

GLOBAL STRING DYNAMICS FROM THE KALB-RAMOND AXION DUALITY

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based on [JCAP10\(2024\)043](#) with K. Saikawa, J. Redondo & A. Vaquero

and work in progress with J. Redondo and I. Rybak

Axions in Stockholm
Nordita, June 30th 2025

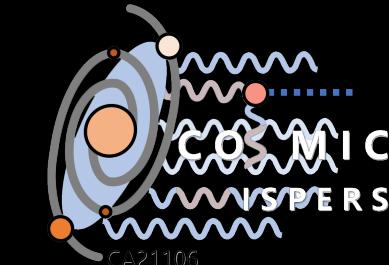
Background: G. Pierobon



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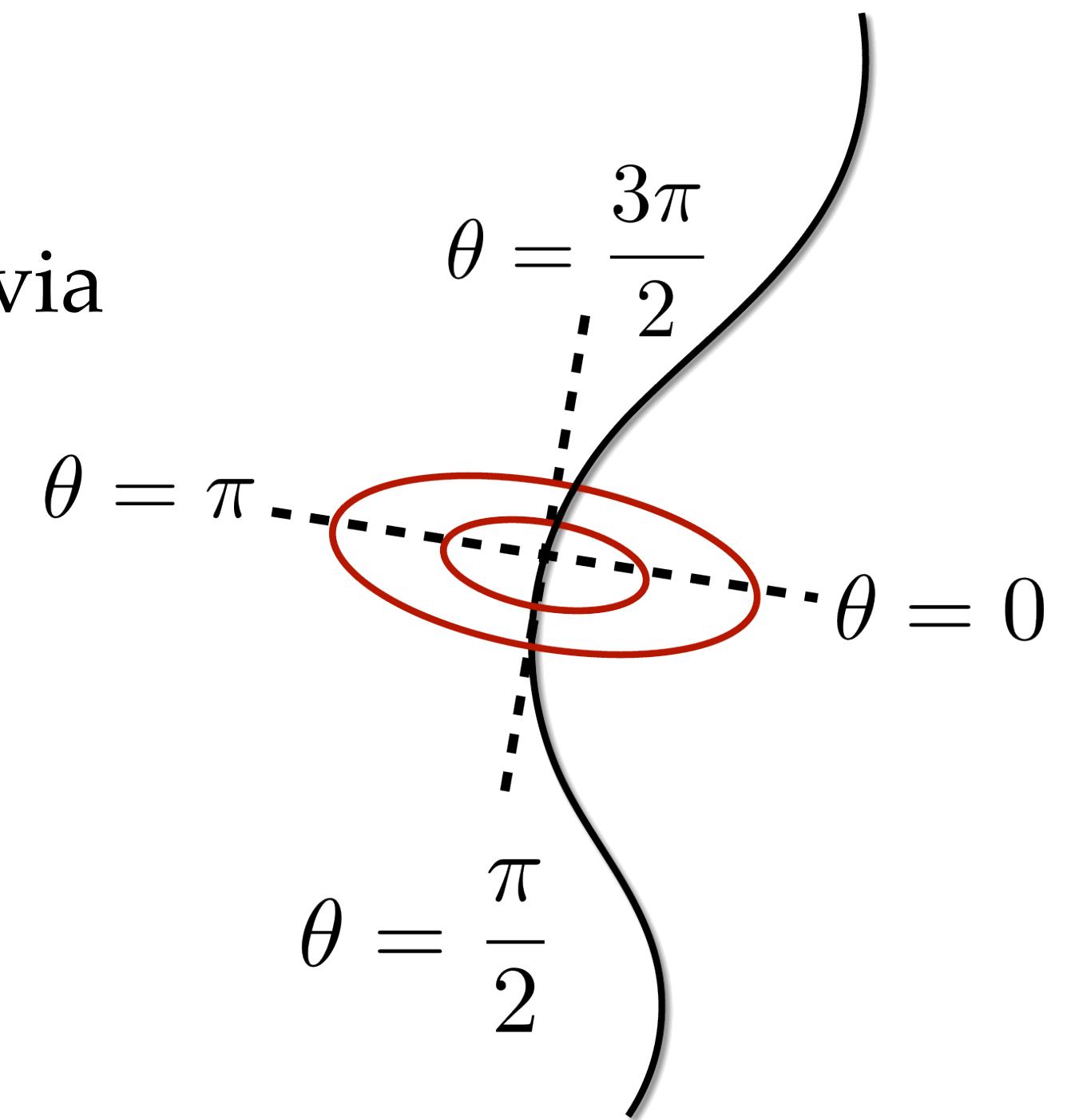
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Axion Strings

- If the PQ symmetry is broken **after inflation**, we can in principle predict the QCD axion dark matter mass, which is a crucial input for direct detection experiments.
- In this scenario, a network of **topological defects** inevitably forms via the Kibble mechanism. Kibble (1976)
- We need to understand how they contribute to the QCD axion DM abundance.
- Strings are characterised by a non-zero energy density (per unit length), the **string tension**:

$$\mu = \hat{\mu} + \pi f_a^2 \int_{c_{\text{UV}}}^{c_{\text{IR}}} \frac{dr}{r} \approx \hat{\mu} + \pi f_a^2 \ln \left(\frac{f_a}{H} \right)$$



How to simulate Axion Strings?

- Solve the classical EoM for a complex scalar field in comoving coordinates, discretised on a (static or more involved*) lattice:

$$\partial_\tau^2 \phi - \nabla^2 \phi + \lambda \phi (|\phi|^2 - \tau^2) = 0$$

- Tricky: Simulations require proper resolution of two very different length scales

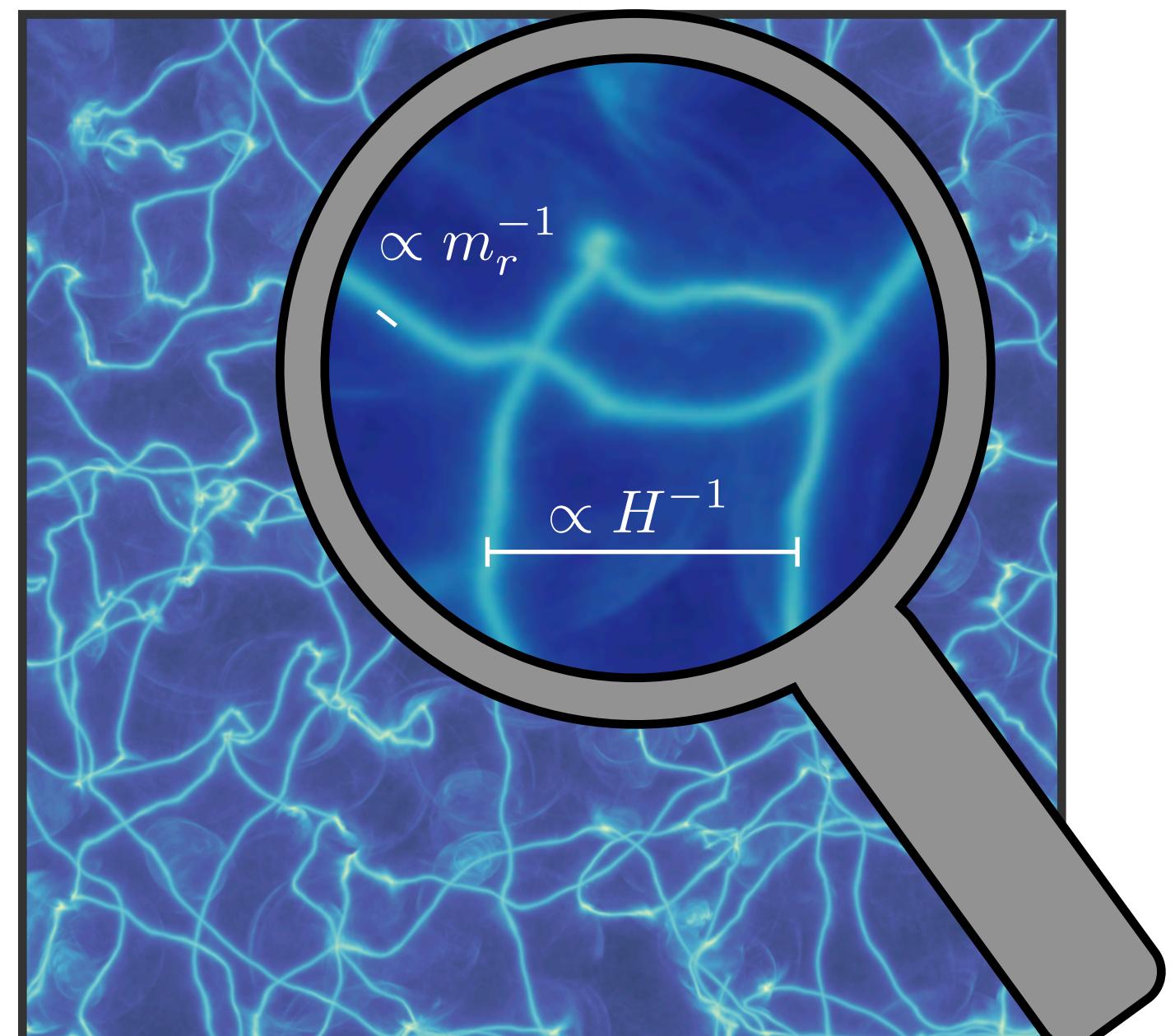
- String core radius

$$\propto \frac{1}{m_r} \propto \frac{1}{f_a}, \text{ where } m_r = \text{radial mass}$$

- String separation given by Hubble radius

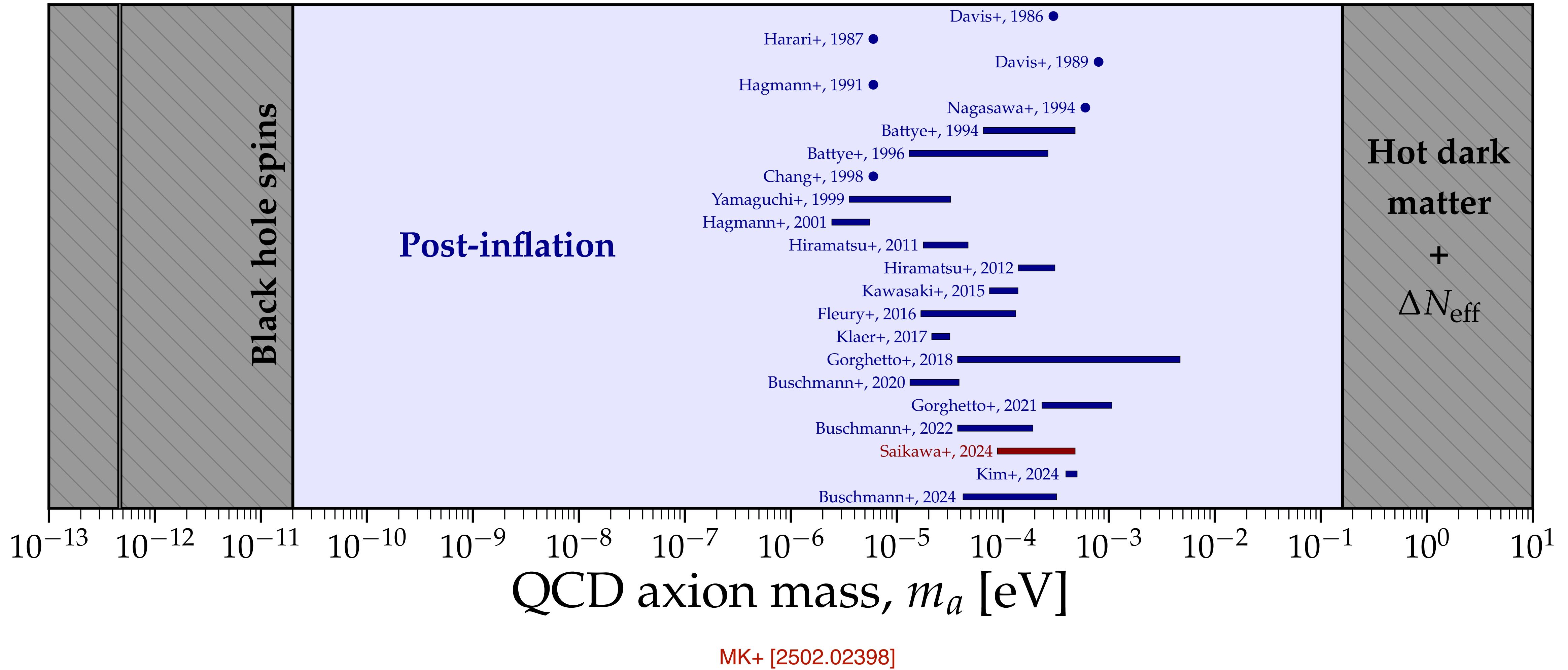
$$\propto \frac{1}{H}$$

- Realistic value: $\frac{f_a}{H_{\text{QCD}}} \approx 10^{30} \implies \log \left(\frac{m_r}{H} \right) \approx 70$



Courtesy of K. Saikawa

Axion DM Mass Predictions from String Simulations



Axion String EFT

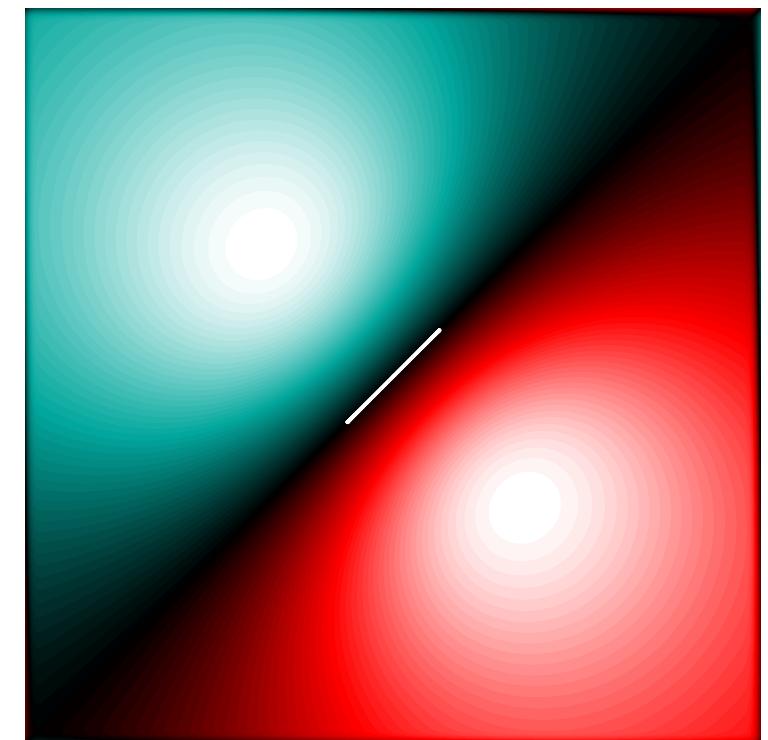
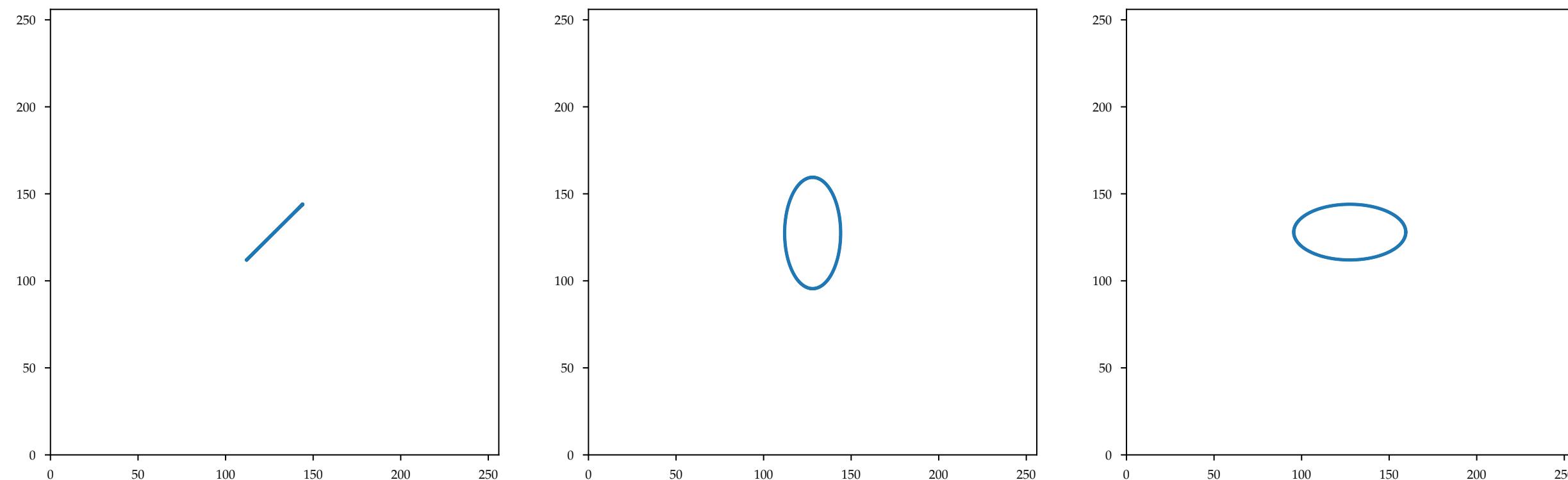
- Effective Theory that allows for an **analytic** treatment of axion strings at high tension, approaching the **Nambu-Goto** limit:

$$f_a \partial_\mu \theta = \epsilon_{\mu\nu\lambda\rho} \partial^\nu B^{\lambda\rho} \equiv \frac{1}{6} \epsilon_{\mu\nu\lambda\rho} H^{\nu\lambda\rho}$$

Shellard & Davis (1988),
Dabholkar & Quaschnock (1990)

- Kalb-Ramond field $B_{\mu\nu}$ describes the behaviour of the axion field around strings:

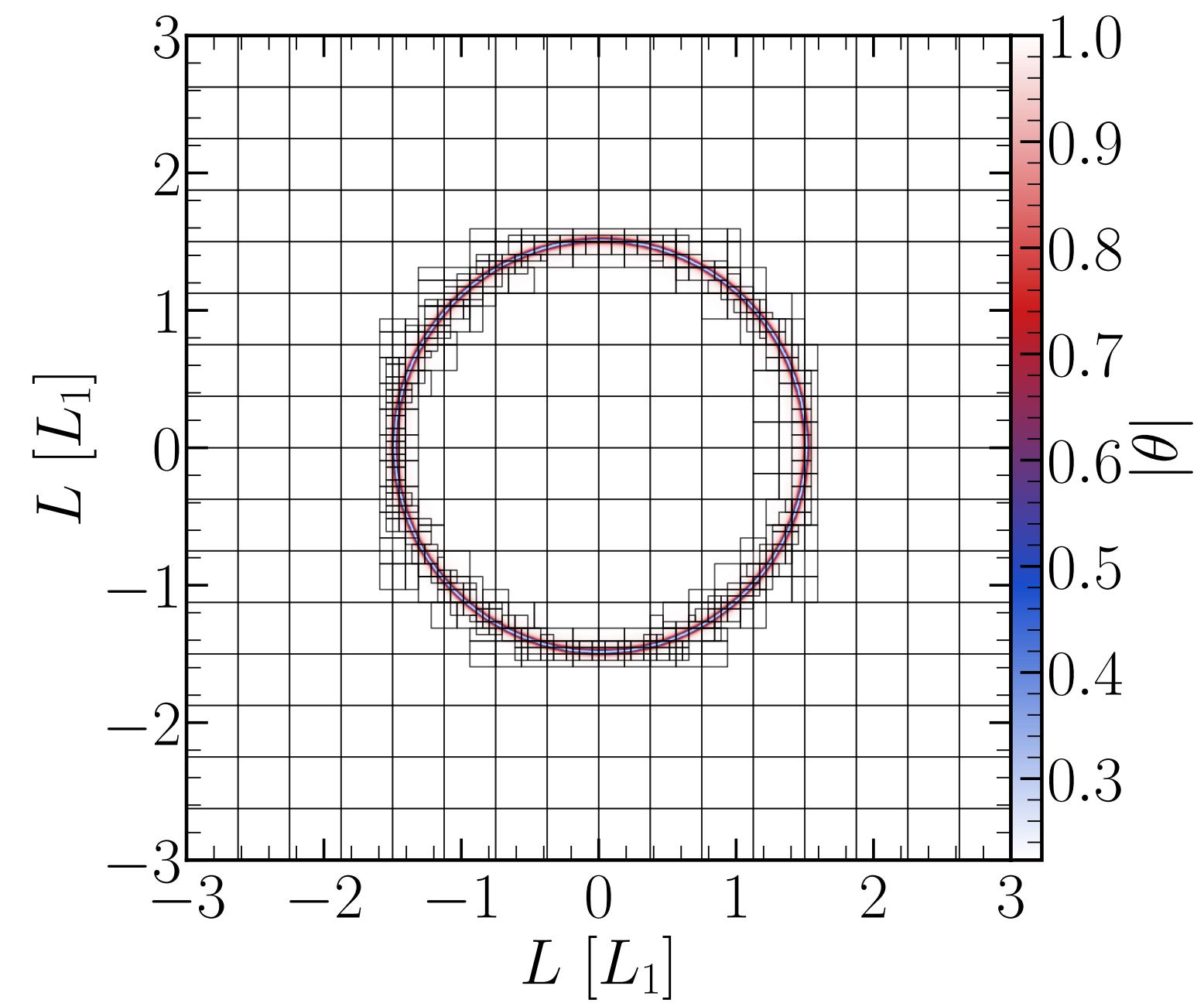
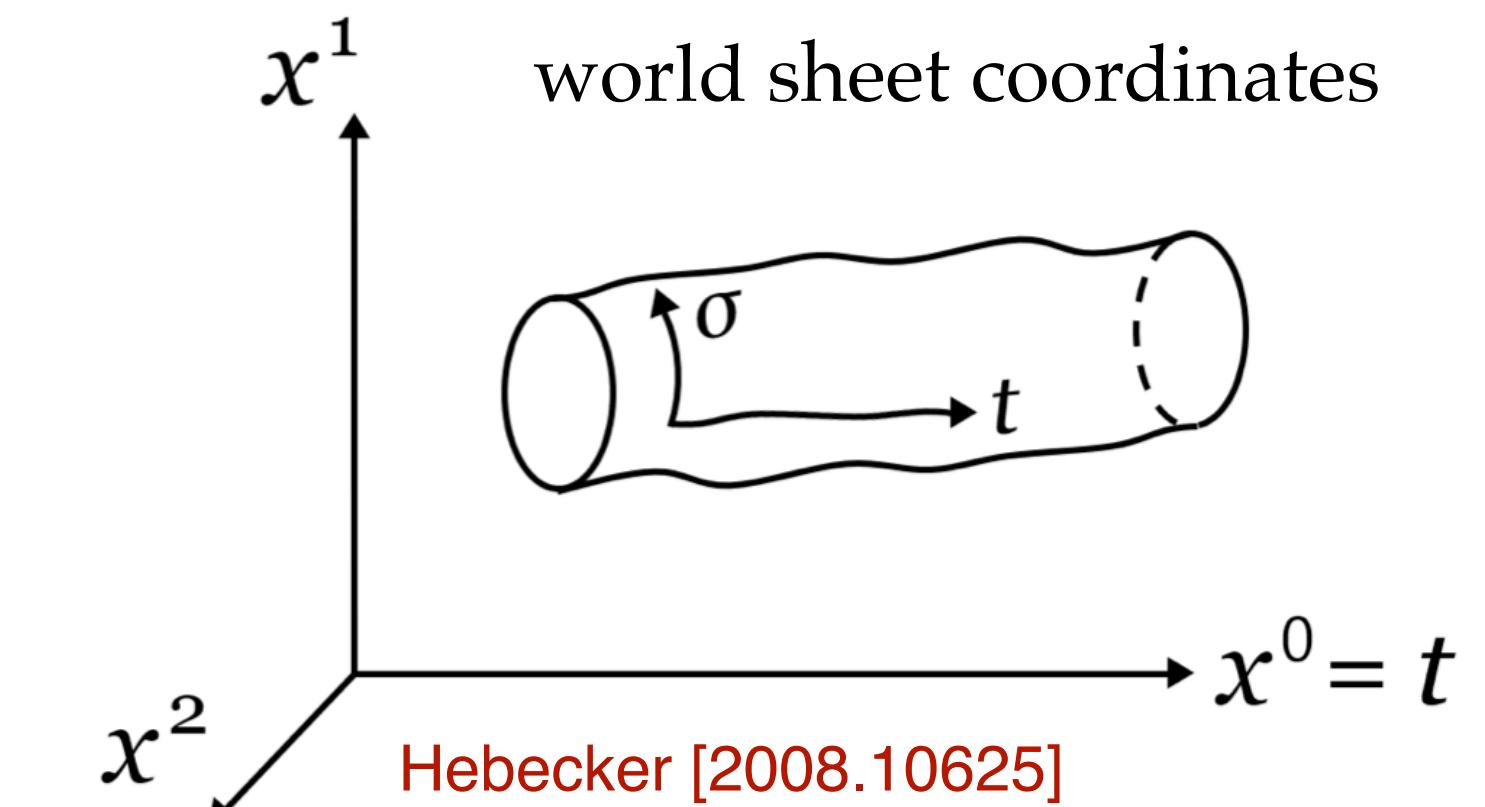
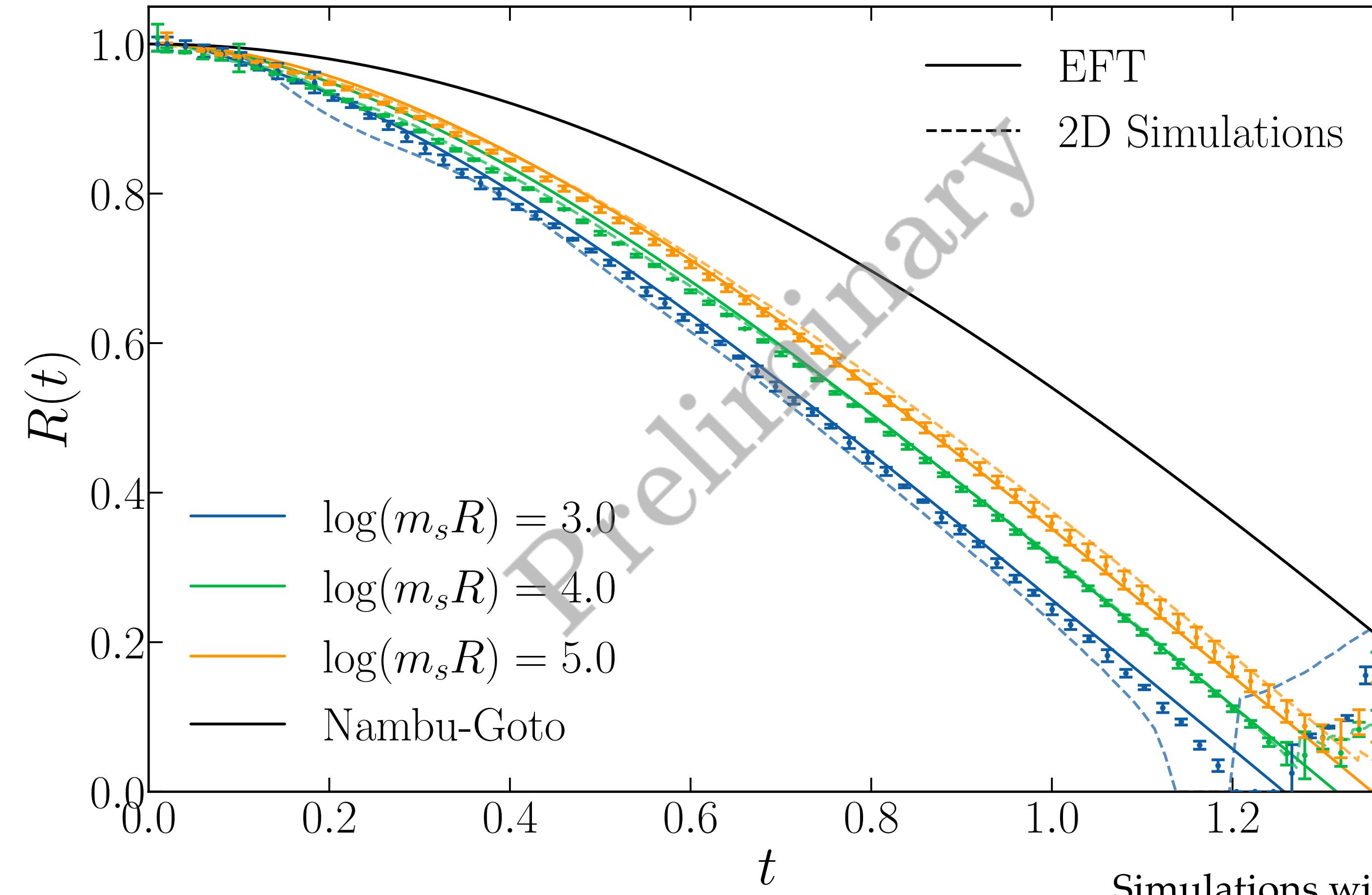
$$\mathcal{S} = -\mu \int \sqrt{-\gamma} d^2\sigma - f_a \int B_{\mu\nu} d\sigma^{\mu\nu} + \frac{1}{6} \int H^2 \sqrt{-g} d^4x$$



Comparison of EFT with Simulations

- First, look at simple example:

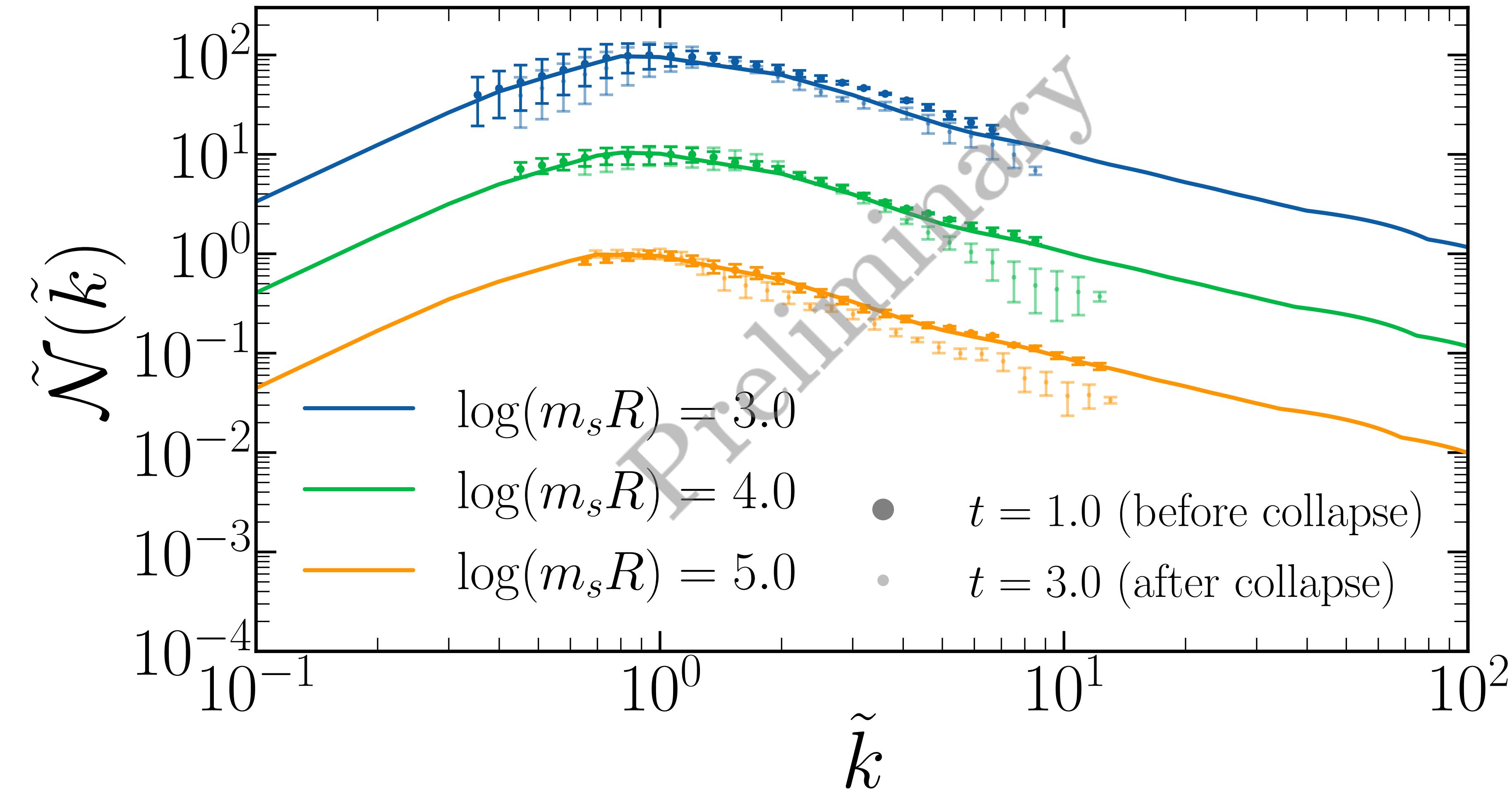
$$X^\mu = \{t, R(t) \cos(\sigma), R(t) \sin(\sigma), 0\}, \quad \sigma \in [0, 2\pi]$$



Simulations with static grids and Adaptive Mesh Refinement (AMR)

Circular Loop Spectrum

- Dimensionless number spectrum: $\mathcal{N}(k) = \frac{1}{Hf_a^2} \frac{\partial \rho(k)}{\partial(k/R)}$



$$\tilde{\mathcal{N}} = \mathcal{N}/\mathcal{N}|_{\text{peak}}$$

+ shift for better visibility

$$\tilde{k} = kL/(kL)|_{\text{peak}}$$

Summary

- The post-inflationary scenario allows for a prediction of the axion dark matter mass, but suffers from the existence of **topological defects**, making simulations very challenging.
- We need to understand the **contribution to the axion DM abundance** from defects as precise as possible.
- Recent literature results are still in disagreement, due to the limited dynamical range of the simulations, no clear answer is in sight ..
- First results of a hybrid approach using **KR EFT and simulations** for particular string configurations show good agreement for higher string tensions and allow us to gain a deeper understanding of the network dynamics and radiation at physically relevant tensions

Summary

- The post-inflationary scenario allows for a prediction of the axion mass, but suffers from the existence of topological defects, making simulations very challenging.
- We need to understand the contribution to the axion abundance from topological defects as precise as possible!
- Recent literature results are still in disagreement, due to the range of the simulations, no clear answer is in sight.
- See you later during the poster session
- First results of a hybrid approach using EFTs and simulations for string configurations are in agreement for higher string tensions, gain a deeper understanding of the network dynamics at physically relevant tensions

Thank you!

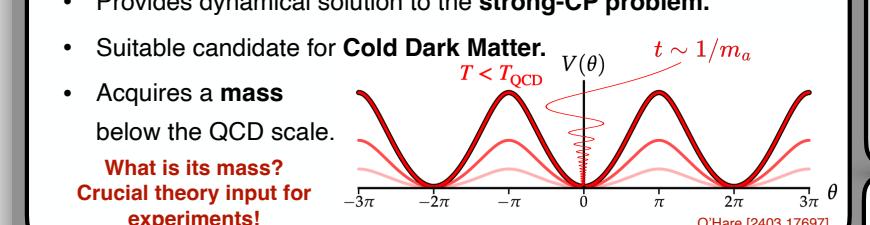
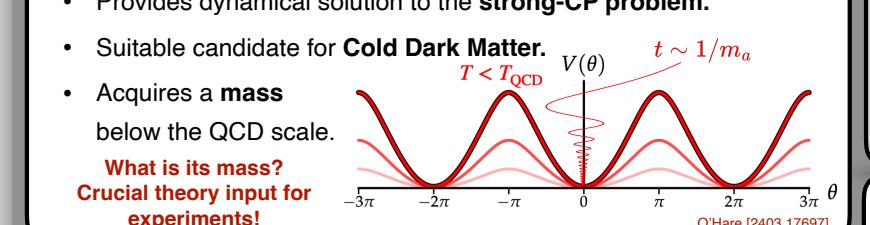
Global String Dynamics from the Kalb-Ramond Axion Duality

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QCD Axion

- (Pseudo-) NG boson associated with the spontaneous breaking of the global PQ-symmetry at high-energy scale
- Provides dynamical solution to the **strong-CP problem**.
- Suitable candidate for **Cold Dark Matter**. 
- Acquires a **mass** below the QCD scale. 

What is its mass?
Crucial theory input for experiments!
O'Hare [2403.17697]

Kalb-Ramond Axion Duality

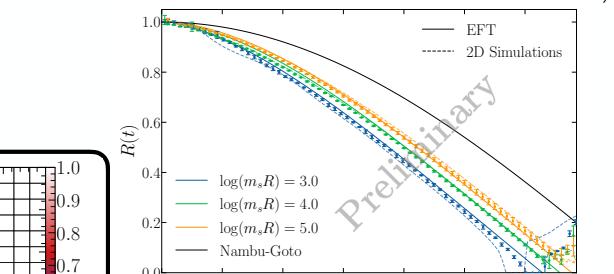
- Effective field theory allowing for an **analytic treatment** of axion strings at **high tension**, approaching the **Nambu-Goto limit** [6.7]:

$$f_a \partial_\mu \theta = \epsilon_{\mu\nu\lambda\rho} \partial^\nu B^{\lambda\rho} \equiv \frac{1}{6} \epsilon_{\mu\nu\lambda\rho} H^{\nu\lambda\rho}$$
- Kalb-Ramond field $B_{\mu\nu}$ describes the behaviour of the axion field around strings:

$$\mathcal{S} = -\mu \int \sqrt{-g} d^2\sigma - f_a \int B_{\mu\nu} d\sigma^{\mu\nu} + \frac{1}{6} \int H^2 \sqrt{-g} d^4x$$

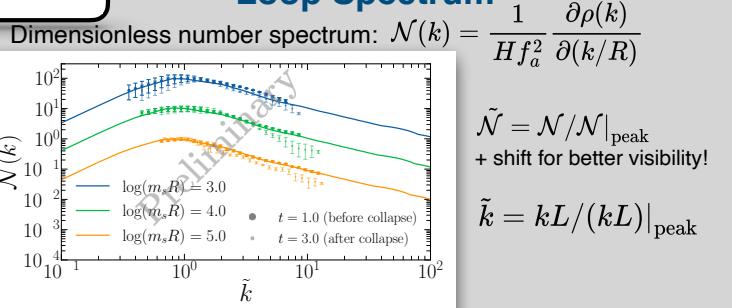
Dynamics of a Circular String Loop

$X^\mu = \{t, R(t) \cos(\sigma), R(t) \sin(\sigma), 0\}, \sigma \in [0, 2\pi]$



- Comparison of EFT (solid) with simulations (points) to study the change of the string dynamics at different tensions

Loop Spectrum



- Final goal: Compute the axion spectrum at different tensions and compare with results for string network simulations.

Conclusions

- The post-inflationary scenario allows for a prediction of the axion dark matter mass, but suffers from the existence of topological defects, making simulations very challenging.
- Recent literature results are still in disagreement, due to the limited dynamical range of the simulations, no clear answer is in sight.
- First results of a hybrid approach using EFTs and simulations for particular string configurations are in agreement for higher string tensions and allow us to gain a deeper understanding of the network dynamics and radiation at physically relevant tensions.

References

- JAXONS code. Scan the QR code to find it on GitHub! 
- B. Schwabe et al., *Phys. Rev. D* 102 (2020) 8, 083518
- M. Buschmann, *Astrophys. J.* 979 (2025) 2, 220
- K. Saikawa, J. Redondo, A. Vaquero, M. Kaltschmidt, *JCAP* 10 (2024) 043
- M. Kaltschmidt, J. Redondo, I. Rybak, *in preparation*
- R. L. Davis & E. P. S. Shellard, *Phys. Lett. B* 214 (1988) 219-222
- A. Dabholkar & J. Quashnock, *Nucl. Phys. B* 333 (1990) 815-832

Simulations of Axion Strings

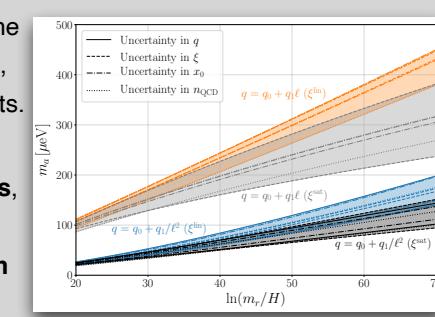
- Simulate complex scalar field $\phi \sim \eta e^{i\theta}$ on a discrete lattice: $\partial_r^2 \phi - \nabla^2 \phi + \lambda \phi (|\phi|^2 - r^2) = 0$
- Problem:** Simulations require proper resolution of two very different length scales, leading to a **limited dynamical range**.
 $\ln\left(\frac{f_a}{H}\right) \approx 70$
- Impossible** to reach, even with modern, highly parallelised codes [1-3].
- Results require **large extrapolations**!
- Advancements in the numerical techniques, such as the use of **Adaptive Mesh Refinement (AMR)** allow for (small) improvements.



Courtesy of K. Saikawa

Spectrum of Global Strings and the Axion Dark Matter Mass [4]

- Lattice simulations to study the **axion spectrum** from strings, with up to $N^3 = 11268^3$ points.
- Systematic study of error sources: **Dependence on ICs**, contaminations due to **oscillations in the spectrum** and **discretisation effects**.
- Mass prediction: $95 \mu\text{eV} \lesssim m_a \lesssim 450 \mu\text{eV}$ **What are the next steps?**

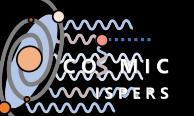


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DE ARAGÓN



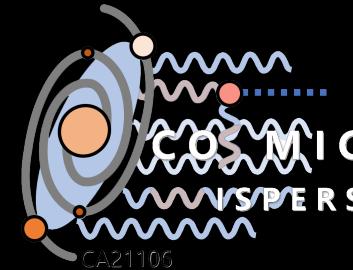
Backup



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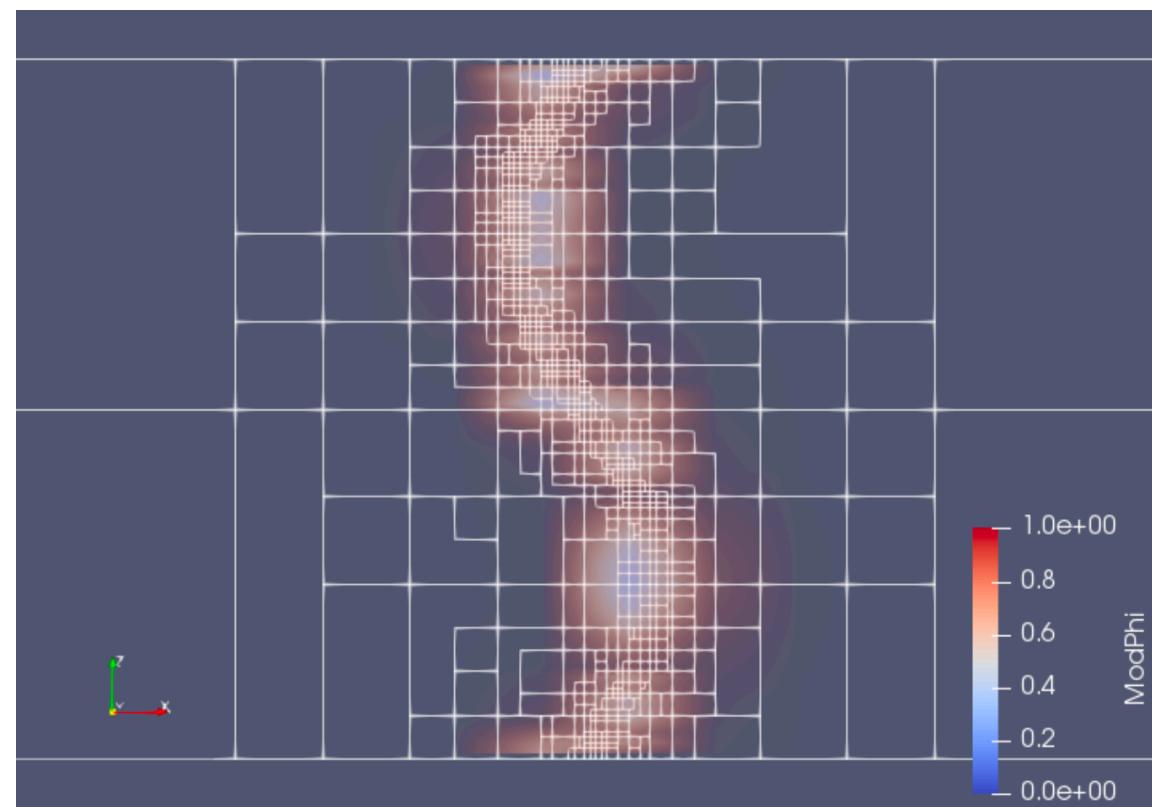


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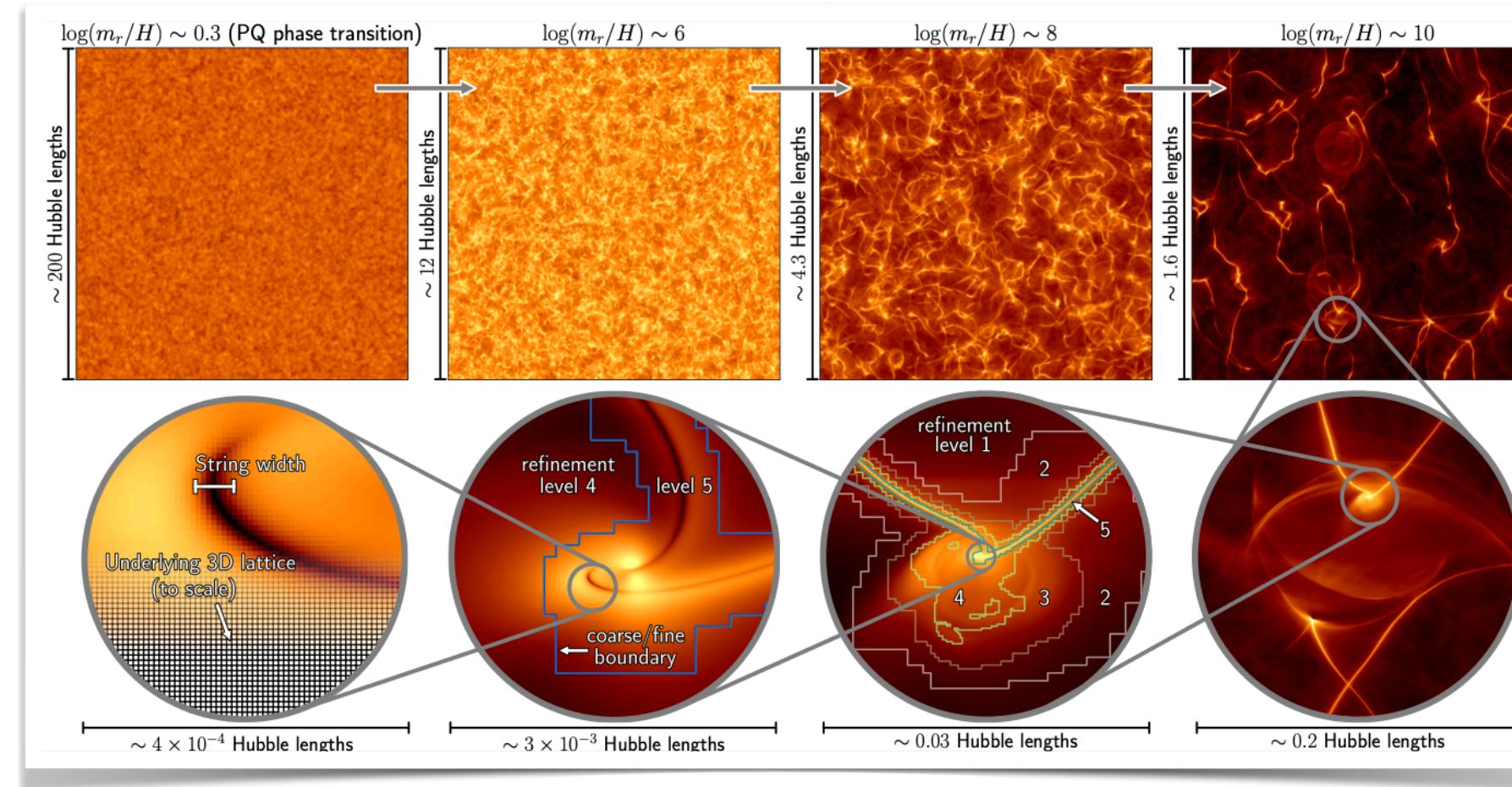


Adaptive Mesh Refinement (AMR)

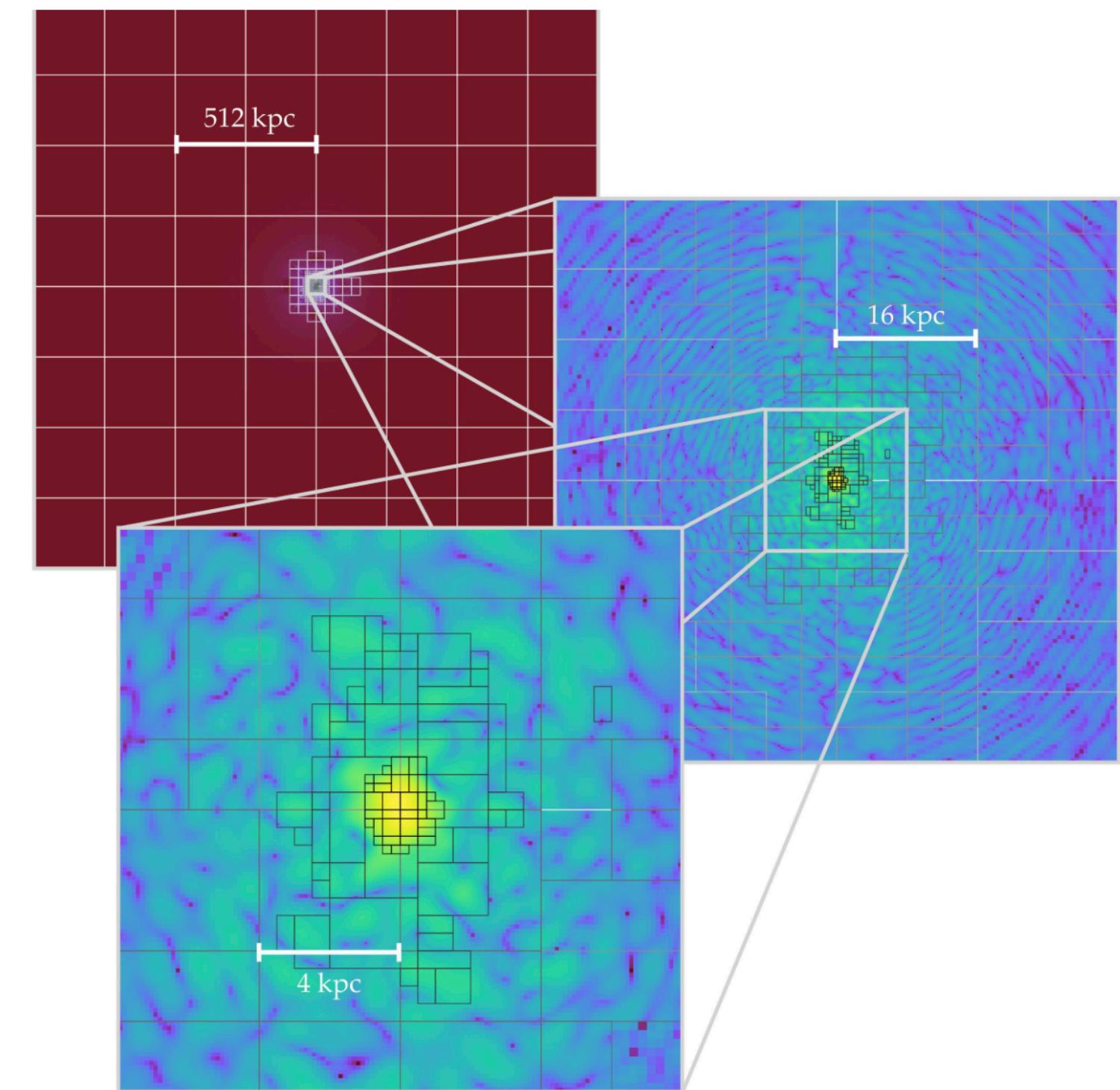
- Idea: Focus computational power on specific parts of the grid
- Nowadays widely used in cosmological simulation codes, numerical relativity **and** in axion string simulations
- Current codes mostly based on [AMReX](#)



Drew & Shellard [1910.01718]
“GRChombo”



Buschmann+ [2412.08699]
“sledgehamr”

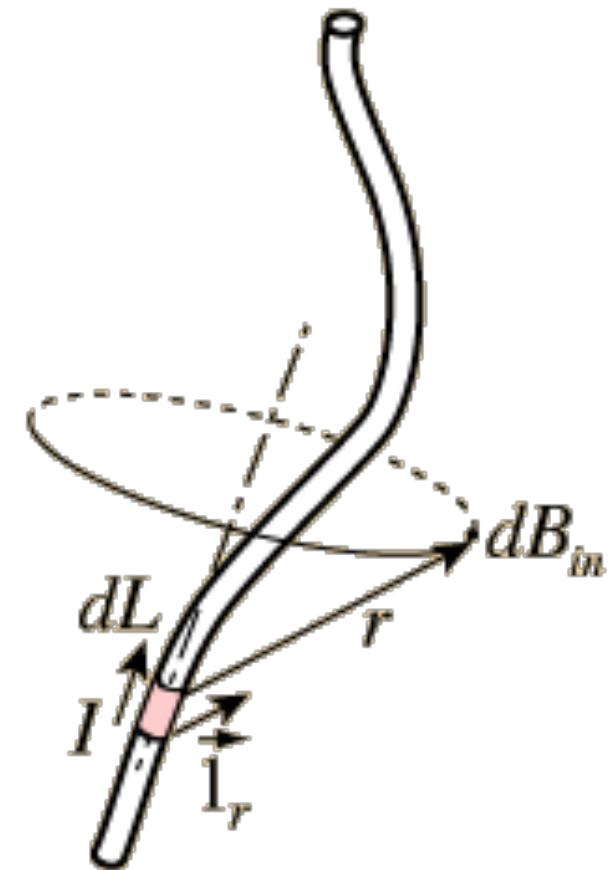


Schwabe+ [2007.08256]
“axioNyx”

Constructing the Axion Field around Strings

- Contribution of a short, straight section of the string to the axionic B -field:

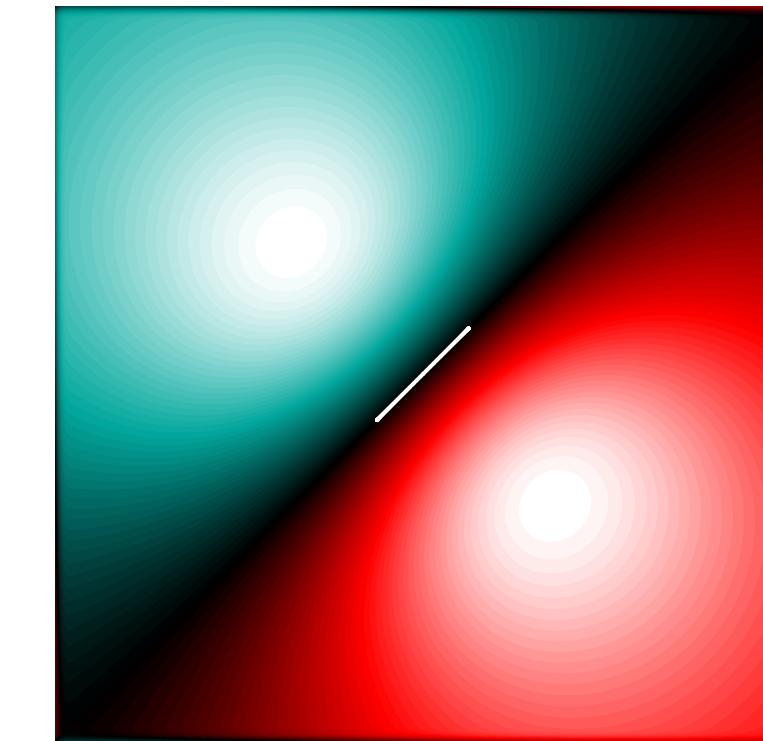
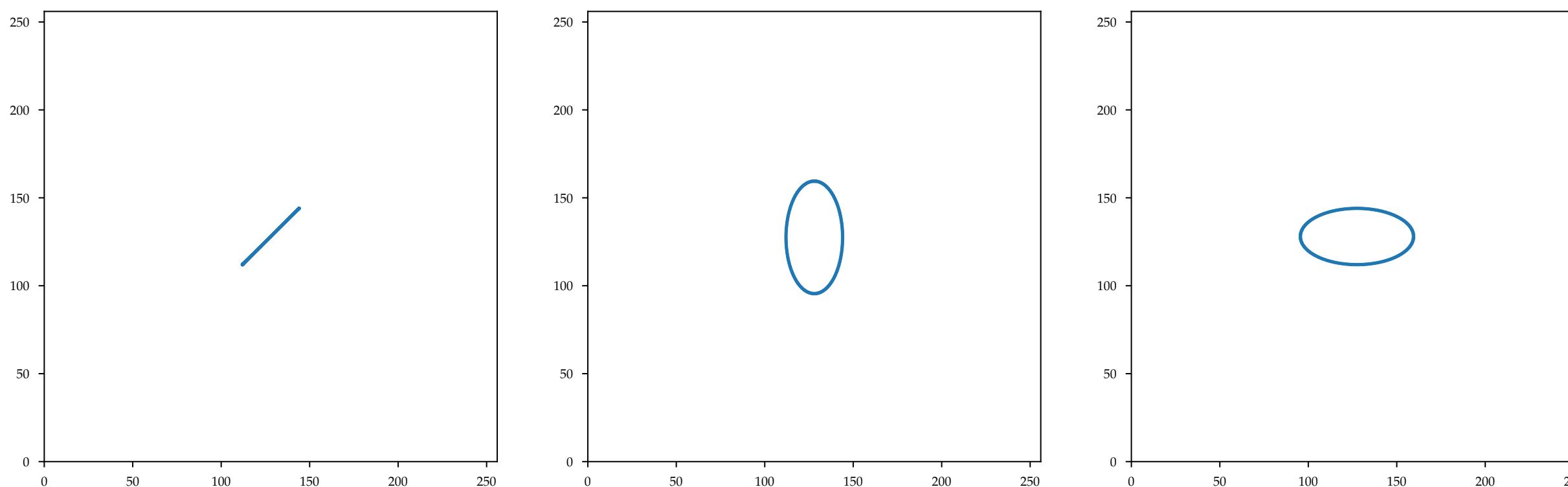
$$\nabla\theta = K \int d\sigma \frac{(\mathbf{x} - \mathbf{X}(\sigma)) \times \mathbf{X}'}{|\mathbf{x} - \mathbf{X}(\sigma)|^3}$$



- Calculate links to construct the axion field in the full plane:

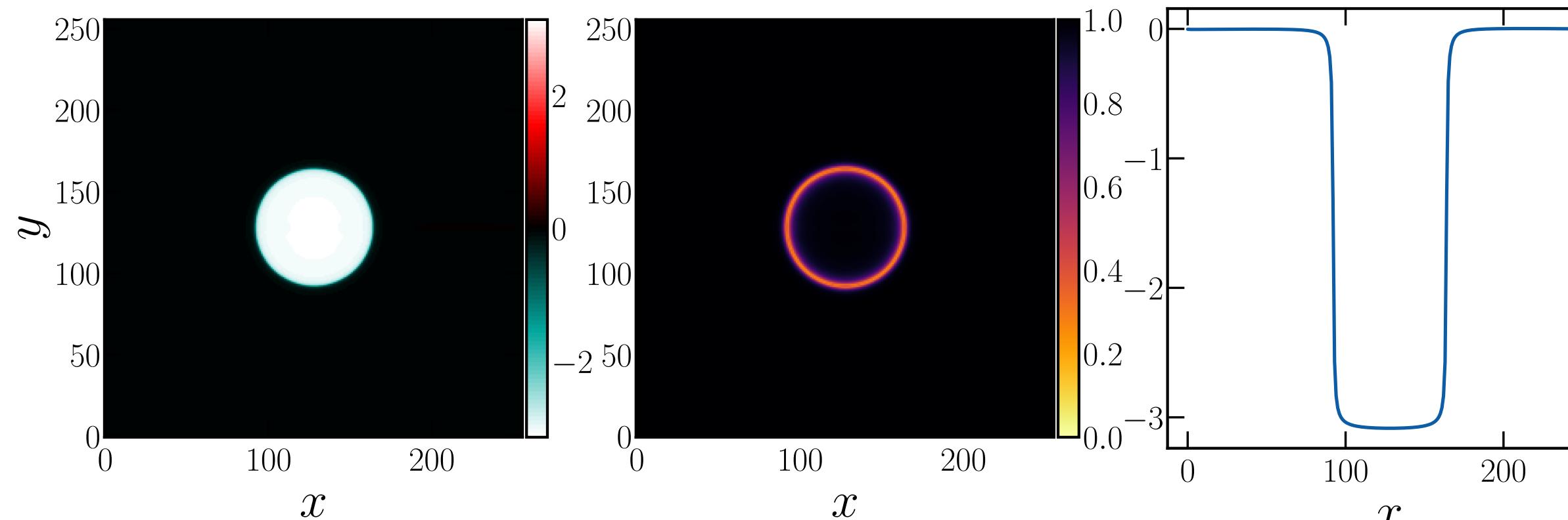
Biot-Savard law ([Link](#))

$$\theta_{\mathbf{x}+\mathbf{d}\mathbf{x}} - \theta_{\mathbf{x}} = \int_x^{x+dx} d^3\mathbf{x} \cdot \nabla\theta = -\frac{1}{2} \int_x^{x+dx} d^3\mathbf{x} \cdot \int d\sigma \frac{(\mathbf{x} - \mathbf{X}(\sigma)) \times \mathbf{X}'}{|\mathbf{x} - \mathbf{X}(\sigma)|^3}$$



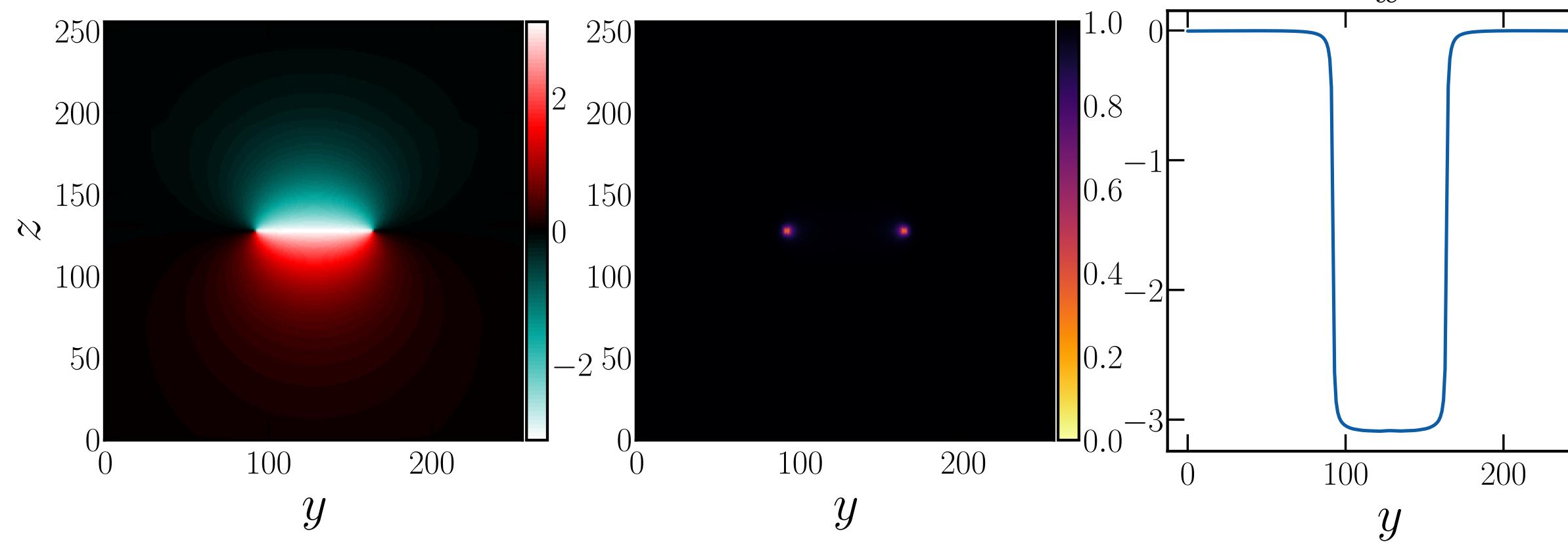
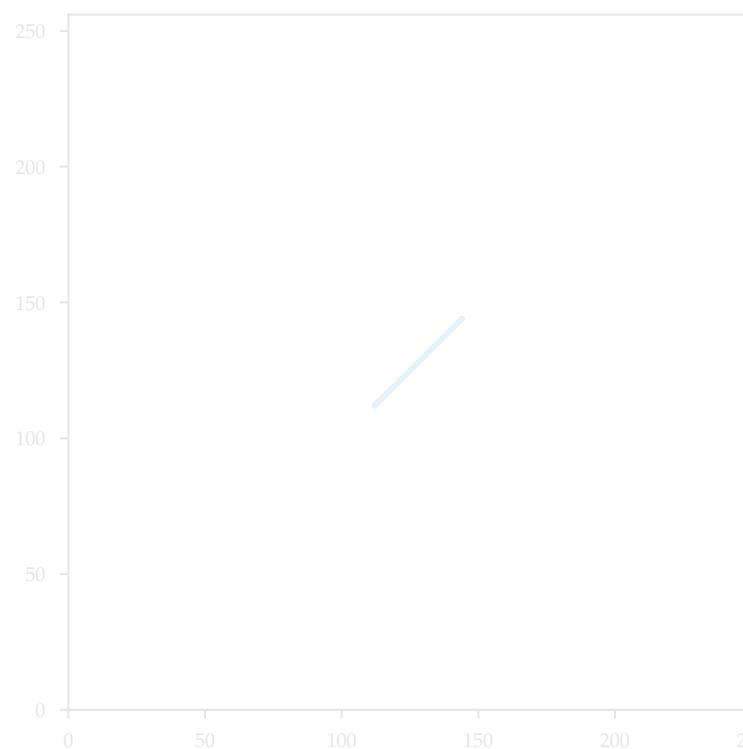
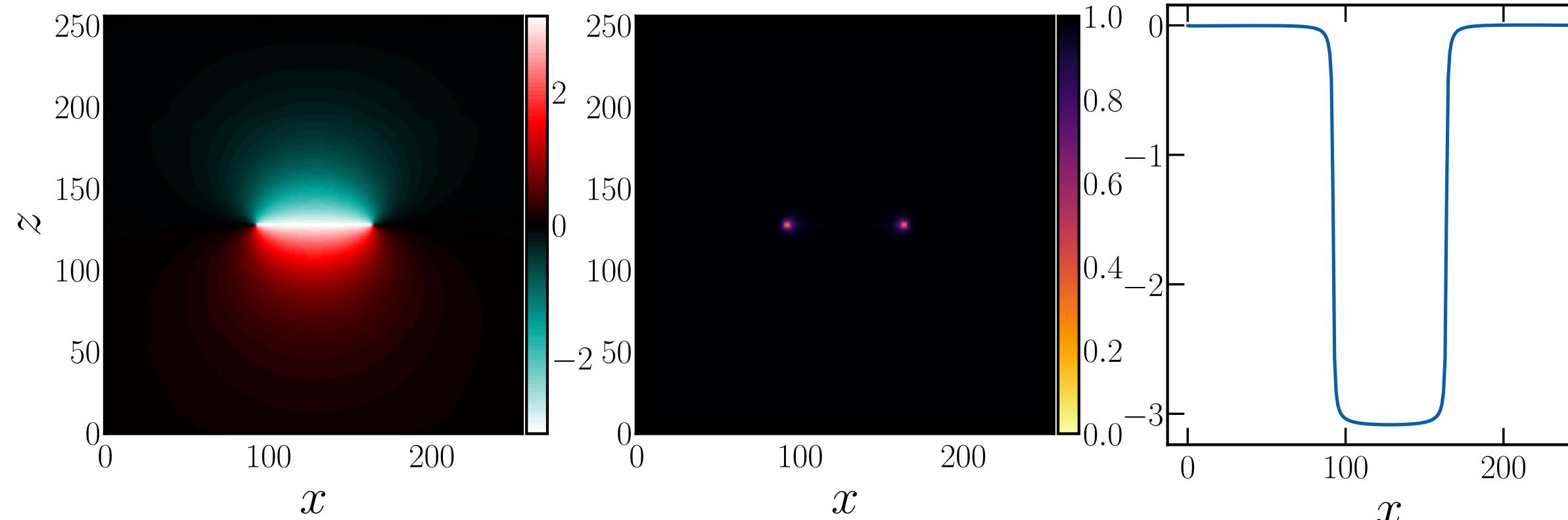
Constructing the Axion Field around Strings

- Contribution of a s



- Calculate links to c

$$\theta_{\mathbf{x}+d\mathbf{x}} - \theta_{\mathbf{x}} =$$



B -field:



Biot-Savard law ([Link](#))

$$\frac{\mathbf{X}(\sigma)) \times \mathbf{X}'}{|\mathbf{X} - \mathbf{X}(\sigma)|^3}$$

