Gravitational Signatures of Domain Walls

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Based on work in progress in collaboration with Simone Blasi, Alberto Mariotti, Miguel Vanvlasselaer

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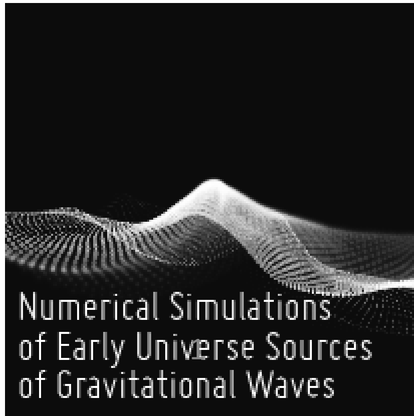

Exciting times

- Era of Gravitational Wave (GW) astronomy
- LIGO-Virgo-KAGRA registers GW events from **binary systems**
- Evidence of **GW background** reported by PTA consortium
- Future **third generation** detectors

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Characterizing with **high precision** GW background from all sources

$\mathcal{C}$osmo$\mathcal{L}$attice

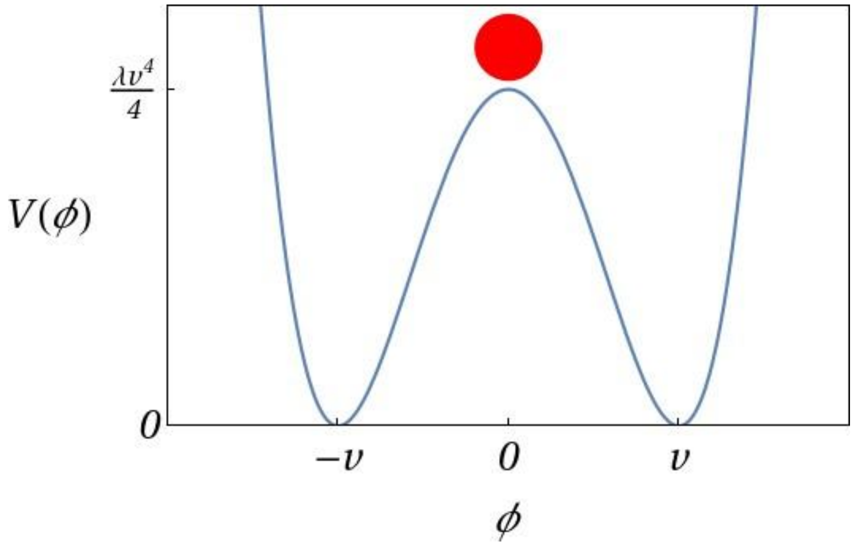
- GW spectrum through **EoM**
- GW spectrum through **equal time correlator** (ETC)
- GW spectrum in different **cosmologies**

Domain walls

Def.: **Topological defects** from spontaneously broken **discrete symmetry**

$\mathbb{Z}_2 : \phi \rightarrow -\phi$

$$V = \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2$$



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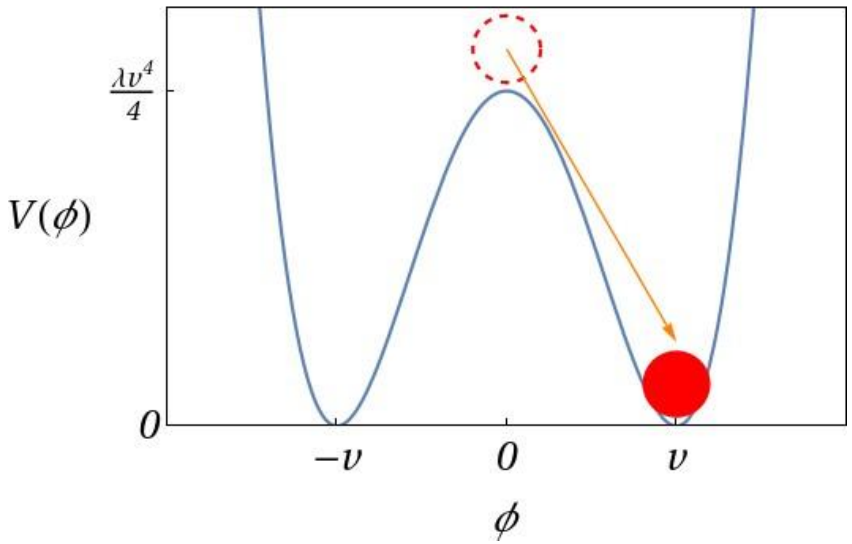
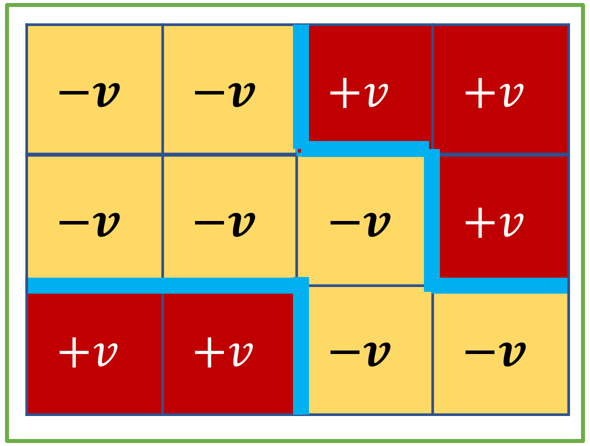


Universe



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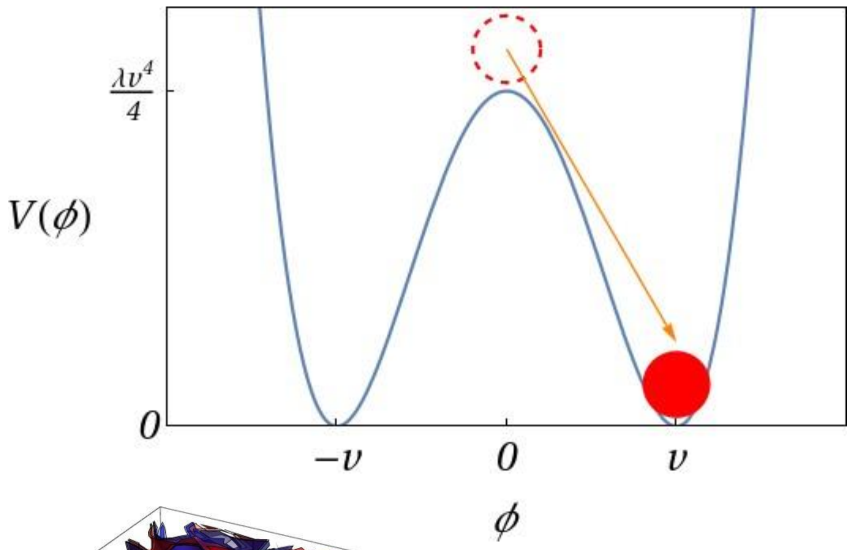

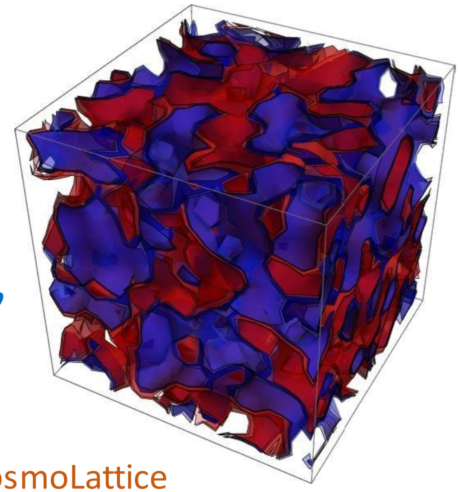

Blue: $-v$

Red: $+v$

Creation of uncorrelated **domains** with **different** vacuum expectation values

credits: CosmoLattice

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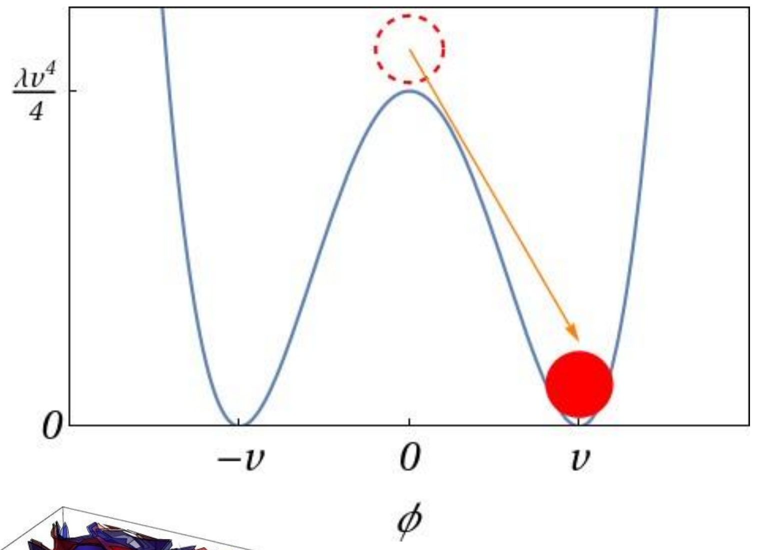

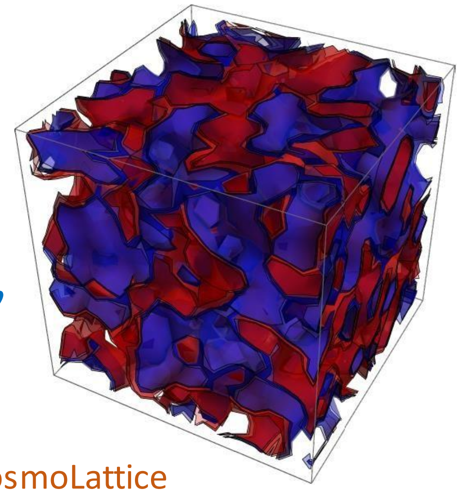

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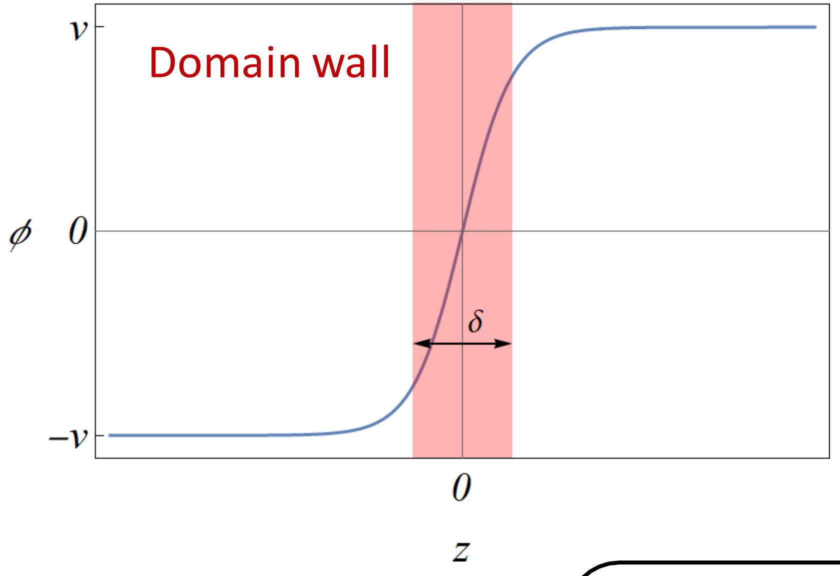
But what is a domain wall?



- Width $\delta \approx \left(\sqrt{\frac{\lambda}{2}}\, v\right)^{-1} \sim m_\phi^{-1}$
- Tension $\sigma \sim m_\phi v^2$

Domain wall:

Large energy density **localized** in 2D **surface**

Model and numerical details

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}(\phi^2 - \eta^2)^2$$

Expanding Universe

$$ds^2 = a^2(\tau)\left(d\tau^2 - dx^2\right)$$

$$a(\tau) = a(\tau_i)\left(1 + \frac{\mathcal{H}_i}{p}(\tau - \tau_i)\right)^p$$

$$\mathcal{H} = a'/a \qquad p = \frac{2}{3(1+w)-2}$$

Equation of State

Equation of Motion

$$\phi = \tilde{\phi}\eta \qquad x = \tilde{x}/m \qquad \tau = \tilde{\tau}/m$$

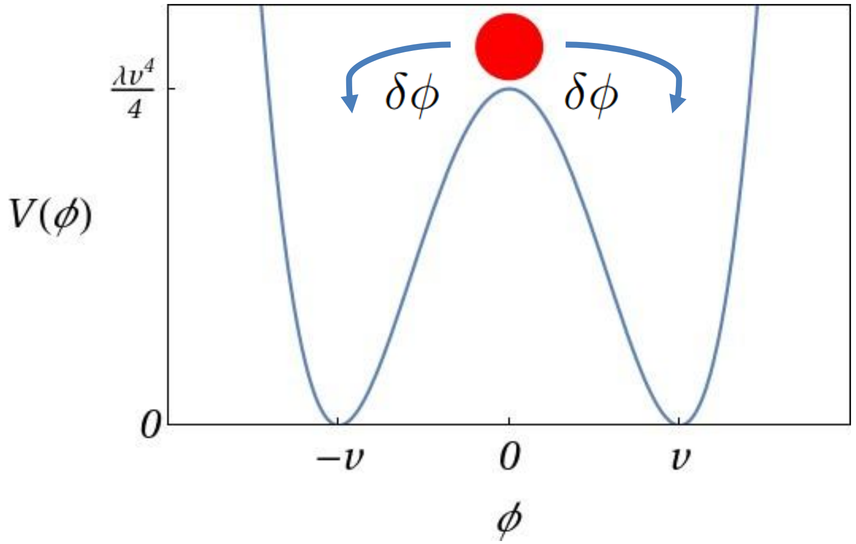
$$\frac{\partial^2\tilde{\phi}}{\partial\tilde{\tau}^2} + 2\left(\frac{\mathcal{H}}{m}\right)\frac{\partial\tilde{\phi}}{\partial\tilde{\tau}} - \tilde{\nabla}^2\tilde{\phi} + \frac{a^2}{2}\tilde{\phi}\left(\tilde{\phi}^2 - 1\right) = 0$$

Only relevant **scale separation**

Model and numerical details

Initial conditions

$$\bar{\phi} = 0$$



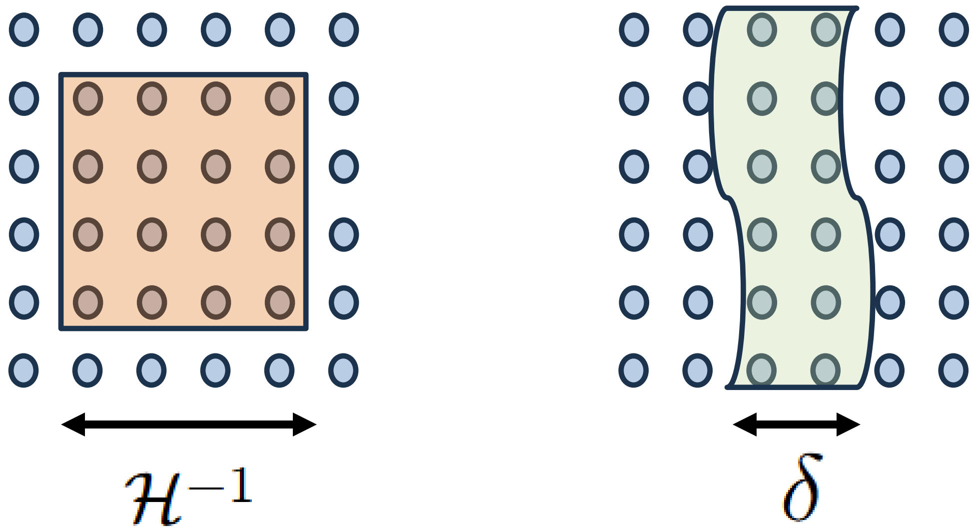
Small Gaussian fluctuations in Fourier space drawn from **white noise** spectrum

$$\langle \delta\phi(\mathbf{k})\delta\phi(\mathbf{q})\rangle \equiv (2\pi)^3 \mathcal{P}_{\delta\phi}(k)\delta(\mathbf{k}-\mathbf{q})$$

$$\sqrt{\langle \delta\phi^2(\mathbf{x})\rangle} = \sqrt{\int_0^{k_{\rm cut}} \frac{k^3}{2\pi^2}\mathcal{P}_{\delta\phi}(k)\, \mathrm{d}\log k} = 0.1\eta$$

Constant

Lattice limitations



At least **one Hubble patch**

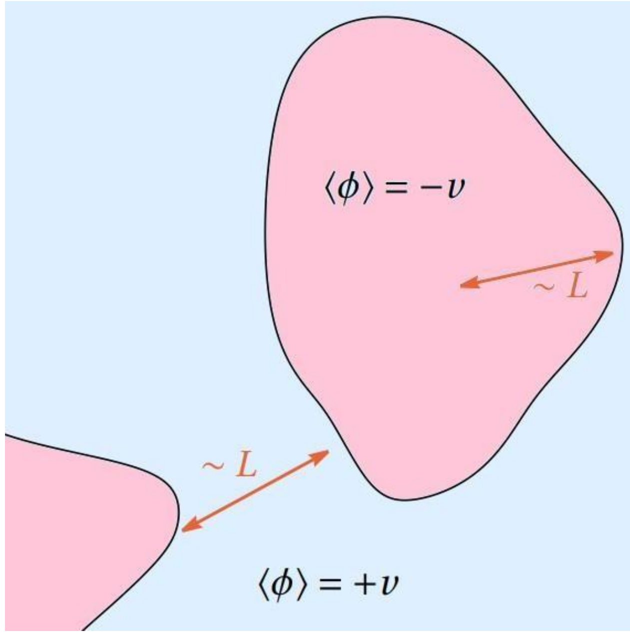
Resolution by **2 grid points**

Determines **final simulation time** and **box size**

Domain wall dynamics: the scaling regime

- **Interplay** between expansion of the Universe and recombination of domain wall network

- Characteristic **length scale** L
  - Curvature radius
  - Average distance



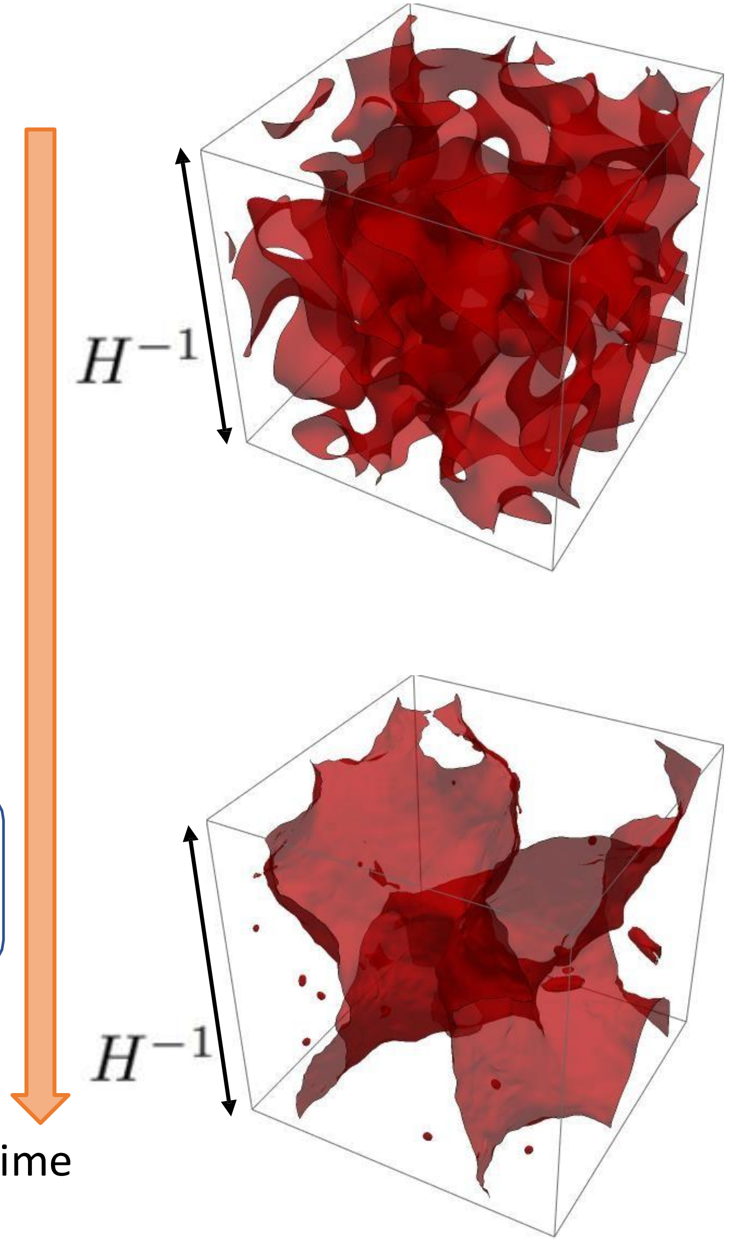
Numerical

- Press et al., Astrophys. J., 1989
- Hindmarsh et al., PRD, 2003
- Avelino et al., PRD, 2005
- Avelino et al., PLB, 2005
- Martins et al., [1110.3486], PRD
- …

Analytical

- Hindmarsh, PRL, 1996
- Hindmarsh, PRD, 2003
- …

Scaling regime $L \sim H^{-1} \sim t$



Domain wall dynamics: the scaling regime

$$\rho_{\rm dw} \sim \frac{\sigma L^2}{H^{-3}} = 2\mathcal{A}\sigma H$$

Number of domain walls per Hubble volume
Scaling ~ O(1)

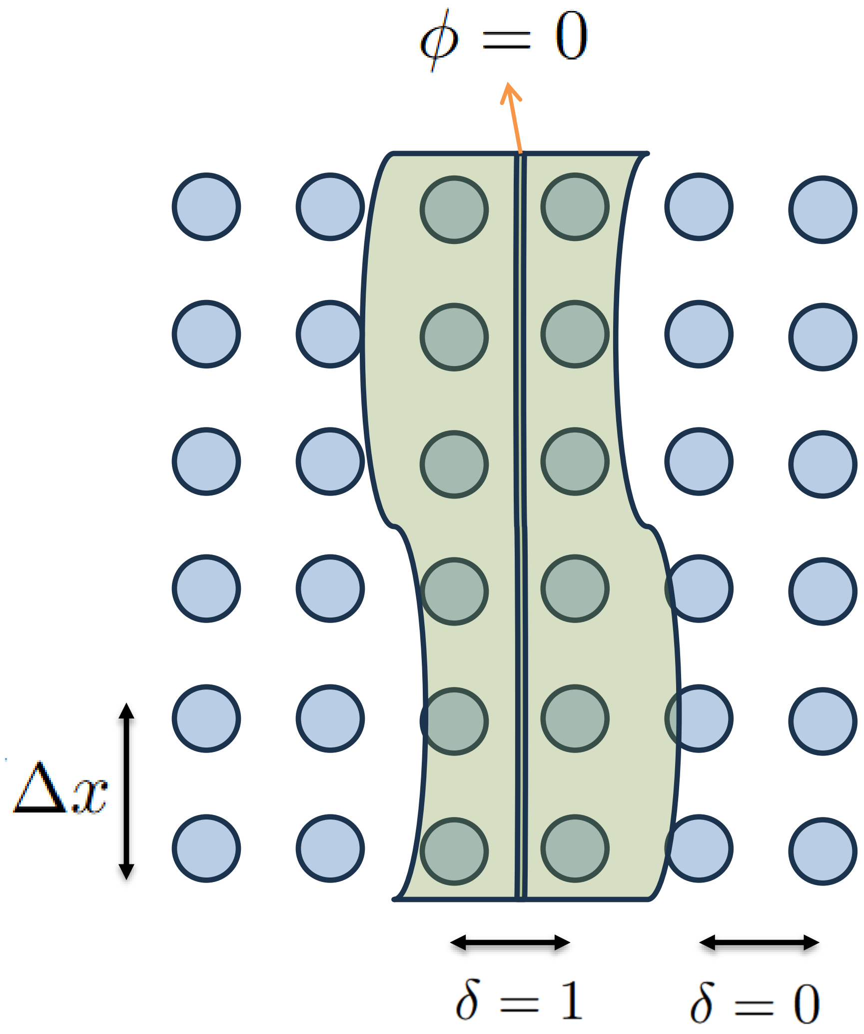
Numerical

$$\rho_{\rm dw} = \frac{\sigma A}{a(\tau)V}$$

$$A = (\Delta x)^2 \sum_{\rm links} \delta \frac{|\nabla\phi|}{\left|\frac{\partial\phi}{\partial x}\right| + \left|\frac{\partial\phi}{\partial y}\right| + \left|\frac{\partial\phi}{\partial z}\right|}$$

Press et al., Astrophys. J., 1989

Curvature



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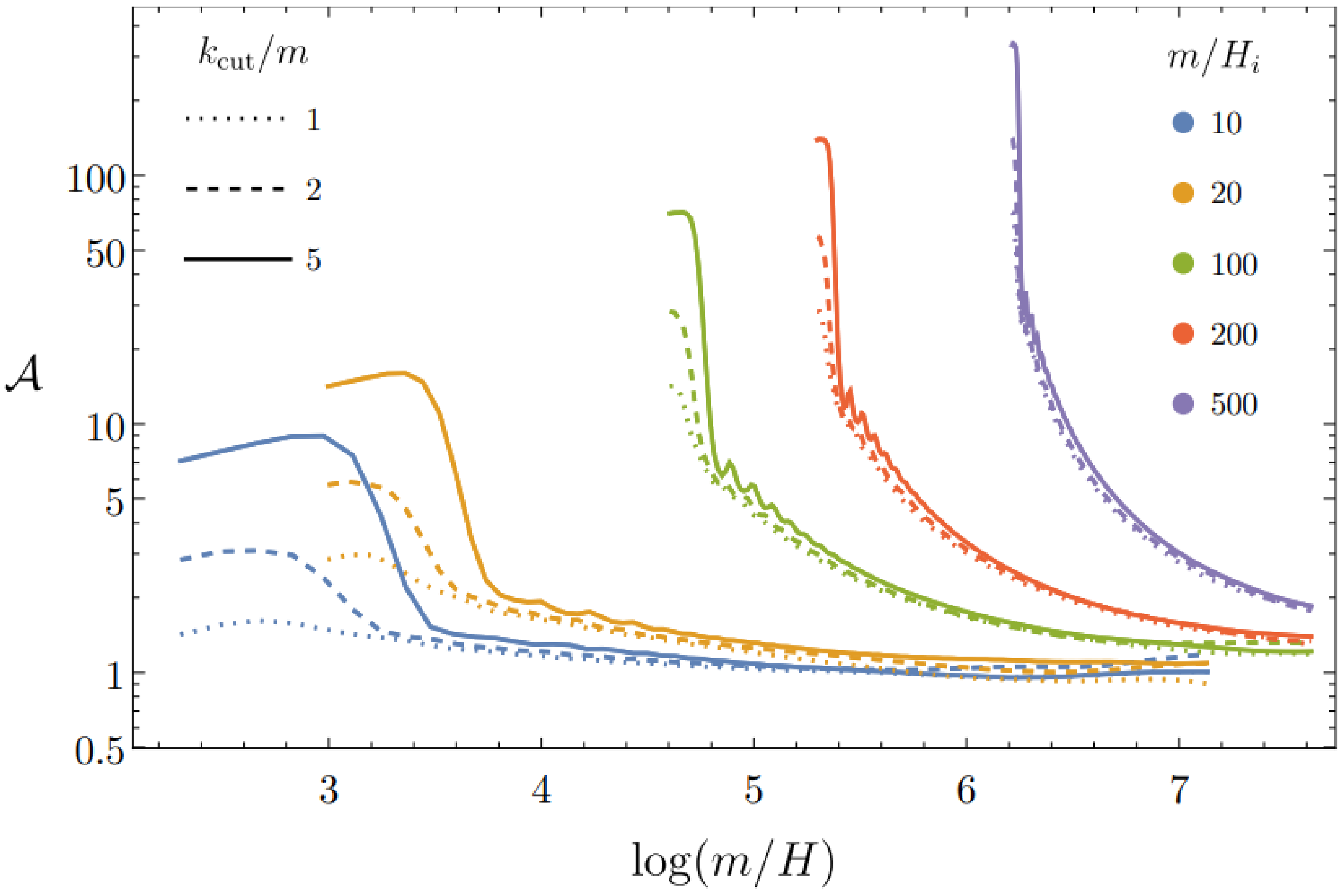


Domain wall dynamics: the scaling regime

Dynamics described by **VOS model**

Chopping parameter

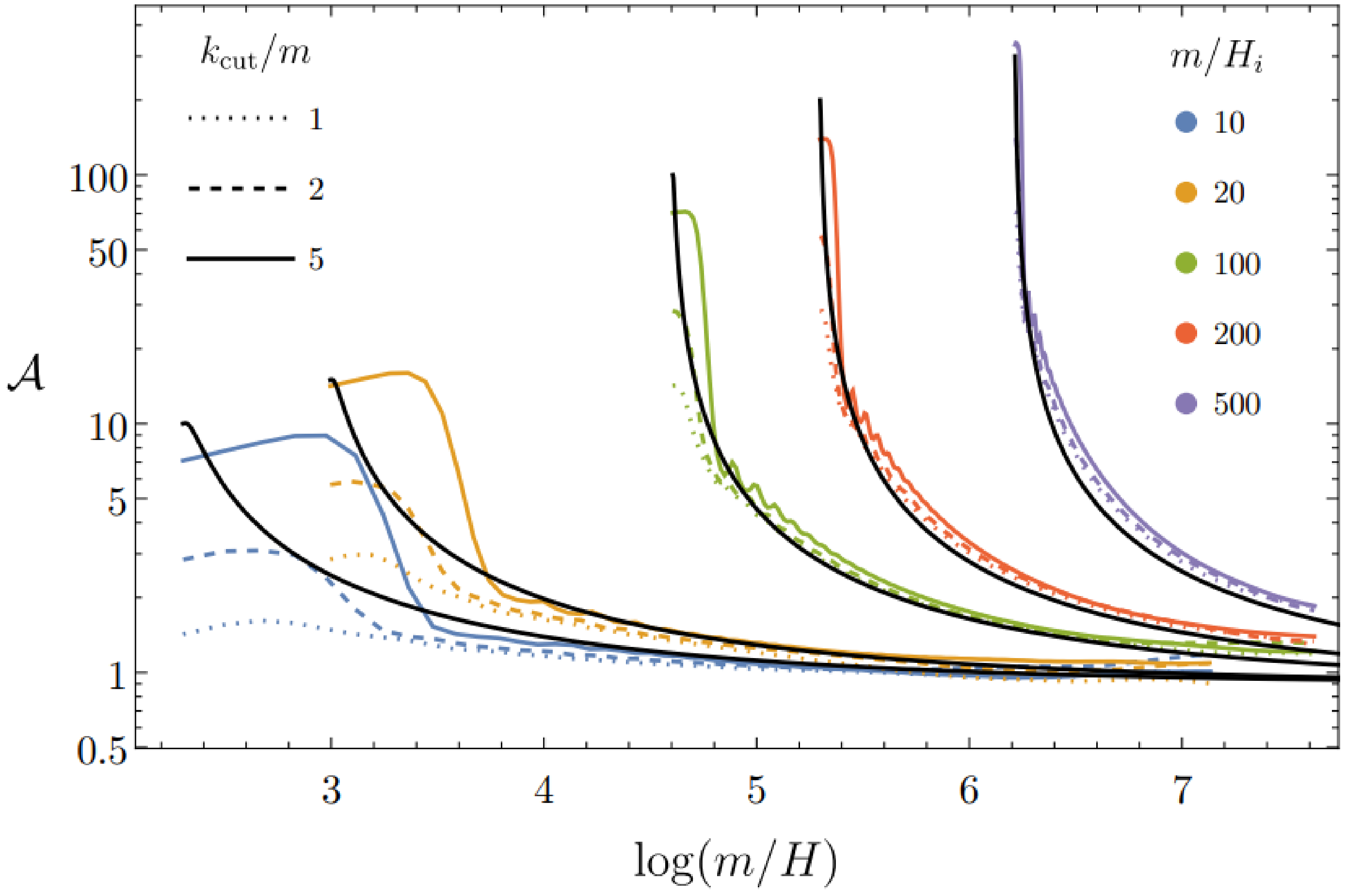
$$\frac{dL}{dt} = (1+3v^2)HL + c_w v$$

$$\frac{dv}{dt} = (1-v^2)\left(\frac{k_w(v)}{L} - 3Hv\right)$$

curvature/momentum parameter

$$\mathcal{A}(t) = (t/t_0)/L(t)$$

Martins et al., PRD, 2016



Gravitational wave spectrum: EoM approach

$\mathcal{C}\mathbf{osmo}\mathcal{L}\mathbf{attice}$: solve the EoM of the GW perturbation

$$\frac{\partial^2 h_{ij}}{\partial \tau^2} + 2\mathcal{H}\frac{\partial h_{ij}}{\partial \tau} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{TT}$$

GW perturbation

$$(\partial_i \phi \partial_j \phi)^{TT}$$

$$\rho_{\rm gw} = \frac{1}{32\pi G a^2(\tau)} \left\langle \frac{\partial h_{ij}}{\partial \tau} \frac{\partial h_{ij}}{\partial \tau} \right\rangle$$

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$\mathcal{C}$osmo$\mathcal{L}$attice

$$\rho_{\rm gw}(t) = \frac{1}{64\pi^3 G}\int\frac{{\rm d}k}{k}k^3 P_{\dot h}(k,t)$$

$$\langle \dot h_{ij}(\mathbf{k},t)\dot h_{ij}(\mathbf{k}',t)\rangle = (2\pi)^3 P_{\dot h}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

$$\Omega_{\rm gw} = \frac{1}{\rho_c}\frac{{\rm d}\rho_{\rm gw}}{{\rm d}\log k} = \frac{1}{\rho_c}\frac{k^3}{64\pi^3 G}P_{\dot h}(k,t)$$

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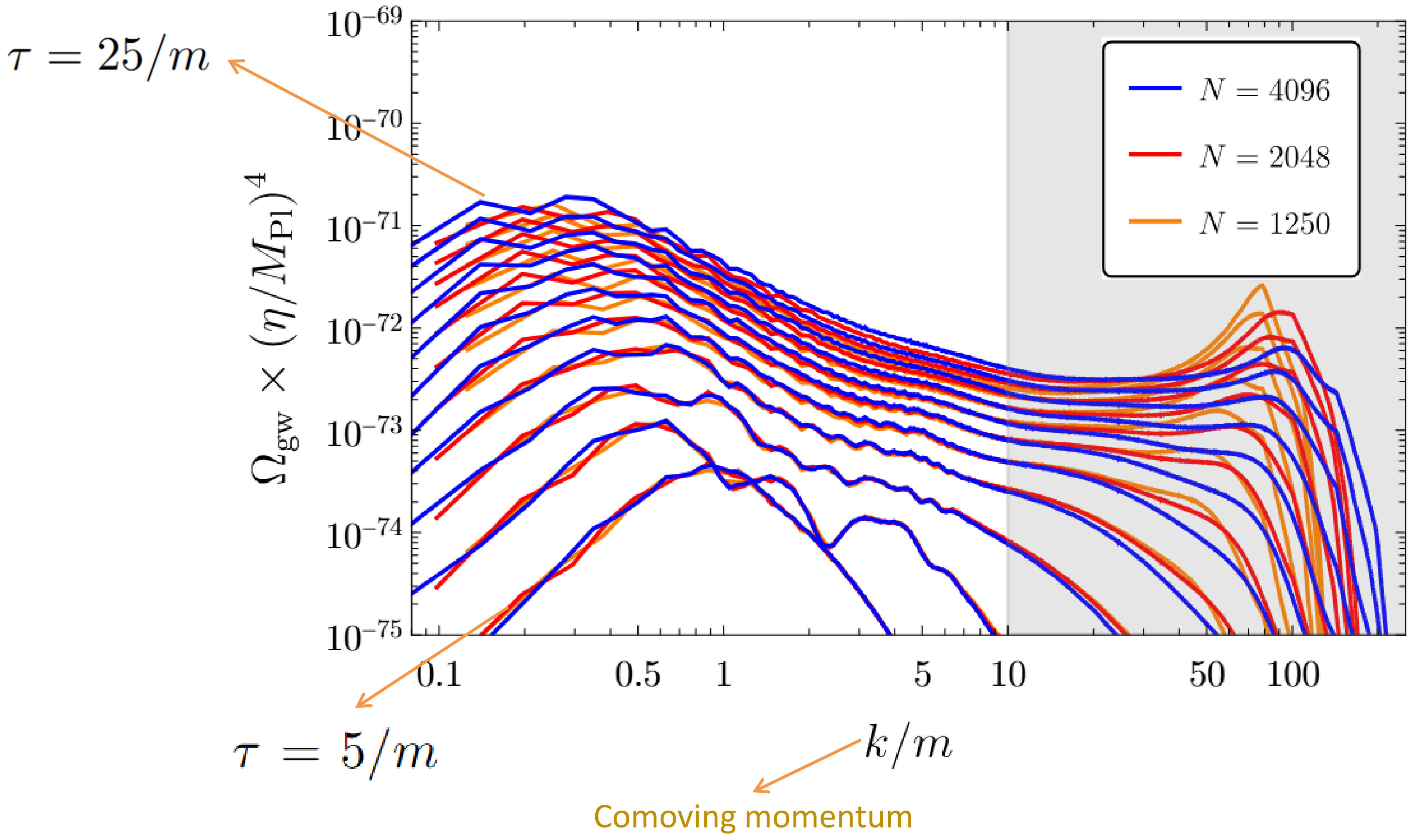
Numerical setup:

- **White noise** spectrum with $k_{\rm cut} = m$
- Initial Hubble size $H_i = m$
- 8 Hubble volumes at the end
- 2 grid point resolution
- **Radiation** dominated Universe

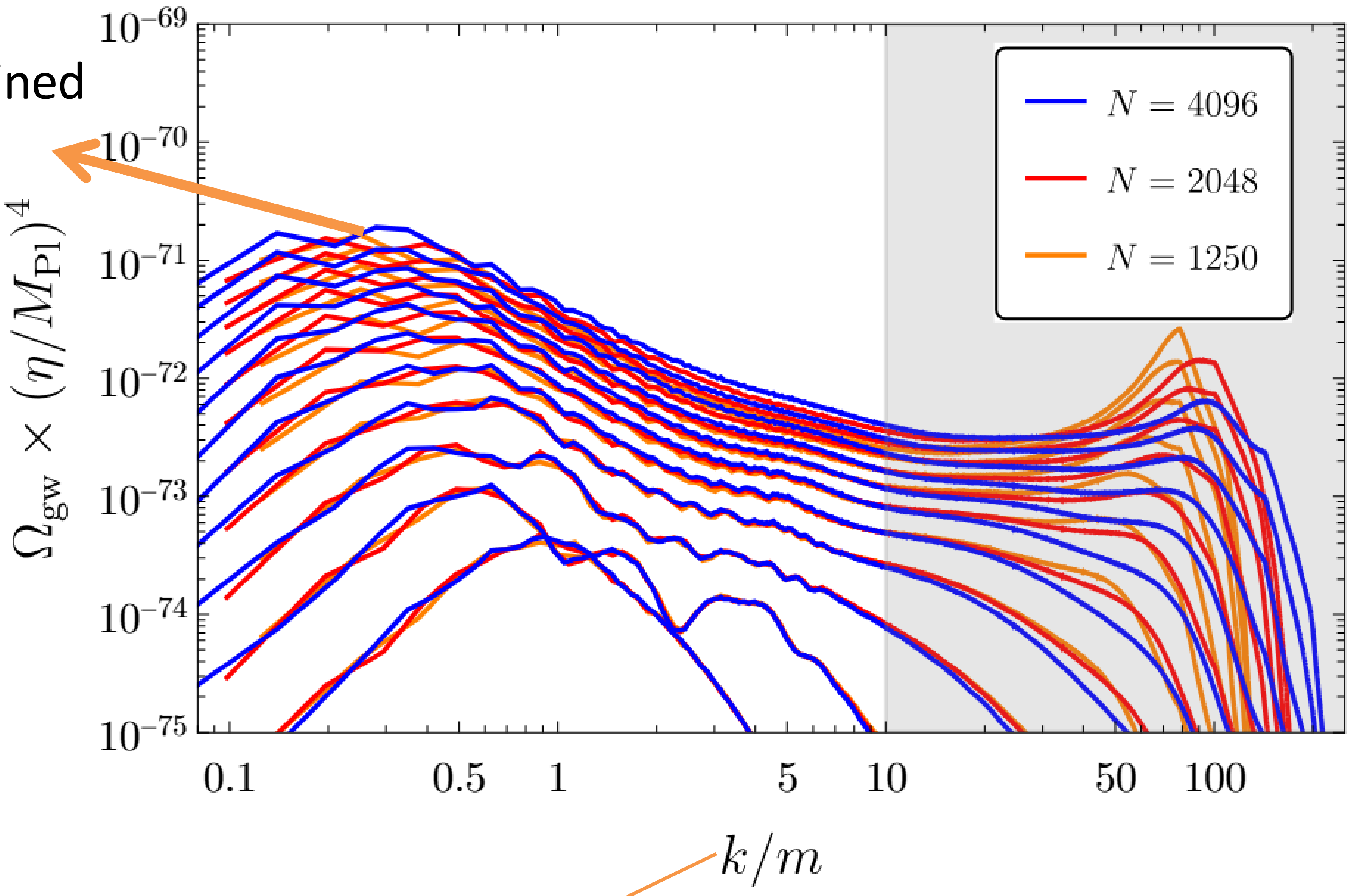
Gravitational wave spectrum: EoM approach



Gravitational wave spectrum: EoM approach

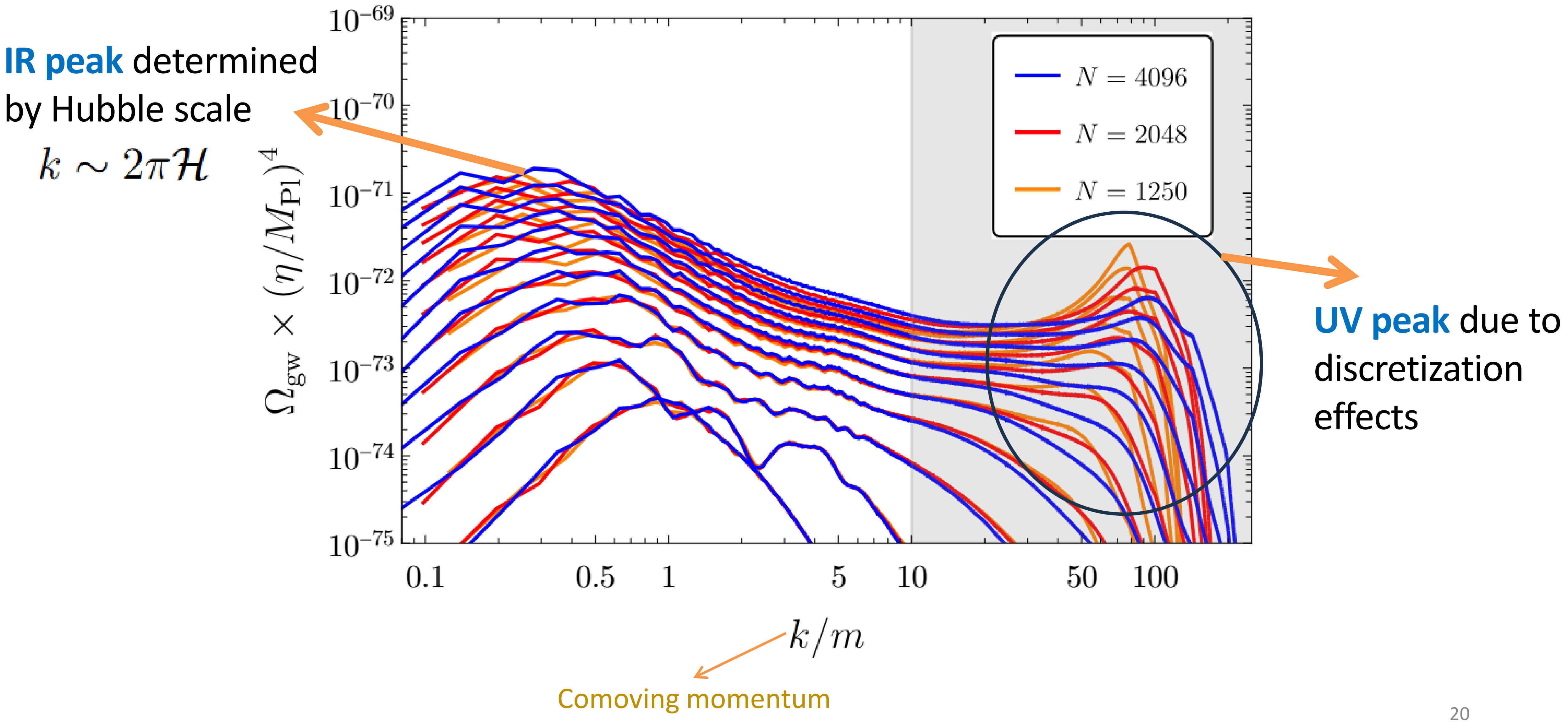
IR peak determined by Hubble scale

$k \sim 2\pi\mathcal{H}$



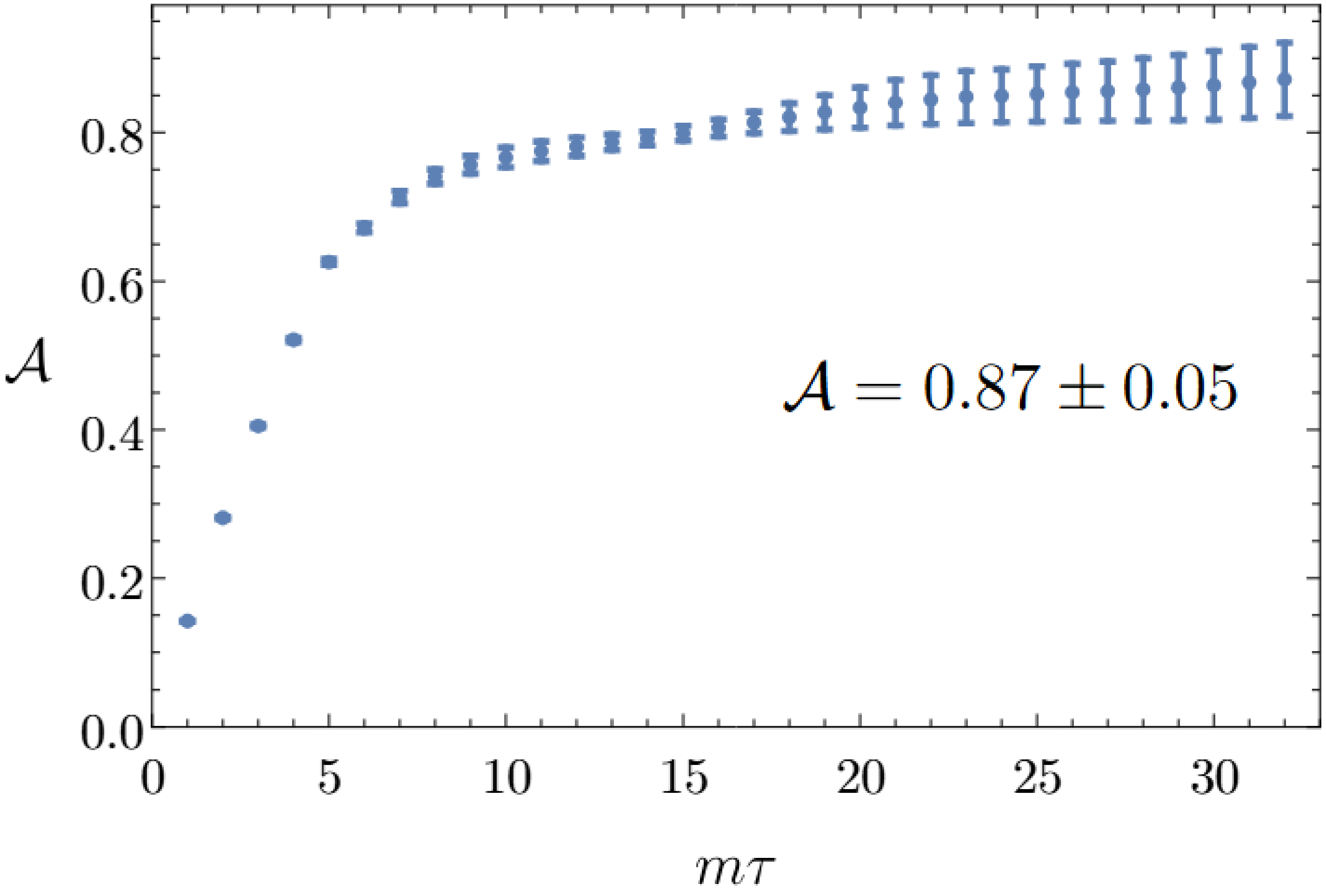
Comoving momentum

Gravitational wave spectrum: EoM approach



Gravitational wave spectrum: EoM approach

- Fit for N = 2048 simulation (5 simulations)



- Scaling reached at the end of the simulation

- Consistent with values obtained in Saikawa et al., JCAP, 2014 and Vikman et al., JCAP, 2024

Gravitational wave spectrum: EoM approach

- Fit for N = 2048 simulation (broken power law)

$$\Omega_{\rm gw}(\tau,k) = \Omega_{\rm gw}^{\rm peak}(\tau) \times \mathcal{S}\left(\frac{k}{2\pi\mathcal{H}}\right)$$

$$\Omega_{\rm gw}^{\rm peak}(\tau) = \frac{\tilde{\epsilon}_{\rm gw} G \mathcal{A}^2 \sigma^2}{\rho_c(\tau)}$$

$$\mathcal{S}(x) \equiv \frac{(3+b)^c}{\left(b\left(\frac{x}{x_{\rm p}}\right)^{-3/c} + 3\left(\frac{x}{x_{\rm p}}\right)^{b/c}\right)^c}$$

- b : subhorizon modes slope
- c : width of the peak
- $x_{\rm p}$: position of the peak
- $\tilde{\epsilon}_{\rm gw}$: GW efficiency

Gravitational wave spectrum: EoM approach

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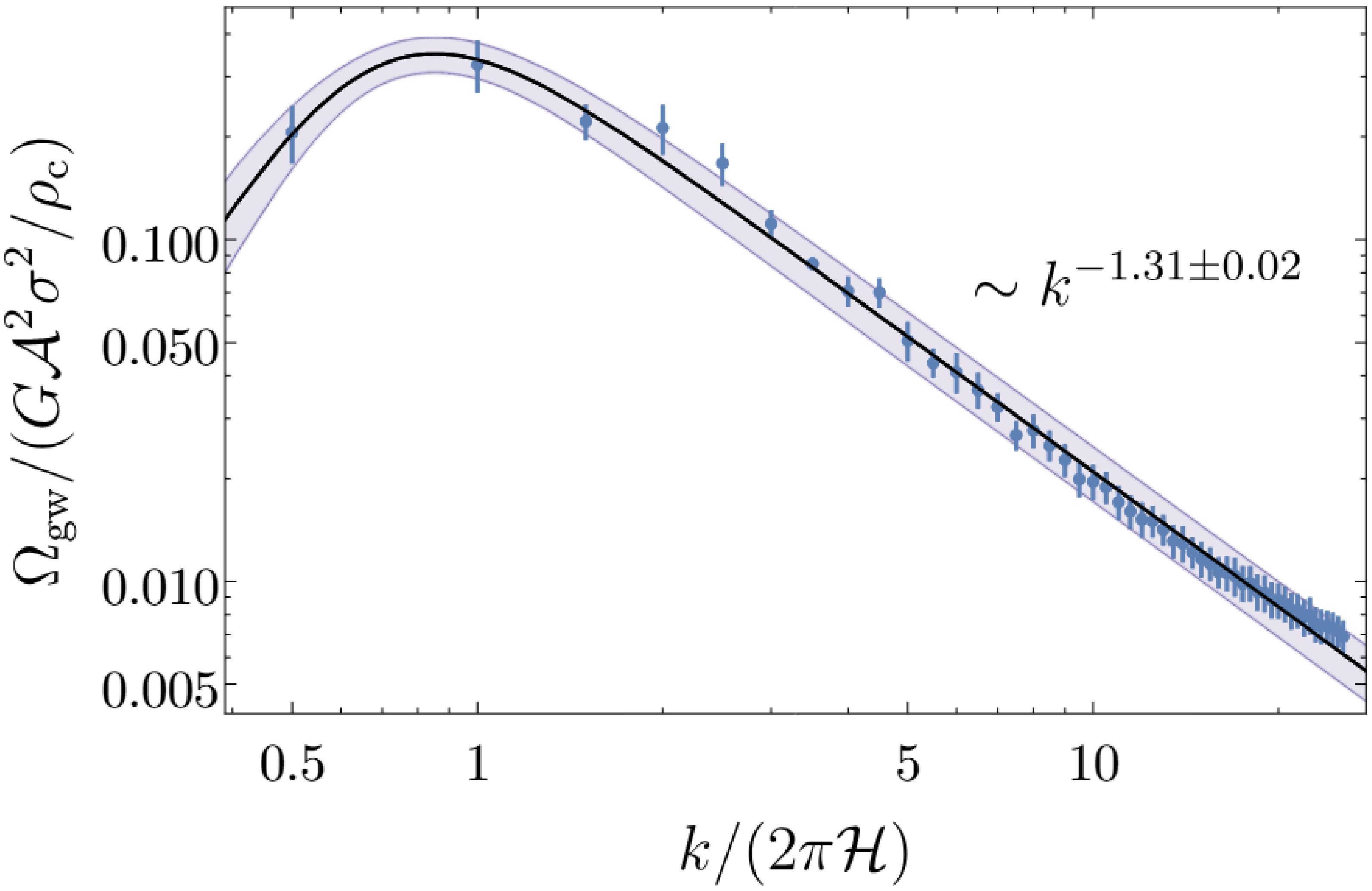

- Fit done at final simulation time
- GW data used up to momentum cutoff

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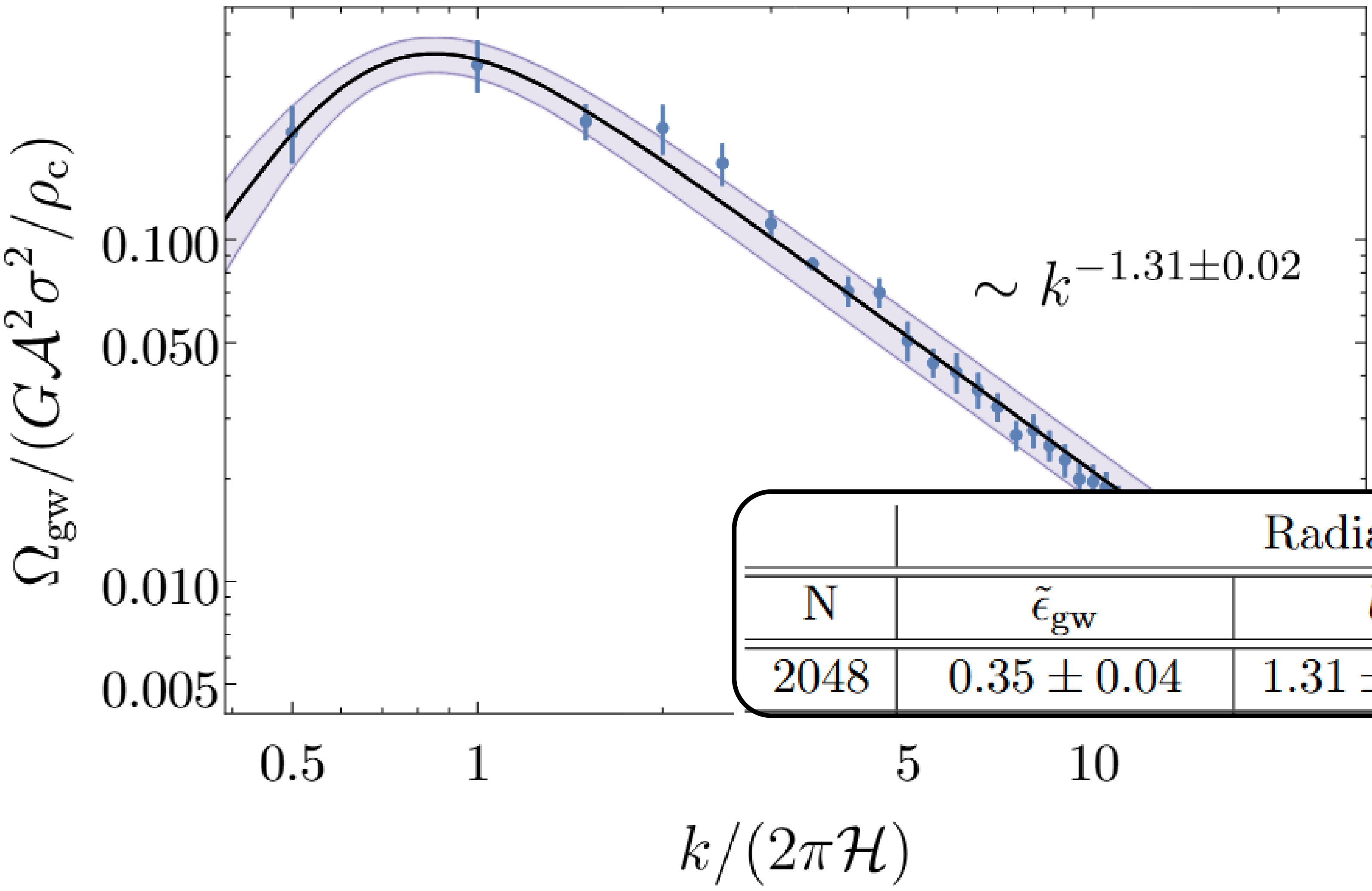

	Radiation domination			
N	$\tilde{\epsilon}_{gw}$	b	c	x_p
2048	0.35 ± 0.04	1.31 ± 0.02	1.15 ± 0.33	0.85 ± 0.04

- Fit done at final simulation time
- GW data used up to momentum cutoff

Gravitational wave spectrum: ETC approach

CosmoLattice **+ modifications**: Equal Time Correlator

$$\frac{\partial^2 h_{ij}}{\partial \tau^2} + 2\mathcal{H}\frac{\partial h_{ij}}{\partial \tau} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{TT}$$

$$\tilde{h}_{ij} = a h_{ij} \qquad T_{ij}^{TT} = a^2 \rho_S \Pi_{ij}^{TT}$$

Anisotropic stress

Caprini et al., PRD, 2009

Energy density of source

Fourier space: $$\tilde{h}''_{ij} + \left(k^2 - \frac{a''}{a}\right)\tilde{h}_{ij} = 16\pi G a^3 \rho_S \Pi_{ij}^{TT}$$

Solution: $$\tilde{h}_{ij}(\mathbf{k},\tau) = 16\pi G \int_{\tau_i}^{\tau} d\tau' \frac{\sin(k(\tau-\tau'))}{k} a^3(\tau')\rho_S(\tau')\Pi_{ij}^{TT}(\mathbf{k},\tau')$$

Gravitational wave spectrum: ETC approach

GW energy density: $$\rho_{\rm gw}(\tau) = \frac{1}{32\pi G a^4(\tau)} \left\langle \left(\tilde{h}'_{ij} - \mathcal{H}\tilde{h}_{ij} \right)^2 (\tau, \mathbf{x}) \right\rangle$$

Subhorizon modes $\tilde{h}'_{ij} \sim k\tilde{h}_{ij} \gg \mathcal{H}\tilde{h}_{ij}$

$$\rho_{\rm gw}(\tau) \approx \frac{1}{32\pi G a^4(\tau)} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{d^3\mathbf{q}}{(2\pi)^3} e^{-i\mathbf{x}\cdot(\mathbf{k}-\mathbf{q})} \langle \tilde{h}'_{ij}(\mathbf{k},\tau) \tilde{h}'^{\star}_{ij}(\mathbf{q},\tau) \rangle$$

Substitute solution

$$\frac{{\rm d}\rho_{\rm gw}}{{\rm d}\ln k}(\tau, k) = \frac{2G}{\pi} \frac{k^3}{a^4(\tau)} \int_{\tau_i}^{\tau} {\rm d}\tau_1 \int_{\tau_i}^{\tau} {\rm d}\tau_2 \; a^3(\tau_1) a^3(\tau_2) \rho_S(\tau_1) \rho_S(\tau_2) \cos\left[k(\tau_1 - \tau_2)\right] \Pi^2(k, \tau_1, \tau_2)$$

Gravitational wave spectrum: ETC approach

$$\frac{d\rho_{\rm gw}}{d\ln k}(\tau,k) = \frac{2G}{\pi}\frac{k^3}{a^4(\tau)}\int_{\tau_i}^{\tau} d\tau_1 \int_{\tau_i}^{\tau} d\tau_2\; a^3(\tau_1)a^3(\tau_2)\rho_S(\tau_1)\rho_S(\tau_2)\cos\left[k(\tau_1-\tau_2)\right]\Pi^2(k,\tau_1,\tau_2)$$

$$\langle \Pi_{ij}^{TT}(\mathbf{k},\tau_1)\Pi_{ij}^{TT\star}(\mathbf{q},\tau_2)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{q})\Pi^2(k,\tau_1,\tau_2)$$

Unequal Time Correlator (UTC)

Gravitational wave spectrum: ETC approach

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Unequal Time Correlator (UTC)

- During **scaling**, the UTC can only depend on k through the variables $x = k\tau$

$$\Pi^2(k,\tau_1,\tau_2) = (\tau_1\tau_2)^{3/2}C^T(x_1,x_2)$$

Gravitational wave spectrum: ETC approach

- For domain wall system during **radiation domination**: $a(\tau) = \sqrt{\Omega^0_{\rm rad}} H_0 \tau$

$$\Omega^{\rm dw}_{\rm gw} \sim \frac{G\mathcal{A}^2\sigma^2}{\rho_c} \sim \tau^4$$

Constant

$$\Omega^{\rm dw}_{\rm gw}(\tau,k) = \frac{64}{3} G^2 \mathcal{A}^2 \sigma^2 \Omega_{\rm rad}(\tau) \Omega^0_{\rm rad} H_0^2 \tau^4 F^T_{\rm dw}(x)$$

$$F^T_{\rm dw}(x) = \frac{1}{x^4} \int_{x_i}^{x} {\rm d}x_1 \int_{x_i}^{x} {\rm d}x_2 \, (x_1 x_2)^{5/2} \cos(x_1 - x_2) C^T_{\rm dw}(x_1, x_2)$$

Determines the spectrum fully

Gravitational wave spectrum: ETC approach

Expected behavior UTC:

- Sharply **peaked along diagonal** $x_1 = x_2$
- **Decay** rapidly away from diagonal
- **Power law** along diagonal

$$C^T_{\rm dw}(x,x) \propto \frac{1}{x^q}$$

Figueroa et al ., JCAP, 2009
Hindmarsh et al, PRD, 2010
Albrecht et al, PRL, 1996
Figueroa et al, PRL, 2013

Assuming correlator strongly localized around diagonal:

$$F^T_{\rm dw}(x)\big|_{x\to\infty} \propto \begin{cases} x^{2-q} & q<6 \\ \dfrac{\log x}{x^4} & q=6 \\ x^{-4} & q>6 \end{cases}$$

Convergence spectrum implies

$$q > 2$$

Gravitational wave spectrum: ETC approach

We were able to compute the **ETC** using CosmoLattice **+ modifications**:

$$\langle T_{ij}^{\mathrm{TT}}(\mathbf{k},\tau)\, T_{ij}^{\mathrm{TT}\star}(\mathbf{q},\tau)\rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}-\mathbf{q})\, T_{\mathrm{dw}}^2(k,\tau,\tau)$$

$$T_{ij}^{TT} = a^2 \rho_S \Pi_{ij}^{TT}$$

Caprini et al., PRD, 2009

$$T_{\mathrm{dw}}^2(k,\tau,\tau) = a^4(\tau)\, \rho_{\mathrm{dw}}^2(\tau)\, \tau^3\, C_{\mathrm{dw}}^T(x,x)$$

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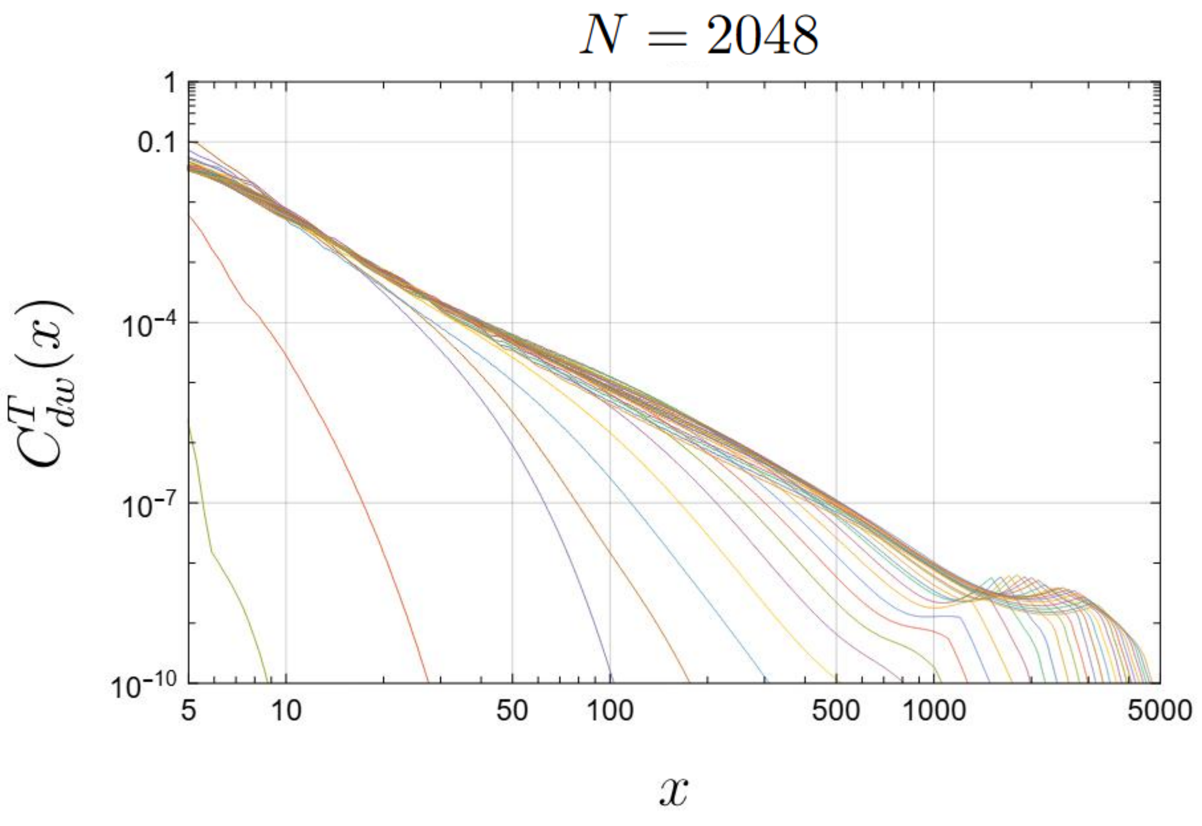
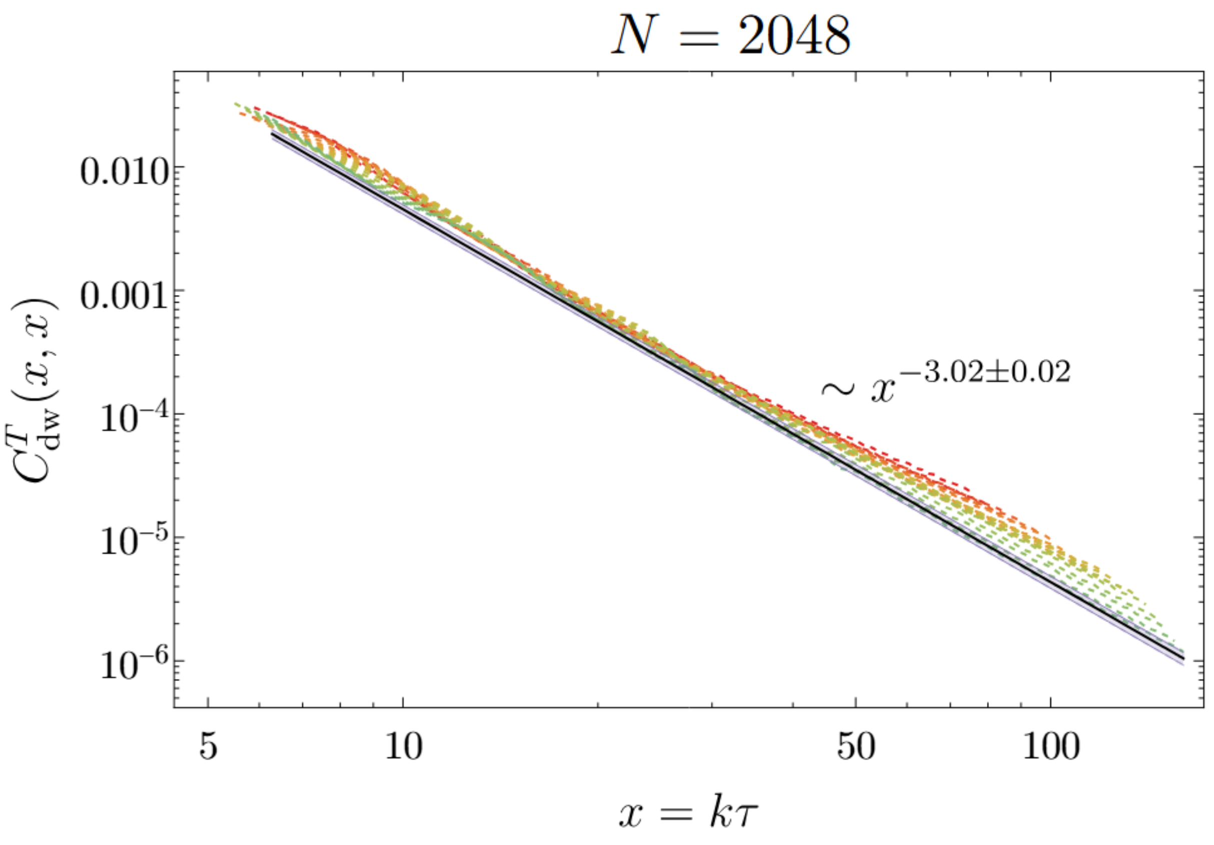
$$T_{\rm dw}^2(k,\tau,\tau) = a^4(\tau)\,\rho_{\rm dw}^2(\tau)\,\tau^3\, C_{\rm dw}^T(x,x)$$

FIT!

Gravitational wave spectrum: ETC approach

We fit up to the same **momentum cutoff** as in the **EoM** case





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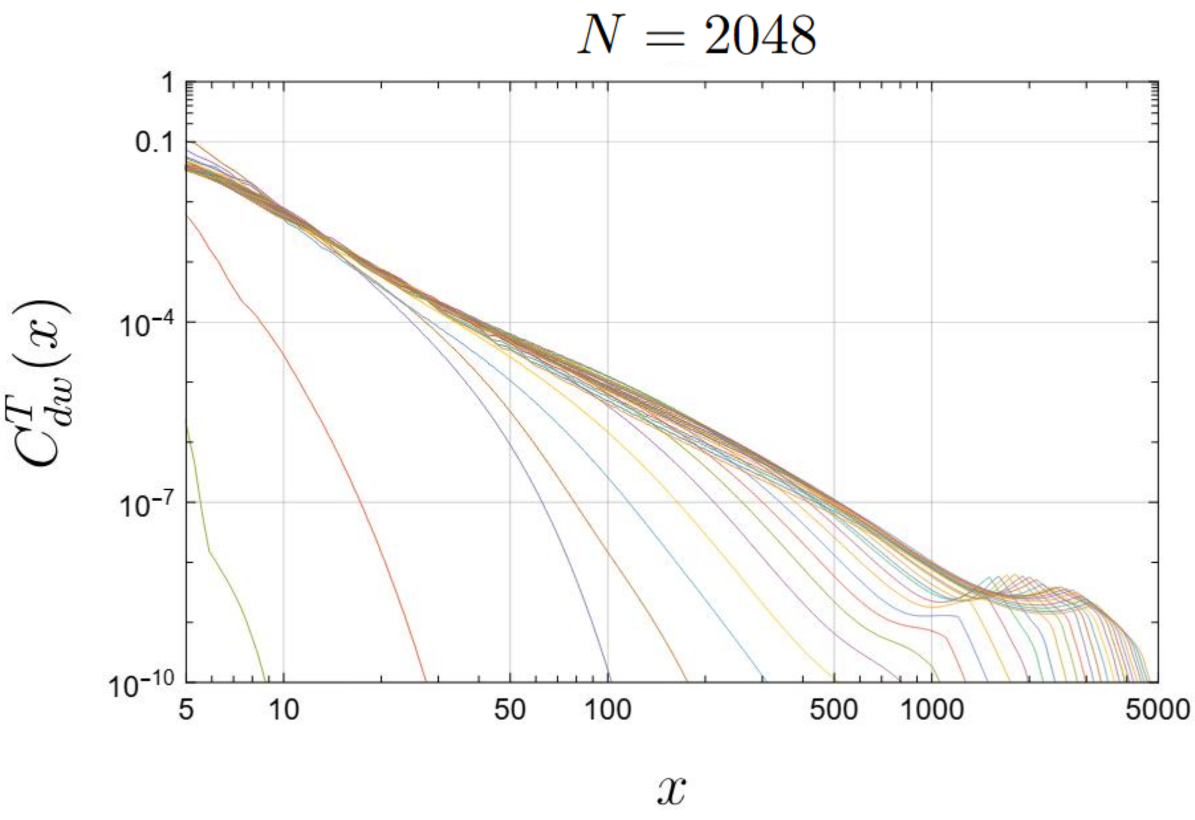

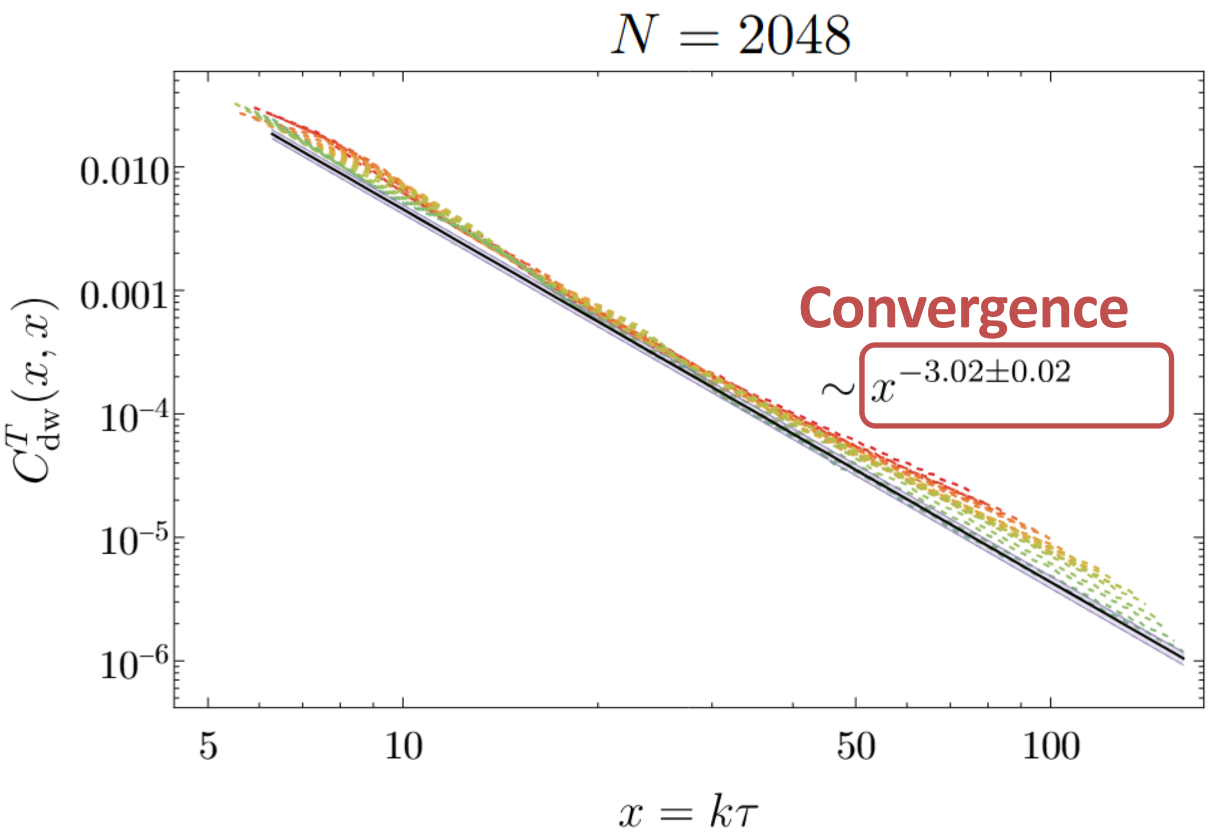

Gravitational wave spectrum: ETC approach

We were able to compute the **ETC** using $\mathcal{C}$osmo$\mathcal{L}$attice **+ modifications**:

$$\langle T_{ij}^{\rm TT}(\mathbf{k},\tau)\, T_{ij}^{\rm TT\star}(\mathbf{q},\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{q})\, T^2_{\rm dw}(k,\tau,\tau)$$

$$T_{ij}^{TT} = a^2 \rho_S \Pi_{ij}^{TT}$$

Caprini et al., PRD, 2009

$$T^2_{\rm dw}(k,\tau,\tau) = a^4(\tau)\,\rho^2_{\rm dw}(\tau)\,\tau^3\, C^T_{\rm dw}(x,x)$$

FIT!

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$$\langle T_{ij}^{\rm TT}(\mathbf{k},\tau)\, T_{ij}^{\rm TT\star}(\mathbf{q},\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{q})\, T_{\rm dw}^2(k,\tau,\tau)$$

$$T_{ij}^{TT} = a^2\rho_S\Pi_{ij}^{TT}$$

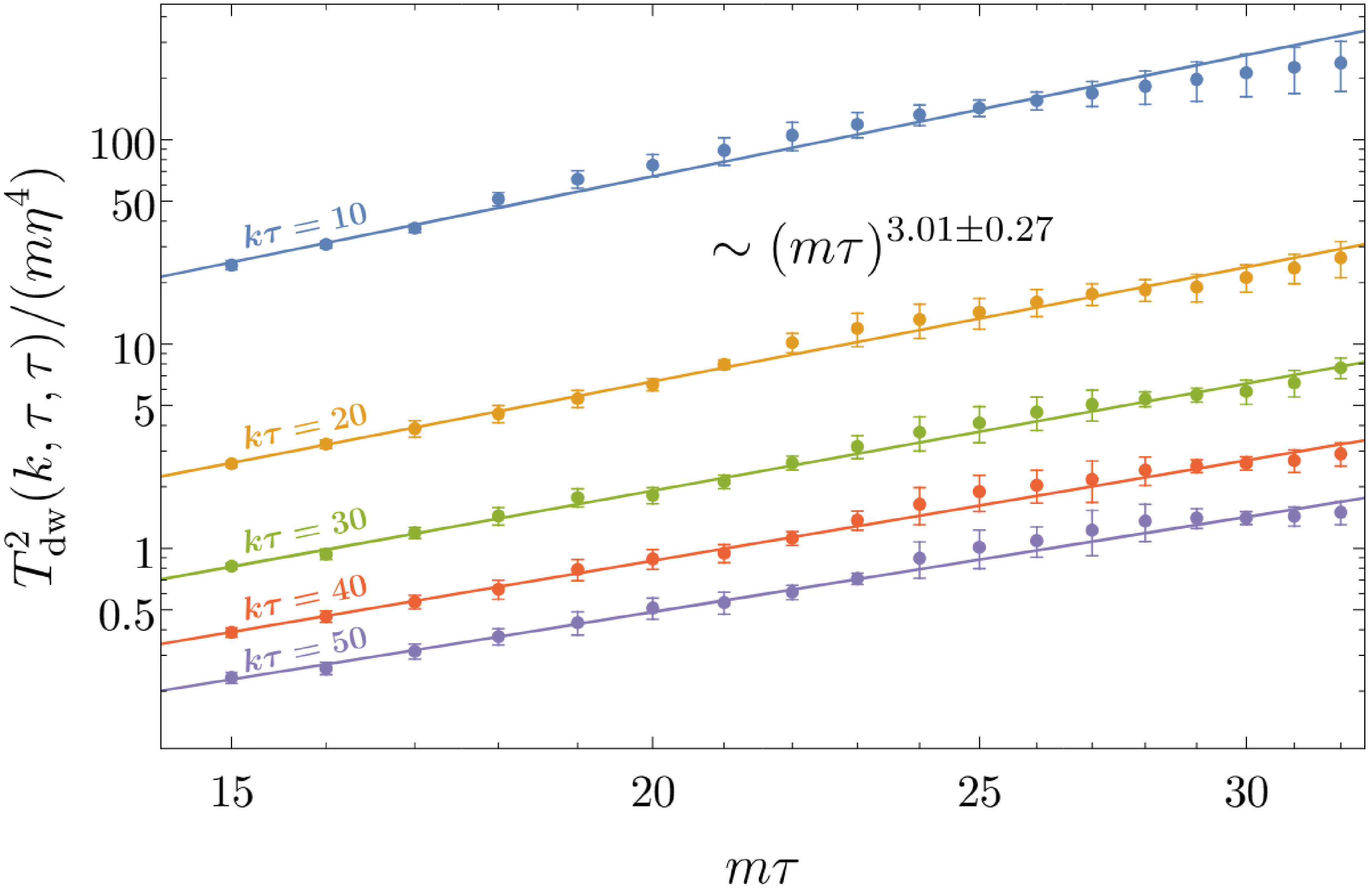
Caprini et al., PRD, 2009

$$T_{\rm dw}^2(k,\tau,\tau) = a^4(\tau)\,\rho_{\rm dw}^2(\tau)\,\tau^3\, C_{\rm dw}^T(x,x)$$

Depends on cosmology **FIT!**

For radiation : $\sim \tau^3$

Gravitational wave spectrum: ETC approach



Gravitational wave spectrum: ETC approach

Relate to GW spectrum and provide **cross-check** of expected spectral shape

→ Use $C^T_{\rm dw}(x,x)$ data/fit to compute $F^T_{\rm dw}(x)$ in one of the following two approximations:

Totally incoherent

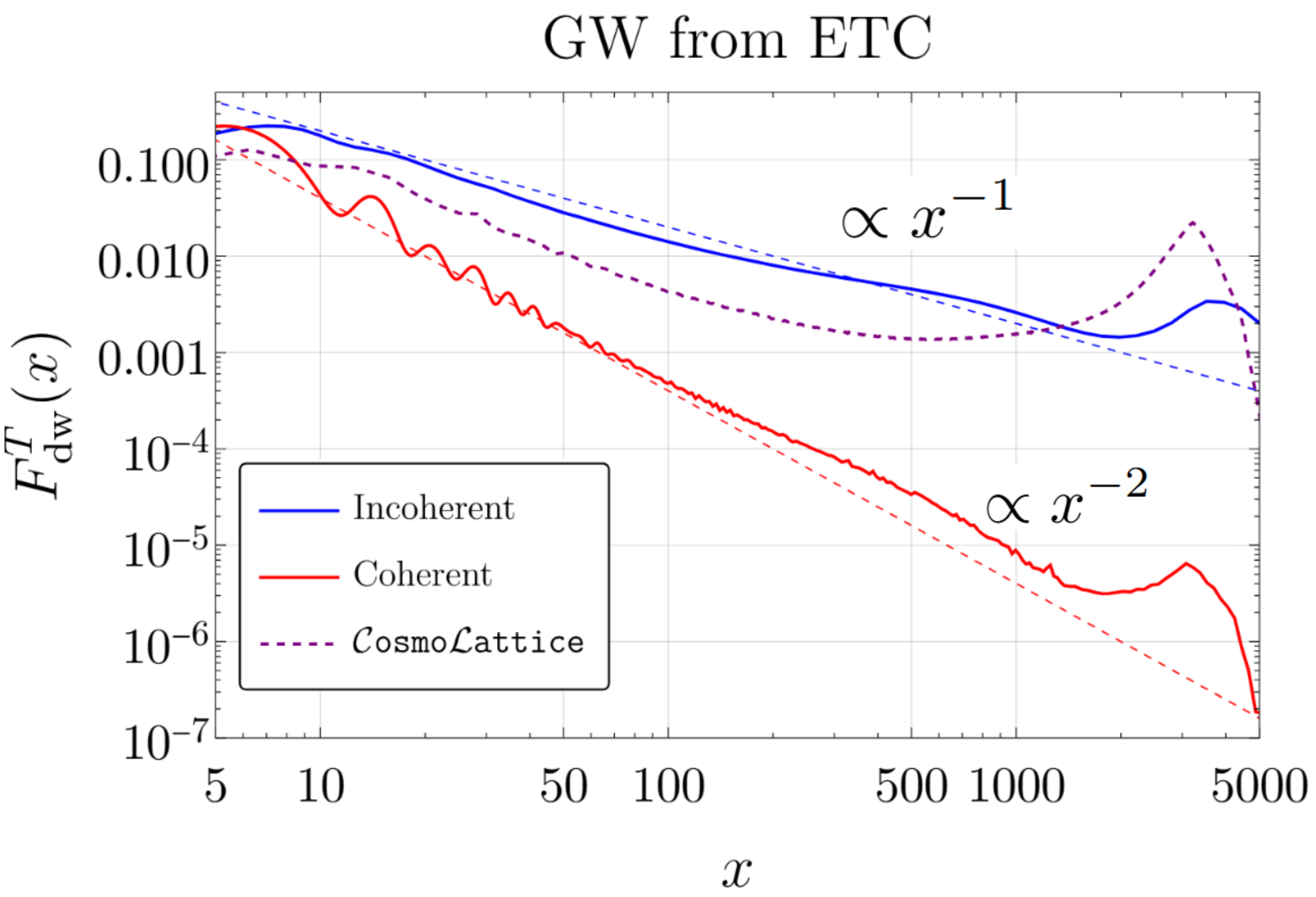
$$\langle \Pi^{TT}_{ij}(\mathbf{k},\tau_1)\Pi^{TT\star}_{ij}(\mathbf{q},\tau_2)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{q})\frac{\delta(\tau_1-\tau_2)}{k}\Pi^2_{\rm dw}(k,\tau_1,\tau_1) \quad \Rightarrow \quad F^T_{\rm dw} \propto x^{-1}$$

Totally coherent

$$\langle \Pi^{TT}_{ij}(\mathbf{k},\tau_1)\Pi^{TT\star}_{ij}(\mathbf{q},\tau_2)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{q})\sqrt{\Pi^2_{\rm dw}(k,\tau_1,\tau_1)}\sqrt{\Pi^2_{\rm dw}(k,\tau_2,\tau_2)} \quad \Rightarrow \quad F^T_{\rm dw} \propto x^{-2}$$

Gravitational wave spectrum: ETC approach

Relate to GW spectrum and provide **cross-check** of expected spectral shape



- **UV peak** due to discretization effects
- No sign of appearance of plateau-like region
- **UTC** to compute rigorously GW signal

Gravitational wave spectrum: other cosmologies

$$T^2_{\rm dw}(k,\tau,\tau) = a^4(\tau)\,\rho^2_{\rm dw}(\tau)\,\tau^3\,C^T_{\rm dw}(x,x)$$

Cosmology dependent **Defect dependent**

Gravitational wave spectrum: other cosmologies

$$T^2_{\rm dw}(k,\tau,\tau) = a^4(\tau)\,\rho^2_{\rm dw}(\tau)\,\tau^3\,C^T_{\rm dw}(x,x)$$

Cosmology dependent **Defect dependent**

Kination: $w = 1$

Exotic: $w = 2/3$

Gravitational wave spectrum: other cosmologies

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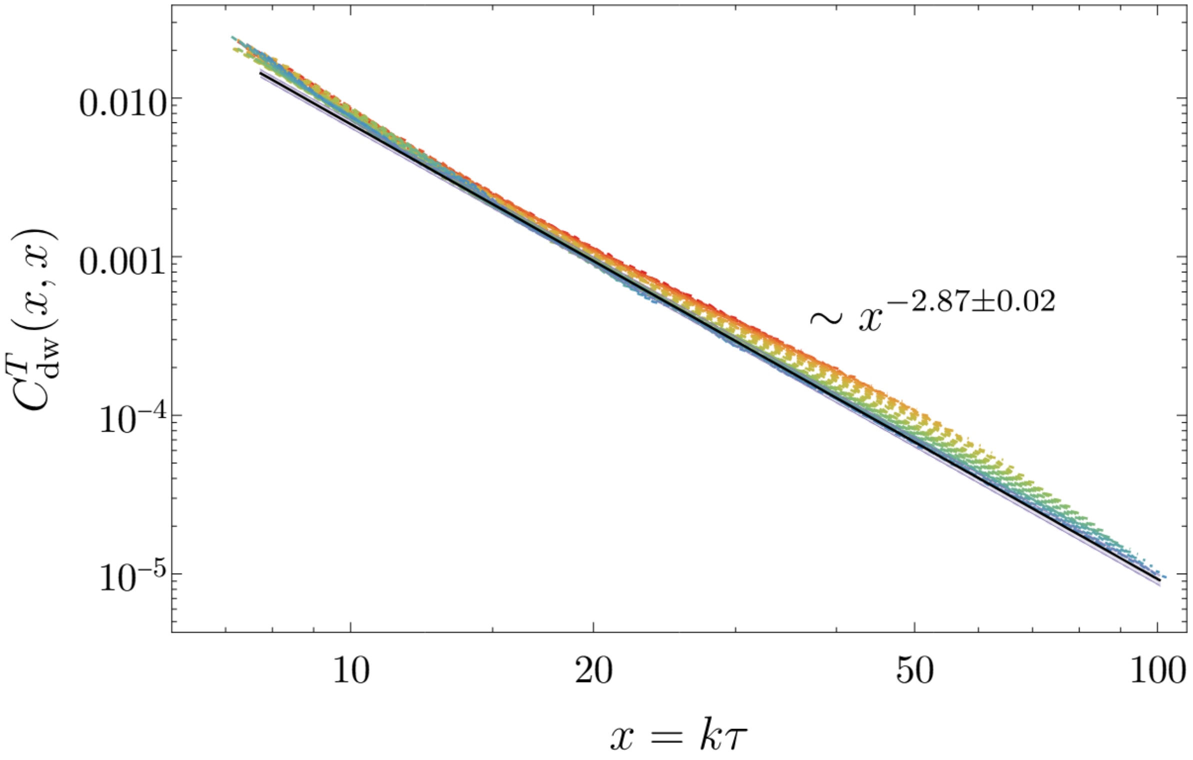

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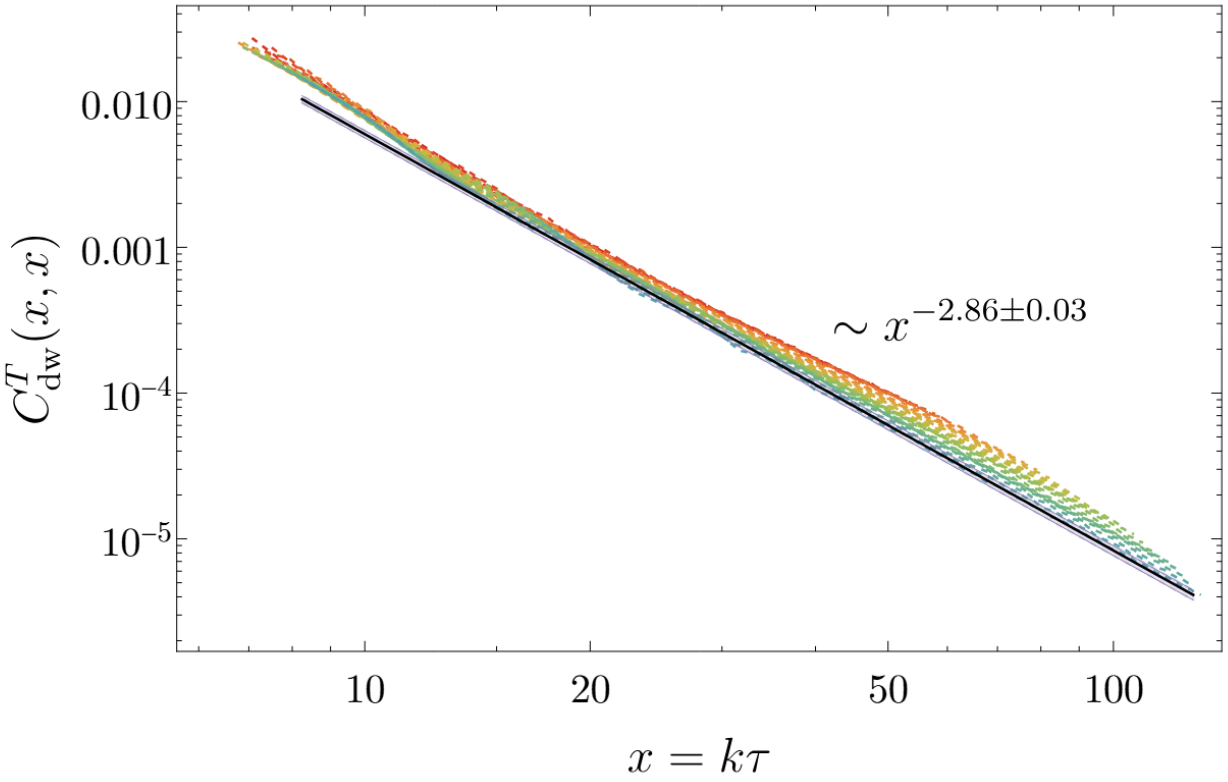

Gravitational wave spectrum: other cosmologies

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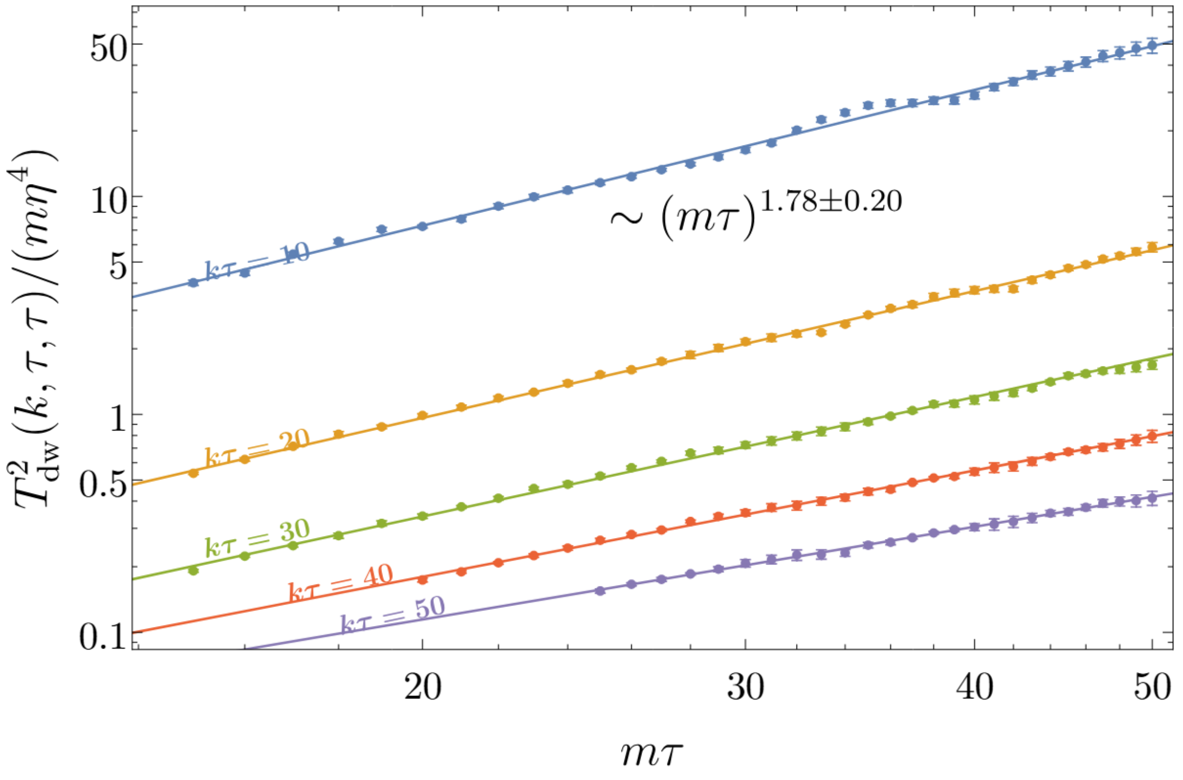
Cosmology dependent **Defect dependent**

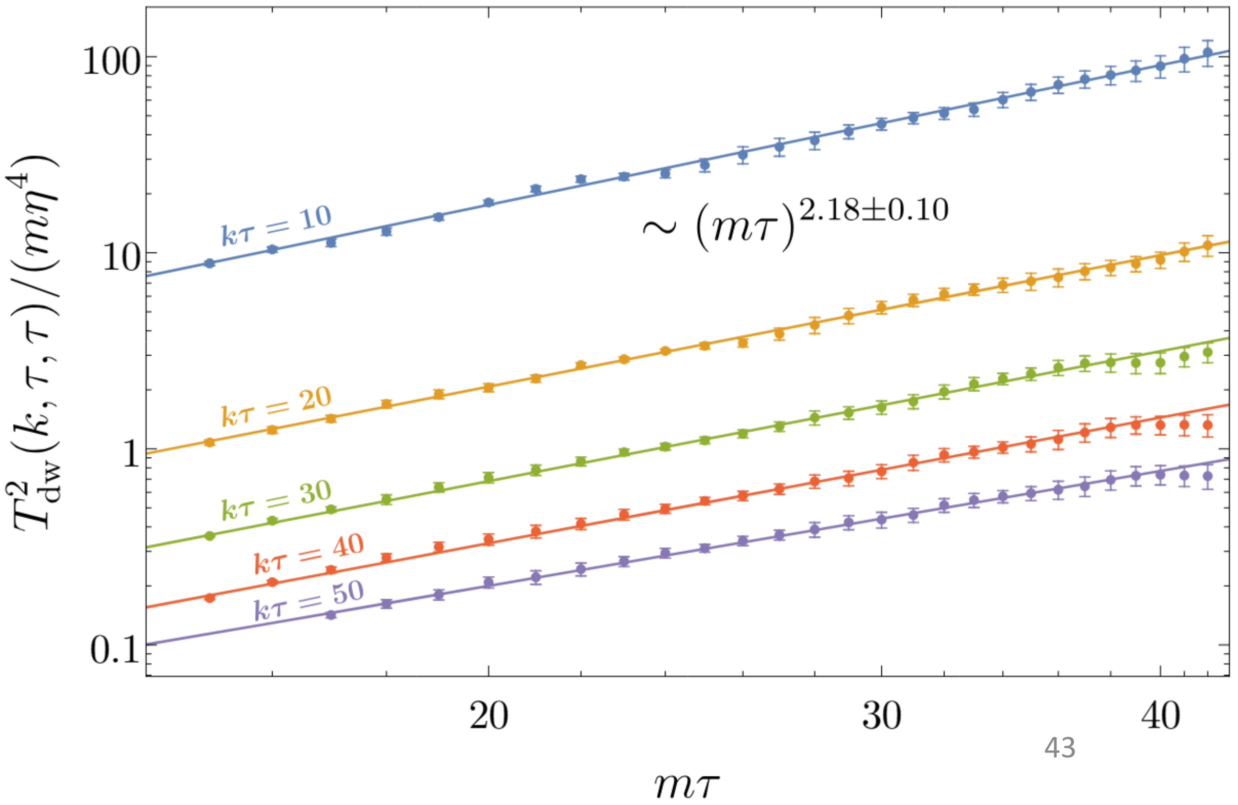
Kination: $w = 1$

Expected: $\sim \tau^2$



Exotic: $w = 2/3$

Expected: $\sim \tau^{2.33}$



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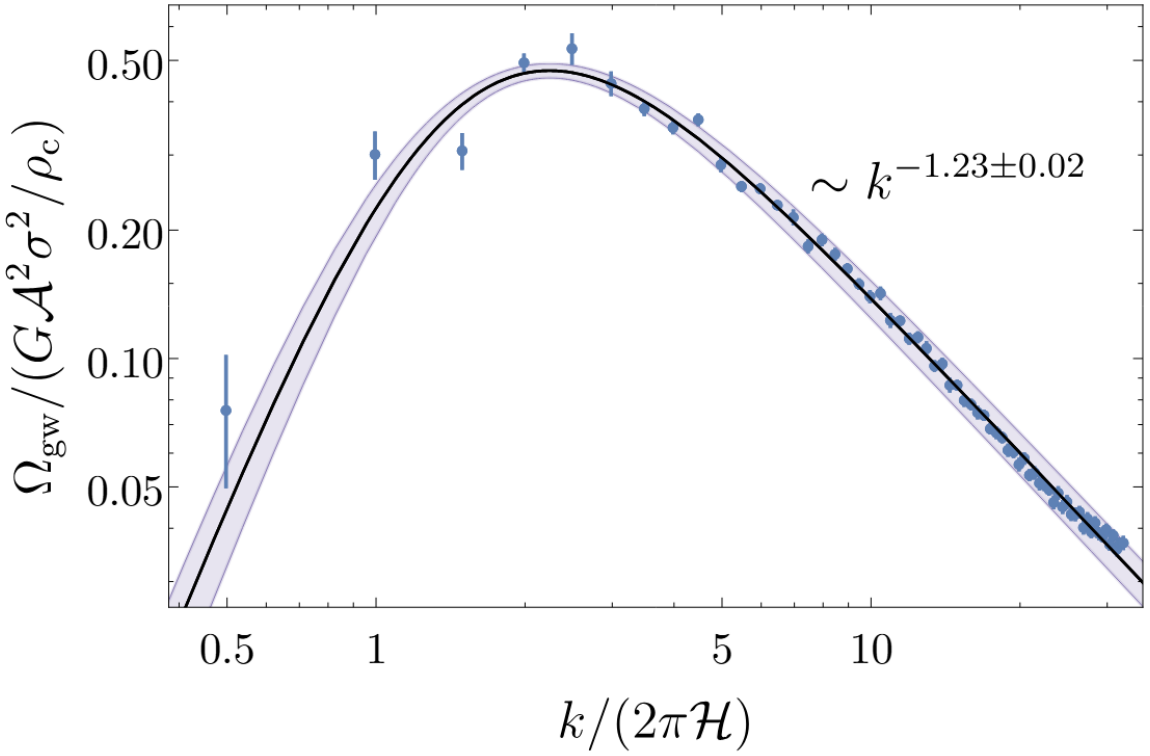

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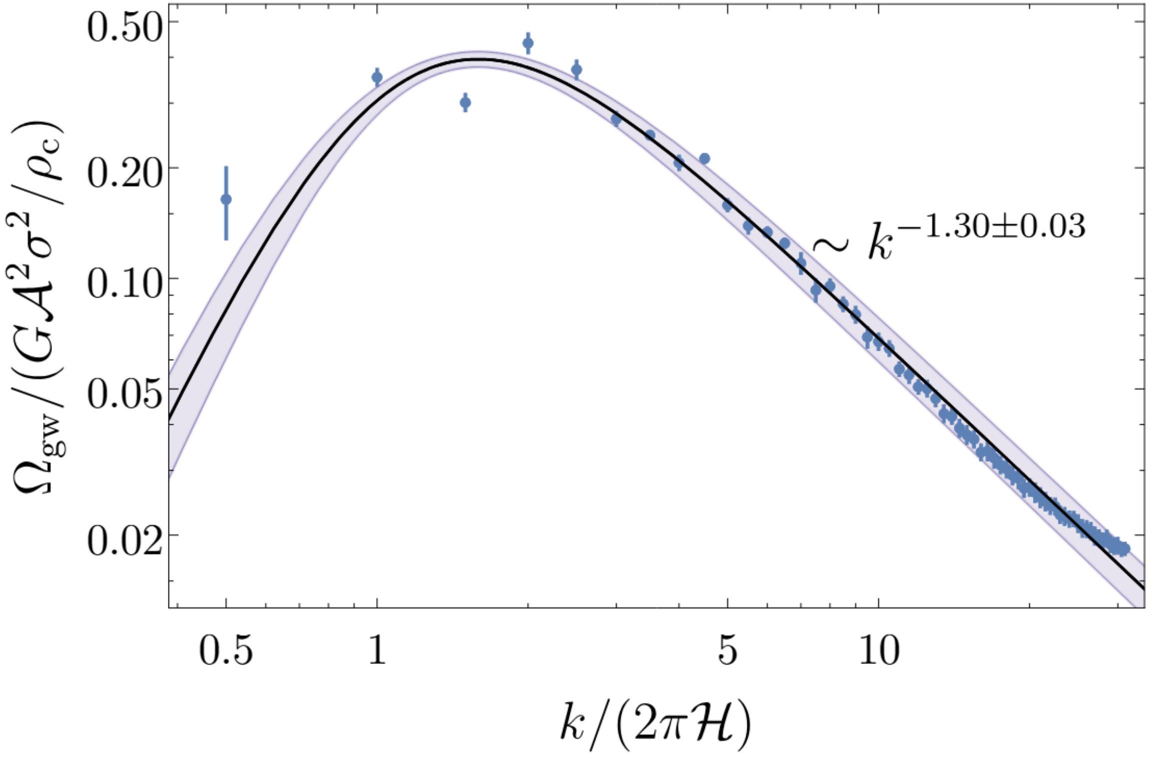


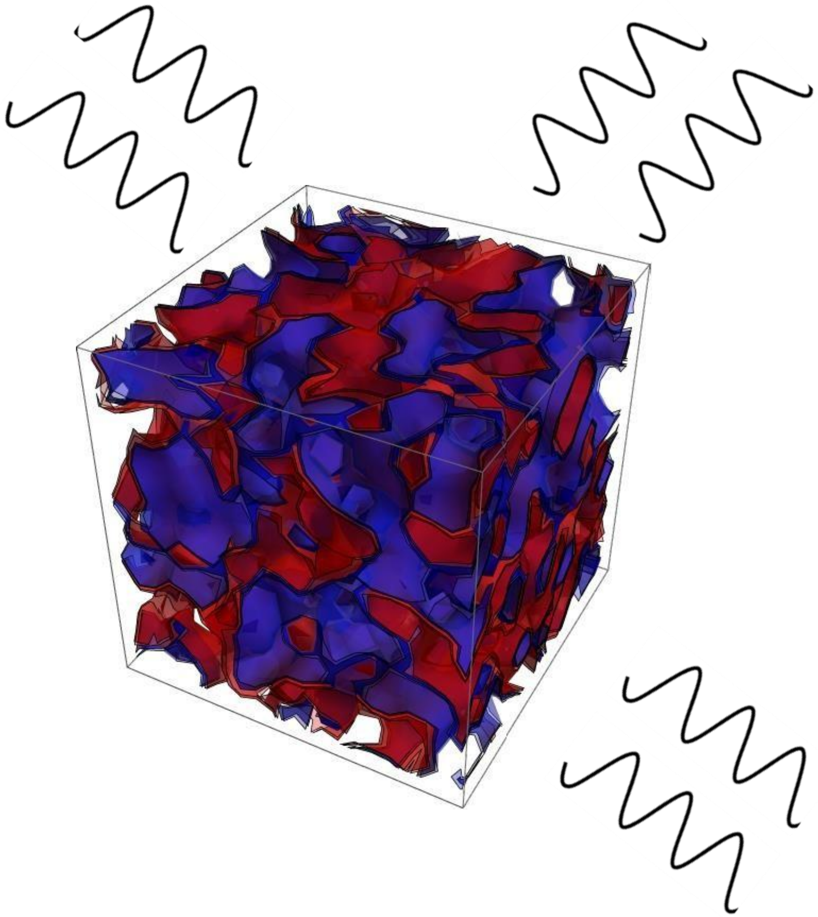
Conclusion

- Domain walls enter **scaling regime**, independent on initial conditions

- GW spectrum through EoM: $\Omega_{\rm gw}^{\rm dw} \sim k^{-1.31\pm0.02}$

- ETC results: $C_{\rm dw}^{T}(x,x) \propto x^{-3.02\pm0.02}$

- Emission of GWs in other cosmologies
  → GW spectrum appears to be **independent on cosmology**

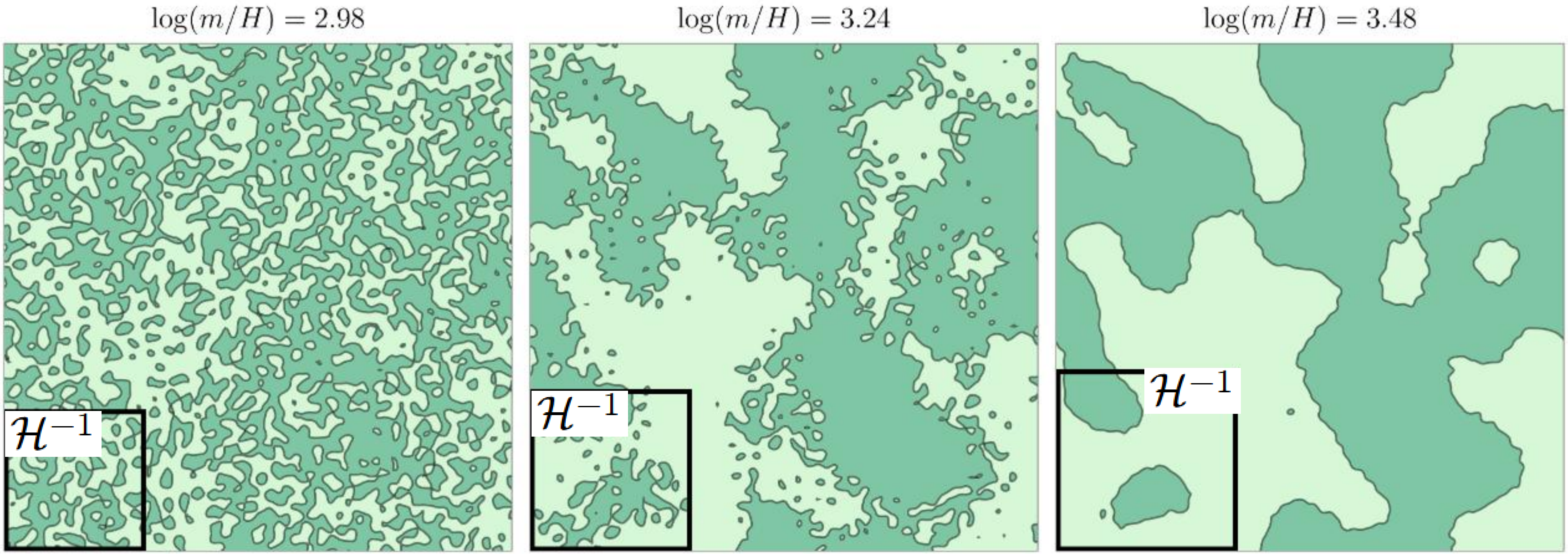


Cosmo	N	w	$\tilde{\epsilon}_{\rm gw}$	b	c	x_p	q
Rad	4096	1/3	0.363 ± 0.004	1.49 ± 0.04	2.09 ± 0.13	1.02 ± 0.01	3 ± 0.04
Rad	2048	1/3	0.35 ± 0.04	1.31 ± 0.02	1.15 ± 0.33	0.85 ± 0.04	3.02 ± 0.02
Kin	2048	1	0.47 ± 0.02	1.23 ± 0.02	1.84 ± 0.25	2.24 ± 0.11	2.87 ± 0.02
Exotic	2048	2/3	0.40 ± 0.02	1.30 ± 0.03	1.79 ± 0.32	1.59 ± 0.08	2.86 ± 0.03

Back up

Domain wall dynamics: the scaling regime

$m/H_i = 10$ and $k_{\text{cut}}/m = 5$ for $N^3 = 256^3$



Gravitational wave spectrum from domain walls

- Production of gravitational waves through domain wall dynamics during scaling

From **dimensional arguments** using quadrupole formula:

$$P_{\rm gw} \sim G\dddot{Q}_{ij}\dddot{Q}_{ij} \qquad\qquad Q_{ij} \sim \sigma L^4$$

Scaling
$$\rho_{\rm gw} \sim \frac{P_{\rm gw} t}{t^3} \sim G\sigma^2$$

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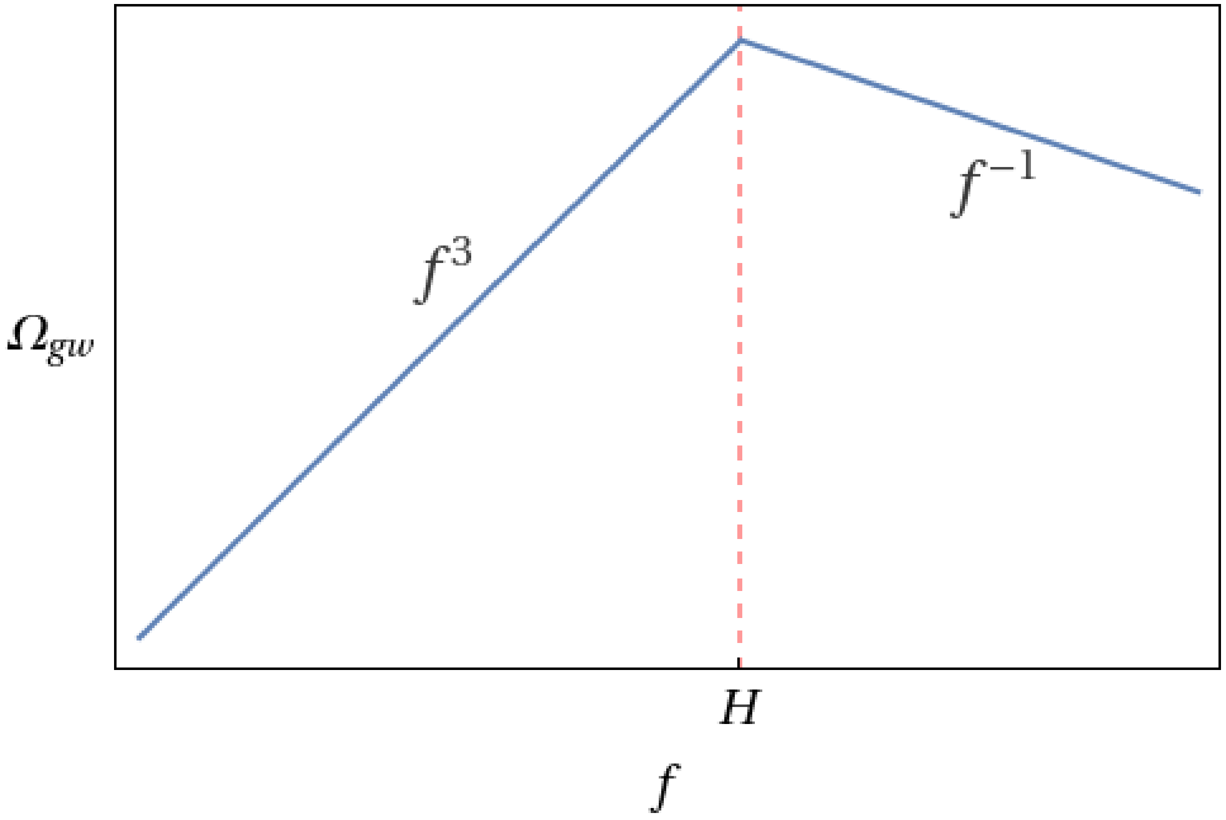
Scaling $$\rho_{\rm gw} \sim \frac{P_{\rm gw} t}{t^3} \sim G\sigma^2$$

From **simulations**: Saikawa et al., JCAP, 2014

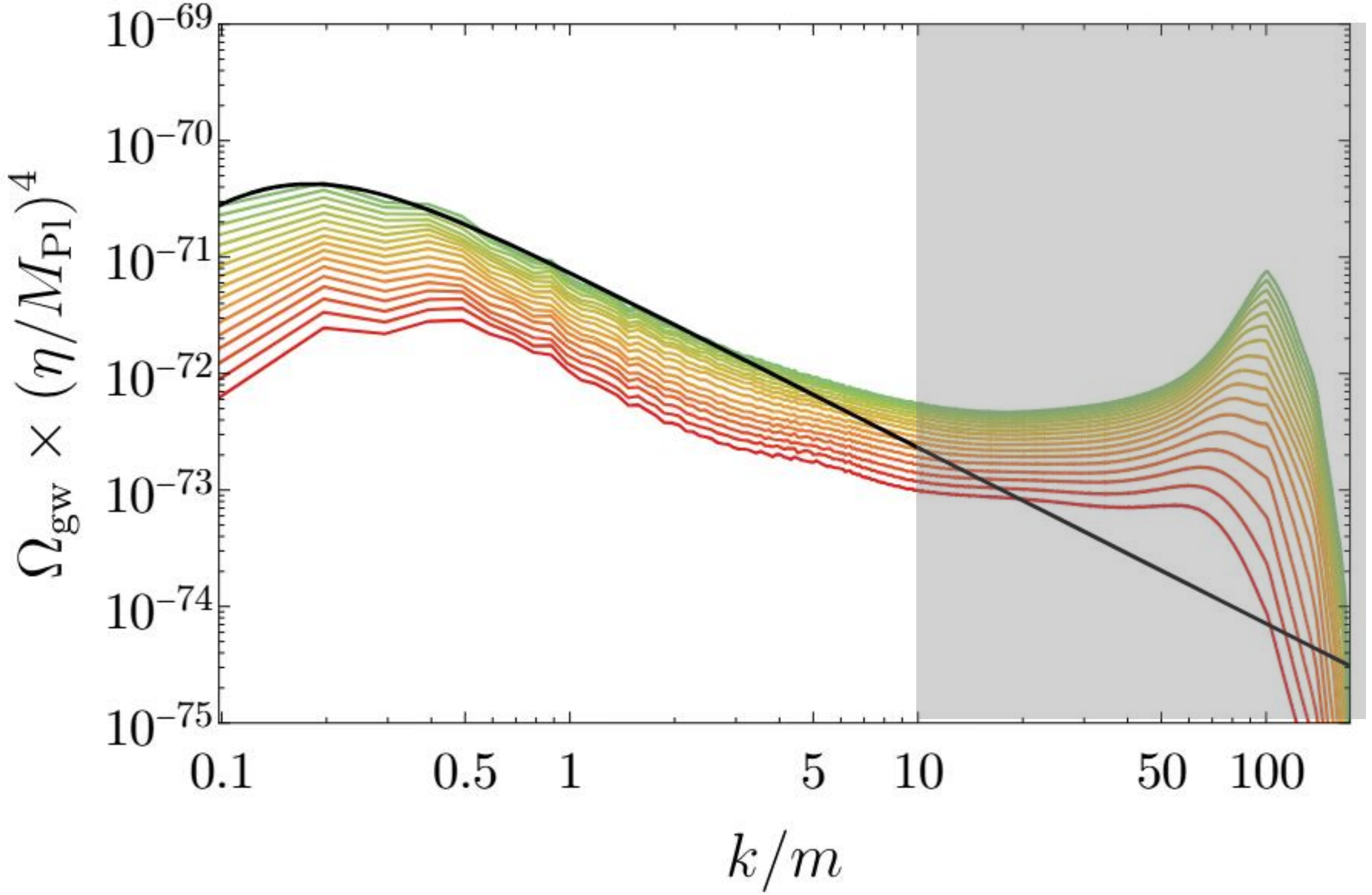
$$\Omega_{\rm gw}(f,t) = \Omega_{\rm gw}^{\rm peak}(t) \begin{cases} \left(\frac{f}{f_{\rm peak}}\right)^3 & f \leq f_{\rm peak} \\ \left(\frac{f}{f_{\rm peak}}\right)^{-1} & f > f_{\rm peak} \end{cases}$$

$$\Omega_{\rm gw}^{\rm peak}(t) = \frac{\tilde{\epsilon}_{\rm gw} \mathcal{A}^2 G\sigma^2}{\rho_c(t)} \qquad f_{\rm peak}(t) = H(t)$$



Gravitational wave spectrum: EoM approach

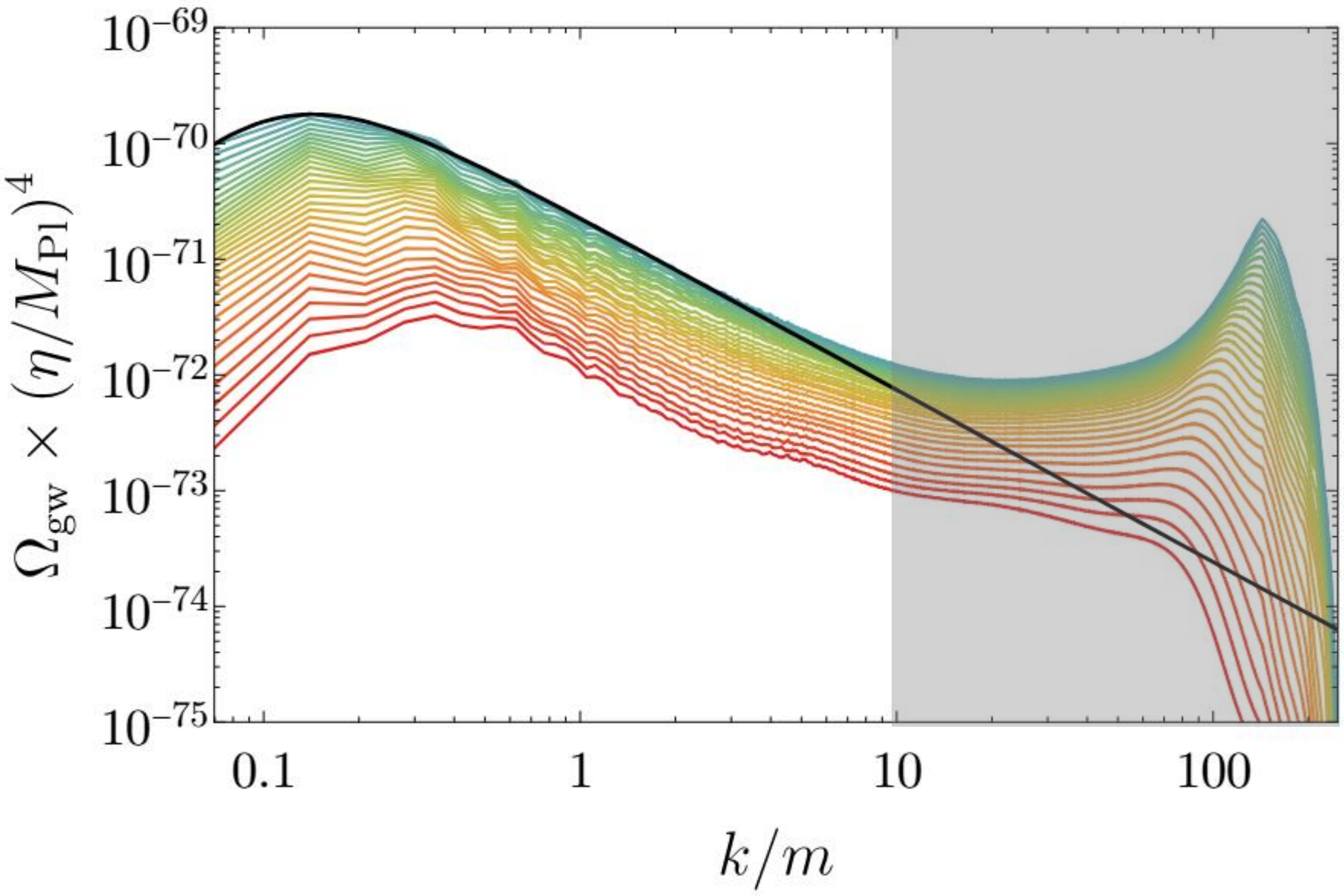
- Fit for N = 2048 simulation (5 simulations)



- Fit done at final simulation time
- GW data used up to momentum cutoff

Gravitational wave spectrum: EoM approach

- Fit for **N = 4096** simulation (**1 simulation**)



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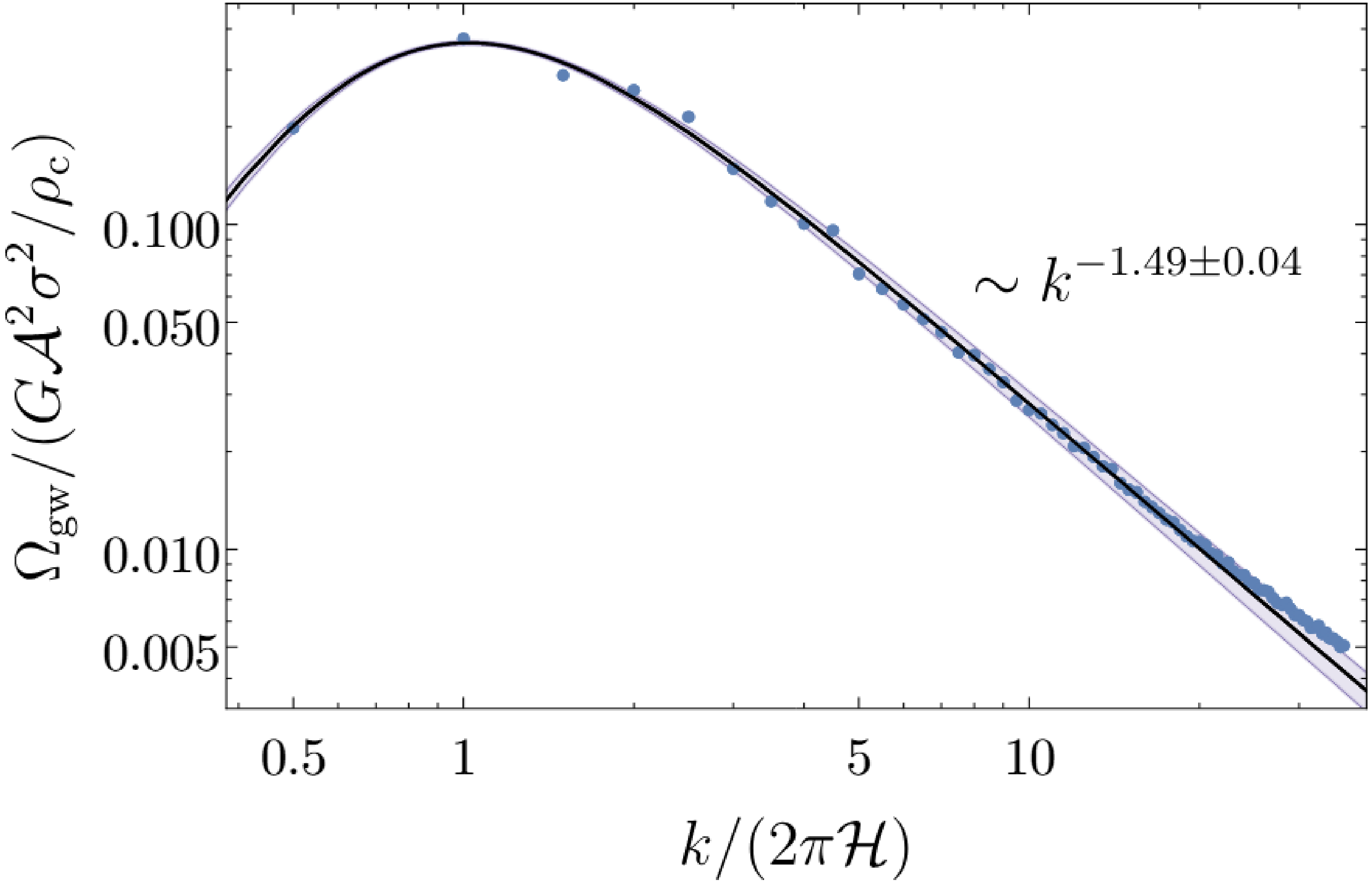

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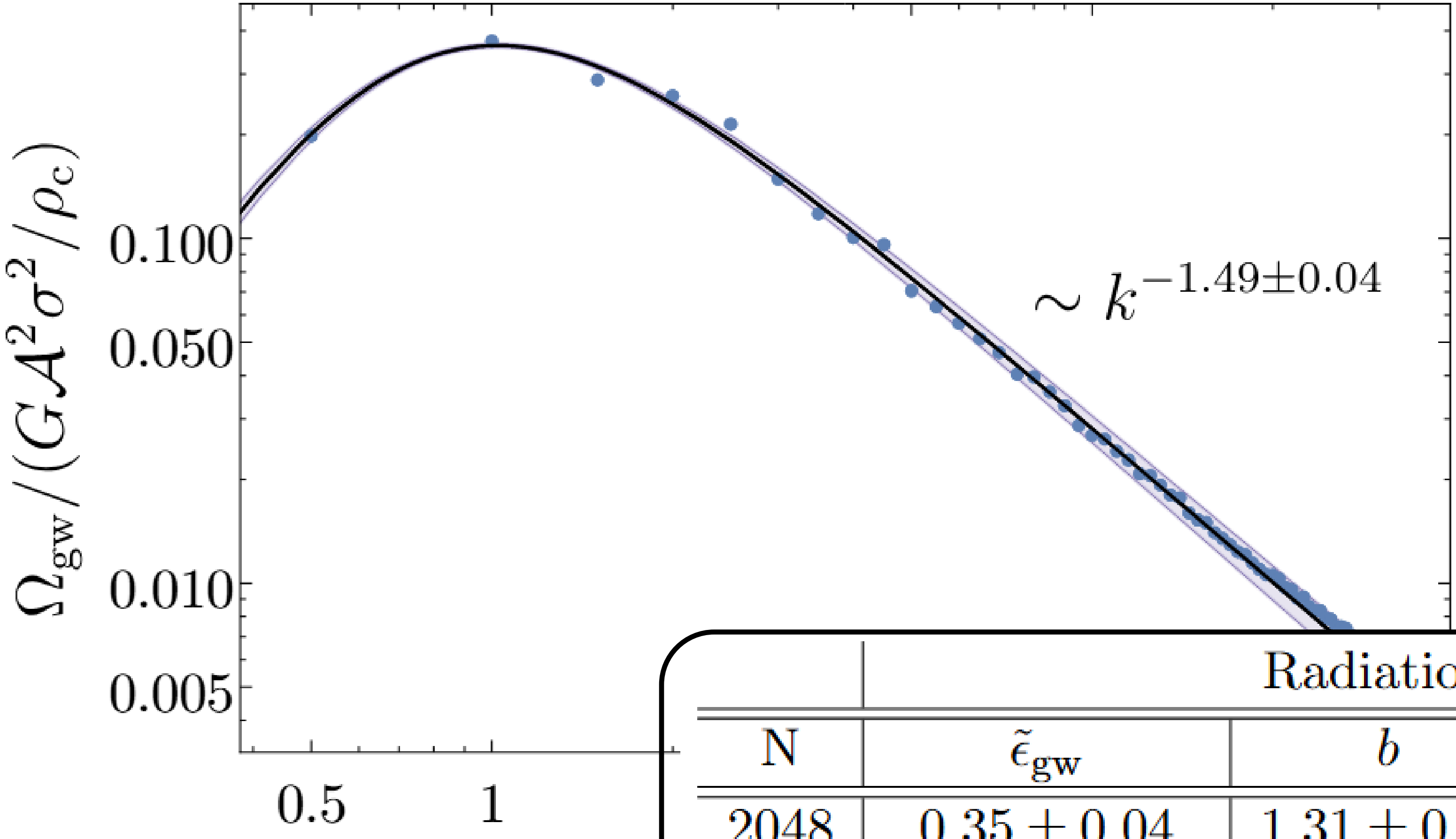

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- GW data used up to momentum cutoff

	Radiation domination			
N	$\tilde{\epsilon}_{gw}$	b	c	x_p
2048	0.35 ± 0.04	1.31 ± 0.02	1.15 ± 0.33	0.85 ± 0.04
4096	0.363 ± 0.004	1.49 ± 0.04	2.09 ± 0.13	1.02 ± 0.01

UTC of domain walls in general cosmology

$$a(\tau) = a_i \left(\frac{\mathcal{H}_i \tau}{p}\right)^p \quad \text{and} \quad \rho_{\rm dw}(\tau) = \frac{\mathcal{A}\sigma H(\tau)}{\tilde{p}}$$

$$\tilde{p} = \frac{2}{3(1+w)}$$

From similar computations as in the radiation dominated case:

$$\Omega_{\rm gw}^{\rm dw}(\tau,k) = \frac{16}{3}\frac{G^2\mathcal{A}^2\sigma^2}{\tilde{p}^2}a_i^2\left(\frac{\mathcal{H}_i}{p}\right)^{2p}\tau^{2+2p}F_{\rm dw}^T(x)\,,$$

$$F_{\rm dw}^T(x) = \frac{1}{x^{4p}}\int_{x_i}^{x}{\rm d}x_1\int_{x_i}^{x}{\rm d}x_2\,(x_1x_2)^{2p+1/2}\cos\left(x_1-x_2\right)C_{\rm dw}^T(x_1,x_2)$$

Spectrum does **not depend on the cosmology** if integral is **not dominated by lower boundary**

Strings and their UTC

$$\frac{\mathrm{d}\rho_{\mathrm{gw}}^{\mathrm{cs}}}{\mathrm{d}\ln k}(k,\tau) = \frac{2G}{\pi}\frac{k^3}{a^4(\tau)}\int_{\tau_i}^{\tau}\mathrm{d}\tau_1\int_{\tau_i}^{\tau}\mathrm{d}\tau_2\; a^3(\tau_1)a^3(\tau_2)\rho_{\mathrm{cs}}(\tau_1)\rho_{\mathrm{cs}}(\tau_2)\cos\left[k(\tau_1-\tau_2)\right]\Pi_{\mathrm{cs}}^2(k,\tau_1,\tau_2)$$

$$a(\tau) = \sqrt{\Omega_{\mathrm{rad}}^0}H_0\tau$$

$$\rho_{\mathrm{cs}} \approx \mu H^2(\tau)$$

String tension

$$\Pi_{\mathrm{cs}}^2(k,\tau_1,\tau_2) = (\tau_1\tau_2)^{3/2}C_{\mathrm{cs}}^T(x_1,x_2)$$

Everything together:

$$\Omega_{\mathrm{gw}}^{\mathrm{cs}}(\tau,k) \equiv \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{\mathrm{gw}}^{\mathrm{cs}}}{\mathrm{d}\ln k} = \frac{16}{3}(G\mu)^2\Omega_{\mathrm{rad}}(\tau)F_{\mathrm{cs}}^T(x)\,,$$

$$F_{\mathrm{cs}}^T(x) \equiv \int_{x_i}^{x}\mathrm{d}x_1\int_{x_i}^{x}\mathrm{d}x_2\sqrt{x_1x_2}\cos\left(x_1-x_2\right)C_{\mathrm{cs}}^T(x_1,x_2)$$

Strings and their UTC

$$\Omega_{\rm gw}^{\rm cs}(\tau,k) \equiv \frac{1}{\rho_c}\frac{{\rm d}\rho_{\rm gw}^{\rm cs}}{{\rm d}\ln k} = \frac{16}{3}(G\mu)^2\Omega_{\rm rad}(\tau)F_{\rm cs}^T(x)\,,$$

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Figueroa et al ., JCAP, 2009

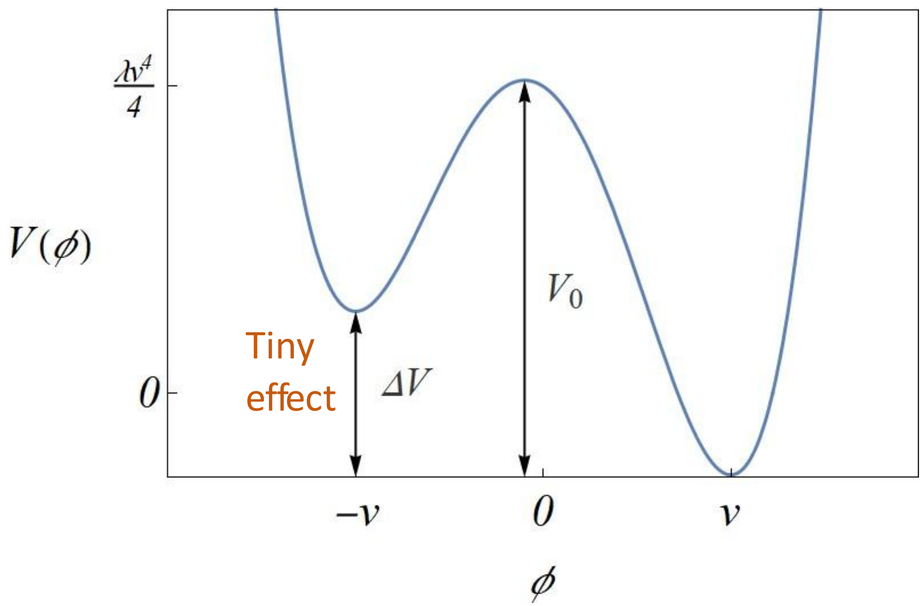
The function $C_{\rm cs}^T(x,x)$ decays faster than x^{-2}

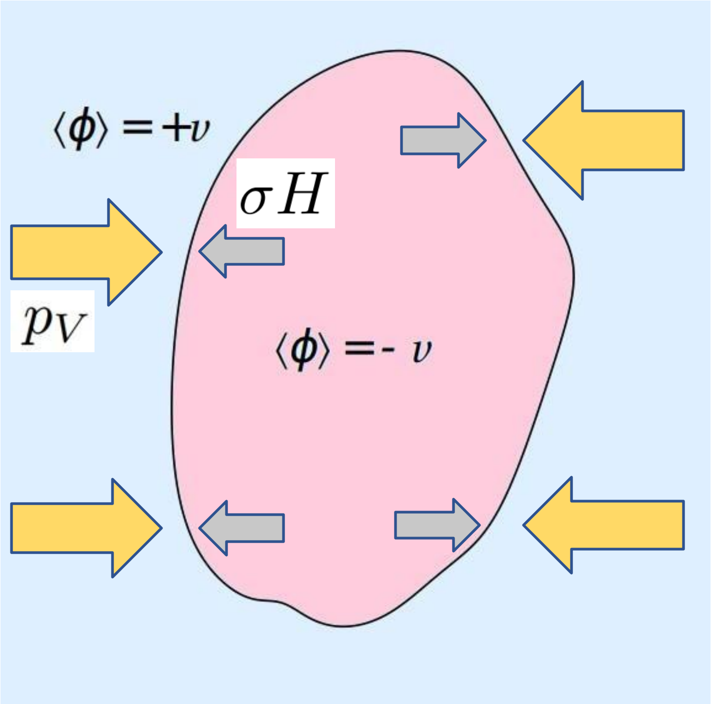
➡ Integral is dominated by lower boundary at large x

$$\Omega_{\rm gw}^{\rm cs}\Big|_{{\rm large}\ k} \approx \frac{16}{3}(G\mu)^2\Omega_{\rm rad}(\tau)F_{\rm cs}^T(x\to\infty) = {\rm const.}$$

Domain wall solution: introducing a bias

- Make symmetry slightly approximate (**energy bias**)





- Creates **volume pressure force**

$$p_V \sim \Delta V$$

Annihilation when

$$\sigma H \lesssim p_V$$

Important time scales

Domination

$$\rho_{\rm dw} = \rho_{\rm rad}$$

$$t_{\rm dom} = \frac{3}{4}\frac{M_p^2}{\sigma}$$

Planck mass

Annihilation

$$\sigma H \lesssim p_V$$

$$t_{\rm ann} = \frac{\sigma}{\Delta V}$$