QCD axion dark matter in the early universe

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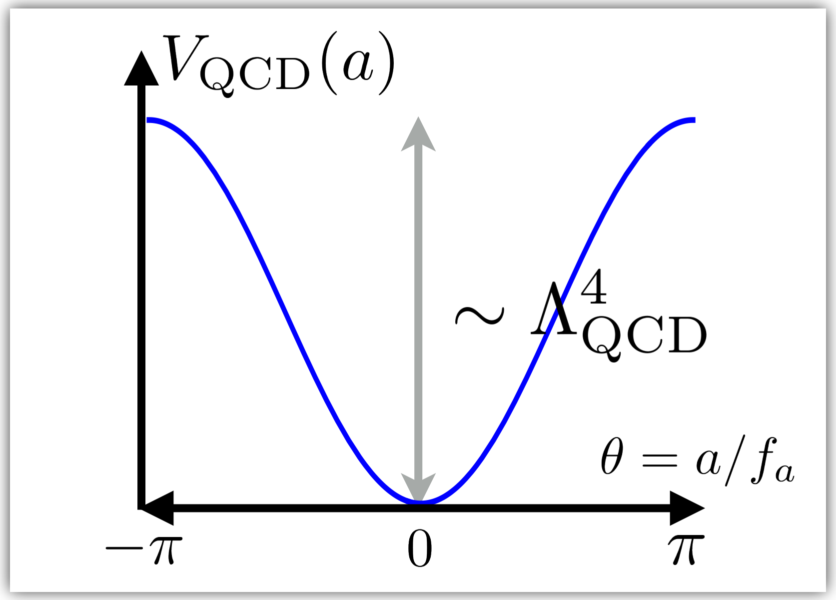
The QCD axion

"*The axion is a very compelling theoretical idea, perhaps the most minimal extension of the standard model.*"

E. W. Kolb and M. S. Turner, The Early Universe

- Pseudo Nambu-Goldstone boson associated with spontaneous breaking of the global Peccei-Quinn (PQ) symmetry at the scale f_a ("axion decay constant").
  - Solution to the strong CP problem
  - Good candidate of cold dark matter
- Acquires a mass below the QCD scale:

$$m_a \simeq 57\,\mu\mathrm{eV}\,\left(\frac{10^{11}\,\mathrm{GeV}}{f_a}\right)$$



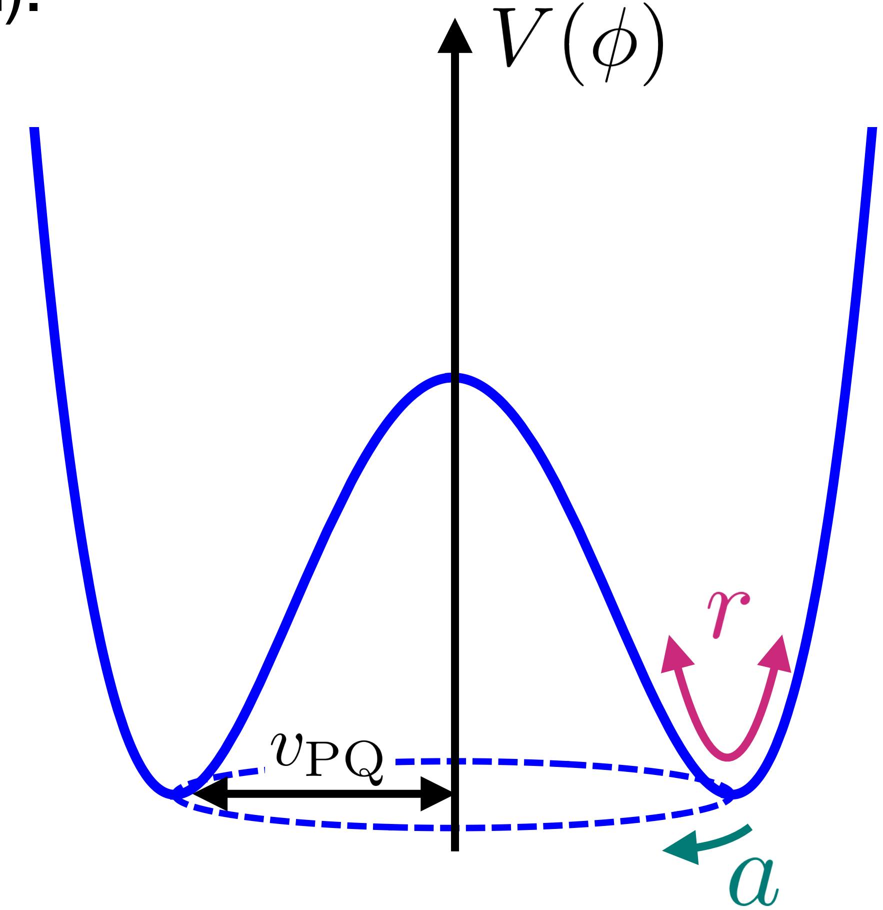
Axions in the early universe

- Dynamics in the early universe can be described by a complex scalar field (the PQ field).

$$\phi(x) = \frac{1}{\sqrt{2}}\left[v_{\mathrm{PQ}} + r(x)\right] e^{ia(x)/v_{\mathrm{PQ}}}$$

Radial field with mass $\sim v_{\mathrm{PQ}} \sim f_a$

Massless angular field (axion)

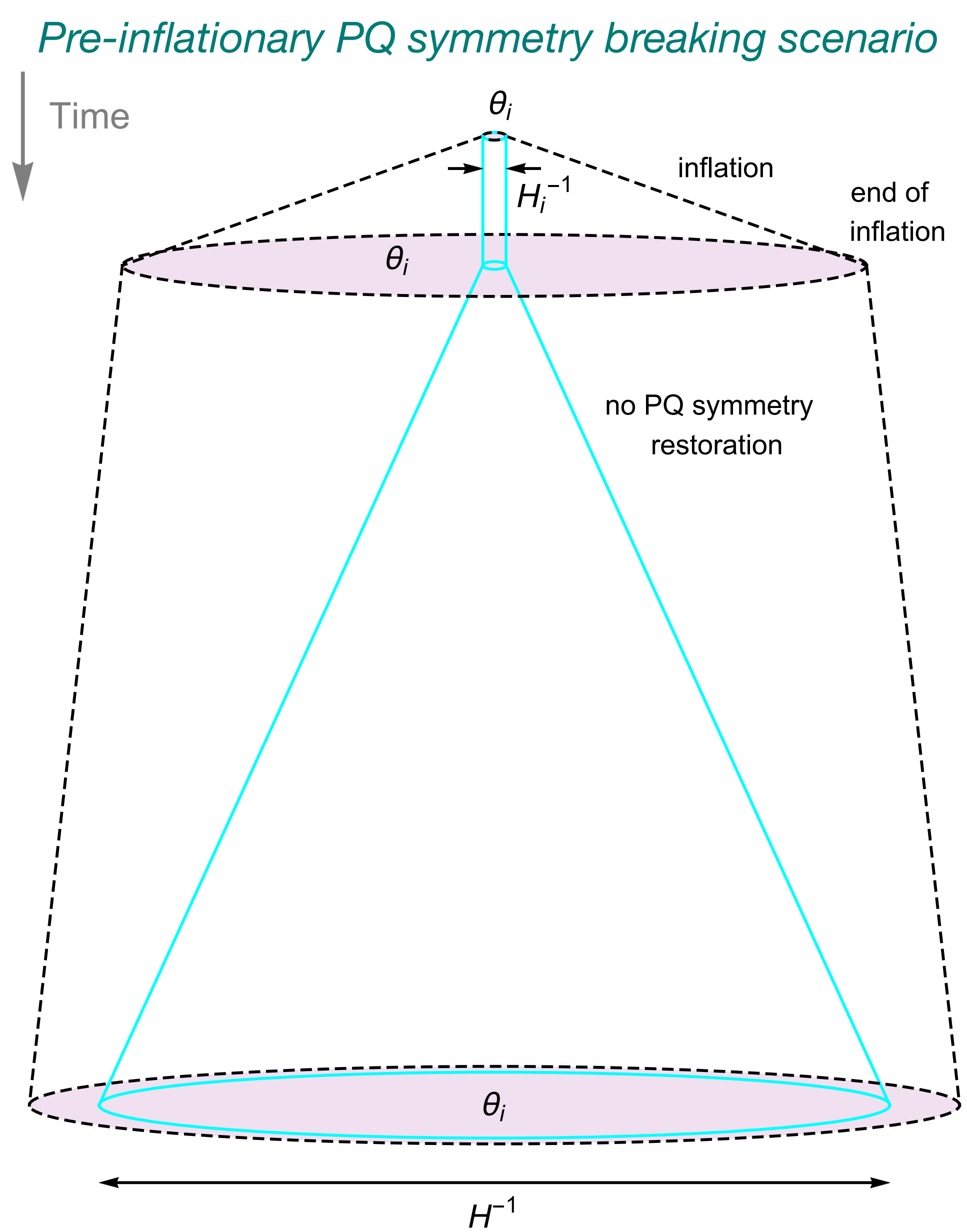


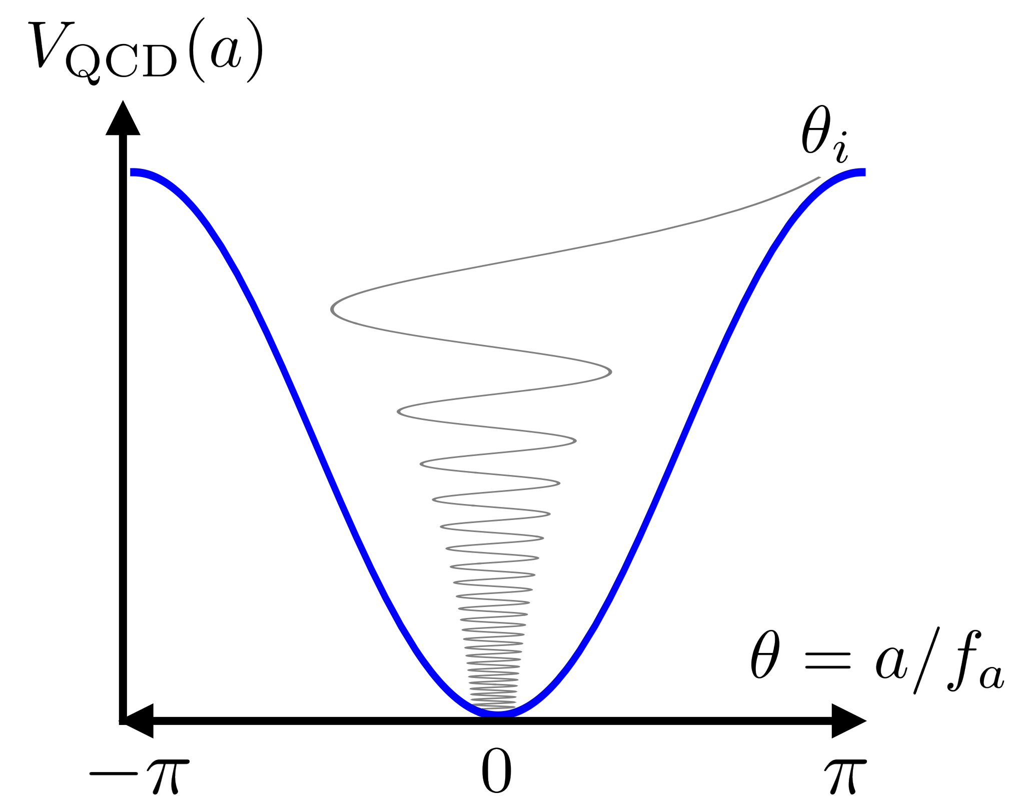
- Two possibilities on the initial condition:
  - Pre-inflationary scenario (PQ symmetry is broken before inflation)
  - Post-inflationary scenario (PQ symmetry is broken after inflation)
- Different scenarios result in varying predictions of axion dark matter mass.

Misalignment mechanism in pre-inflationary scenario

Axion starts oscillating, turns into pressureless matter at $m_a \approx 3H$.

[Preskill, Wise, Wilczek (1983); Abbott, Sikivie (1983); Dine, Fischler (1983)]





$$\Omega_a h^2 \simeq 0.14 \theta_i^2 \left(\frac{f_a}{10^{12}\,\mathrm{GeV}} \right)^{1.17}$$

Uncertainty associated with the unknown initial angle θ_i.

Post-inflationary scenario

- Topological defects
  - Strings

    formed at the PQ phase transition
    ($T \lesssim f_a$)

  - Domain walls

    formed at around the QCD phase transition
    ($T_{\rm QCD} \sim 1\,{\rm GeV}$)

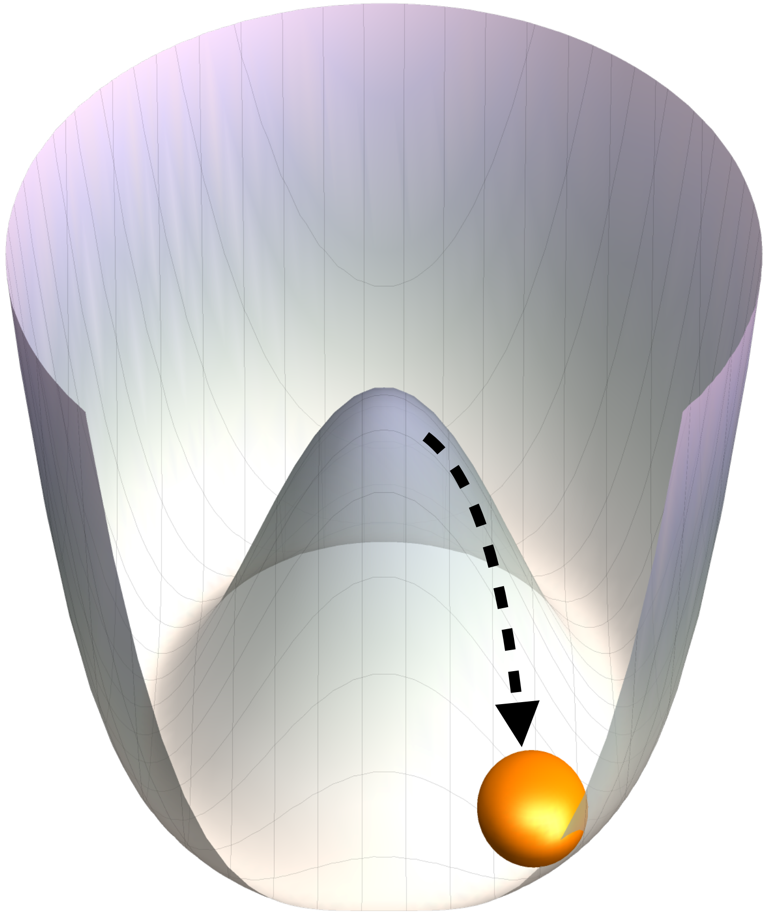
- Inhomogeneity at

$$L = \frac{R_0}{R_{\rm QCD} H_{\rm QCD}} = 0.036\,{\rm pc} \left(\frac{50\,\mu{\rm eV}}{m_a} \right)^{0.167}$$

➡ Seeds of miniclusters

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi)$$

$$V(\phi) = \lambda \left(|\phi|^2 - \frac{v_{\rm PQ}^2}{2} \right)^2$$



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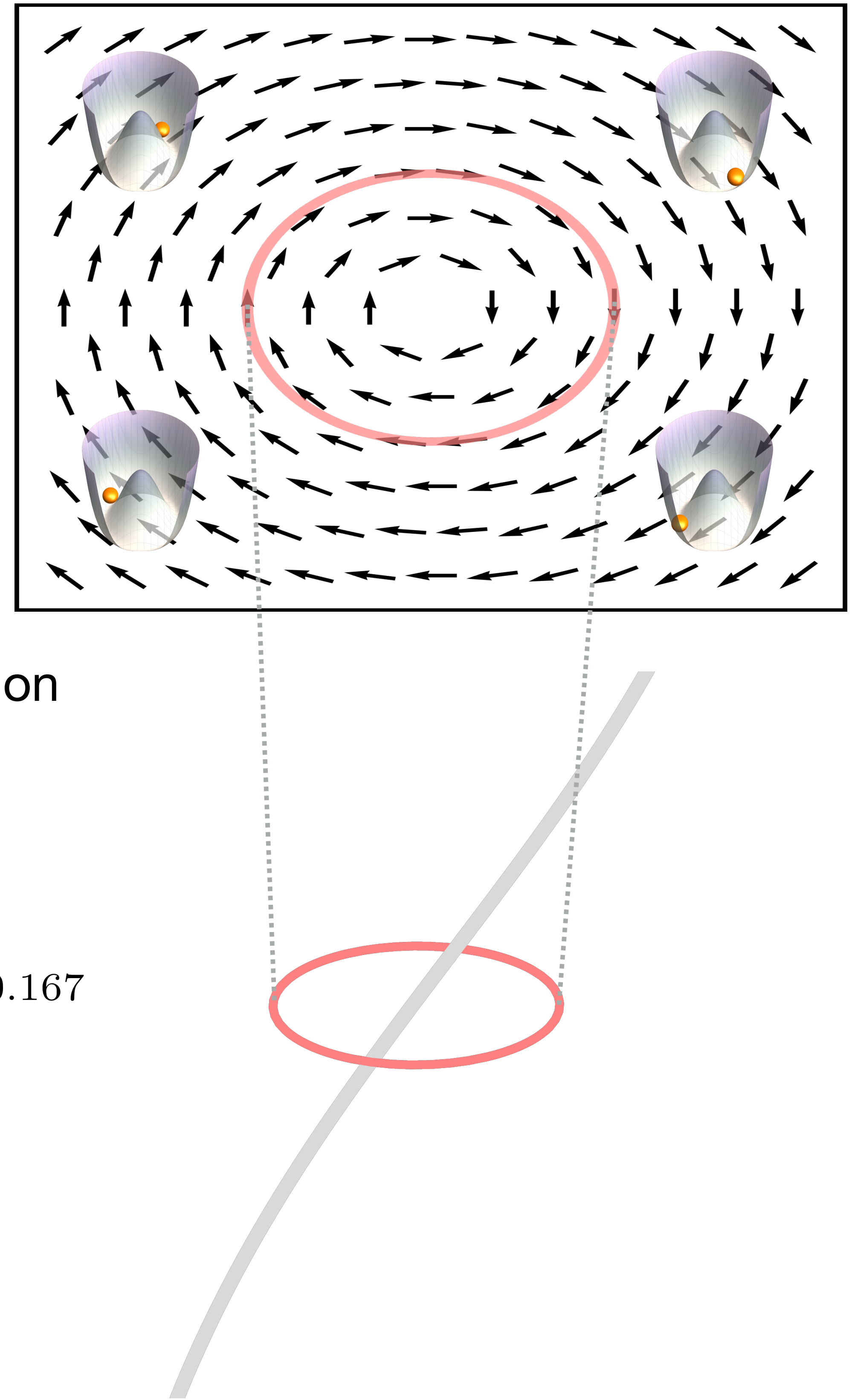
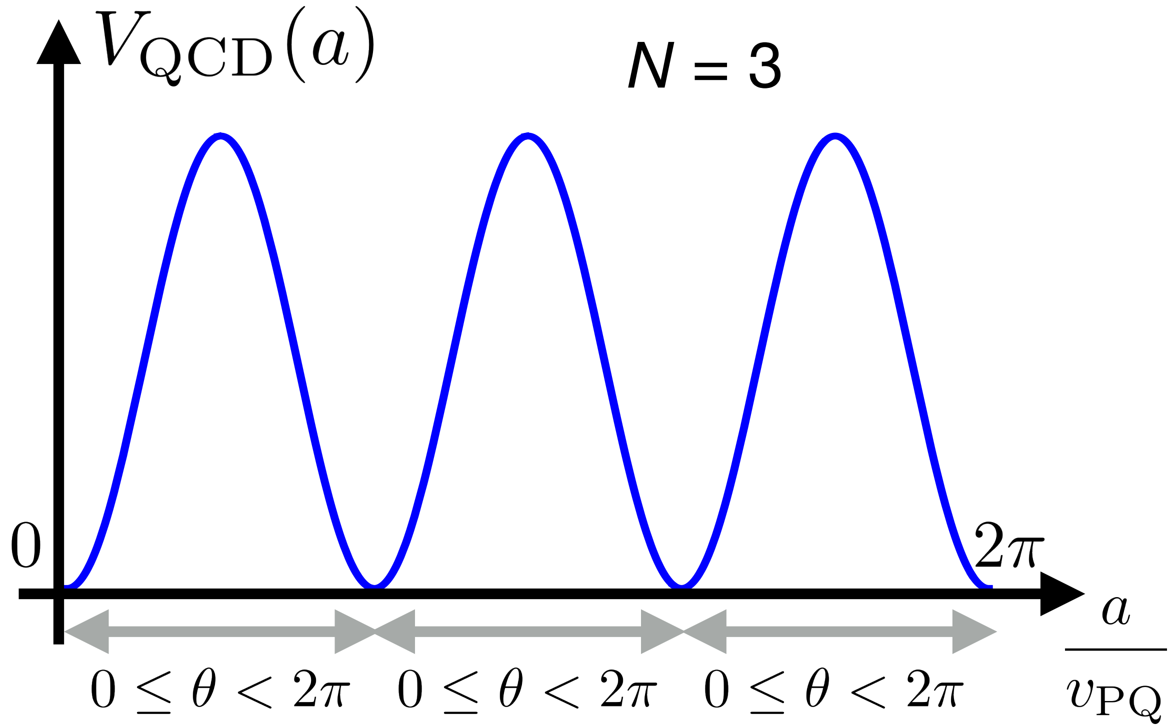


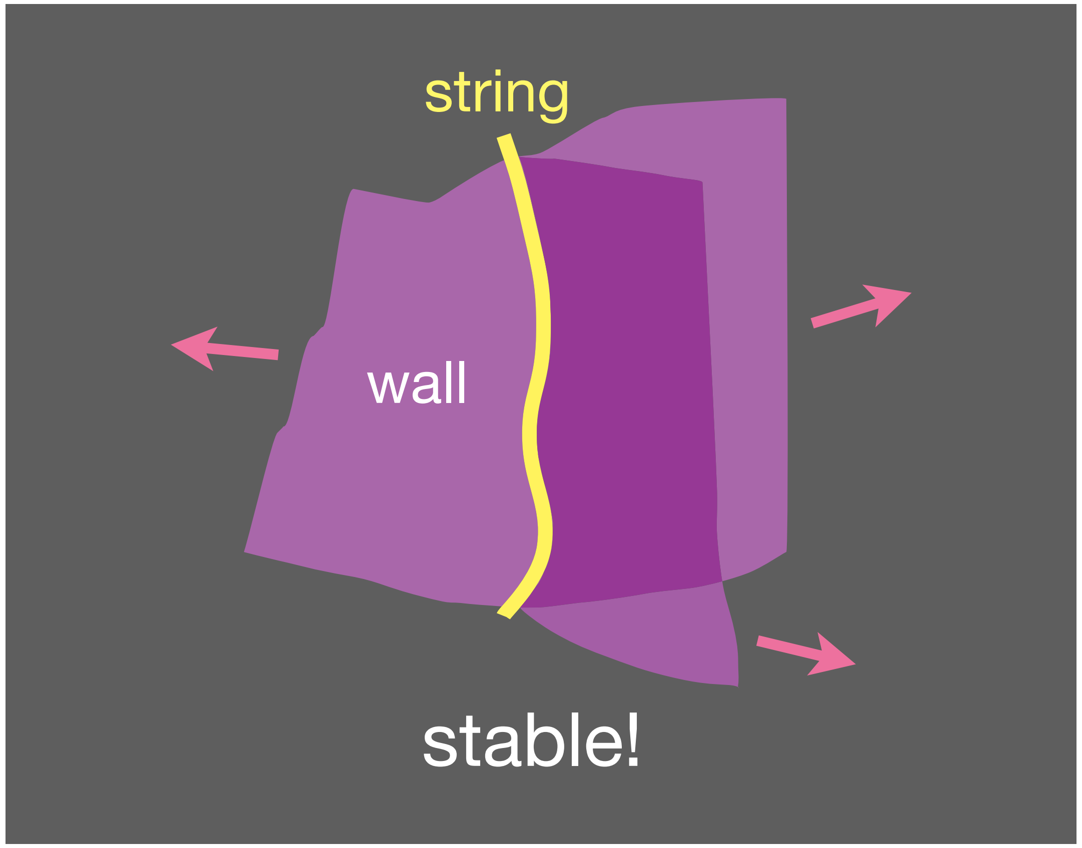
Domain wall problem

[Sikivie (1982)]



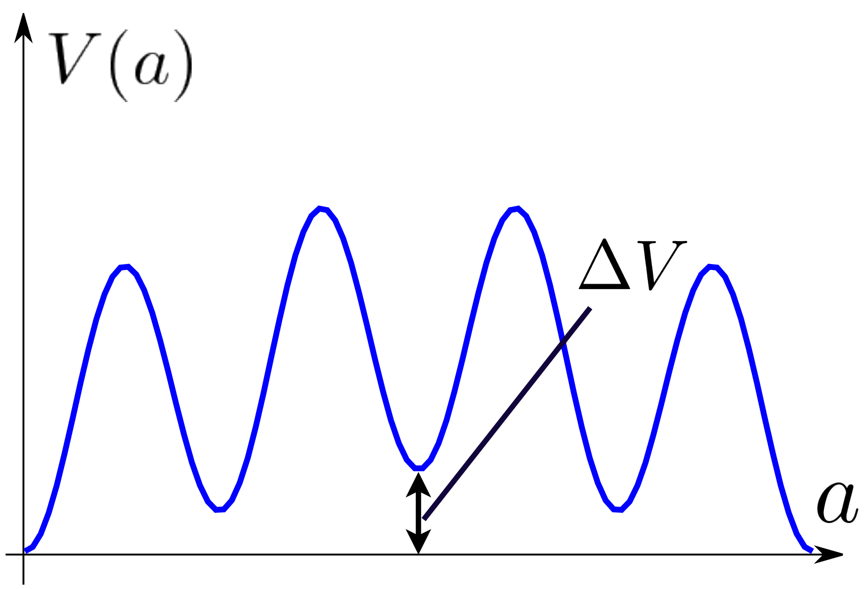
v_{PQ} : symmetry breaking scale

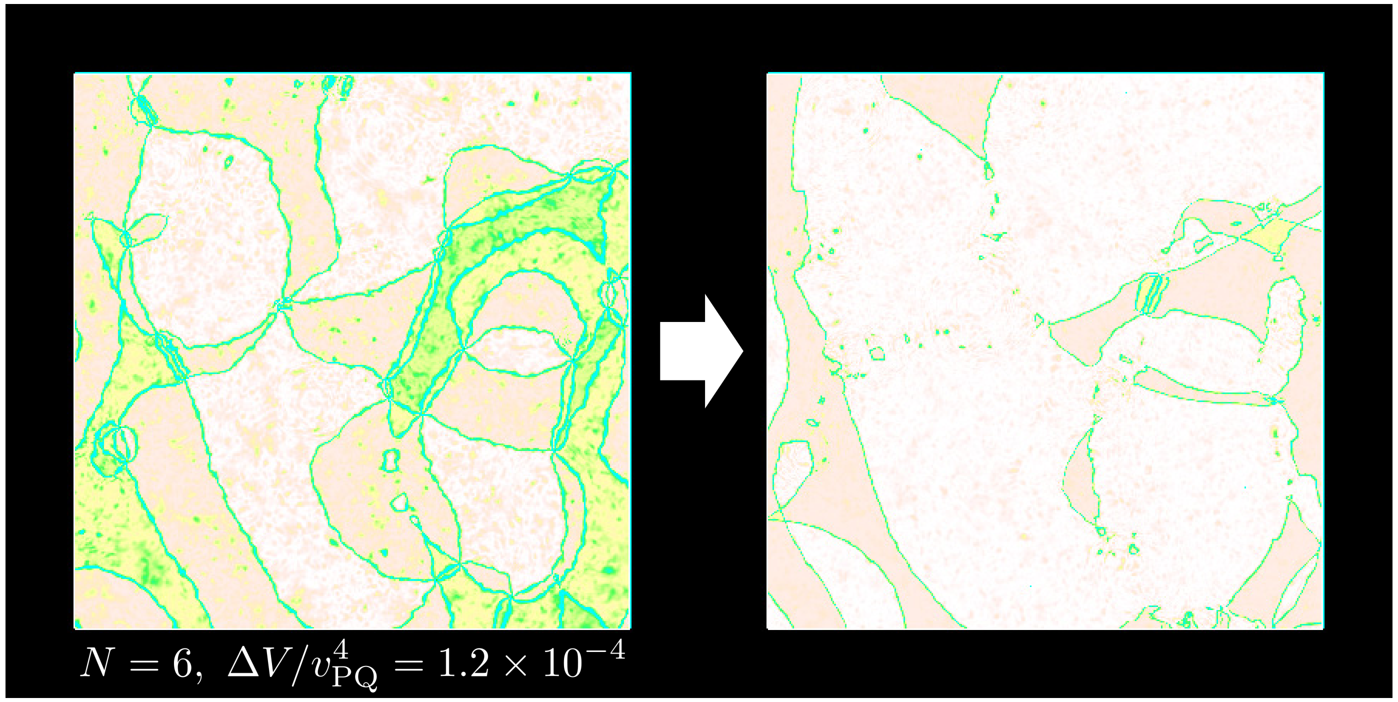
N = 3



- The low-energy axion potential has N degenerate minima, where the "domain wall number" N is a model-dependent integer.
- Models with $N > 1$ are basically ruled out since domain walls are stable and come to overclose the universe.

Axion cosmology with $N > 1$: biased domain walls





[Kawasaki, KS, Sekiguchi, 1412.0789]

- The domain wall problem can be avoided if there exists an additional term which explicitly breaks the PQ symmetry and lifts degenerate minima, making walls unstable.
- Unstable but long-lived domain walls may lead to higher dark matter mass due to an enhancement in the dark matter abundance.

[Kawasaki, KS, Sekiguchi, 1412.0789; Ringwald, KS, 1512.06436; Gorghetto, Hardy, 2212.13263]

- Possibility of primordial black hole formation.

[Ferrer, Masso, Panico, Pujolas, Rompineve, 1807.01707; Gelmini, Simpson, Vitagliano, 2207.07126]

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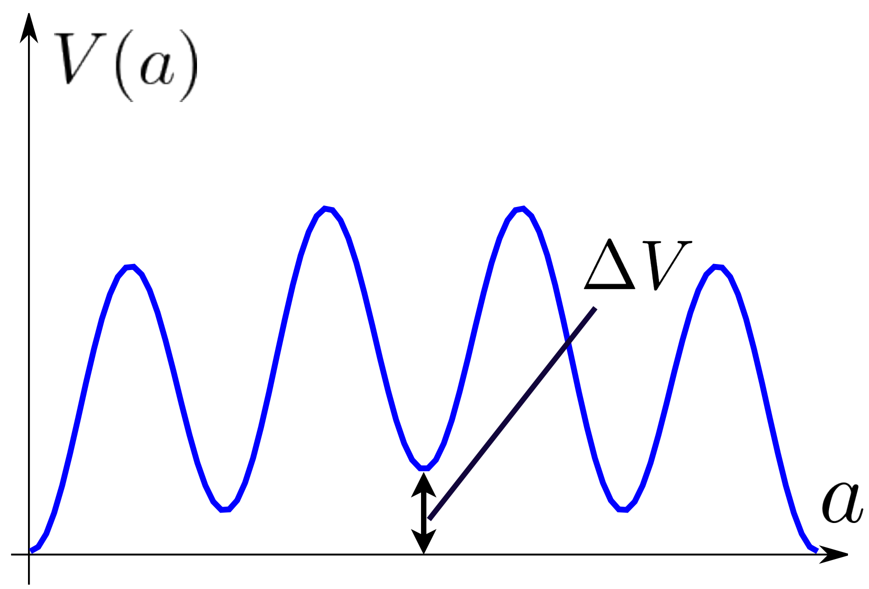

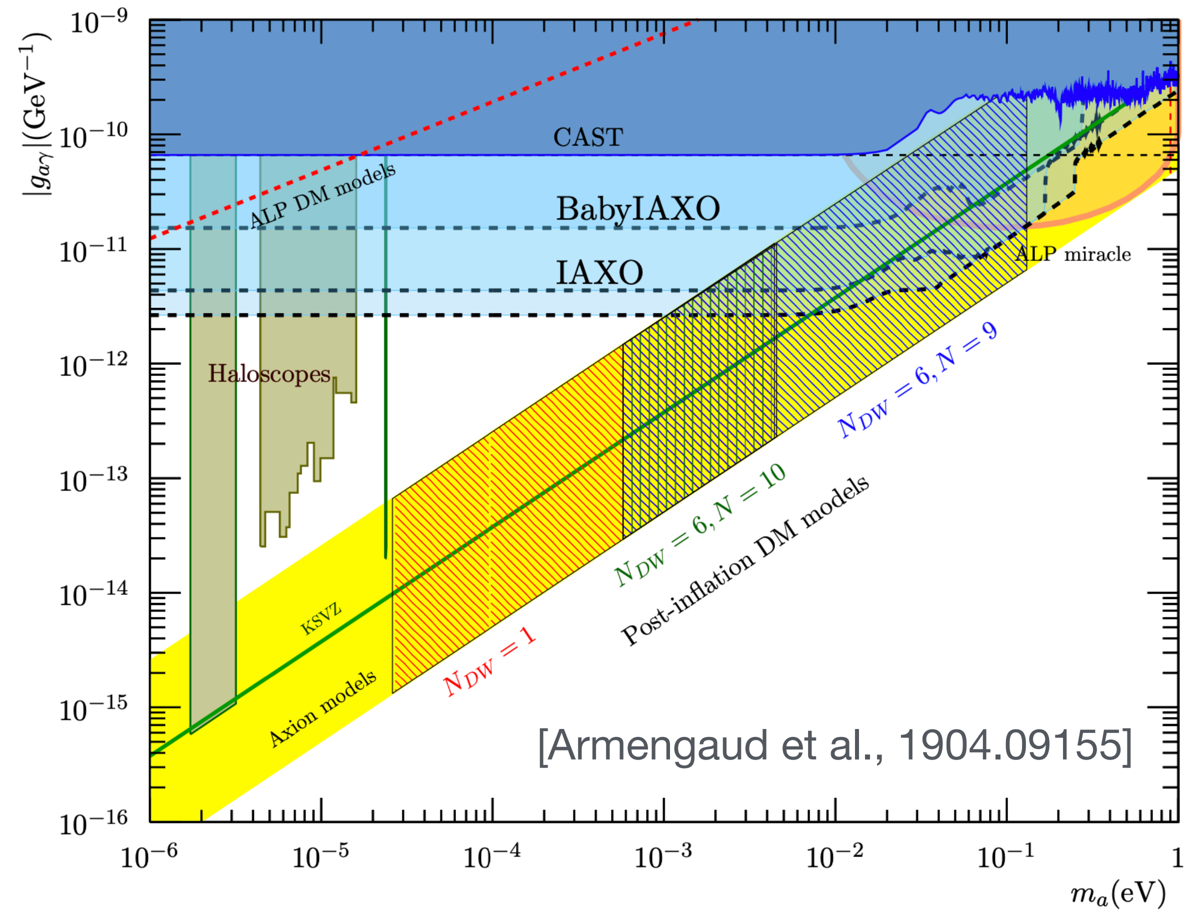

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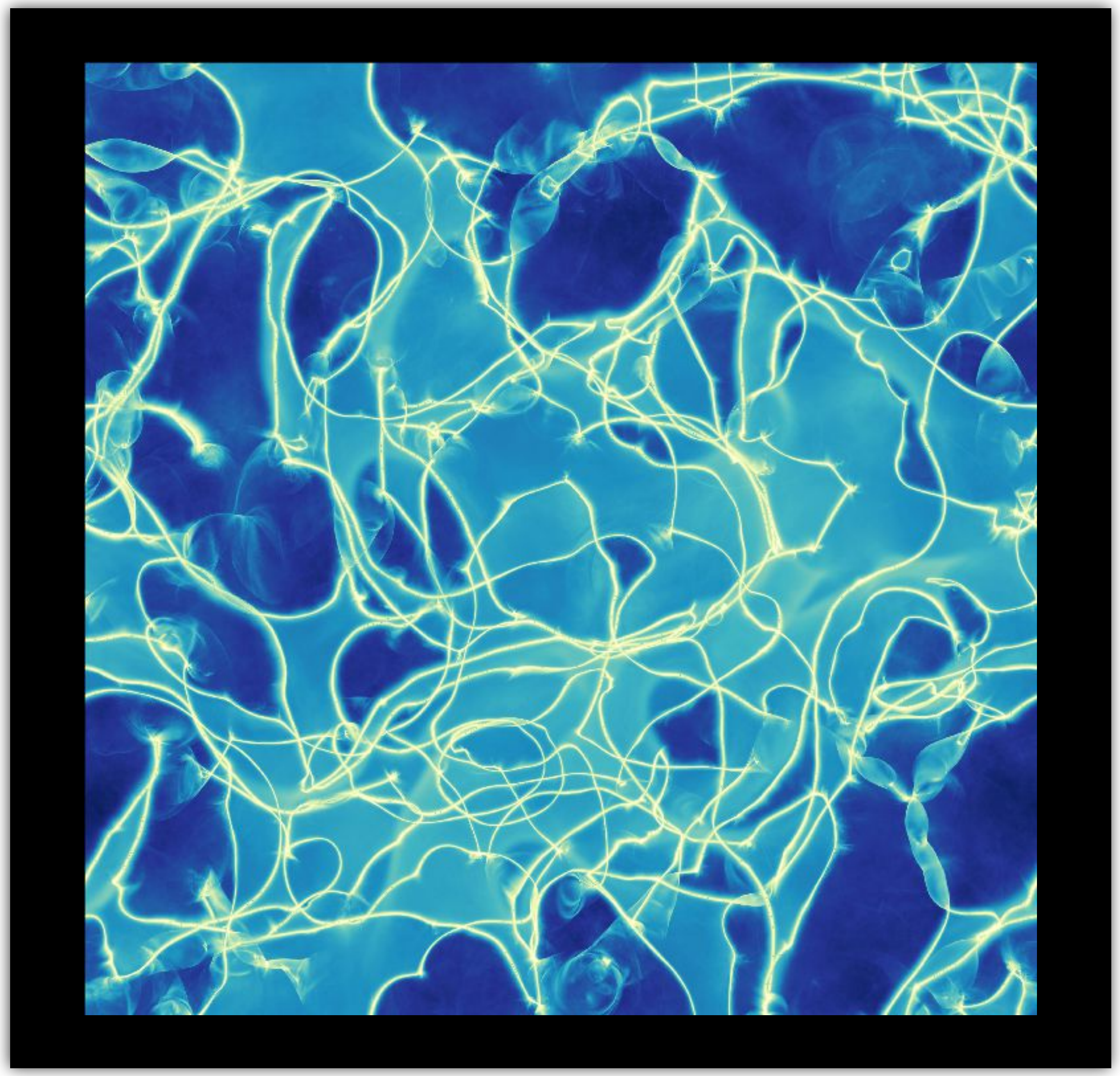
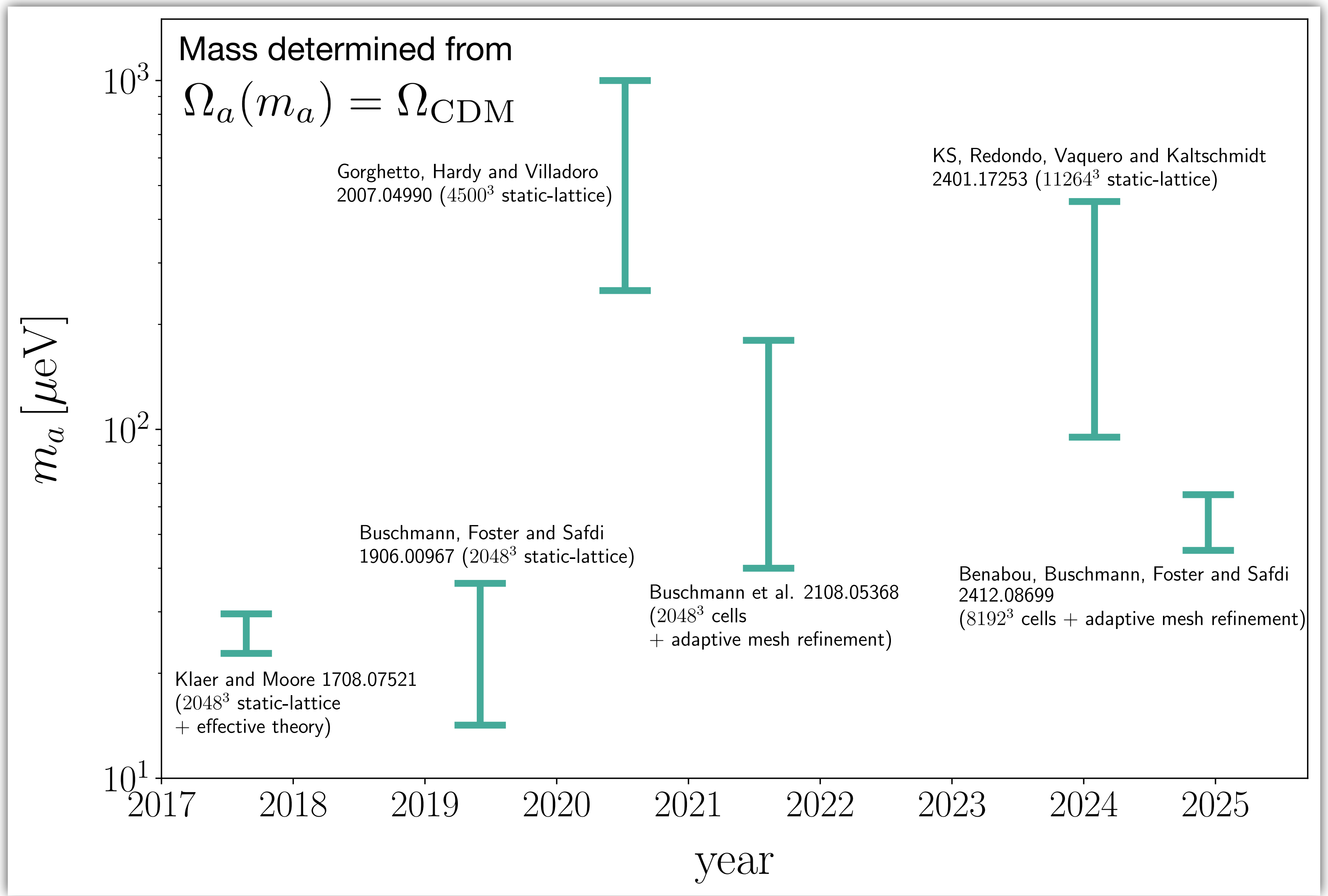
$N = 1$: a predictive scenario?



- No domain wall problem.
- One should be able to predict the dark matter mass uniquely (only one free parameter, f_a).
- Controversy on the interpretation of results from axion string simulations.

$N = 1$: a predictive scenario?



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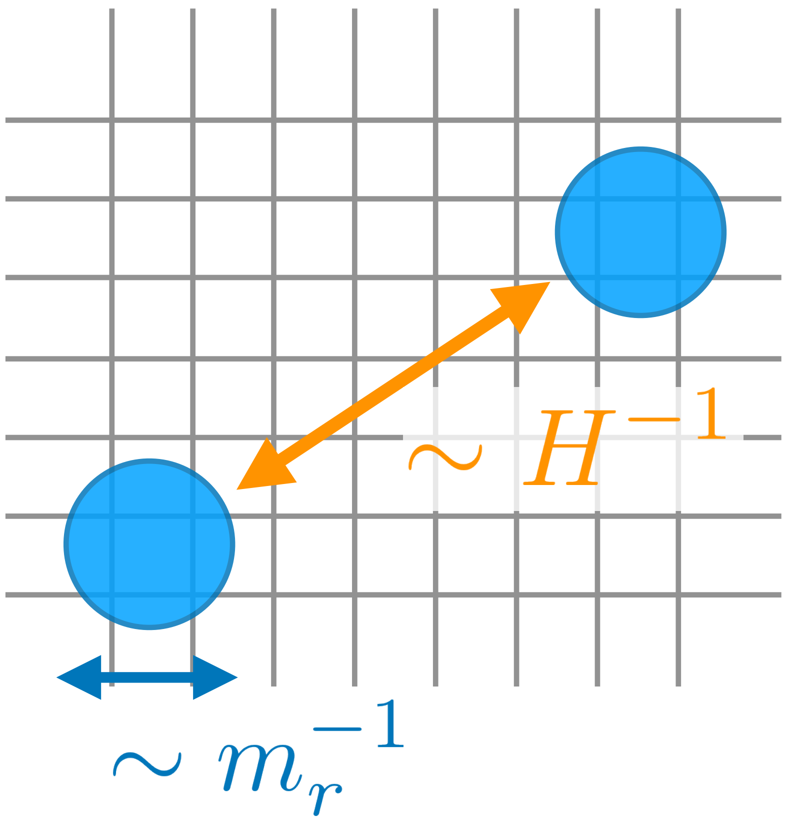
How to simulate axion strings

- Solve the classical EOM for a complex scalar field (PQ field) in comoving coordinates, discretized as lattice.

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{R^2}\nabla^2\phi + \lambda\phi\left(2|\phi|^2 - f_a^2\right) = 0$$

R : scale factor



- Simulation requires a proper resolution of two different length scales.
  - String core radius

$$\sim m_r^{-1} \sim f_a^{-1}$$

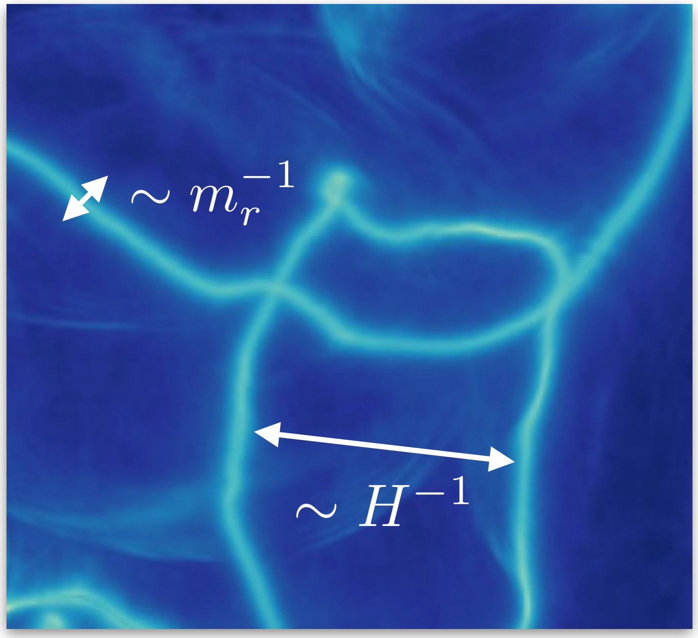
$m_r = \sqrt{2\lambda} f_a$: mass of the radial direction

  - Hubble radius

$$\sim H^{-1}$$

Difficulty in string dynamics

- Existence of two extremely different length scales



- String tension acquires a logarithmic correction.

$$\mu = \frac{\text{energy}}{\text{length}} \simeq \pi f_a^2 \ln\left(\frac{m_r}{H}\right)$$

- Realistic value

$$f_a / H_{\rm QCD} \sim 10^{30} \quad \Rightarrow \quad \ln(m_r/H) \sim 70$$

- Impossible to reach it in simulations with limited dynamical ranges. One has to perform extrapolation by many orders of magnitude.

Strategy for extrapolation

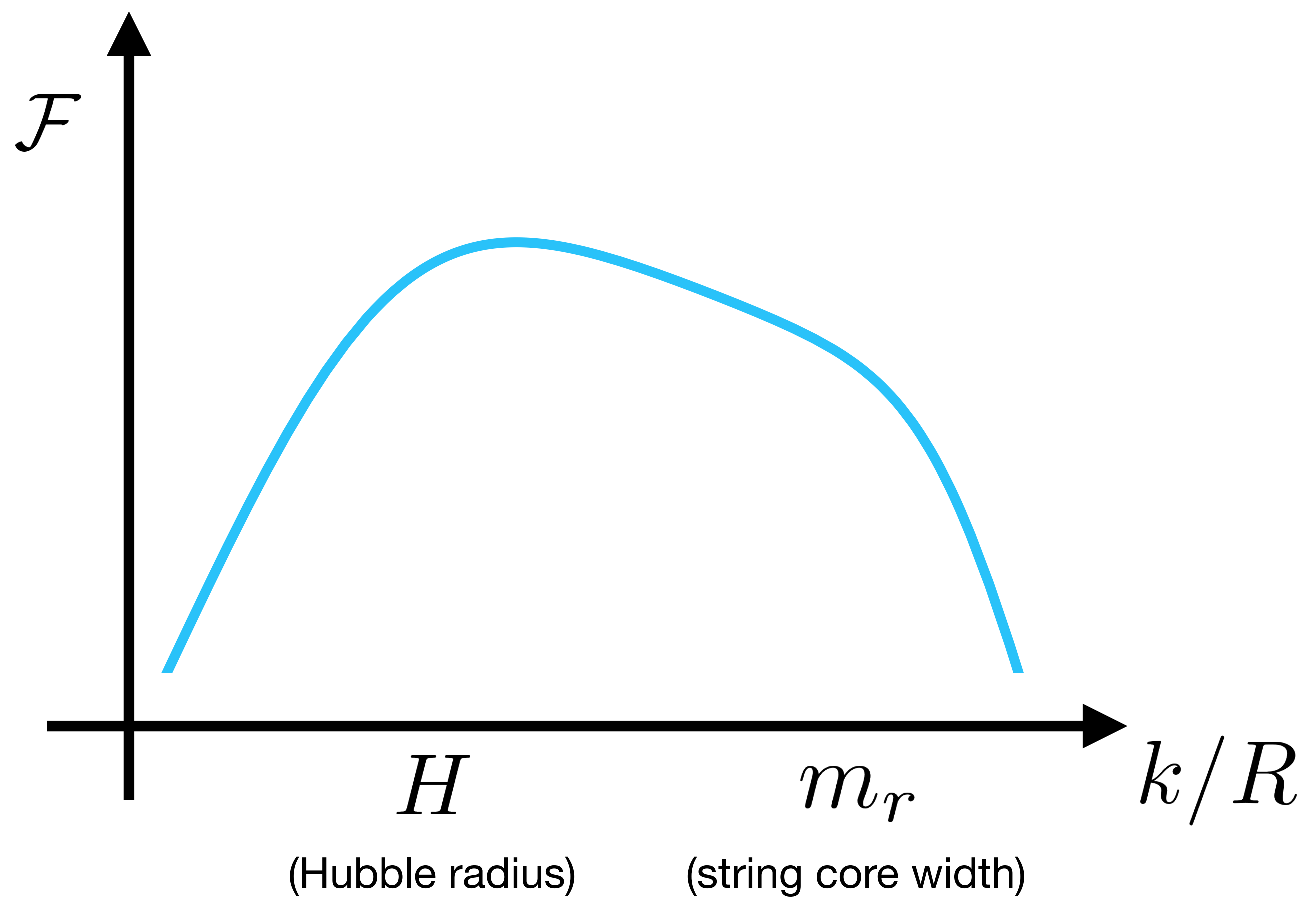
1. Find the attractor (scaling) solution of the string network.
2. Characterize the spectrum of axions on the attractor by introducing some parameters (e.g. string density ξ, spectral index q, ...).
3. Measure how the parameters evolve with $\ell \equiv \ln(m_r/H)$ in the simulation (as accurate as possible).
4. Extrapolate to large ℓ.

Spectrum of radiated axions

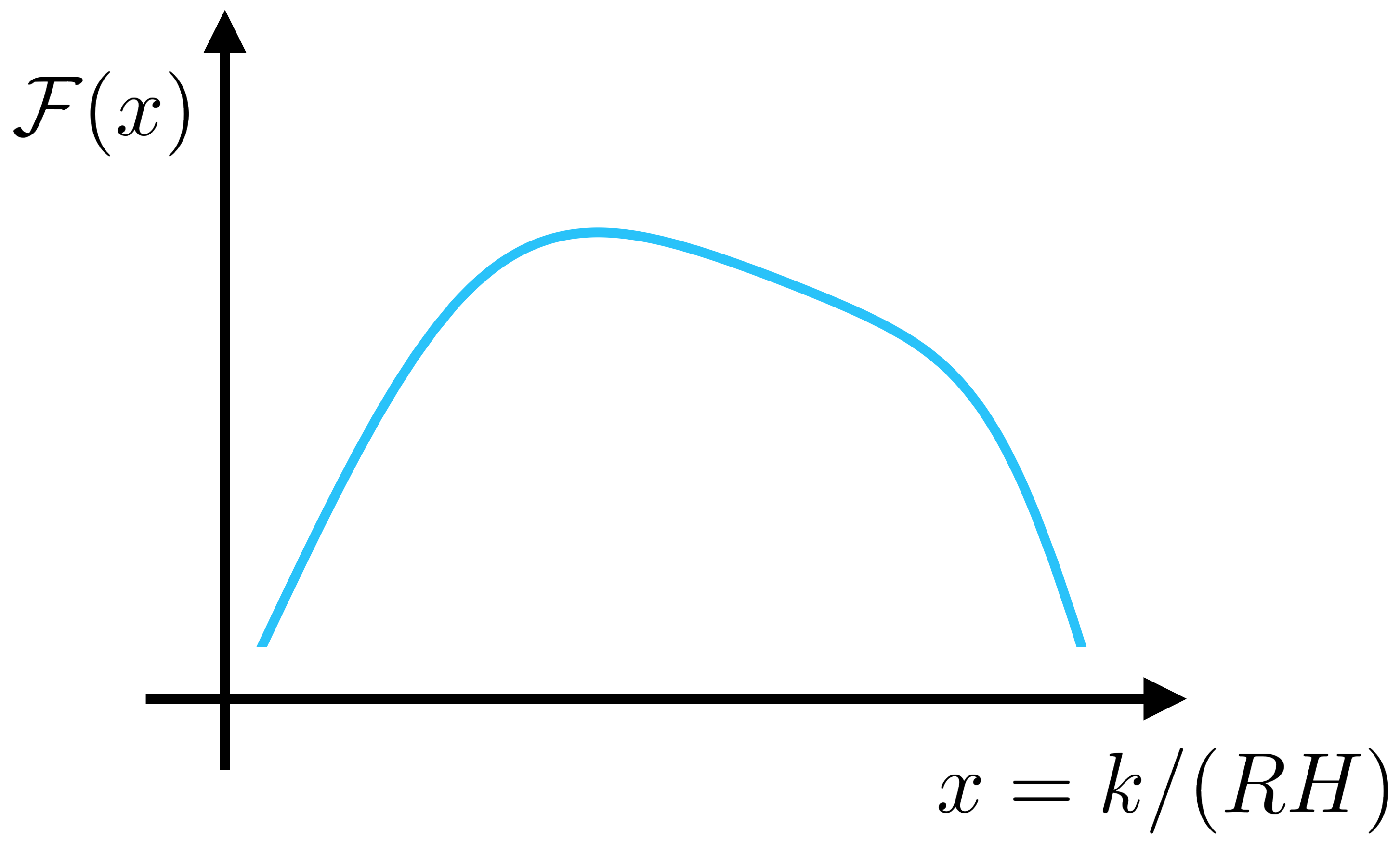
- Instantaneous emission spectrum [Gorghetto, Hardy, Villadoro, 1806.04677]

$$\mathcal{F}\left(\frac{k}{RH}, \frac{m_r}{H}\right) \equiv \frac{1}{(f_a H)^2} \frac{1}{R^3} \frac{\partial}{\partial t} \left(R^4 \frac{\partial \rho_a}{\partial k}\right)$$

- $\mathcal{F}$ seems to be well approximated by a simple power law, with IR and UV cutoffs at $k/R \sim H$ and $k/R \sim m_r$.



Characterizing the emission spectrum



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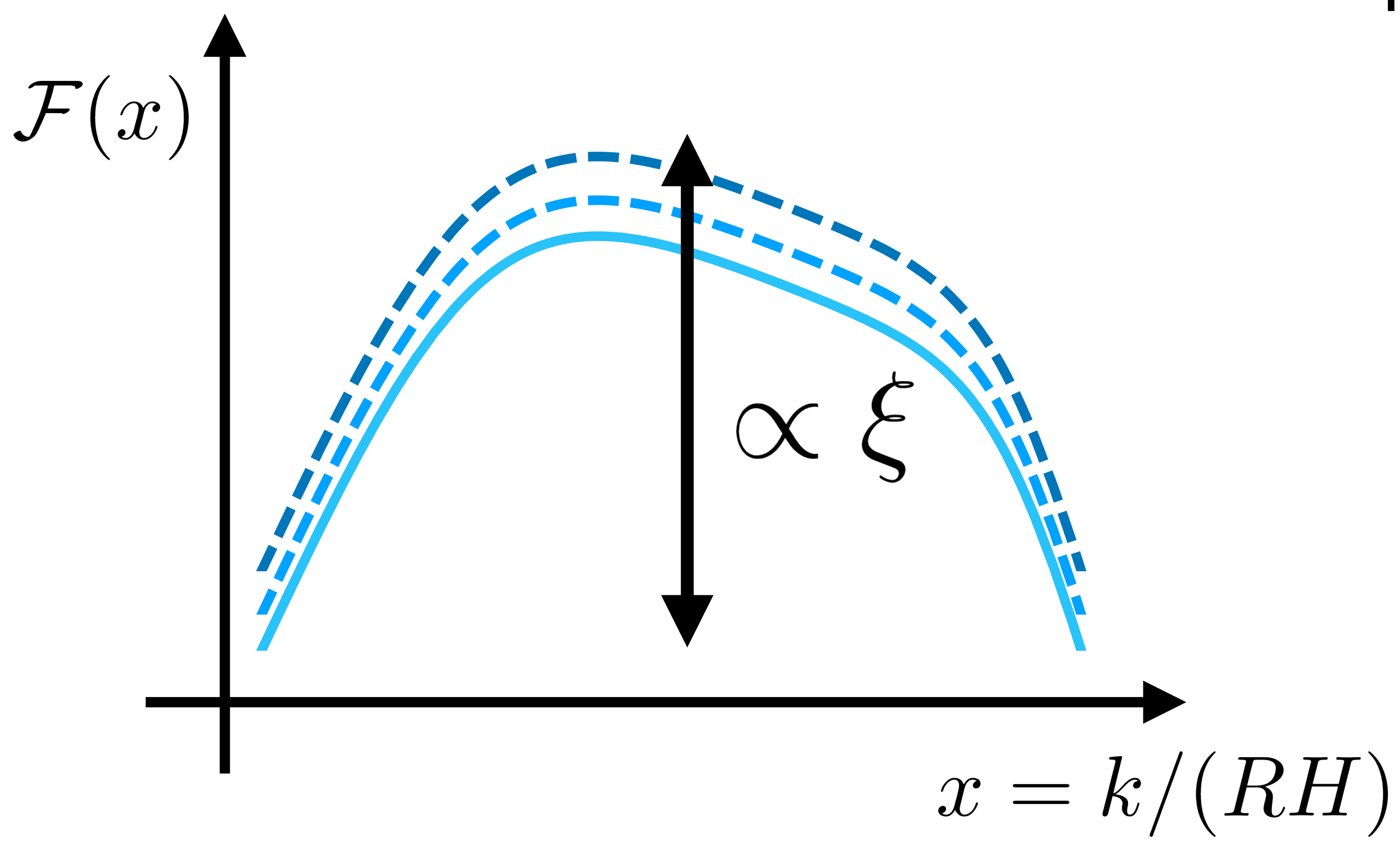


1. String density ξ

$$\xi \equiv \frac{\ell_s}{\mathcal{V}} t^2$$

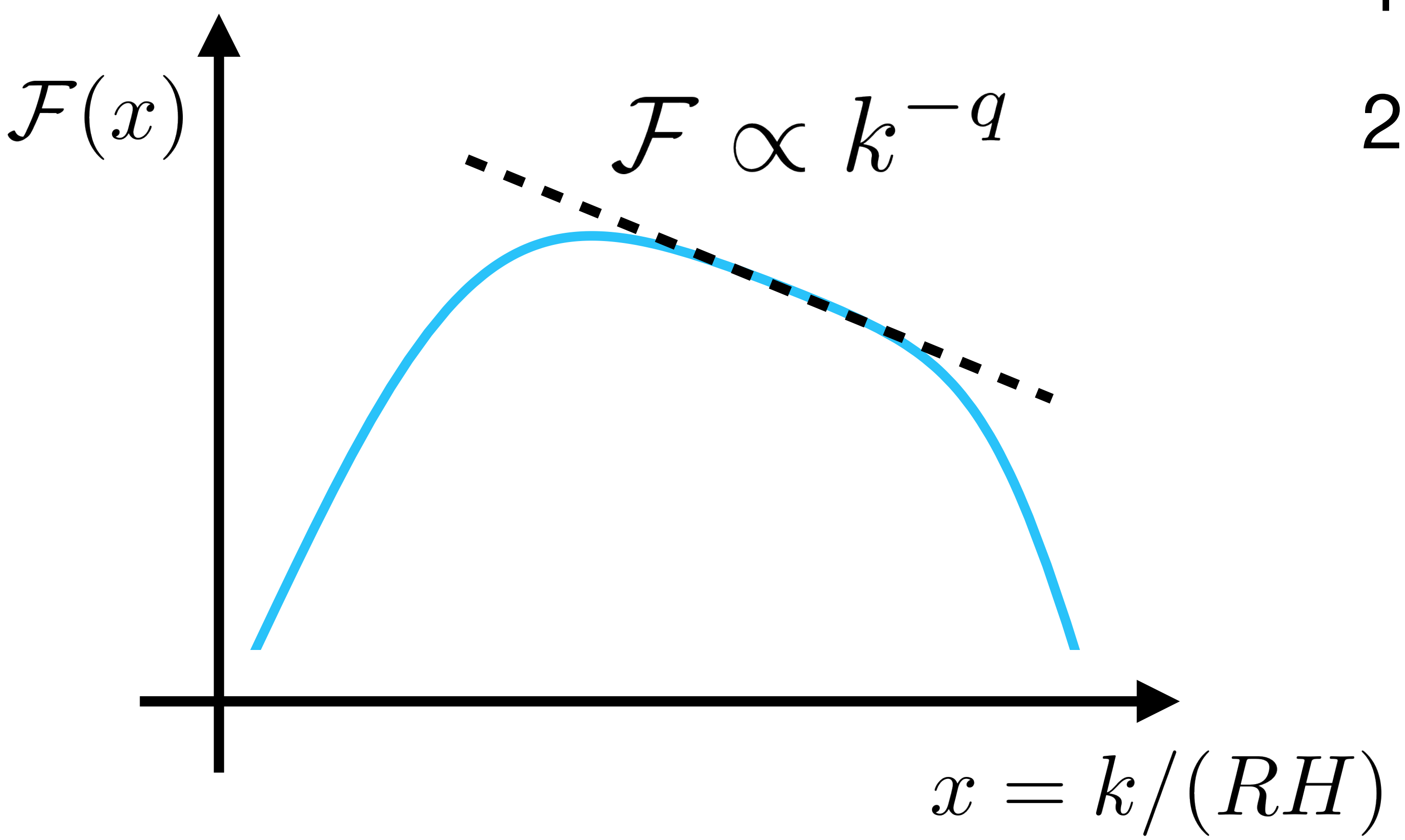
ℓ_s : string length

$\mathcal{V}$: spatial volume

Related to the energy density of strings as

$$\rho_{\rm string} = \xi \frac{\mu}{t^2}$$

Characterizing the emission spectrum



1. String density ξ
2. Spectral index q

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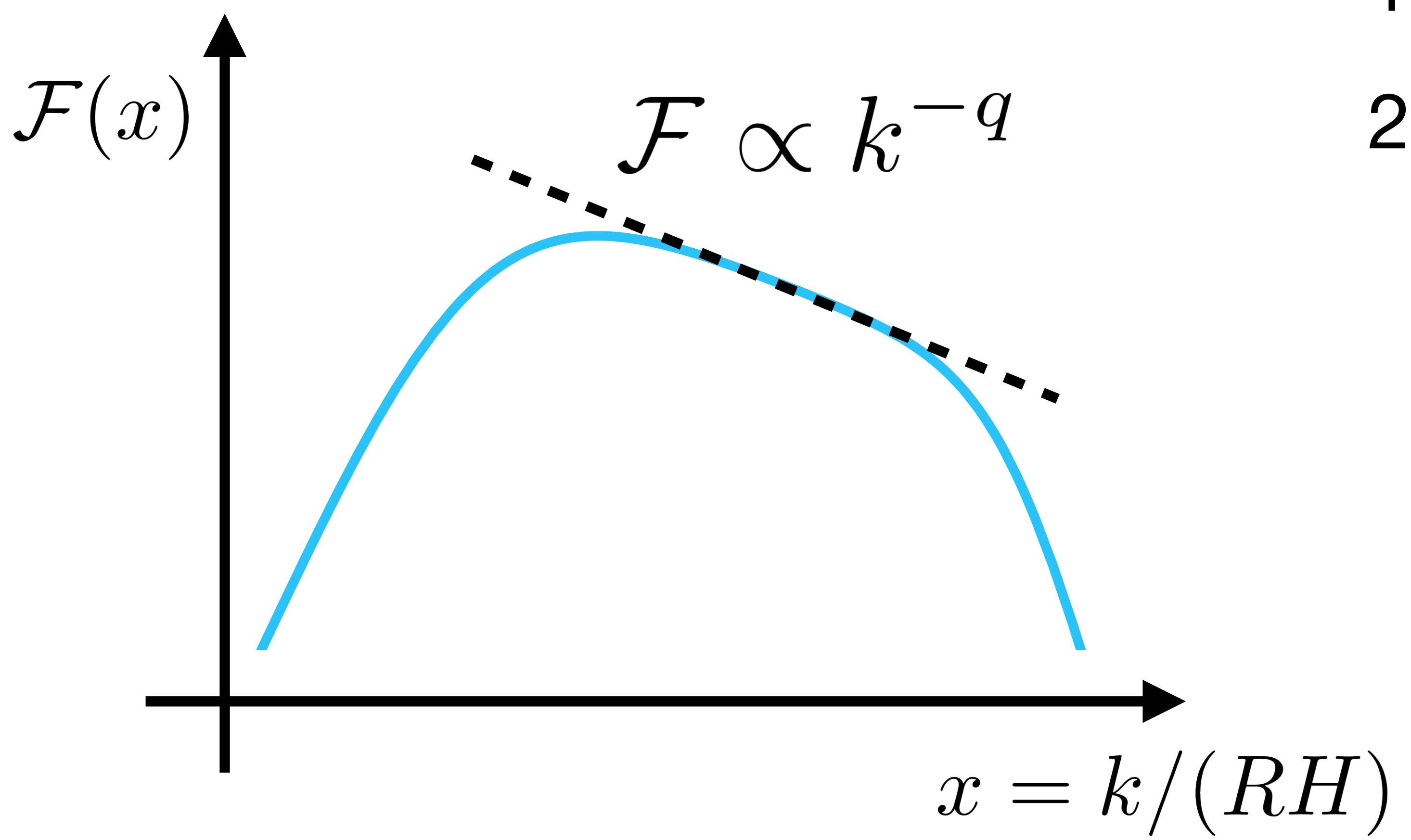

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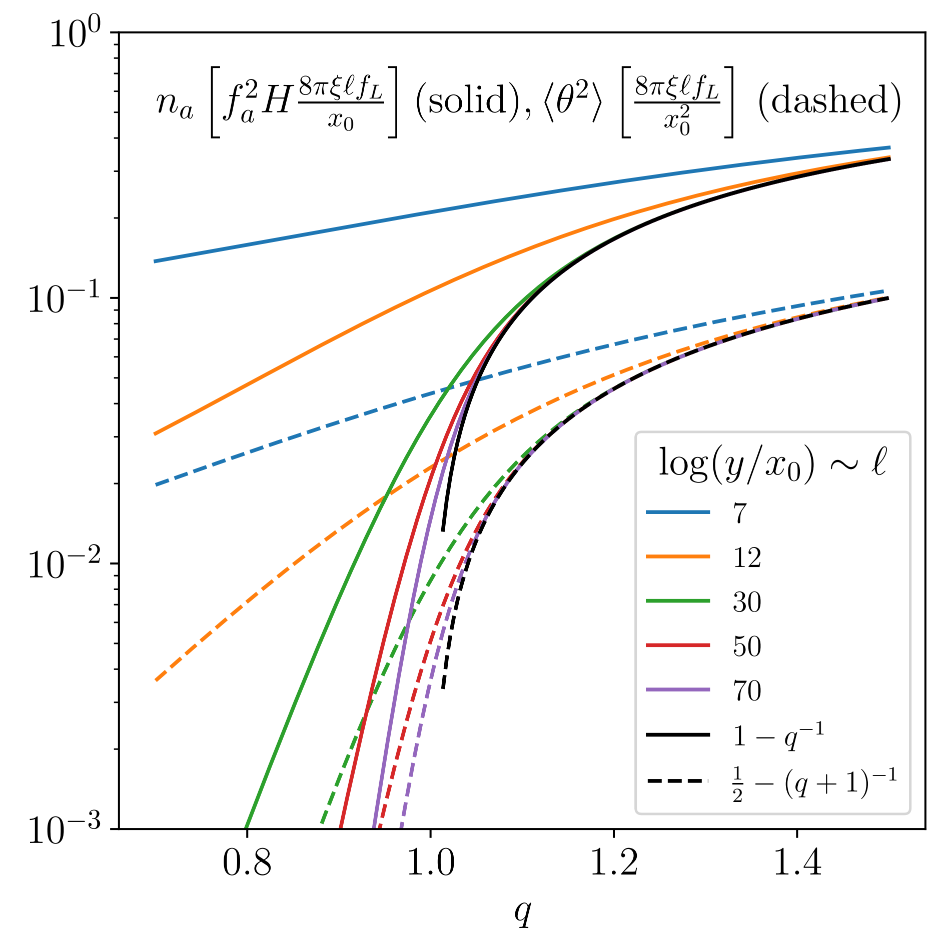
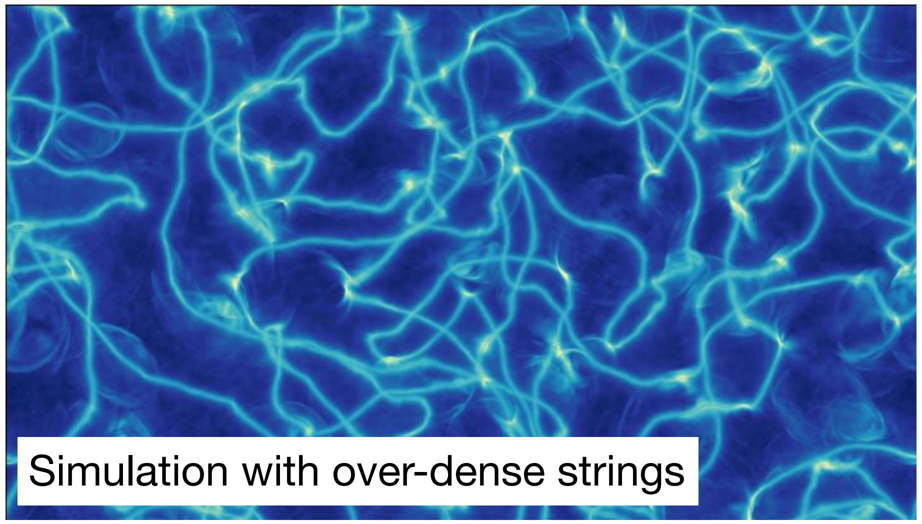
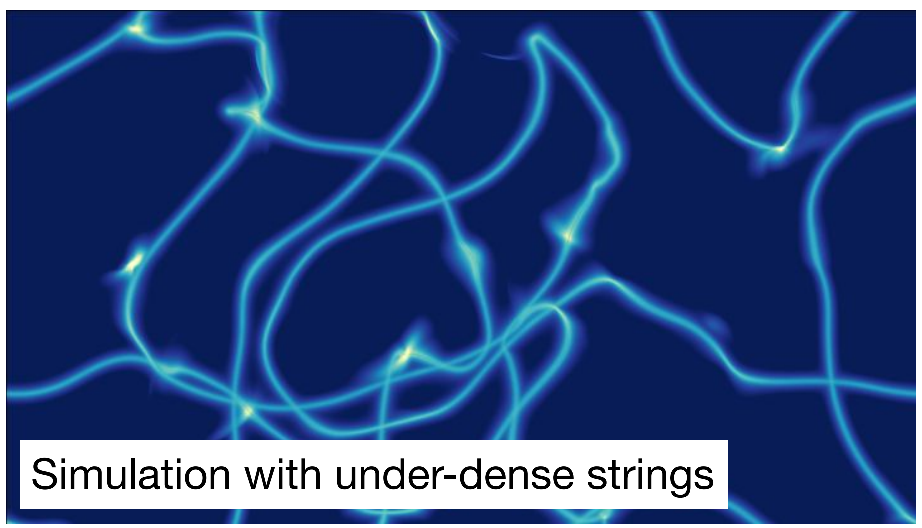
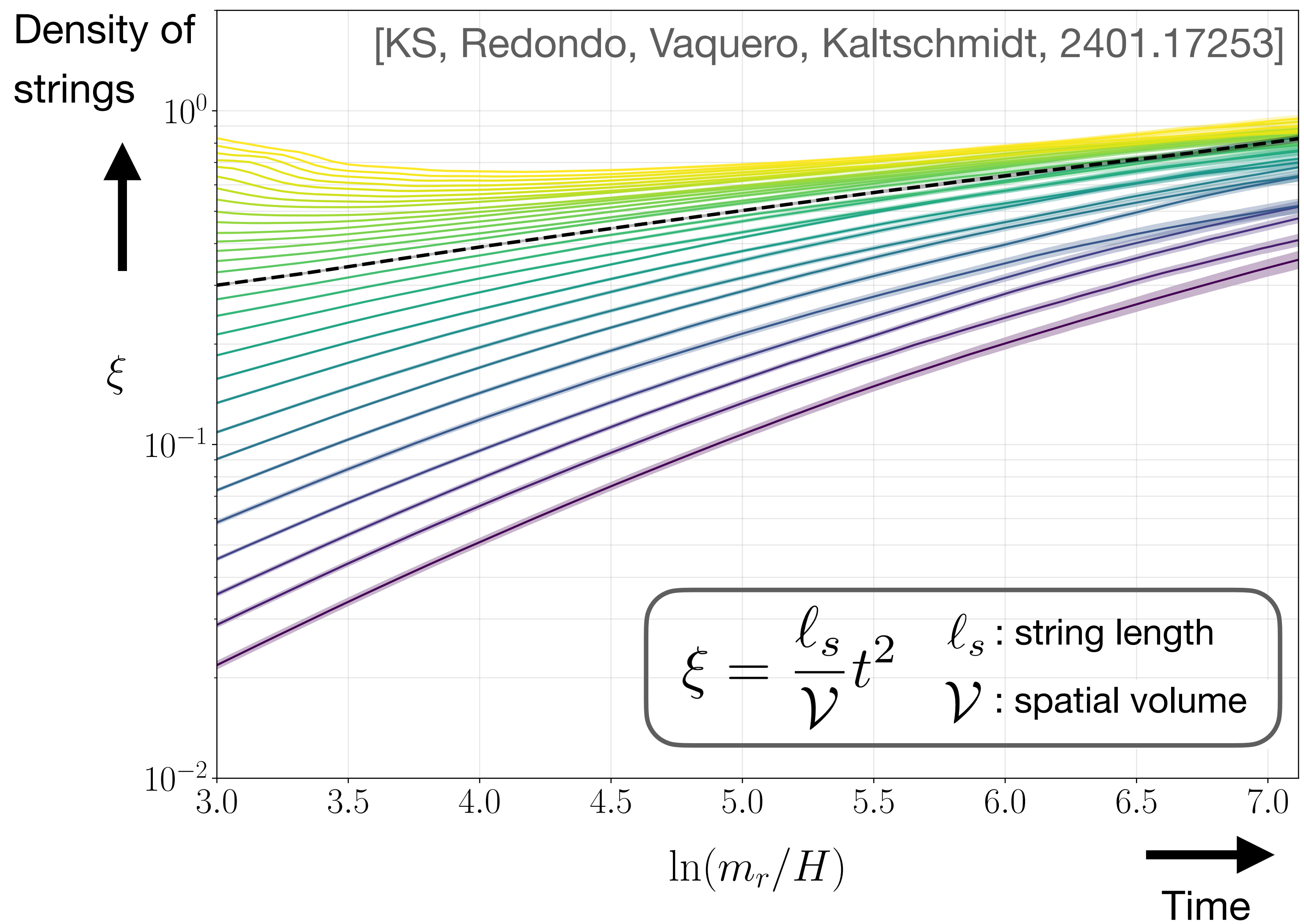


Crucial to see if $q > 1$ or $q \lesssim 1$.

String density and attractor







- Different initial conditions appear to converge.
- Recent simulations observe a logarithmic growth of string density on the attractor.

[Fleury et al. 1509.00026; Gorghetto et al. 1806.04677; Kawasaki et al. 1806.05566; Gorghetto et al. 2007.04990; Buschmann et al. 2108.05368; KS et al. 2401.17253; Kim et al. 2402.00741; Benabou et al. 2412.08699]

- However, full consensus has not been reached.

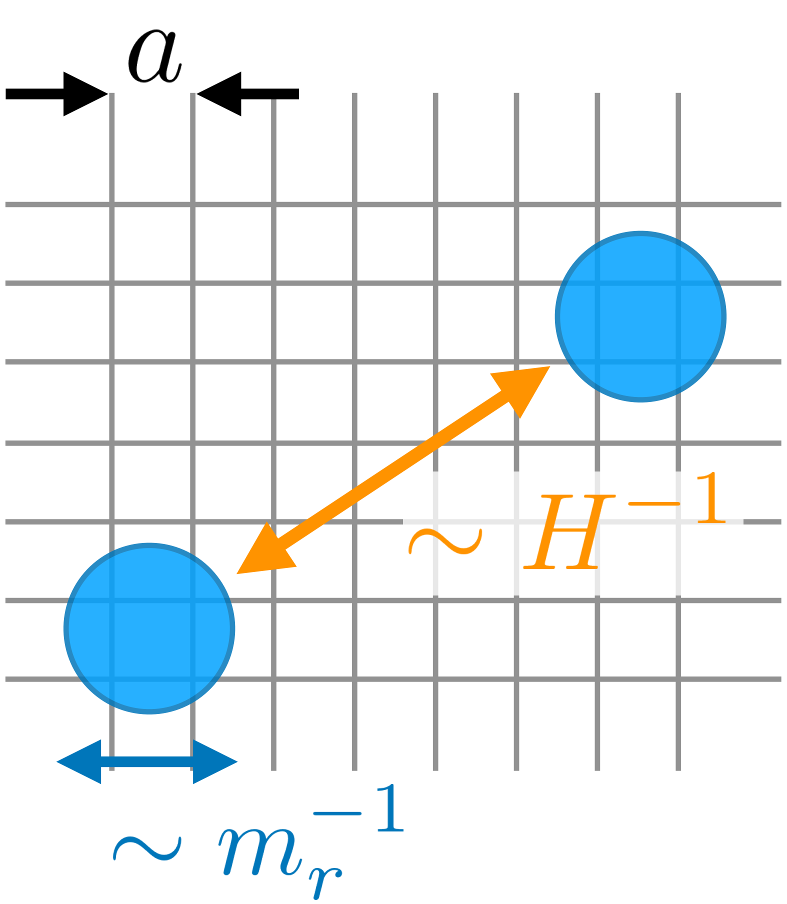
cf. [Hindmarsh et al. 1908.03522; Hindmarsh et al. 2102.07723; Correia et al. 2410.18064]

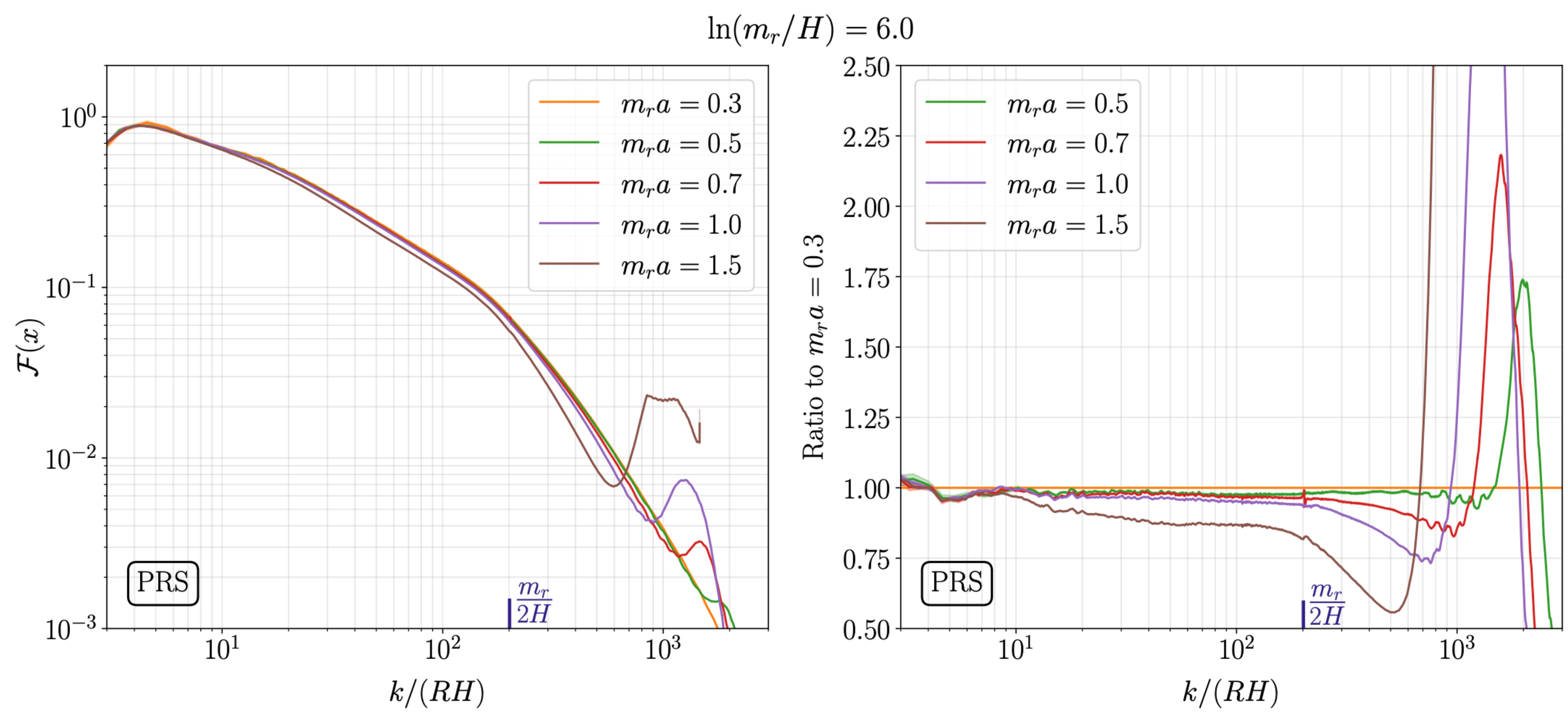
Axion emission spectrum and discretization effects

- The resolution of the string core parameterized by $m_r a$.

$a = R(t)L/N$: lattice spacing

- Larger $m_r a$ can lead to a distortion of the axion spectrum.





[KS, Redondo, Vaquero, Kaltschmidt, 2401.17253]

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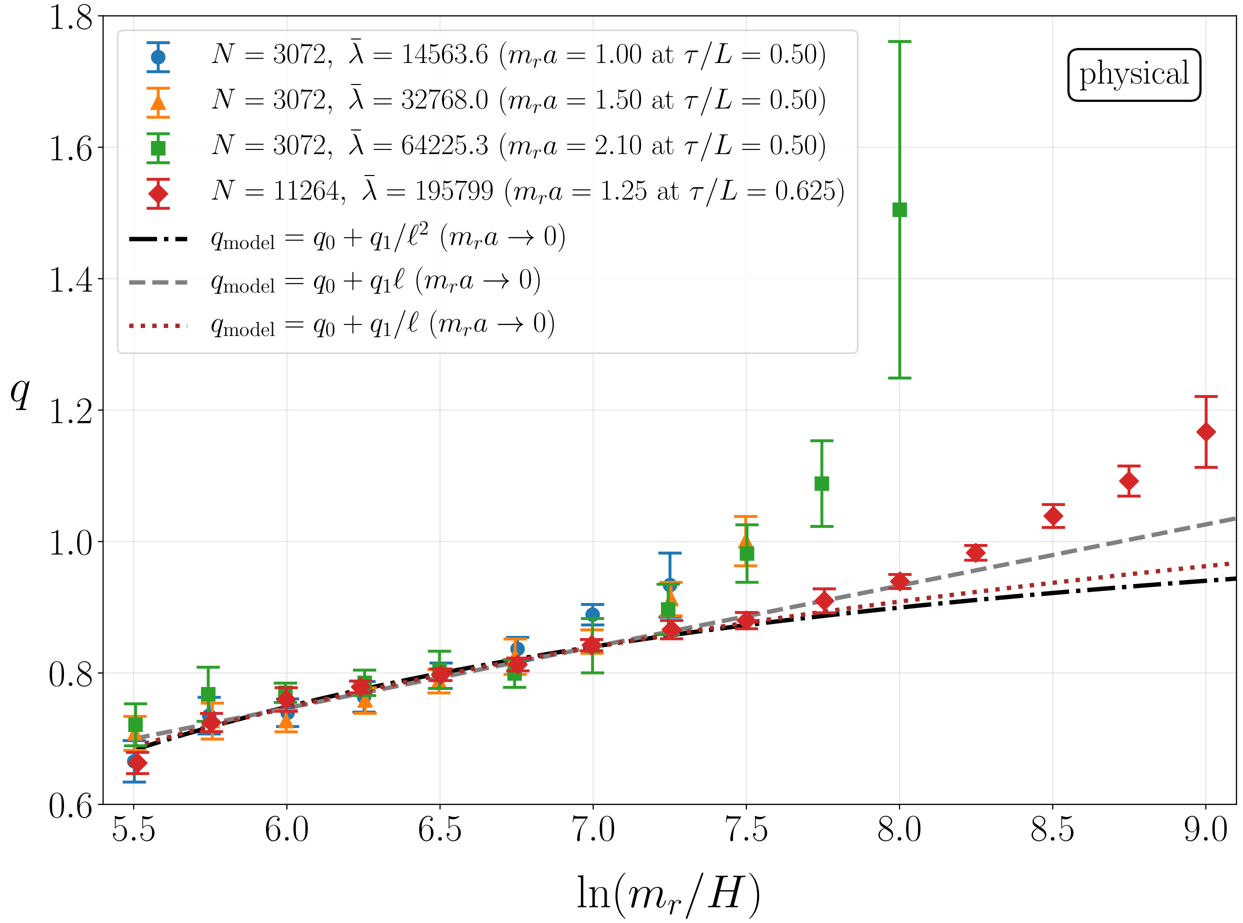
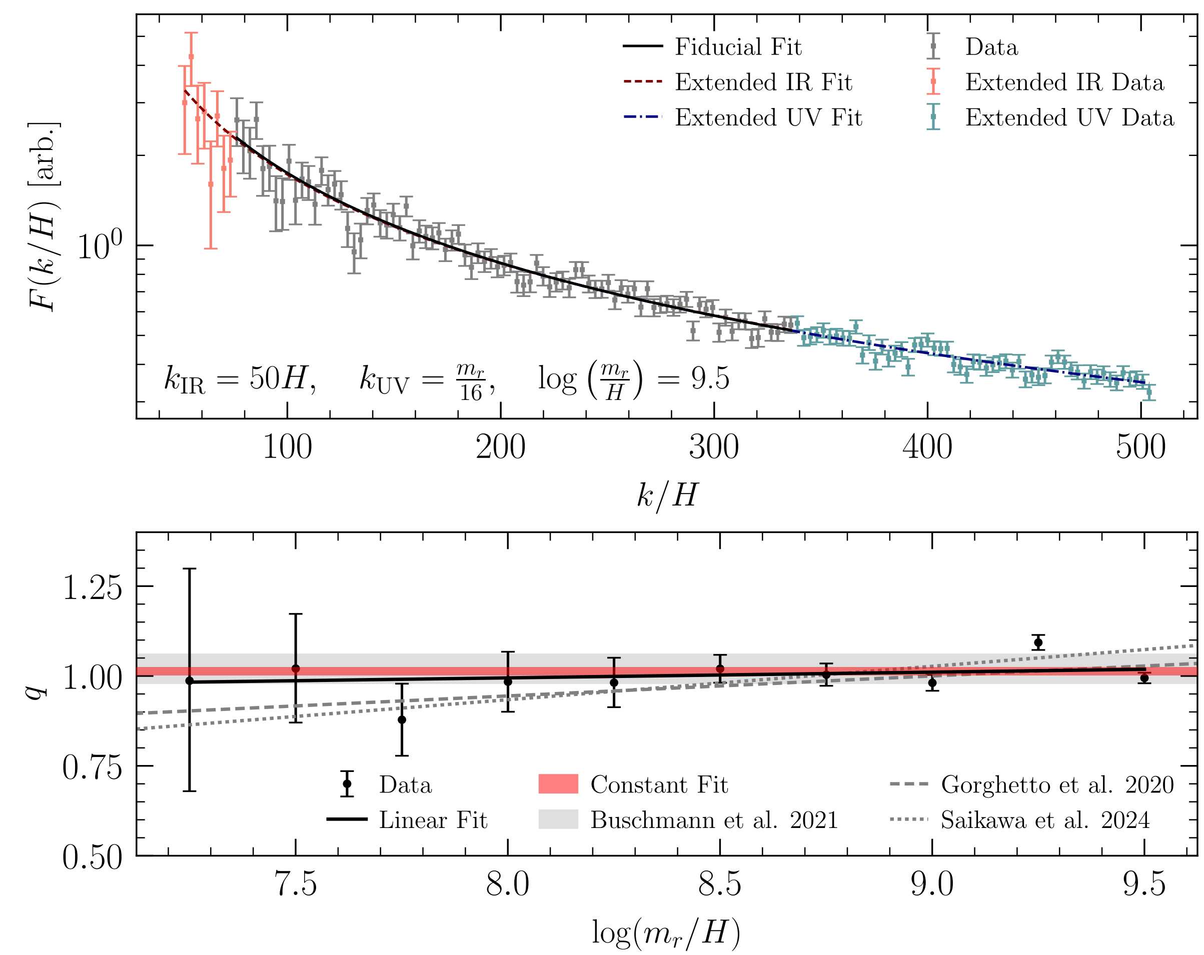
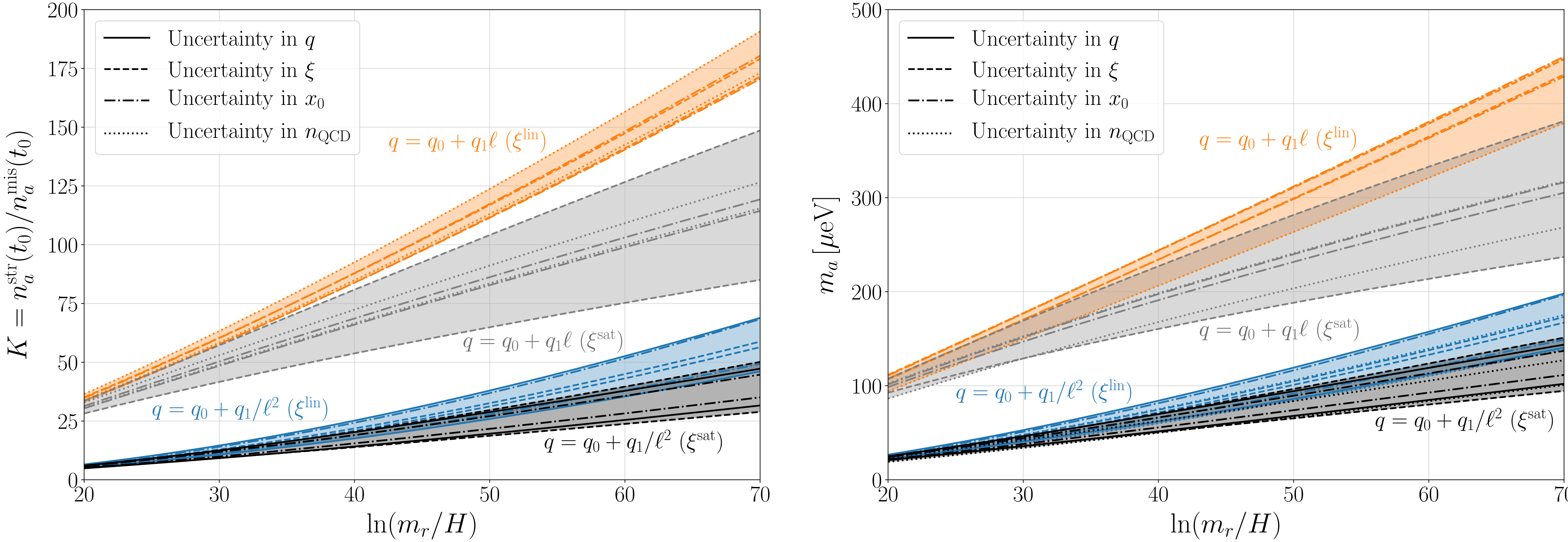


[Benabou, Buschmann, Foster, Safdi, 2412.08699]



- Large $m_r a$ biases q towards larger values.
- The data show a logarithmic increase in q at $\ln(m_r/H) \lesssim 7$, but the behavior at larger $\ln(m_r/H)$ remains uncertain.

Extrapolation and dark matter mass prediction



[KS, Redondo, Vaquero, Kaltschmidt, 2401.17253]

Extrapolating with different models results in different dark matter mass predictions.

- Does ξ continue to increase or saturate at some point?
- Does q continue to increase or saturate at some point?

Emerging opportunities and challenges

- **Opportunities from numerical advancements.**
  Main problem is dynamical range. Developing a more advanced numerical scheme is the principled way to handle even broader dynamical ranges.

  - Largest static lattice size simulated: $16{,}384^3$ [Correia et al., 2410.18064]
  - Employing the adaptive mesh refinement (AMR) technique:
    [Drew, Shellard, 1910.01718; 2211.10184; Buschmann et al., 2102.07723; Drew, Kinowski, Shellard, 2312.07701]

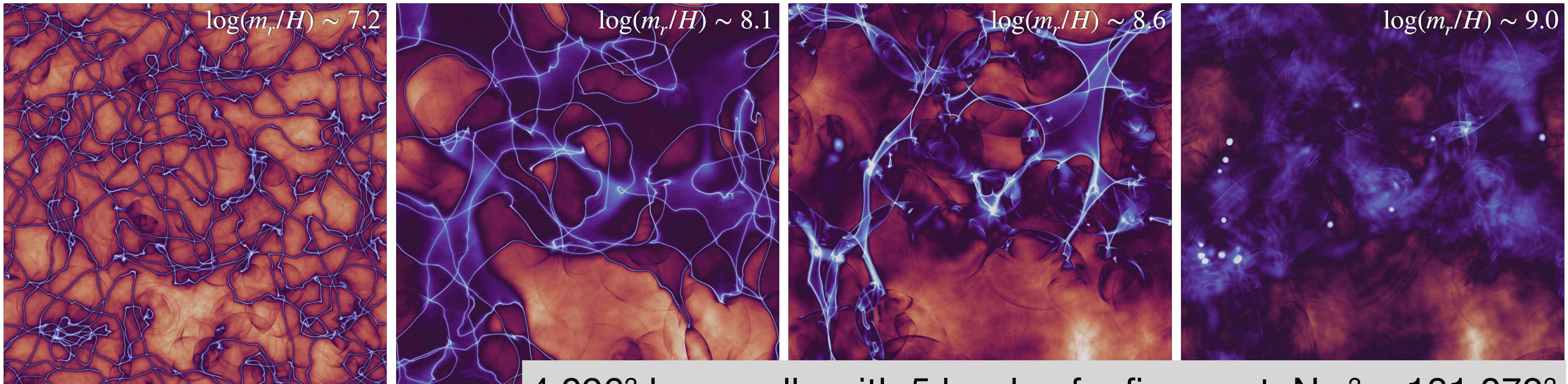
    8192^3 cells with 5 levels of 2x refinement = $262{,}144^3$ effective grid points
    [Benabou et al., 2412.08699]

- **Contribution from domain walls.**
  It received less attention compared to that from strings, but quantifying its relative importance remains necessary. [Benabou et al., 2412.08699]

- **Alternative simulation methods.**
  Use of modified (unphysical) field equations to effectively describe the system under large string tension, at the cost of physical fidelity.
  [Klaer, Moore, 1707.05566; 1912.08058]

Simulations of the network collapse due to domain walls

[Benabou, Buschmann, Foster, Safdi, 2412.08699]



$4{,}096^3$ base cells with 5 levels of refinement: $N_{\mathrm{eff}}^3 = 131{,}072^3$

Figure S10. 3D→2D projection of the total axion energy density $\rho_a = \frac{1}{2}\dot{a}^2 + \frac{1}{2}(\nabla a)^2 + V(a)$ from a simulation of the QCD phase transition. From left to right: (1) Axion string network before the QCD phase transition. (2) The beginning of the phase transition. The axion mass starts growing and domain walls form. (3) The extra tension from the domain walls causes the network to collapse. (4) The QCD phase transition is complete. All strings have vanished and just a few oscillons are visible as bright point-like spots, though we do not investigate these features in this work. An animated version of this simulation can

- Adding the QCD potential (temperature-dependent axion mass) leads to the formation of domain walls and their collapse.

$$V(\phi) = \lambda \left(|\phi|^2 - \frac{f_a^2}{2} \right)^2 + m_a(T)^2 f_a^2 \left[1 - \frac{\sqrt{2}|\phi|}{f_a} \cos \mathrm{Arg}(\phi) \right]$$

- Recently it has been claimed that accounting for domain wall production could increase the dark matter abundance by an additional factor ~ 5.

Alternative methods to study the effect of string tension

- Simulation method to effectively increase the string tension by adding more fields. [Klaer, Moore, 1707.05566; 1708.07521]

$$\mu \simeq 2\pi v^2 \quad \text{with} \quad f_a = \frac{v}{\sqrt{q_1^2 + q_2^2}}$$

$$\Rightarrow \frac{\mu}{\pi f_a^2} \simeq 2(q_1^2 + q_2^2) \gg 1$$

$$\begin{aligned} -\mathcal{L} = &\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+ |(\partial_\mu - iq_1 e A_\mu)\phi_1|^2 \\ &+ |(\partial_\mu - iq_2 e A_\mu)\phi_2|^2 \\ &+ \lambda\left[\left(|\phi_1|^2 - \frac{v^2}{2}\right)^2 + \left(|\phi_2|^2 - \frac{v^2}{2}\right)^2\right] \end{aligned}$$

- Simulation method to maintain constant string tension by introducing a time dependence of the coupling parameter.

[Klaer, Moore, 1912.08058]

$$V(\phi) = \lambda(t)\left(|\phi|^2 - \frac{f_a^2}{2}\right)^2, \quad \lambda(t) = \lambda_0\left(\frac{R_0}{R(t)}\right)^4$$

$$m_r = \sqrt{2\lambda(t)}f_a \propto R(t)^{-2} \Rightarrow \frac{\mu}{\pi f_a^2} \simeq \ln(m_r/H) = \text{const.}$$

"conformal" string

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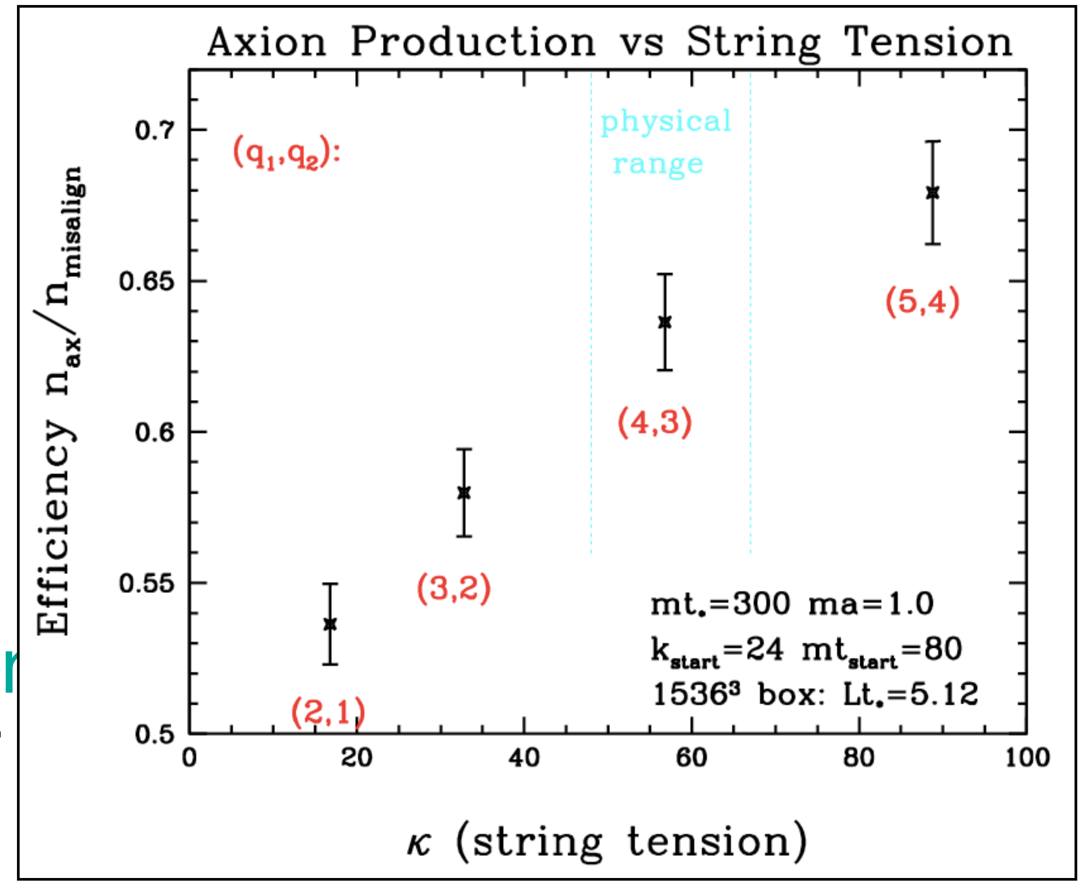

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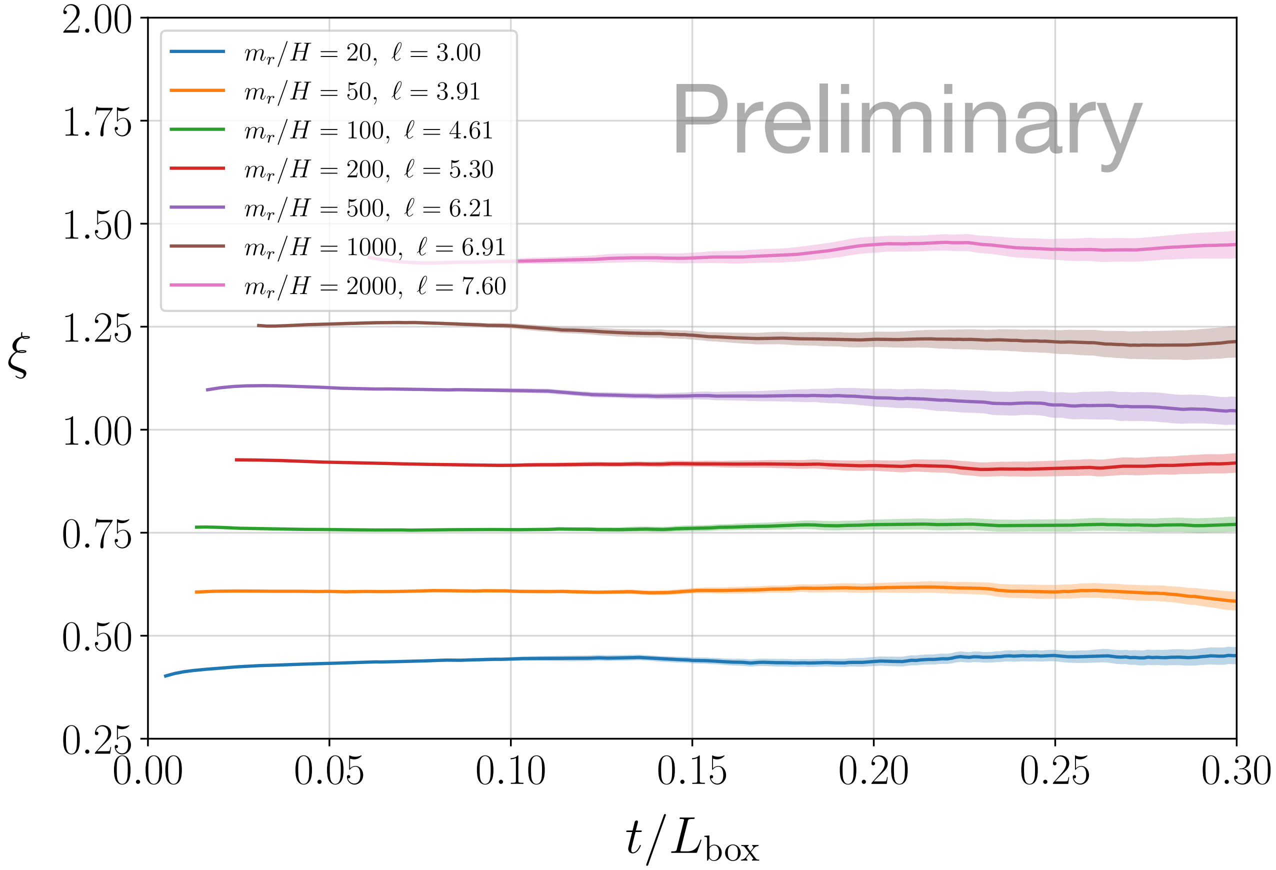
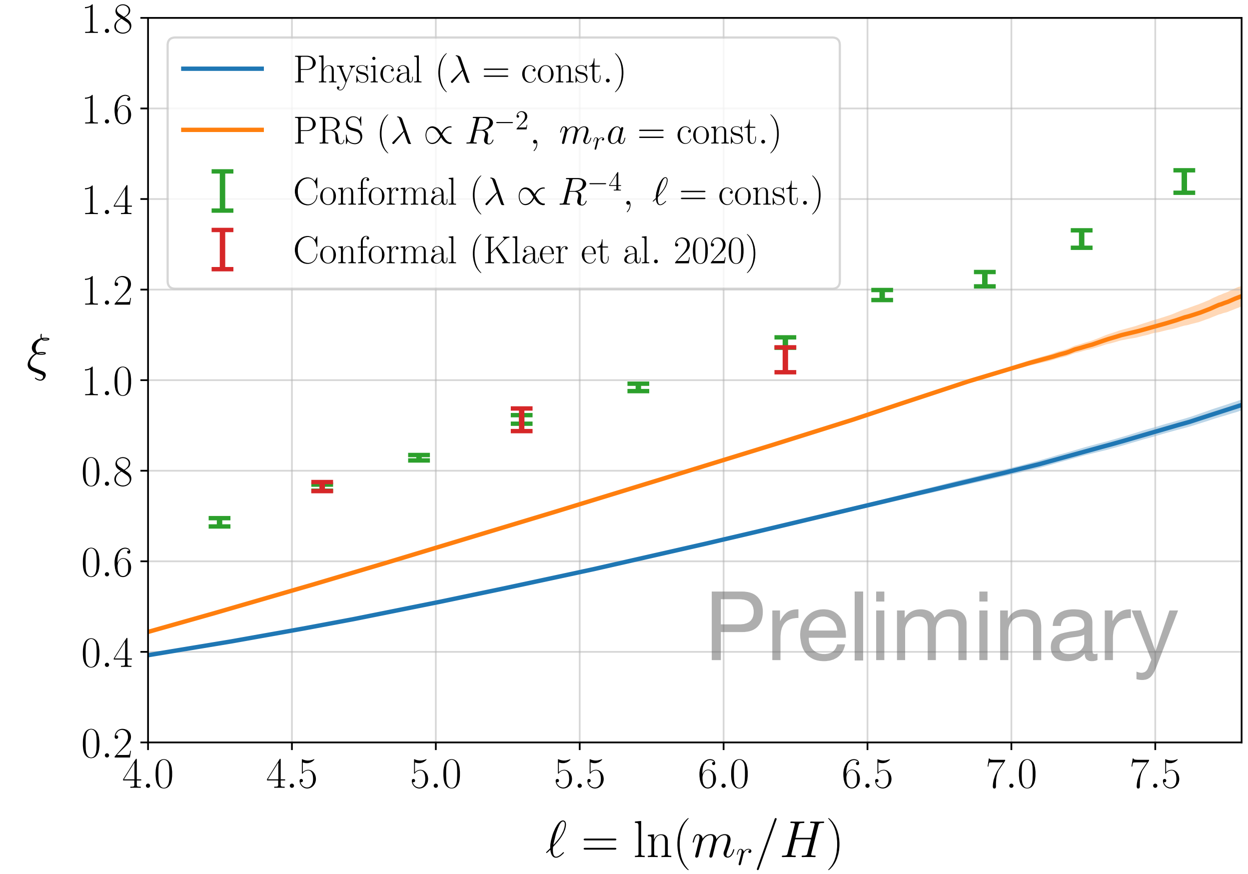
"conformal" string

Simulation of "conformal" string networks

[Klaer, Moore, 1912.08058; KS, Redondo, work in progress]





- Existence of a "perfect" scaling solution: $\xi \simeq \text{const.} \equiv \xi_c(\ell)$
- The behavior can be described by a simple semi-analytical model.

[KS, Redondo, Vaquero, Kaltschmidt, 2401.17253]

$$\frac{d\xi}{dt} = \frac{C}{t}\left(\xi_c(\ell(t)) - \xi(t)\right)$$

$$\Rightarrow \quad \xi \simeq \xi_c(\ell) - \frac{f}{C}\frac{d\xi_c}{d\ell} \qquad f = \begin{cases} 1 & \text{(physical)} \\ 1/2 & \text{(PRS)} \end{cases}$$

tracking solution

Conclusions

- Properties of the axion dark matter are tied to its production scenarios in the early universe.
- The post-inflationary scenario (with $N = 1$) can provide a sharp prediction for the axion dark matter mass, but uncertainties remain in interpreting simulation results of cosmic strings and domain walls.
    - Modelling of the axion string evolution
    - Precise calculation of the axion emission spectrum
    - Understanding the role of domain walls
- Further development in computational approaches is essential to anticipate and support upcoming experimental findings.