Global String Dynamics from the Kalb-Ramond Axion Duality

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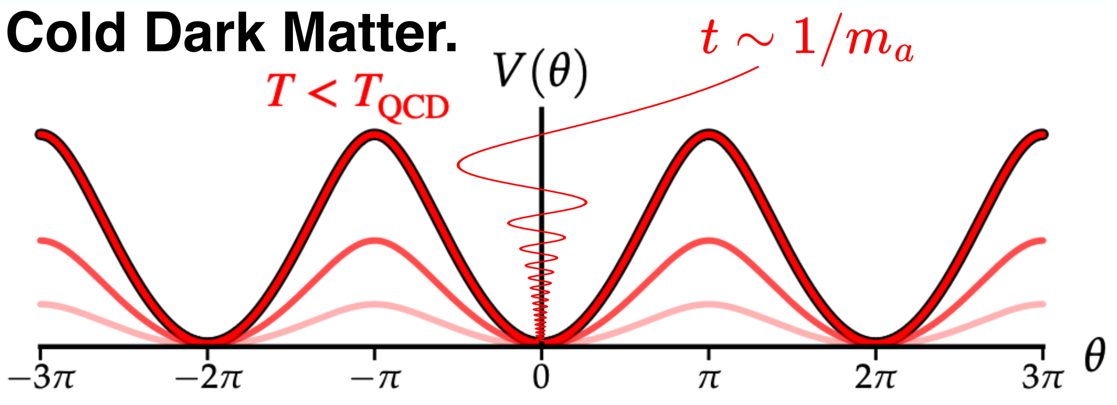
QCD Axion

- (Pseudo-) NG boson associated with the spontaneous breaking of the global PQ-symmetry at high-energy scale
- Provides dynamical solution to the **strong-CP problem.**
- Suitable candidate for **Cold Dark Matter.**
- Acquires a **mass** below the QCD scale.

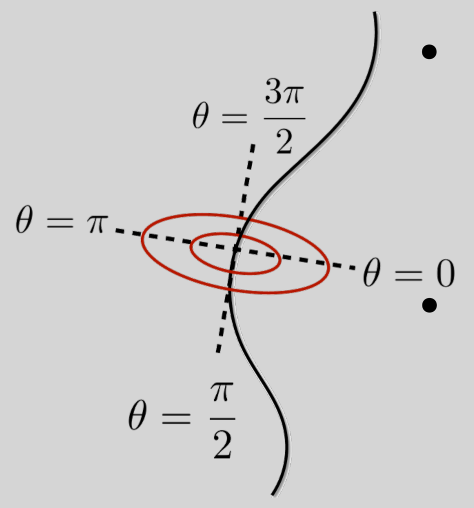
What is its mass? Crucial theory input for experiments!



O'Hare [2403.17697]

Topological Defects (Axion Strings + Domain Walls)

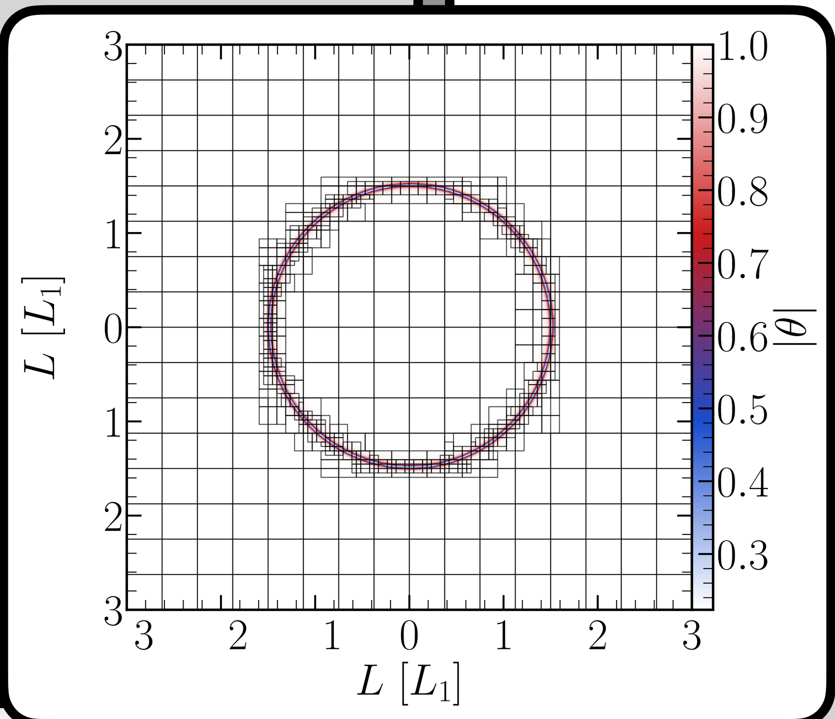
In principle, the post-inflationary scenario allows for a clear prediction, but …



- If the PQ symmetry is broken **after inflation,** a network of **topological defects** inevitably forms via the Kibble mechanism.
- Strings are characterised by a non-zero energy density (per unit length), the **string tension:**

$$\mu = \hat{\mu} + \pi f_a^2 \int_{c_{\rm UV}}^{c_{\rm IR}} \frac{\mathrm{d}r}{r} \approx \hat{\mu} + \pi f_a^2 \ln\left(\frac{f_a}{H}\right)$$

We need to understand the contribution from topological defects to the axion abundance!



Simulations of Axion Strings

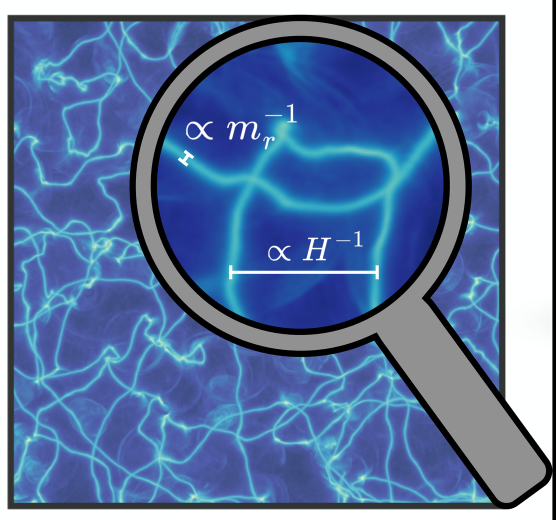
- Simulate complex scalar field $\phi \sim \eta e^{i\theta}$ on a discrete lattice:

$$\partial_\tau^2 \phi - \nabla^2\phi + \lambda\phi\left(|\phi|^2 - \tau^2\right) = 0$$

- **Problem:** Simulations require proper resolution of two very different length scales, leading to a **limited dynamical range**.

$$\ln\left(\frac{f_a}{H}\right) \sim 70$$

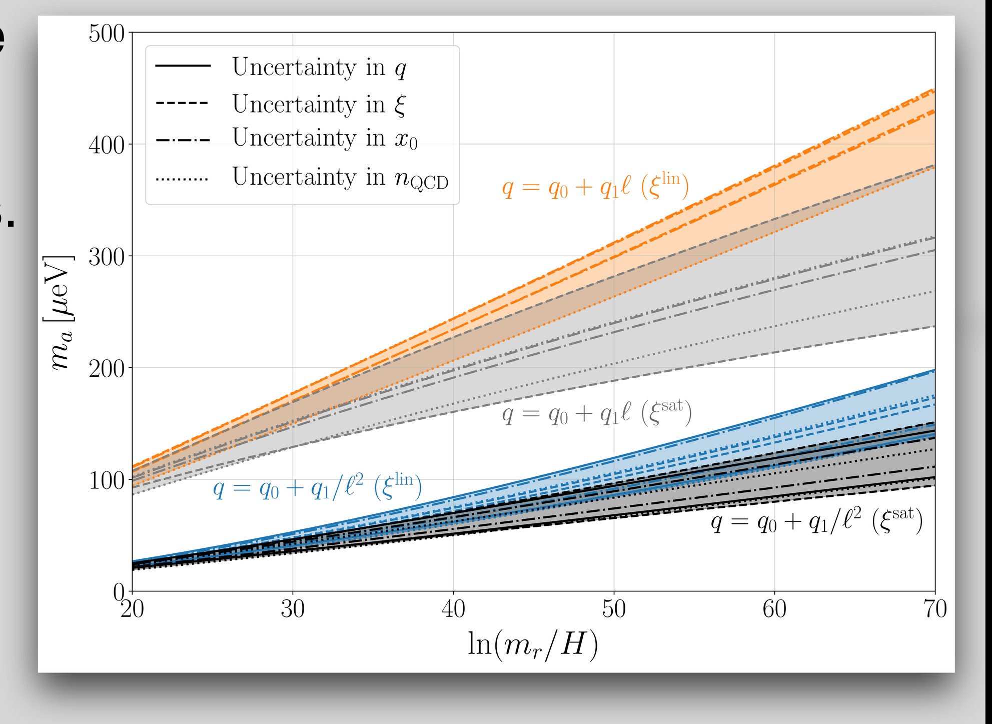
- **Impossible** to reach, even with modern, highly parallelised codes [1-3].
- Results require **large extrapolations!**
- Advancements in the numerical techniques, such as the use of **Adaptive Mesh Refinement** (AMR) allow for (small) improvements.



Courtesy of K. Saikawa

Spectrum of Global Strings and the Axion Dark Matter Mass [4]

- Lattice simulations to study the **axion spectrum** from strings, with up to $N^3 = 11268^3$ points.
- Systematic study of error sources: **Dependence on ICs,** contaminations due to **oscillations in the spectrum** and **discretisation effects**.



- Mass prediction: $95\mu\mathrm{eV} \lesssim m_a \lesssim 450\mu\mathrm{eV}$ **What are the next steps?**

Kalb-Ramond Axion Duality

- Effective field theory allowing for an **analytic** treatment of axion strings at **high tension**, approaching the **Nambu-Goto** limit [6,7]:

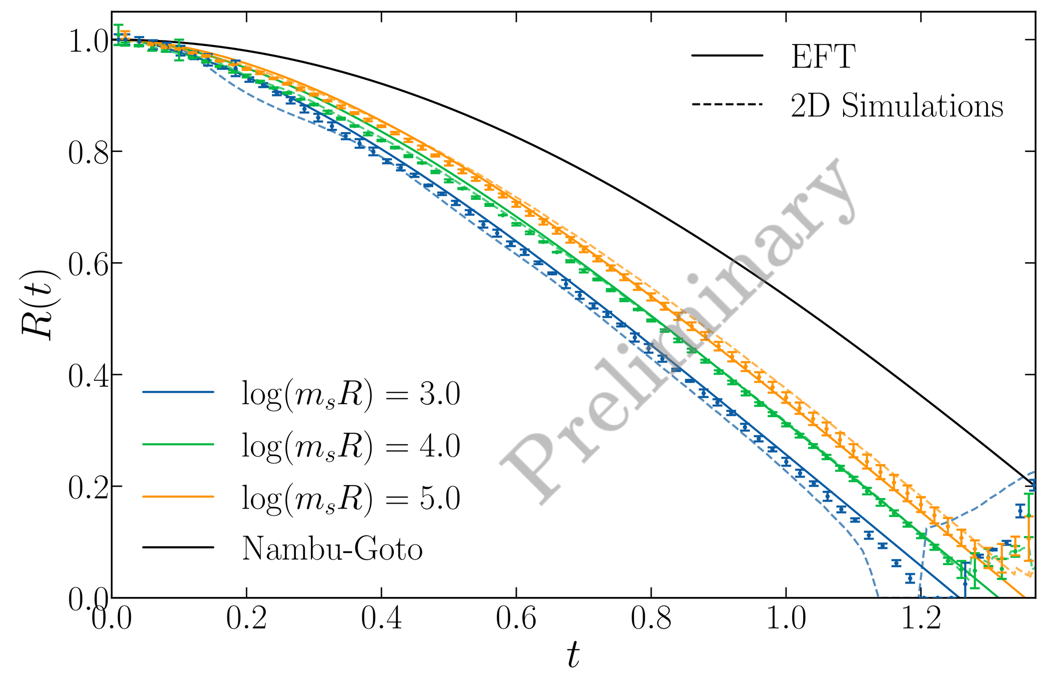
$$f_a\partial_\mu\theta = \epsilon_{\mu\nu\lambda\rho}\partial^\nu B^{\lambda\rho} \equiv \frac{1}{6}\epsilon_{\mu\nu\lambda\rho}H^{\nu\lambda\rho}$$

- Kalb-Ramond field $B_{\mu\nu}$ describes the behaviour of the axion field around strings:

$$\mathcal{S} = -\mu\int\sqrt{-\gamma}\mathrm{d}^2\sigma - f_a\int B_{\mu\nu}\mathrm{d}\sigma^{\mu\nu} + \frac{1}{6}\int H^2\sqrt{-g}\,\mathrm{d}^4x$$

Dynamics of a Circular String Loop

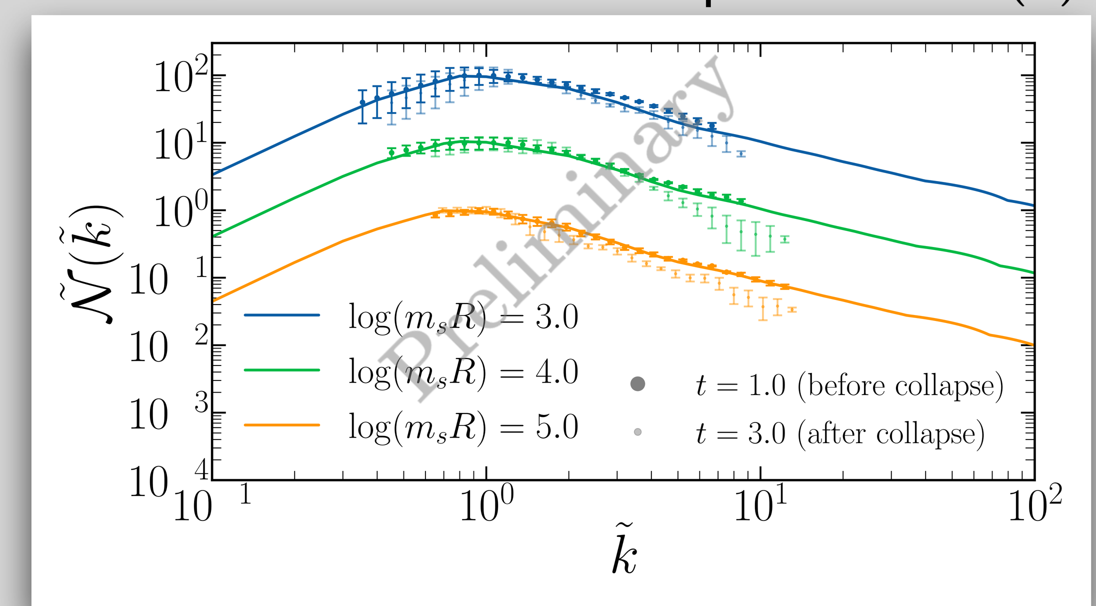
$$X^\mu = \{t, R(t)\cos(\sigma), R(t)\sin(\sigma), 0\}, \quad \sigma\in[0,2\pi]$$



- Comparison of EFT (solid) with simulations (points) to study the change of the string dynamics at different tensions

Loop Spectrum

- Dimensionless number spectrum: $\mathcal{N}(k) = \dfrac{1}{Hf_a^2}\dfrac{\partial\rho(k)}{\partial(k/R)}$



$\tilde{\mathcal{N}} = \mathcal{N}/\mathcal{N}|_{\rm peak}$
+ shift for better visibility!

$\tilde{k} = kL/(kL)|_{\rm peak}$

- **Final goal:** Compute the axion spectrum at different tensions and compare with results for string network simulations.

Conclusions

- The post-inflationary scenario allows for a prediction of the axion dark matter mass, but suffers from the existence of topological defects, making simulations very challenging.
- Recent literature results are still in disagreement, due to the limited dynamical range of the simulations, no clear answer is in sight.
- First results of a hybrid approach using EFTs and simulations for particular string configurations are in agreement for higher string tensions and allow us to gain a deeper understanding of the network dynamics and radiation at physically relevant tensions.

References

[1] JAXIONS code. Scan the QR code to find it on GitHub!
[2] B. Schwabe et al., *Phys.Rev.D* 102 (2020) 8, 083518
[3] M. Buschmann, *Astrophys.J.* 979 (2025) 2, 220
[4] K.Saikawa, J. Redondo, A. Vaquero, **M.Kaltschmidt**, *JCAP* 10 (2024) 043
[5] **M. Kaltschmidt**, J. Redondo, I. Rybak, *in preparation*
[6] R. L. Davis & E. P. S. Shellard, *Phys.Lett.B* 214 (1988) 219-222
[7] A. Dabholkar & J. Quashnock, *Nucl.Phys.B* 333 (1990) 815-832

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