Vortices and rotating solitons in ultralight dark matter

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arXiv: 2501.02297, 2502.12100

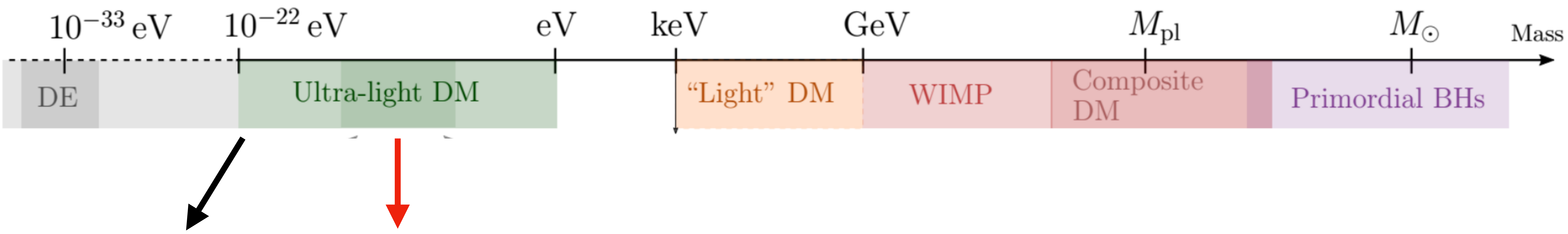
Axions in Stockholm, July 2nd, 2025

Galaxy-scale dynamics:

Formation of DM halos with a flat core

R. Galazo Garcia et al., 2024, arXiv: 2304.1022

I- ULTRA-LIGHT MATTER



FuzzyDM   This talk: SFDM, ALPs $10^{-22}\,\mathrm{eV} < m < 1\,\mathrm{eV}$

$$S_\phi = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right].$$

$$V(\phi) = \frac{1}{2}m^2\phi^2 + V_{\rm I}(\phi)$$

Dominant quadratic term ➡ behaves like dark matter: $\bar{\rho} \propto a^{-3}$

$$V_{\rm I}(\phi) = \frac{\lambda_4}{4}\phi^4$$

Small repulsive quartic self-interaction ➡ important on small-scales, gives an effective pressure $P \propto \rho^2$

J. Fan, 2016

➡ Contrary to the FuzzyDM case, we assume that on small scales we are in the Thomas-Fermi regime: the self-interaction pressure dominates over the quantum pressure.

The de Broglie wavelength is much smaller than the size of the system.

II- NON-RELATIVISTIC REGIME

On the scale of the galactic halo we are in the nonrelativistic regime: the frequencies and wave numbers of interest are much smaller than m and the metric fluctuations are small. We can also neglect the Hubble expansion on galactic scales.

A) From Klein-Gordon eq. to Schrödinger eq.:

Decompose the real scalar field ϕ in terms of a complex scalar field ψ

$$\phi = \frac{1}{\sqrt{2m}}(e^{-imt}\psi + e^{imt}\psi^\star)$$

factorizes (removes) the fast oscillations of frequency m

$\psi(x,t)$ evolves slowly, on astrophysical or cosmological scales. $\dot{\psi} \ll m\psi, \quad \nabla\psi \ll m\psi$

Instead of the Klein-Gordon eq., it obeys a (non-linear) Schrödinger eq.:

$$i\dot{\psi} = -\frac{\nabla^2\psi}{2m} + m(\Phi_{\rm N} + \Phi_{\rm I})\psi$$

Newtonian gravity

Self-interactions

Which is complemented by the Poisson equation and the expression of the self-interaction pressure:

$$\nabla^2\Phi_{\rm N} = 4\pi\mathcal{G}\rho \qquad \Phi_I = \frac{\rho}{\rho_a}$$

$$\rho = m|\psi|^2$$

$$\rho_a = \frac{4m^4}{3\lambda_4}$$

B) From Schrödinger eq. to Hydrodynamical eqs (Madelung transformation):

Madelung 1927, Chavanis 2012, ...

One can map the Schrödinger eq. to hydrodynamical eqs.: $\psi = \sqrt{\frac{\rho}{m}} e^{is}$ $\vec{v} = \frac{\nabla s}{m}$

The real and imaginary parts of the Schrödinger eq. lead to the continuity and Euler eqs.:

$$\dot{\rho} + \nabla \cdot (\rho \vec{v}) = 0$$

conservation of probability for ψ → conservation of matter for ρ

$$\dot{\vec{v}} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla(\Phi_{\rm Q} + \Phi_{\rm N} + \Phi_{\rm I})$$

Self-interactions

$$\Phi_{\rm I} = \frac{\rho}{\rho_a}$$

effective pressure $P_{\rm eff} \propto \rho^2$

$\gamma = 2$

« quantum pressure » $\Phi_{\rm Q} = -\frac{\nabla^2 \sqrt{\rho}}{2m^2 \sqrt{\rho}}$

comes from part of the kinetic terms in ψ

In the following, we neglect the « quantum pressure » (which dominates for FDM) large-*m* limit

III- SOLITON (boson star, ground state): HYDROSTATIC EQUILIBRIUM

As compared with CDM, the self-interactions allow the formation of hydrostatic equilibrium solutions, with a balance between gravity and the effective pressure:

$$\dot{\vec{v}} + (\vec{v}\cdot\nabla)\vec{v} = -\nabla(\Phi_{\rm Q} + \Phi_{\rm N} + \Phi_{\rm I}) \rightarrow \nabla(\Phi_{\rm N} + \Phi_{\rm I}) = 0 \rightarrow \rho(r) = \rho_0 \frac{\sin(r/r_a)}{(r/r_a)}$$

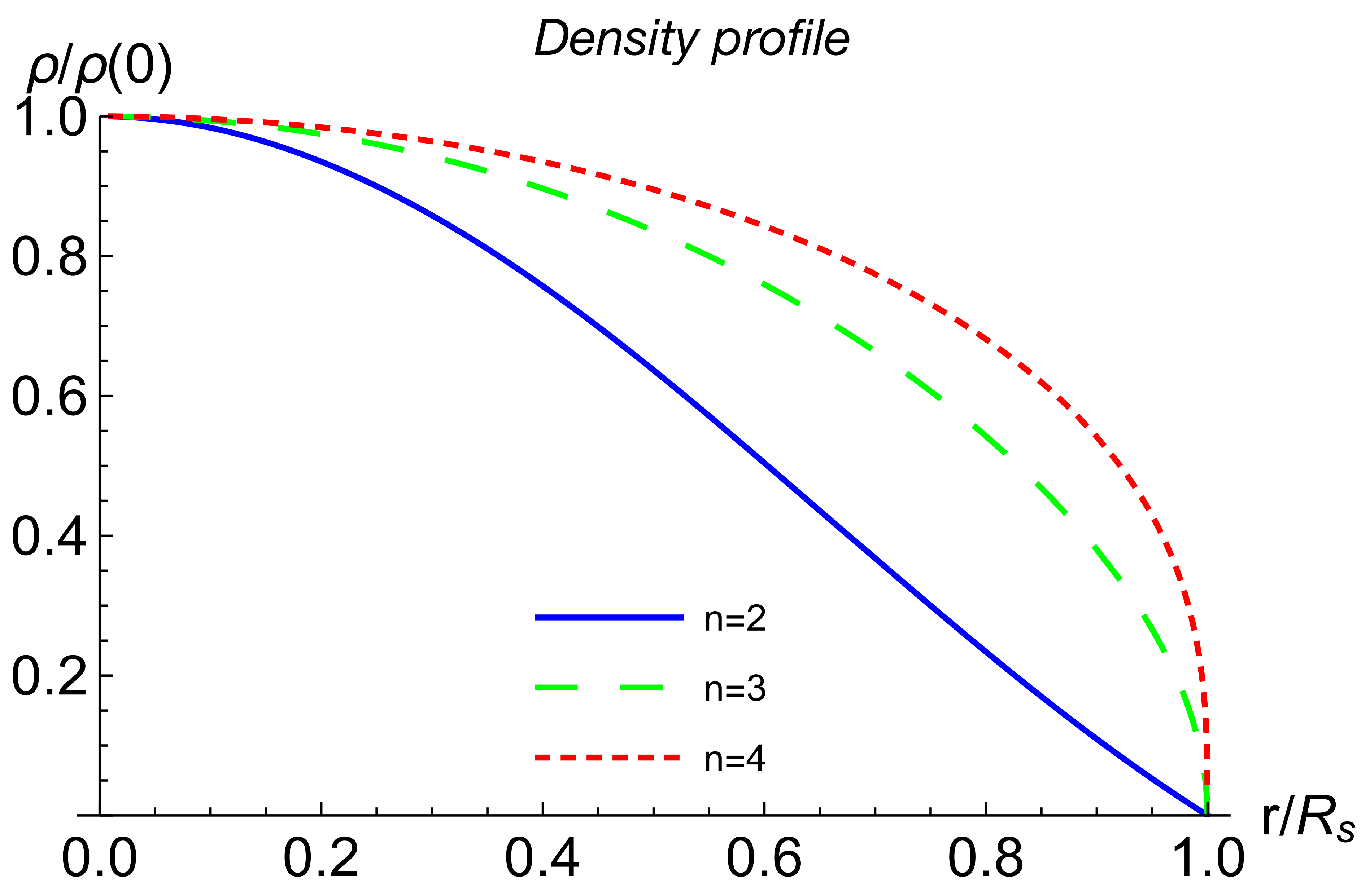
$$R_{\rm sol} \simeq \pi r_a$$

→ Finite-size halo, called « soliton » or « boson star »

$$\rho_a = \frac{4m^4}{3\lambda_4}, \quad r_a = \frac{1}{\sqrt{4\pi\mathcal{G}\rho_a}}$$

P. Brax, J. Cembranos, PV, 1906.00730

Ruffini and Bonazolla 1969,
Chavanis 2011,
Schiappacasse and Hertzberg 2018, ...



$$V_{\rm I}(\phi) = \Lambda^4 \frac{\lambda_{2n}}{2n}\frac{\phi^{2n}}{\Lambda^{2n}}$$

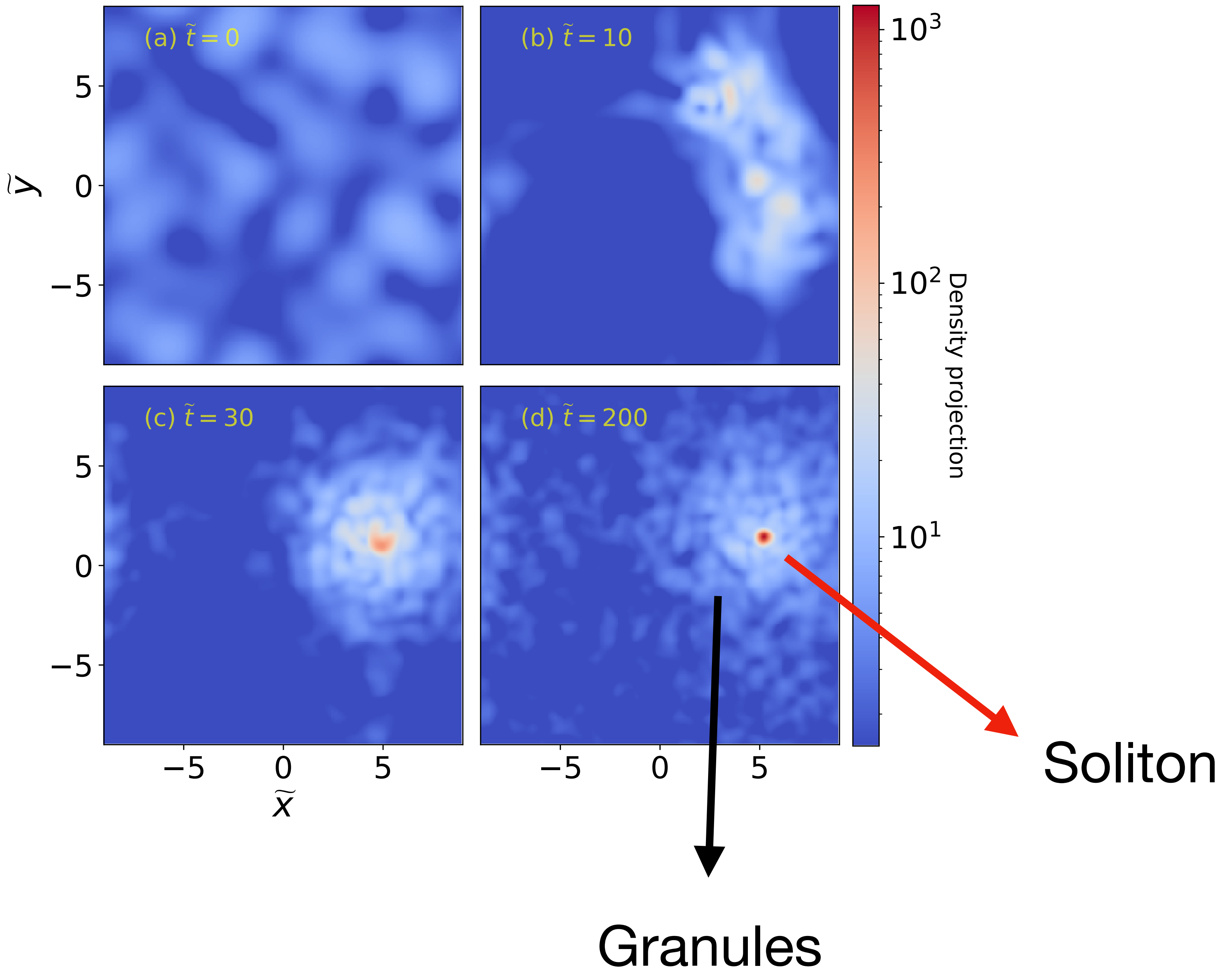
$m \gg 10^{-18}\mathrm{eV}$: galactic soliton governed by the balance between the repulsive self-interaction and self-gravity.

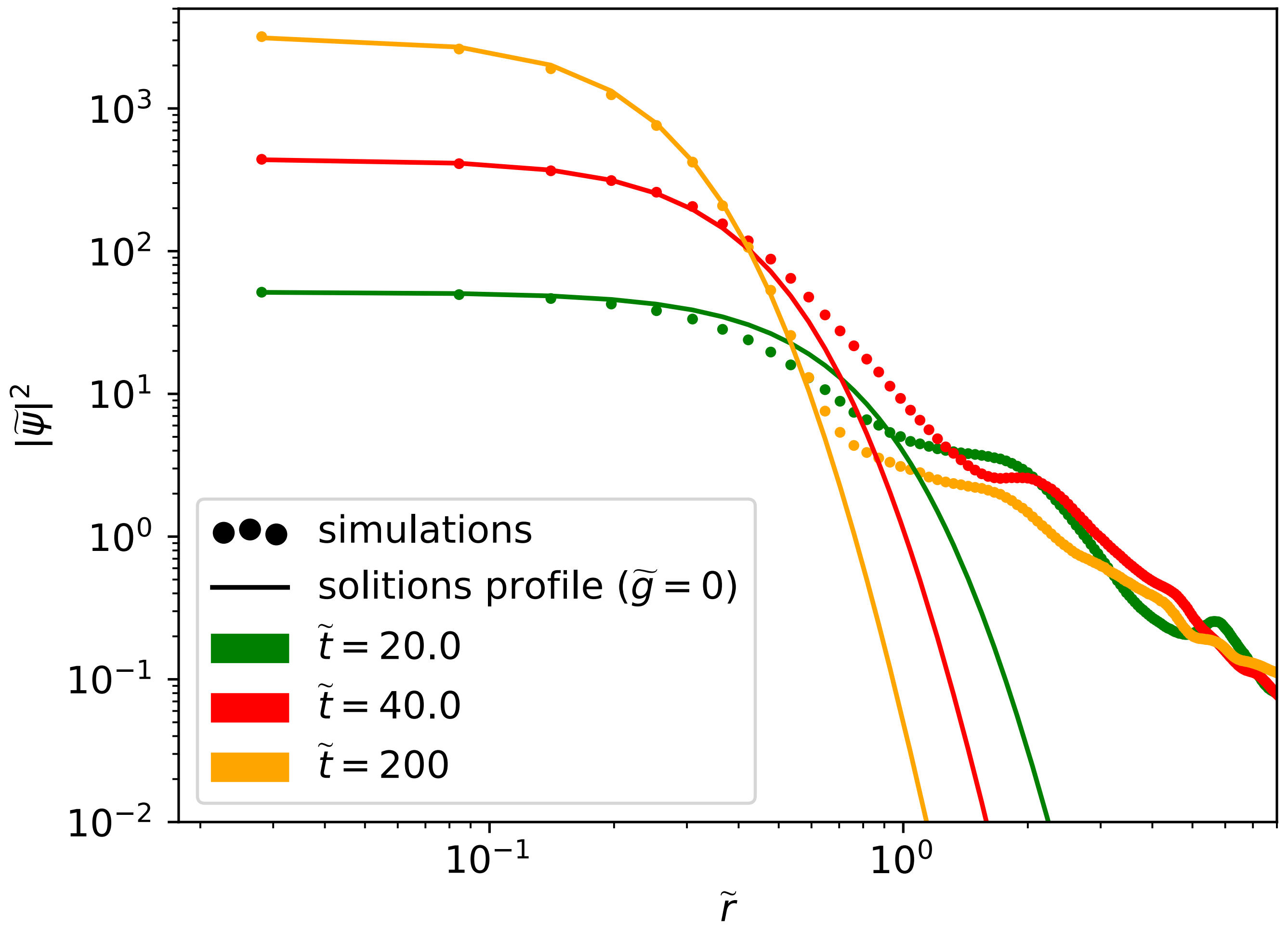
$m \sim 10^{-21}\mathrm{eV}$: Fuzzy Dark Matter (de Broglie wavelength of galactic size): galactic soliton governed by the balance between the quantum pressure and self-gravity.

Numerical simulations of FDM indeed find that solitons form, from gravitational collapse, within an extended NFW-like out-of-equilibrium halo.

Chen et al. 2020

Schive et al. 2014,
Veltmaat et al. 2018,
Mocz et al. 2019,
Amin and Mocz 2019, ...





IV- SOLITON FORMATION IN THE THOMAS-FERMI REGIME

(Self-interactions dominate over the quantum pressure in the soliton)

A) Numerical simulations

Initial conditions: halo (+ central soliton): $\psi_{\rm initial} = \psi_{\rm sol} + \psi_{\rm halo}$ $\quad \rho_{\rm sol}(r) = \rho_{0\rm sol} \frac{\sin(\pi r/R_{\rm sol})}{\pi r/R_{\rm sol}}, \quad \hat{\psi}_{\rm sol}(r) = \sqrt{\rho_{\rm sol}(r)}$

Stochastic halo: sum over eigenmodes of the target gravitational potential with random coefficients

$$\psi_{\rm halo}(\vec{x},t) = \sum_{n\ell m} a_{n\ell m} \hat{\psi}_{n\ell m}(\vec{x}) e^{-iE_{n\ell}t/\epsilon}$$

$a_{n\ell m} = a(E_{n\ell}) e^{i\Theta_{n\ell m}}$ (random phase)

$$-\frac{\epsilon^2}{2}\nabla^2\hat{\psi}_E + \bar{\Phi}\hat{\psi}_E = E\hat{\psi}_E \qquad \bar{\Phi}(r) = \bar{\Phi}_N(r), \quad \nabla^2\bar{\Phi}_N = 4\pi\bar{\rho}$$

$\langle\rho_{\rm halo}\rangle = \sum_{n\ell m} a(E_{n\ell})^2 |\hat{\psi}_{n\ell m}|^2$ → Choose $a(E)$ so as to recover the target density profile $\rho(r)$

With the WKB approximation we can relate this system to a classical system defined by a phase-space distribution $f(E)$

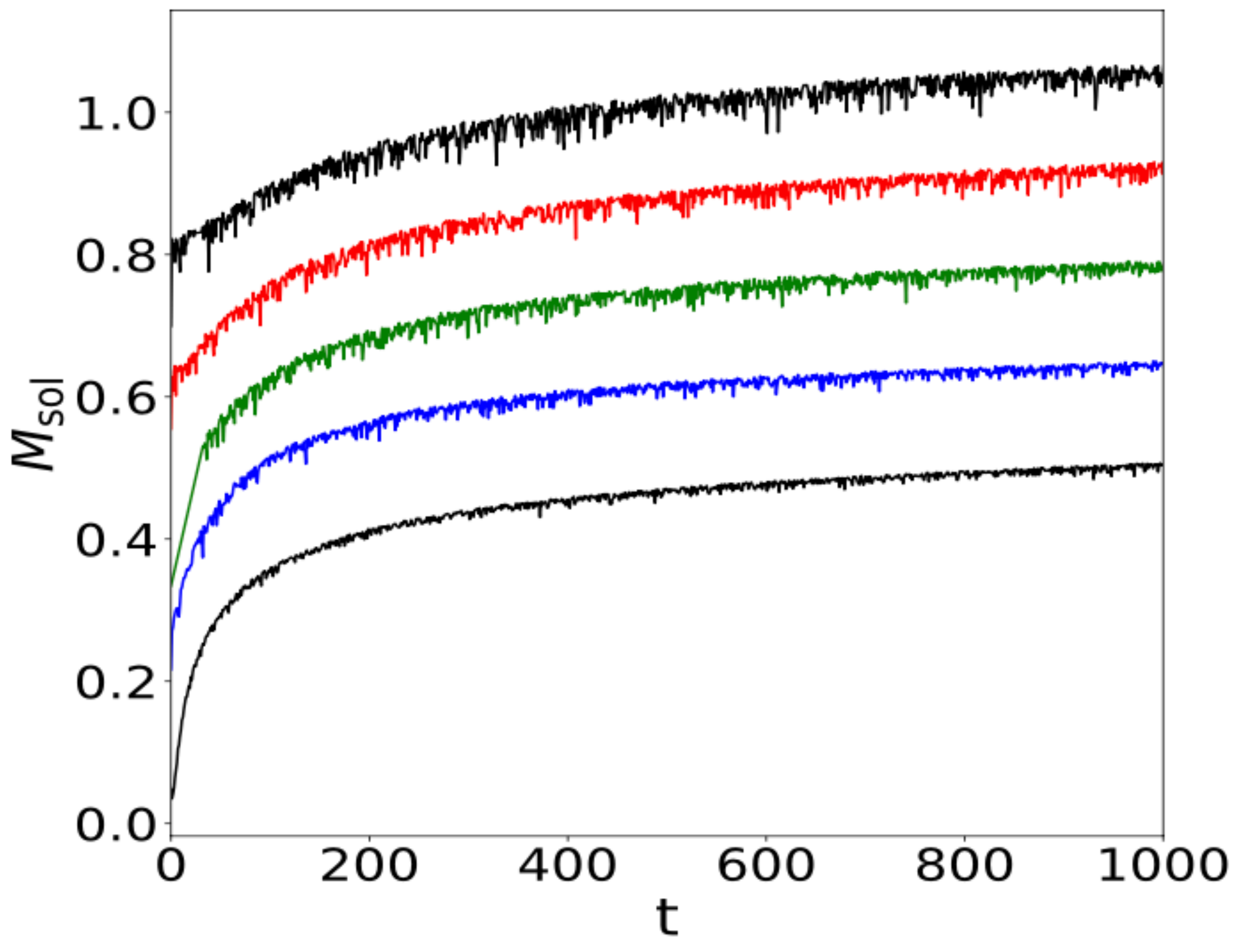
→ take $a(E)^2 = (2\pi\epsilon)^3 f(E)$

$$f(E) = \frac{1}{2\sqrt{2}\pi^2}\frac{d}{dE}\int_E^0 \frac{d\Phi_N}{\sqrt{\Phi_N - E}}\frac{d\rho_{\rm classical}}{d\Phi_N}$$ (Eddington formula)

- At t ~ 8, the soliton is formed with Rsol ~ 0.5 and it contains ~ 50% of the total mass.
- The system reaches a quasi-stationary state.
- Afterwards, the mass of the soliton slowly grows.

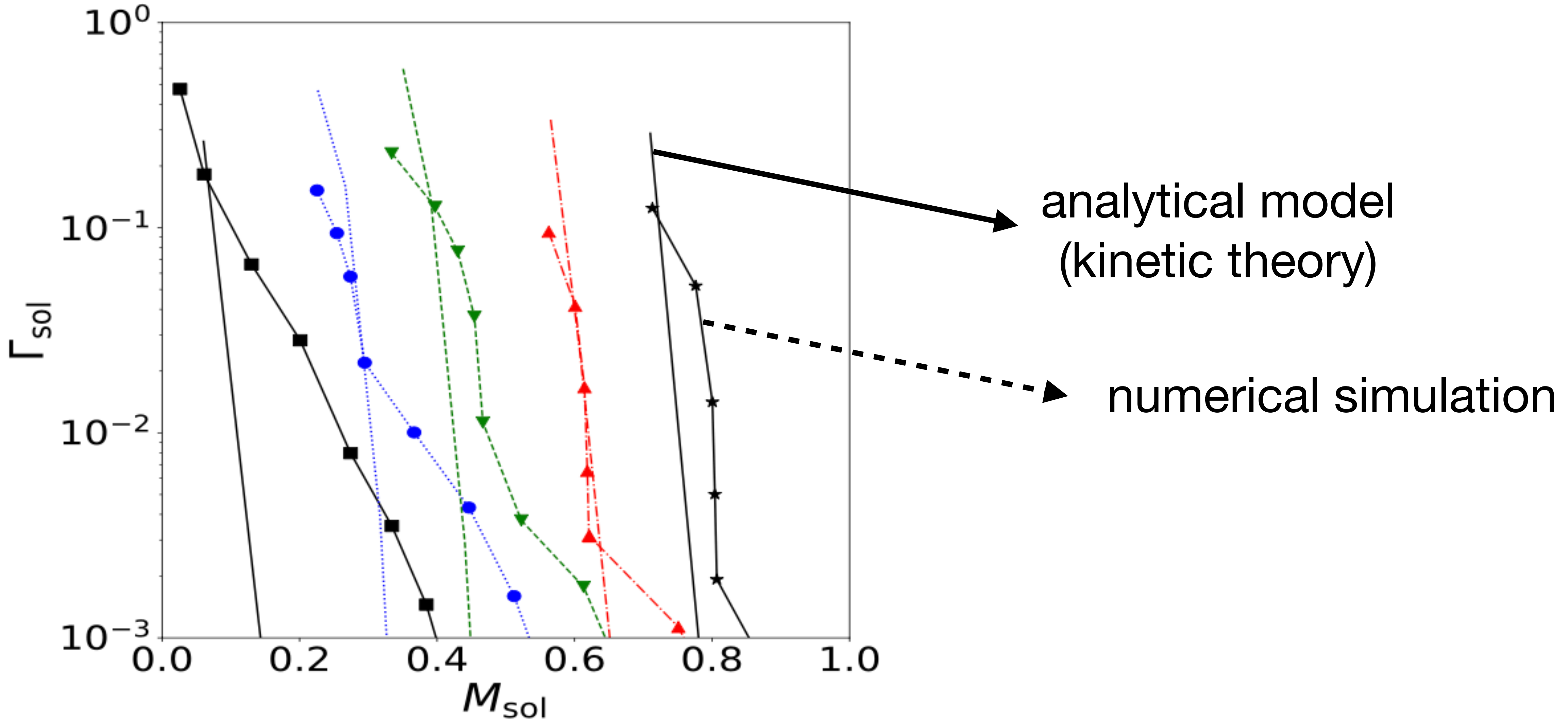
2) Dependence of the soliton mass on the formation history

Growth with time of the soliton mass



Growth rate as a function of the soliton mass, for several initial conditions

$$\Gamma_{\rm sol} = \frac{\dot{M}_0}{M_0}$$



- The soliton always forms and grows, with a growth rate that decreases with time.
- Its mass can reach 50% of the total mass of the system.
- There is no sign of a scaling regime, where the growth rate would be independent of initial conditions.

Probably no well-defined halo-mass/soliton mass relation

Solitons always form at the center of virialized halos.

For large self-interactions, the soliton forms in a few dynamical times.

For small self-interactions, the soliton formation can take a long time, until stochastic density peaks reach densities that are large enough to trigger the formation of the soliton.

The soliton keeps growing until the end of our simulations, making from 10% to 80% of the total mass.

The growth rate of the soliton does not seem to obey a scaling regime.
It seems to depend on the formation history of the system.

In the cosmological context, there should be a large scatter for the soliton mass as a function of the halo mass, depending on the assembly history ?

It is not clear how to derive simple but accurate analytical predictions for the soliton mass.

Vortex lines and rotating soliton

(What happens when a collapsing halo has a nonzero angular momentum)

Ph. Brax and P. Valageas, 2025, arXiv: 2501.02297, 2502.12100

I- Vortices

Nondimensional units (rescaled to the typical size and mass of the system)

$$i\epsilon\frac{\partial\psi}{\partial t} = -\frac{\epsilon^2}{2}\Delta\psi + (\Phi_N + \Phi_I)\psi$$

$$\epsilon = \lambda_{dB}/L$$

Thomas-Fermi regime: $\epsilon \ll 1$

$$\Delta\Phi_N = 4\pi\rho, \quad \Phi_I = \lambda\rho, \quad \rho = |\psi|^2$$

➡ Gross-Pitavskii equation: similar to BEC and superfluids at low temperature, where the external confining potential is replaced by the self-gravity.

Hydrodynamical picture: $\psi = \sqrt{\rho}e^{iS}, \quad \vec{v} = \epsilon\vec{\nabla}S$ ➡ Curl-free velocity field

No longer true if the phase is not regular: at locations where the density vanishes this mapping is ill-defined !

➡ Appearance of vortices/vortex lines that carry the vorticity of the system (BEC, superfluids), associated with singularities of the phase and of the velocity field.

This is observed in cold atoms experiments:

Abo-Shaer et al. 2001

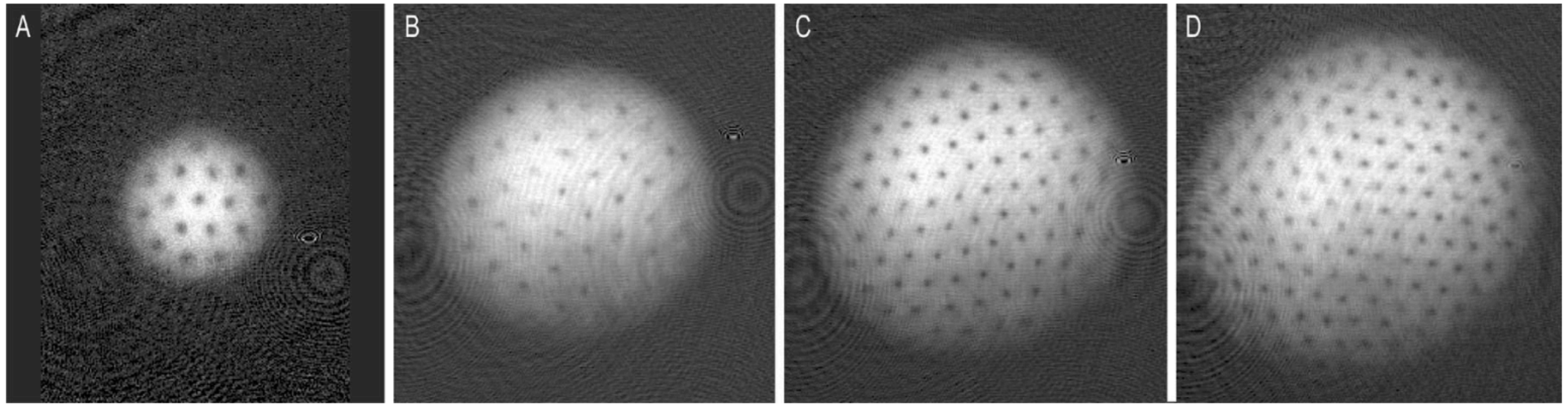


Fig. 1. Observation of vortex lattices. The examples shown contain approximately (**A**) 16, (**B**) 32, (**C**) 80, and (**D**) 130 vortices. The vortices have "crystallized" in a triangular pattern. The diameter of the cloud in (D) was 1 mm after ballistic expansion, which represents a magnification of 20. Slight asymmetries in the density distribution were due to absorption of the optical pumping light.

The vortices correspond to troughs of the density field.

10^7 Na atoms

Thomas-Fermi radius= $29\mu m$

Healing length $\xi = 0.2\mu m$

(ballistic expansion after the trap is switched off)

The spatial distribution of the density is obtained by resonant absorption imaging.

Rotation of the BEC is produced by the dipole force exerted by laser beams.

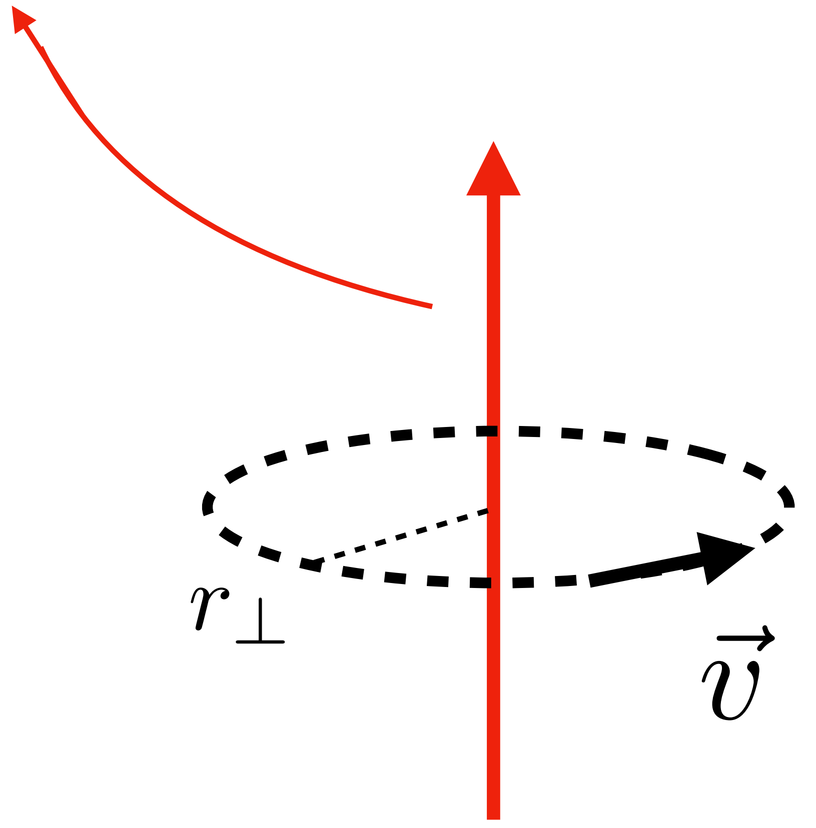
The spatial distribution of the density is obtained by resonant absorption imaging.

One observes a regular lattice of vortices. Such Abrikosov lattices were first predicted for quantised magnetic flux lines in type-II superconductors. Abrikosov 1957

In our case, there is no external container.

The rotation will be generated by the initial rotation of the dark matter halo.

Vortex line aligned with the vertical z-axis of spin σ



$$\psi(\vec{r},t) = e^{-i\mu t/\epsilon}\sqrt{\rho_0} f(r_\perp) e^{i\sigma\varphi}, \quad \rho(\vec{r}) = \rho_0 f^2(r_\perp)$$

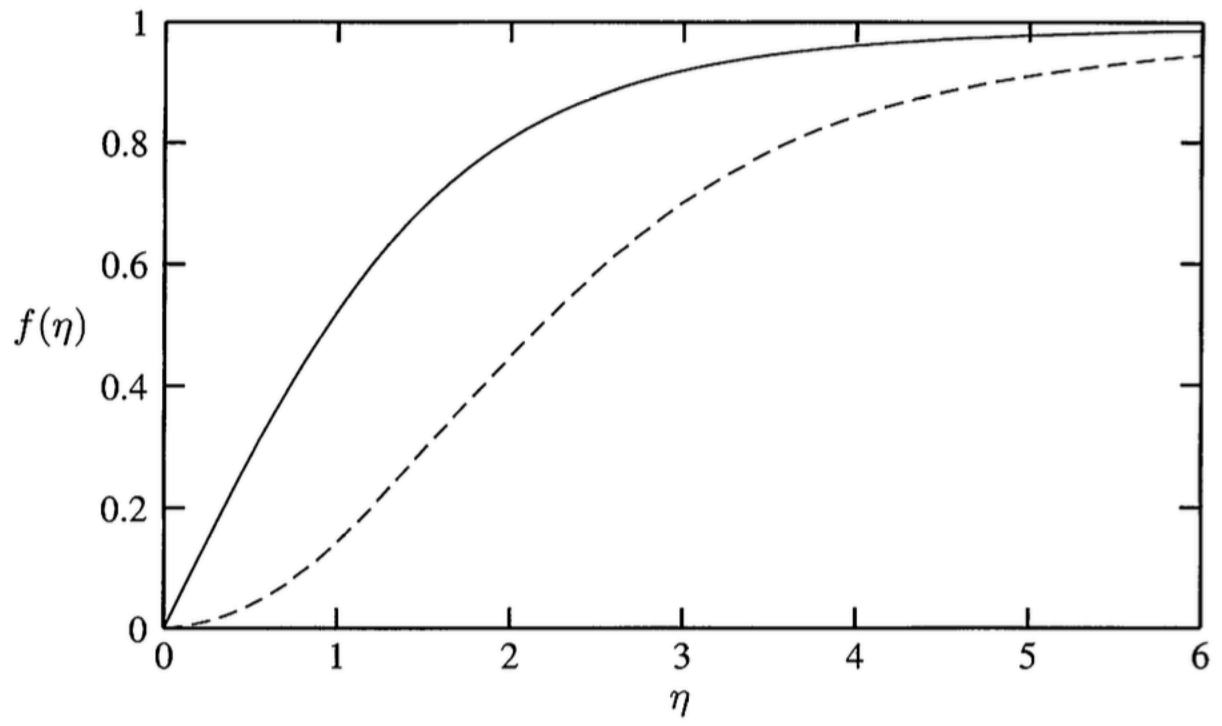


FIG. 5.2. Vortical solutions ($s = 1$, solid line; $s = 2$, dashed line) of the Gross–Pitaevskii equation as a function of the radial coordinate r/ξ. The density of the gas is given by $n(\mathbf{r}) = nf^2$, where n is the density of the uniform gas

Pitaevski 2003

Healing length: $\xi = \dfrac{\epsilon}{\sqrt{2\lambda\rho_0}}$ $\quad r_\perp \ll \xi: \quad f \propto (r/\xi)^{|\sigma|}, \quad r_\perp \gg \xi: \quad f \simeq 1 - \dfrac{\sigma^2\xi^2}{2r^2} + \ldots$

Velocity field: $\vec{v} = \dfrac{\epsilon\sigma}{r_\perp}\vec{e}_\varphi = \epsilon\sigma\dfrac{\vec{e}_z \times \vec{r}_\perp}{r_\perp^2}, \quad v_{r_\perp} = v_z = 0, \quad v_\varphi = \dfrac{\epsilon\sigma}{r_\perp}$

The vorticity is carried by the vortices

Vorticity: $\vec{\omega} = \vec{\nabla} \times \vec{v} = 2\pi\epsilon\sigma\delta_D^{(2)}(\vec{r}_\perp)\vec{e}_z$

The vorticity and circulation are quantized

$$\Gamma(r_\perp) = \oint_C \vec{v}\cdot d\vec{\ell} = \int_S \vec{\omega}\cdot d\vec{S} = 2\pi\epsilon\sigma$$

Excess energy (as compared with the static soliton) for a vortex of spin σ: $\Delta E_\sigma \sim \sigma^2 \pi \rho_0 \epsilon^2 \ln[R_0/(|\sigma|\xi)]$

Excess energy for N_v vortices of unit spin: $\Delta E_{N_v} \sim N_v \pi \rho_0 \epsilon^2 \ln[R_0/\xi] + N_v^2 \frac{\pi}{4} \rho_0 \epsilon^2$

➡ It is energetically favorable for a large-spin vortex to break up into unit-spin vortices.

In the numerical simulations we only find unit-spin vortices.

II- Many vortices

For a collection of N_v vortices: $\psi(\vec{r},t) = \sqrt{\rho}e^{is}\prod_{j=1}^{N_v} e^{i\sigma_j\varphi_j}$ $\varphi_j(\vec{r}) = (\widehat{\vec{e}_x, \vec{r}_\perp - \vec{r}_{\perp j}})$

As in classical hydrodynamics of ideal fluids, the vortices move with the matter along the flow generated by the other vortices and the background curl-free velocity

$$\dot{\vec{r}}_i = \vec{v}(\vec{r}_i), \quad \vec{v} = \epsilon\vec{\nabla}s + \sum_{j=1}^{N_v}\vec{v}_j$$

$$\vec{v}_j(\vec{r}) = \epsilon\sigma_j\vec{e}_z \times \frac{\vec{r}_\perp - \vec{r}_{\perp j}}{|\vec{r}_\perp - \vec{r}_{\perp j}|^2}$$

The system is again described by the continuity and Euler equations, but the velocity field is no longer curl-free:

$$\vec{\omega} = \vec{\nabla}\times\vec{v} = 2\pi\epsilon\vec{e}_z\left(\sum_j \sigma_j\delta_D^{(2)}(\vec{r}_\perp - \vec{r}_{\perp j})\right)$$

III- Continuum limit

The Gross-Pitaevskii equation conserves the mass, the momentum and the energy, as well as the angular momentum.

Rotating soliton: we look for a minimum of the energy at fixed mass and angular momentum: $\delta^{(1)}\left(E-\mu M-\Omega J_z\right)=0$

Lagrange multipliers

→ Solid-body rotation: $\vec{v}=\vec{\Omega}\times\vec{r}$ $\qquad \vec{\Omega}=\Omega\,\vec{e}_z$

At leading order for a slow rotation, we obtain the density profile and the soliton surface:

$$\rho(r,\theta)=\left(\rho_0-\frac{\Omega^2}{2\pi}\right)j_0\left(\frac{\pi r}{R_0}\right)-\frac{5\pi\Omega^2}{12}j_2\left(\frac{\pi r}{R_0}\right)P_2(\cos\theta)+\frac{\Omega^2}{2\pi},$$

$$R_\Omega(\theta)=R_0\left(1+\frac{\Omega^2}{2\pi\rho_0}\right)-R_0\frac{5\Omega^2}{4\pi\rho_0}P_2(\cos\theta)$$

→ Oblate shape: $R_z=R_0\left(1-\frac{3\Omega^2}{4\pi\rho_0}\right)$ $\qquad R_{xy}=R_0\left(1+\frac{9\Omega^2}{8\pi\rho_0}\right)$ $\qquad R_{xy}>R_z$

→ Dynamical stability for $|\Omega|\lesssim\sqrt{\rho_0}$ $\qquad \Phi_{\rm rot}\lesssim\Phi_N\sim\Phi_I$

IV- Numerical simulations

A) Initial conditions

Initial conditions: virialized collisionless halo = sum over eigenmodes of the target gravitational potential with coefficients with a random phase.

$$\psi(\vec{r}) = \sum_{n\ell m} a_{n\ell m} \hat{\psi}_{n\ell m}(\vec{r}), \quad \hat{\psi}_{n\ell m}(\vec{r}) = \mathcal{R}_{n\ell}(r) Y_\ell^m(\theta,\varphi)$$

$$a_{n\ell m} = |a_{n\ell m}| e^{i\Theta_{n\ell m}}$$

random phase

$$\left[-\frac{\epsilon^2}{2} \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} \right) + \frac{\epsilon^2 \ell(\ell+1)}{2r^2} + \Phi_N \right] \mathcal{R}_{n\ell} = E_{n\ell} \mathcal{R}_{n\ell}$$

$$\langle \rho \rangle = \langle |\psi|^2 \rangle = \sum_{n\ell m} |a_{n\ell m}|^2 \mathcal{R}_{n\ell}^2 |Y_\ell^m|^2$$

→ Choose $|a_{n\ell m}|$ so as to recover the target density profile $\rho(r)$

With the WKB approximation we can relate this system to a classical system defined by a phase-space distribution

→ Take $|a_{n\ell m}|^2 = (2\pi\epsilon)^3 f(E_{n\ell}, L_z), \quad L_z = \epsilon m,$

For instance: $f(E, L_z) = f_0(E) + f_-(E, L_z)$ with $f_-(E, L_z)$ odd over L_z

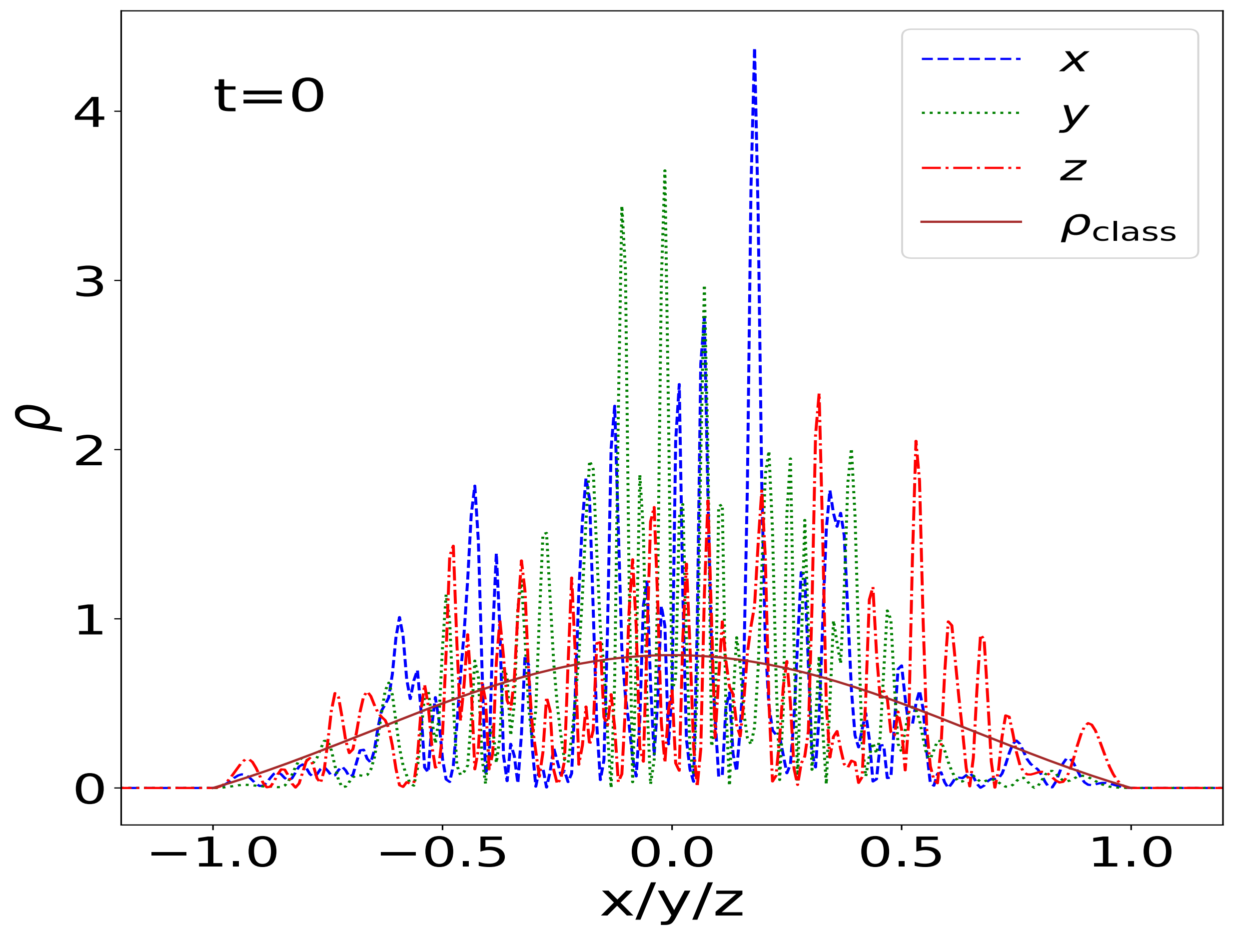
$$\langle \rho(r) \rangle = 4\pi \int_{\Phi_N(r)}^{\infty} dE \sqrt{2(E - \Phi_N(r))} f_0(E)$$

$$\langle J_z \rangle = 16\pi^2 \int_0^R dr\, r \int_{\Phi_N}^{\infty} dE \int_0^{r\sqrt{2(E-\Phi_N)}} dL_z\, L_z\, \text{Arccos}\left(\frac{L_z}{r\sqrt{2(E-\Phi_N)}} \right) f_-(E, L_z)$$

Initial angular momentum and rotation

$J_{z\mathrm{init}} \simeq 0.17$

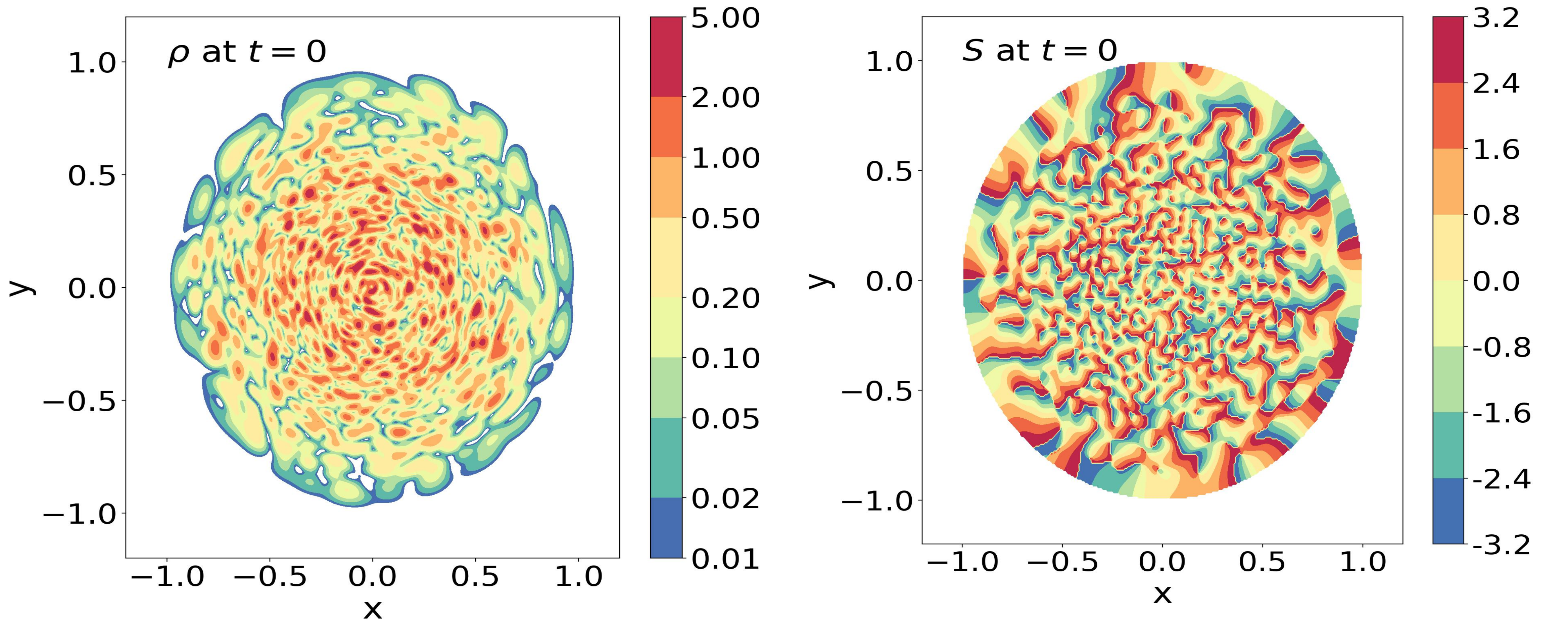
Density along the x/y/z axis



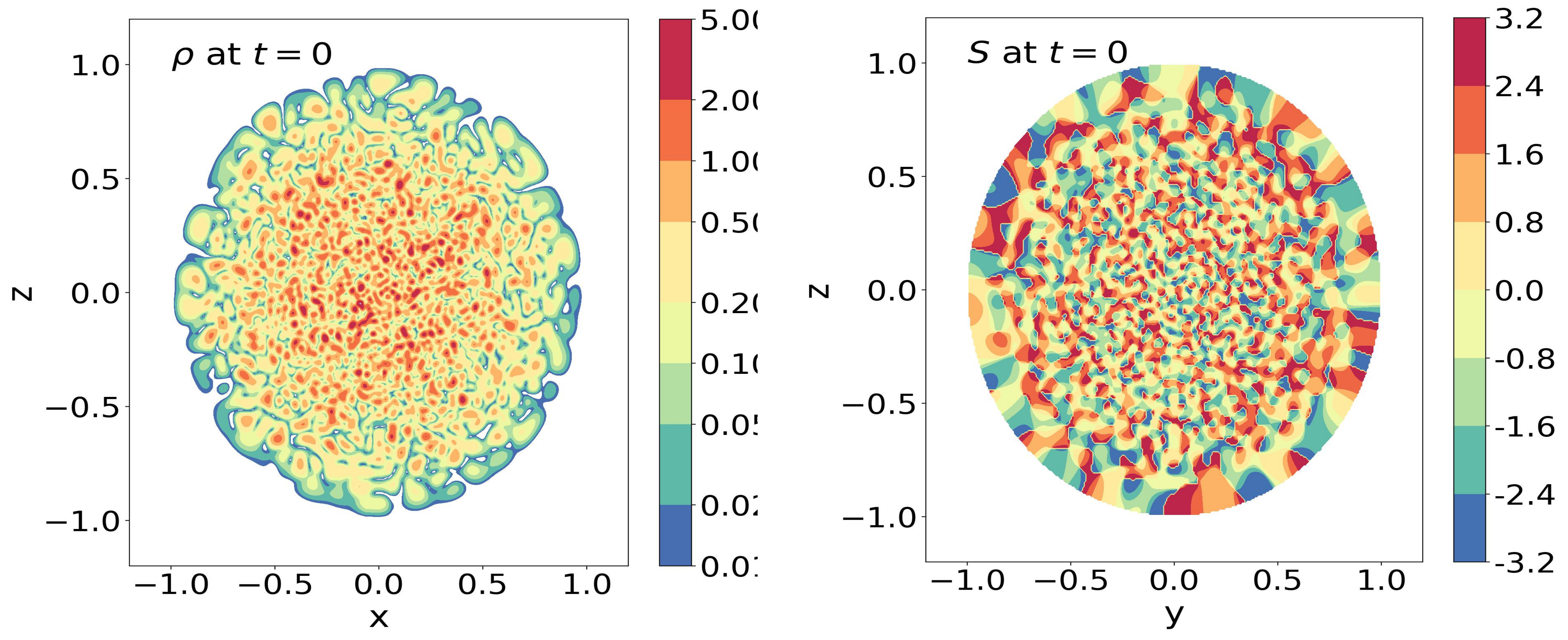
Density

Phase

Equatorial (x,y) plane



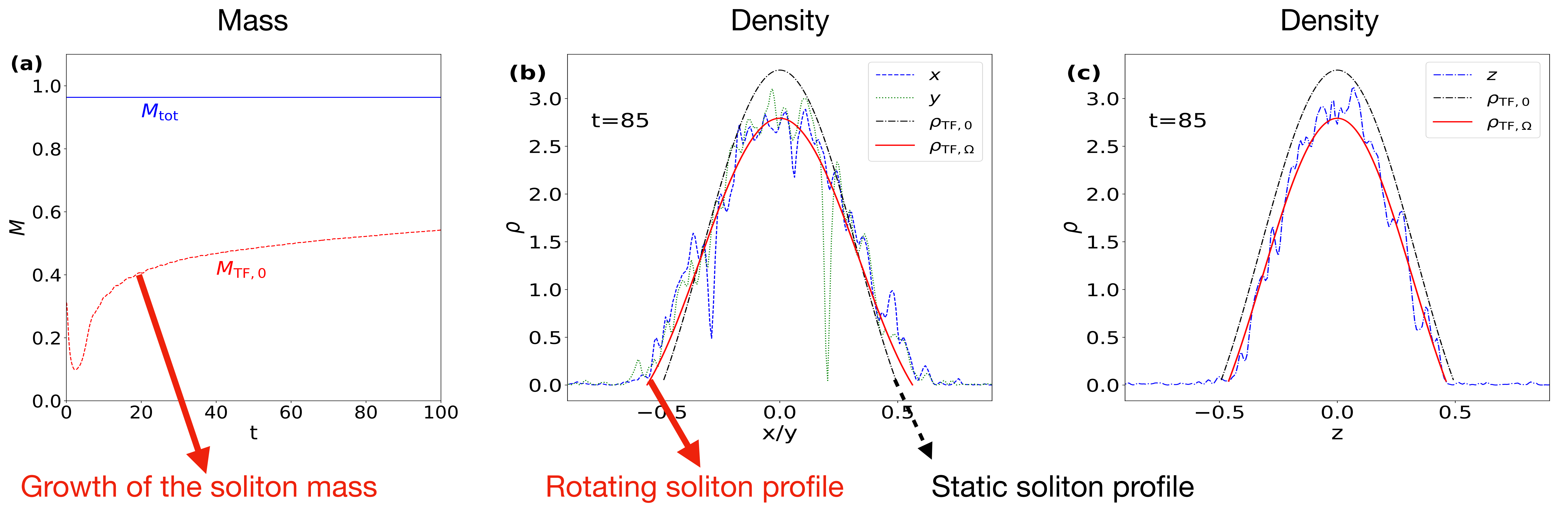
Vertical (x,z) plane



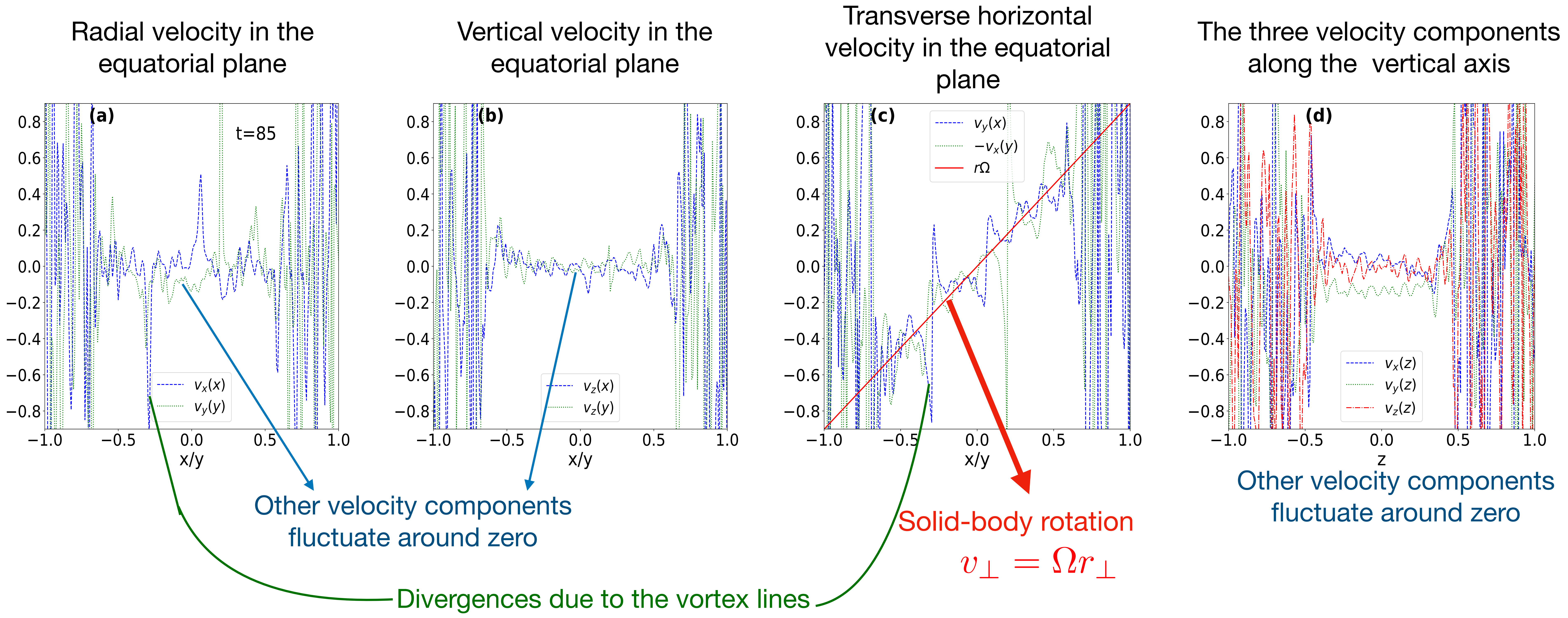
B) Formation of a rotating soliton in a few dynamical times

Mass and Density

Oblate shape



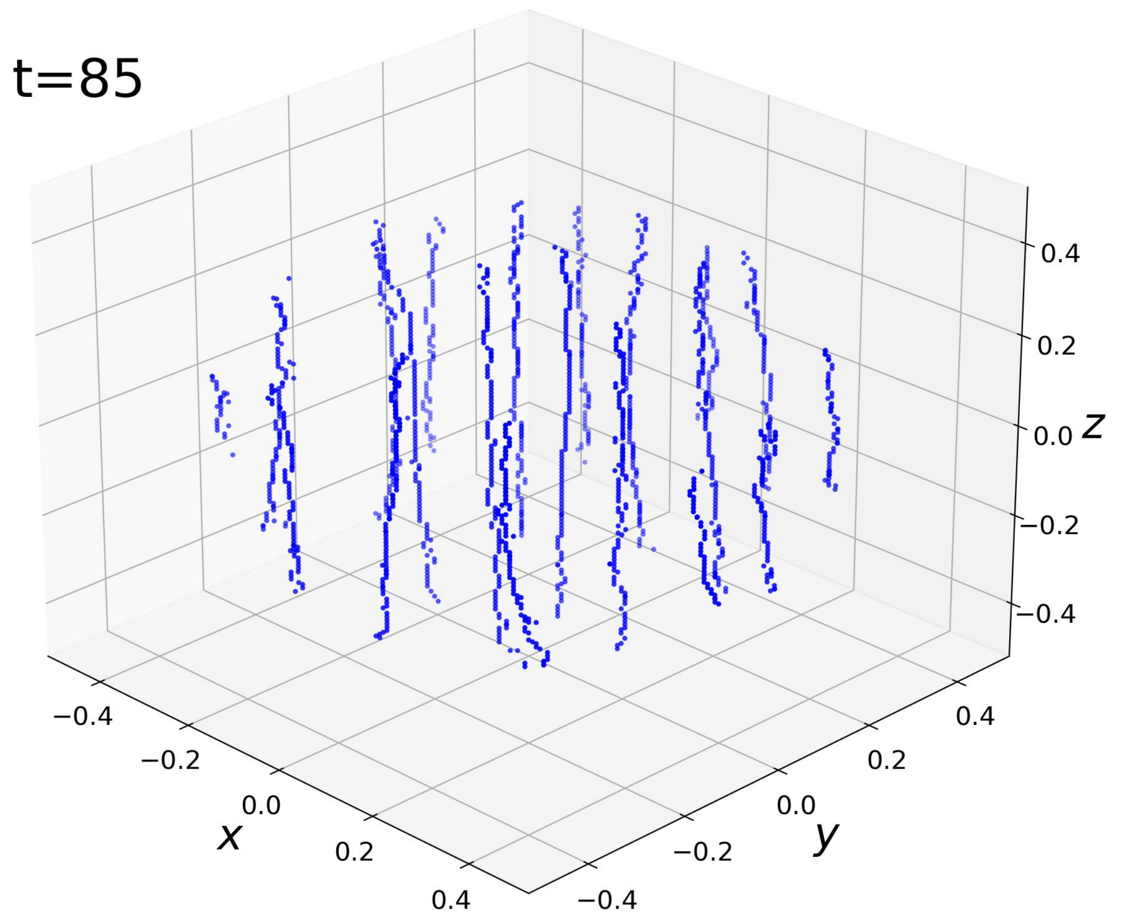
Velocity



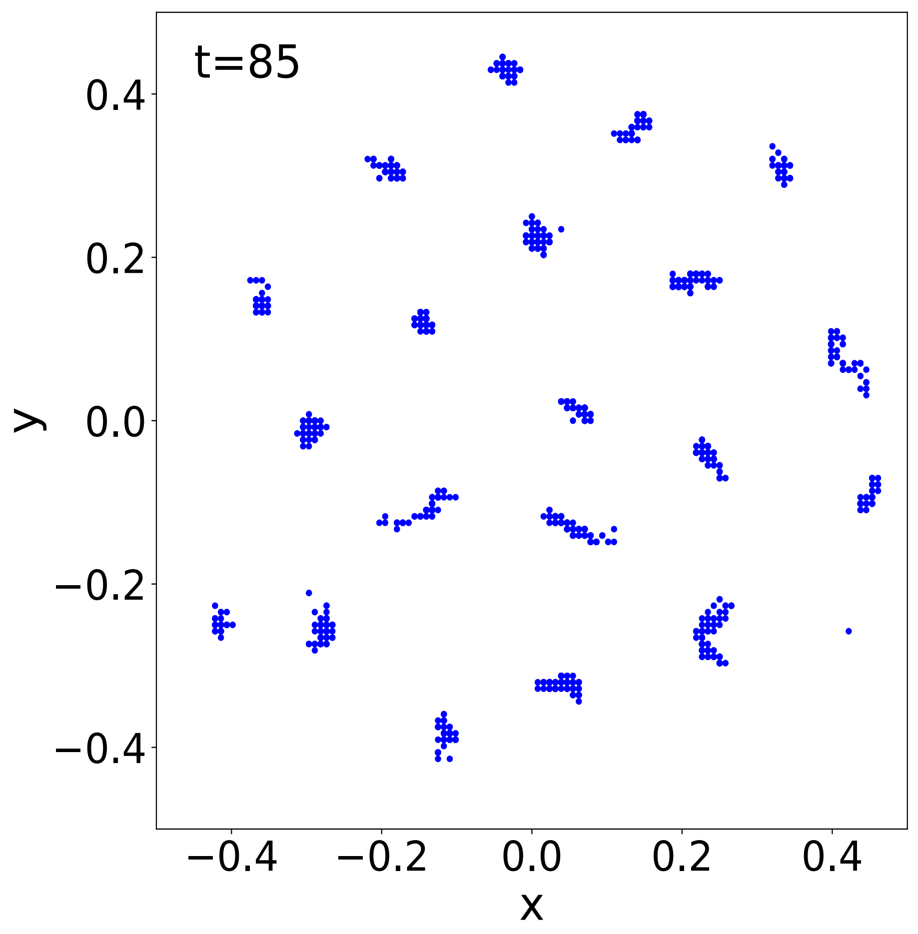
Circular section of the soliton
Lattice of velocity troughs
Density
Phase
Winding number
Random vortices of +/- spin in the outer virialized halo
Regular lattice of vortex lines inside the soliton
Equatorial (x,y) plane
Vertical (x,z) plane
Divergent velocity on the vortices
Solid-body rotation inside the soliton
Normalised velocity
Streamlines
Velocity amplitude
Oblate shape of the rotating soliton
Trace of a vertical vortex line
Equatorial (x,y) plane
Velocity fluctuates around zero inside the vertical plane
Vertical (x,z) plane

C) Formation of a lattice of vertical vortex lines

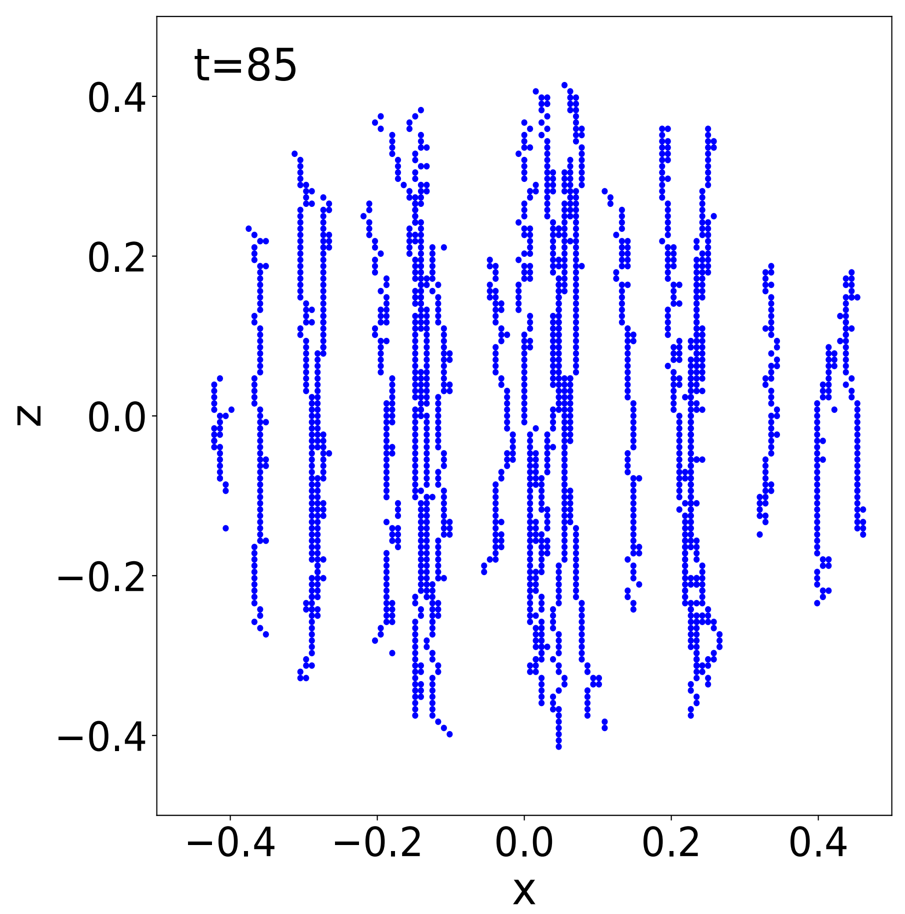
Vertical vortex lines inside the soliton



Projection onto the equatorial plane

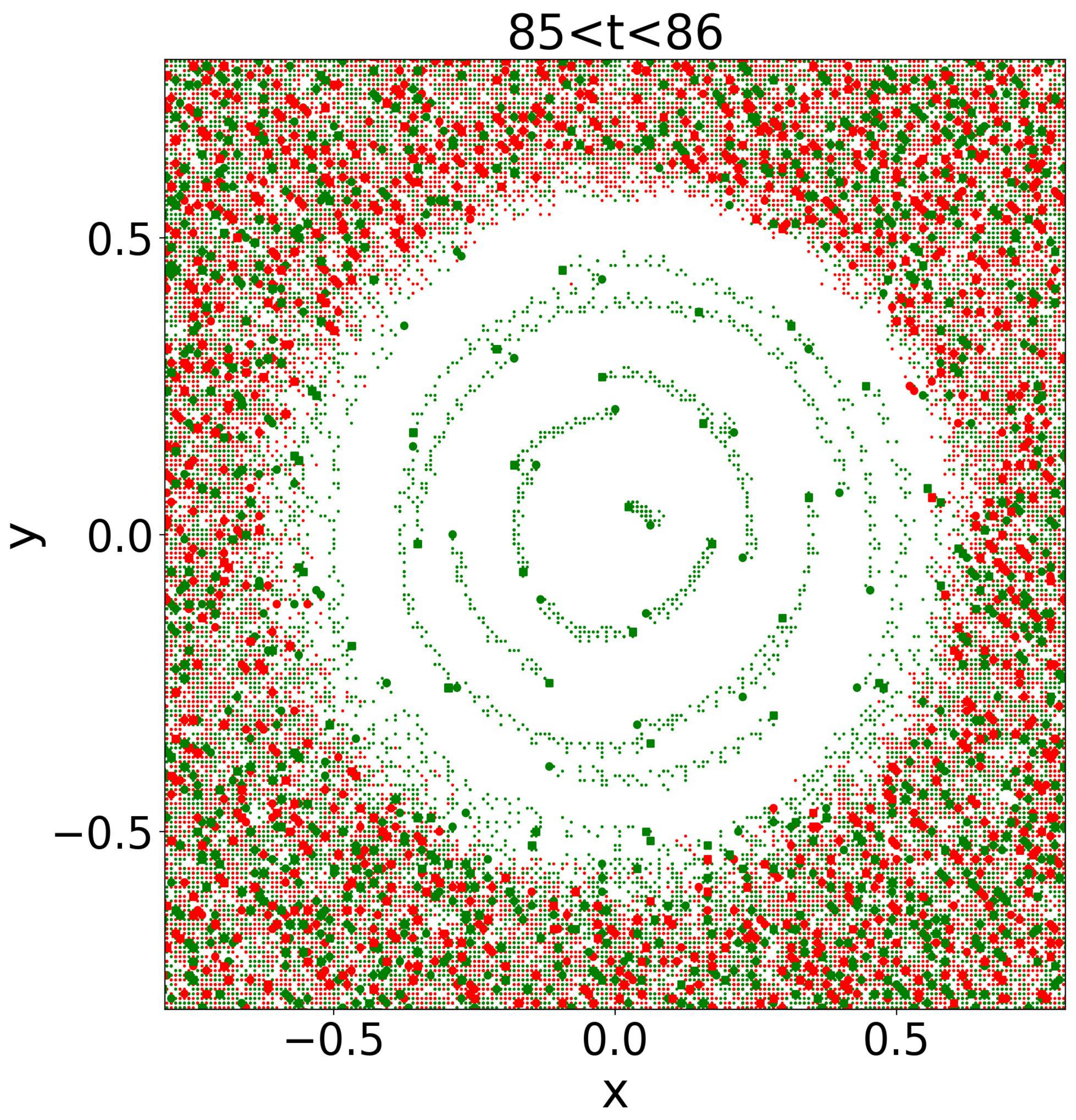


Projection onto the vertical plane

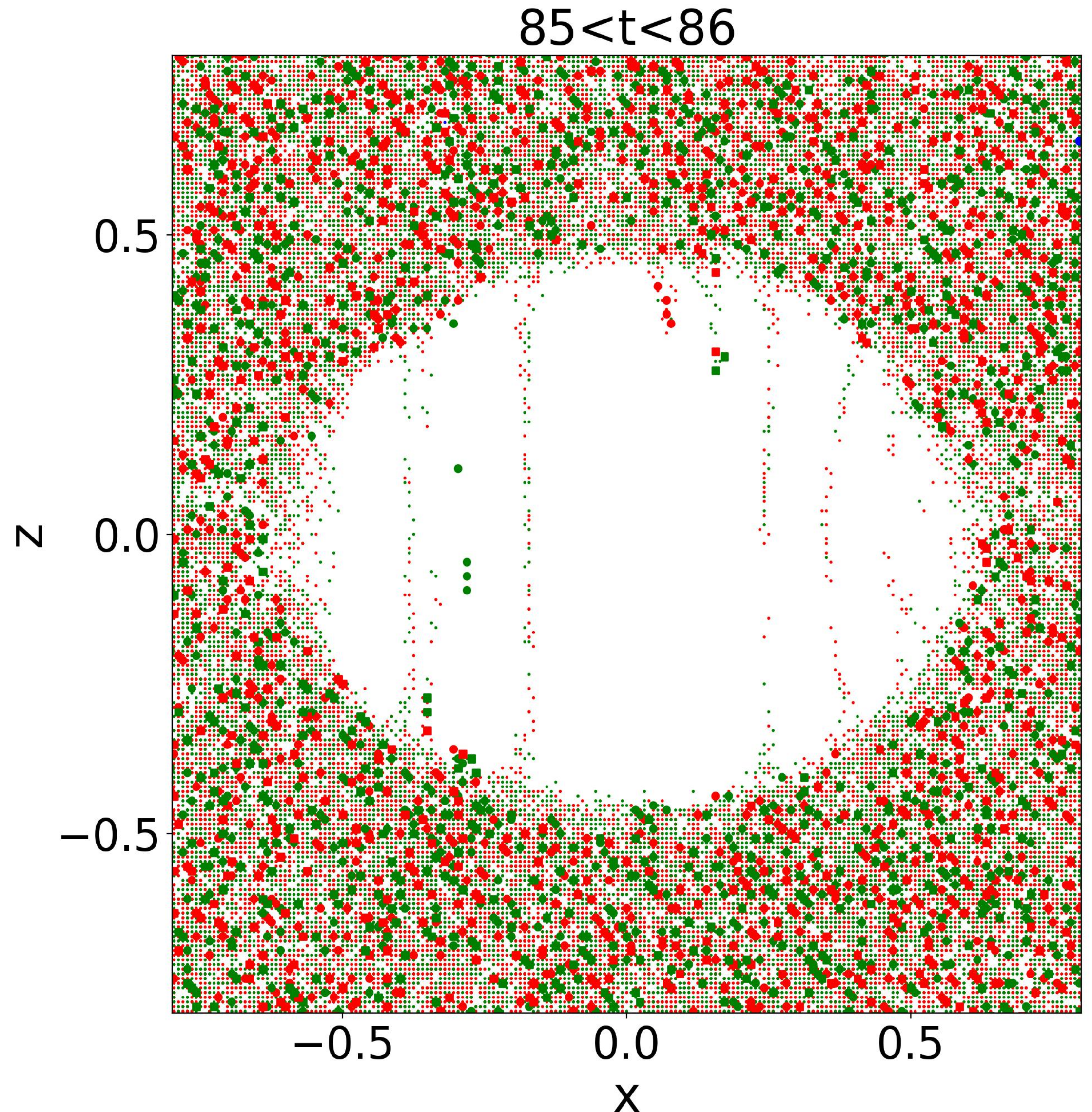


Stacking of the vortex lines (winding number maps) over many times

Solid-body rotation of the vortex lines in the equatorial plane



Trace of vertical vortex lines passing through the vertical plane



V- Conclusion

→ Halos with a nonzero angular momentum form stable rotating solitons with an oblate shape, for $\epsilon \ll 1$

These rotating solitons are not high angular momentum eigenstates of the Schrödinger equation with a vanishing central density

$$\psi_{\ell m}(\vec{x},t) = e^{-i\mu t/\epsilon} f(r) Y_\ell^m(\theta,\varphi) \qquad \ell \gg 1, \quad |m| \gg 1$$

Instead, they have a maximum central density and display a solid-body rotation that is supported by a regular lattice of vortex lines, aligned with the initial angular momentum of the system.

The number of vortex lines grows linearly with the soliton angular momentum.

- Cosmic web of vortex lines along filaments, linking collapsed halos ?
- Connection with spinning filaments ?
- → Relativistic regime ? Frame dragging effects on baryons ?
- Detection of such DM substructures by lensing ?
- Impact on the distribution of the gas ?

Alvarez-Rios et al. 2025