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Thanks to the organizers!!
I won’t say it, I only have 5 minutes

Cosmology of preferred axion models

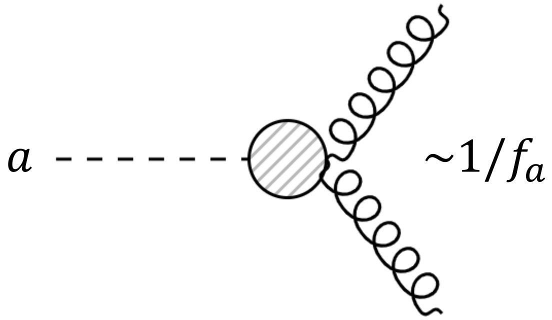
AXIONS IN STOCKHOLM, CONFERENCE (WEEK 2), JUNE/JULY 2025

Andrew Cheek, TDLI, SJTU, Shanghai, China
Based on *JCAP* 03 (2024) 061, *JCAP* 03 (2025) 014 and arXiv:2505.04614
With J. Osinski, L. Roszkowski, U. Min, A. Ghoshal and D. Paul

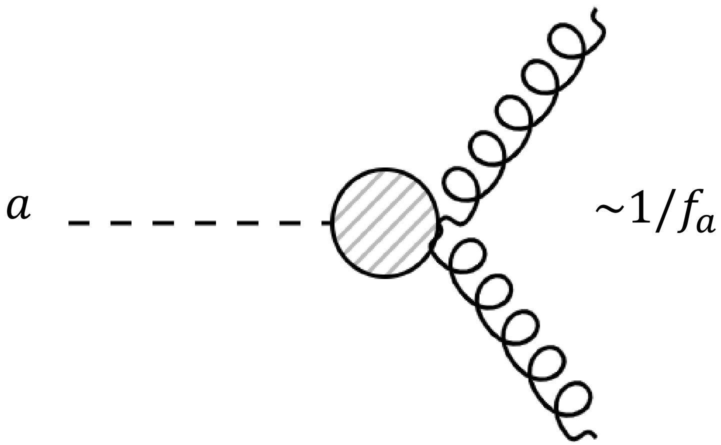
Axion dark matter: a simple solution

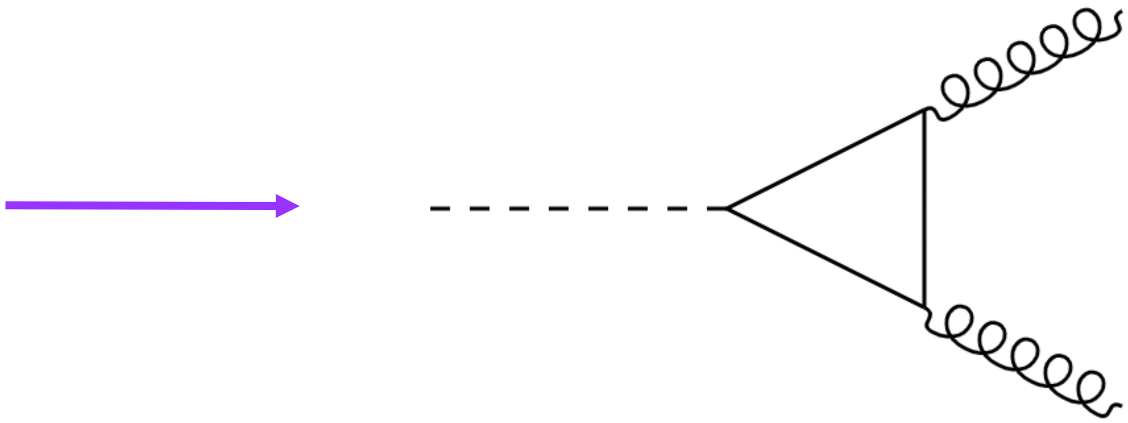
- The **QCD** axion provides an elegant solution to the strong CP and dark matter problems
- All comes from the spontaneously broken $U(1)_{\mathrm{PQ}}$ and the anomaly term

$$\mathcal{L}^a_{\mathrm{eff}} = \frac{a\,(t,x)}{f_a}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$

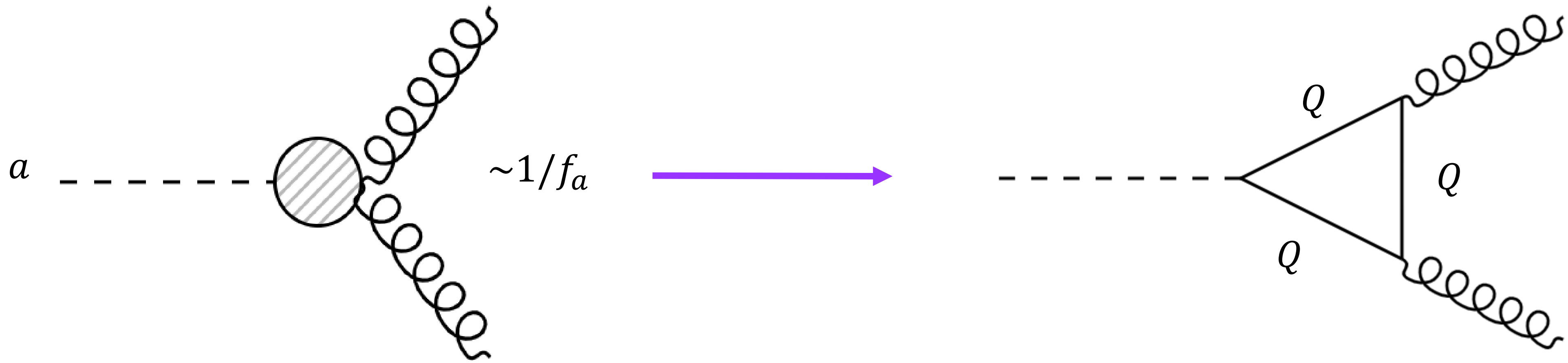


Axion models: not so simple





Axion models: not so simple



- **KSVZ** axion models generate this term by introducing a ‘heavy quark’.
- This setup can have charge configurations that **avoid** the domain wall problem.

Preferred axion models

Models where

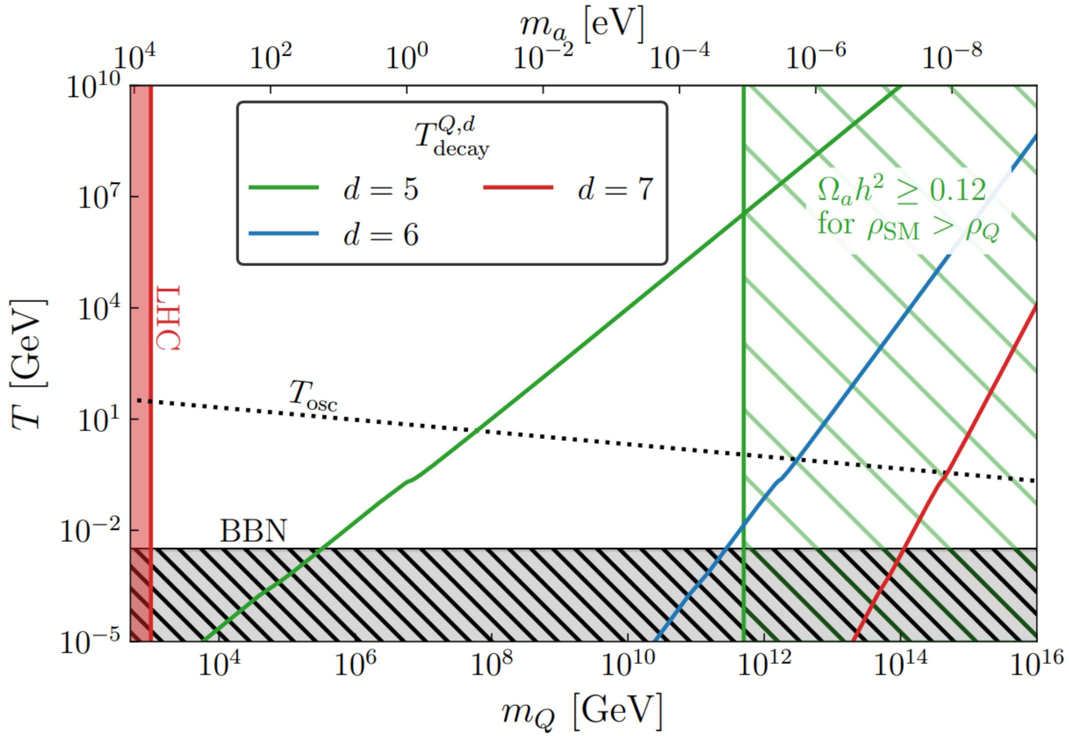
1) Misalignment doesn't overproduce axions
2) Q decay occurs before BBN

(Important in post-inflationary PQ breaking)

↓ ↓ ↓ ↓ ↓ ↓ ↓

Preferred axion models **decay via <u>dimension 5</u> at most!**

$$\mathcal{L}_{Qq} = \mathcal{L}_{Qq}^{d\leq 4} + \frac{1}{\Lambda^{(d-4)}}\mathcal{O}^{d>4} + \text{h.c.}$$

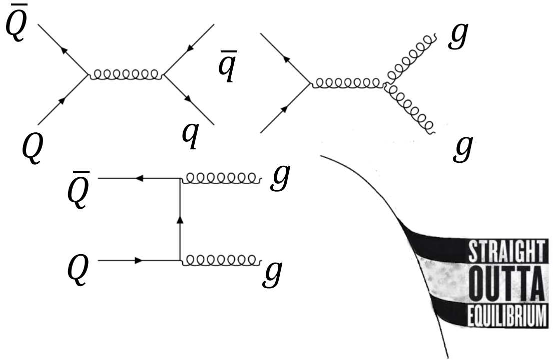


Put forward by Luzio, Mescia and Nardi in [*PRL* 118 (2017) 3, 031801] and [*PRD* 96 (2017) 7, 075003].

$\Delta N_{\rm eff}$ constraints of these models studied in **AC** + Ui Min [arXiv:2411.17320]

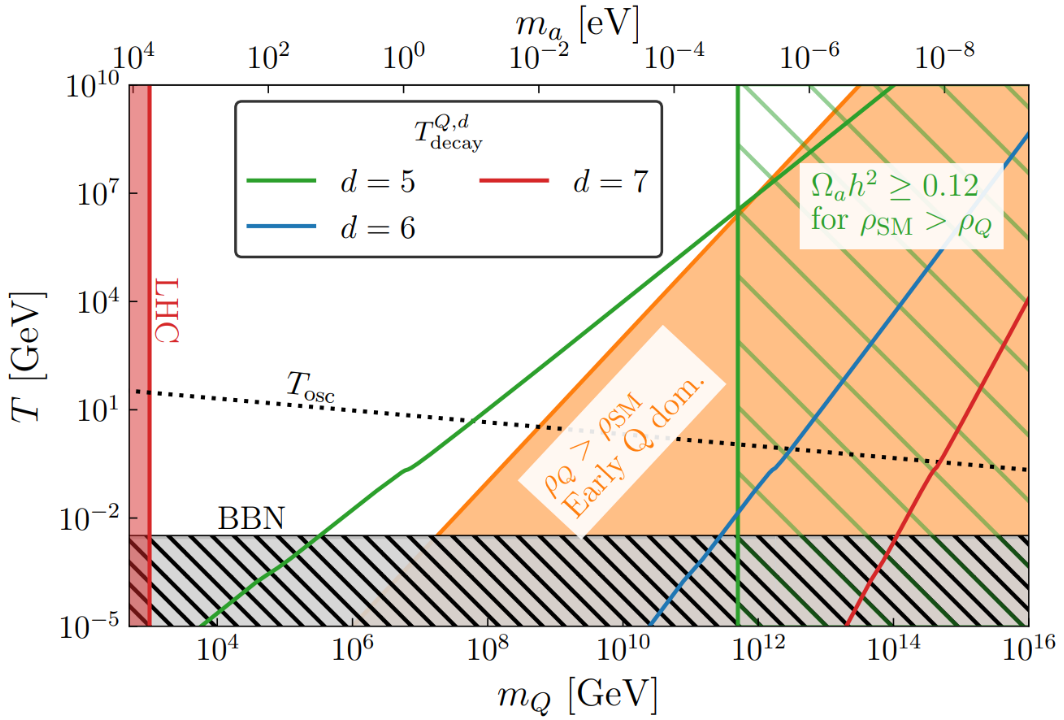
Preferred axion models too restrictive

- Constraints on heavy quark decay terms assumed standard cosmology and ignored impact of heavy quarks themselves on the cosmology



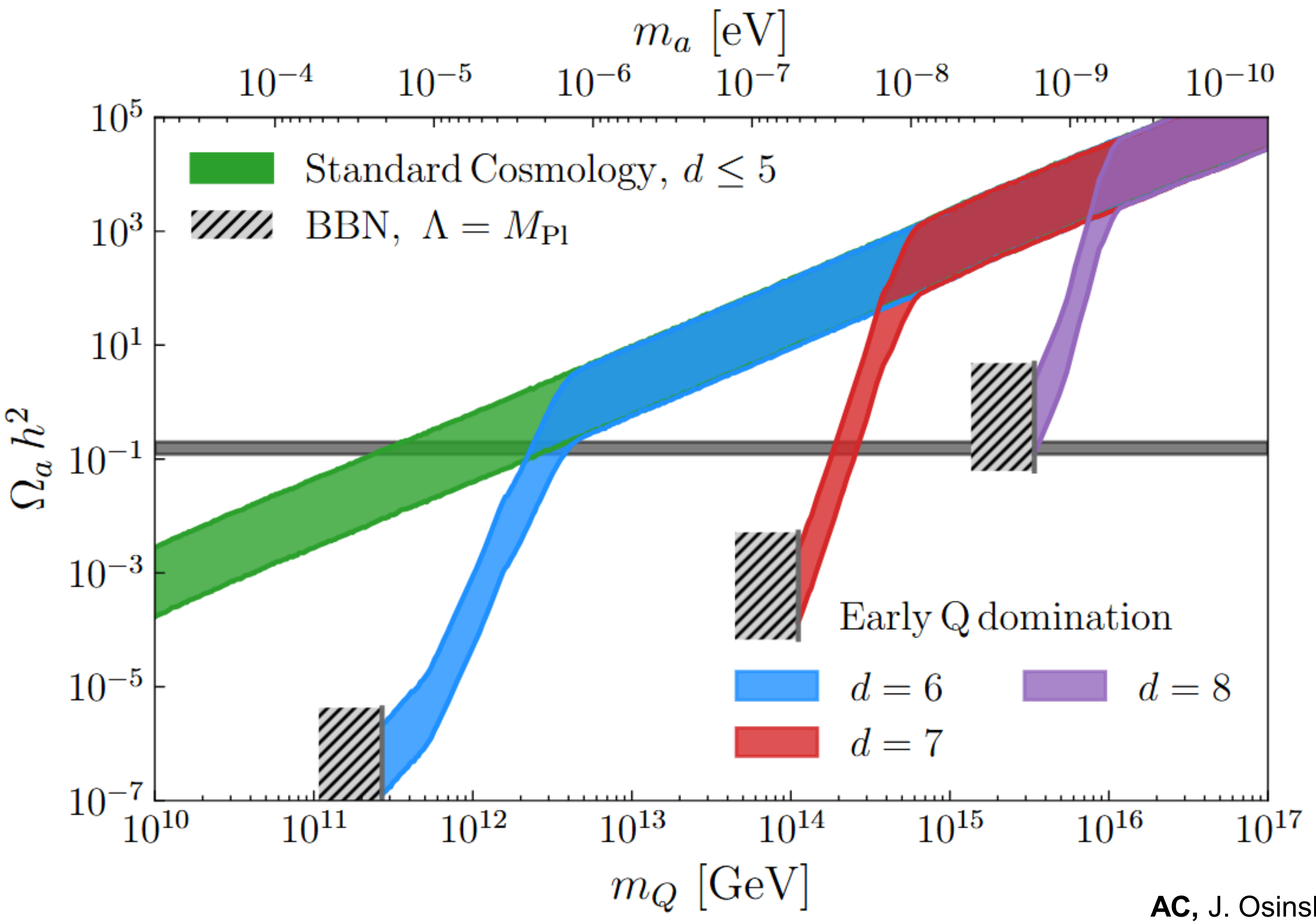
$$\frac{\rho_Q}{\rho_R^{SM}} \sim 10^{10} \left(\frac{m_Q}{10^{12}\,\mathrm{GeV}}\right)^2 \left(\frac{1\,\mathrm{MeV}}{T}\right)$$

$m_Q \geq 10^7$ GeV, quarks dominate before BBN



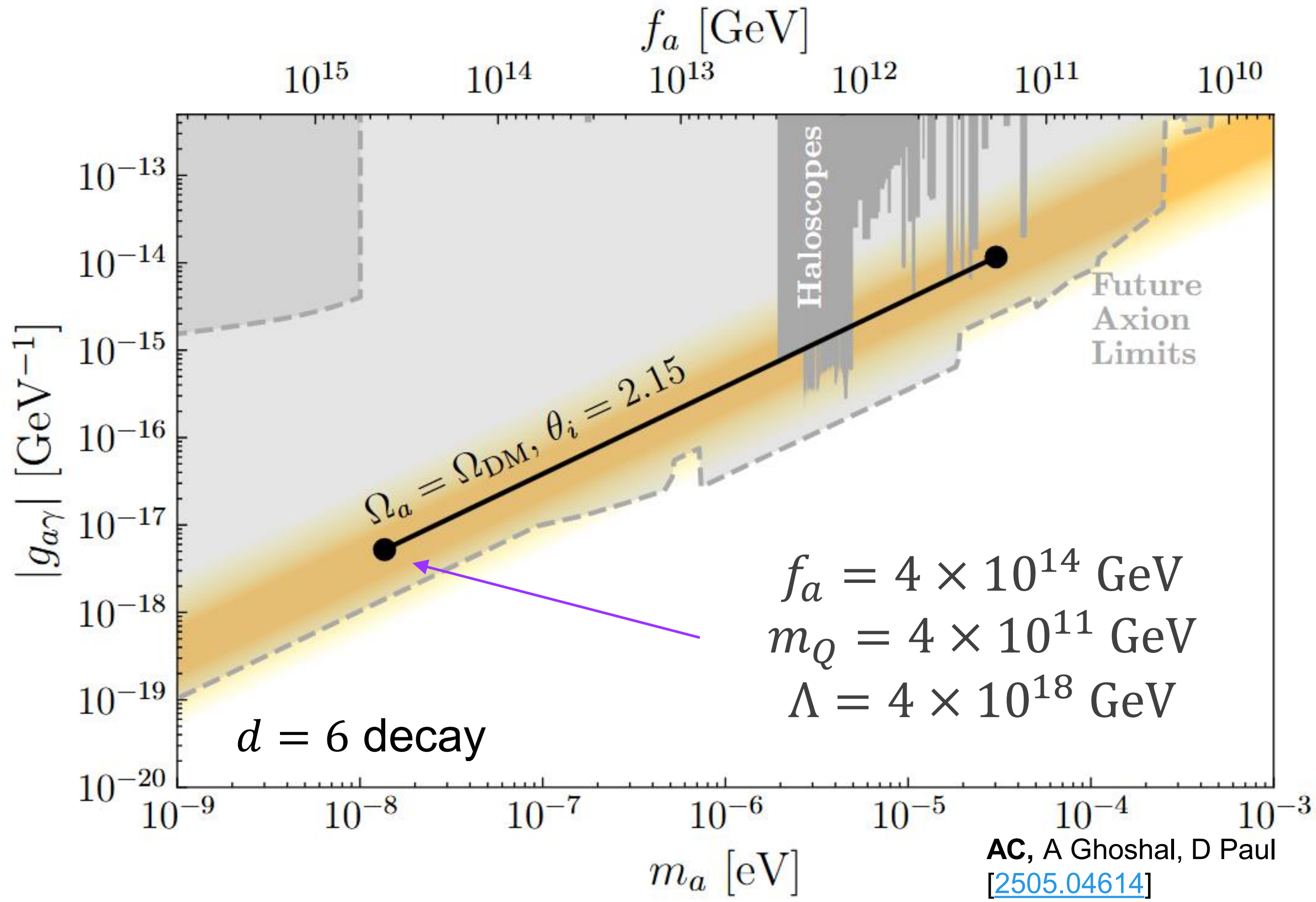
AC, J. Osinski, L. Roszkowski [2310.16087]

Heavy quark domination dilutes Ω_a



AC, J. Osinski, L. Roszkowski [2310.16087]

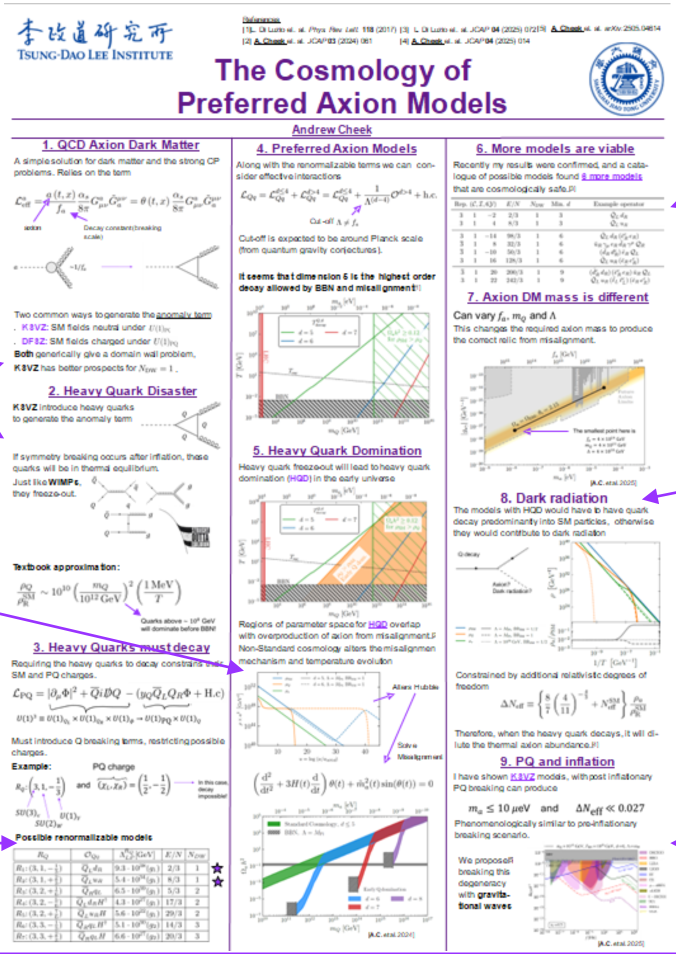
Axion DM mass can be lower



AC, A Ghoshal, D Paul [2505.04614]

4 models at this dimension don't have a domain wall problem.

More physics on my poster



From 2 to 8 models with domain wall problem

More details

CMB constraints

Full numerical calculation

Possible charges for quark decay

Gravitational waves as a probe of heavy quark domination

Back-up slides

谁让你非要问的！

QCD axion as dark matter

Misalignment production in standard cosmology

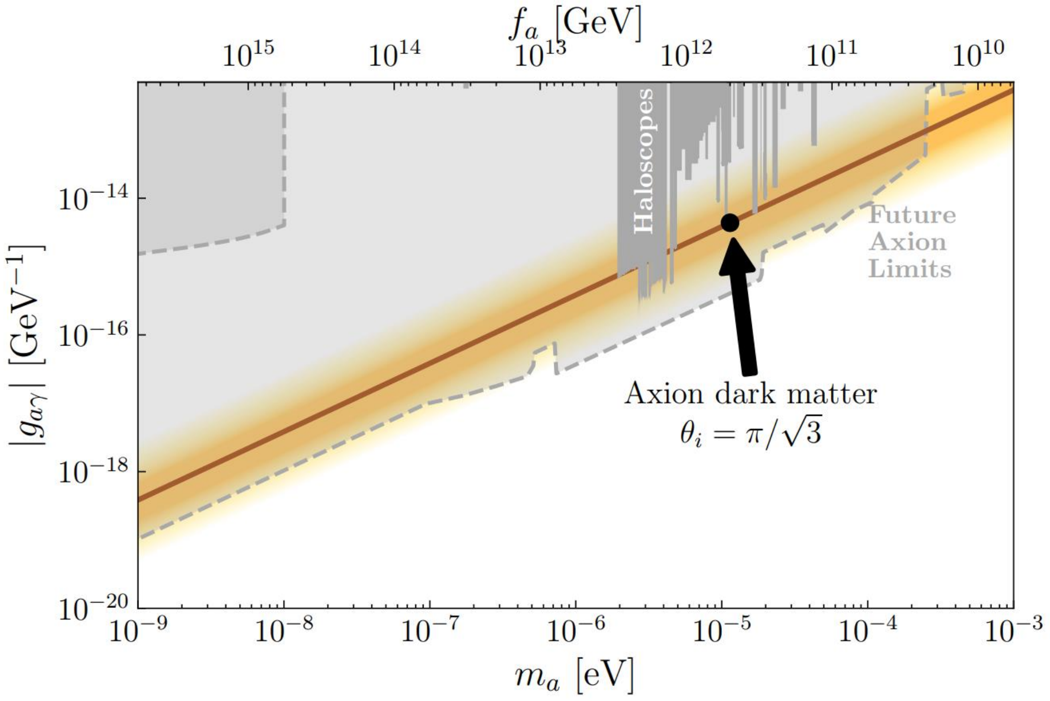
$$\Omega_a h^2 \approx 0.12 \left(\frac{\theta_{\rm i}}{2.15}\right)^2 \left(\frac{28\ \mu\text{eV}}{m_a}\right)^{7/6}$$

In the post-inflationary breaking you expect random θ_i in range $[-\pi, \pi)$.

Take random values in each Hubble patch

$$\theta_i \equiv \sqrt{\langle \theta_i^2 \rangle} = \frac{\pi}{\sqrt{3}} \simeq 1.81 \longrightarrow \approx 2.15$$

Anharmonic corrections

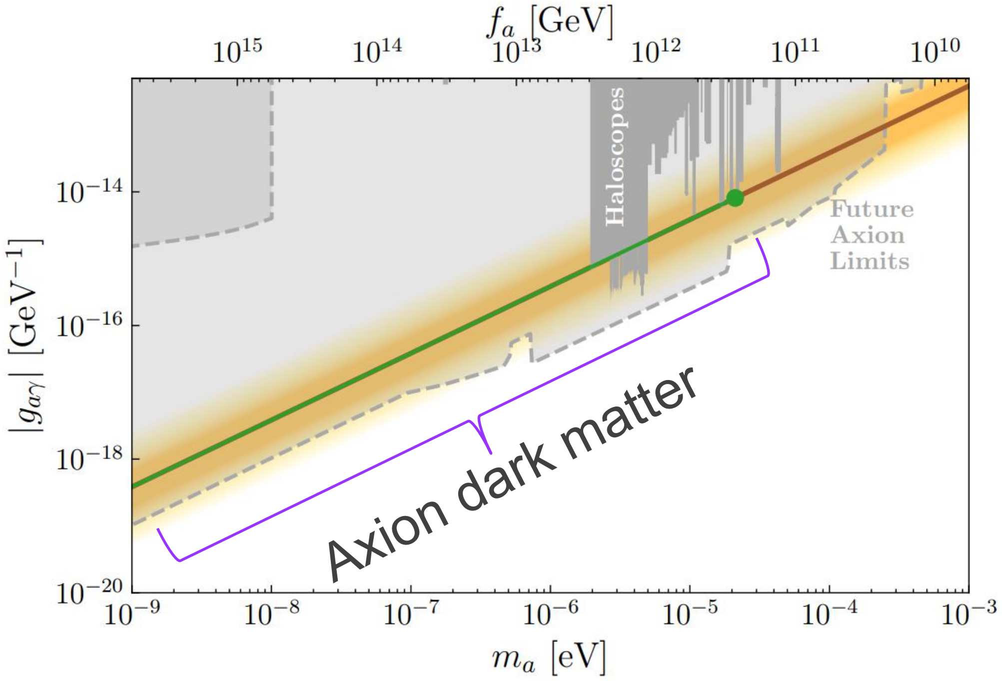


$$m_a = 5.7 \times 10^{-5}\,\text{eV} \left(\frac{10^{11}\,\text{GeV}}{f_a}\right)$$

PQ breaking and inflation

- If PQ symmetry is broken before inflation, the whole observable universe has the same initial angle θ_i, QCD axion could be much lighter.

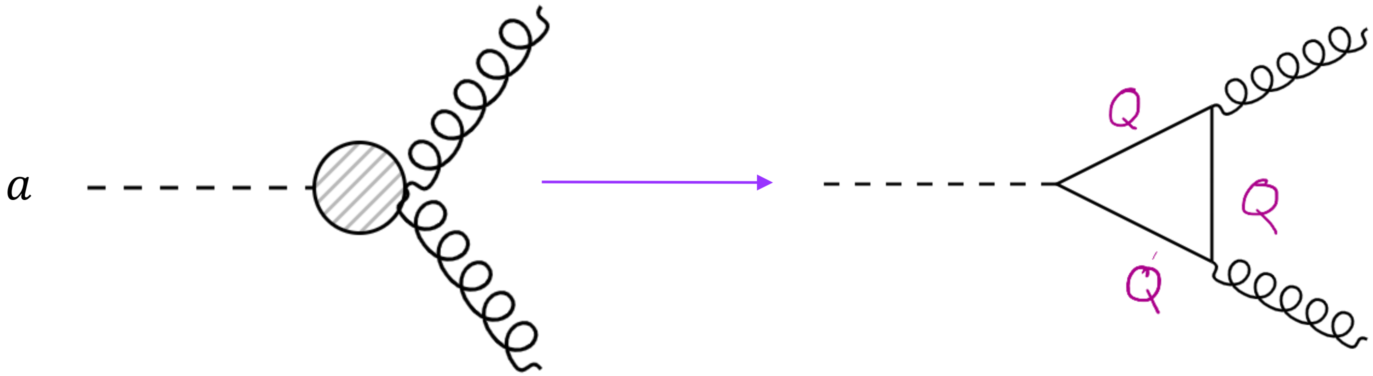
$$\Omega_a h^2 \approx 0.12 \left(\frac{\theta_{\rm i}}{2.15}\right)^2 \left(\frac{28\ \mu{\rm eV}}{m_a}\right)^{7/6}$$



- The discovery of a light axion would be an indication of pre-inflationary PQ breaking.
- Other phenomenological considerations,
  - Thermal axion contributions to dark radiation.
  - Isocurvature bounds on scale of inflation.

Complicated by completions

$$\mathcal{L}^a_{\rm eff} = \frac{a\,(t,x)}{f_a}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$



UV completions must involve strongly coupled particles.

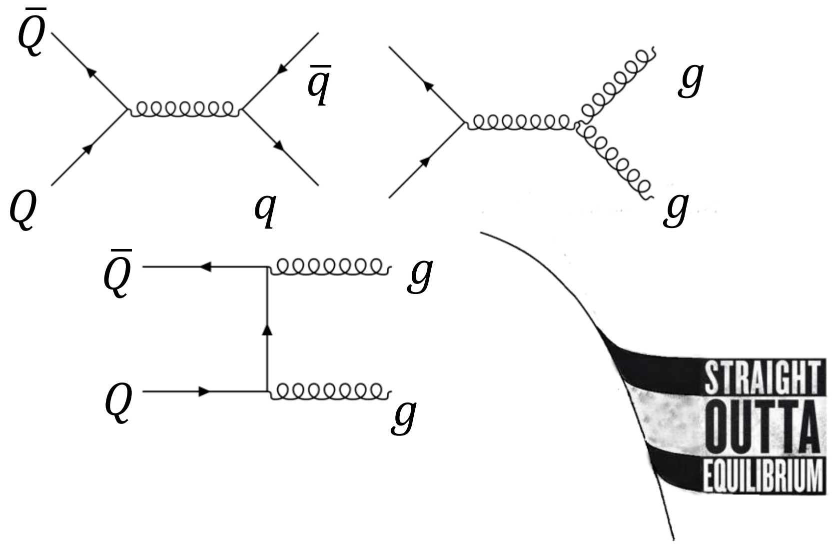
KSVZ: SM fields are $U(1)_{\rm PQ}$ neutral → Introduce heavy quarks which need to decay ✦

DFSZ: SM fields are charged under $U(1)_{\rm PQ}$ → suffer from a domain wall problem ✦

✦ These problems can be avoided with pre-inflationary PQ breaking

Heavy quark disaster

These new strongly interacting massive particles undergo thermal freeze-out in a similar way to weak-scale dark matter, but now they are more massive and overproduced!



Assuming stable Q,

$$Y_Q^\infty \approx \frac{x_f}{\lambda} \approx \frac{10\, H(m_Q)}{m_Q^3 \langle \sigma v \rangle}$$

leads to

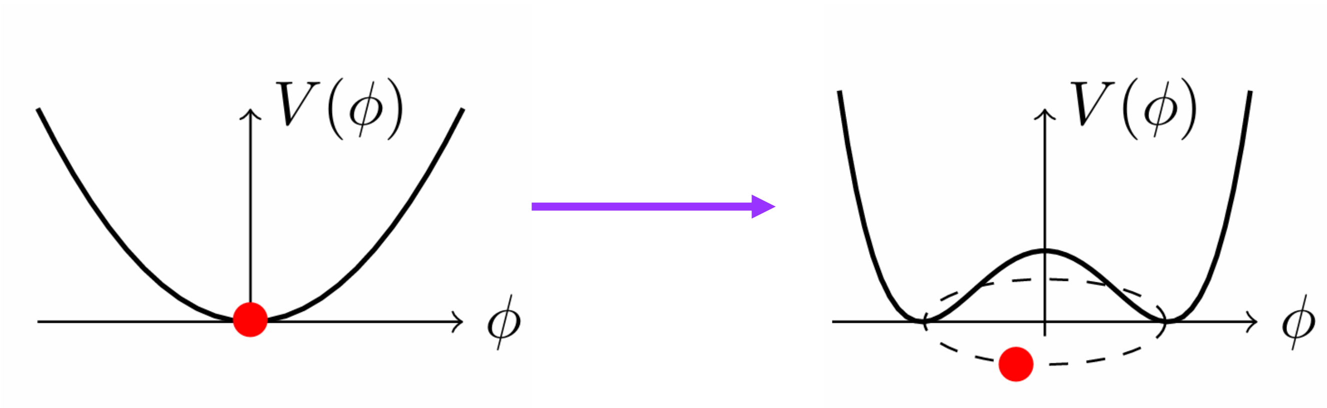
$$\frac{\rho_Q}{\rho_{\rm R}^{\rm SM}} \sim 10^{10} \left(\frac{m_Q}{10^{12}\,{\rm GeV}} \right)^2 \left(\frac{1\,{\rm MeV}}{T} \right)$$

$m_Q \geq 10^7$ GeV, quarks dominate before BBN

How heavy is the quark?

The **KSVZ** quark gets its mass from $U(1)_{\mathrm{PQ}}$ symmetry breaking

$$\mathcal{L}_{\mathrm{PQ}} = |\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q \; - (y_Q \overline{Q}_L Q_R \Phi + \mathrm{H.c})$$



$\Phi \to (v_\varphi/\sqrt{2}) e^{ia/f_a}$ so $m_Q = y_Q f_a/\sqrt{2}$, typical choice is $y_Q \approx 1$.

So $m_Q \sim f_a$

Heavy quarks must decay

- If such heavy quarks will be overabundantly produced via freeze-out they must decay,

$$\mathcal{L}_{\rm PQ} = |\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q - (y_Q \overline{Q}_L Q_R \Phi + \text{H.c})$$

$$U(1)^3 \equiv U(1)_{Q_L} \times U(1)_{Q_R} \times U(1)_\phi \rightarrow U(1)_{\rm PQ} \times U(1)_Q$$

- Must introduce Q-breaking term
- This is only possible for some charge assignments
- For example,

$$R_Q: \left(3, 1, -\frac{1}{3}\right) \quad \text{and} \quad \overbrace{(\chi_L, \chi_R)}^{\text{PQ charge}} = \left(\frac{1}{2}, -\frac{1}{2}\right) \longrightarrow$$

In this case, decay impossible!

($3 \to SU(3)_c$, $1 \to SU(2)_W$, $-\frac{1}{3} \to U(1)_Y$)

Not many choices for SM charges

Sticking with only renormalizable terms is already quite restrictive, especially if $N_{\mathrm{DW}} = 1$

R_Q	$\mathcal{O}_{Qq}$	$\Lambda_{LP}^{R_Q}$[GeV]	E/N	N_{DW}
R_1: $(3, 1, -\frac{1}{3})$	$\overline{Q}_L d_R$	$9.3 \cdot 10^{38} (g_1)$	$2/3$	1
R_2: $(3, 1, +\frac{2}{3})$	$\overline{Q}_L u_R$	$5.4 \cdot 10^{34} (g_1)$	$8/3$	1
R_3: $(3, 2, +\frac{1}{6})$	$\overline{Q}_R q_L$	$6.5 \cdot 10^{39} (g_1)$	$5/3$	2
R_4: $(3, 2, -\frac{5}{6})$	$\overline{Q}_L d_R H^\dagger$	$4.3 \cdot 10^{27} (g_1)$	$17/3$	2

R_Q	$\mathcal{O}_{Qq}$	$\Lambda_{LP}^{R_Q}$[GeV]	E/N	N_{DW}
R_5: $(3, 2, +\frac{7}{6})$	$\overline{Q}_L u_R H$	$5.6 \cdot 10^{22} (g_1)$	$29/3$	2
R_6: $(3, 3, -\frac{1}{3})$	$\overline{Q}_R q_L H^\dagger$	$5.1 \cdot 10^{30} (g_2)$	$14/3$	3
R_7: $(3, 3, +\frac{2}{3})$	$\overline{Q}_R q_L H$	$6.6 \cdot 10^{27} (g_2)$	$20/3$	3

L. Di Luzio et. al.
[arXiv:1610.07593]

From here, can determine distinct models from PQ charges

$$\mathcal{O}_4^M = M_d \overline{Q}_L d_R, \qquad \text{for } (\chi_L, \chi_R) = (0, -1), \qquad \text{Model A,}$$
$$\mathcal{O}_4^H = y_{1,q} H \overline{q}_L Q_R, \qquad \text{for } (\chi_L, \chi_R) = (1, 0), \qquad \text{Model B,}$$
$$\mathcal{O}_4^\Phi = y_{2,d} \Phi \overline{Q}_L d_R, \qquad \text{for } (\chi_L, \chi_R) = (1, 0), \qquad \text{Model B,}$$
$$\mathcal{O}_4^{\Phi^\dagger} = y_{3,d} \Phi^\dagger \overline{Q}_L d_R, \qquad \text{for } (\chi_L, \chi_R) = (-1, -2), \qquad \text{Model C.}$$

Not many choices for SM charges

Sticking with only renormalizable terms is already quite restrictive, especially if $N_{\rm DW} = 1$

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{LP}$[GeV]	E/N	N_{DW}
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R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{LP}$[GeV]	E/N	N_{DW}
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R_7: $(3, 3, +\frac{2}{3})$	$\overline{Q}_R q_L H$	$6.6 \cdot 10^{27}(g_2)$	$20/3$	3

L. Di Luzio et. al. (PRL) [arXiv:1610.07593]

Can also go to the non-renormalizable level to determine the limit?

$$\mathcal{L}_{Qq} = \mathcal{L}_{Qq}^{d\leq 4} + \mathcal{L}_{Qq}^{d>4} = \mathcal{L}_{Qq}^{d\leq 4} + \frac{1}{\Lambda^{(d-4)}}\mathcal{O}^{d>4} + \text{h.c.}$$

This leads to decays which are suppressed by powers of $\Lambda \neq f_a$

$$\Gamma_{d,n_f} = \frac{m_Q}{4\,(4\pi)^{2n_f-3}\,(n_f-1)!\,(n_f-2)!}\left(\frac{m_Q^2}{\Lambda^2}\right)^{d-4}$$

Can explore *preferred* models

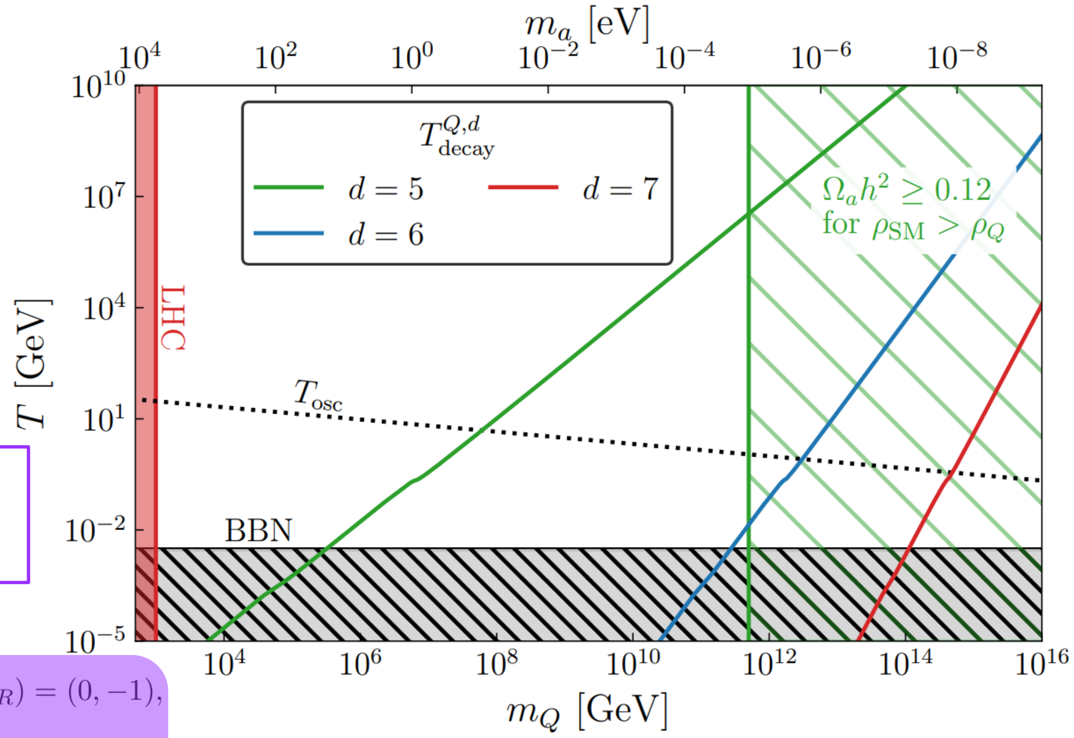
Preferred axion models **decay via dimension 5 at most!**

$$\text{KSVZ–I} : (3,\ 1,\ -1/3),\ \text{or}$$
$$\text{KSVZ–II} : (3,\ 1,\ +2/3).$$

$$\mathcal{O}_4^M = M_d \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$
$$\mathcal{O}_4^H = y_{1,d} H \bar{d}_L Q_R, \quad \text{for } (\chi_L, \chi_R) = (1, 0),$$
$$\mathcal{O}_4^\Phi = y_{2,d} \Phi \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (1, 0),$$
$$\mathcal{O}_4^{\Phi^\dagger} = y_{3,d} \Phi^\dagger \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (-1, -2),$$
$$\mathcal{O}_5^\Phi = \frac{\lambda_{2,d}}{\Lambda} \Phi^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (2, 1),$$
$$\mathcal{O}_5^{\Phi H} = \frac{\lambda'_{2,d}}{\Lambda} \overline{Q}_R q_L H^\dagger \Phi, \quad \text{for } (\chi_L, \chi_R) = (2, 1),$$
$$\mathcal{O}_5^{\Phi^\dagger} = \frac{\lambda_{3,d}}{\Lambda} (\Phi^\dagger)^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (-2, -3),$$

5 models for each KSVZ model type

$$\mathcal{O}_5^{|H|^2} = \frac{\lambda_d}{\Lambda} |H|^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$
$$\mathcal{O}_5^{|\Phi|^2} = \frac{\lambda'_d}{\Lambda} |\Phi|^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$
$$\mathcal{O}_5^H = \frac{\lambda_{1,d}}{\Lambda} \Phi H \bar{d}_L Q_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$



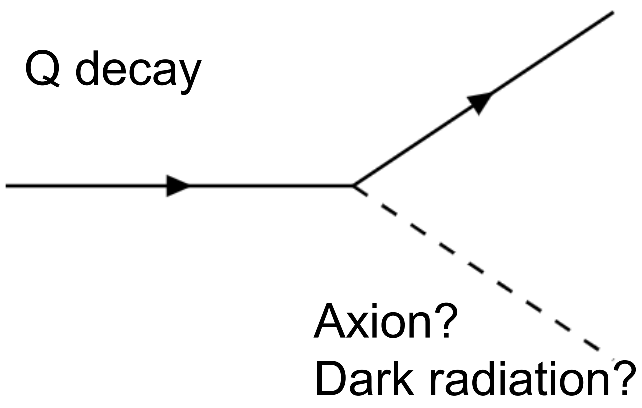
[arXiv:2411.17320]
AC + Ui Min

Decay of heavy quarks

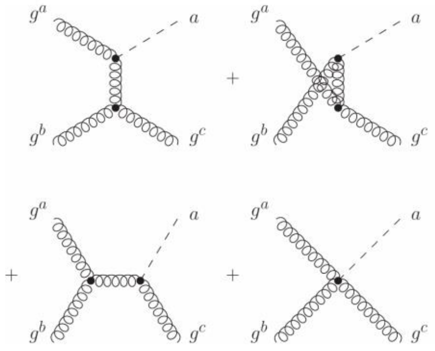
- Heavy quark decay products may leave a trace in the early universe.

$$\frac{\mathrm{d}\rho_a}{\mathrm{d}t} = -4H\rho_a + \mathrm{BR}_{\mathrm{axion}}\Gamma_Q\rho_Q + \langle E^{\mathrm{axion}}_{\mathrm{scat}}\rangle\gamma_a\left(1 - \frac{n_a}{n_a^{\mathrm{eq}}}\right),$$

$$\frac{\mathrm{d}n_Q}{\mathrm{d}t} = -3Hn_Q - \Gamma_Q n_Q - \langle\sigma\, v\rangle\left[n_Q^2 - (n_Q^{\mathrm{eq}})^2\right].$$



$T^{\mathrm{decoupling}}_{\mathrm{axion}} < T^{\mathrm{decay}}_{Q}$?



P. Graf et. al. (PRD) [arXiv:1008.4528] + more

Light remnants of heavy quarks

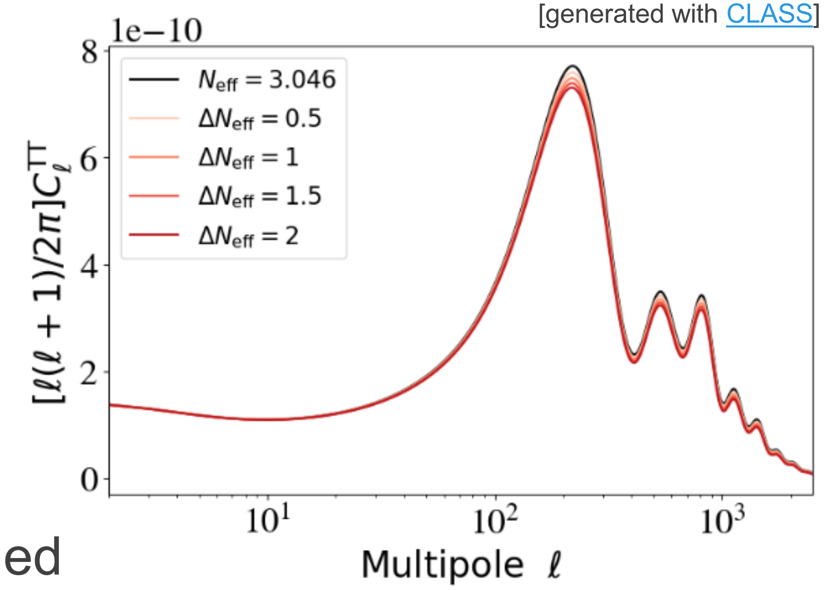
In the standard picture of the Big Bang, we have two particles species that remain relativistic until recombination.

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu$$

$$\rho_{\rm rad} = \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{\rm eff}\right]$$

$$(N_{\rm eff})_{\rm P18} = 2.88^{+0.44}_{-0.42}$$

[generated with CLASS]



Consistent with 3 light neutrinos, additions constrained

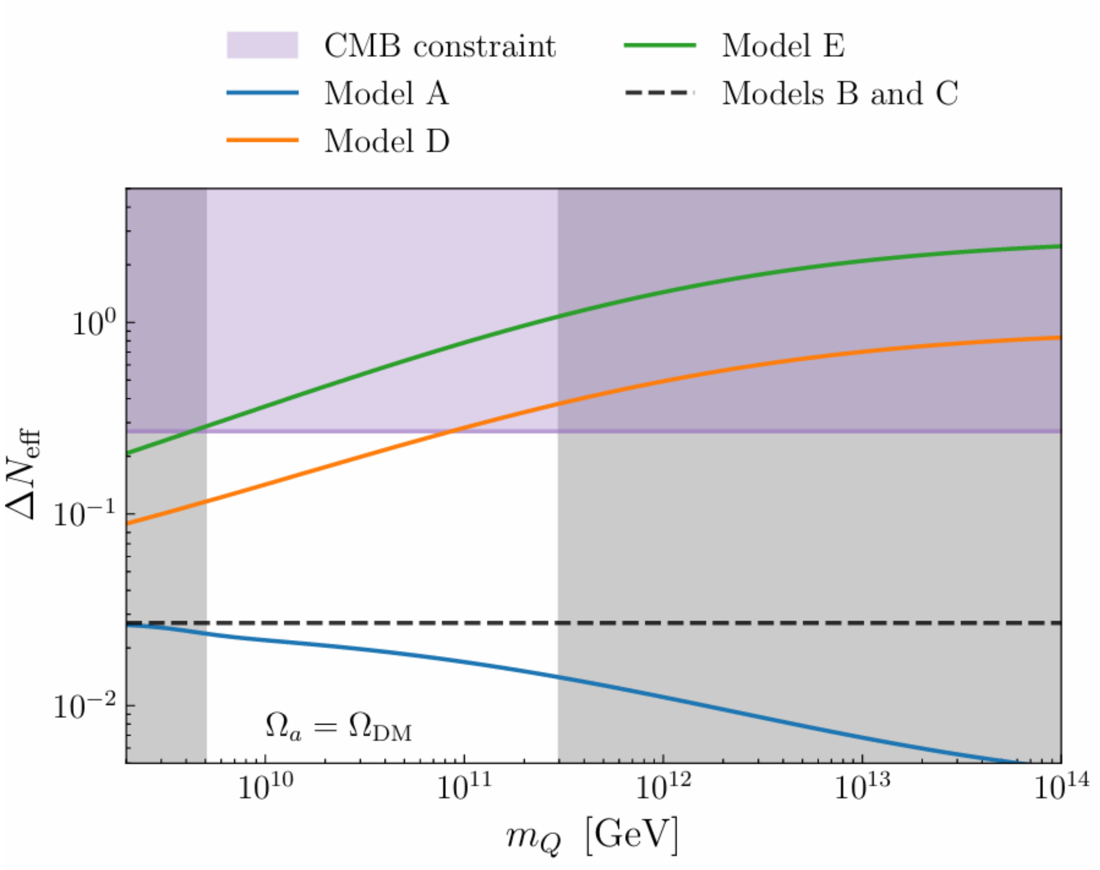
$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu + \rho_{\rm DR} \longrightarrow \Delta N_{\rm eff} \equiv \left\{\frac{8}{7}\left(\frac{4}{11}\right)^{-\frac{4}{3}} + N_{\rm eff}^{\rm SM}\right\}\frac{\rho_a}{\rho_{\rm R}^{\rm SM}}$$

$$\Delta N_{\rm eff} \leq 0.276$$

CMB disfavors some models

Decay	Model A	Model B	Model C	Model D	Model E
$\Gamma(Q \to a\, d) = \frac{3}{32\pi}\mathcal{C}^2 m_Q$	—	$\frac{y_{2,d}}{\sqrt{2}}$	$\frac{y_{3,d}}{\sqrt{2}}$	$\frac{\lambda_{2,d} f_a}{\Lambda}$	$\frac{\lambda_{3,d} f_a}{\Lambda}$
$\Gamma(Q \to H\, q_L) = \frac{3}{32\pi}\mathcal{C}^2 m_Q$	$\frac{\lambda_{1,q} f_a}{\Lambda}$	$y_{1,q}$	—	$\frac{\lambda_{2,q} f_a}{\Lambda}$	—
$\Gamma(Q \to a\, a\, d) = \frac{1}{256\pi^3}\mathcal{C}^2 m_Q^3$	—	$\frac{y_{2,d}}{2 f_a}$	$\frac{y_{3,d}}{2 f_a}$	—	$\frac{\lambda_{3,d}}{\Lambda}$
$\Gamma(Q \to H\, H\, d) = \frac{1}{256\pi^3}\mathcal{C}^2 m_Q^3$	$\frac{\lambda_d}{\Lambda}$	—	—	—	—
$\Gamma(Q \to H\, q_L\, a) = \frac{1}{512\pi^3}\mathcal{C}^2 m_Q^3$	$\frac{\lambda_{1,q}}{\Lambda}$	—	—	$\frac{\lambda_{2,q}}{\Lambda}$	—
M_d	M_d	$\frac{y_{2,d} f_a}{\sqrt{2}}$	$\frac{y_{3,d} f_a}{\sqrt{2}}$	$\frac{\lambda_{2,d} f_a^2}{2\Lambda}$	$\frac{\lambda_{3,d} f_a^2}{2\Lambda}$

[arXiv:2411.17320]
AC + Ui Min (JCAP)

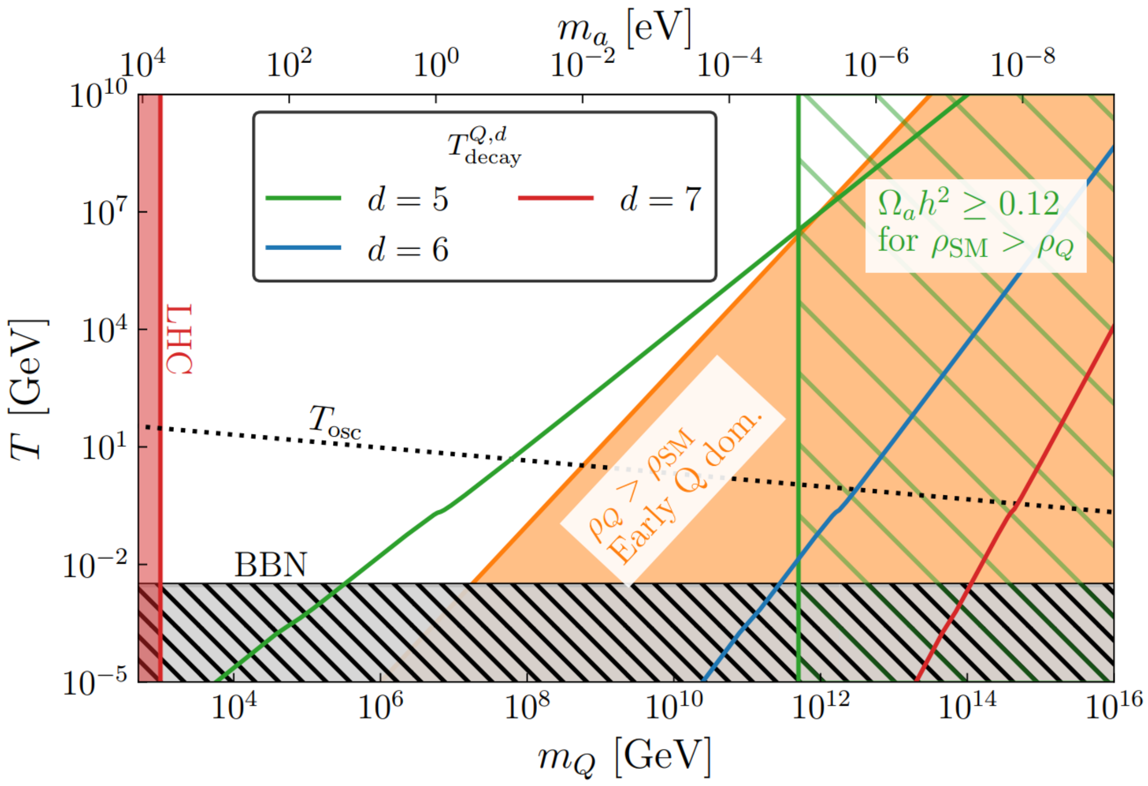


2

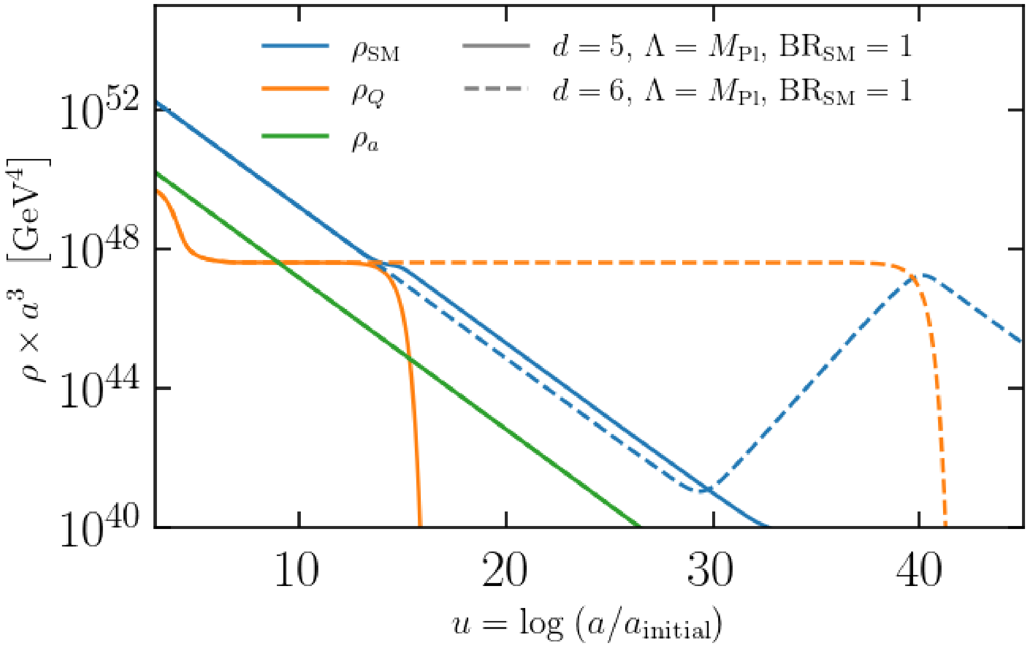
Heavy quark domination

- For these higher dimensional Q decay models, the heavy quarks will dominate the early universe.
- This alters the misalignment mechanism, has been known for decades *Steinhart* et. al. (1984) + *Lazarides* et. al. (1990)
- We show $T_{\rm osc}$, temperature when axion field oscillations begin

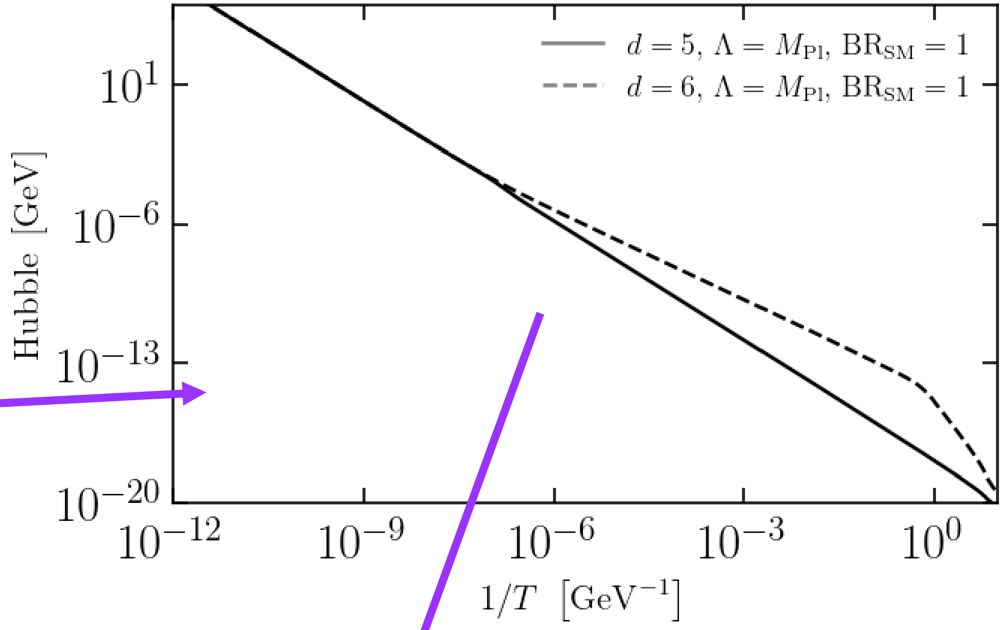
$$3H\left(T_{\rm osc}\right) = \tilde{m}_a\left(T_{\rm osc}\right)$$



First you approximate, then you solve



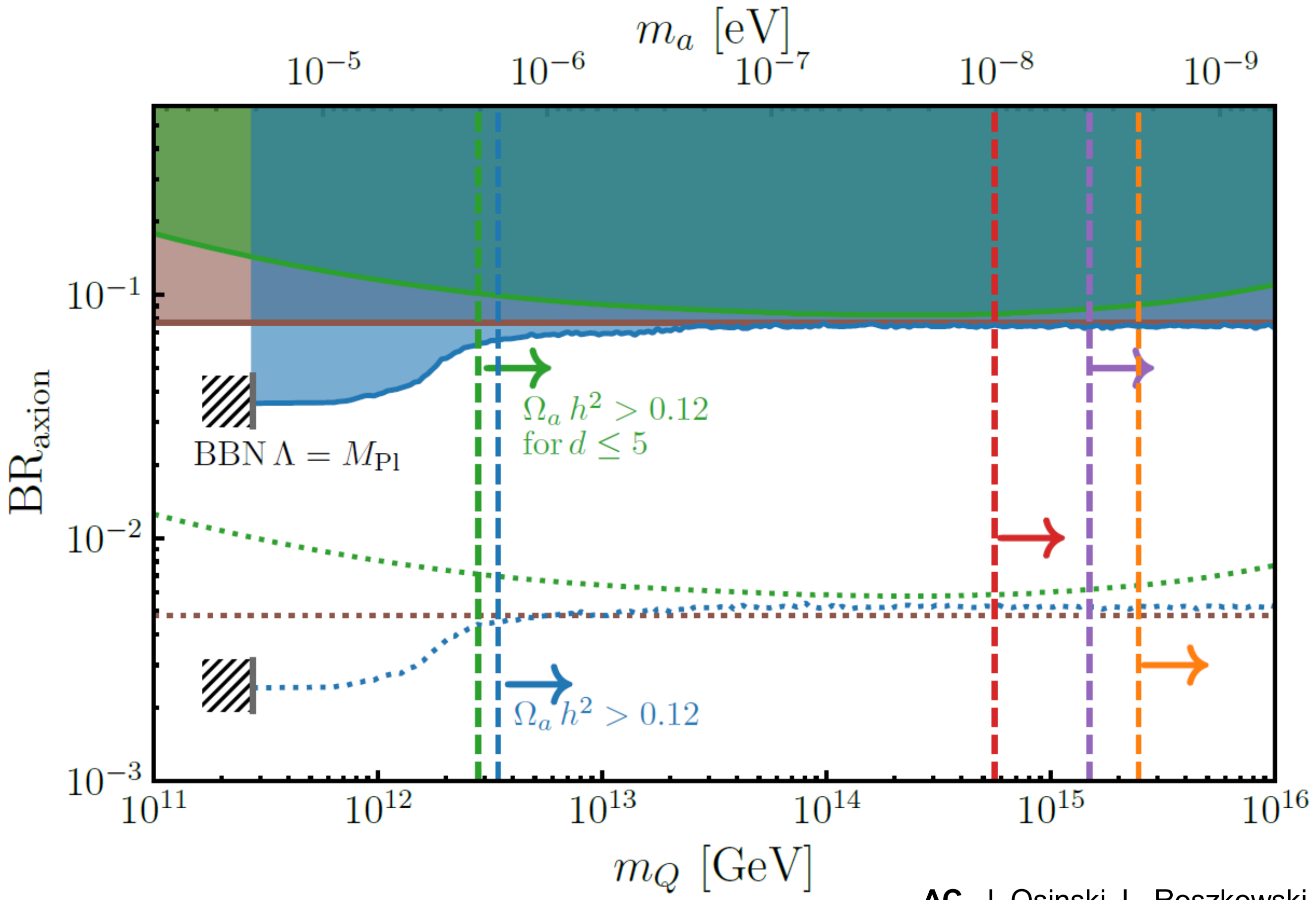
Scale factor



$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} + 3H(t)\frac{\mathrm{d}}{\mathrm{d}t}\right)\theta(t) + \tilde{m}_a^2(t)\sin(\theta(t)) = 0$$

[MiMes] misalignment solver

Constraints from dark radiation



Current measurements of CMB give

$$\mathrm{BR}_a \leq 0.077$$

Future measurements of CMB project

$$\mathrm{BR}_a \leq 0.0048$$

AC, J. Osinski, L. Roszkowski [2310.16087]

More models without domain walls

Recently, Di Luzio et. al. confirmed my findings and catalogued higher dimensional models

L. Di Luzio et. al. [arXiv:2412.17896]

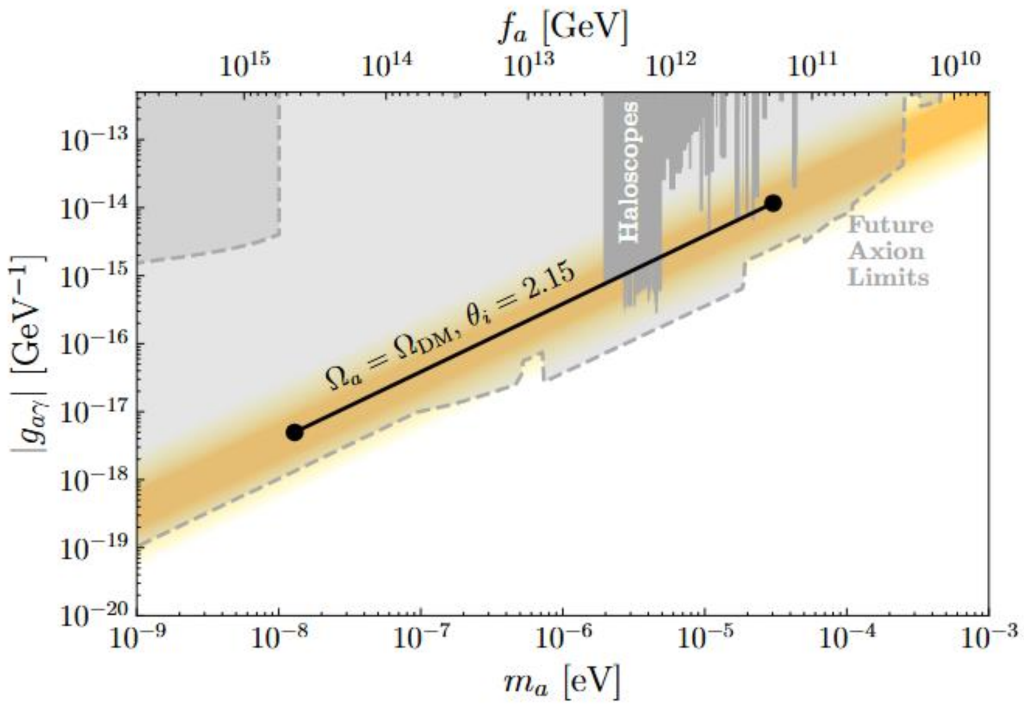
Rep. $(\mathcal{C}, \mathcal{I}, 6\mathcal{Y})$			E/N	N_{DW}	Min. d	Example operator	LP [GeV]
3	1	-2	$2/3$	1	3	$\bar{\mathcal{Q}}_L\, d_R$	2.0×10^{39}
3	1	4	$8/3$	1	3	$\bar{\mathcal{Q}}_L\, u_R$	6.8×10^{35}
3	1	-14	$98/3$	1	6	$\bar{\mathcal{Q}}_L\, d_R\, (\bar{e}^c_R\, e_R)$	2.2×10^{22}
$\overline{3}$	1	8	$32/3$	1	6	$\bar{u}_R\, \gamma_\mu\, e_R\, \bar{d}_R\, \gamma^\mu\, \mathcal{Q}_R$	3.0×10^{28}
$\overline{3}$	1	-10	$50/3$	1	6	$(\bar{d}_R\, d^c_R)\, \bar{e}_R\, \mathcal{Q}_L$	6.4×10^{25}
3	1	16	$128/3$	1	6	$\bar{\mathcal{Q}}_L\, u_R\, (\bar{e}_R\, e^c_R)$	1.8×10^{21}
$\overline{3}$	1	20	$200/3$	1	9	$(\bar{d}^c_R\, d_R)\, (\bar{e}^c_R\, e_R)\, \bar{u}_R\, \mathcal{Q}_L$	6.2×10^{19}
3	1	22	$242/3$	1	9	$\bar{\mathcal{Q}}_L\, u_R\, (\bar{\ell}_L\, \ell^c_L)\, (\bar{e}_R\, e^c_R)$	2.0×10^{19}

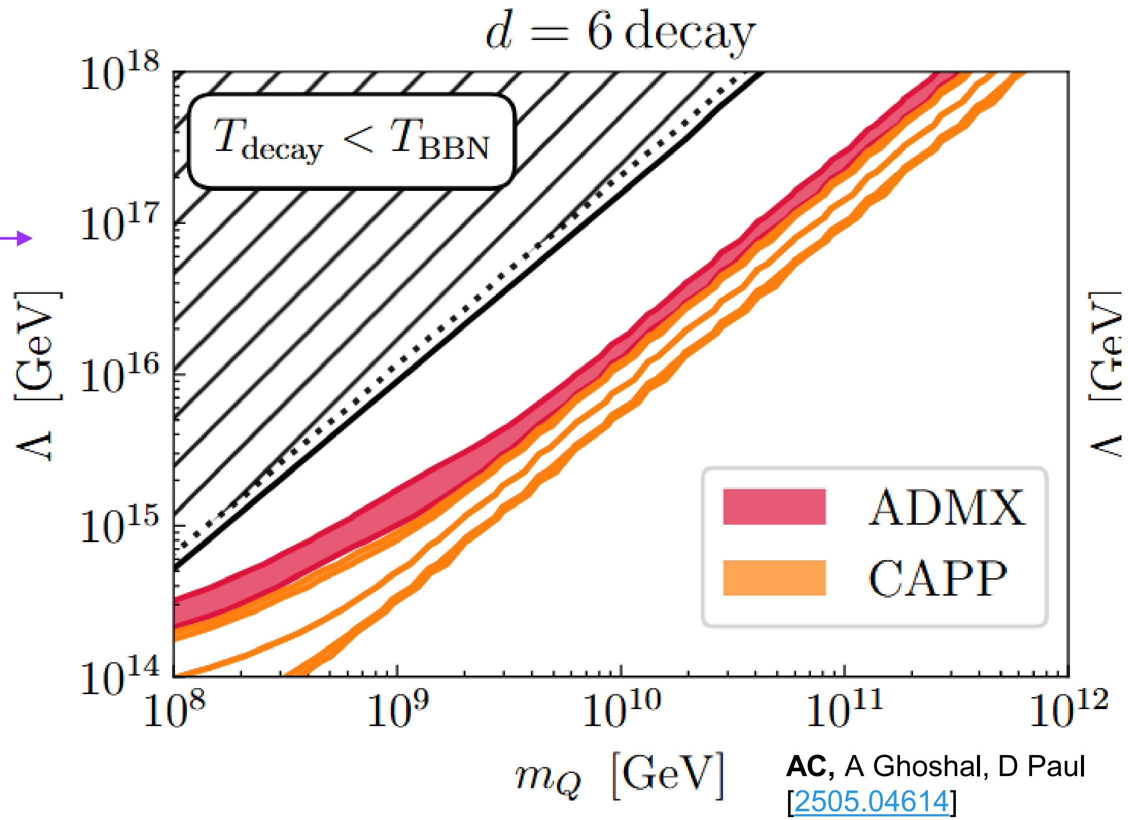
$\text{Br}_{\text{SM}} \approx 1$

$\therefore \Delta N_{\text{eff}} \ll 0.027$

$$g_{a\gamma} \equiv \frac{\alpha}{2\pi} \frac{1}{f_a} \left(\frac{E}{N} - 1.92(4) \right)$$

GUT-scale PQ breaking & $N_{\mathrm{DW}} = 1$





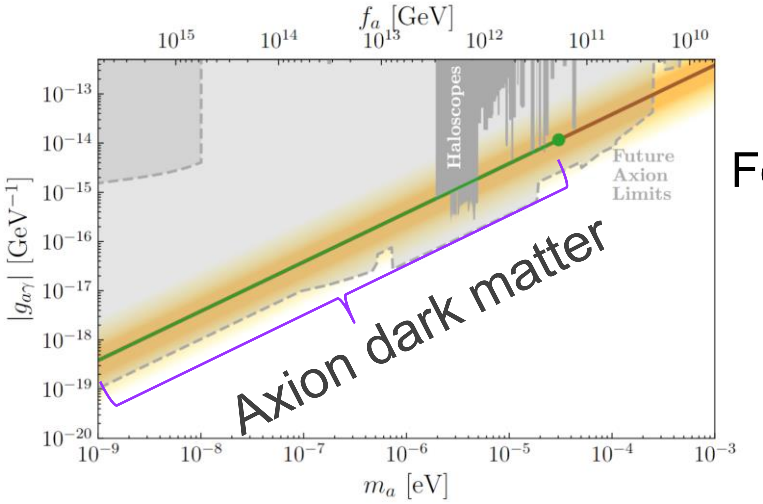
AC, A Ghoshal, D Paul [2505.04614]

When does PQ break?

BEFORE INFLATION

Can have $m_a \leq 10\ \mu\text{eV}$

No detectable dark radiation component from axion.



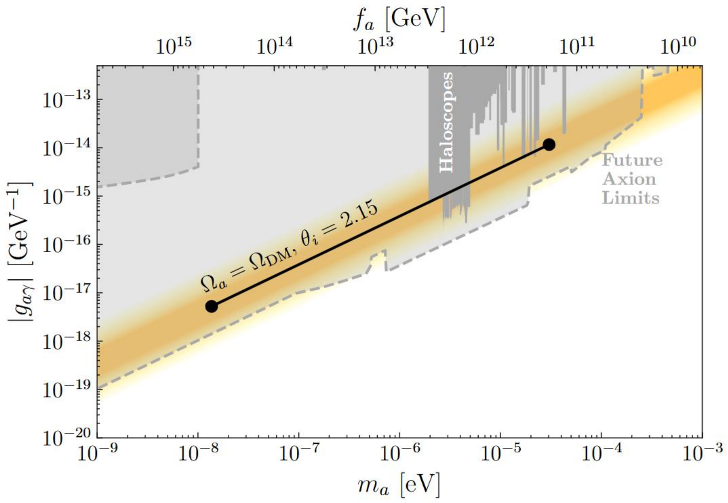
For $m_a = 10^{-8}$ eV

$\theta_i = 10^{-2}$

AFTER INFLATION

Now can have $m_a \leq 10\ \mu\text{eV}$ with HQD

The only models that survive with have no detectable dark radiation component from axion.



Both scenarios have the same phenomenological output.

We perform a full numerical treatment

To assess the effect, we solve the coupled Friedmann-Boltzmann equations

$$\frac{3H^2 M_{\rm Pl}^2}{8\pi} = \rho_{\rm R}^{\rm SM} + \rho_{\rm DR}^{\rm axion} + \rho_Q$$

Entropy density evolution:

$$\frac{{\rm d}s_{\rm R}^{\rm SM}}{{\rm d}t} = -3H s_{\rm R}^{\rm SM} + \frac{{\rm BR}_{\rm SM}\Gamma_Q}{T}\rho_Q \,,$$

Thermal axion energy density evolution:

$$\frac{{\rm d}\rho_a}{{\rm d}t} = -4H\rho_a + {\rm BR}_{\rm axion}\Gamma_Q\rho_Q + \langle E_{\rm scat}^{\rm axion}\rangle\gamma_a\left(1 - \frac{n_a}{n_a^{\rm eq}}\right) \,,$$

Heavy quark energy density evolution:

$$\frac{{\rm d}n_Q}{{\rm d}t} = -3Hn_Q - \Gamma_Q n_Q - \langle\sigma\, v\rangle\left[n_Q^2 - (n_Q^{\rm eq})^2\right] \,.$$

We perform a full numerical treatment

To fully assess the effect we solve the coupled Friedmann-Boltzmann equations

$$\frac{3H^2 M_{\rm Pl}^2}{8\pi} = \rho_{\rm R}^{\rm SM} + \rho_{\rm DR}^{\rm axion} + \rho_Q$$

Entropy density evolution:

$$\frac{{\rm d}s_{\rm R}^{\rm SM}}{{\rm d}t} = -3H s_{\rm R}^{\rm SM} + \frac{{\rm BR}_{\rm SM}\Gamma_Q}{T}\rho_Q \,,$$

“Simplified” branching ratio
If only $Q \to a + q$ the only decay channel, ${\rm BR}_{\rm SM} = {\rm BR}_{\rm axion} = 1/2$

Thermal axion energy density evolution:

$$\frac{{\rm d}\rho_a}{{\rm d}t} = -4H\rho_a + {\rm BR}_{\rm axion}\Gamma_Q \rho_Q + \langle E_{\rm scat}^{\rm axion}\rangle \gamma_a \left(1 - \frac{n_a}{n_a^{\rm eq}}\right) \,,$$

Heavy quark energy density evolution:

$$\frac{{\rm d}n_Q}{{\rm d}t} = -3Hn_Q - \Gamma_Q n_Q - \langle \sigma\, v\rangle \left[n_Q^2 - (n_Q^{\rm eq})^2\right] \,.$$

We perform a full numerical treatment

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$$\frac{3H^2 M_{\rm Pl}^2}{8\pi} = \rho_{\rm R}^{\rm SM} + \rho_{\rm DR}^{\rm axion} + \rho_Q$$

γ_a: Thermal axion production rate more on this later.

$$\left\langle E_{\rm scat}^{\rm axion} \right\rangle \sim 3T_{\rm SM}$$

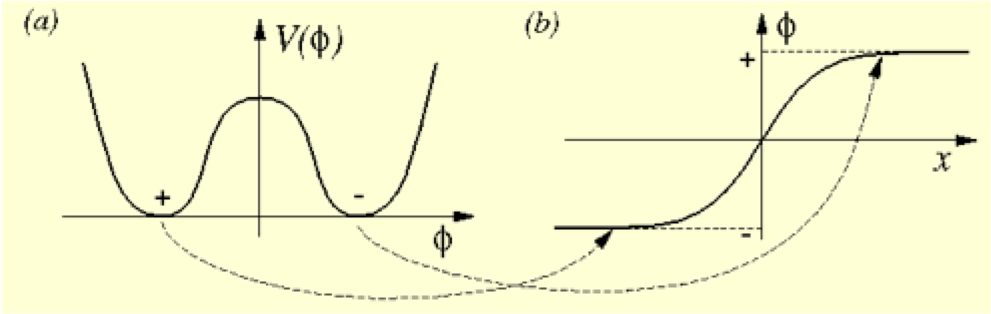
Entropy density evolution: $$\frac{{\rm d}s_{\rm R}^{\rm SM}}{{\rm d}t} = -3Hs_{\rm R}^{\rm SM} + \frac{{\rm BR_{SM}}\Gamma_Q}{T}\rho_Q \,,$$

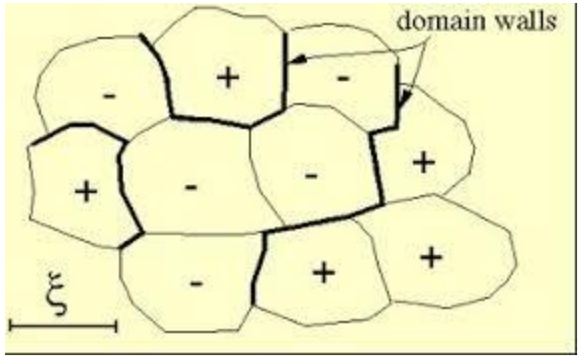
Thermal axion energy density evolution: $$\frac{{\rm d}\rho_a}{{\rm d}t} = -4H\rho_a + {\rm BR_{axion}}\Gamma_Q\rho_Q + \langle E_{\rm scat}^{\rm axion}\rangle\gamma_a\left(1 - \frac{n_a}{n_a^{\rm eq}}\right) \,,$$

Heavy quark energy density evolution: $$\frac{{\rm d}n_Q}{{\rm d}t} = -3Hn_Q - \Gamma_Q n_Q - \langle\sigma v\rangle\left[n_Q^2 - (n_Q^{\rm eq})^2\right] \,.$$

Domain walls

- Topological defect where two distinct vacua are separated by a potential





- Stable domain walls scale like a^{-2} so can quickly dominate.
- Much axion model building effort has gone into getting these things to decay or be destroyed.
- I think its interesting to first explore models where this isn't a problem.