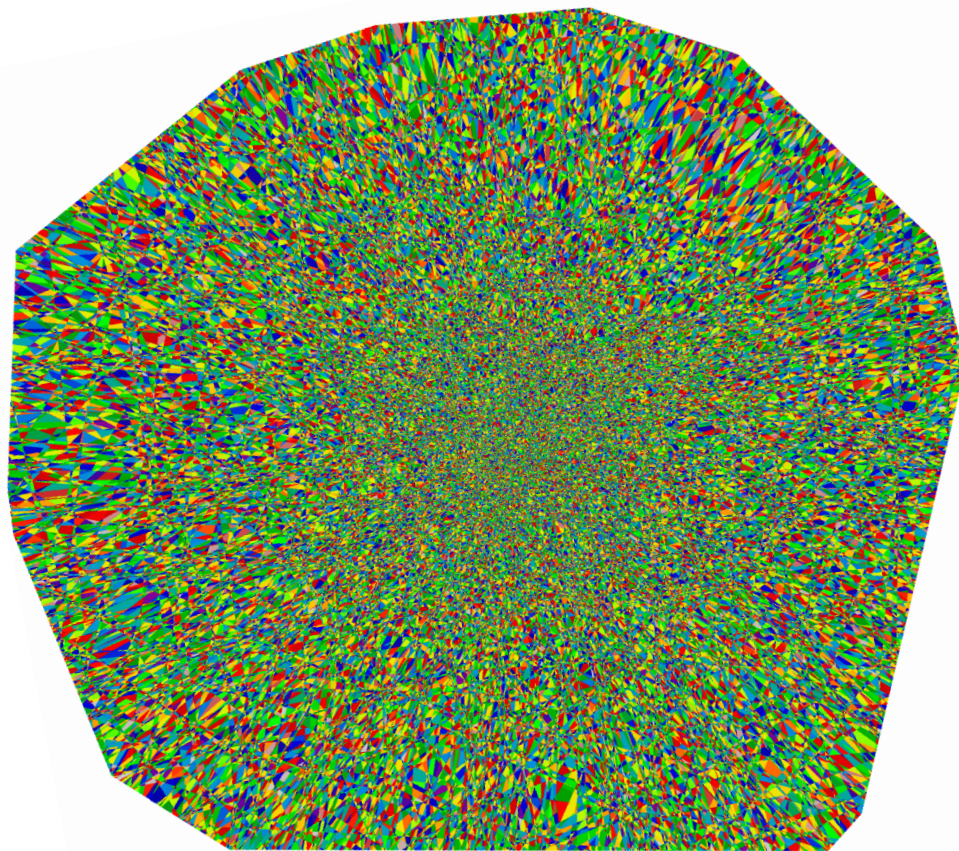


# The Many Axions of String Theory



Liam McAllister  
Cornell

*Axions in Stockholm, July 1, 2025*

# The Many Axions of String Theory

Why does string theory have axions?

Antisymmetric tensor fields in six extra dimensions

Why *many* axions? How many?

Rich topology of extra dimensions.  $N \sim 100$  to 100,000.

What are their properties?

Couplings scale with  $N$ .

Why does it matter what happens in string theory?

Axion couplings depend on nature of quantum gravity.

# The Many Axions of String Theory

String theory has many axions because of the topological complexity of the six extra dimensions,

and the number  $N$  of axions *matters*:

large- $N$  theories are qualitatively different from small- $N$  theories.

# The Many Axions of String Theory

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and the number  $N$  of axions *matters*:

large- $N$  theories are qualitatively different from small- $N$  theories.

We have, for the first time, constructed large- $N$  theories in actual solutions of string theory.

We are in the process of discovering the properties of these theories.



# Summary

Research program: computing the **string axiverse**, the landscape of many-axion effective theories in string theory.

We have made advances in computational geometry allowing study of theories with  $N \gg 1$  axions.

We have **constructed part of the axiverse**, in type IIB string theory on Calabi-Yau threefolds.

We find geometric hierarchies involving powers of  $N$ , and corresponding hierarchies in the low-energy couplings.

Consequences for strong CP, DM abundance, superradiance, axion-photon couplings.

We can already exclude many string models.

# Based on

Demirtas, Long, L.M., Stillman 2018

‘Kreuzer-Skarke Axiverse’:  $C_4$  axions in type IIB on  $CY_3$

topological complexity  
 $\Rightarrow$  hierarchies

Mehta, Demirtas, Long, Marsh, L.M., Stott 2021

Black hole superradiance

Demirtas, Gendler, Long, L.M., Moritz 2021

Strong CP problem

Gendler, Marsh, L.M., Moritz 2023

Axion-photon couplings

Sheridan, Carta, Gendler, Jain, Marsh, L.M., Righi, Rogers, Schachner 2024

Fuzzy dark matter

Bellas, Halverson, L.M., Vander Ploeg Fallon, Zhu 2025

Axions in F-theory

many more axions  
 $\Rightarrow$  stronger effects

—consequences—



# Plan

- I. Axions in string theory
- II. Geometric hierarchies at  $N \gg 1$
- III. Hierarchies in axion couplings

# Setting

We start with superstring theory,  
for which the fundamental solution is  $\mathbb{R}^{9,1}$ .

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for which the fundamental solution is  $\mathbb{R}^{9,1}$ .

We study *compactifications* in the *geometric regime*:

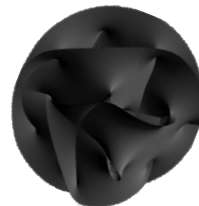
$$ds_{10}^2 = g_{\mu\nu}^{(3+1)} dx^\mu dx^\nu + ds_{X_6}^2$$

with  $X_6$  a compact six-manifold that is large compared to a string.



4d spacetime

x



6d compact space



# Geometry of the extra dimensions

Einstein equations in vacuum: **Ricci** = 0

$$R_{\mu\nu} = 0 \quad \text{and} \quad R_{mn} = 0$$

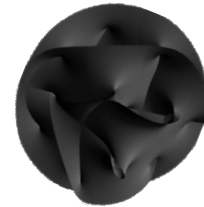
$$\mu, \nu \in \{0, 1, 2, 3\}$$

$$m, n \in \{4, 5, 6, 7, 8, 9\}$$



4d spacetime

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0123

456789

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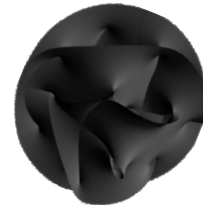
$$R_{\mu\nu} = 0 \quad \text{and} \quad R_{mn} = 0$$

Vacuum solution of string theory =  
 $\mathcal{M}^{3,1} \times$  [compact Ricci-flat six-manifold]  
[e.g. ‘Calabi-Yau threefold’]



4d spacetime

x



6d compact space

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456789

# Kaluza-Klein reduction

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + R^2 d\vartheta^2 \qquad \vartheta \cong \vartheta + 2\pi$$

Given a scalar  $\Phi$  in 5d:

$$\Phi = \sum_n c_n \Phi_n(x^\mu, \vartheta)$$

$$\Phi_n(x^\mu, \vartheta) = \phi_n(x^\mu) \cdot \varphi_n(\vartheta) \qquad \varphi_n(\vartheta) = e^{in\vartheta}$$

$$0 = \square_5 \Phi_n(x^\mu, \vartheta) \Leftrightarrow \left( \square_4 + \frac{n^2}{R^2} \right) \phi(x^\mu) = 0$$

Massless 5d scalar  $\Phi$  gives Kaluza-Klein tower of 4d scalars  $\phi_n$ .  
Massless 4d scalar from **zero-mode**  $\varphi_0$ .

In string theory it's the same, but with more extra dimensions.

# Axions from extra dimensions

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + R^2 d\vartheta^2 \qquad \vartheta \cong \vartheta + 2\pi$$

Given a gauge field (1-form)  $A_M$  in 5d:

$$S_5 \supset - \int d^5x F_{MN} F^{MN}, \qquad M, N \in 0, \dots, 4$$

$$\theta := \int_0^{2\pi} d\vartheta A_4$$

$$S_4 \supset -\frac{1}{2} \int d^4x f^2 \partial_\mu \theta \partial^\mu \theta, \qquad f^2 = \frac{2}{\pi R^2}$$

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A 1-form  $A_1$   
integrated over a 1-manifold  
yields an axion.

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + ds_{\text{CY}_3}^2$$

$$S_{10} \supset - \int d^{10}x F_{MNPQR} F^{MNPQR}, \quad M, N \in 0, \dots, 9$$

$$\theta_i := \int_{\Sigma_i} [dx^4 dx^5 dx^6 dx^7] A_4$$

$$S_4 \supset -\frac{1}{2} \int d^4x \sum_i f_i^2 \partial_\mu \theta_i \partial^\mu \theta_i, \quad f_i^2 \propto \text{Vol}(\text{CY}_3)^{-4/3}$$

A 4-form  $A_4$   
integrated over a 4-manifold  
yields an axion.

In typical Calabi-Yau threefolds there are **many submanifolds**  
 $\Rightarrow$  many axion fields.



# Setting

A **QGEFT** is an effective theory, of gravity and other fields, that results from an ultraviolet completion of gravity, such as a compactification of string theory.

Axion fields and couplings in QGEFT depend on topology, geometry, fields of extra dimensions.

We aim to understand axions in QGEFTs.

Strategy: enumeration of compactifications on Calabi-Yau threefolds ( $CY_3$ ).

# Context

Topologically simple compactifications yield simple QGEFTs.

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- No large dimensionless parameters. NDA works.
- Planck-suppressed operators have expected size:

$$\mathcal{L} \supset c_{\Delta} \frac{\mathcal{O}_{\Delta}}{M_{\text{pl}}^{\Delta-4}} \text{ with } c_{\Delta} \gtrsim \mathcal{O}(1)$$

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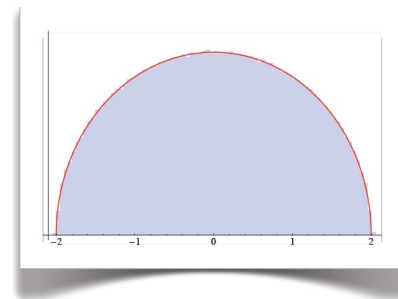
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M.C.D. Marsh, L.M., Wrase 2011



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Enumerate compactifications of string theory at large  $N$ .

# Context

Obstacle:

Until recently, topologically complex Calabi-Yau compactifications — those with  $N \gg 1$  axions — were too complex to analyze.

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Kreuzer and Skarke, 2000

Cost of many steps is  $\sim e^N$  in Sage/TOPCOM/Instanton/etc.

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We have overcome this problem.

Long, L.M., McGuirk 14

Long, L.M., Stout 16

Braun, Long, L.M., Stillman, Sung 17

Demirtas, Long, L.M., Stillman 18

Demirtas, Kim, L.M., Moritz 19

Demirtas, L.M., Rios-Tascon 20

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Mehta, Demirtas, Long, Marsh, L.M., Stott 21

Demirtas, Kim, L.M., Moritz, Rios-Tascon 21

Demirtas, Gendler, Long, L.M., Moritz 21

Demirtas, L.M., Rios-Tascon 22

Demirtas, Kim, L.M., Moritz, Rios-Tascon 23



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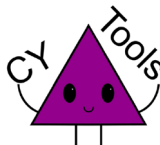
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Cost of many steps is  $\sim e^N$  in Sage/TOPCOM/Instanton/etc.

We have overcome this problem.

We found polynomial-time algorithms for ‘everything’, and implemented them in a software package, CYTools.

Demirtas, L.M., Rios-Tascon 22



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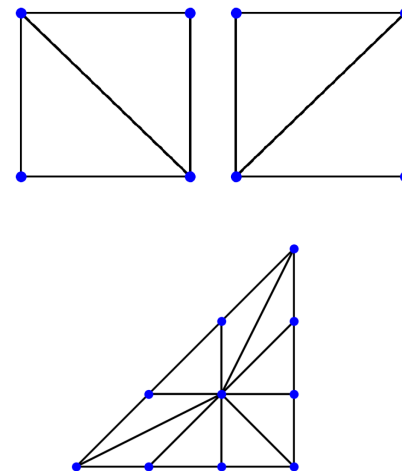
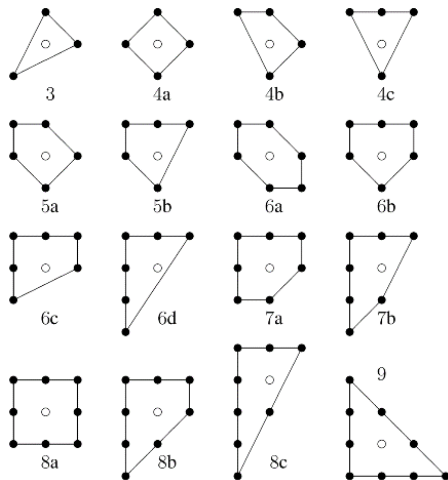
Demirtas, Kim, L.M., Moritz, Rios-Tascon 23

# Combinatoric geometry

Key to unlocking large  $N$ :

Constructing  $CY_3$  as hypersurfaces in **toric varieties**,  
and exploiting combinatoric structures.

Toric varieties correspond to triangulations of polytopes.



473,800,776 4d reflexive polytopes

Kreuzer and Skarke 2000

$< 10^{428} CY_3$

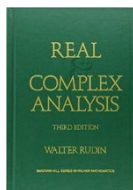
Demirtas, L.M., Rios-Tascon 2020

# Combi-na-toric geometry

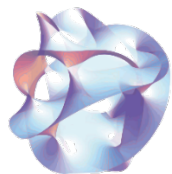
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analysis



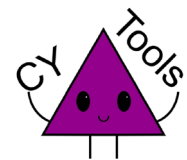
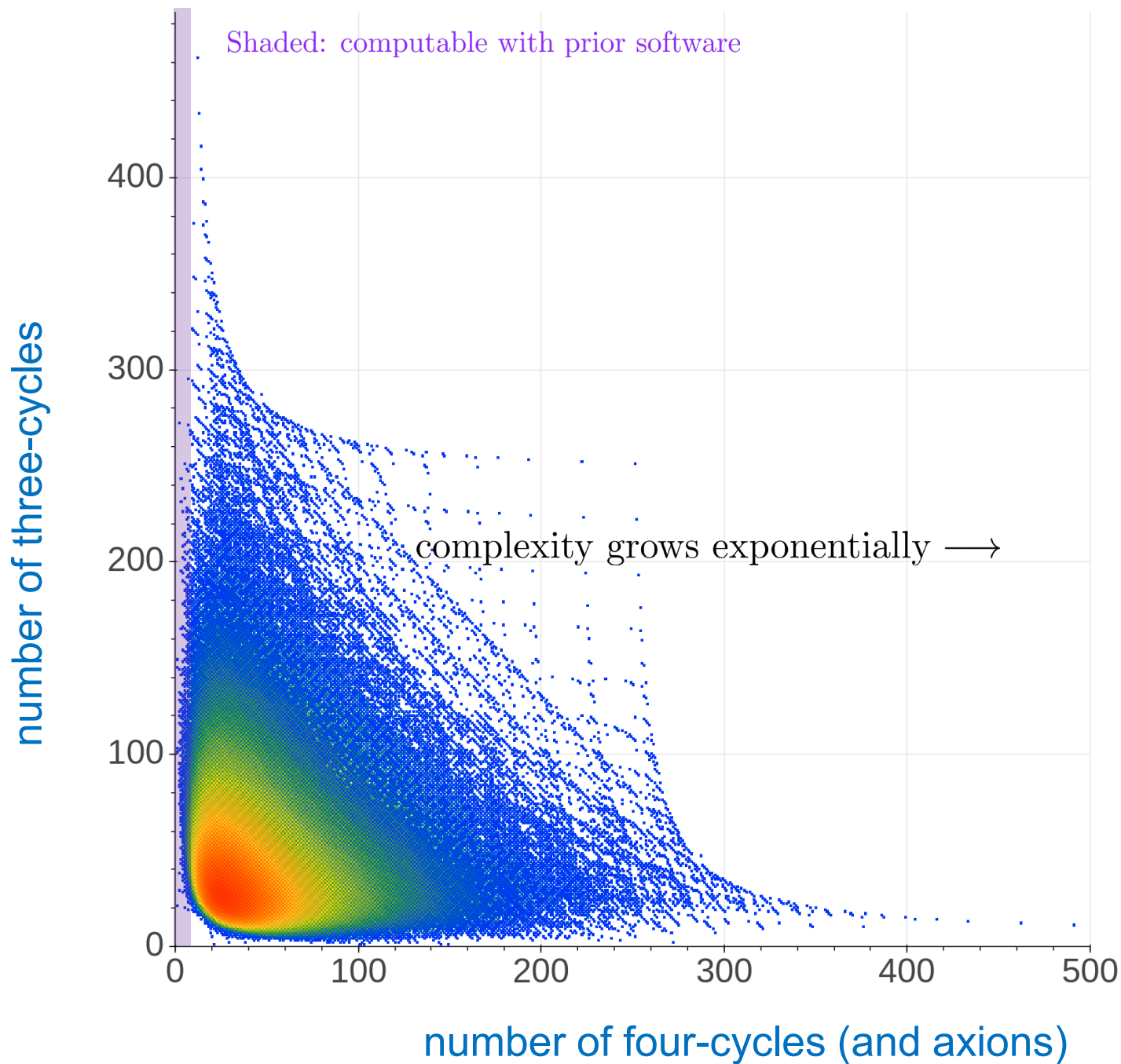
algebraic geometry



combinatorics



easier to automate

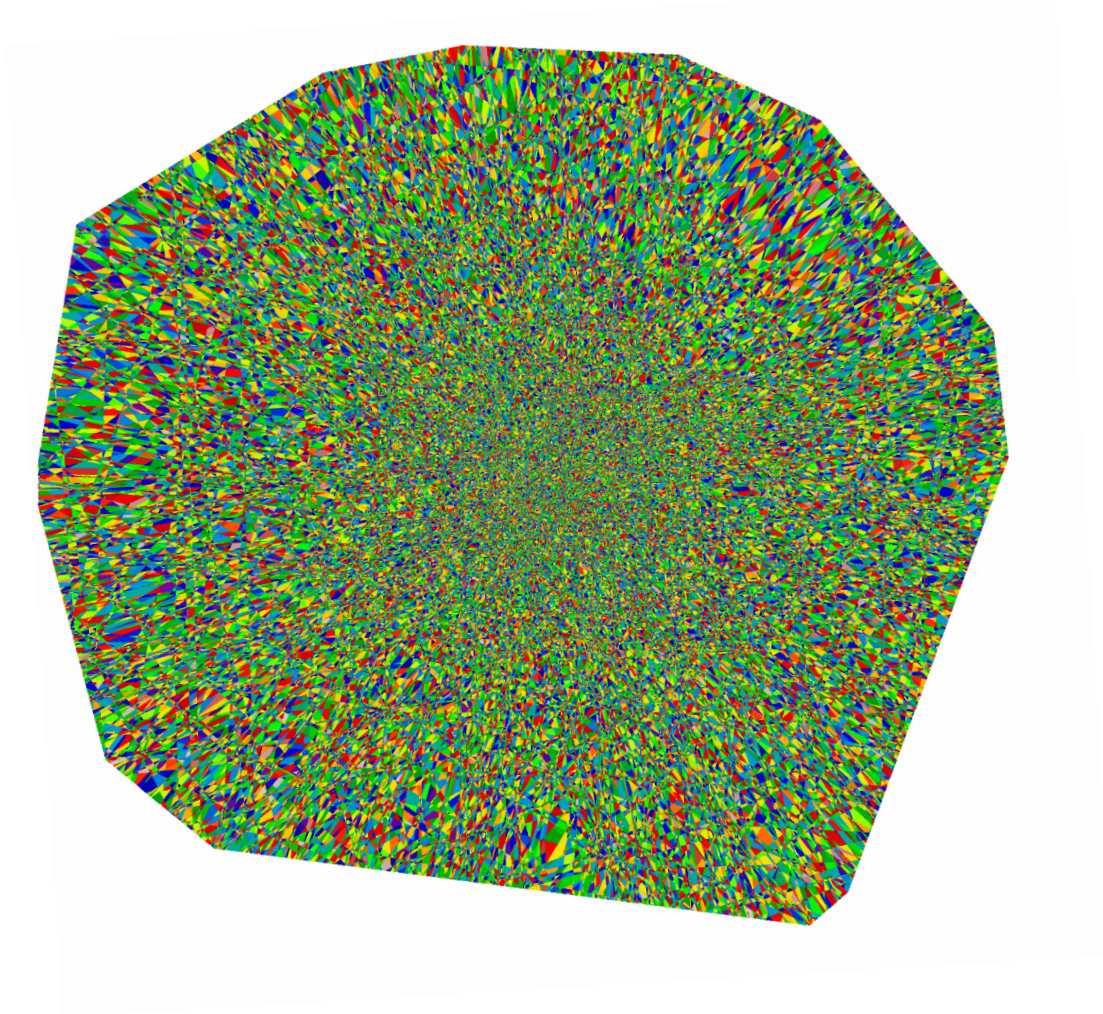


<http://cy.tools>

Demirtas, L.M., Rios-Tascon

# The Kreuzer-Skarke Axiverse

Using `CYTools` we generated millions of  $CY_3$  hypersurfaces.



# The Kreuzer-Skarke Axiverse

Using `CYTools` we generated millions of  $\text{CY}_3$  hypersurfaces.

Compactifying type IIB string theory, we constructed an ensemble of  $N$ -axion QGEFTs,  $1 \leq N \leq 491$ .

These are incarnations of the string axiverse.

Arvanitaki, Dimopoulos, Dubovsky, Kaloper, March-Russell 09

Null hypothesis: isotropic internal space, one length scale  $L$ .

Our result: hierarchies of cycle sizes, polynomial in  $N$ .

Demirtas, Long, L.M., Stillman 2018

Topologically complex CY compactifications are anisotropic.

Consequence: **hierarchical axion couplings**.

# Plan

- I. Axions in string theory
- II. Geometric hierarchies at  $N \gg 1$
- III. Hierarchies in axion couplings



# Cycle sizes

Curvature expansion: a fundamental expansion in string theory.

Coupling  $\ell_s/L$ .     $\ell_s$ : string length  
                               $L$ : typical length

Can compute QGEFT when  $\ell_s \ll L$ .

Strategy: ensure that appropriate cycles are large in units of  $L$ .

Specifically: arrange that all 2-cycles have volume  $> \ell_s^2$ .

Let's see how to achieve this.

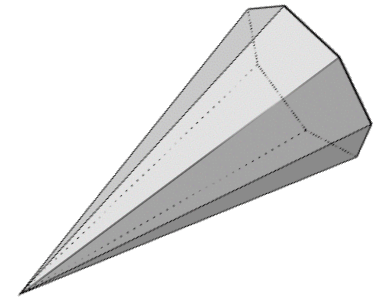


# The Kähler cone

The number of 2-cycles is the number of axions,  $N$ .

So moduli space of 2-cycle sizes is  $\subset \mathbb{R}^N$ .

In fact it's a cone:



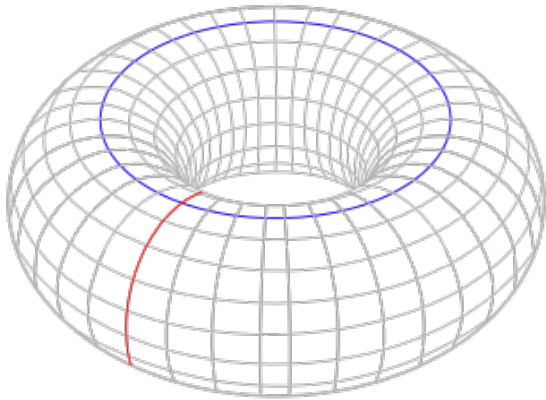
The *Kähler cone*  $\mathcal{K} \subset \mathbb{R}^N$

is the region where all 2-cycles have volume  $\geq 0$ .

On the walls of the cone, one or more 2-cycles have zero size.  
Inside the cone, all (holomorphic) 2-cycles have positive size.

The **Stretched Kähler cone**

is the subregion of  $\mathcal{K}$  where all 2-cycles have volume  $\geq 1$ .



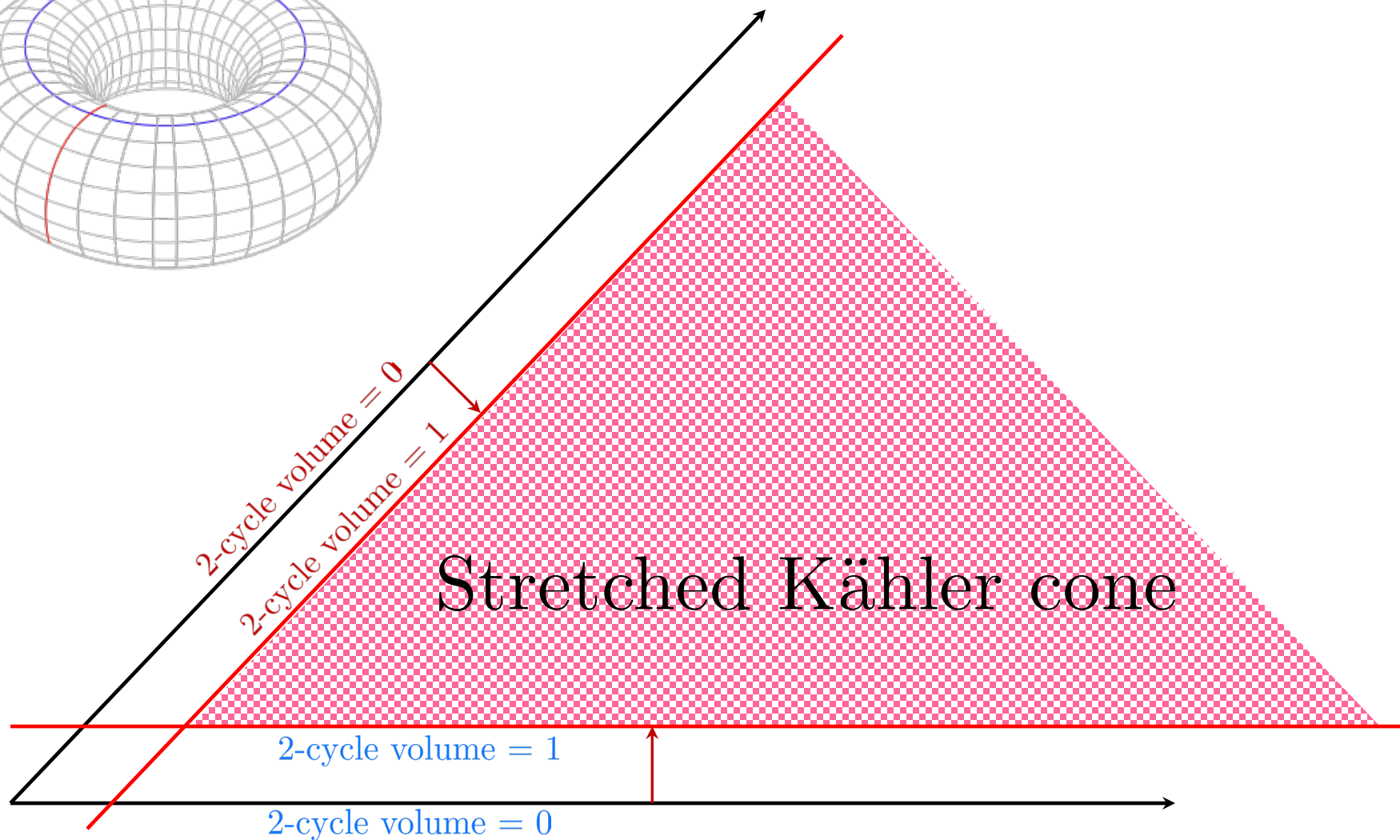
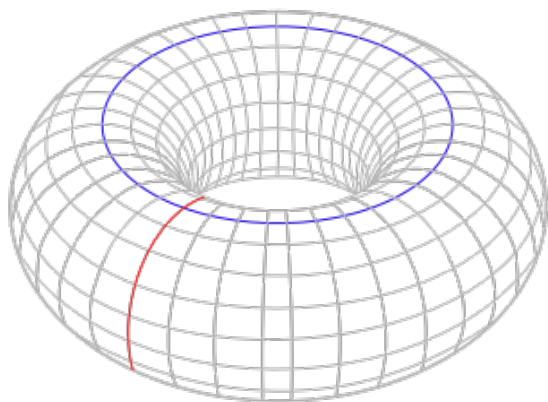
2-cycle volume = 0

2-cycle volume  $> 0$

2-cycle volume  $> 0$

Kähler cone

2-cycle volume = 0



We work in the stretched Kähler cone,  
where the curvature expansion is plausibly controlled.

# Key observation

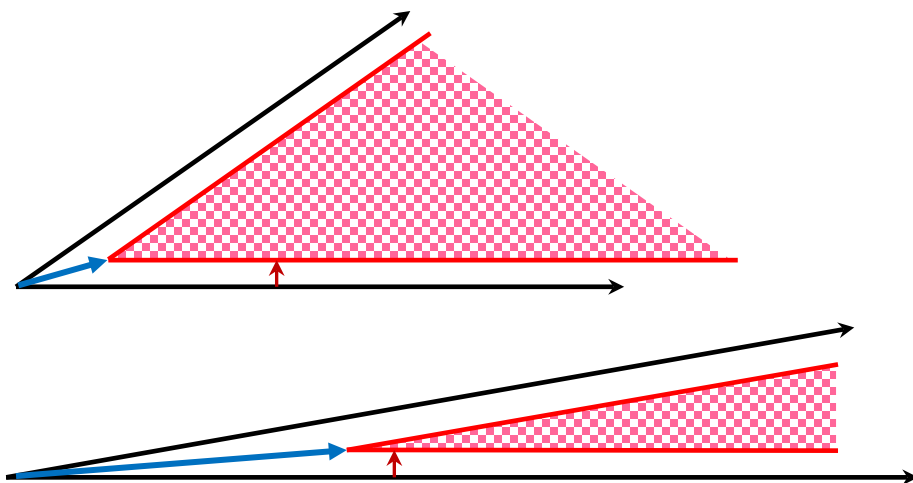
In a narrow cone with many walls,  
the subcone that is distance  $\geq 1$  from every wall  
has its apex far from the origin.

Fact: for  $N \gg 1$ , Kähler cones are narrow and have many walls.

So for  $N \gg 1$ , stretched Kähler cones have apex far from the origin.

This means that some cycles are large.

Their size is a power of  $N$ .



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So for  $N \gg 1$ , stretched Kähler cones have apex far from the origin.

This means that some cycles are large.

Their size is a power of  $N$ .

But other cycles have volumes of order unity.

A main result: hierarchies of cycle sizes, by powers of  $N$ .

‘Topologically complex CY compactifications are anisotropic’.

# Key observation

Consider a spherical cow of radius  $R_{\text{cow}}$ .

Q: find minimum  $R_{\text{cow}}$  s.t. all its spots have size  $R_{\text{spot}} > 1$  meter.



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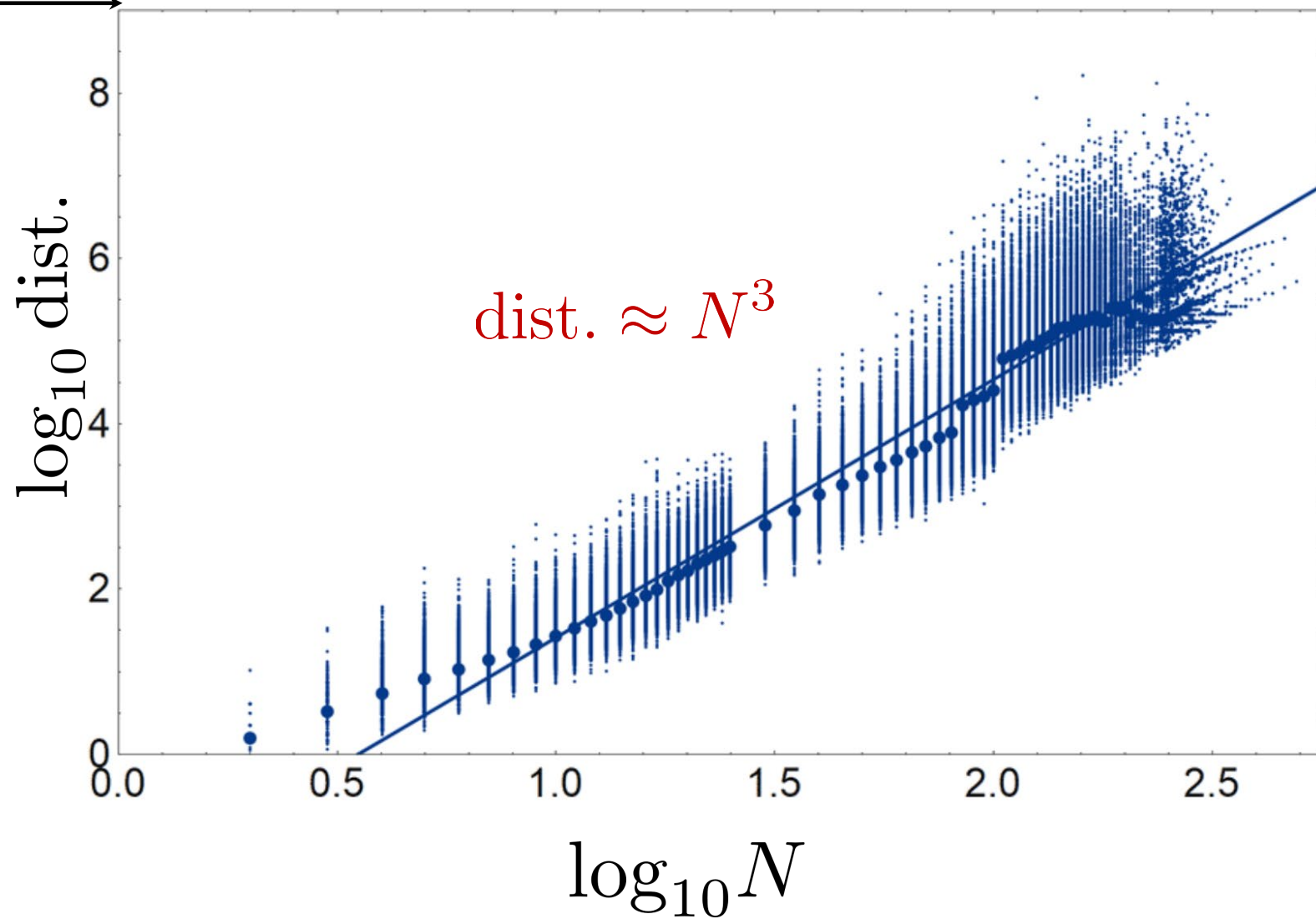
A2: For an *anisotropic cow with many spots of widely varied size*, one has  $R_{\text{cow},\text{min}} \gg 1$ .



CY<sub>3</sub> are anisotropic, with many cycles of widely varied size.



# Distance to tip of cone



# Story so far

We constructed a landscape of string compactifications, and studied the resulting many-axion theories.

We found geometric hierarchies that are powers of the number of axions,  $N$ .

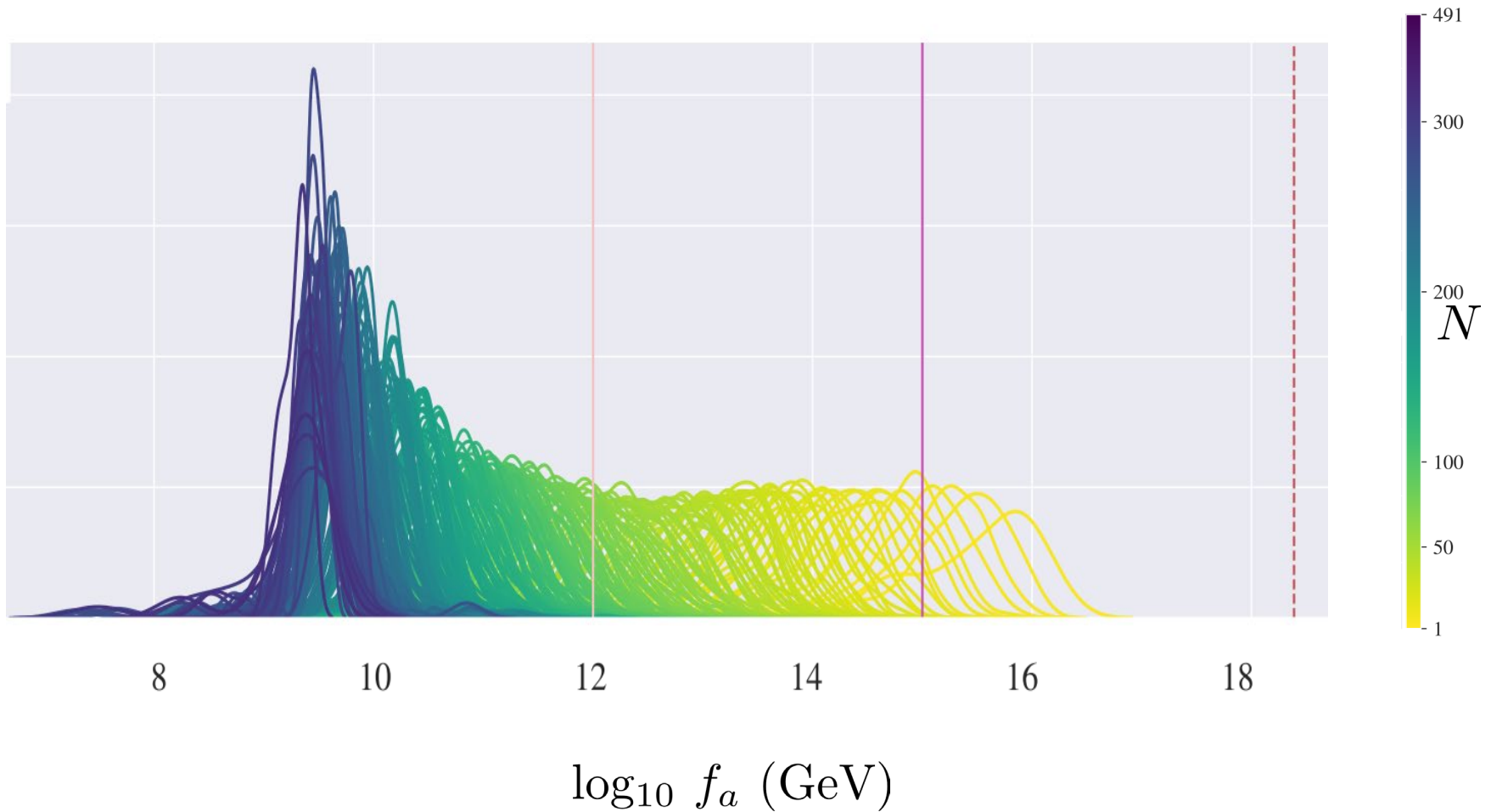
Next: the geometric hierarchies lead to hierarchies in the low-energy couplings, with consequences for phenomenology.

Warning: not clear that these extra-dimensional axions can form axion strings.

## Consequence #1:

Decay constants spread over wide range,  
and diminish with  $N$ .

# Decay constants $f_a$



## Consequence #2:

Fuzzy axion dark matter is extremely rare  
for  $N \gtrsim 15$ .

# Fuzzy axion dark matter

Could dark matter consist of ultralight axions?

For  $m_a \lesssim 10^{-18}$  eV, chance of future tests.

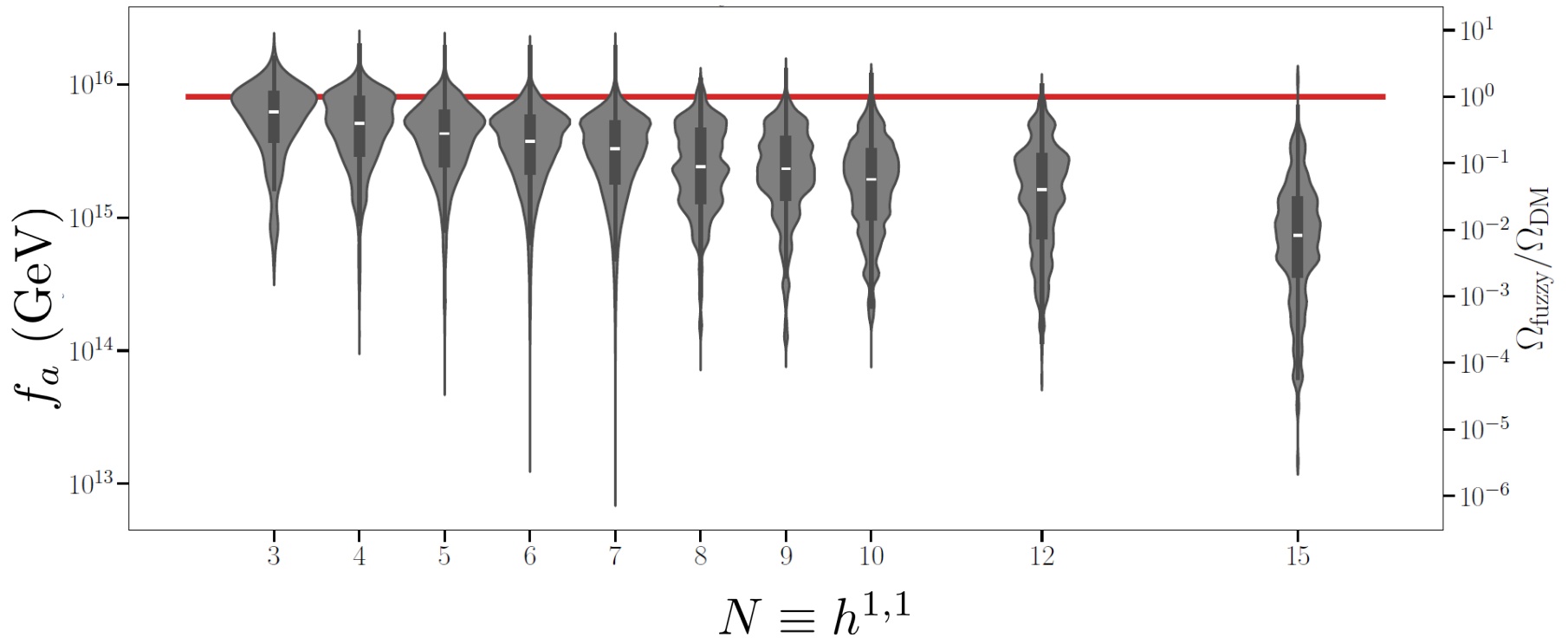
For  $m_a \sim 10^{-21}$  eV, significant constraints.

Misalignment abundance  $\Omega_a \propto f_a^2 \sqrt{m_a}$ .

$\Rightarrow$  need  $f_a \gtrsim 10^{16}$  eV for abundant fuzzy axion DM.

We find  $f_a$  decreases with  $N$ ,  
and is typically too small for fuzzy DM for  $N \gtrsim 15$ .

# Fuzzy axions at small N



$f_a$  typically too small for fuzzy DM for  $N \gtrsim 15$ .

Fuzzy DM generally accompanied by overabundance of heavier-axion DM. Exceptions: fibration, or tuned cosmology.

## Consequence #3:

Strong CP problem solved by PQ mechanism,  
without a PQ quality problem,  
for  $N \gtrsim 15$ .



# PQ quality problem

The Peccei-Quinn solution of the strong CP problem is sensitive to Planck-scale CP-breaking, and so requires UV completion. This is the *PQ quality problem*.

High-scale physics, even Planck-scale physics, can easily break PQ symmetry badly enough to ruin mechanism.

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If there is another term that breaks the PQ shift symmetry,

$$\text{e.g.} \quad V_{\text{breaking}} = \Lambda^4 \cos(\bar{\theta} + \psi) \quad \psi \in [0, 2\pi)$$

$$\text{then} \quad \langle \bar{\theta} \rangle \sim \frac{\Lambda^4}{\Lambda_{\text{QCD}}^4} \lesssim 10^{-10} \quad \Leftrightarrow \quad \Lambda^4 \lesssim 6 \times 10^{-14} \text{ GeV}^4$$

Write in terms of  $\Lambda^4 \equiv M_{\text{pl}}^4 e^{-S}$  as

$$S \gtrsim 200.$$

So *every instanton carrying PQ charge* must have  $S \gtrsim 200$ .

# PQ quality problem

Every instanton carrying PQ charge must have  $S \gtrsim 200$ .

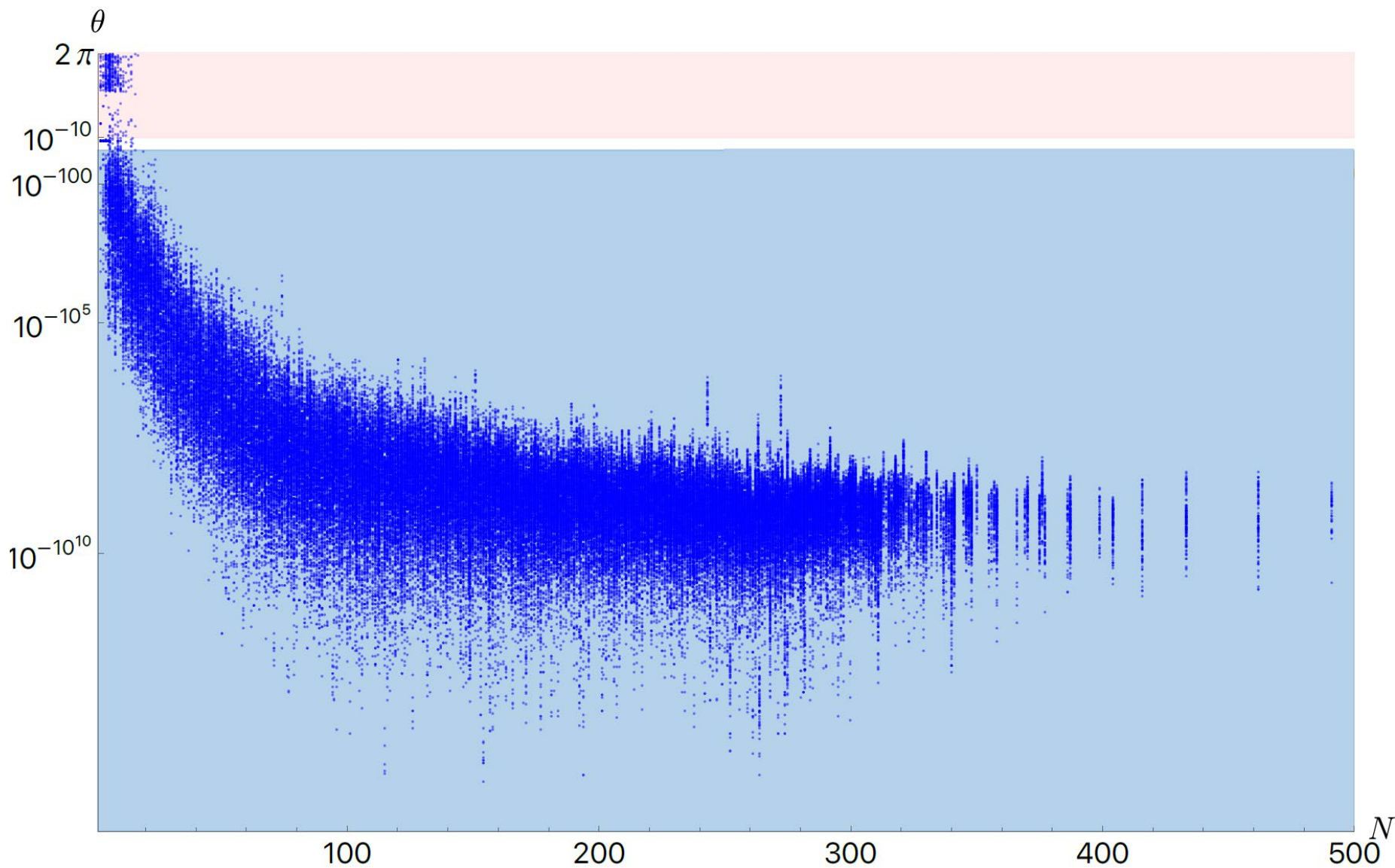
This dramatic sensitivity to Planck-scale physics is an opportunity for string theory: the effects in question can be computed.

In our ensemble this breaking comes from certain instantons.

We explicitly compute the leading such instantons.

When  $N \gtrsim 15$ , the Planck-scale breaking is negligible, and the quality problem is solved.

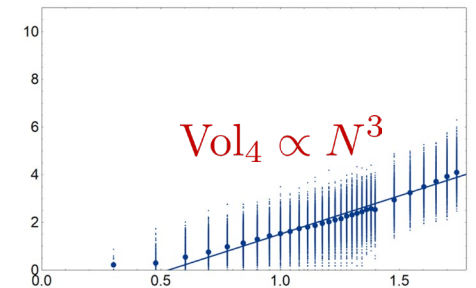
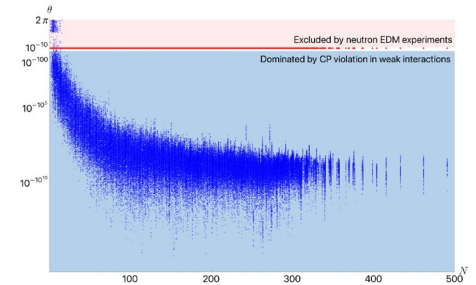
# PQ quality



# PQ quality problem in string theory

What's going on:

$$\Delta\theta \propto e^{-2\pi \text{Vol}_4}, \text{ for leading 4-cycle.}$$



Anisotropy of topologically complex space

$\Rightarrow$  large 4-cycles

$\Rightarrow$  small  $\theta$ -angle.

## Consequence #4:

Photon couplings to ultralight axions  
are hierarchically suppressed,  
if there is a stringy instanton on the QED cycle.

# Axion-photon couplings

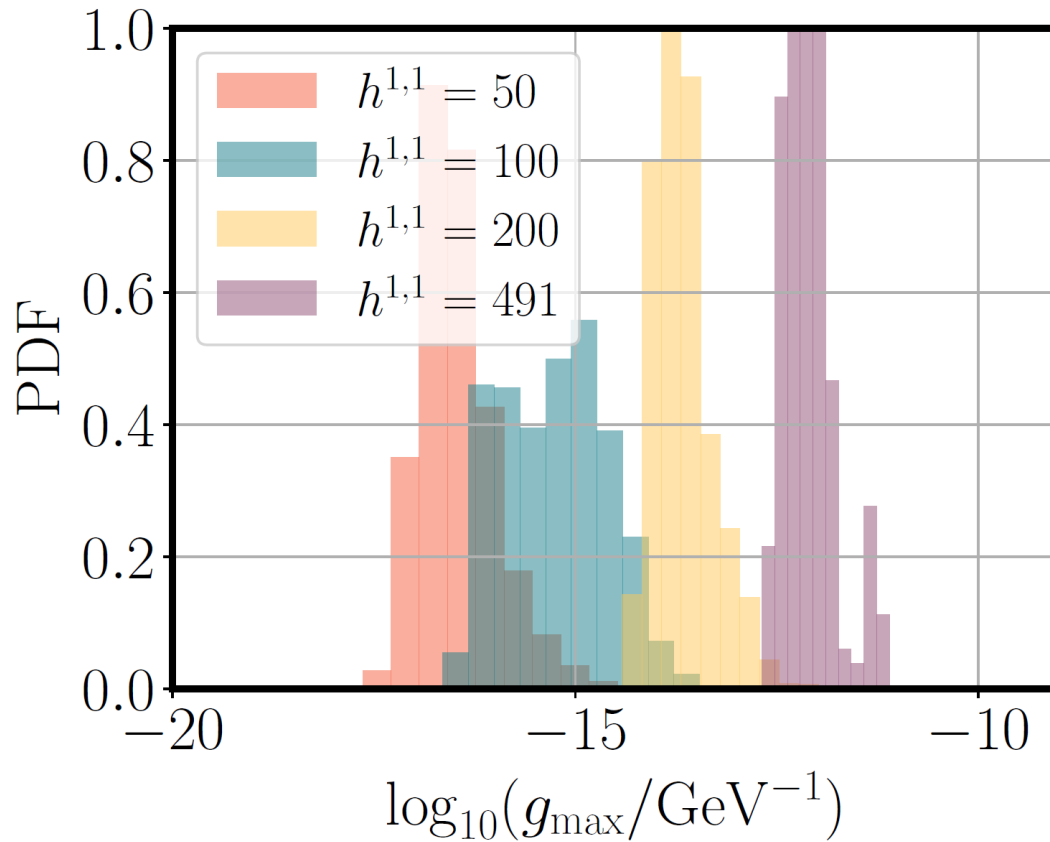
$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} g_{a\gamma\gamma} \varphi^a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

We have not built actual SM.

QED proxy: 4-cycle  $\Sigma_{\text{QED}}$  that ‘could’ host QED.

$g_{a\gamma\gamma} \sim 1/f \Rightarrow$  coupling **increases** with  $N$

# Axion-photon couplings



$g_{a\gamma\gamma} \sim 1/f \Rightarrow$  coupling **increases** with  $N$



# Light threshold

Gauge group with  $\theta$ -angle  $\mathcal{L}_{\text{gauge}} = \theta_a F_{\mu\nu} \tilde{F}^{\mu\nu}$

and potential  $\mathcal{L} \supset \Lambda^4 \cos(\theta_a)$   $\varphi_a = \theta_a / f_a, \quad a = 1, \dots, N$

has negligible coupling to axions  $\varphi_j$  with  $m_j \ll m_{\text{gauge}} \equiv \Lambda^2 / f_a$ .

QED does not have ordinary instantons of its own.

But there can be stringy instantons!

Then the photon has negligible couplings to axions  $j$  with

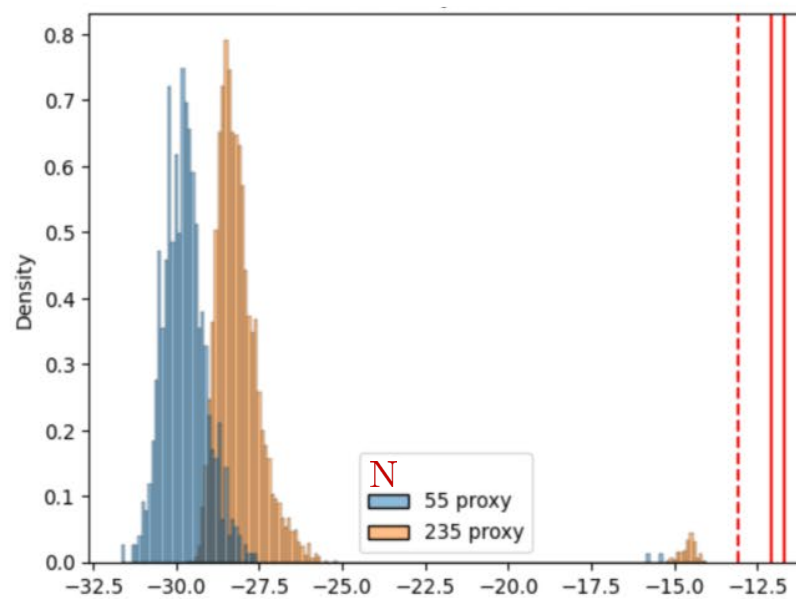
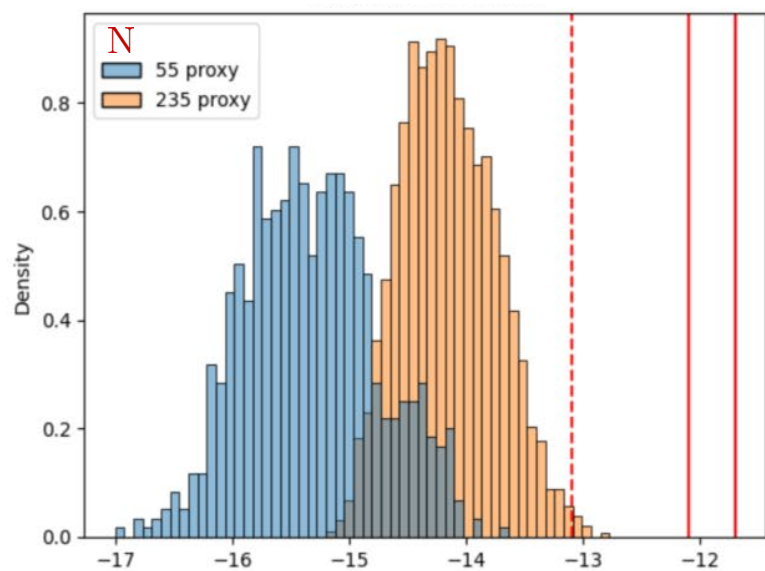
$$m_j \ll m_{\text{light}} \equiv f_{\text{QED}}^{-1} e^{-2\pi \text{Vol}(\text{QED})}$$

**Light threshold:** a mass scale set by stringy instantons on  $\Sigma_{\text{QED}}$ .

Axions with  $m \ll m_{\text{light}}$  have negligible photon couplings.

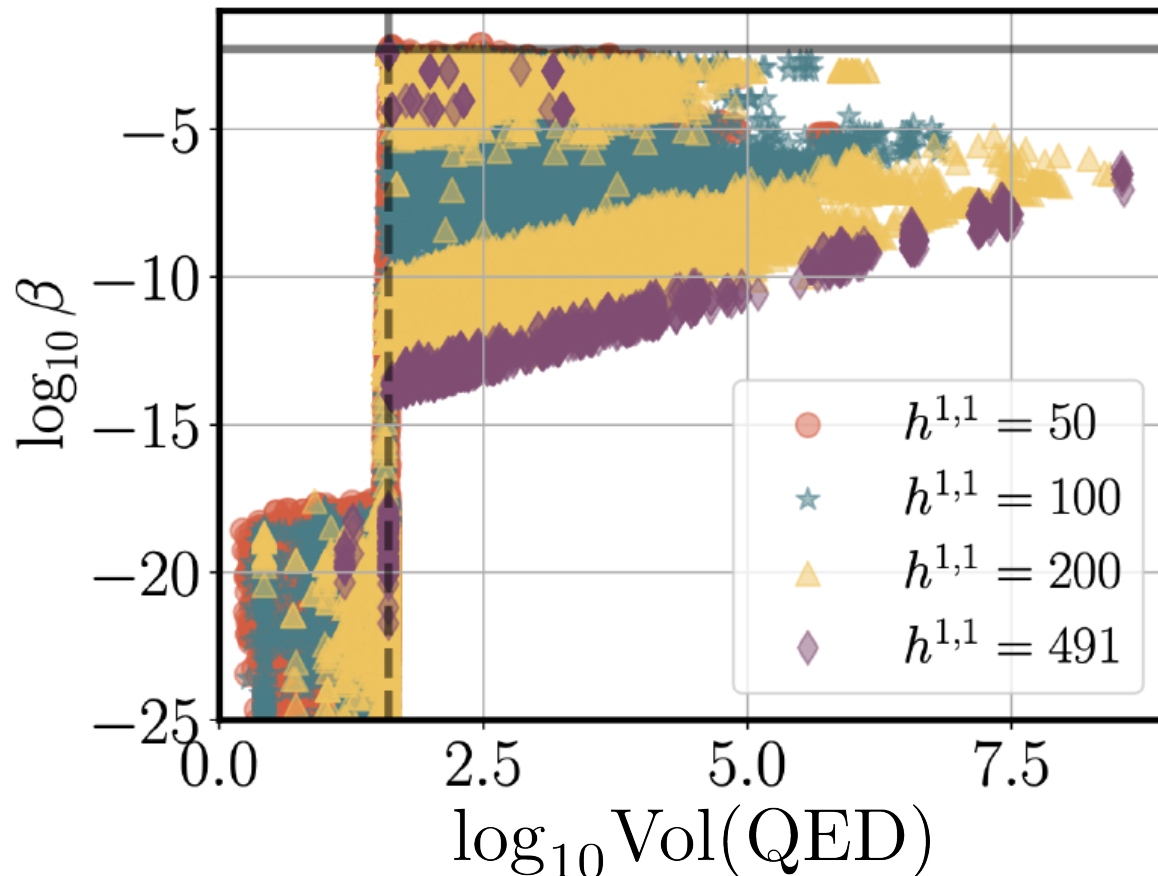
Makes ultralight axions invisible.

# Light threshold



# Birefringence prospects

Maximum birefringence



$\beta \approx 0.3^\circ$  possible for  $\text{Vol}(\text{QED}) \approx 40$ .

Minami, Komatsu 20

Coincidentally,  $\alpha^{-1} \approx \frac{1}{40}$  at GUT scale in SM.

# F-theory axiverse

So far, the string axiverse. Now, a work in progress: F-theory.

F-theory = type IIB string theory with strong + varying coupling

Compactify F-theory on a  $CY_4$  that is a fibration over a base  $B_3$ .

In general  $B_3$  is **not** a  $CY_3$ .

Consequence: much wider range of possible topologies.

String theory on  $CY_3$ :  $N \leq 491$ .

F-theory on  $CY_4$ :  $N \leq 181,820$ . Wang 2020

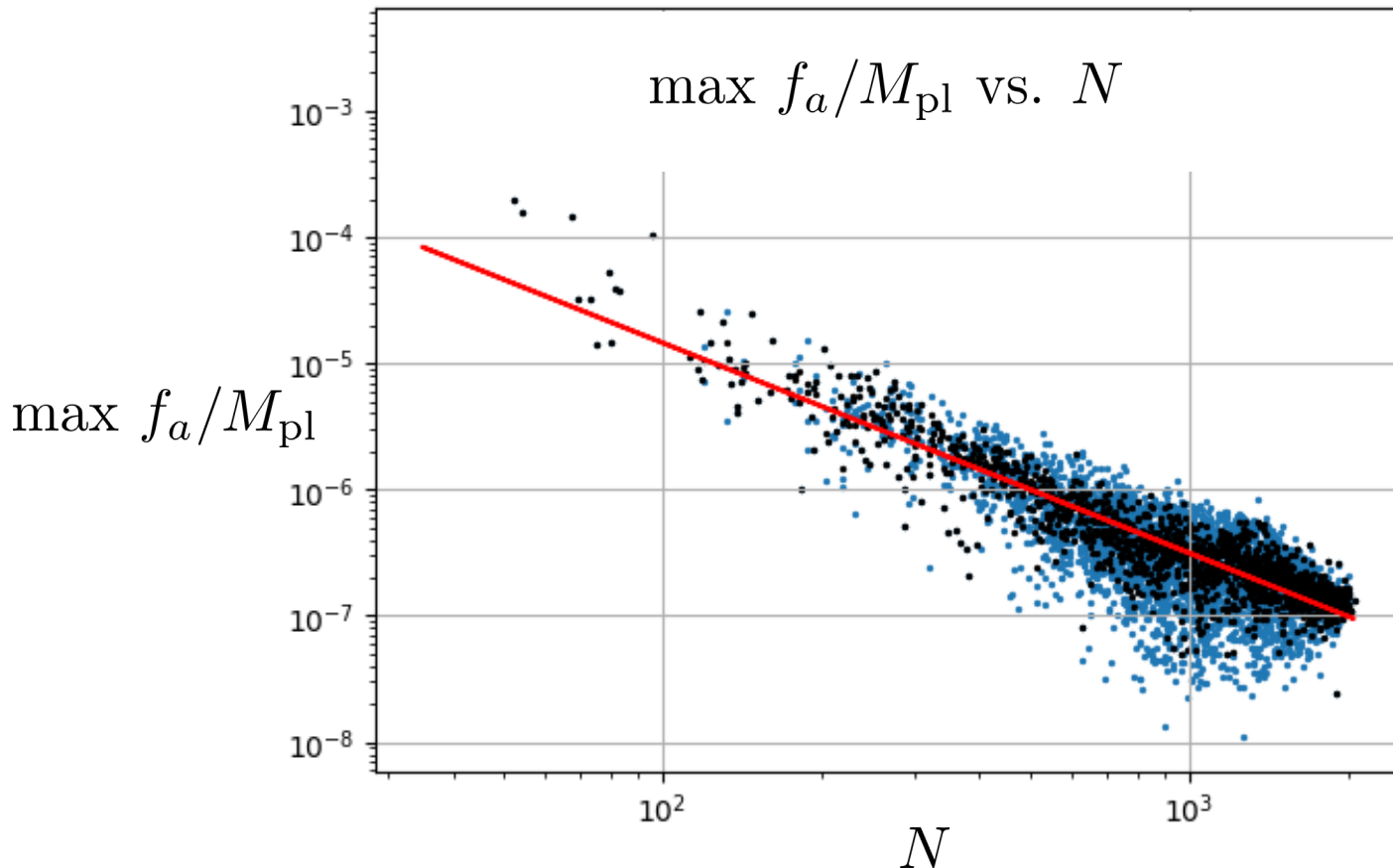
best-understood ensemble:  $N \leq 2,591$  Halverson, Long, Sung 17

# F-theory axiverse

‘Tree’ ensemble, of toric threefold bases.  $N \leq 2,591$

*cf.* Halverson, Long, Sung 17

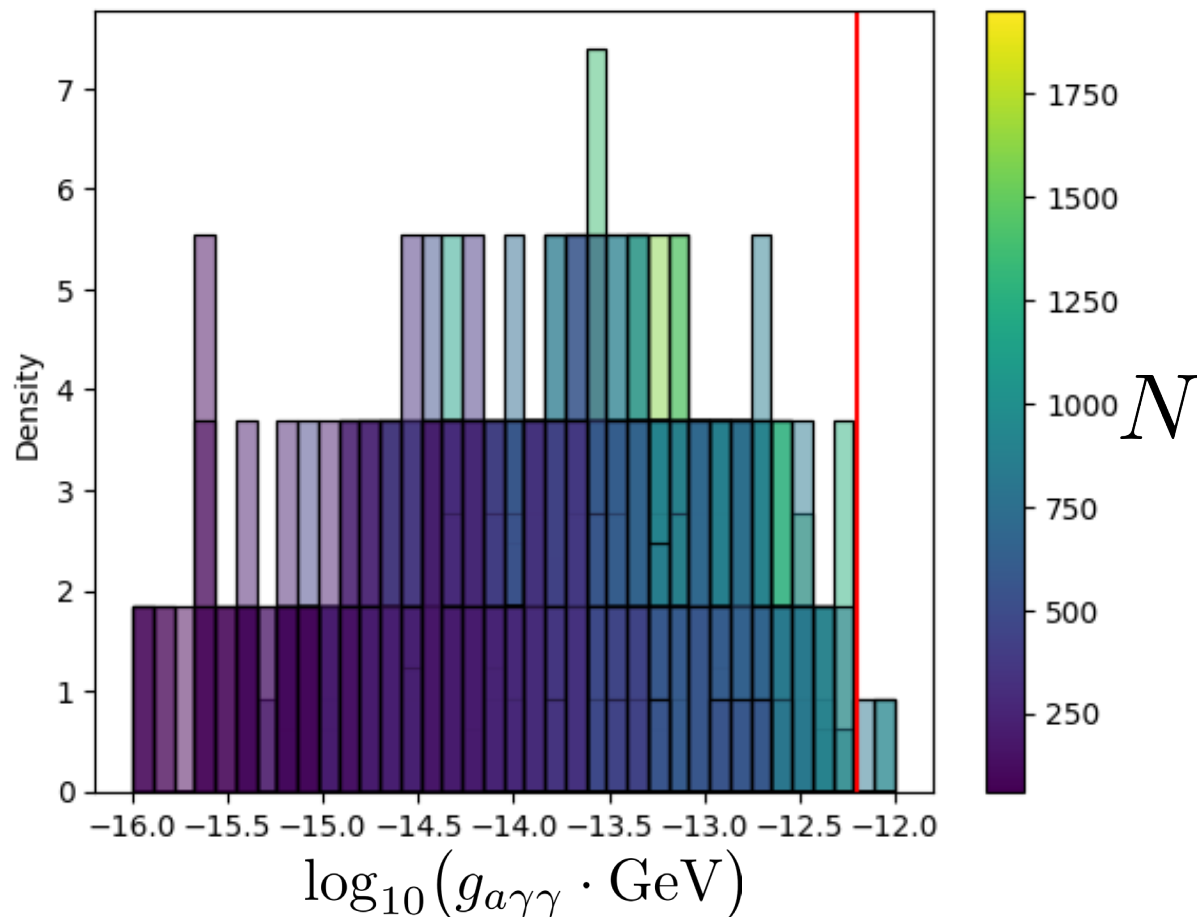
Result: trends seen in  $CY_3$  ( $N \leq 491$ ) continue for  $N \leq 2,591$ .



# F-theory axiverse

Result:  $g_{a\gamma\gamma}$  increases with  $N$ .

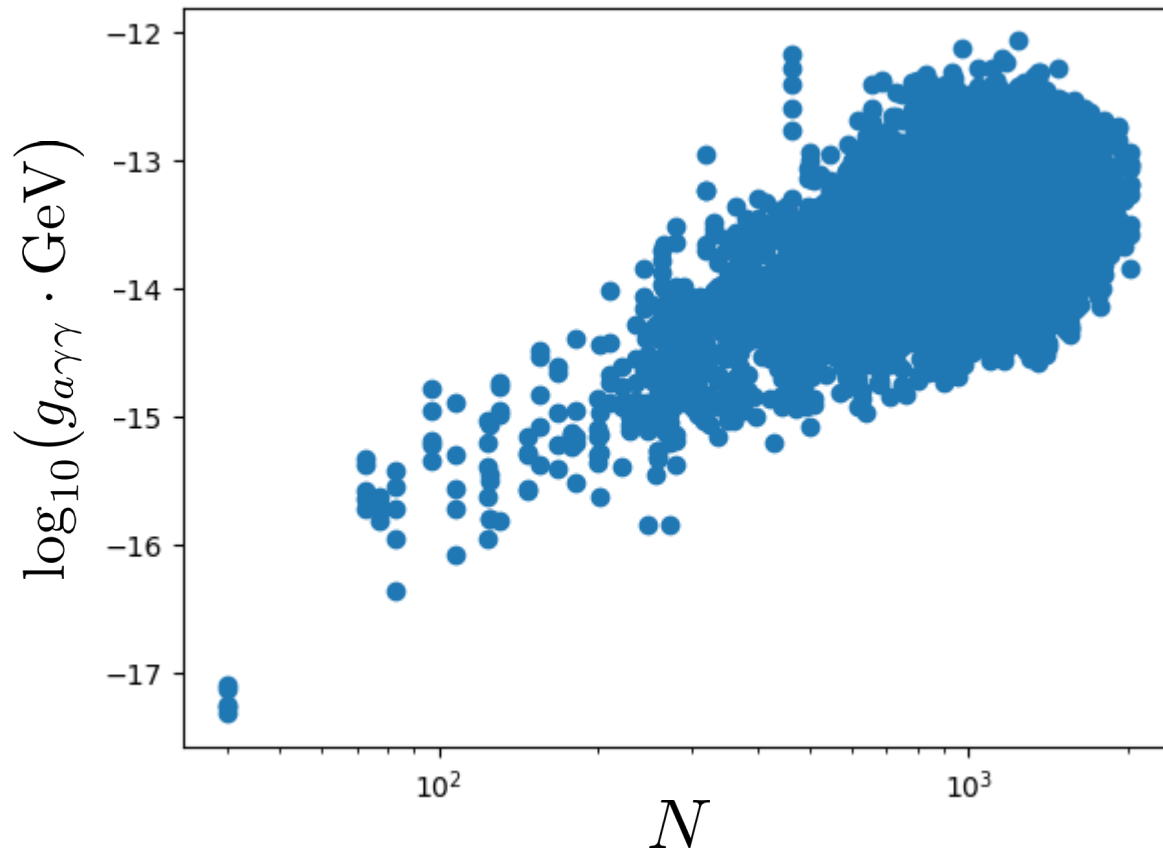
Here: **preliminary** first sample up to  $N \leq 2,591$ .



# F-theory axiverse

Result:  $g_{a\gamma\gamma}$  increases with  $N$ .

Here: **preliminary** first sample up to  $N \leq 2,591$ .



Models exist up to  $N = 181,820$ .

# Conclusions and Outlook

Advances in computational geometry have allowed us to construct parts of the string axiverse, with  $N \gg 1$ .

Geometric hierarchies from control of curvature expansion lead to hierarchies of couplings in the axion EFT.

Expect progress in:

- explicit SM constructions and couplings
- other string theories, other compactifications
- F-theory, with  $N$  up to 180,000
- moduli stabilization
- models of inflation and reheating
- detailed constraints



Thanks!



# Axion couplings

$$N \text{ axions: } \theta^a := \int_{a^{th} \text{ 4-cycle, } \Sigma_a} C_4 \quad a = 1, \dots, N$$

$$\mathcal{L}_{\text{axion}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{gauge}}$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2} \mathcal{K}_{ab} \partial \theta^a \partial \theta^b$$

metric  $\mathcal{K}_{ab}$ : volumes,  
intersection numbers

$$\mathcal{L}_{\text{pot}} = - \sum_{\text{instantons, } I} \Lambda_I^4 \left[ 1 - \cos(Q^I_a \theta^a) \right]$$

scales  $\Lambda_I$ : 4-cycle volumes  
charges  $Q^I_a$ : topology

$$\mathcal{L}_{\text{gauge}} = \sum_{\text{gauge groups, } A} C_a^A \theta^a (F_{\mu\nu} \tilde{F}^{\mu\nu})_A$$

gauge groups: D-branes  
couplings  $C_a^A$ : topology

Computable in terms of topology and point in moduli space.

# Turn the crank

1. Construct ensemble of  $CY_3$  hypersurface topologies.
2. Sample moduli space inside stretched Kähler cone.
3. Specify (or model) D-brane configuration.
4. Identify contributing instantons.
5. Compute axion couplings.
6. Express in terms of mass+kinetic eigenstates.

# Canonical axion couplings

$$\mathcal{L}_{\text{axion}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{gauge}} + \dots$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2} \sum_a (\partial \varphi^a)^2$$

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \sum_a m_a^2 (\varphi^a)^2$$

$$m_a^2 = \frac{\Lambda_a^4}{f_a^2}$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \sum_{\text{groups, A}} g_{a\gamma^A\gamma^A} \varphi^a (F_{\mu\nu} \tilde{F}^{\mu\nu})_A$$

# Fuzzy axion abundance

