

Axions and non-Abelian gauge fields dynamics during inflation

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- PRD 110 (2024) 10, 103542 - Dimastrogiovanni, AP, Fasiello
- 2504.17750 - Dimastrogiovanni, AP, Fasiello, Zenteno Gatica
- 2506.11853 - Bhattacharya, Dimastrogiovanni, AP, Fasiello



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Axion Inflation - Theoretical motivation

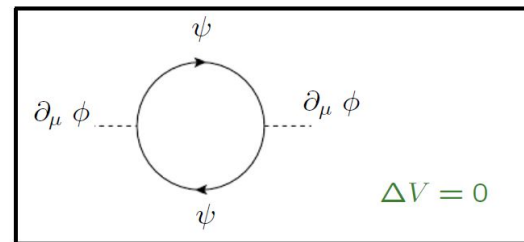
It is generally hard to preserve the flatness of the inflaton potential for $\Delta\phi > M_p$

- **Shift symmetry** comes to the rescue $\phi \rightarrow \phi + C$
- Axions arise ubiquitously in string theory [Svrcek, Witten '06](#)

E.g. Axion (natural) inflation
[Freese, Friedman Olinto '90](#)

The most general dimension 5 operators that couple the axion to other fields are

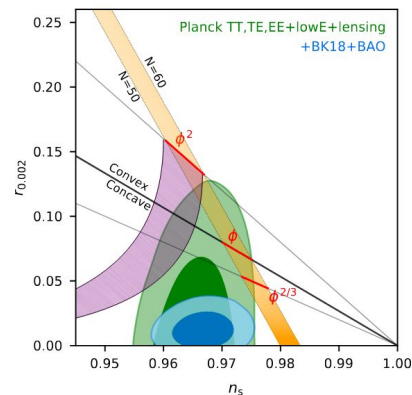
$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 + V_{\text{shift}}(\phi) + \frac{c_\psi}{f} \partial_\mu \phi \bar{\psi} \gamma^\mu \gamma_5 \psi + \frac{c_A}{f} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}$$



The simplest model has a sinusoidal potential arising from instanton effects

$$V(\phi) = \Lambda^4 \left[1 + \cos \left(\frac{\phi}{f} \right) \right]$$

[Planck & Bicep '21](#)



Axion Inflation - Phenomenological motivation

Such a coupling was originally studied for magnetogenesis

Turner, Widrow, '88

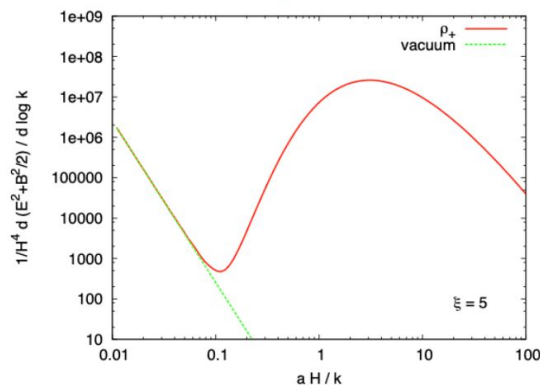
Assuming a hidden U(1) gauge field, the operator **breaks parity** and modifies the dispersion relation

$$A''_{\pm} + \left(k^2 \mp k c_A \frac{\phi'}{f} \right) A_{\pm} = 0$$

Garretson, Field, Carroll '92
Anber, Sorbo '09

- The unphysical vacuum energy is renormalized away
Ballardini, Braglia, Finelli, Marozzi, Starobinsky '19
- One of the two polarizations becomes tachyonic just before horizon crossing
- The maximum amplitude reached is of the order

$$A_+ \propto e^{\pi \xi} \quad , \quad \xi \equiv \frac{c_A \dot{\phi}}{2Hf} \simeq \frac{M_p c_A \sqrt{\epsilon_H}}{\sqrt{2}f}$$



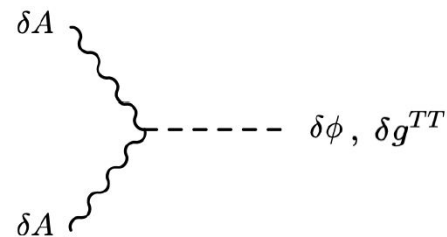
Axion Inflation - Phenomenological motivations

Amplified vector fields in turn produce scalar and tensor perturbations nonlinearly

Barnaby, Peloso '10
Barnaby, Pajer, Peloso '11

- The sourced signal adds up incoherently compared to the vacuum modes

$$\langle \phi^2 \rangle = \langle \phi_{\text{vacuum}}^2 \rangle + \langle \phi_{\text{sourced}}^2 \rangle$$



- Rich phenomenology
 - Sourced, chiral GWs at large and small scales - Sorbo '11, Garcia-Bellido, Peloso, Unal '16
 - non-Gaussianity - Barnaby, Namba, Peloso '11
 - Leptogenesis - Caldwell, Devulder '17, AP, Peloso '16
 - Magnetogenesis - Adshead, Giblin Jr., Scully, Sfakianakis '16, Brandenburg, Ilyashenko, Sfakianakis, Sharma '24
 - PBH production - Garcia-Bellido, Peloso, Unal '17



Chromo-Natural Inflation

Phenomenology of the Strong Backreaction regime

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} (\partial\chi)^2 - V(\chi) - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{\lambda\chi}{4f} F_{\mu\nu}^a \tilde{F}^{a\mu\nu} \right]$$

Isotropic gauge field background is possible $A_0^a = 0$, $A_i^a = a(t) Q(t) \delta_i^a$

The isotropic vev is an emergent feature of the model

Maleknejad, Sheikh-Jabbari '11

Leads to additional friction already at the background level!

Domcke, Mares, Muia, Pieroni '18
Wolfson, Maleknejad, Komatsu '20

$$\ddot{\chi} + 3H\dot{\chi} + U'(\chi) + \frac{3\lambda g}{f} Q^2 (\dot{Q} + HQ) = 0$$

$$\ddot{Q} + 3H\dot{Q} + (\dot{H} + 2H^2)Q + gQ^2 \left(2gQ - \frac{\lambda\dot{\chi}}{f} \right) = 0$$

Chromo-Natural Inflation - Perturbations

- There are two additional “tensor” modes that mix with the GWs at the linearized level as a consequence of the isotropic vev.
- There are two additional scalar modes that mix with the axion perturbation.

The gauge “tensor” modes eoms, have their dispersion relation modified due to the axion gauge coupling.

$$\hat{t}_{R,L}'' + \left[1 + \frac{2}{x^2} (m_Q \xi \pm x(m_Q + \xi)) \right] \hat{t}_{R,L} = 0 \quad x \equiv -k\tau$$

Particle Production parameters: $m_Q \equiv \frac{gQ}{H}$, $\Lambda \equiv \frac{\lambda Q}{f}$, $\xi \equiv \frac{\lambda \dot{\chi}}{2Hf}$

Chromo-Natural Inflation - Strong Backreaction

$$\ddot{\chi} + 3H\dot{\chi} + V_{\chi} + \frac{3g\lambda}{f} Q^2 (\dot{Q} + HQ) + \mathcal{T}_{BR}^{\chi} = 0,$$

$$\ddot{Q} + 3H\dot{Q} + (\dot{H} + 2H^2) Q + gQ^2 \left(2gQ - \frac{\lambda\dot{\chi}}{f} \right) + \mathcal{T}_{BR}^Q = 0$$

Hubble friction

Magnetic drift “friction”

Tensor backreaction “friction”

Entering the strong backreaction regime is inevitable in CNI

$$\mathcal{T}_{BR}^{\chi} = -\frac{\lambda}{2fa^3} \int \frac{d^3k}{(2\pi)^3} \frac{d}{dt} \left[(agQ + k) |\hat{t}_R|^2 + (agQ - k) |\hat{t}_L|^2 \right]$$

$$\mathcal{T}_{BR}^Q = \frac{g}{3a^2} \int \frac{d^3k}{(2\pi)^3} \left[\left(\frac{\lambda\dot{\chi}}{2f} + \frac{k}{a} \right) |\hat{t}_R|^2 + \left(\frac{\lambda\dot{\chi}}{2f} - \frac{k}{a} \right) |\hat{t}_L|^2 \right]$$

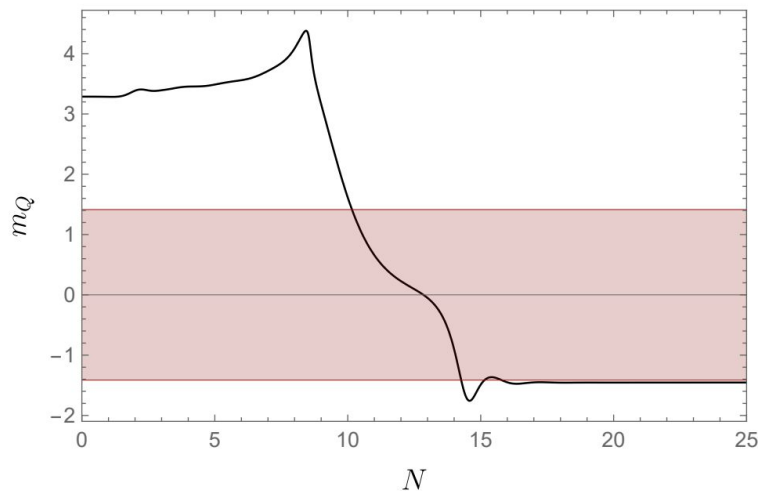
Strong backreaction in the homogeneous approximation requires solving a set of integrodifferential equations.

Chromo-Natural Inflation - Strong Backreaction Regime

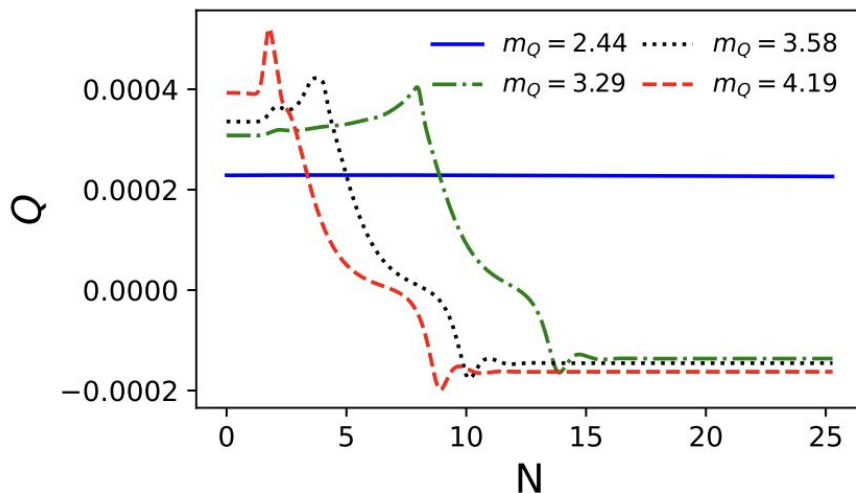
The strong backreaction regime first explored recently [Larygina, Sfakianakis, Sharma, Brandenburg '23](#)

We have recreated the results using our own independent code in [Julia](#)

[Dimastrogiovanni, Fasiello, AP '24](#)



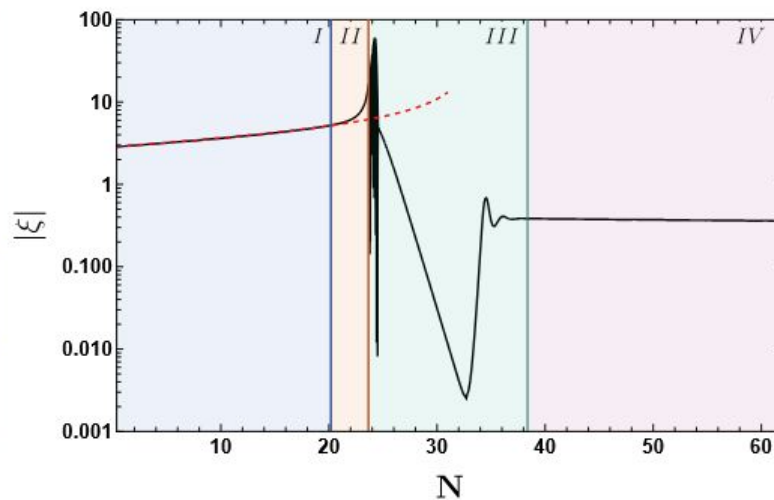
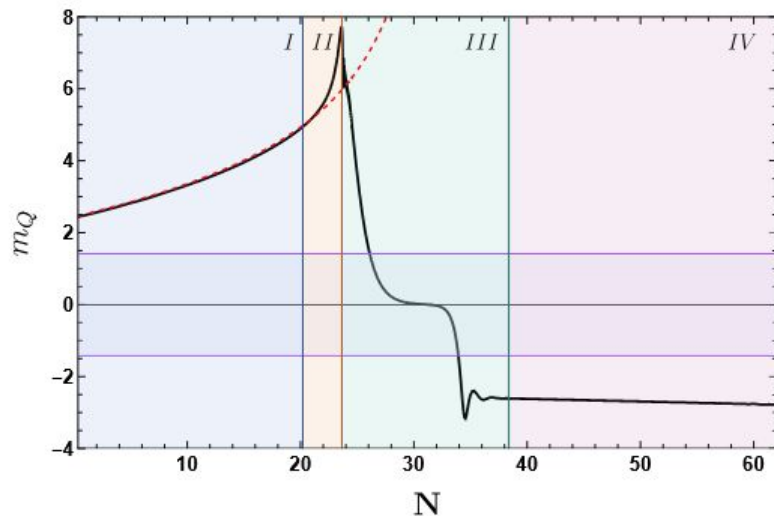
- Early time attractor $m_Q \equiv \frac{gQ}{H} = \xi + 1/\xi$
- Late time attractor $m_Q \equiv \frac{gQ}{H} = \frac{-1}{\xi}$



[New work on perturbativity constraints](#)

[Dimastrogiovanni, Fasiello, Michelotti, Ozsoy '24](#)

Chromo-Natural Inflation - Numerical Analysis



This is alarming because there is a strong instability in the scalar perturbations for $|m_Q| < \sqrt{2}$

Dimastrogiovanni, Peloso '12

Chromo-Natural Inflation - Scalar Tachyonic Instability

$$\ddot{\hat{X}} + H\dot{\hat{X}} + \left[-2 + \epsilon_H + 3\eta_\chi + \frac{k^2 (k^2 + m_Q^2 (2 + \Lambda) a^2 H^2)}{a^2 H^2 (k^2 + 2m_Q^2 a^2 H^2)} \right] H^2 \hat{X} + \frac{\sqrt{2}m_Q^2 \Lambda a H}{\sqrt{k^2 + 2m_Q^2 a^2 H^2}} H \dot{\hat{\phi}} + \frac{m_Q \Lambda (k^4 + 3m_Q^2 k^2 a^2 H^2 + 4m_Q^4 a^4 H^4)}{a H (k^2 + 2m_Q^2 a^2 H^2)^{3/2}} \sqrt{\frac{2\epsilon_E}{\epsilon_B}} H^2 \hat{\phi} - \sqrt{2}m_Q \Lambda H \dot{\hat{Z}} - 2m_Q^2 \Lambda \sqrt{\frac{2\epsilon_E}{\epsilon_B}} H^2 \hat{Z} = 0,$$

$$\ddot{\hat{\phi}} + H\dot{\hat{\phi}} + \left[\frac{6m_Q^4 k^2 a^2 H^2}{(k^2 + 2m_Q^2 a^2 H^2)^2} \frac{\epsilon_E}{\epsilon_B} + \frac{k^4 + 2m_Q(3m_Q - \xi)k^2 a^2 H^2 + 4m_Q^4 a^4 H^4}{(k^2 + 2m_Q^2 a^2 H^2) a^2 H^2} \right] H^2 \hat{\phi} - \frac{2(m_Q - \xi) \sqrt{k^2 + 2m_Q^2 a^2 H^2}}{a H} H^2 \hat{Z} - \frac{\sqrt{2}m_Q^2 \Lambda a H}{\sqrt{k^2 + 2m_Q^2 a^2 H^2}} H \dot{\hat{X}} + \frac{\sqrt{2}m_Q^2 \Lambda a H}{\sqrt{k^2 + 2m_Q^2 a^2 H^2}} \left[1 + \frac{k^4}{m_Q (k^2 + 2m_Q^2 a^2 H^2) a^2 H^2} \sqrt{\frac{\epsilon_E}{\epsilon_B}} \right] H^2 \hat{X} = 0,$$

$$\ddot{\hat{Z}} + H\dot{\hat{Z}} + \left[\frac{k^2}{a^2 H^2} + 2m_Q (m_Q - 2\xi) \right] H^2 \hat{Z} + \sqrt{2}m_Q \Lambda H \dot{\hat{X}} - \sqrt{2}m_Q \Lambda H^2 \hat{X} - \frac{2(m_Q - \xi) \sqrt{k^2 + 2m_Q^2 a^2 H^2}}{a H} H^2 \hat{\phi} = 0.$$

The amplitude of the scalars grows exponentially:

$$e^{iS(x_{\text{late}})} = e^{i \int_{x_{\text{inst}}}^{x_{\text{late}}} \Omega_{\text{slow}}} = e^{\sqrt{\frac{2-m_Q^2}{3m_Q^2}} \sqrt{2-m_Q^2} \Lambda}$$

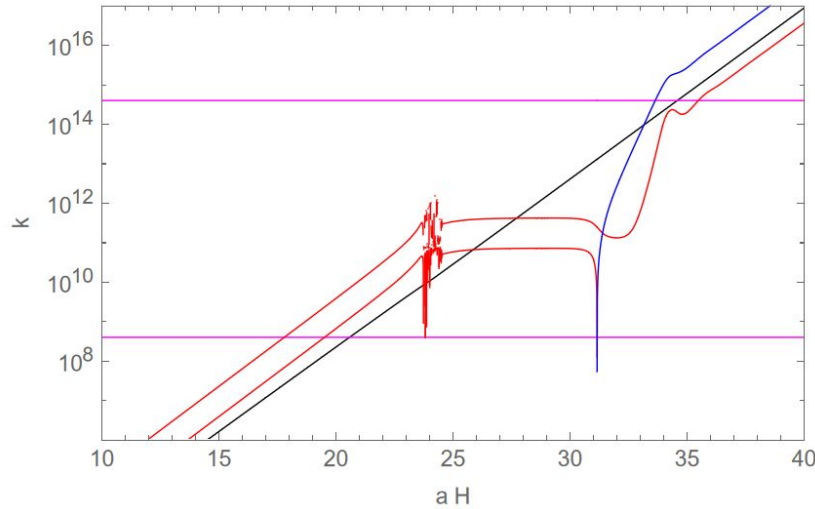
If Λ is large, there is a possibility for strong scalar backreaction regime

$$B_{BR}^X = \frac{V^{(3)}(\chi)}{2a^2} \int \frac{d^3 k}{(2\pi)^3} |\hat{X}(\tau, k)|^2, \\ B_{BR}^Q = \frac{2g\Lambda^2 m_Q}{3a^2 H} \int \frac{d^3 k}{(2\pi)^3} \frac{k^2 (k^2 + a^2 H^2 m_Q^2)}{(k^2 + 2a^2 H^2 m_Q^2)^2} |\hat{X}(\tau, k)|^2$$

Ongoing work with
Dimastrogiovanni, Fasiello
and Mattia Cielo

Chromo-Natural Inflation - Instabilities revisited

$$\hat{t}_{R,L}'' + \left[1 + \frac{2}{x^2} (m_Q \xi \pm x(m_Q + \xi)) \right] \hat{t}_{R,L} = 0$$



The instability in the Left handed mode moves from inside to outside the horizon during the transition.

$$x_{L,\pm} = m_Q + \xi \pm \sqrt{m_Q^2 + \xi^2}$$

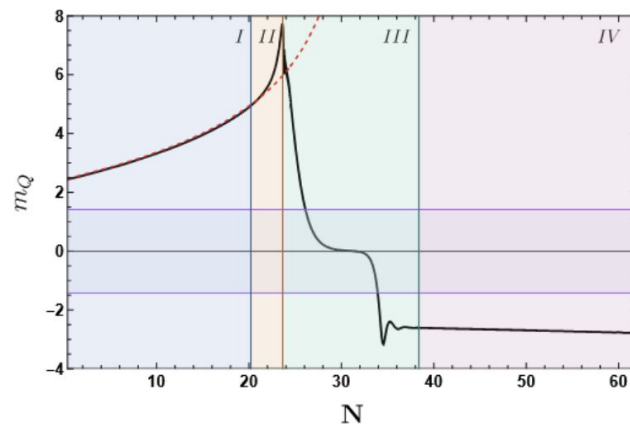
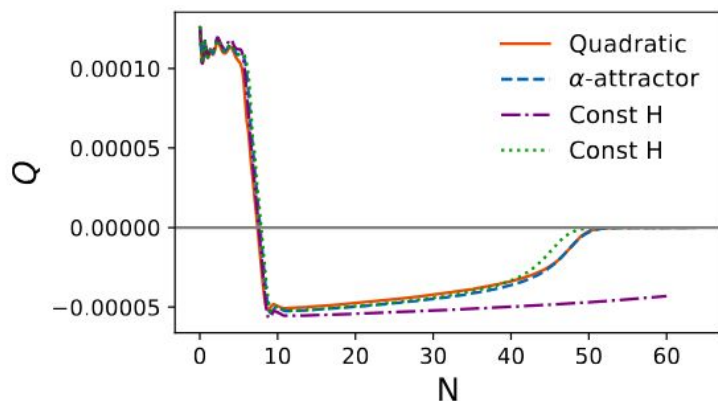
$$x_{R,\pm} = -m_Q - \xi \pm \sqrt{m_Q^2 + \xi^2}$$

The instability in the Right handed mode moves inside the horizon in the strong backreaction regime.

Chromo-Natural Inflation - A possible novel regime?

If m_Q grows enough in the negative direction, it might be possible to enter a new strong backreaction regime from the right handed tensor modes.

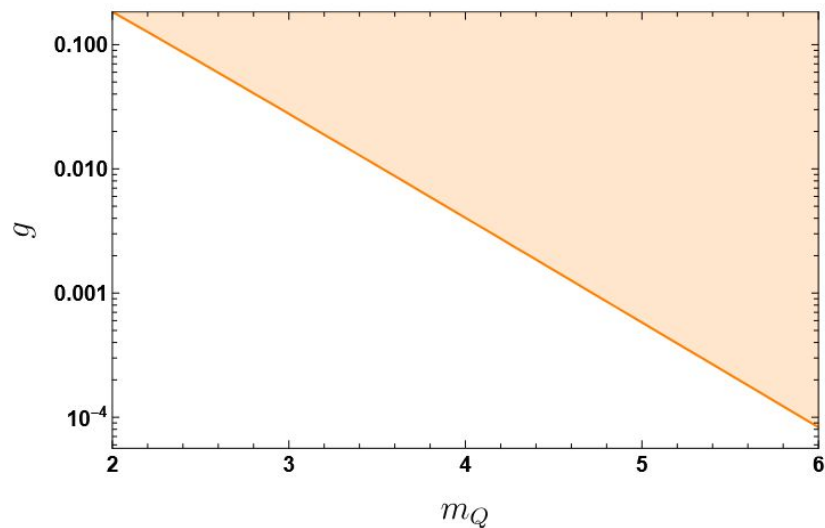
This is unlikely to play a role in the spectator model because Q typically decays.



Chromo-Natural Inflation - Backreaction limits

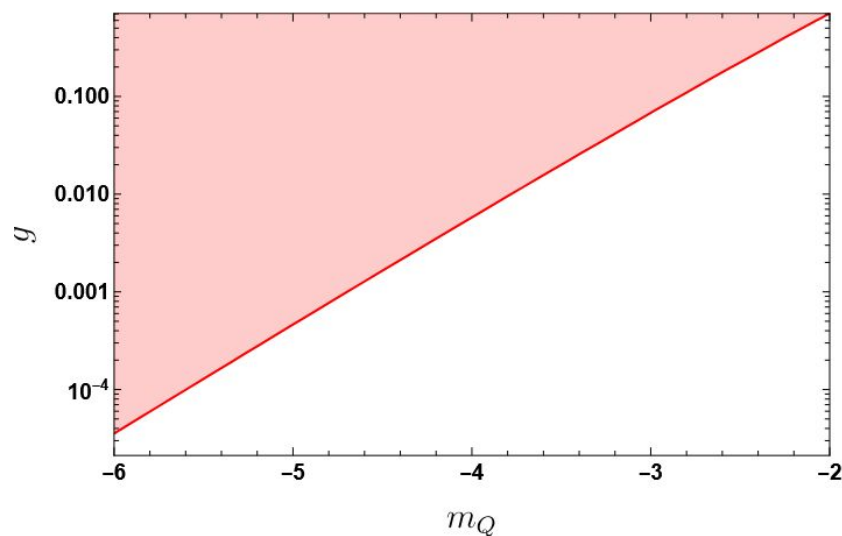
Weak backreaction attractor

$$m_Q \equiv \frac{gQ}{H} = \xi + 1/\xi$$



Strong backreaction attractor

$$m_Q \equiv \frac{gQ}{H} = \frac{-1}{\xi}$$



Chromo-Natural Inflation - Potential

Nomura, Watari, Yamazaki 17'
Nomura, Yamazaki 17'

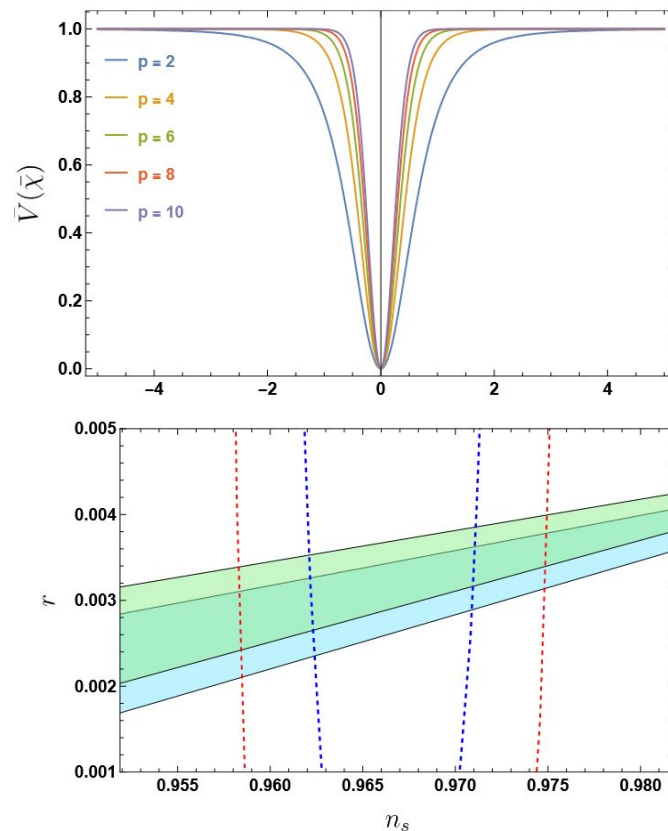
For concreteness we choose the “Pure Natural Inflation” model which features a plateau-like region

$$V_{\text{PNI}}(\chi) = M^4 \left[1 - \left(1 + (\chi/F)^2 \right)^{-p} \right]$$

If the axion inflation couples to pure SU(N) Yang-Mills the cosine is in general not correct in the large N limit with the 't Hooft coupling held fixed.

The multi-valued nature of the potential allows a single branch not to respect the periodicity under

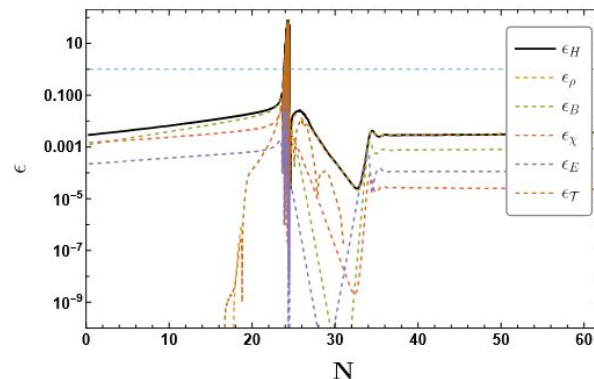
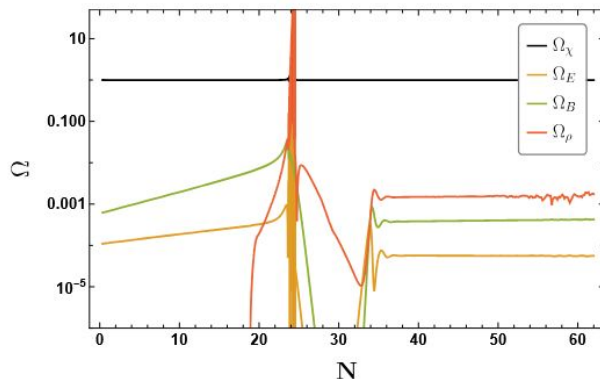
$$\phi \rightarrow \phi + 2\pi f$$



Particle Production Channels

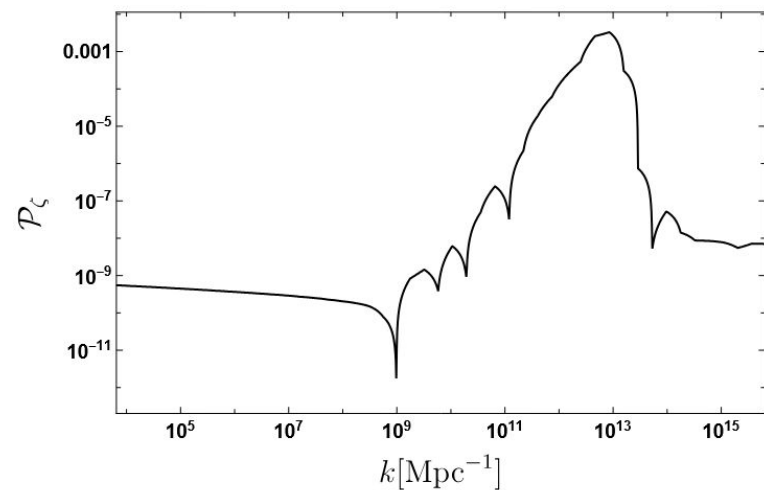
- Linear sourcing of GWs by gauge field left handed perturbations in the weak backreaction regime
- Scalar perturbation production during the transition
 - Due to the scalar instability
 - Due to the axion effectively ultra slow-rolling
- Linear sourcing of GWs by gauge field right handed perturbations in the strong backreaction regime

$$\zeta = \frac{V_\chi(\chi)}{6\epsilon_H H^2 M_P^2} \delta\chi$$

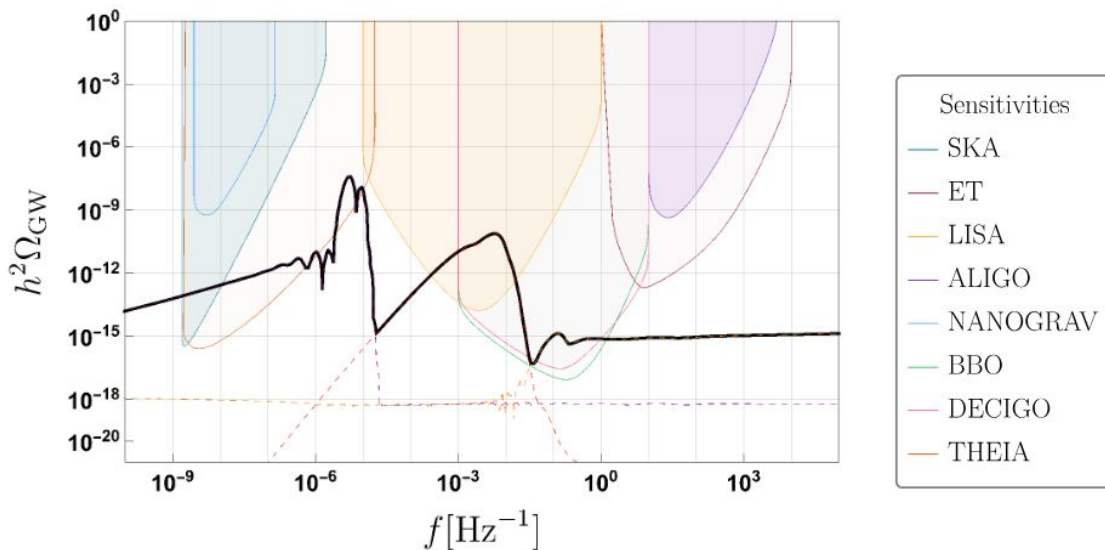


Phenomenology

Density perturbation



Gravitational waves





Axion-SU(2) Inflation

Warm or Cold?

Axion-SU(2) Inflation - Warm or Cold?

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} (\partial\chi)^2 - V(\chi) - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{\lambda\chi}{4f} F_{\mu\nu}^a \tilde{F}^{a\mu\nu} \right]$$

Chromo-Natural Inflation

- Assumes a gauge field background
- Perturbations non-thermal and controlled by the linear approximation

Adshead, Wyman '12

Minimal Warm Inflation

- Zero gauge field background
- Perturbations assumed to be thermal

Berghaus, Graham, Kaplan '23

Only recently have these two ideas been considered in tandem

Mukuno, Soda '24
Kamali, Ramos '24

See “Thermalized Axion Inflation” for the U(1) case
Ferreira, Notari '17

Chromo-Natural Inflation - Initial Conditions

It is important to understand the phase space of the model and delineate the parameter space of **warm** vs **cold** inflation.

Perhaps the initial conditions play a role?

What if we assume the absence of a gauge field background and perturbations in their adiabatic vacuum?

$$Q_{\text{in}} = 0 \quad , \quad \delta A \simeq \frac{1}{\sqrt{2k}} e^{-ik\tau}$$

Both scenarios are emerging

Emerging Chromo-Natural Inflation

Domcke, Mares, Muia, Pieroni '18

Domcke, Sandner '19

Warming up cold Inflation

DeRocco, Graham, Kalia '21

Minimal Warm Inflation

Berera '95
Yokoyama, Linde '99
Bastero-Gil, Berera, Ramos, Rosa '16
Kamali '19

Gauge field perturbations form a thermal bath during inflation

$$\ddot{\phi} + 3H\dot{\phi} + \Upsilon(T)\dot{\phi} + V'(\phi) = 0$$

$$H^2 - \frac{1}{3M_{\text{pl}}^2} \left(\rho_R + \frac{\dot{\phi}^2}{2} + V(\phi) \right) = 0$$

$$\dot{\rho}_R + 4H\rho_R - \Upsilon(T)\dot{\phi}^2 = 0$$

Friction coefficient in the presence
of rapid sphaleron transitions

$$\Upsilon(T) = \frac{\Gamma_{\text{sph}}}{2Tf^2} \left(\frac{\Gamma_{\text{ch}}}{\Gamma_{\text{ch}} + \frac{24T_R^2}{d_R T^3} \Gamma_{\text{sph}}} \right)$$

$$\Gamma_{\text{sph}} \sim N_c^5 \alpha^5 T^4$$

McLerran, Mottola, Shaposhnikov '91
Laine, Proccacci '21

How to form the VEV?

In the absence of the gauge field VEV, the non-Abelian perturbations in the linearized limit at three copies of U(1) gauge fields

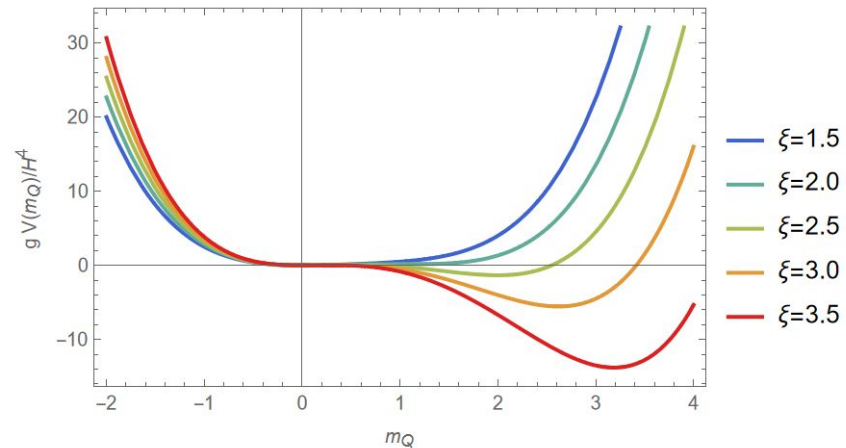
$$\frac{d^2 T_{\mathcal{AB},\pm}^a(\vec{k}, \tau)}{dx^2} + \left[1 \mp \frac{2\xi}{x}\right] T_{\mathcal{AB},\pm}^a(\vec{k}, \tau) = 0, \quad x \equiv -k\tau.$$

Solution: $T_{\mathcal{AB},+}^a(\tau, k) \simeq \frac{e^{\frac{\pi}{2}\xi}}{\sqrt{2k}} W_{-i\xi, 1/2}(-2ix)$

The gauge field background has an effective potential

$$V_{\text{eff}}(Q) = H^2 Q^2 - \frac{2}{3} g H \xi Q^3 + \frac{g^2}{2} Q^4$$

Domcke, Mares, Muia, Pieroni '18



Can the VEV form via tunneling?

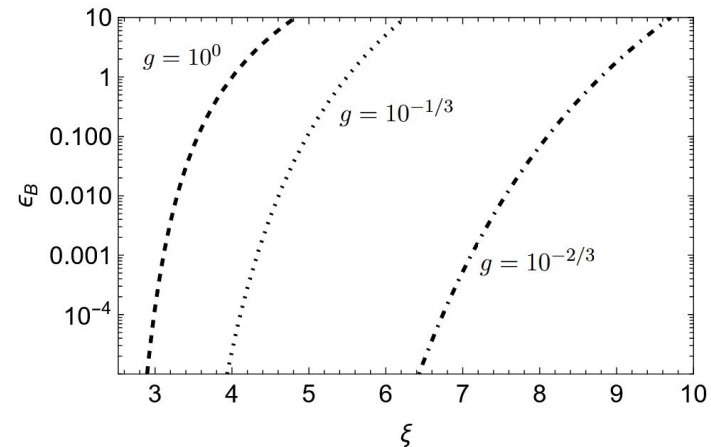
Need to compute the bubble nucleation efficiency of the transition from the false to the true vacuum

$$\epsilon_B = \frac{\Delta V}{H^4} e^{-S_{E4}}$$

Polynomial potentials of fourth order have been computed numerically and tabulated in [Adams '93](#)

$$V(\phi) = \lambda\phi^4 - a\phi^3 + b\phi^2 + c\phi + d$$

$$S_{E4} = \frac{\pi^2}{3\lambda} (2 - \delta)^{-3} [\alpha_1\delta + \alpha_2\delta^2 + \alpha_3\delta^3]$$



How does the thermal bath form?

Conditions for thermalization:

1. Nonlinear interactions of the perturbations whose rate is faster than Hubble kickstart at some point during inflation. **We check this condition**
2. The spectrum of fluctuations transitions from cold to warm obeying the Bose-Einstein distribution while the universe is expanding. **We don't check this condition**
3. Assuming the thermal distribution has been achieved, the thermalization rate must be greater than Hubble. **We check this condition**

First condition: rapid self-interactions

Nonlinearity gives rise to thermalization

Thermalization happens when modes exchange energy efficiently

Exchange of energy will lead to thermalization because that is configuration maximizes entropy (2nd Law of Thermodynamics)

See Ferreira, Notari '17

Nonlinear interactions of the model:

$$\mathcal{L}^{(3)} \supset -\frac{\lambda}{f} \delta\chi \left[\frac{g}{2} (aQ T_{ab} T_{ab})' - f^{abc} T_{ia}' \partial_b T_{ic} \right] + \left[\frac{g^2 a^2 Q^2}{-\partial^2 + 2g^2 a^2 Q^2} \delta\chi \right] \partial_b (f^{abd} T_{ia}' T_{ic})$$

$$\mathcal{L}^{(3)} \supset -g f^{abc} T_{ai} T_{bj} \partial_i T_{cj} - \frac{g\xi}{3\tau} f^{abc} \epsilon^{ijk} T_{ai} T_{bj} T_{ck} - \frac{gm_Q}{\tau} T_{ij} T_{jk} T_{ki}$$

$$\mathcal{L}^{(4)} \supset -\frac{1}{4} g^2 f^{abc} f^{ade} T_{bi} T_{cj} T_{di} T_{ej}.$$

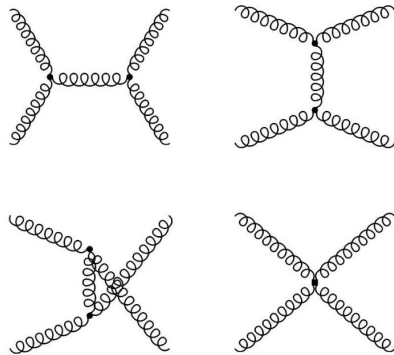
First condition: rapid self-interactions

Nonlinear interactions modify the equation of motion of perturbations

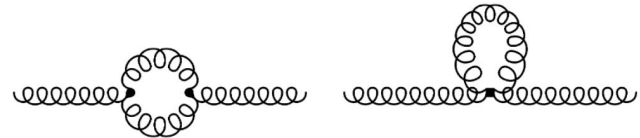
$$\frac{d^2 T_{\pm}(\vec{k}, \tau)}{k^2 d\tau^2} + \left[1 + \frac{2m_Q \xi}{(k\tau)^2} \mp \frac{2(\xi + m_Q)}{(-k\tau)} \right] T_{\pm}(\vec{k}, \tau) = \mathcal{S}^{(3)}(\vec{k}, \tau) + \mathcal{S}^{(4)}(\vec{k}, \tau)$$

Two ways to address nonlinearities:

Particle scattering rate:



Non-perturbativity:



First condition: rapid self-interactions

Ferreira, Notari '17

Ferreira, Ganc, Norea, Sloth '15

Peloso, Sorbo, Unal '16

Particle scattering:

- Valid for subhorizon modes
- We consider a Boltzmann-like equation for the particle number
- We compute tree level diagrams

$$N_k(\tau) + 1/2 \equiv \frac{k^2 |T_k|^2 + |T'_k|^2}{2E_k}$$

$$\frac{dN_k}{d\tau} = \frac{2|G(k, \tau)|}{k^2 + |G(k, \tau)|^2} (k^2 - \omega^2(\tau))(N_k(\tau) + 1/2) + S_{++}(k)$$

$$S_{++}(k) = \frac{1}{2E_k} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} (2\pi)^4 \delta^{(4)}(k + p_1 - p_2 - p_3) \\ \times |\mathcal{M}|^2 \times \mathcal{B}(k, p_1, p_2, p_3),$$

Non-perturbativity

- Valid for all modes
- We use the in-in formalism
- Does not treat quartic interactions at the same level as cubic

$$\langle \hat{T}_k(\tau) \hat{T}_{k'}(\tau) \rangle_{1\text{-loop}}^{(3)} = - \int_{-\infty}^{\tau} d\tau' \int_{-\infty}^{\tau'} d\tau'' \langle [[\hat{T}_k(\tau) \hat{T}_{k'}(\tau), \hat{H}^{(3)}(\tau')], \hat{H}^{(3)}(\tau'')] \rangle$$

$$\mathcal{R}_T \equiv \left| \frac{P_+^{(1)}(k, \tau)}{P_+^{(0)}(k, \tau)} \right|$$

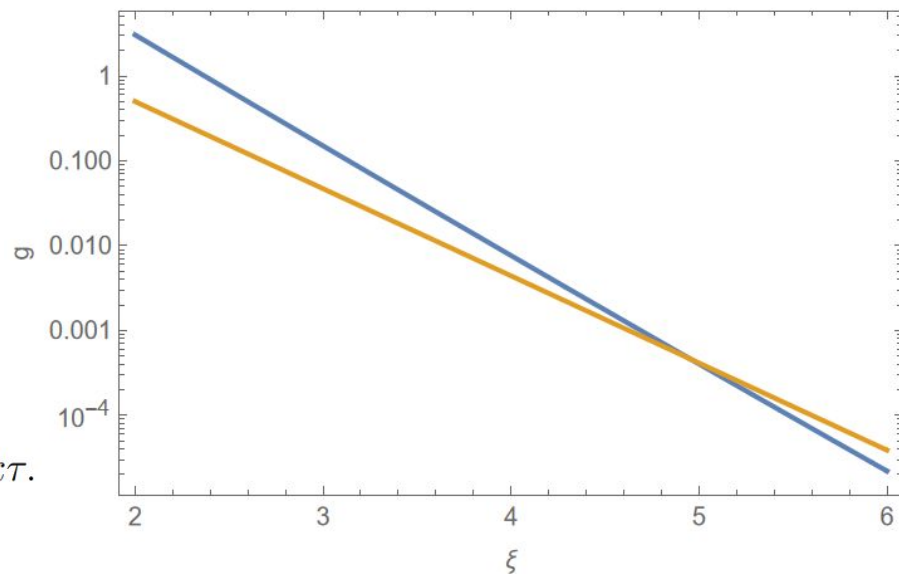
First condition: rapid self-interactions

The two tests of nonlinearity yield nearly the same bound

Inflationary trajectories are horizontal lines that move from left to right. When these limits are reached we have to go beyond linear approximation

~~$$\frac{d^2 T_{AB,\pm}^a(\vec{k}, \tau)}{dx^2} + \left[1 \mp \frac{2\xi}{x}\right] T_{AB,\pm}^a(\vec{k}, \tau) = 0, \quad x \equiv -k\tau.$$~~

~~$$\frac{d^2 T_{\pm}(\vec{k}, \tau)}{dx^2} + \left[1 + \frac{2m_Q \xi}{x^2} \mp \frac{2(\xi + m_Q)}{x}\right] T_{\pm}(\vec{k}, \tau) = 0,$$~~



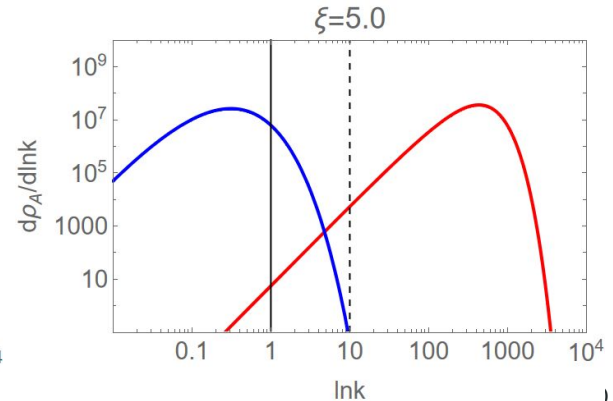
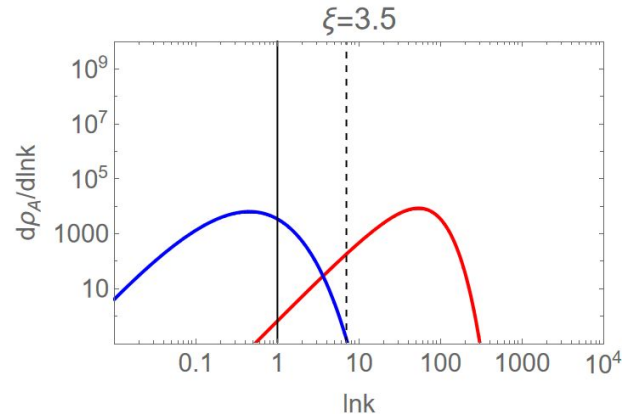
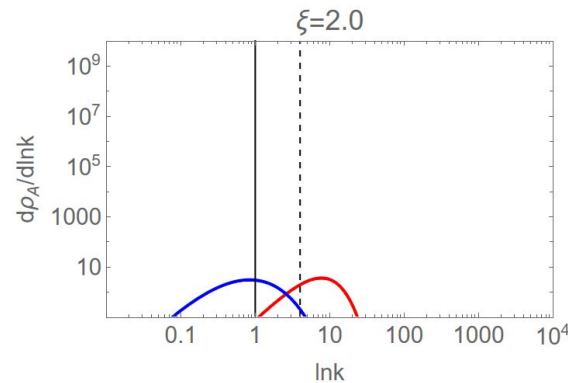
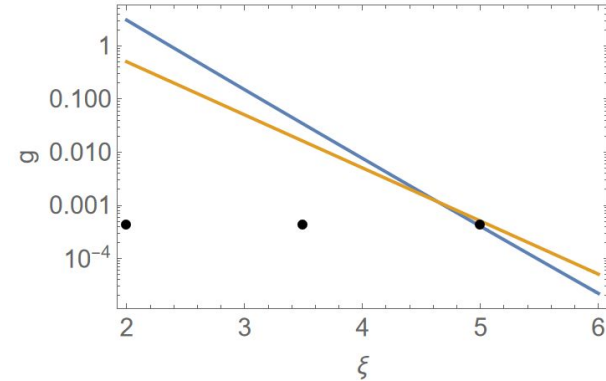
Second condition: Smooth transition from cold to warm

Energy of fluctuations: $\rho_A \simeq 1.4 \times 10^{-4} g_\epsilon H^4 \frac{e^{2\pi\xi}}{\xi^3}$

Thermal spectrum: $\rho_{\text{thr}} = \frac{\pi^2}{30} g_\epsilon T^4$

$$\frac{d\rho_A}{d \ln k}$$

$\rho_A \simeq \rho_{\text{thr}}$, (instantaneous thermalization)



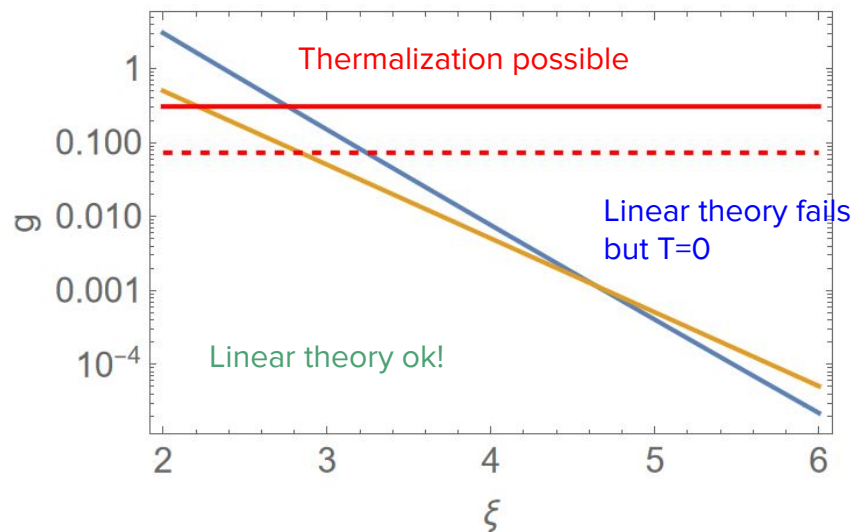
Third condition: Conservative thermalization

Can we rule out thermalization?

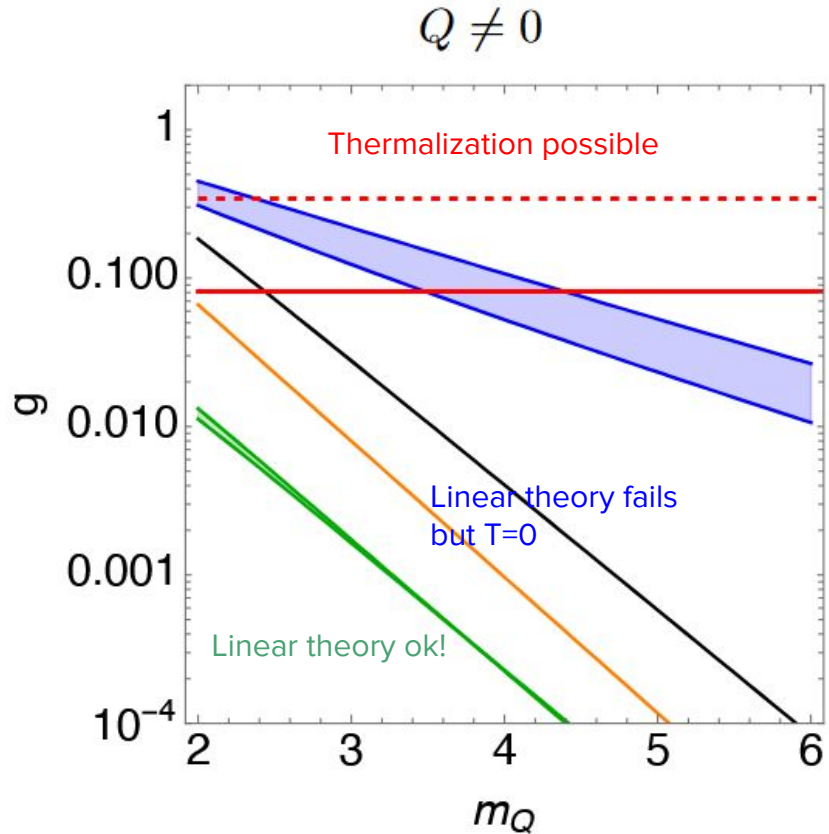
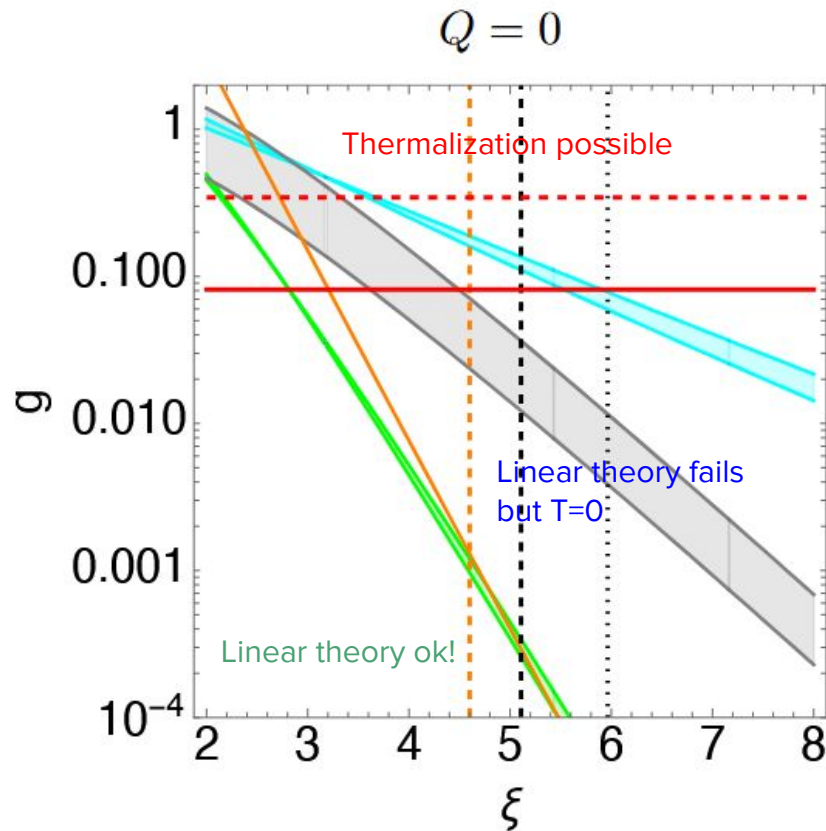
Assuming thermalization happens, we can compute the self-thermalization rate of non-Abelian gauge fields from field theory

$$\Delta \approx 10N_c^2 \alpha^2 T, \quad \text{Laine Procacci '21}$$

$$\frac{\pi^2}{30} g_* \left(\frac{H_{\text{inf}}}{10N_c^2 \left(\frac{g^2}{4\pi} \right)^2} \right)^4 \leq \gamma \times 3H_{\text{inf}}^2 M_{\text{Pl}}^2.$$



Overview of all constraints



Axion-SU(2) Lattice simulations?

We have seen a tremendous effort on lattice simulations of Axion-U(1)

1. Caravano, Komatsu, Lozanov, Weller '22
2. Figueroa, Lizarraga, Urio, Urrestilla '23
3. Caravano, Peloso '24
4. Figueroa, Lizarraga, Loayza, Urio, Urrestilla '24
5. Sharma, Brandenburg, Subramanian, Vikman '24
6. Lizarraga, López-Mediavilla, Urio '25
7. Iarygina, Sfakianakis, Brandenburg '25

Yet nothing on Axion-SU(2)...

- Can the **gauge field background** emerge in the presence of nonlinearities?
- Is the **Strong Backreaction** regime accessible in the presence of nonlinearities?
- How much do nonlinearities modify the **spectrum of fluctuations**?
- Is there a limit in which the **weak backreaction attractor** is valid?

Conclusions

- Axion inflation is well motivated from UV complete theories such as string theory
- Shift symmetry protects the flatness of the potential from large radiative corrections
- The strong backreaction regime of Chromo-Natural Inflation features many novel features that are now beginning to be explored
- The transition from weak to strong backreaction leaves characteristic signatures in the form of GWs and density perturbations
- Nonlinearities are important for Chromo-Natural Inflation. We will need lattice simulations to get the full picture
- Unclear what is a mechanism for the gauge field VEV to form

Thank you