Global String Dynamics from the Kalb-Ramond Axion Duality

Mathieu Kaltschmidt

mkaltschmidt@unizar.es

based on JCAP10(2024)043 with K. Saikawa, J. Redondo & A. Vaquero

and work in progress with J. Redondo and I. Rybak

Axions in Stockholm
Nordita, June 30th 2025

Background: G. Pierobon

Departamento de Física Teórica
Universidad Zaragoza

CAPA Centro de Astropartículas y Física de Altas Energías
Universidad Zaragoza

GOBIERNO DE ARAGON

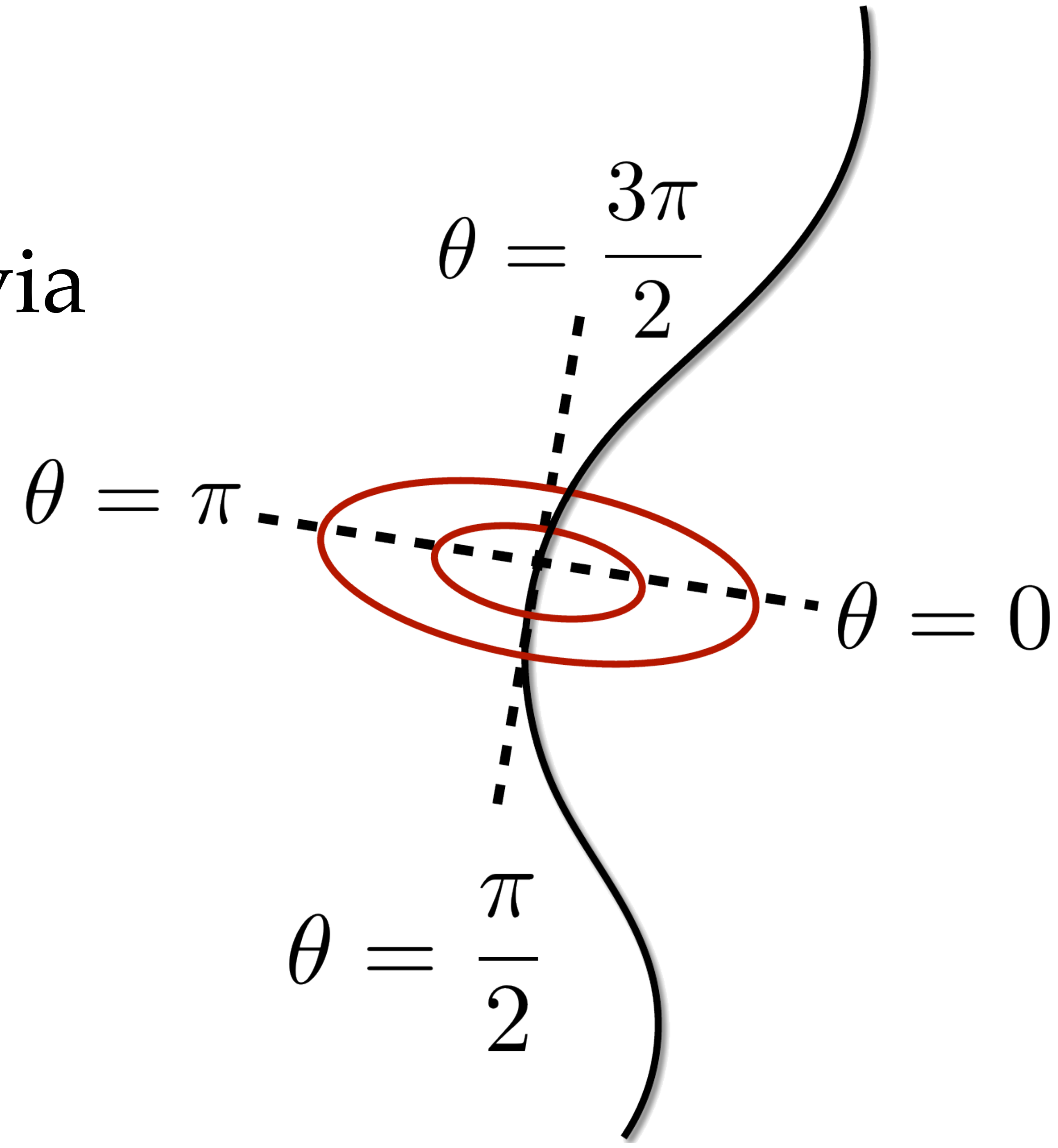
COSMIC WISPERS

cost EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

Axion Strings

- If the PQ symmetry is broken **after inflation,** we can in principle predict the QCD axion dark matter mass, which is a crucial input for direct detection experiments.
- In this scenario, a network of **topological defects** inevitably forms via the Kibble mechanism. Kibble (1976)
- We need to understand how they contribute to the QCD axion DM abundance.
- Strings are characterised by a non-zero energy density (per unit length), the **string tension:**

$$\mu = \hat{\mu} + \pi f_a^2 \int_{c_{\mathrm{UV}}}^{c_{\mathrm{IR}}} \frac{\mathrm{d}r}{r} \approx \hat{\mu} + \pi f_a^2 \ln\left(\frac{f_a}{H}\right)$$



How to simulate Axion Strings?

- Solve the classical EoM for a complex scalar field in comoving coordinates, discretised on a (static or more involved*) lattice:

$$\partial_\tau^2 \phi - \nabla^2 \phi + \lambda \phi \left(|\phi|^2 - \tau^2 \right) = 0$$

- **Tricky:** Simulations require proper resolution of two very different length scales

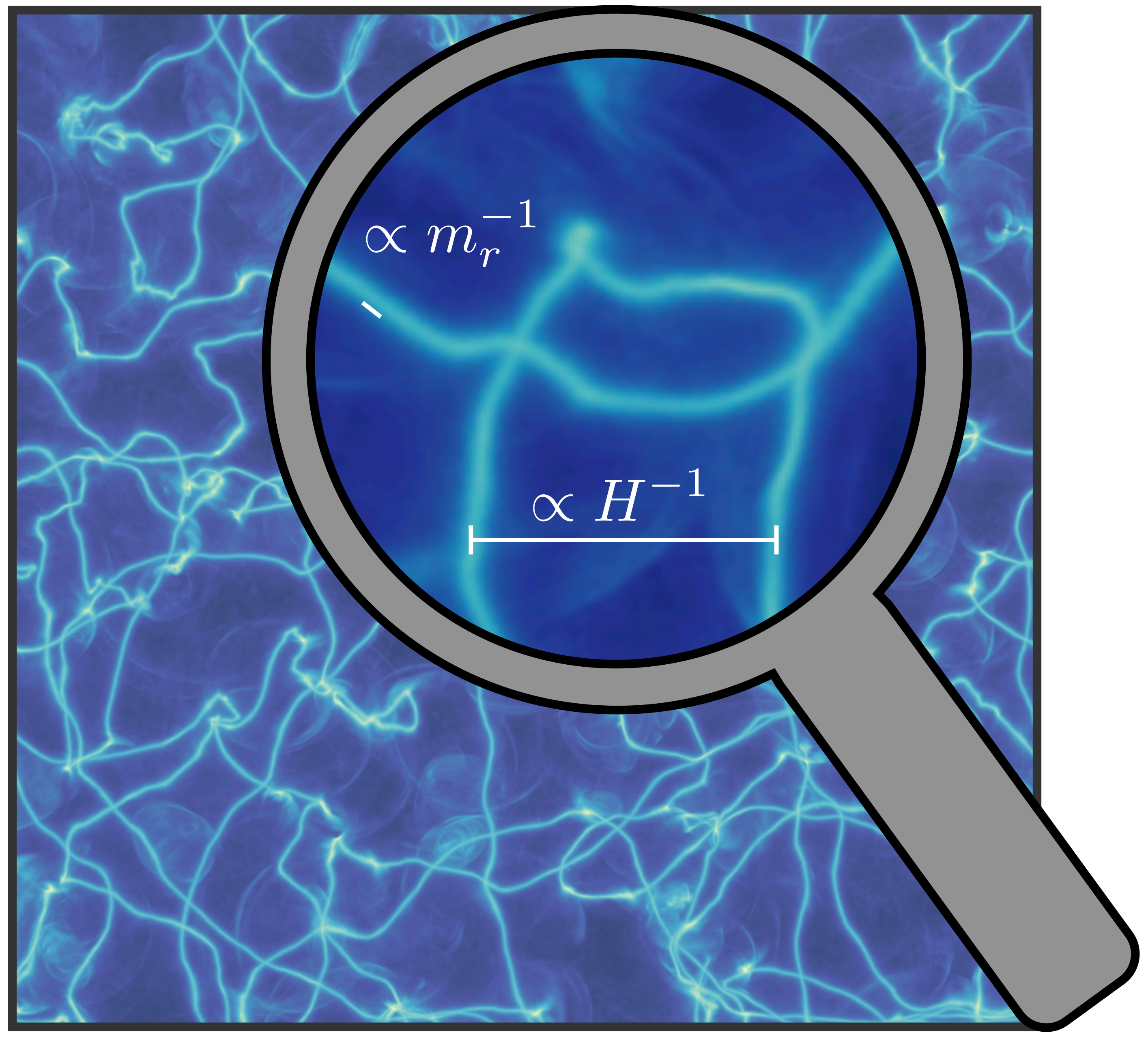
  - String core radius

    $$\propto \frac{1}{m_r} \propto \frac{1}{f_a}, \text{ where } m_r = \text{radial mass}$$

  - String separation given by Hubble radius

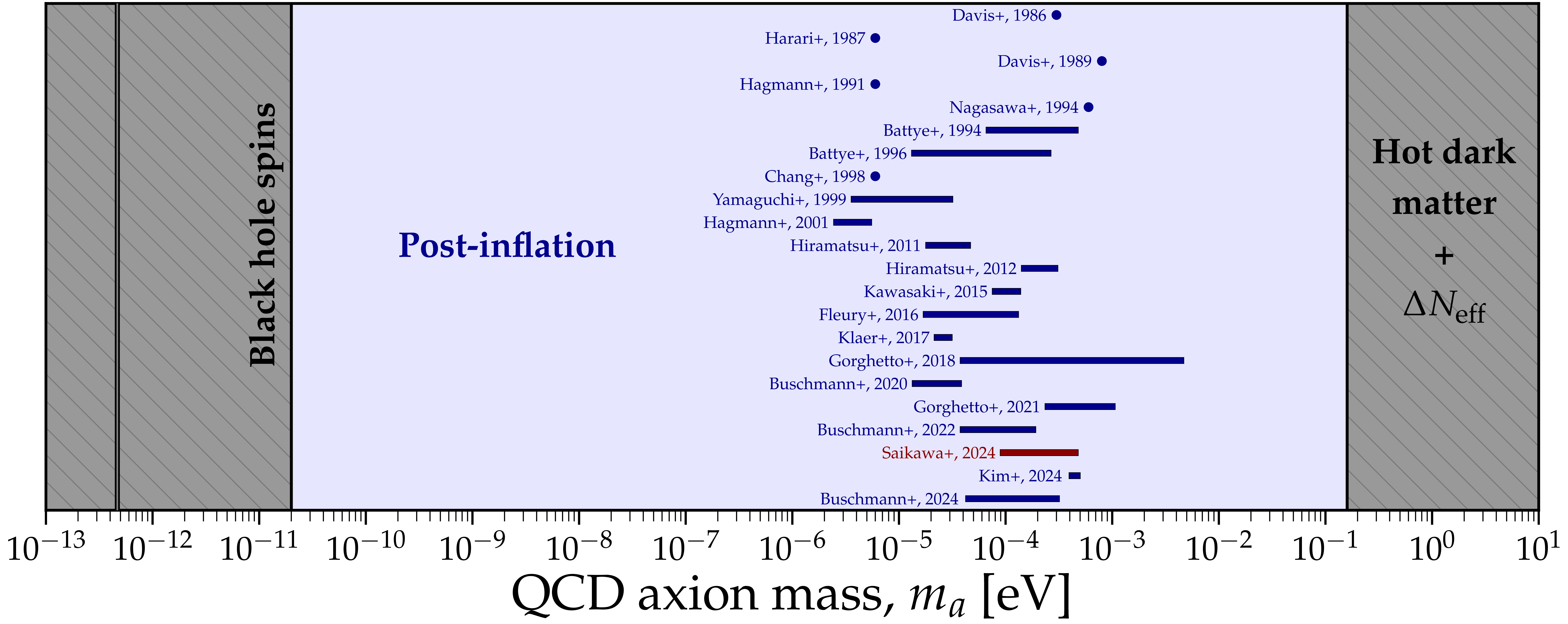
    $$\propto \frac{1}{H}$$

  - Realistic value: $\frac{f_a}{H_{\mathrm{QCD}}} \approx 10^{30} \implies \log\left(\frac{m_r}{H}\right) \approx 70$



Courtesy of K. Saikawa

Axion DM Mass Predictions from String Simulations



MK+ [2502.02398]

Axion String EFT

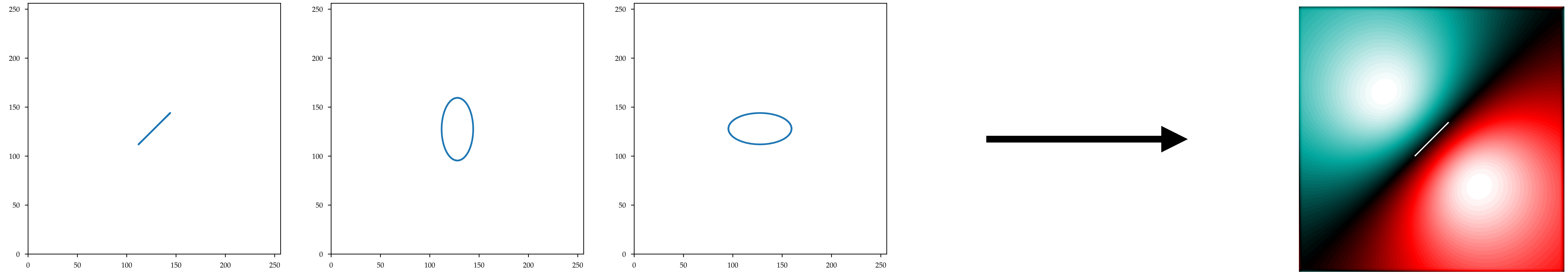
- Effective Theory that allows for an **analytic** treatment of axion strings at high tension, approaching the **Nambu-Goto** limit:

$$f_a \partial_\mu \theta = \epsilon_{\mu\nu\lambda\rho} \partial^\nu B^{\lambda\rho} \equiv \frac{1}{6} \epsilon_{\mu\nu\lambda\rho} H^{\nu\lambda\rho}$$

Shellard& Davis (1988),
Dabholkar & Quaschnock (1990)

- Kalb-Ramond field $B_{\mu\nu}$ describes the behaviour of the axion field around strings:

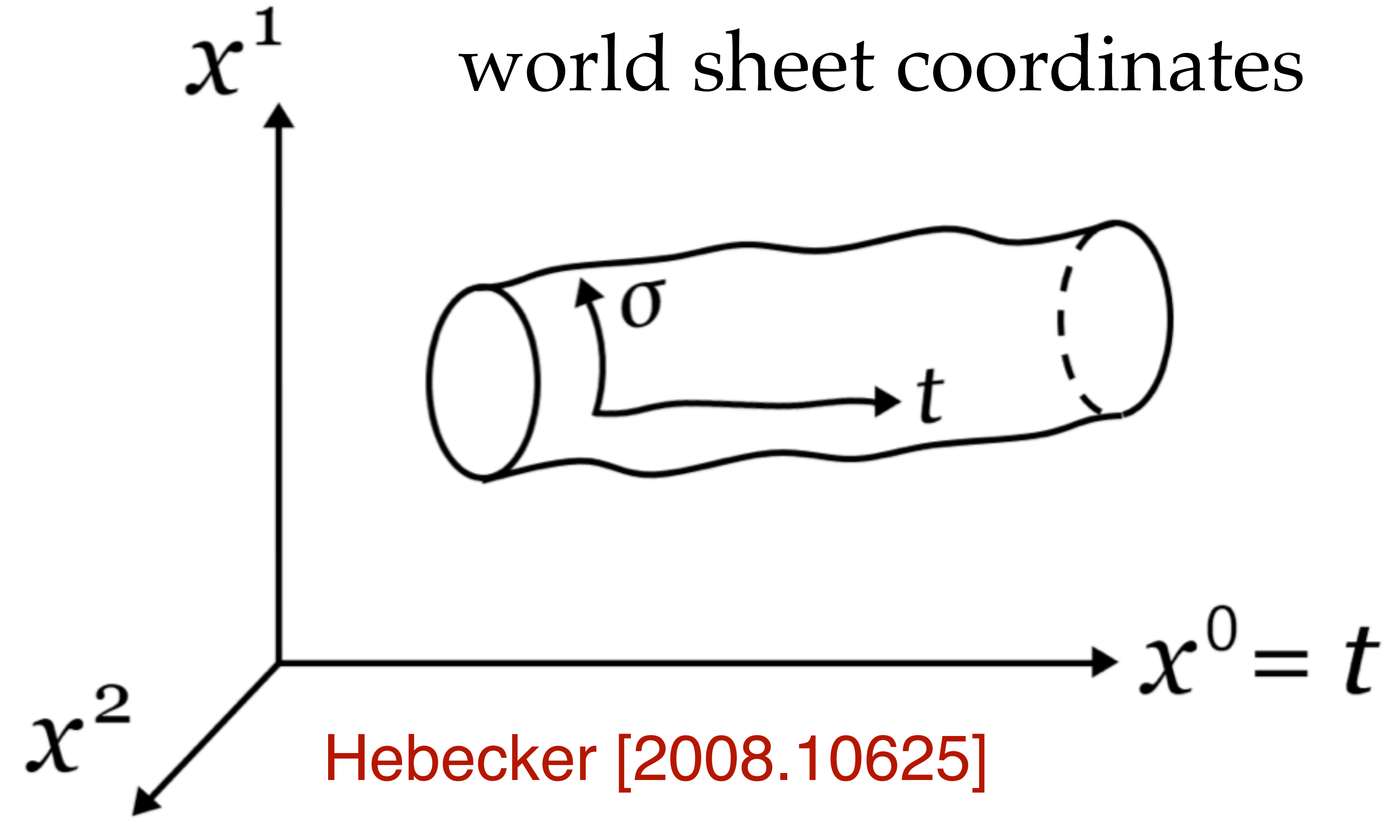
$$\mathcal{S} = -\mu \int \sqrt{-\gamma} \mathrm{d}^2\sigma - f_a \int B_{\mu\nu} \mathrm{d}\sigma^{\mu\nu} + \frac{1}{6} \int H^2 \sqrt{-g}\, \mathrm{d}^4 x$$

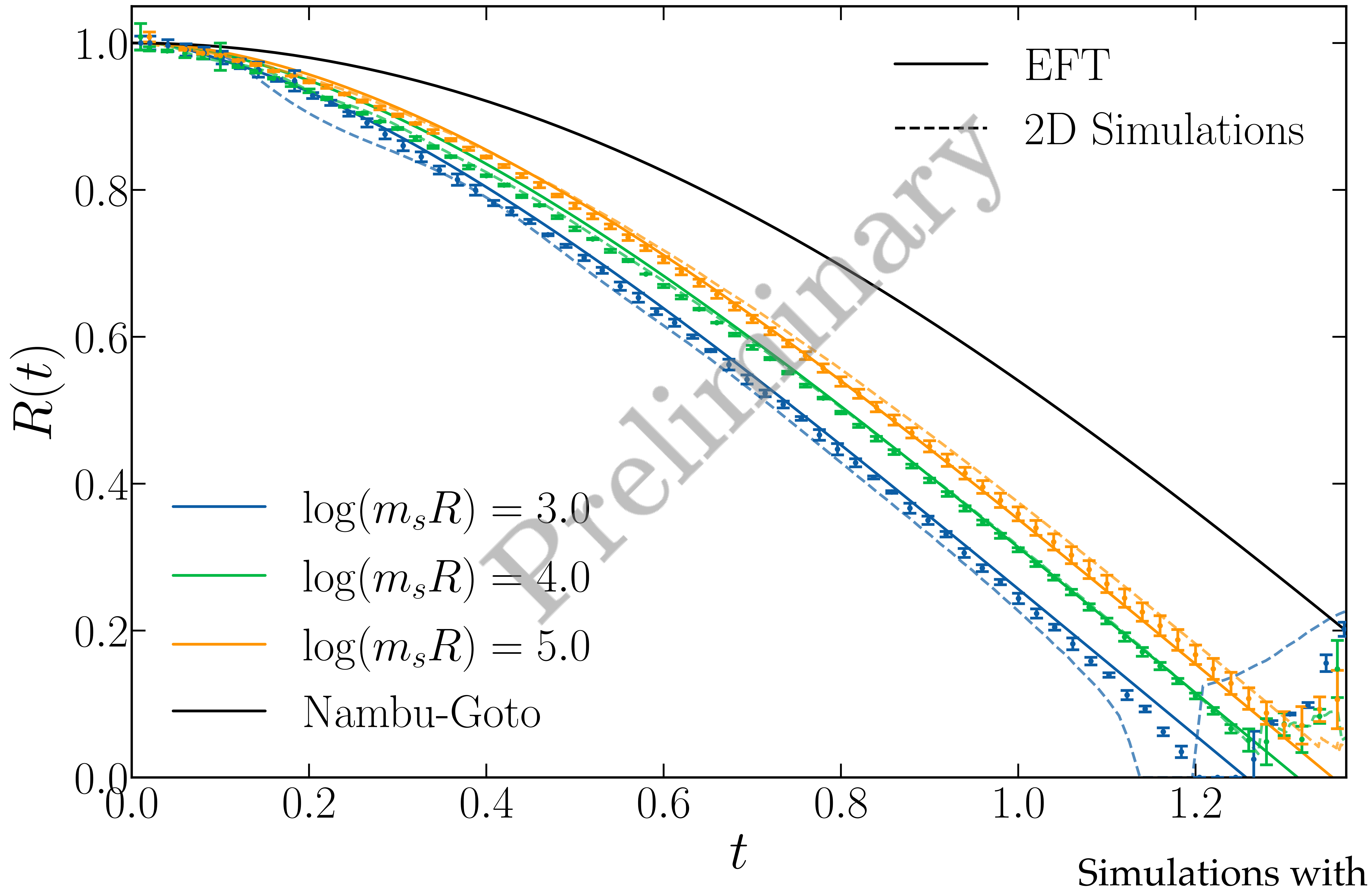


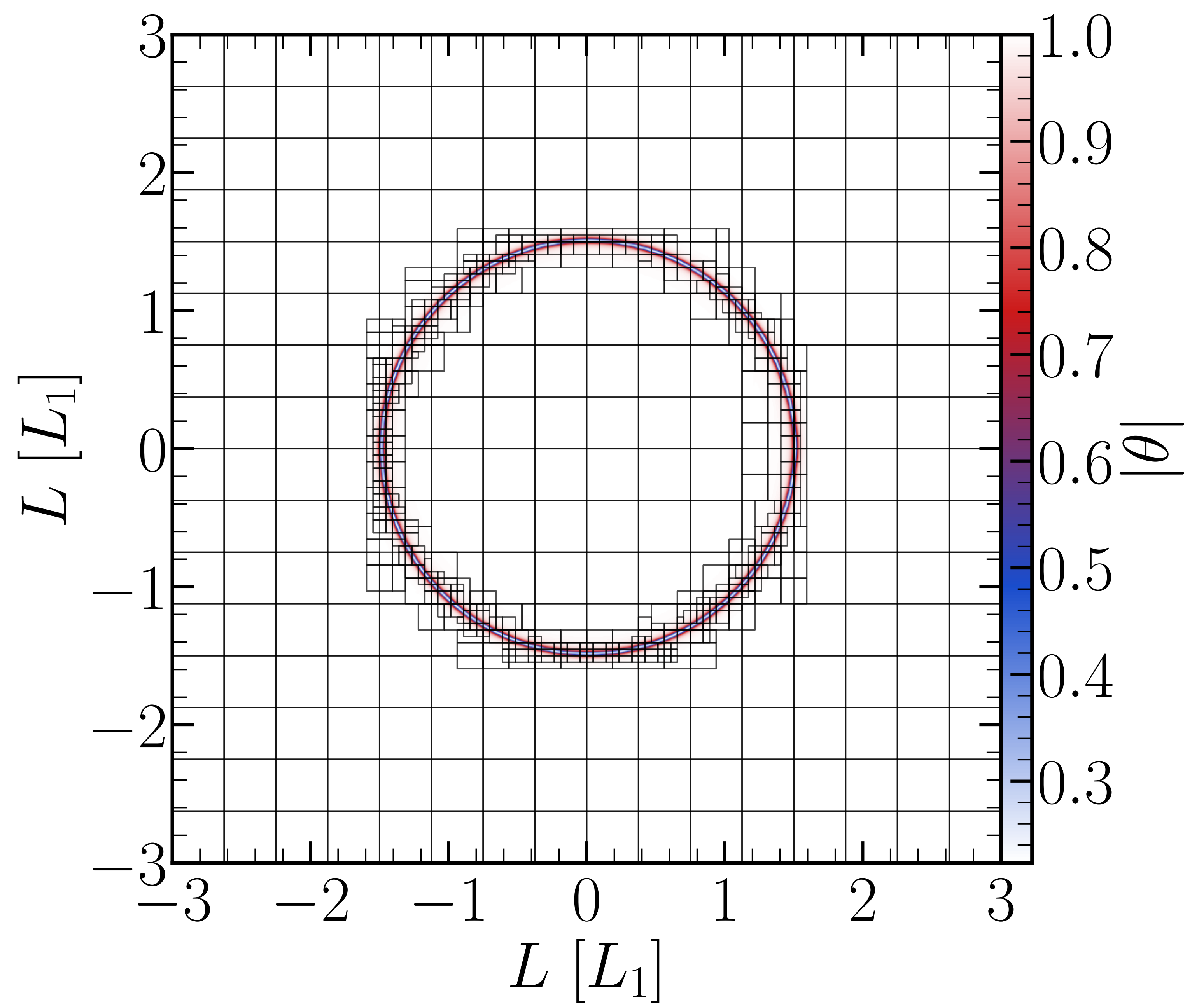
Comparison of EFT with Simulations

- First, look at simple example:

$$X^\mu = \{t, R(t)\cos(\sigma), R(t)\sin(\sigma), 0\}, \quad \sigma \in [0, 2\pi]$$



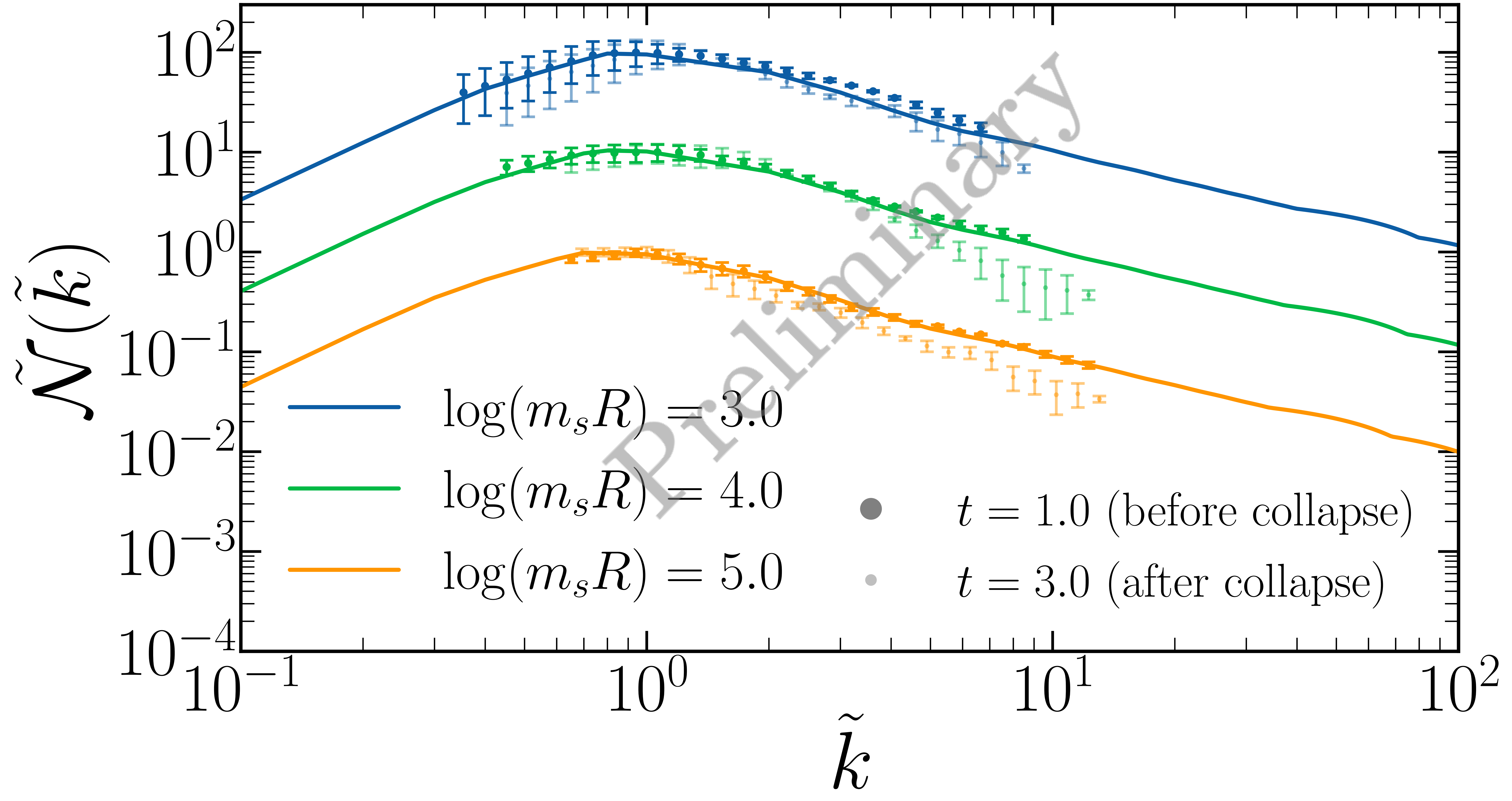




Simulations with static grids and **Adaptive Mesh Refinement** (AMR)

Circular Loop Spectrum

- Dimensionless number spectrum: $\mathcal{N}(k) = \frac{1}{Hf_a^2}\frac{\partial\rho(k)}{\partial(k/R)}$



$\tilde{\mathcal{N}} = \mathcal{N}/\mathcal{N}|_{\text{peak}}$

+ shift for better visibility

$\tilde{k} = kL/(kL)|_{\text{peak}}$

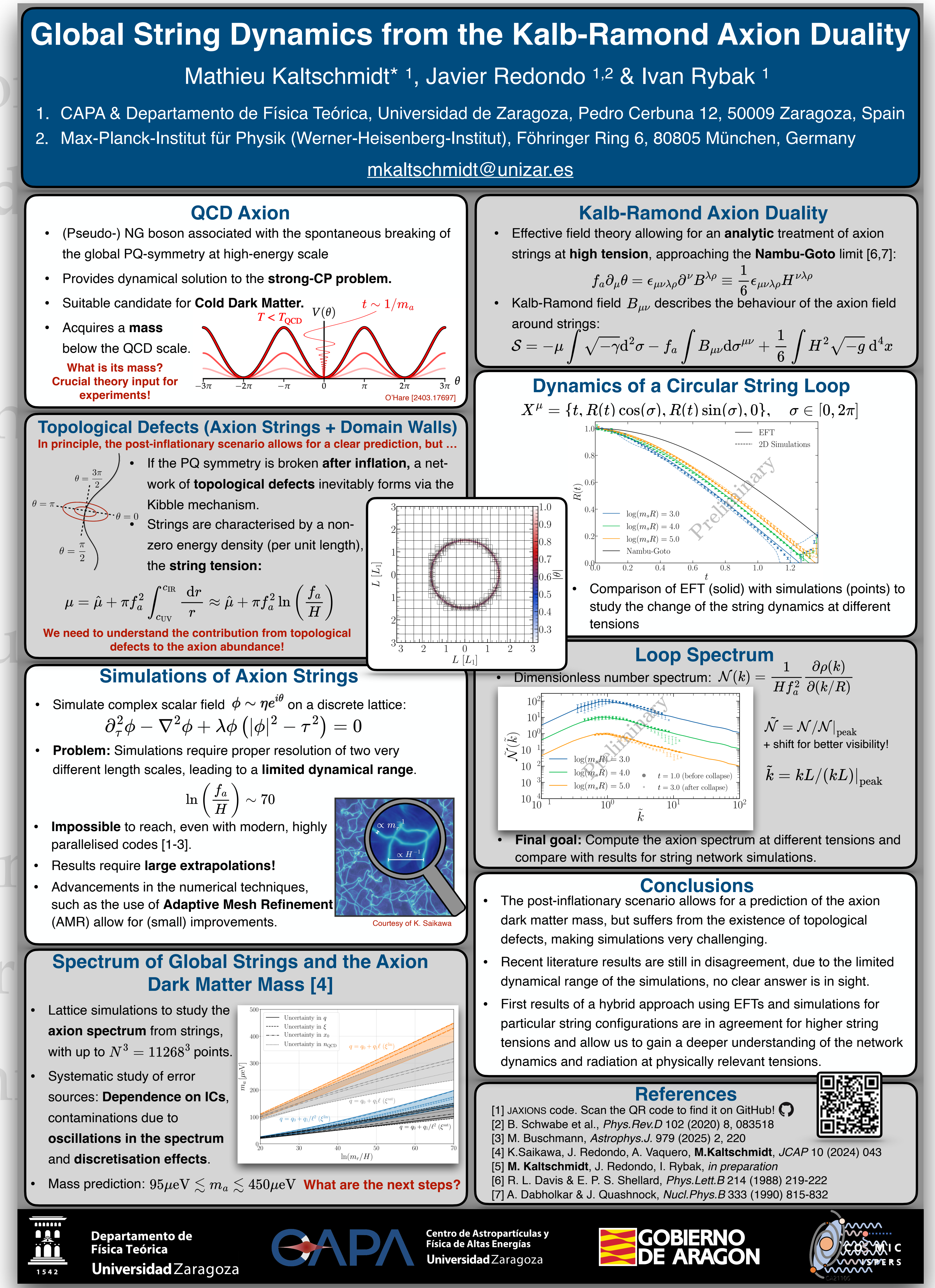
Summary

- The post-inflationary scenario allows for a prediction of the axion dark matter mass, but suffers from the existence of **topological defects**, making simulations very challenging.
- We need to understand the **contribution to the axion DM abundance** from defects as precise as possible.
- Recent literature results are still in disagreement, due to the limited dynamical range of the simulations, no clear answer is in sight ..
- First results of a hybrid approach using **KR EFT and simulations** for particular string configurations show good agreement for higher string tensions and allow us to gain a deeper understanding of the network dynamics and radiation at physically relevant tensions

- The post-inflationary scenario allows for a predictio mass, but suffers from the existence of **topological d** very challenging.
- We need to understand the contribution to the axion defects as precise as possible.
- Re range of the simulations, no clear answer is in sight ment, du
- First results of a hybrid approach using EFTs and si string configurations are in agreement for higher str gain a deeper understanding of the network dynam physically relevant tensions

Thank you!

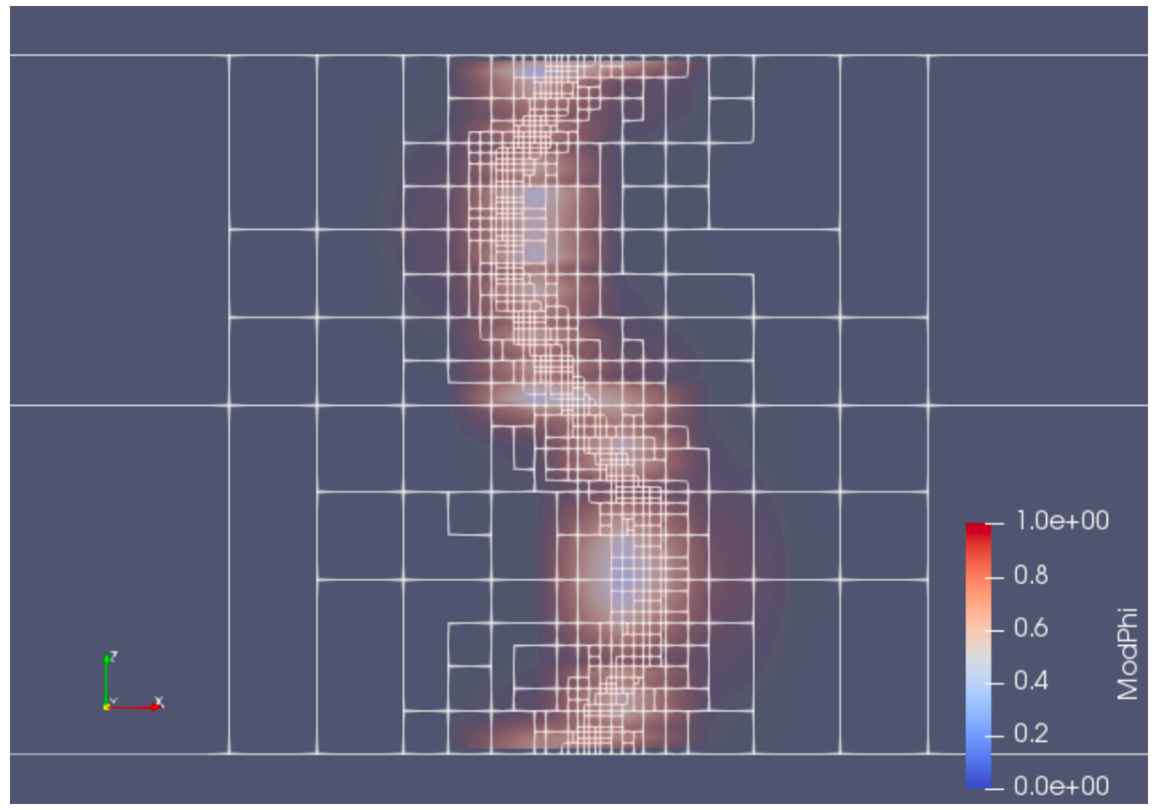
See you later during the poster session



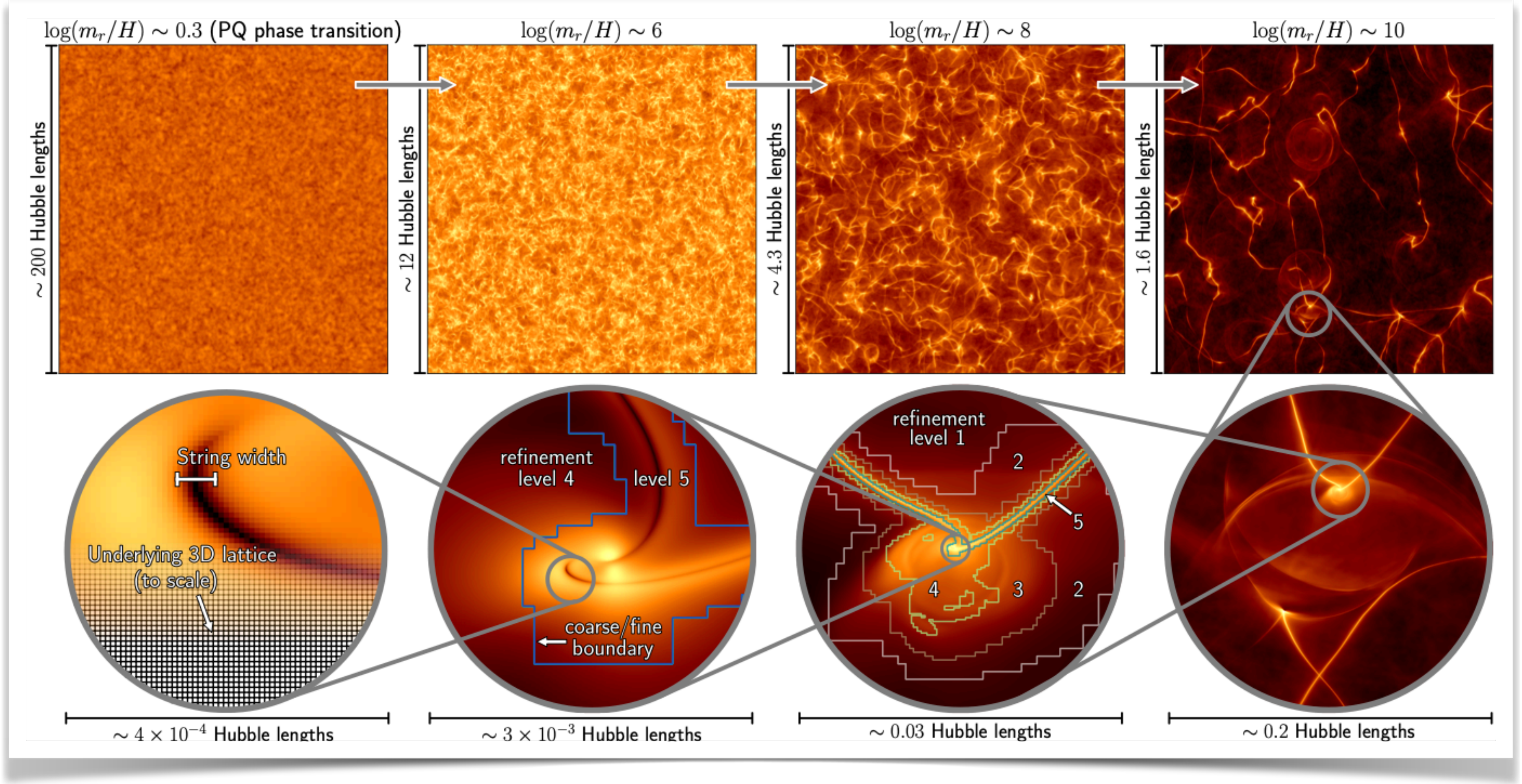
Backup

Adaptive Mesh Refinement (AMR)

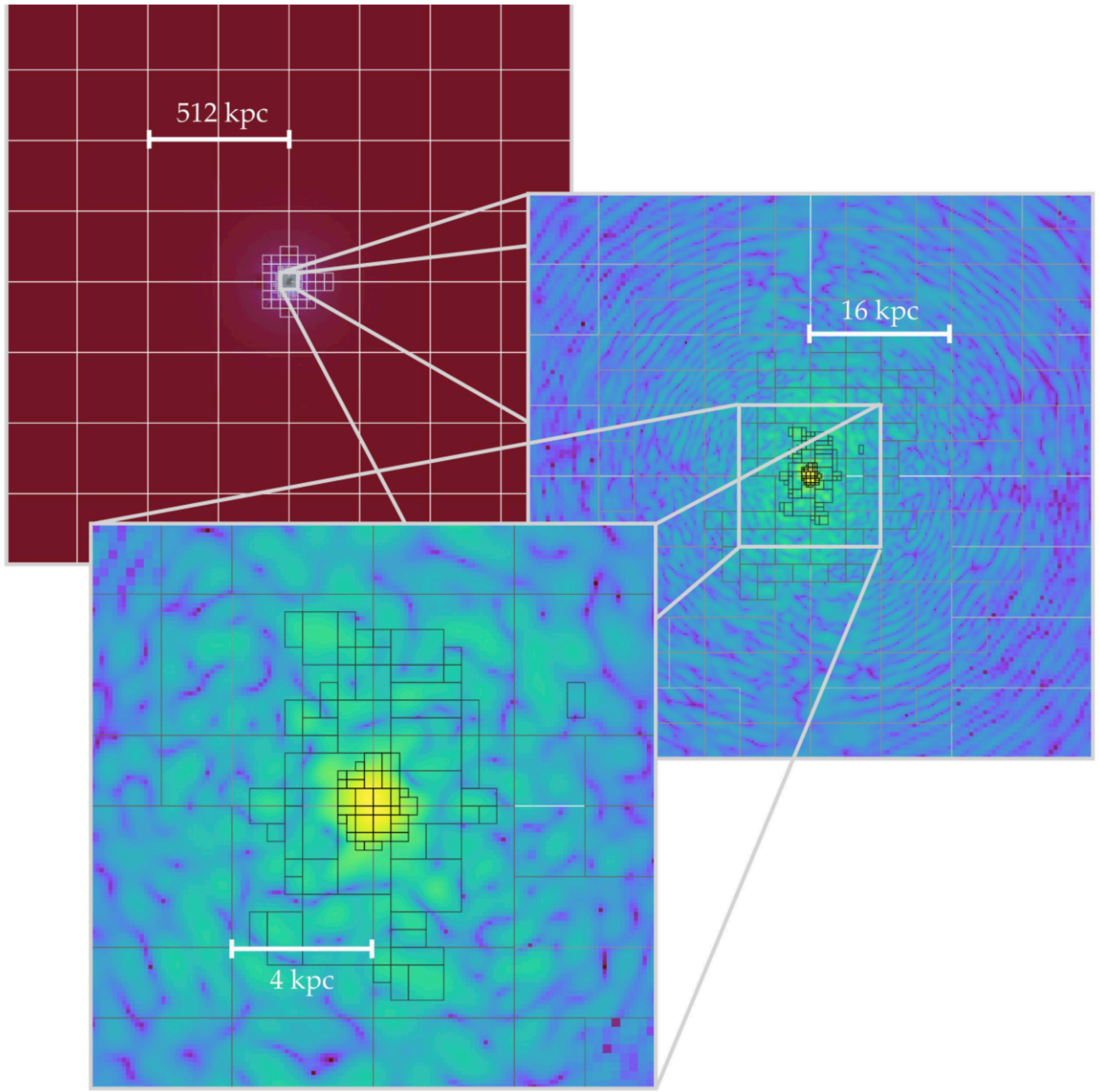
- **Idea:** Focus computational power on specific parts of the grid
- Nowadays widely used in cosmological simulation codes, numerical relativity **and** in axion string simulations
- Current codes mostly based on AMReX



Drew & Shellard [1910.01718]
"GRChombo"



Buschmann+ [2412.08699]
"sledgehamr"

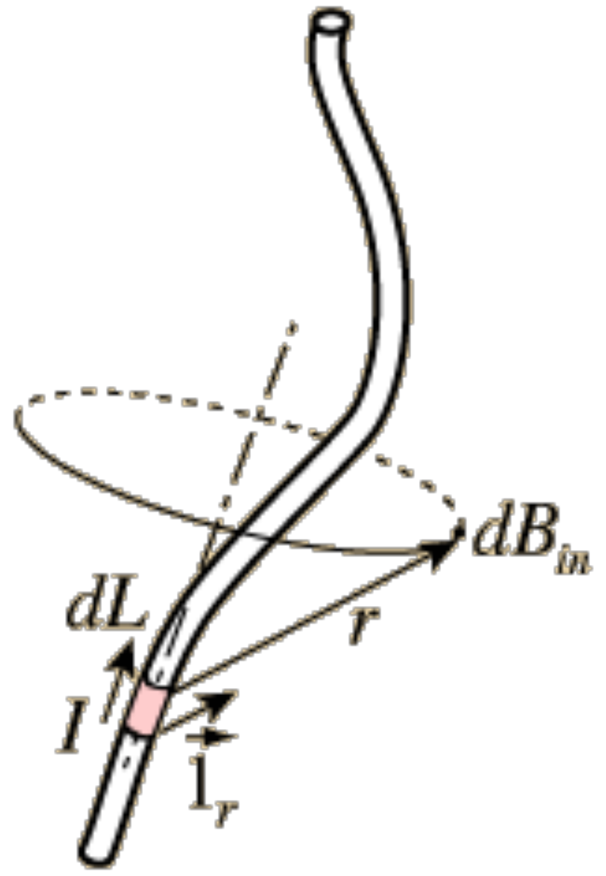


Schwabe+ [2007.08256]
"axioNyx"

Constructing the Axion Field around Strings

- Contribution of a short, straight section of the string to the **axionic *B*-field**:

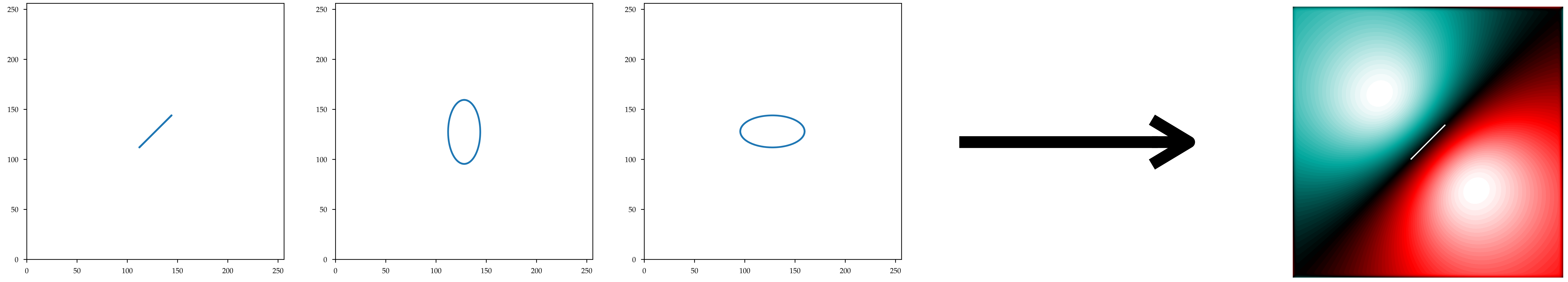
$$\nabla\theta = K \int d\sigma \frac{(\mathbf{x} - \mathbf{X}(\sigma)) \times \mathbf{X}'}{|\mathbf{x} - \mathbf{X}(\sigma)|^3}$$



Biot-Savard law (Link)

- Calculate links to construct the axion field in the full plane:

$$\theta_{\mathbf{x}+\mathbf{d}x} - \theta_{\mathbf{x}} = \int_x^{x+dx} d^3\mathbf{x} \cdot \nabla\theta = -\frac{1}{2}\int_x^{x+dx} d^3\mathbf{x} \cdot \int d\sigma \frac{(\mathbf{x} - \mathbf{X}(\sigma)) \times \mathbf{X}'}{|\mathbf{x} - \mathbf{X}(\sigma)|^3}$$



Constructing the Axion Field around Strings

- Contribution of a s … B-field:

Biot-Savard law (Link)

- Calculate links to c …

$\theta_{\mathbf{x}+\mathrm{d}x} - \theta_{\mathbf{x}} =$ … $- \mathbf{X}(\sigma)) \times \mathbf{X}'$ … $\mathbf{x} - \mathbf{X}(\sigma)|^3$