What is the maximum temperature ever reached in the universe?

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in collaboration with

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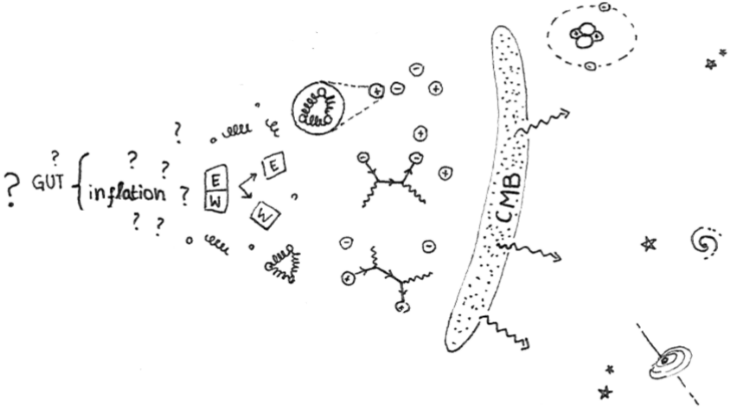
Numerical Simulations of Early Universe Sources of Gravitational Waves

Nordita, 2025

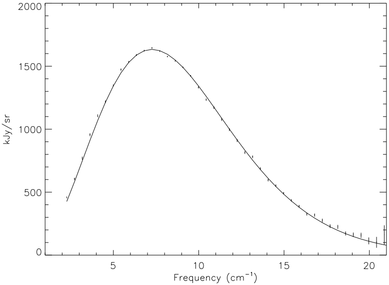
GWs could probe the early universe... but so many models!



is there something we are already sure about? yes!

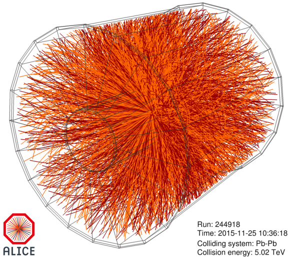
Cosmic Microwave Background originates from a thermal system

the CMB temperature spectrum
as a perfect blackbody



D.J. Fixsen *et al.*, Astrophys.J. 473 (1996) 576

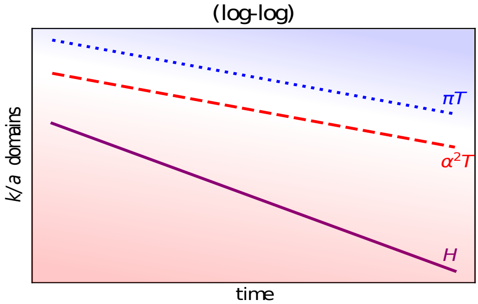
collider searches test
the primordial plasma



tracks from a lead-lead collision
recorded by the ALICE TPC at CERN

and anisotropies in a thermal plasma can source GWs!

$$\Big(\partial_t^2 + \underbrace{3H}_{\sim T^2/m_{\rm pl}} \partial_t + \frac{k^2}{a^2} \Big) h_{ij}^{\tt tt}(t,\mathbf{k}) = \frac{8\pi}{m_{\rm pl}^2} \overbrace{\mathbb{L}_{ij}{}^{mn}(\hat{\mathbf{k}})}^{\text{tt-projector}} \underbrace{T_{mn}(t,\mathbf{k})}_{\text{parametrized by temperature } T}$$



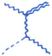
* $T \sim 1/a$ during radiation-domination
* micro/macro scopic sourcing process depending on k/a domain

shape of the corresponding GW spectrum is well understood

$$\mathcal{P}_t(t,k) \supset \frac{(32)^2 k^3}{m_{\mathrm{pl}}^4} \int_{t_e}^{t} \mathrm{d}t_1 \int_{t_e}^{t} \mathrm{d}t_2 \overbrace{G_{\mathrm{R}}(t,t_1,k)}^{\text{Green's function}} G_{\mathrm{R}}(t,t_2,k) \times \underbrace{\mathbb{L}^{ijmn} \langle T_{ij}(t_1,\mathbf{k})\, T^*_{mn}(t_2,\mathbf{k}) \rangle}_{\text{source spectrum } \mathcal{P}_{\Pi}(t_1,t_2,k)}$$

* $k/a \to 0$ corresponds to hydrodynamic fluctuations 

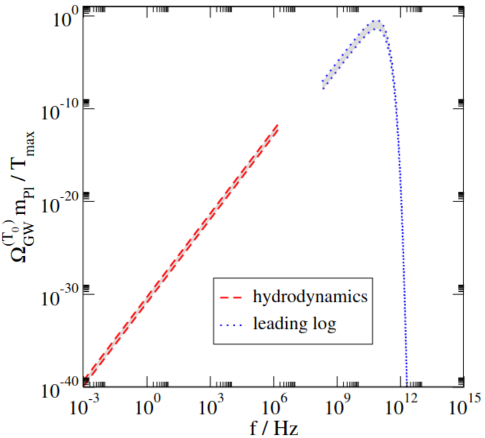
$$(\partial_t^2 + 3H\partial_t) G_{\mathrm{R}} \approx 0\,, \quad \mathcal{P}_{\Pi} \to \text{thermal noise} \quad \Rightarrow \quad \sim k^3 \text{ growth}$$

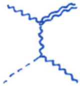
* $k/a \to \infty$ resolves microscopic interactions 

follow Boltzmann distribution $\Rightarrow$ rapid decrease

accurate predictions for a Standard Model plasma[1]

from J. Ghiglieri and M. Laine, JCAP 07 (2015) 022





hydrodynamic fluctuations

$$\frac{\mathrm{d}\Omega_{\mathrm{gw}}}{\mathrm{d}\ln f} \sim \underbrace{\hat{\eta}}_{\text{viscosity}}\, T_{\max} \times f^3$$

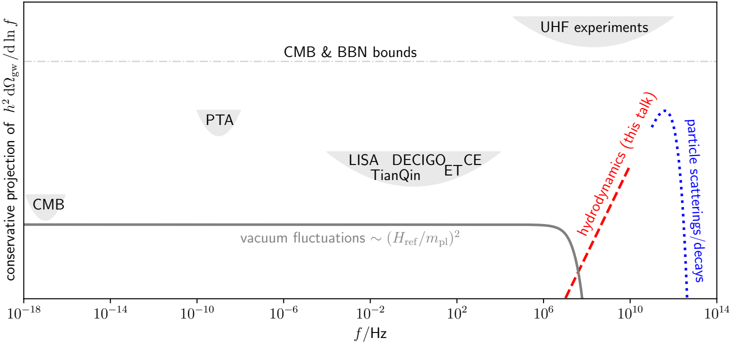
particle scatterings

$$\frac{\mathrm{d}\Omega_{\mathrm{gw}}}{\mathrm{d}\ln f} \sim T_{\max} \times f^4 \underbrace{n_{\mathrm{B}}(f/T_{\max})}_{\text{Boltzmann distr.}}$$

[1] J. Ghiglieri, G. Jackson, M. Laine and Y. Zhu, JHEP 07 (2020) 092,
J. Ghiglieri, M. Laine, J. Schütte-Engel and E. Speranza, JCAP 04 (2024) 062.

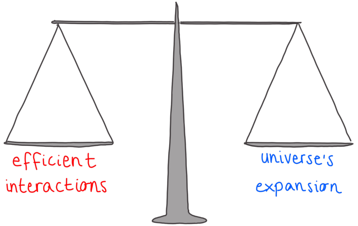
observational prospects depend on the maximal temperature

$$h^2 \frac{d\Omega_{gw}^{hydro}}{d\ln f} \approx \underbrace{10^{-29} \times \left(\frac{f}{kHz}\right)^3}_{\text{model-independent}} \times \underbrace{\frac{\overbrace{\hat{\eta}}^{\text{known in SM}}\, T_{max}}{m_{pl}}}_{\text{model-dependent}}$$



what do we know about the maximal temperature?[2]

Big Bang Nucleosynthesis



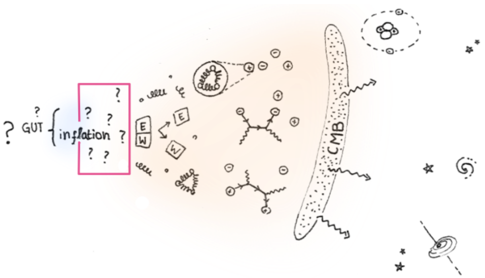
$$\color{red}{\Gamma(T)} \qquad \color{blue}{H^2 \sim 2e_\gamma + N_{\text{eff}}\, e_\nu + \ldots}$$

$$\color{red}{\searrow} \qquad \color{blue}{\swarrow}$$

$$T_{\texttt{max}} \in [\,\color{red}{10^{-2}},\ \color{blue}{10^{18}}\,]\ \texttt{GeV}$$

[2] J. Ghiglieri, G. Jackson, M. Laine and Y. Zhu, JHEP 07 (2020) 092,
J. Ghiglieri, M. Laine, J. Schütte-Engel and E. Speranza, JCAP 04 (2024) 062.

can we predict the maximal temperature?



* thermal state required down to $t_{\text{BBN}} \sim 10\,\text{s}$

* early vacuum state assumed at $t \sim 10^{-32}\,\text{s}$

* $T_{\max}$ reached in transition period [3]

[3] H. Kolesova, M. Laine and S. Procacci, JHEP 05 (2023) 239

background evolution before radiation-domination[4]

early vacuum-domination parametrized by φ

$$\ddot{\bar{\varphi}} + 3H\dot{\bar{\varphi}} + V_{,\varphi} \approx \underbrace{-\Upsilon\dot{\bar{\varphi}}}_{\text{energy transfer}}$$

$$\dot{\bar{e}}_r + 3H(\bar{e}_r + \bar{p}_r) \approx \overbrace{\Upsilon\dot{\bar{\varphi}}^2}$$

radiation plasma at equilibrium temperature T emerges

$$\Rightarrow \quad (T\,\bar{s}_r)_{\max} \approx \left(\frac{\Upsilon\dot{\bar{\varphi}}^2}{3H}\right)_{\max}$$

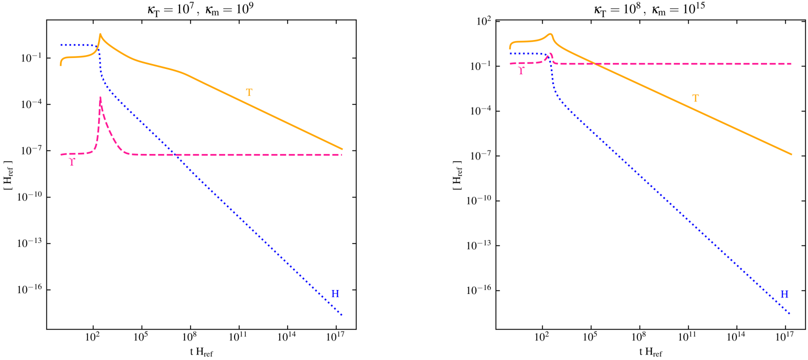
[4] H. Kolesova, M. Laine and S. Procacci, JHEP 05 (2023) 239.

$T_{\max}$ depends on energy transfer $\varphi \leftrightarrow$ plasma

* the universe expands at rate $H \sim \sqrt{e_\varphi + e_{\rm rad}}$

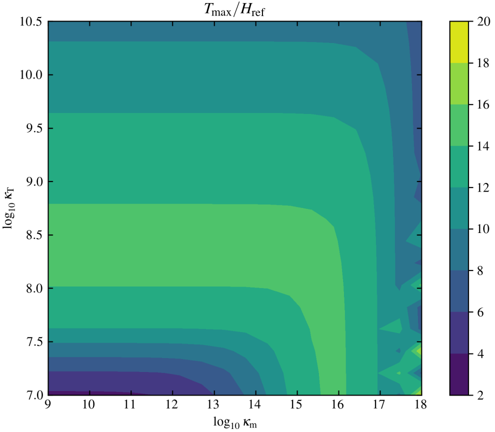
* $\Upsilon = \frac{\kappa_{\rm T} T^3 + \kappa_{\rm m} m_\varphi^3}{(4\pi)^3 f_a^2}$ transfers energy to the plasma[5]

* plasma at T dominates at late times



[5] M. Laine, L. Niemi, S. Procacci and K. Rummukainen, JHEP 11 (2022) 126, M. Laine, S. Procacci, A. Rogelj, JCAP 10 (2024) 040, and [2507.12849].

a model-dependent upper bound for $T_{\max}$

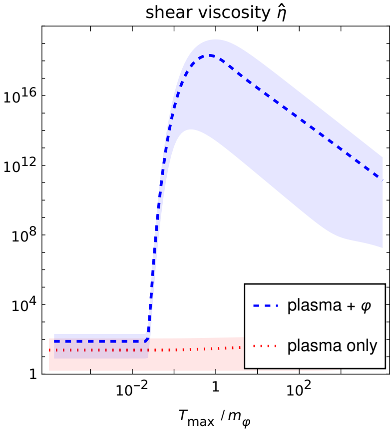


* $\kappa_{\rm m}$ and $\kappa_{\rm T}$ determine the reheating mechanism[6]

* $\sim H_{\rm ref}$ sets the energy scale of inflation

[6] M. Laine, S. Procacci and A. Rogelj, JCAP 10 (2024) 040, and [2507.12849].

viscosity $\hat{\eta}$ enhanced by weakly-interacting extensions[7]



* new degrees of freedom at high energies?

* weaker interactions $\Rightarrow$ higher viscosity $\sim \lambda_{\rm free}$

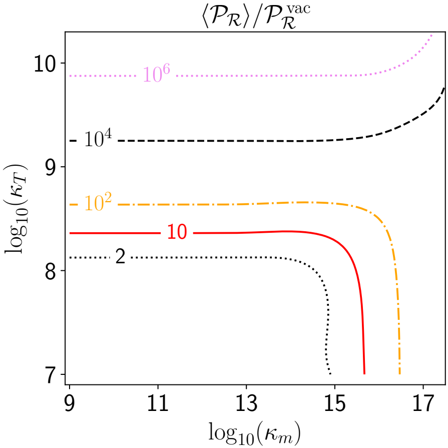
[7] P. Klose, M. Laine and S. Procacci, JCAP 05 (2022) 021.

is $T_{\max}$ observable?

$$h^2 \frac{d\Omega_{gw}^{\mathsf{hydro}}}{d\ln f} \approx \underbrace{10^{-29} \times \left(\frac{f}{\text{kHz}}\right)^3}_{\text{model-independent}} \times \underbrace{\frac{\overbrace{\hat{\eta}}^{\text{known in SM}} \; T_{\max}}{m_{\text{pl}}}}_{\text{model-dependent}}$$

* SM extensions could make the signal visible at high f
* but then $T_{\max}$ degenerates with $\hat{\eta}$...
* what are other observational inputs?

scalar perturbations are affected by high $T_{\max}$ as well[8]



... more in Mikko's talk on Thursday!

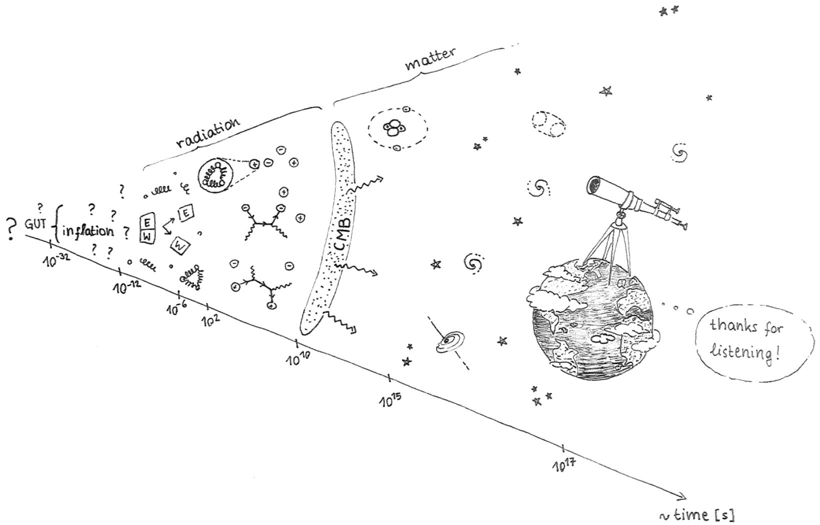
8 M. Laine, S. Procacci and A. Rogelj, JCAP 10 (2024) 040 and [2507.12849].

What is the maximum temperature ever reached in the universe?

let's keep asking this question!

it's of great interest for

stochastic gravitational wave background
large (and less large)-scale structures
thermalization models
fundamental physics at high energies


radiation
matter
GUT
inflation
CMB
10^{-32}
10^{-12}
10^{-6}
10^{2}
10^{10}
10^{15}
10^{17}
~ time [s]
thanks for listening!

backup

more about the hydrodynamic fluctuation spectrum

at linear order in perturbations and gradients,

$$T_{\mu\nu}^{\texttt{hydro}} = (e+p)\, u_\mu u_\nu + p\, g_{\mu\nu} + a^2 \Pi_{\mu\nu}$$

only some viscous corrections contribute,

$$\Pi_{\mu\nu} = \underbrace{\Sigma_{\mu\nu}}_{\substack{\texttt{shear}\\ \texttt{term}}} + \underbrace{Z_{\mu\nu}}_{\substack{\texttt{bulk}\\ \texttt{term}}} + \underbrace{S_{\mu\nu}}_{\substack{\texttt{thermal}\\ \texttt{noise}}}$$

and the fluctuation-dissipation relation implies

$$\langle S_{ij}\, S_{mn} \rangle \sim T \left[\underset{\texttt{shear}}{\eta} \left(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm} \right) + \left(\underset{\texttt{bulk}}{\zeta} - \frac{2\eta}{3} \right) \delta_{ij}\delta_{mn} \right]$$

$$\Rightarrow \quad \mathbb{L}^{ijmn} \langle T_{ij}^{\texttt{hydro}}\, T_{mn}^{\texttt{hydro}*} \rangle \sim T\eta$$

more on the shear viscosity $\hat{\eta}$

* gauge plasma with coupling g,[9]

$$T\,\eta_g \overset{N_c=3}{\underset{N_f\approx 0}{\approx}} \frac{27.126T^4}{g^4 \ln\left(\frac{2.765T}{m_D}\right)}\,, \quad m_D \equiv \frac{g^2T^2}{3}$$

* scalar field weakly coupled via Υ,[10]

$$T\,\eta_\varphi = \lim_{\omega,k\to 0} \frac{T\,\mathrm{Im}G^R_{xy;xy}(\omega,k)}{\omega} \overset{T\ll m}{\approx} \frac{T^5}{\Upsilon}\left(\frac{2\pi m}{T}\right)^{\frac{3}{2}} e^{-m/T}$$

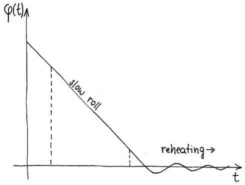
[9] P.B. Arnold, G.D. Moore and L.G. Yaffe, JHEP 11 (2000)
[10] P. Klose, M. Laine and S. Procacci, JCAP 05 (2022) 021.

weakly-coupled inflaton in medium

$$\mathcal{L} = -\tfrac{1}{2}\,\partial^\mu\varphi\,\partial_\mu\varphi - V(\varphi) - \varphi J + \mathcal{L}_{\texttt{bath}}$$

$$\downarrow$$

$$\ddot{\bar\varphi} + 3H\dot{\bar\varphi} + \partial_\varphi V(\bar\varphi) + \underbrace{\langle J(t)\rangle}_{\text{medium response}} = 0$$



expect: $$\ddot{\bar\varphi} + (3H + \Upsilon)\dot{\bar\varphi} + \partial_\varphi V(\bar\varphi, m_{\rm T}) \approx 0$$

as effective evolution equation at the end of inflation[11]

[11] M. Laine and S. Procacci, JCAP 06 (2021) 031.

$\langle J(t) \rangle$: response of medium to small perturbation

Hamiltonian: $\hat{H} = \hat{H}_{\texttt{bath}} + \bar{\varphi} J$

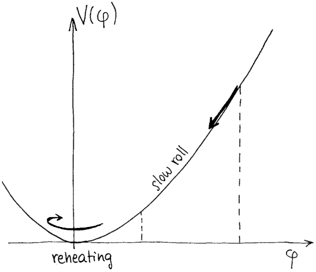
Heat bath density matrix: $\rho(t)$, $[\hat{H}_{\texttt{bath}}, \rho(0)] = 0$

$$i\partial_t \dot{\rho}(t) = [\hat{H}(t), \rho(t)]$$

$$\overset{\text{linear}}{\underset{\text{response}}{\Rightarrow}} \quad \langle J(t) \rangle = -\int_0^t dt_1\, \bar{\varphi}(t_1) \underbrace{G_{\mathrm{R}}(t-t_1)}_{\substack{\text{retarded} \\ \text{correlator}}} + \mathcal{O}(J^3)$$

$$\equiv \theta(t-t_1)\, \langle i[J(t), J(t_1)] \rangle_0$$

$\overset{\texttt{eom}}{\curvearrowright} \quad \overset{t\to\omega}{\curvearrowright} \quad \varphi(t)\theta(t) \approx \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega t}\, \mathcal{G}[\omega, \varphi^{(n)}(0)]}{\omega^2 + 3iH\omega - m^2 + G_{\mathrm{R}}(\omega)}$



$$V(\bar{\varphi}) \approx \frac{1}{2} m^2 \bar{\varphi}^2 + \mathcal{O}(\bar{\varphi}^4)$$

thermal corrections relevant at $\omega \sim m \Rightarrow G_{\mathrm{R}} \to G_{\mathrm{R}}(m)$

evolution equations

inflaton:

$$\ddot{\bar{\varphi}} + (3H + \Upsilon)\dot{\bar{\varphi}} + \partial_\varphi V(\bar{\varphi}, m_{\mathtt{T}}) \approx 0$$

$$\Upsilon \approx \frac{\mathtt{Im} G_{\mathtt{R}}(m)}{m} \,, \qquad m_{\mathtt{T}}^2 \approx m^2 - \mathtt{Re} G_{\mathtt{R}}(m)$$

medium ("radiation"):

$$\dot{e}_r + 3H(e_r + p_r - T\partial_T V) - T\partial_T \dot{V} = \Upsilon \dot{\bar{\varphi}}^2$$

parametrize $e_r = e_r(T)$, $p_r = p_r(T)$

benchmark: non-Abelian axion-like inflation[12]

$$\mathcal{L} = -\frac{1}{2}\,\partial^\mu\varphi\,\partial_\mu\varphi - V(\varphi) - \varphi J + \mathcal{L}_{\texttt{bath}}$$

* topological interaction term: $J = \frac{\alpha}{16\pi f_a}\epsilon^{\mu\nu\rho\sigma}F^c_{\mu\nu}F^c_{\rho\sigma}$

  f_a decay const. , $c \in \{1, \ldots, N_c^2 - 1\}$

* periodic potential: $V(\varphi) = m^2 f_a^2 \left[1 - \cos\left(\frac{\varphi}{f_a}\right)\right]$

$\Rightarrow$ $\varphi \to \varphi + 2\pi f_a$ symmetry, corrections are non-perturbative

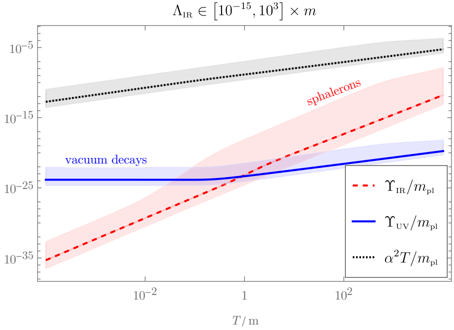
12 e.g. W. De Rocco, P.W. Graham and S. Kalia, JCAP 11 (2021) 011

friction coefficient $\Upsilon \approx \Upsilon_{\text{IR}} + \Upsilon_{\text{UV}}$

* $m \lesssim \alpha N_c T$: non-perturbative sphaleron dynamics[13]

$$\Upsilon_{\text{IR}} \sim \frac{\alpha^5 (N_c T)^3}{f_a^2} \left[1 + \left(\frac{m}{c_{\text{IR}}\alpha^2 N_c^2 T}\right)^2\right] \left[1 + \left(\frac{m}{c_{\text{M}}\alpha^2 N_c^2 T}\right)^2\right]^{-1}$$

* $m \gg \pi T$: perturbative decays $\varphi \to gg$,[14,15] $\Upsilon_{\text{UV}} \sim \frac{\alpha^2 m^3}{f_a^2}$

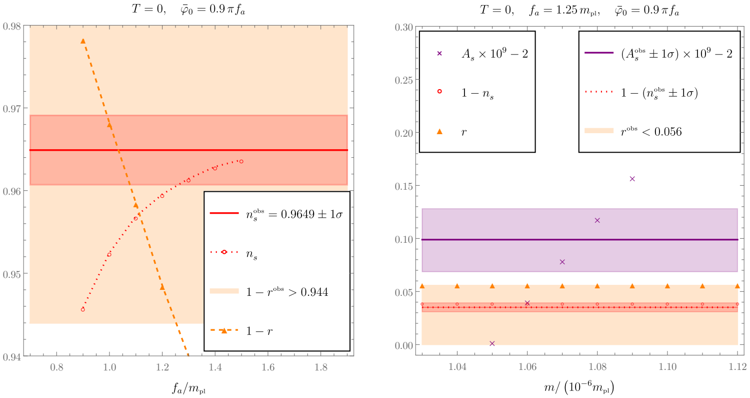


[13] M. Laine, L. Niemi, S. Procacci and K. Rummukainen, JHEP 11 (2022) 126

[14] S. Caron-Huot, Phys. Rev. D 79 (2009) 125009

[15] A. L. Kataev, N. V. Krasnikov and A. A. Pivovarov, Nucl. Phys. B 198 (1982) 508

inflaton mass m and decay rate f_a from CMB constraints



cold, non-Abelian axion-like inflation in agreement with Planck data within 2σ[16]

[16] Y. Akrami et al. [Planck], Astron. Astrophys. 641 (2020), A10