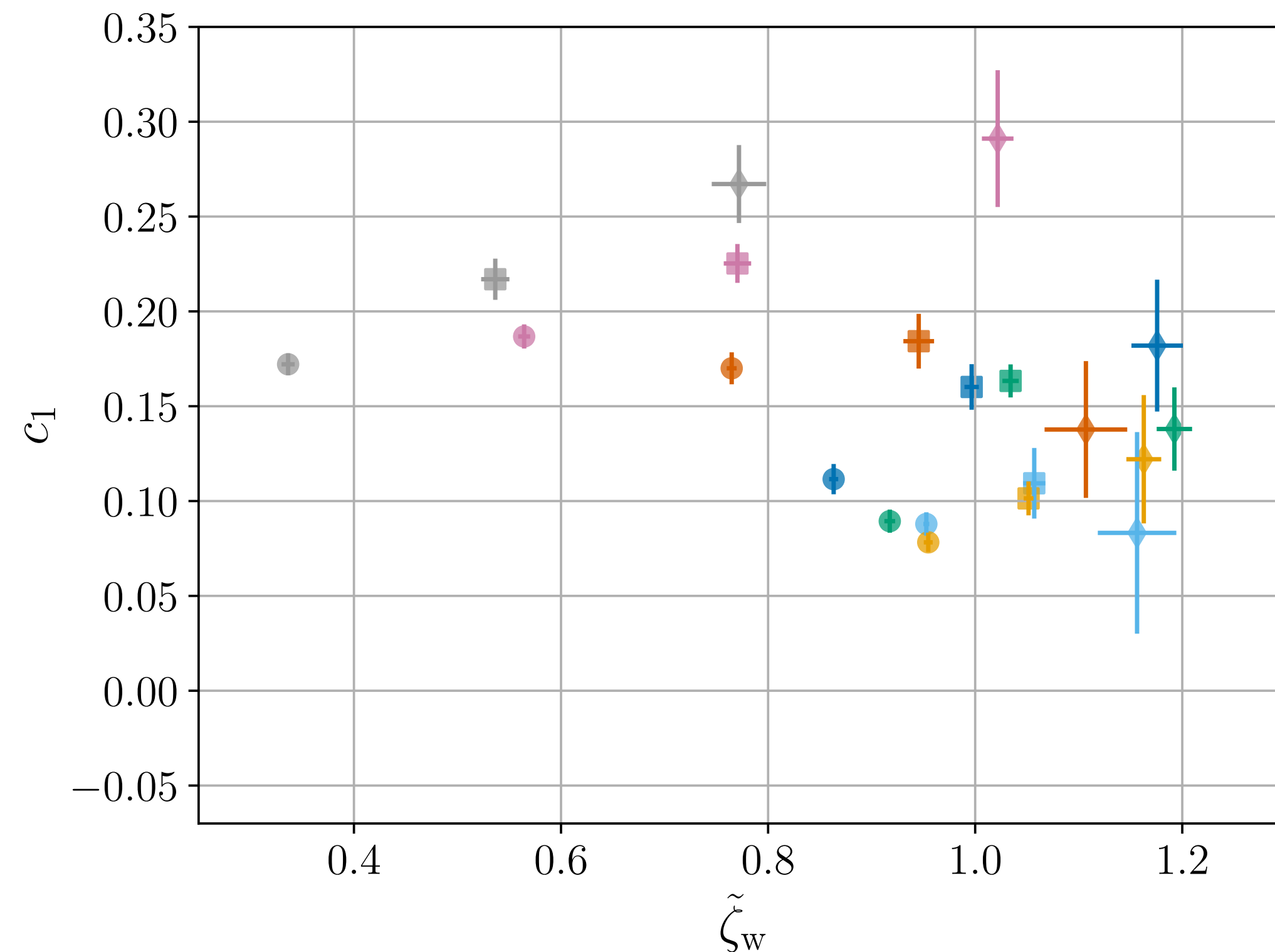
Long Term Growth Analysis

- Flattening out of curves can be seen more easily if we pick out a specific resolution / dynamic range (16k)
- This choice of variables will highlight the **initial conformal time range**, where evolution is most **sensitive** to **initial conditions**. **Scaling “compressed”** in the **last e-fold**. Again we repeat **velocity masks convergence of ζ_W** .



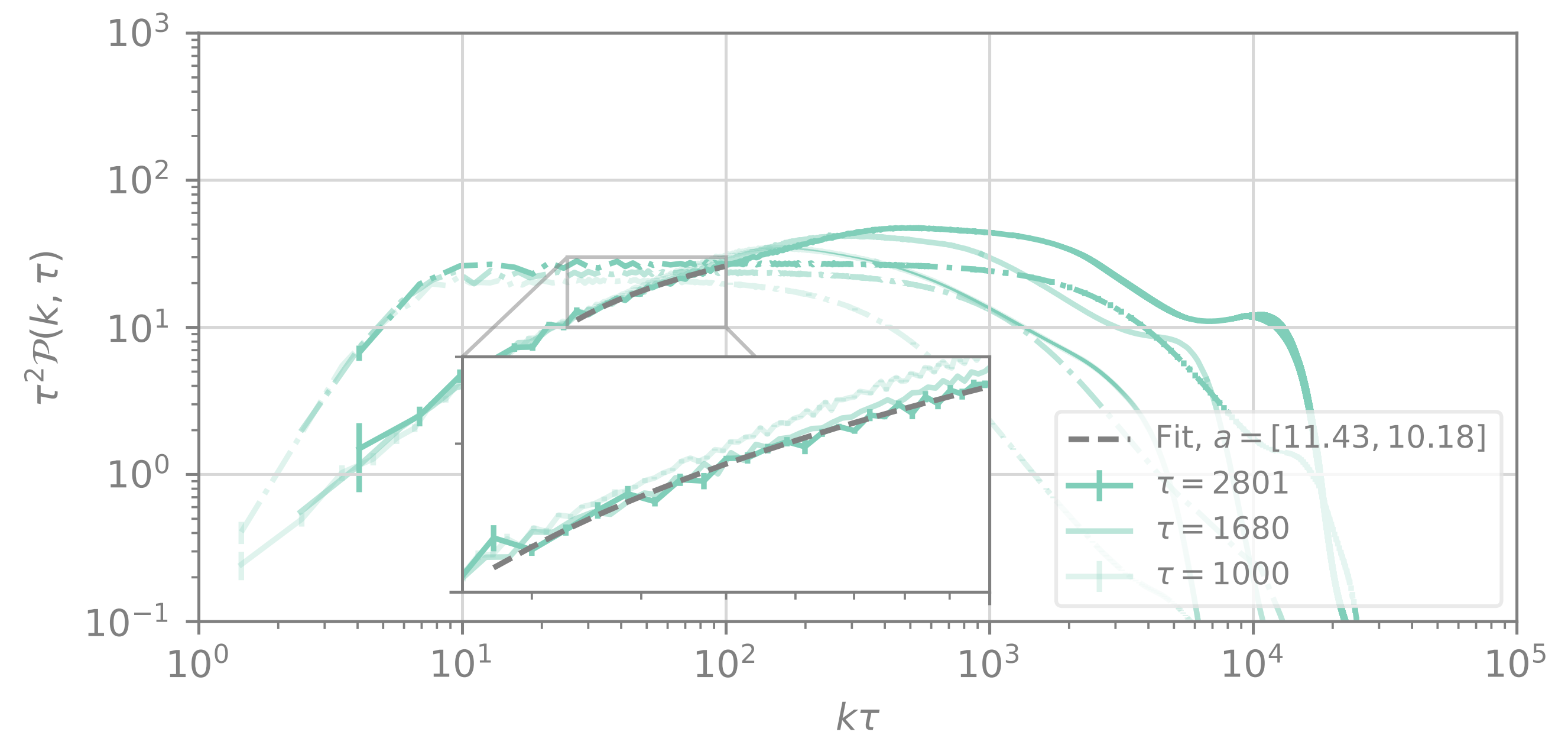
Long Term Growth Analysis

- We will attempt fits of the form $c_1 \log(t) + c_0$, in different intervals of logarithm of cosmic time $[6, 7]$, $[7, 8]$ and $[8, 9]$ for both $\zeta_{r,W}$
- We can observe **strict convergence** in ζ_r in $5 \leq l_\phi \leq 20$; although ζ_W **convergence masked** by **velocity** evolution
 - If the **logarithmic growth** were the correct late time behaviour, slopes would **increase with time. This is not observed.**



The spectrum of axions in a scaling string network

In Preparation
CSC project: 462000084



The spectrum of axions in a scaling axion string network

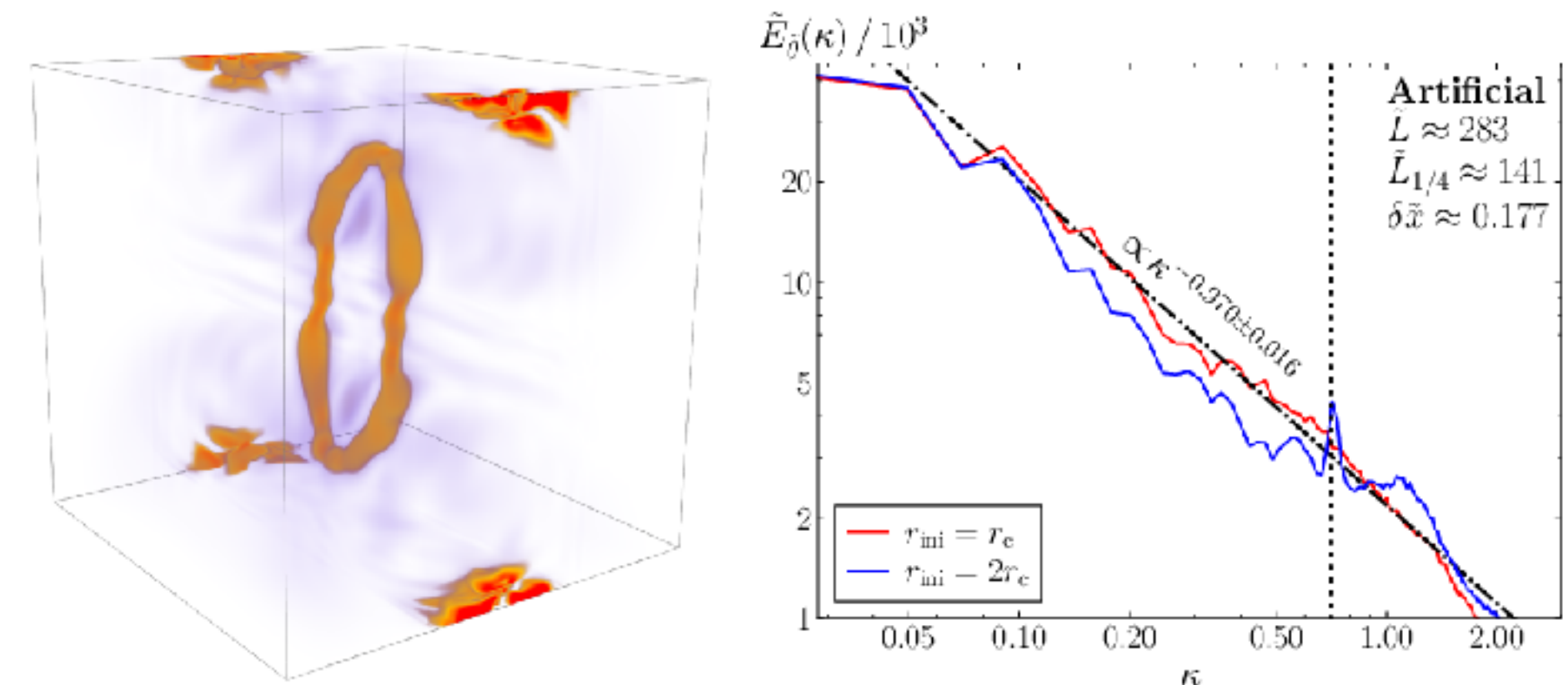
- Another source of controversy / way to make friends :)

$$\mathcal{P} \propto \ln(k/k^*) \quad \text{or} \quad \mathcal{P} \propto (k/k_*)^{1-q}$$

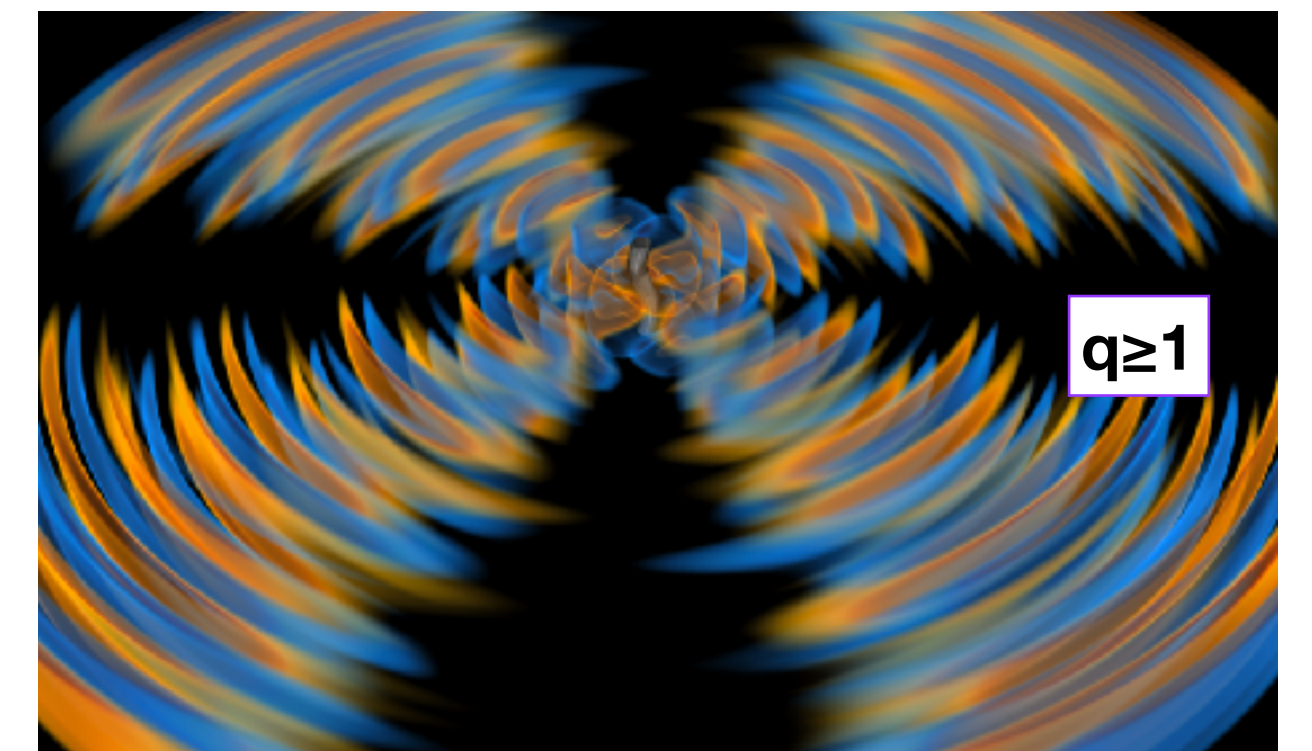
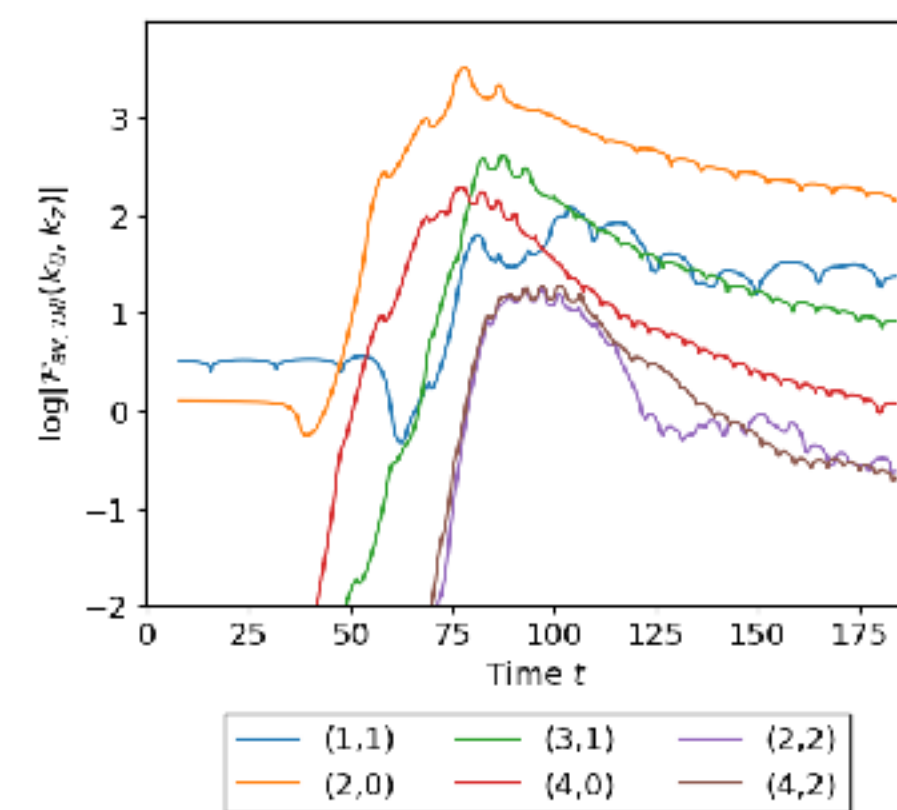
$$q = 1 \text{ [Hariri et al, 1986]} \quad \text{or} \quad q \geq 1 \text{ [Davis et al, 1987]}$$

- **Caveats:**

- There is currently no model for production of axions from strings for the whole network (**long strings + loops**);
- Results depend on fitting range, approach to scaling (or definition of it :p), masking functions, lattice spacing, etc. From networks:
 - For smaller lattices, [Gorghetto et al, 2018][Kim et al, 2024] ($q \geq 1$) (and even older papers...)
 - See Malte's talk [Benabou et al, 2025] ($q \approx 1$)
 - Also [Saikawa et al, 2024] ($q \approx 1$ or $q \geq 1$)



Saraubh et al, 2020; Baeza-Ballasteros et al 2023



Battye et al, 1998; Drew et al 2022

The spectrum of axions in a scaling axion string network*

- Currents:**

- $J_0 = \mathfrak{I}(\phi\Pi)$ - used by most groups
- $J_i = \mathfrak{I}(\phi D_i \phi)$ - should be less “polluted”

- Fourier quantities denoted by $\sim$. In Fourier Space, let's define some projected observables,

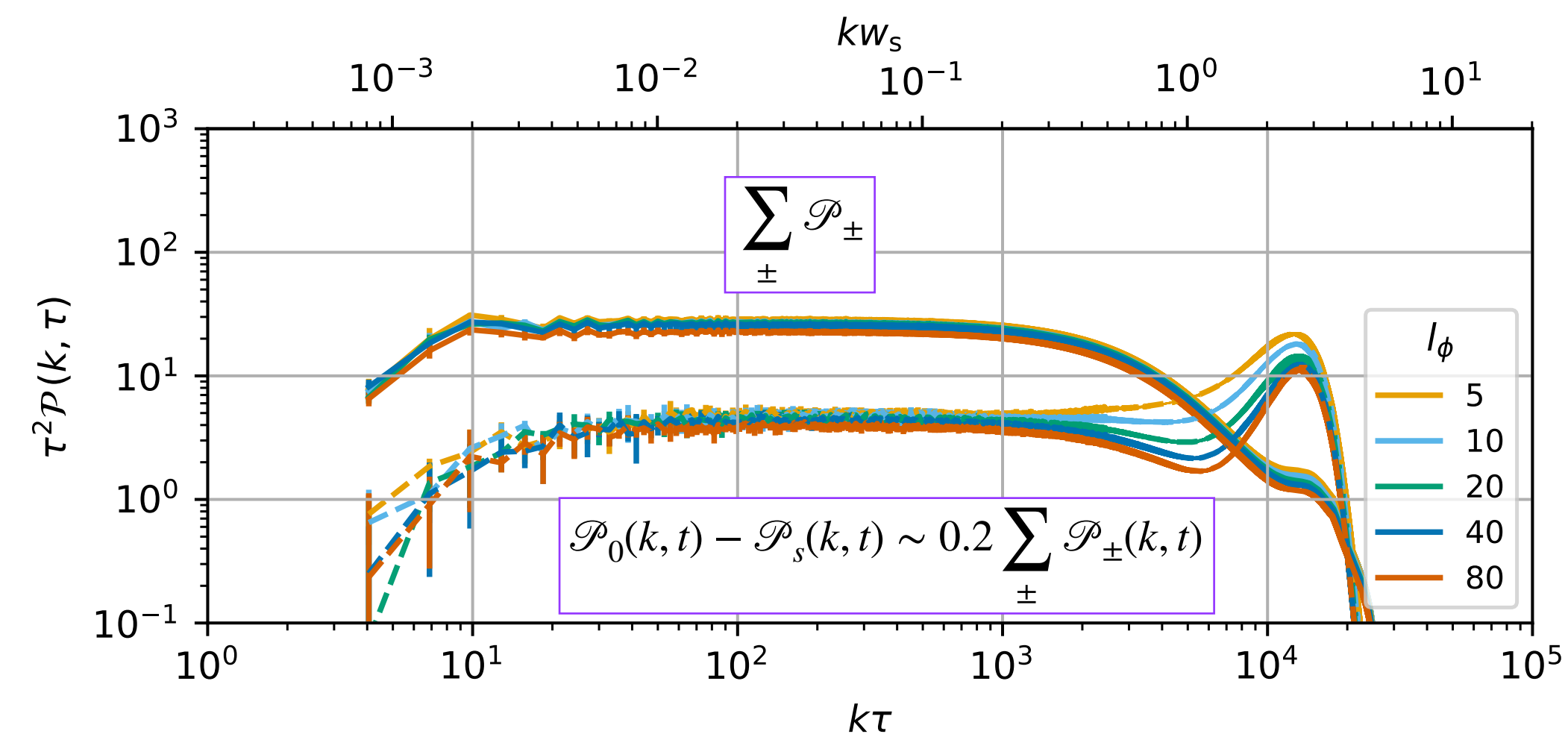
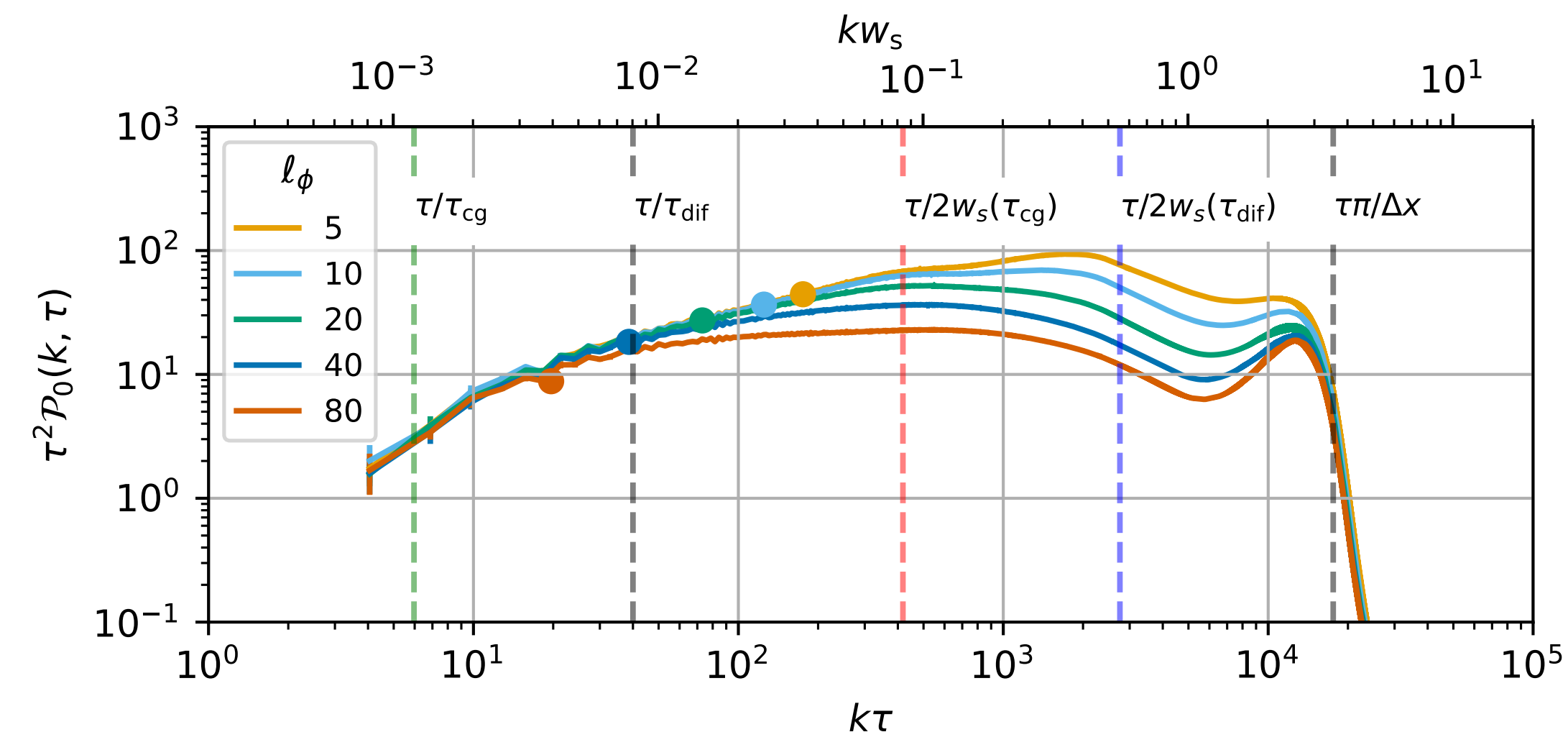
- $\tilde{J}_s = \tilde{J}_i e_s^i$ where $e_s^i = k^i$

- $\tilde{J}_\pm = \tilde{J}_i e_\pm^i$ where $i\epsilon_{ijk} e_s^i e_\pm^k = \pm e_\pm^j$

- **Axion** production - $\tilde{J}_0$ contains string $\mathcal{O}(v^2)$, $\tilde{J}_s$ $\mathcal{O}(v^4)$

- **Strings** only - $\tilde{J}_\pm$

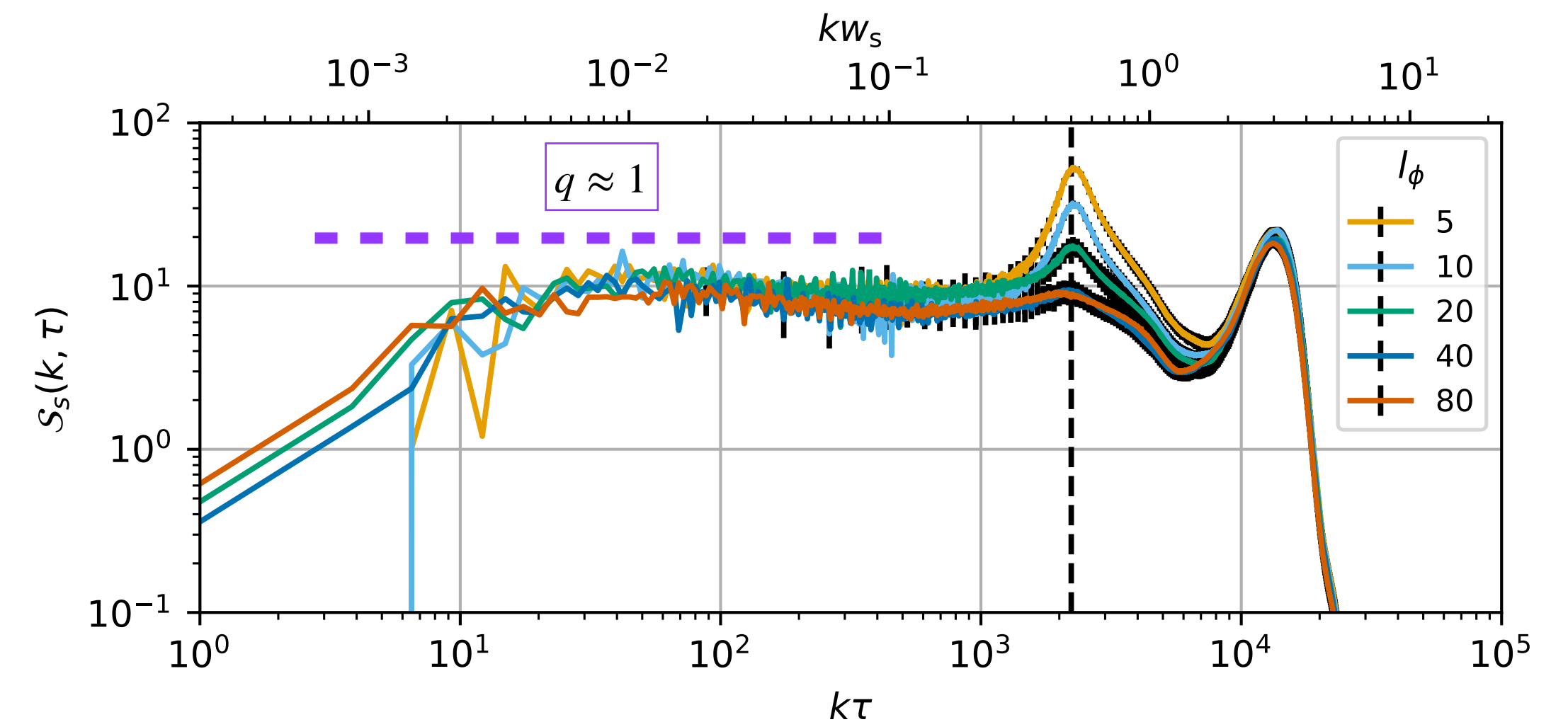
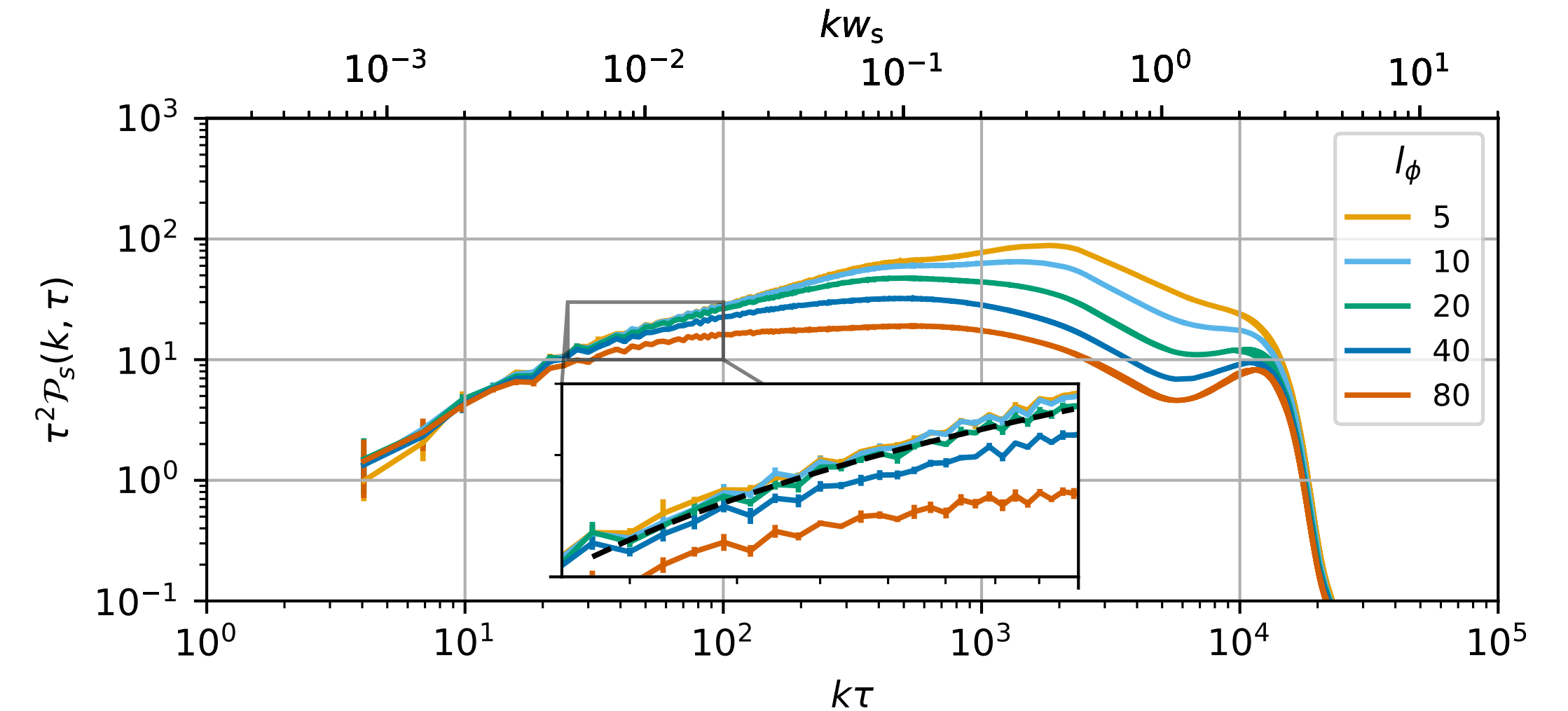
- **Power Spectra:** $\mathcal{P} = \frac{k^3}{2\pi} \langle |\tilde{J}(k, \tau)|^2 \rangle$



*Correia et al, in preparation

The spectrum of axions in a scaling axion string network^{*}

- **Power Spectra:** $\mathcal{P} = \frac{k^3}{2\pi} \langle |\tilde{J}(k, \tau)|^2 \rangle$
 - Fitting $\mathcal{P} = a_0 \ln(k\tau/a_1)$ in the range $25 < k\tau < 100$
 - Range is not a unique choice, modes are chosen to, well within the horizon, never inside string
- **Rate of Change of Power Spectra:** $\mathcal{S} = \tau \frac{d}{d\tau} \left(\tau^2 \mathcal{P}(k, \tau) \right)$
 - Fits to power law $\mathcal{S} = (k\tau)^{1-q}$
 - q broadly distributed around 1, thus $q \approx 1$
 - Agreement with [Saikawa et al, 2024][Benabou et al, 2024]



The spectrum of axions in a scaling axion string network*

- **Unequal time Correlator:**

$$U_{aa}(k, \tau_1, \tau_2) = \mathcal{V}^{-1} \langle \tilde{J}_a^*(k, \tau_1) \tilde{J}_a(k, \tau_2) \rangle$$

- **Decoherence functions:**

$$D_a(k, \tau_1, \tau_2) = \frac{U_{aa}(k, \tau_1, \tau_2)}{\sqrt{P_a(k, \tau_1) P_a(k, \tau_2)}}$$

- Axion observables (D_s , D_0) decorrelate like waves; looks a lot like compressional modes in a fluid
- String observables ($D_{\pm}$) decorrelate like vortices; looks a lot like vortical modes in a fluid: Kraichnan sweeping! [Kraichnan, 1964]
 - [Davis & Shellard, 1989] showed equivalence of global strings as vortices in a superfluid

