# The Schwinger effect in axion inflation

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Axions in Stockholm 2025:
Axions and gauge fields in the early and late universe

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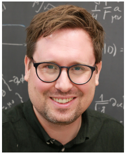
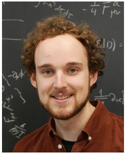
# Collaborators

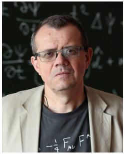
Universität Münster



Prof. Kai Schmitz



Richard von Eckardstein



Prof. Stanislav Vilchinskii

Prof. Eduard Gorbar

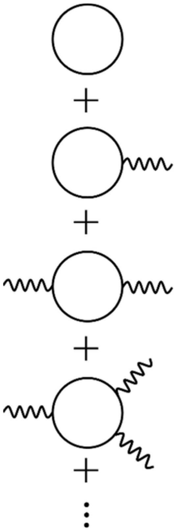
Prof. Andrii Momot

Anastasia Lysenko

Talk is based on **[2408.16538]** and [1909.10332], [2004.12664], [2109.01651], [2111.04712], [2311.15981], [25xx.yyyyy]

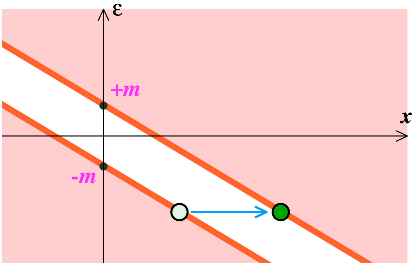
# Overview

1. Schwinger effect in flat spacetime
2. Why early Universe?
3. Schwinger effect in de Sitter spacetime
4. Axion inflation
5. Tools and numerical results
6. Phenomenology: GW's from axion inflation
7. Beyond Ohm's law
8. Conclusions

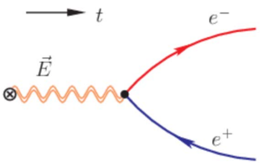


# What is the Schwinger effect?

- Spontaneous creation of charged particle–antiparticle pairs from vacuum in extreme electric fields.
- Foundations:
  - Sauter (1931): tunneling picture
  - Heisenberg and Euler (1936): effective action approach
  - Schwinger (1951): full QED derivation
- Requires near-critical field: $E_c \sim 10^{18}\ \mathrm{V/m}$




$$E_{\mathrm{th}} \triangleq \frac{2m_e c^3}{e\hbar} \simeq 10^{18}\ \mathrm{V/m}.$$




Credit: F. Gelis & N. Tanji, Prog. Part. Nucl. Phys. **87**, 1 (2016).

# Schwinger pair production rate

- In a constant **electric** field [Heisenberg & Euler'36, Weisskopf'36, Schwinger'51]

$$\boxed{\frac{\Gamma}{V} = g\frac{(eE)^2}{8\pi^3}\exp\Big(-\frac{\pi m^2}{|eE|}\Big),}$$

where $g = 1$ for scalars and $g = 2$ for spin-$1/2$ fermions.

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- In collinear constant **electric & magnetic** fields [Schwinger'54, Nikishov'69]

$$\boxed{\frac{\Gamma}{V} = g\frac{(eE)(eB)}{8\pi^2}f\left(\pi\frac{B}{E}\right)\exp\left(-\frac{\pi m^2}{|eE|}\right),}$$

where

$$f(x) = \begin{cases} 1/\sinh(x), & \text{for scalars;} \\ \coth(x), & \text{for spin-1/2 fermions.} \end{cases}$$

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- In **non-collinear** electric and magnetic fields — boost to the collinear frame and compute $\Gamma/V$ there.

# Induced current

- Production of charged particles, which are than accelerated by the electric field, generates electric current ($t \gg m/|eE|, 1/\sqrt{|eE|}$):

$$\partial_t j = 2e \times \frac{\Gamma}{V} \operatorname{sign}(eE) = \mathrm{const.}$$

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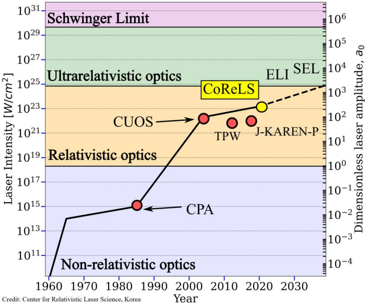
$$f(x) = \begin{cases} 1/\sinh(x), & \text{for scalars;} \\ \coth(x), & \text{for spin-1/2 fermions.} \end{cases}$$

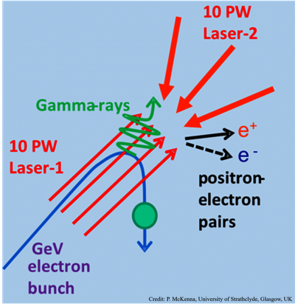
- So, the current can **grow indefinitely** in time since new and new pairs are constantly produced.

# Experimental observation of the Schwinger effect in QED

- Still **unobserved** due to huge field strength
- Future advanced laser systems approaching near-critical intensities (boosted light in the UR electron's reference frame, colliding PW laser beams, light boosting by plasma mirrors) [Turcu et al.'19]:
  - Extreme Light Infrastructure (ELI), Romania — 10 PW
  - Station of Extreme Light (SEL), China — 100 PW
- Dynamically assisted Schwinger effect: strong/slow + fast/weak pulses [Schützhold et al.'08, Dunne et al.'09, Torgrimsson et al.'16]



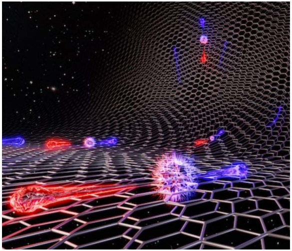
Credit: Center for Relativistic Laser Science, Korea



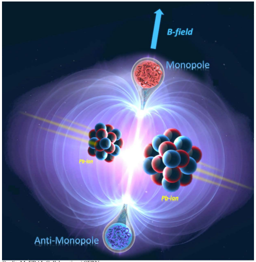
Credit: P. McKenna, University of Strathclyde, Glasgow, UK

# Other possibilities

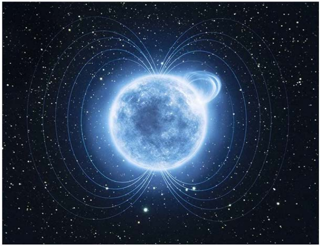
- Analogues in condensed-matter systems with Dirac spectrum: graphene with $E_c \sim 10^7\,\mathrm{V/m}$ [Berdyugin et al.'22, Schmitt et al.'23]
- Heavy-ion collisions:
  - QED effects ($e^- e^+$ production if total charge of nuclei exceeds the critical one) [Gershtein & Zeldovich'70, Rafelski et al.'71]
  - QCD effects (parton fragmentation, $q\bar{q}$ production by the chromoelectric field) [Casher et al.'79, Kajantie et al.'85, Andersson et al.'83]
- Astrophysics: near magnetars [Kim et al.'21], black holes [Treves&Turolla'99]
- Early Universe: induced by primordial gauge fields



Credit: M. Ceccanti and S. Cassandra




Credit: MoEDAL Collaboration / CERN



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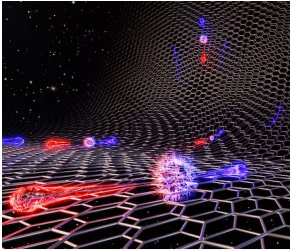

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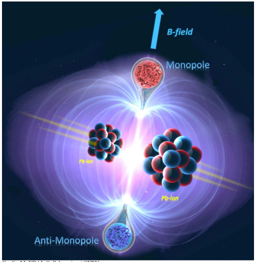


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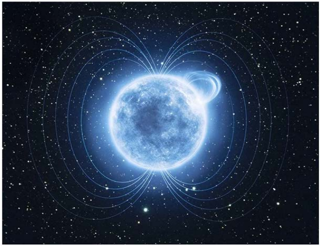

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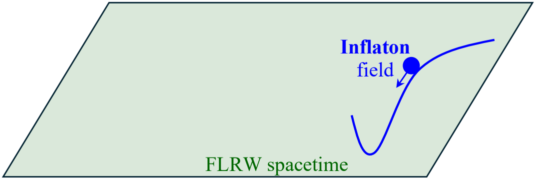
# Schwinger effect during inflation




[Starobinsky'80]

[Guth'81]

[Linde'82]

[Starobinsky'82]

[Albrecht'82]

[Linde'83]

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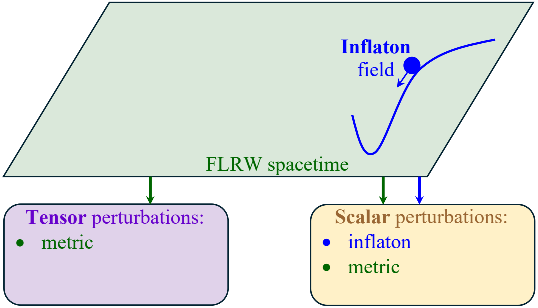


[Mukhanov'81]

[Mukhanov'82]

[Guth'82]

[Hawking'82]

[Bardeen'82]

[Grishchuk'74]

[Starobinsky'79]

[Rubakov'82]

[Fabbri'83]

[Abbott'84]

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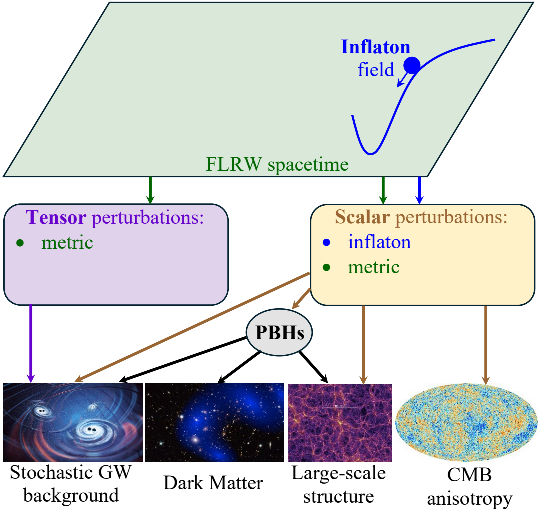


Credit: O. Shmahalo, D. Harvey, R. Massey, H. Ebeling,J.-P. Kneib, Millenium Simulation Project, NASA, ESA, Planck Collaboration

[Planck'18]
[BICEP/Keck'21]
[Reid'10]
[Cabass'22]

[NANOGrav'23]
[EPTA/InPTA'23]
[Reardon'23]
[Xu'23]

[Hawking'71]
[Carr'74]
[Carr'16]
[Carr'22]

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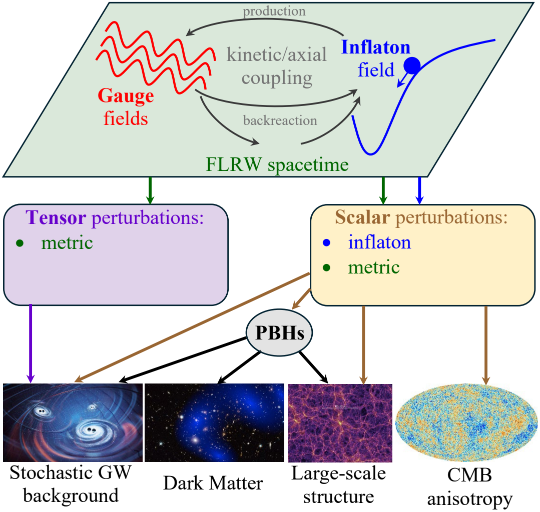


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[Turner'88]
[Ratra'91]
[Garretson'92]
[Anber'06]
[Martin'08]
[Durrer'13]

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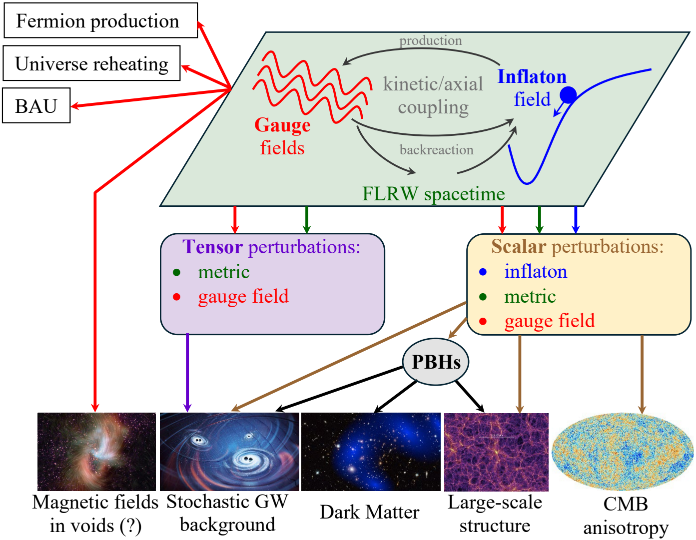


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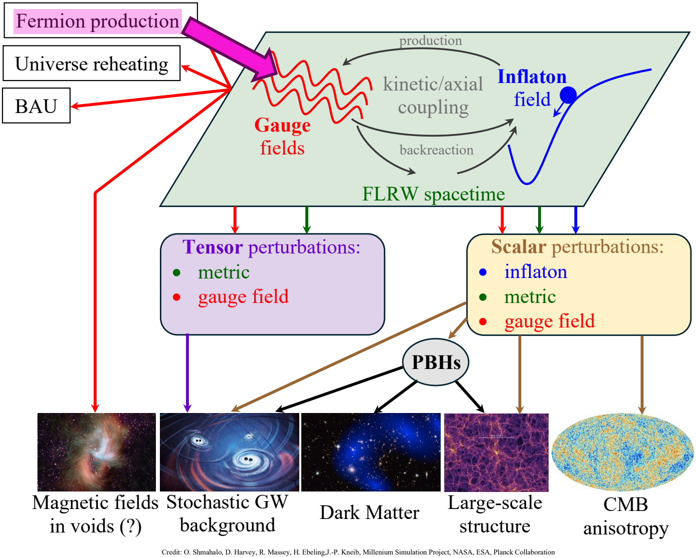
[Neronov'10]
[Taylor'11]

[Anber'15]
[Adshead'16]
[Domcke'19]
[Domcke'23]

[Cuissa'19]
[Adshead'20]
[Figueroa'23]

[Kobayashi'14]
[Hayashinaka'16]
[Domcke'18]
[Sobol'20]




**Our aim:**
To consistently describe the Schwinger pair production and its impact on the evolution of the gauge field.

# Schwinger effect in dS in a constant electric field

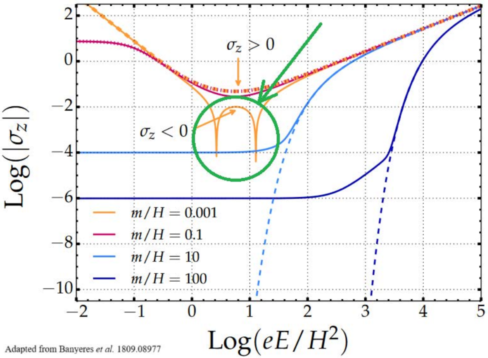
- First-principles approach: exact solution for the quantum scalar/fermion field in de Sitter with a classical constant electric field [Kobayashi&Afshordi'14, Hayashinaka et al.'16, Bavarsad et al.'16, Stahl et al.'16]
- Peculiarities in the weak-field regime $|eE| \ll H^2$:

**Negative current for light particles**:

$$j \propto e^2 E \left[ \ln \frac{m^2}{H^2} + (\text{terms} > 0) \right]$$

This term may be absorbed into the running of the gauge coupling $e(m) \to e(H)$.

[Banyeres et al.'18]



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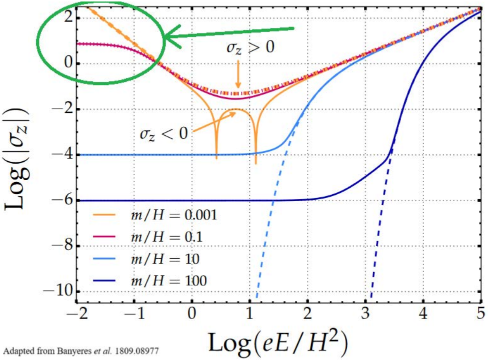
- Peculiarities in the weak-field regime $|eE| \ll H^2$:

**Infrared hyperconductivity for light scalars**:

$$j \propto 1/E, \text{ for } m \ll |eE|/H$$

This comes from the higgsing of the gauge field

$$M_A^2 = e^2\langle\chi^*\chi\rangle = \frac{3e^2}{4\pi^2}\frac{H^4}{m^2 + \frac{(eE)^2}{H^2}}$$




Adapted from Banyeres *et al.* 1809.08977

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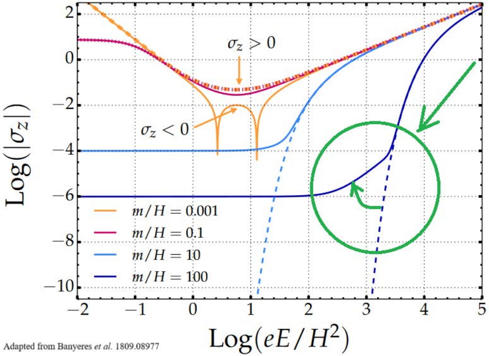
- Peculiarities in the weak-field regime $|eE| \ll H^2$:

**Terms, not suppressed exponentially at large $m$:**

$$j \propto EH^3/m^2,\; E^3H/m^4,\; \ldots$$

Originate from nonlinear corrections to Maxwell's action (Euler&Heisenberg) [Banyeres et al.'18]. Do not describe pair creation.



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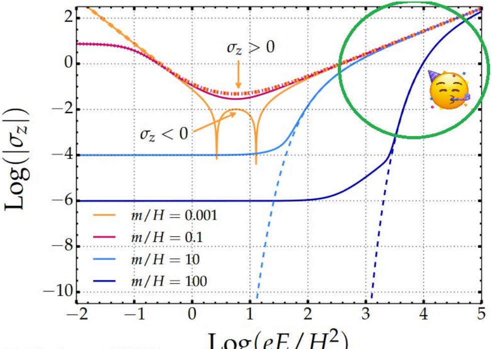
- Familiar features in the strong-field regime $|eE| \gg H^2$:

**Exponential suppression**:

$$j \propto \exp\left(-\frac{\pi m^2}{|eE|}\right)$$

Similar behavior to the case of Minkowski spacetime.

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# Induced current in the strong-field regime

- Constant electric field in Minkowski spacetime

[Warringa'12, Gavrilov et al.'08]

$$j = g\frac{e^3E^2}{4\pi^3}\exp\left(-\frac{\pi m^2}{|eE|}\right)\mathrm{sign}(eE)\times t$$

- Collinear electric and magnetic fields in Minkowski spacetime

[Warringa'12, Gavrilov et al.'08]

$$j = g\frac{e^3EB}{4\pi^2}f\left(\pi\frac{B}{E}\right)\exp\left(-\frac{\pi m^2}{|eE|}\right)\mathrm{sign}(eE)\times t$$

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- Constant electric field in de Sitter spacetime

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$$j = g\frac{e^3 E^2}{4\pi^3}\exp\left(-\frac{\pi m^2}{|eE|}\right)\mathrm{sign}(eE)\times\frac{1}{3H}$$

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**Produced particles only inside the Hubble horizon contribute to the current.**

# Question:

**How to maintain the constant electric and magnetic fields during inflation?**

# Axion inflation

- For successful slow-roll inflation we need the inflaton potential to be **sufficiently flat**. However, the **radiative corrections** may **break** this **flatness** and spoil inflation.
- This usually happens unless the flatness of the potential is **protected by a shift symmetry** $\phi \to \phi + \text{const}$.
  E.g., natural inflation model [Freese et al., PRL **65** (1990)]
- Interaction terms with matter fields should also be shift-symmetric. The simplest choice for the gauge field is [Garretson et al., PRD **46** (1992)]

$$S_{GF} = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\beta}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \right].$$

- Such scalar field $\phi$ is often called **axion** (or axion-like field).

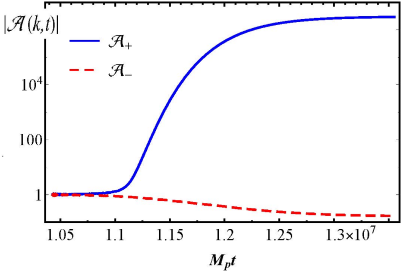
# Gauge-field generation during axion inflation

$$S=\int d^4x\sqrt{-g}\Big[\underbrace{\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi-V(\phi)}_{\text{pseudoscalar inflaton}}-\underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{free gauge field}}-\underbrace{\frac{\beta}{4M_{\rm P}}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\text{axion coupling of GF to inflaton}}+\underbrace{\mathcal{L}_{ch}(A_\nu,\chi)}_{\text{charged field (Schwinger eff.)}}\Big]$$

W/o the Schwinger effect, EoM for the mode function with circular polarization $\lambda=\pm$

$$\mathcal{A}''_\lambda(\eta,k)+[k^2-\lambda k I'_\eta]\mathcal{A}_\lambda(\eta,k)=0.$$

**Only one** of the two polarizations is amplified. Therefore, the generated MF will be **helical**!




$$\mathcal{H}\sim\int k^3(|\mathcal{A}_+|^2-|\mathcal{A}_-|^2)dk\neq 0.$$

# Equations of motion

$$S=\int d^4x\sqrt{-g}\Big[\underbrace{\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi-V(\phi)}_{\text{pseudoscalar inflaton}}-\underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{free gauge field}}-\underbrace{\frac{\beta}{4M_{\rm P}}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\text{axion coupling of GF to inflaton}}+\underbrace{\mathcal{L}_{ch}(A_\nu,\chi)}_{\text{charged field (Schwinger eff.)}}\Big]$$

Equations of motion:

- Friedmann eq.: $H^2=\dfrac{1}{3M_{\rm P}^2}\Big[\dfrac{1}{2}\dot{\phi}^2+V(\phi)+\dfrac{1}{2}\langle\boldsymbol{E}^2+\boldsymbol{B}^2\rangle+\rho_\chi\Big]$
- Klein-Gordon eq.: $\ddot{\phi}+3H\dot{\phi}+V'(\phi)=\dfrac{\beta}{M_{\rm P}}\langle\boldsymbol{E}\cdot\boldsymbol{B}\rangle$
- Maxwell equations:

$$\dot{\boldsymbol{E}}+2H\boldsymbol{E}-\frac{1}{a}\mathrm{rot}\,\boldsymbol{B}+\frac{\beta}{M_{\rm P}}\dot{\phi}\,\boldsymbol{B}+\boldsymbol{j}=0,$$

$$\dot{\boldsymbol{B}}+2H\boldsymbol{B}+\frac{1}{a}\mathrm{rot}\,\boldsymbol{E}=0,\quad \mathrm{div}\,\boldsymbol{E}=0,\quad \mathrm{div}\,\boldsymbol{B}=0.$$

- Eq. for charged particles: $\dot{\rho}_\chi+4H\rho_\chi=\boldsymbol{j}\cdot\boldsymbol{E}$.

# Question:

**How to incorporate the current**

$$\boxed{j = \frac{|e|^3}{6\pi^2}\frac{|B|E}{H}\coth\Big(\frac{\pi|B|}{|E|}\Big)\exp\Big(-\frac{\pi m^2}{|eE|}\Big)}$$

**into EoM's for axion inflation?**

# Schwinger induced current: different representations

1) **Electric picture**:

$$j = \sigma_E E, \qquad \sigma_E = \frac{|e|^3}{6\pi^2}\frac{|B|}{H}\coth\left(\frac{\pi|B|}{|E|}\right)\exp\left(-\frac{\pi m^2}{|eE|}\right);$$

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2) **Magnetic picture**:

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3) **Mixed picture:**

$$j = \sigma_E E + \sigma_B B = \frac{|e|^3}{6\pi^2}\frac{|B|E}{H}\coth\left(\frac{\pi|B|}{|E|}\right)\exp\left(-\frac{\pi m^2}{|eE|}\right).$$

# Schwinger induced current: different representations

1) **Electric picture:**

$$j = \sigma_E E, \qquad \sigma_E = \frac{|e|^3}{6\pi^2}\frac{|B|}{H}\coth\left(\frac{\pi|B|}{|E|}\right)\exp\left(-\frac{\pi m^2}{|eE|}\right);$$

2) **Magnetic picture:**

$$j = \sigma_B B, \qquad \sigma_B = \frac{|e|^3}{6\pi^2}\frac{|E|}{H}\coth\left(\frac{\pi|B|}{|E|}\right)\exp\left(-\frac{\pi m^2}{|eE|}\right)\mathrm{sign}\,(EB);$$

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$$j = \sigma_E E + \sigma_B B = \frac{|e|^3}{6\pi^2}\frac{|B|E}{H}\coth\left(\frac{\pi|B|}{|E|}\right)\exp\left(-\frac{\pi m^2}{|eE|}\right).$$

No difference at the classical level and for collinear electric and magnetic fields.

# Questions:

1. What is the expression if the electric and magnetic fields are **not collinear**?

2. How to incorporate this **classical** current into the Maxwell equation for the **quantum** gauge field?

# Schwinger induced current: different representations

Typical approach: current is **linear in gauge-field operators** with **conductivities being classical functions** (depend on mean fields). Then, three pictures appear to be **inequivalent**!

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1) **Electric picture** [Kobayashi, JHEP **10** (2014); Gorbar, PRD **104** (2021)]:

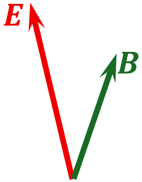
$$\hat{\boldsymbol{j}} = \sigma_E \hat{\boldsymbol{E}}, \qquad \sigma_E = \frac{|e|^3}{6\pi^2}\frac{|B|}{H}\coth\left(\frac{\pi|B|}{|E|}\right)\exp\left(-\frac{\pi m^2}{|eE|}\right);$$

2) **Magnetic picture** [Domcke, JHEP **11** (2018); JHEP **02** (2020)]:

$$\hat{\boldsymbol{j}} = \sigma_B \hat{\boldsymbol{B}}, \qquad \sigma_B = \frac{|e|^3}{6\pi^2}\frac{|E|}{H}\coth\left(\frac{\pi|B|}{|E|}\right)\exp\left(-\frac{\pi m^2}{|eE|}\right)\mathrm{sign}\,(EB);$$

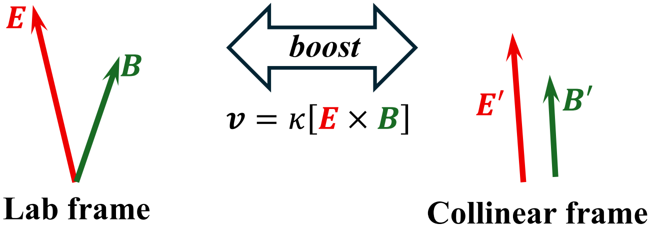
3) **Mixed picture** [von Eckardstein, JHEP **02** (2025)]:

$$\hat{\boldsymbol{j}} = \sigma_E \hat{\boldsymbol{E}} + \sigma_B \hat{\boldsymbol{B}} \qquad \sigma_E,\ \sigma_B - ?$$




**Lab frame**

# Schwinger induced current: vector decomposition



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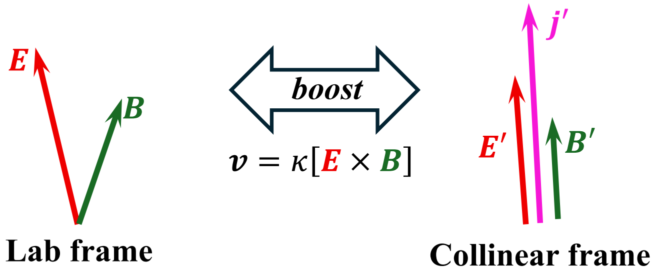

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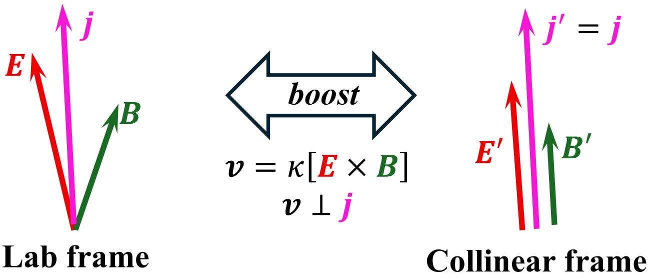

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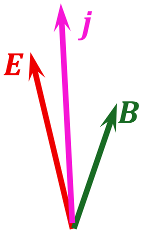
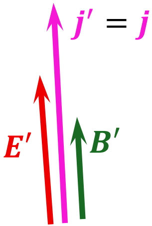



**Lab frame**


$$\boldsymbol{v} = \kappa[\boldsymbol{E} \times \boldsymbol{B}]$$
$$\boldsymbol{v} \perp \boldsymbol{j}$$




**Collinear frame**

$$\hat{\boldsymbol{j}} = \sigma_E \hat{\boldsymbol{E}} + \sigma_B \hat{\boldsymbol{B}}$$

$$\sigma_E = \left[\frac{|e|^3}{6\pi^2}\frac{|B'|}{H}\coth\left(\frac{\pi|B'|}{|E'|}\right)\exp\left(-\frac{\pi m^2}{|eE'|}\right)\right]\gamma(1-\kappa\boldsymbol{B}^2)\,;$$

$$\sigma_B = \left[\frac{|e|^3}{6\pi^2}\frac{|B'|}{H}\coth\left(\frac{\pi|B'|}{|E'|}\right)\exp\left(-\frac{\pi m^2}{|eE'|}\right)\right]\gamma\kappa\,\boldsymbol{E}\cdot\boldsymbol{B}\,.$$

# Question:

**Does the Schwinger suppression of the gauge-field Fourier modes depend on their momentum scale?**

# Relevant scales

1) **Tachyonic instability scale:**

$$k_h(t) = \max_{t' \leq t} \left( \max_k k : \Omega^2(k, t') < 0 \right).$$

Maximal momentum of the Fourier mode which undergoes (or underwent in the past) **tachyonic instability**: $\mathcal{A}_k'' + \Omega^2(k, t)\mathcal{A}_k = 0$, $\Omega^2 < 0$.

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2) **Pair-creation scale**:

$$k_{\mathrm{S}}(t) = a(t)\sqrt{|eE'(t)|}$$

Modes with wavelengths much shorter than $\lambda_{\mathrm{S}} \sim 1/k_{\mathrm{S}}$ cannot feel the presence of a conducting medium.

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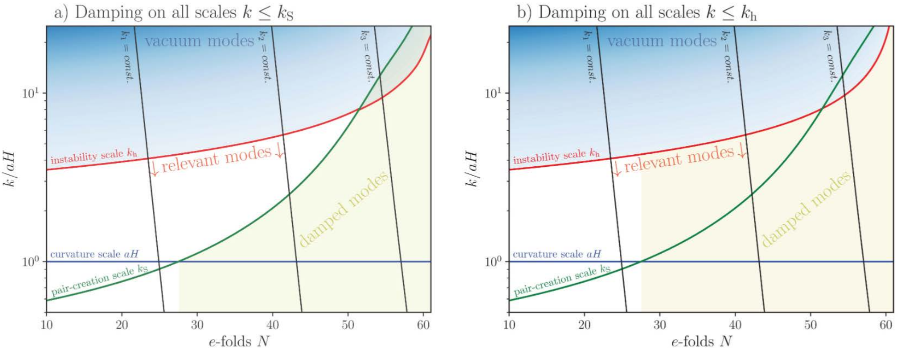
3) **Curvature (Hubble) scale**:

$$k_H(t) = a(t)H(t)$$

If $k_{\mathrm{S}} \ll k_H$, i.e., $|eE'| \ll H^2$, the Schwinger pair production is not effective, but also irrelevant for the gauge-field evolution.

# Scale-dependent damping




One must track the evolution of all relevant scales and “turn on” the Schwinger conductivities in the right moments of time (depending on the momentum).

# Gradient-expansion formalism

[Gorbar, Schmitz, OS, Vilchinskii, PRD **104** (2021); PRD **105** (2022)]

We introduce an infinite set of quantities:

$$\mathcal{E}^{(n)} = \frac{1}{a^n}\langle \boldsymbol{E}\cdot \mathrm{rot}^n \boldsymbol{E}\rangle\,, \quad \mathcal{G}^{(n)} = -\frac{1}{a^n}\langle \boldsymbol{E}\cdot \mathrm{rot}^n \boldsymbol{B}\rangle\,, \quad \mathcal{B}^{(n)} = \frac{1}{a^n}\langle \boldsymbol{B}\cdot \mathrm{rot}^n \boldsymbol{B}\rangle\,.$$

They satisfy the following chain of equations ($\xi \equiv \beta\dot{\phi}/(2HM_p)$):

$$\dot{\mathcal{E}}^{(n)} + (n+4)H\,\mathcal{E}^{(n)} - 4H\xi\,\mathcal{G}^{(n)} + 2\mathcal{G}^{(n+1)} = [\dot{\mathcal{E}}^{(n)}]_b,$$

$$\dot{\mathcal{G}}^{(n)} + (n+4)H\,\mathcal{G}^{(n)} - \mathcal{E}^{(n+1)} + \mathcal{B}^{(n+1)} - 2H\xi\,\mathcal{B}^{(n)} = [\dot{\mathcal{G}}^{(n)}]_b,$$

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$$\dot{\mathcal{G}}^{(n)} + [(n+4)H + \sigma_E]\, \mathcal{G}^{(n)} - \mathcal{E}^{(n+1)} + \mathcal{B}^{(n+1)} - [2H\xi + \sigma_B]\, \mathcal{B}^{(n)} = [\dot{\mathcal{G}}^{(n)}]_b,$$

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$$\dot{\mathcal{B}}^{(n)} + (n+4)H\,\mathcal{B}^{(n)} - 2\mathcal{G}^{(n+1)} = [\dot{\mathcal{B}}^{(n)}]_b.$$

Thus, we trade an **infinite number of Fourier-modes** for an **infinite set** of scalar **functions** in the coordinate space – **what's the gain?**

**The chain can be truncated!**

# Boundary terms

Any function $X^{(n)}$ has the following spectral decomposition:

$$X = \int_0^{k_{\rm h}(t)} \frac{dk}{k} \frac{dX}{d\ln k}.$$

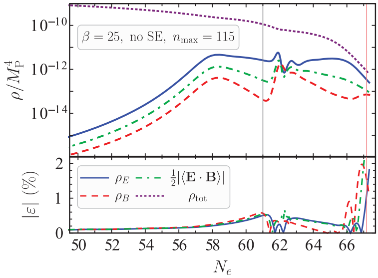
There are two sources of time dependence:

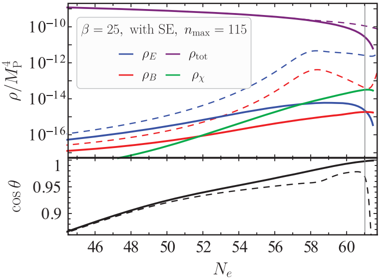
- The spectral density depends of $\mathcal{A}_\lambda(k, t)$ and its derivatives.
- The upper integration limit $k_{\rm h}(t)$ is time dependent!
  E.g., w/o Schwinger effect, $k_{\rm h}(t) = 2a(t)H(t)|\xi(t)|$.

Boundary terms describe the latter time dependence, i.e., they take into account the fact that the **number of physically relevant modes grows in time** during inflation.

$$(\dot{X})_b = \left.\frac{dX}{d\ln\ k}\right|_{k=k_{\rm h}} \cdot \frac{d\ln k_{\rm h}}{dt}.$$

They are expressed in terms of Whittaker functions.



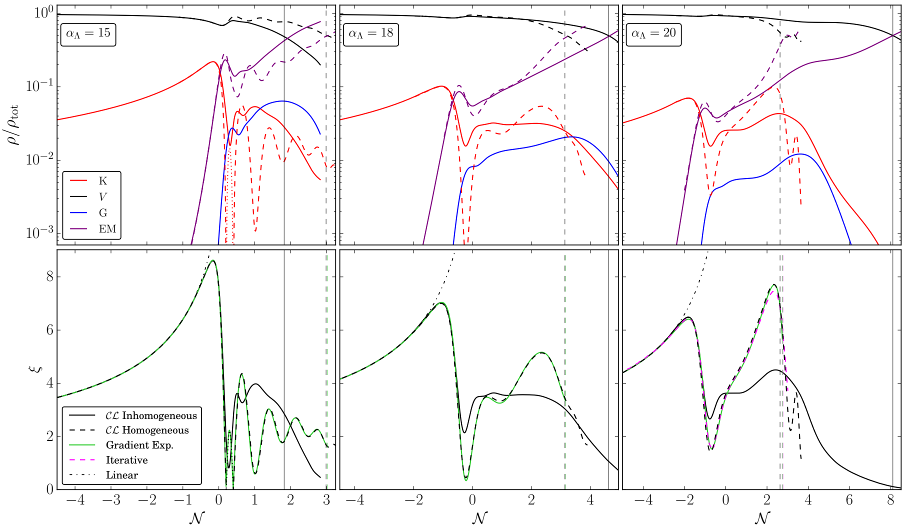




- A benchmark model for numerical analysis:

$$V(\phi) = \frac{m^2\phi^2}{2}, \qquad I(\phi) = \beta\frac{\phi}{M_p}, \qquad \beta \text{ – free parameter.}$$

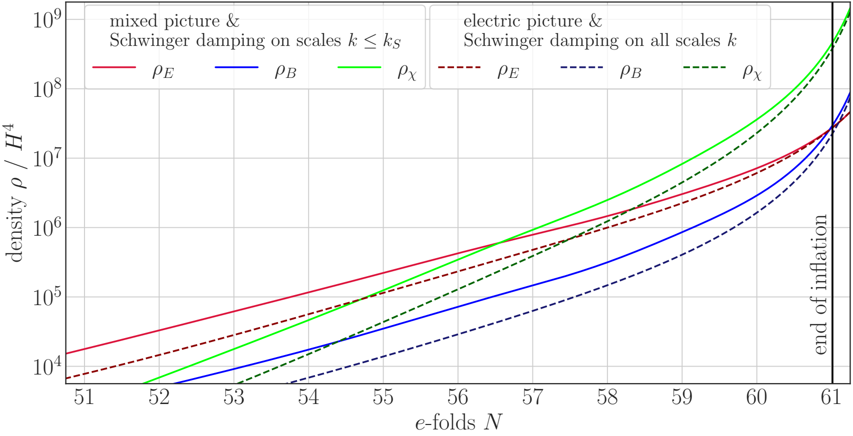
- The backreaction becomes important for the large value of axion–vector coupling $\beta \sim 15 - 25$.
- The Schwinger effect **suppresses** the produced gauge field by a few orders of magnitude and **lifts the backreaction**!

# A comment on the backreaction regime

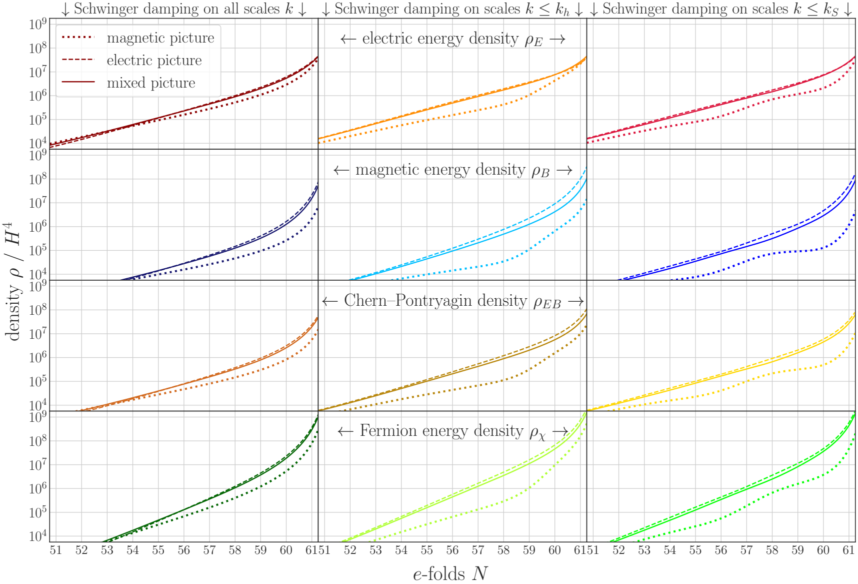



Lattice simulations by [Figueroa et al., arXiv:2303.17436].

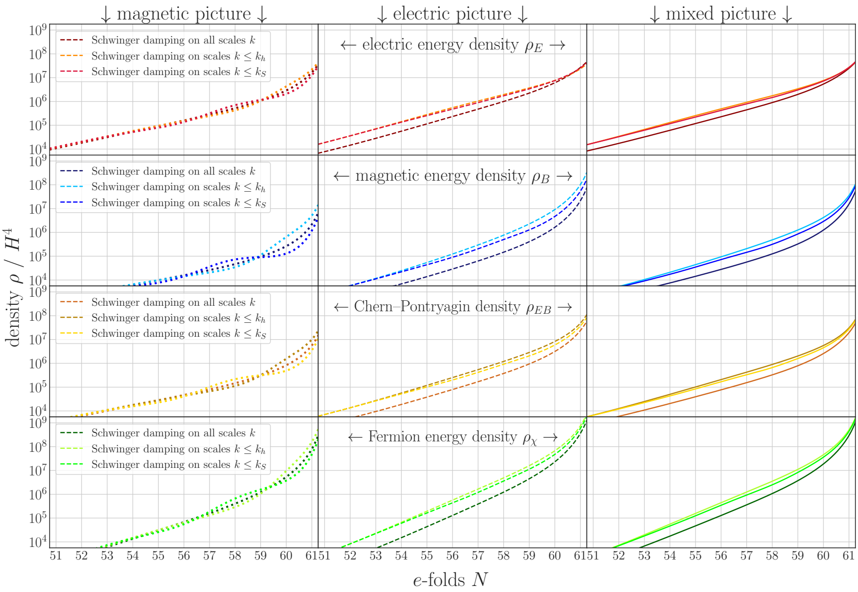
**"Old" approach**: $\boldsymbol{j} = \sigma_E \boldsymbol{E}$ (electric), damping of **all** gauge-field modes.
**"New" approach**: $\boldsymbol{j} = \sigma_E \boldsymbol{E} + \sigma_B \boldsymbol{B}$ (mixed), damping only for $k \leq k_{\rm S}(t)$.



# Comparison of different approaches: three pictures



# Comparison of different types of scale-dependence



# Tensor perturbations induced by a $U(1)$ gauge field

- Transverse-traceless perturbations on top of the FLRW metric:

$$ds^2 = a^2(\eta)\{\mathrm{d}\eta^2 - [\delta_{ij} + h_{ij}^{TT}(\eta, \boldsymbol{x})]\mathrm{d}x^i\mathrm{d}x^j\}.$$

- The gauge field introduces anisotropic stresses

$$T^i_j = -p\delta^i_j - \Sigma^i_j, \qquad p = \frac{1}{6}(E^2 + B^2), \qquad \boxed{\Sigma_{ij} = -(E_iE_j + B_iB_j).}$$

- The Einstein equations imply the following EoM for tensor perturbations:

$$\boxed{\left(\frac{\partial^2}{\partial\eta^2} + 2\frac{a'}{a}\frac{\partial}{\partial\eta} - \nabla^2\right)h_{ij}^{TT}(\eta, \boldsymbol{x}) = -2\frac{a^2}{M_{\mathrm{P}}^2}(E_iE_j + B_iB_j)^{TT}.}$$

- Thus, the gauge field may source primordial gravitational waves!

# Tensor power spectrum and GW abundance

- For the mode of perturbations with momentum $\boldsymbol{k}$ and polarization $\lambda$:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}\eta^2} + 2\frac{a'}{a}\frac{\mathrm{d}}{\mathrm{d}\eta} + \boldsymbol{k}^2\right) h_\lambda(\eta, \boldsymbol{k}) = -2\frac{a^2}{M_{\mathrm{P}}^2}\Pi_\lambda^{ij}(\boldsymbol{k})(E_i E_j + B_i B_j)^{TT}.$$

- Two statistically independent contributions in the power spectrum:
  - vacuum $\mathcal{P}_{T,\lambda}^{\mathrm{vac}}(\eta, k) = \frac{k^2}{\pi^2}|h_\lambda^{\mathrm{vac}}(\eta, k)|^2$;
  - induced:

$$\mathcal{P}_{T,\lambda}^{\mathrm{ind}}(\eta, k) = \frac{k^3}{2\pi^2 M_{\mathrm{P}}^4}\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\sum_{\alpha,\beta=\pm1}\left|1 + \lambda\alpha\frac{\boldsymbol{k}\cdot\boldsymbol{p}}{kp}\right|^2\left|1 + \lambda\alpha\frac{(\boldsymbol{k}-\boldsymbol{p})\cdot\boldsymbol{p}}{|\boldsymbol{k}-\boldsymbol{p}|p}\right|^2$$

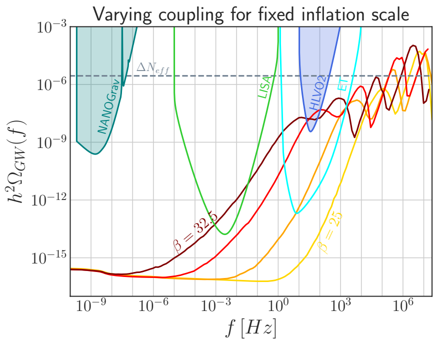
$$\times\left|\int_{-\infty}^{\eta}\mathrm{d}\eta'\,\frac{G_{\boldsymbol{k}}(\eta,\eta')}{a^2(\eta')}\left[\mathcal{A}'_\alpha(p,\eta')\mathcal{A}'_\beta(|\boldsymbol{k}-\boldsymbol{p}|,\eta') + \alpha\beta p|\boldsymbol{k}-\boldsymbol{p}|\mathcal{A}_\alpha(p,\eta')\mathcal{A}_\beta(|\boldsymbol{k}-\boldsymbol{p}|,\eta')\right]\right|^2.$$

- GW spectrum (frequency $f = k/2\pi a_0$):

$$\Omega_{\mathrm{GW}}(f) = \frac{\pi^2 f^2}{3H_0^2}|T_{\mathrm{GW}}(f)|^2\mathcal{P}_T(k,\eta_k) = \Omega_{\mathrm{GW}}^{\mathrm{vac}}(f) + \Omega_{\mathrm{GW}}^{\mathrm{ind}}(f).$$

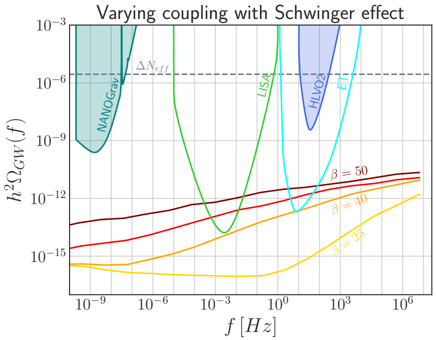
# Numerical results for GW spectra (preliminary)

**"Sterile" GF (no SE):**




- Violation of $\Delta N_{\rm eff}$ bound
- Oscillatory region is not entirely reliable

**With the Schwinger effect:**




- No violation of $\Delta N_{\rm eff}$ bound
- Larger couplings allowed

**What about GW's induced by the charged particles?**

- Energy density of the produced particles is greater than the gauge-field energy density at the end of inflation: is it the same for the anisotropic stress?
- Can one effectively represent $\Sigma_{ij}^{\text{ch. part.}}$ in terms of the gauge field (the same idea as with the electric current)?
- How to compute $\Sigma_{ij}^{\text{ch. part.}}$ from the first principles?

**To be continued...**

# Question:

**How can the Schwinger effect be described if the growth of the gauge field is not adiabatically slow?**

Let us consider the case where the magnetic field is subdominant and only the electric field is important (e.g., in the dilatonic coupling model

$$\mathcal{L} = -(1/4)f^2(\phi)F_{\mu\nu}F^{\mu\nu}$$

[Gorbar, Momot, OS, Vilchinskii (2019)]

## Kinetic approach

Boltzmann kinetic equation:

$$\frac{\partial \mathcal{F}}{\partial t}+eE\frac{\partial \mathcal{F}}{\partial p_{\parallel}}-H\boldsymbol{p}\frac{\partial \mathcal{F}}{\partial \boldsymbol{p}}=\mathcal{S}[\mathcal{F}]+\mathcal{C}[\mathcal{F}]$$

Schwinger source term (constructed phenomenologically):

$$\mathcal{S}[\mathcal{F}]=(1\pm 2\mathcal{F})\sqrt{|eE|}\exp\left[-\pi\frac{\boldsymbol{p}^2+m^2}{|eE(t)|}\right]$$

Collision integral ($\tau$-approximation):

$$\mathcal{C}[\mathcal{F}]=-\frac{1}{\tau}\left(\mathcal{F}-\mathcal{F}^{(\mathrm{eq})}\right)$$

The induced current:

$$j=2g\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\left[\frac{p_{\parallel}}{\epsilon_{\boldsymbol{p}}}\mathcal{F}+\epsilon_{\boldsymbol{p}}\frac{\mathcal{S}[\mathcal{F}]}{eE(t)}\right].$$

## Hydrodynamical approach

EoM for the number density

$$\frac{dn}{dt}+3Hn=2g\Gamma-\frac{n-n_{(\mathrm{eq})}}{\tau},$$

the energy density

$$\frac{d\rho}{dt}+4H\rho=eE(j_{\mathrm{cond}}+j_{\mathrm{pol}}),$$

and the induced current

$$\frac{dj_{\mathrm{cond}}}{dt}+\left(3H+\frac{1}{\tau}\right)j_{\mathrm{cond}}=eE\frac{n^2-j_{\mathrm{cond}}^2}{\rho}$$

*Advantages*: ODEs, simplicity.

*Disadvantages*: approximate truncation rule, does not take into account quantum statistics.

# Schwinger source from the first principles

- The kinetic approach can be improved by deriving the Schwinger source *from the first principles* using QFT methods. [OS et al.'20]
- The mode function of a charged **scalar** field (whose particles are generated by the Schwinger effect) satisfies EoM

$$\ddot{\chi}_{\boldsymbol{k}}(t) + \Omega_{\boldsymbol{k}}^2(t)\chi_{\boldsymbol{k}}(t) = 0, \quad \Omega_{\boldsymbol{k}}^2(t) = m^2 + \frac{(\boldsymbol{k} - e\boldsymbol{A}(t))^2}{a^2} - \frac{H^2}{4} - \frac{\dot{H}}{2}.$$

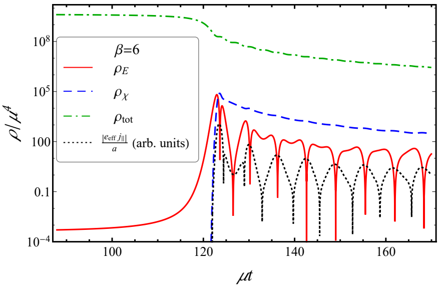
This is the equation of an oscillator with variable frequency.

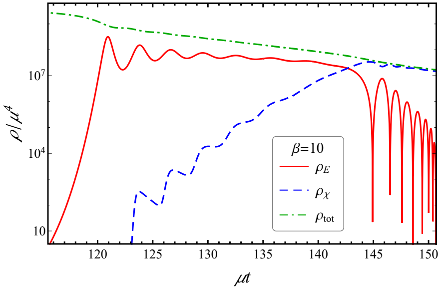
- Using the formalism of **Bogolyubov coefficients**, we can find the equation for the distribution function of produced particles:

$$\left[\frac{\partial}{\partial t} + (e\boldsymbol{E} - H\boldsymbol{p})\frac{\partial}{\partial \boldsymbol{p}}\right]\mathcal{F}(t,\boldsymbol{p}) = Q(t,\boldsymbol{p})\mathcal{G}(t,\boldsymbol{p}),$$

where $Q(t,\boldsymbol{p}) = \dot{\Omega}/\Omega$, and the source $\mathcal{G}(t,\boldsymbol{p})$ can be determined *from another two kinetic equations*.

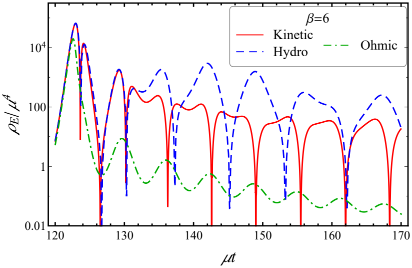
# Physical implications




- In a variable electric field, the current has a **non-Markovian character** and is not described by Ohm's law.
- A qualitatively new effect is that the current **is retarded** w.r.t. the electric field, which leads to **oscillatory behavior** of both quantities (fig. on the left).
- The energy density of the generated charged particles can constitute a significant fraction of the total energy density – **Schwinger reheating** (fig. on the right).

# Comparison of different approaches



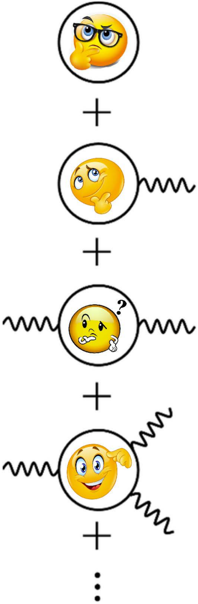

- The Schwinger current in Ohmic form (local in time), $\boldsymbol{j} = \sigma(E) \cdot \boldsymbol{E}$ – leads to a fast damping of the electric field (green curve).

- Hydrodynamical approach: predicts oscillatory behavior of the electric field (blue curve), but does not take into account the effects of quantum statistics (underestimates the current for scalars).
- Kinetic approach: the Schwinger source is nonlocal in time and momentum space; takes into account the effects of quantum statistics and the expansion of the universe (red curve). The current **is retarded** w.r.t. the electric field, **oscillatory behavior** of both quantities.

**Note, that such a treatment for the axion inflation is still missing!**

# Conclusions

1. The Schwinger effect is a strong-field QED phenomenon of great interest.
2. Although it has not been observed in lab, it may be important for the physics of the early Universe.
3. In axion inflation with Abelian gauge fields, the Schwinger effect is extremely efficient, suppressing gauge-field production to a large extent.
4. A proper description in an evolving inflationary background is still missing.
5. Further directions of study:
   - dynamics of created particles;
   - induced scalar and tensor perturbations;
   - thermalization of particles;
   - chiral asymmetry production and evolution;
   - ...



# Thank you very much for your attention!

Peace to all of us!