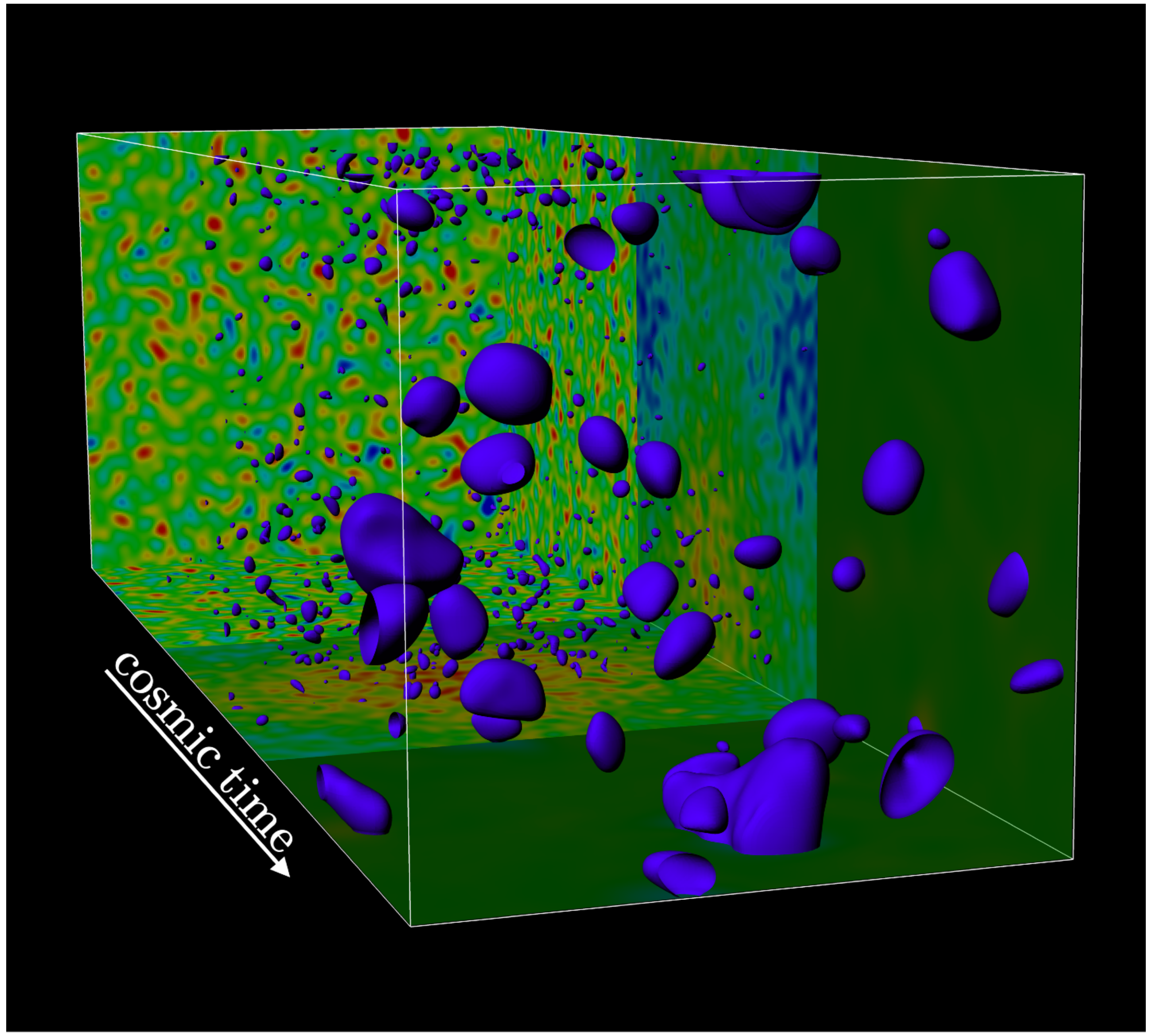
Numerical relativity for axion cosmology

Katy Clough, Queen Mary
University of London



J. Aurrekoetxea, KC, F Muia
Oscillon formation during inflationary preheating with general relativity
Phys.Rev.D 108 (2023) 2, 023501

Cosmology using Numerical Relativity

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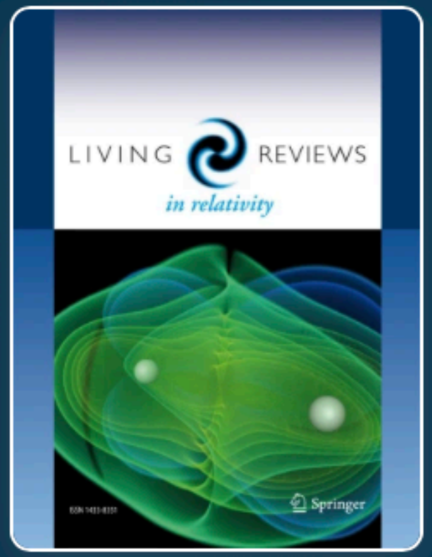
Cosmology using numerical relativity

Review Article | Open access | Published: 23 June 2025

Volume 28, article number 5, (2025) Cite this article



Living Reviews in Relativity

Aims and scope →

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Josu C. Aurrekoetxea, Katy Clough & Eugene A. Lim

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Part of a collection:

Numerical Relativity

Physical Cosmology

Sections	Figures	References

Abstract

This review is an up-to-date account of the use of numerical relativity to study dynamical, strong-gravity environments in a cosmological context. First, we provide a gentle introduction into the use of numerical relativity in solving cosmological spacetimes, aimed at both cosmologists and numerical relativists. Second, we survey the present body of work, focusing on general relativistic simulations, organised according to the cosmological history—from cosmogenesis, through the early hot Big Bang, to the late-time evolution of the universe. We discuss the present state-of-the-art, and suggest directions in which future work can be fruitfully pursued.

Josu Aurrekoetxea

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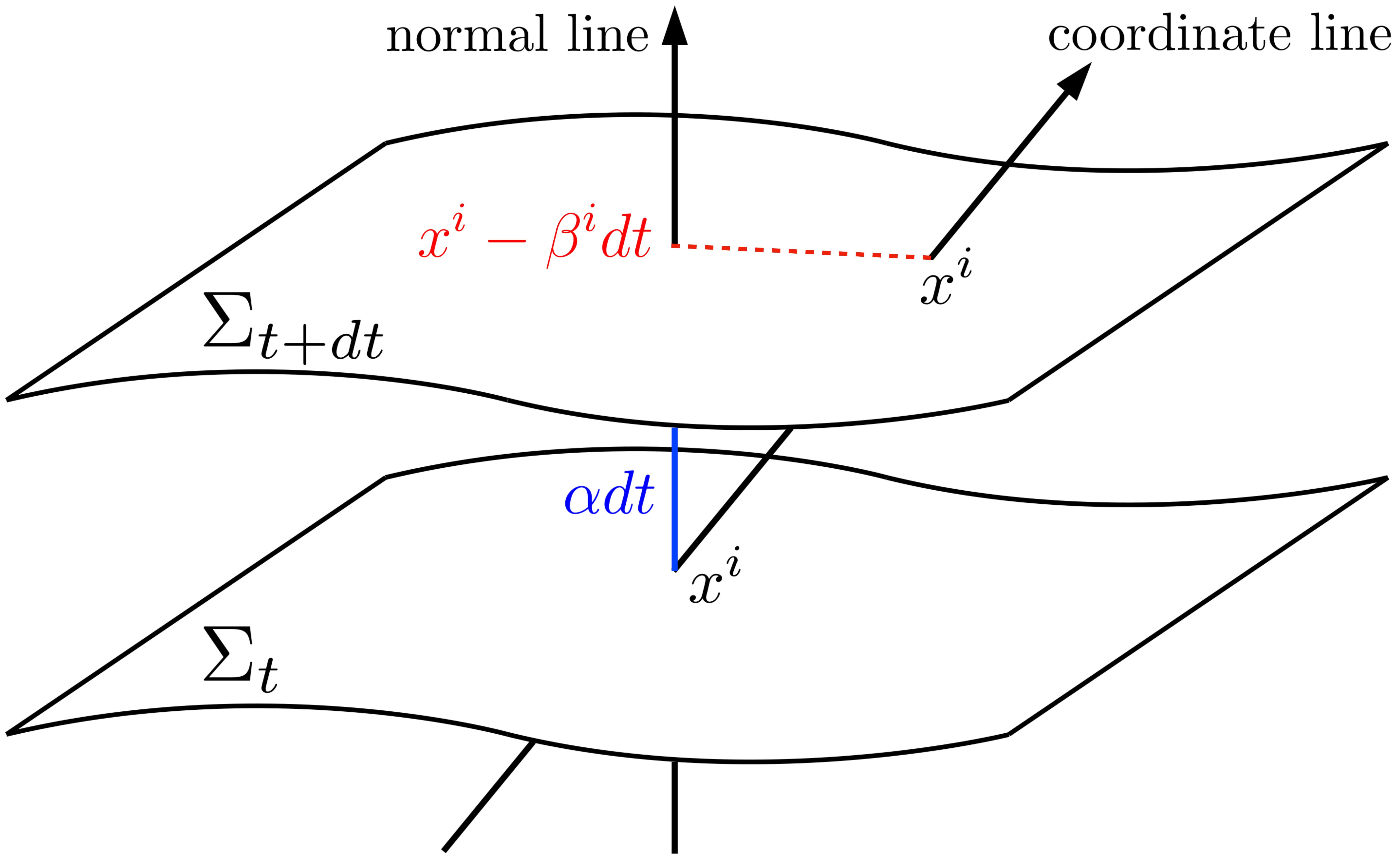
https://link.springer.com/article/10.1007/s41114-025-00058-z

What is numerical relativity?



ADM decomposition

- Decompose the 4D spacetime metric as:

$$ds^2 = -\left(\alpha^2 - \beta^i\beta_i\right)\, dt^2 + 2\beta_i\, dx^i dt + \gamma_{ij}\, dx^i dx^j$$

- Compare to FLRW:

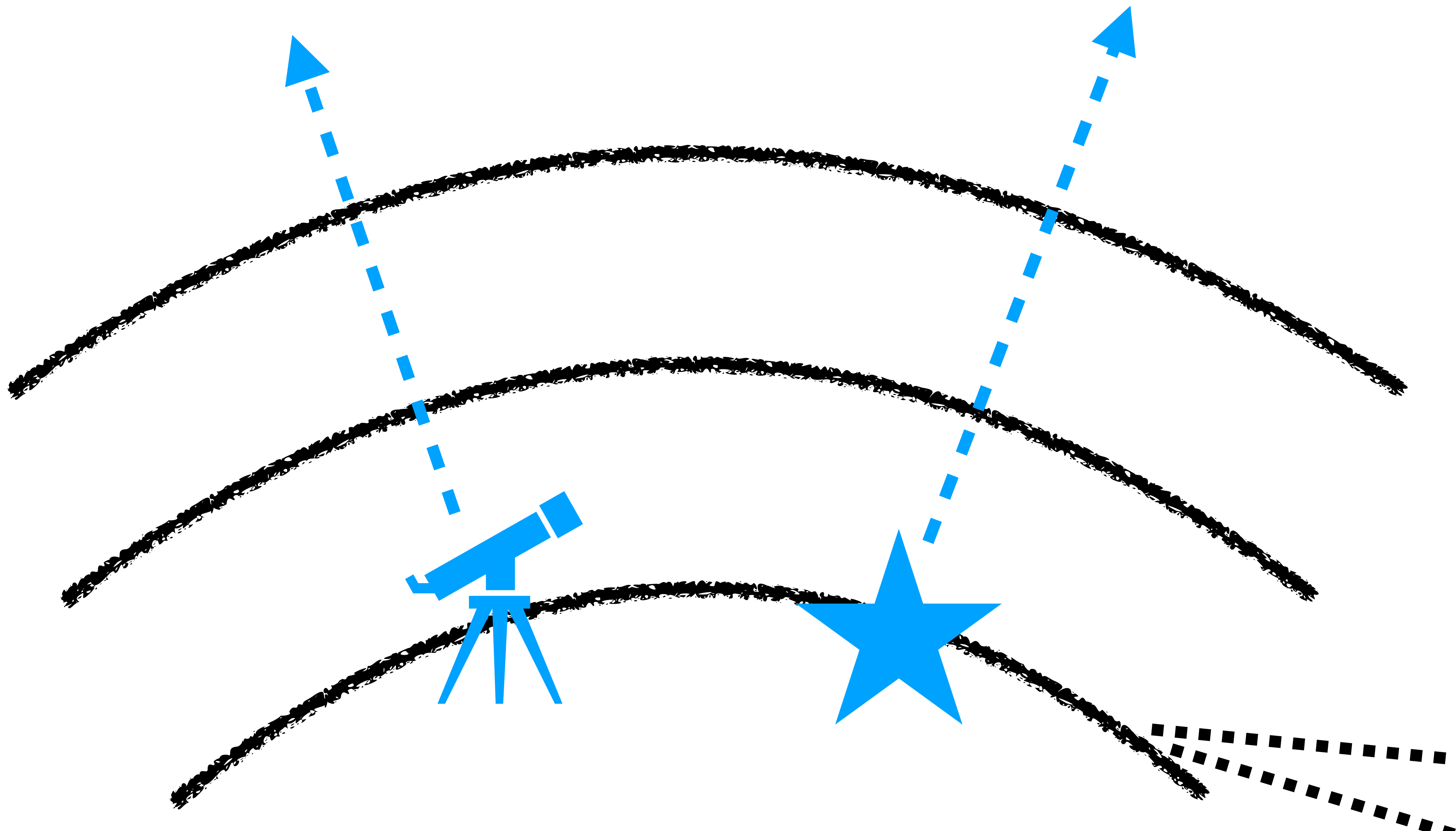
$$ds^2 = -a^2(\tau)d\tau^2 + a^2(\tau)\delta_{ij}dx^i dx^j$$

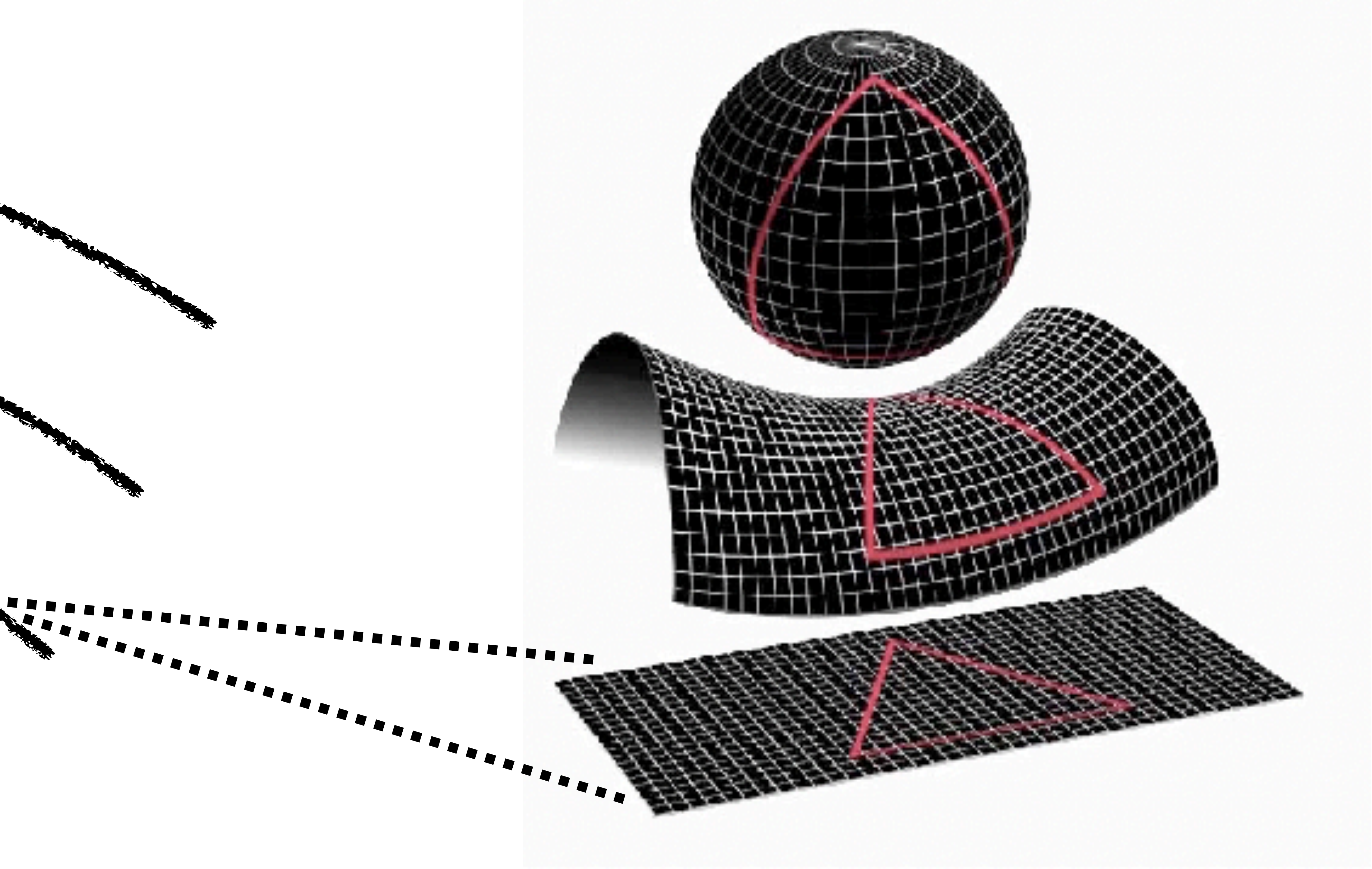
Extrinsic curvature

- Related to the time derivative of the spatial metric

$$H = \frac{\dot{a}}{a}$$

$$K_{ij} \sim -\frac{\partial_t \gamma_{ij}}{2\alpha}$$





1 dimensional "time" is curved

3 dimensional "space" is (roughly) flat

Constraints and evolution

- Hamiltonian constraint

$$H^2 = \frac{8\pi\rho}{3M_{pl}^2} \qquad \mathcal{H} = R + K^2 - K_{ij}K^{ij} - 16\pi\rho = 0$$

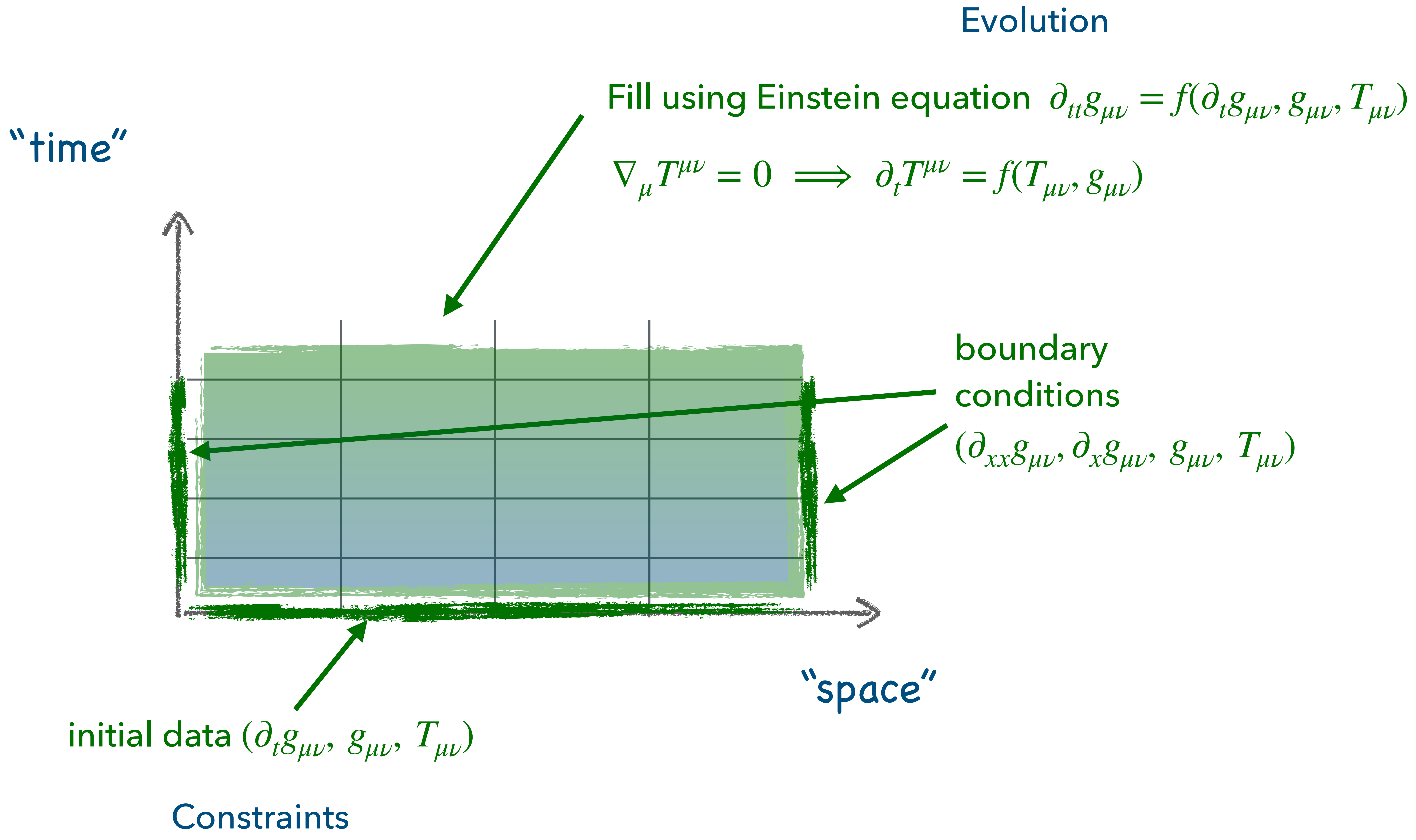
- Momentum constraint

$$isotropy \qquad \mathcal{M}_i = D^j(\gamma_{ij}K - K_{ij}) - 8\pi S_i = 0$$

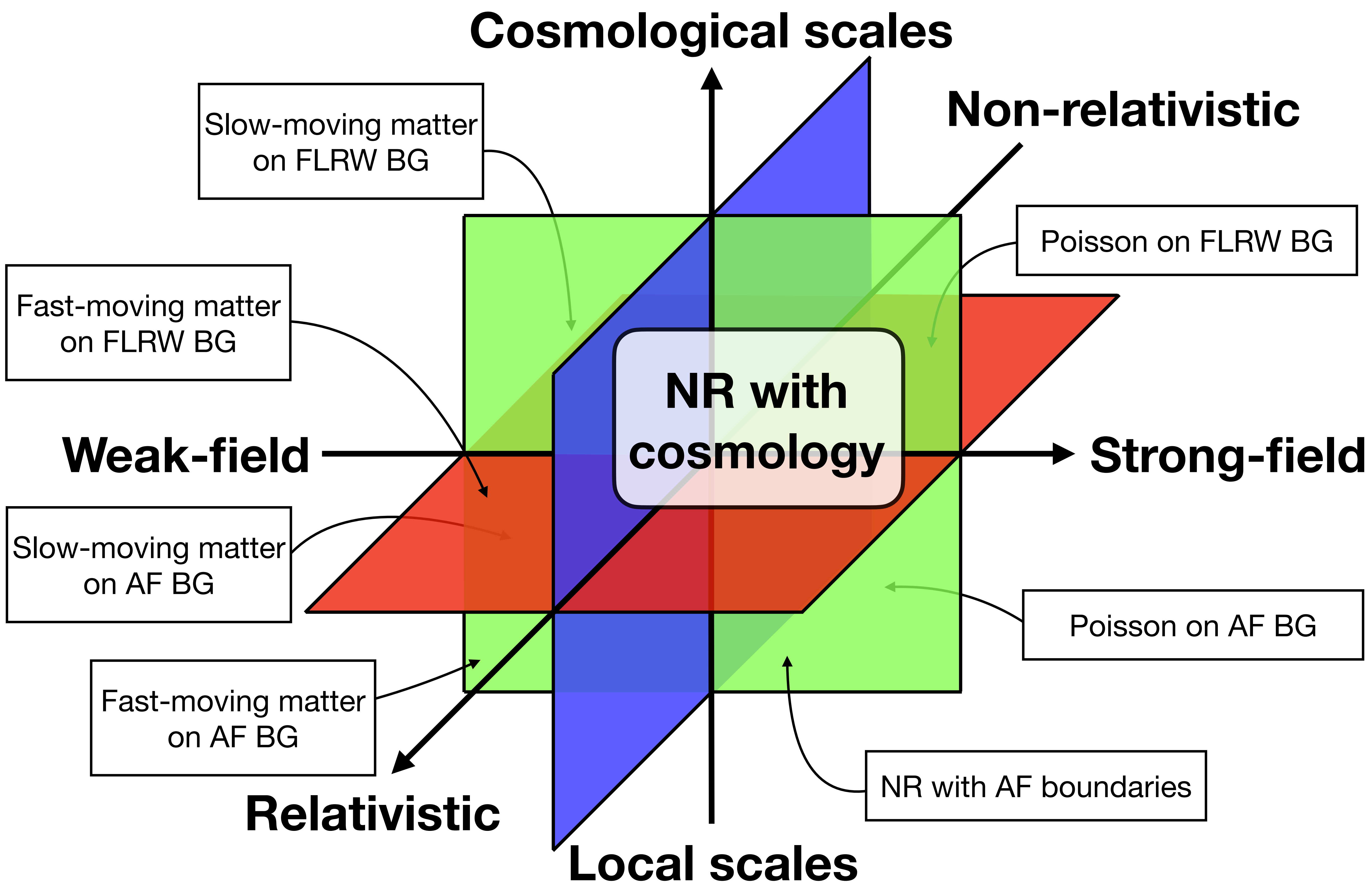
- Evolution eqn for extrinsic curvature

$$\frac{\ddot{a}}{a} = -\frac{4\pi(\rho + 3P)}{3M_{pl}^2} \qquad \partial_t K_{ij} = f(\alpha, \beta^i, \gamma_{ij}, K_{ij}, matter)$$

Constraints and evolution



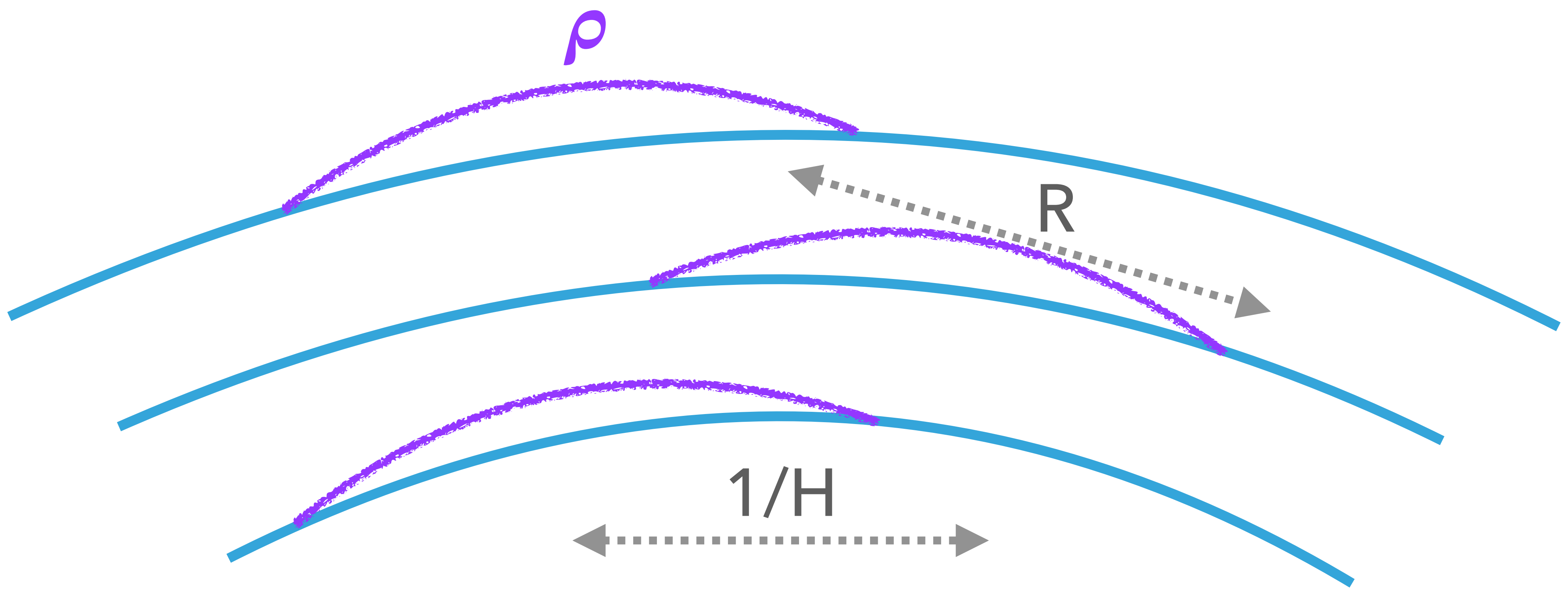
When do we need numerical relativity for cosmology?



When do we need numerical relativity for cosmology?

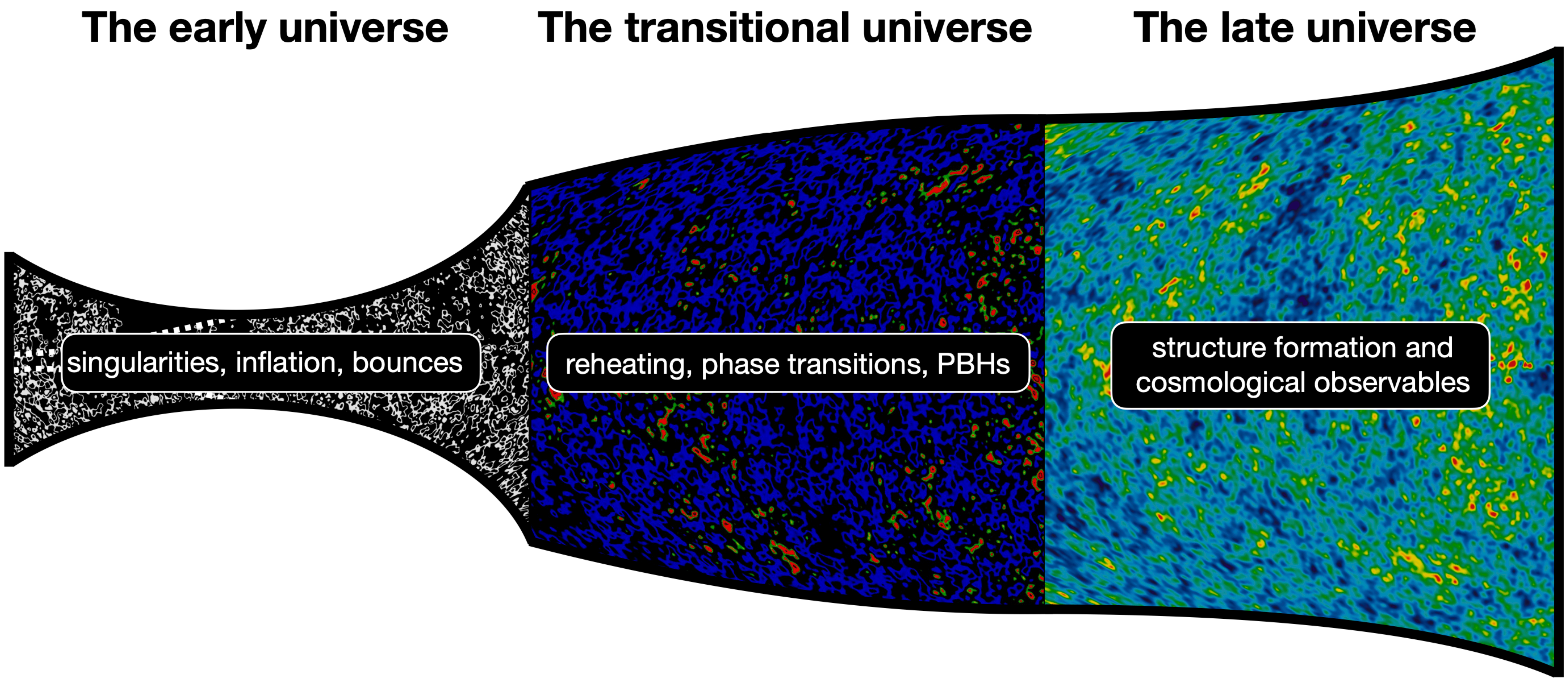
DYNAMICAL
SPACETIME

NO PERTURBATIVE
EXPANSION



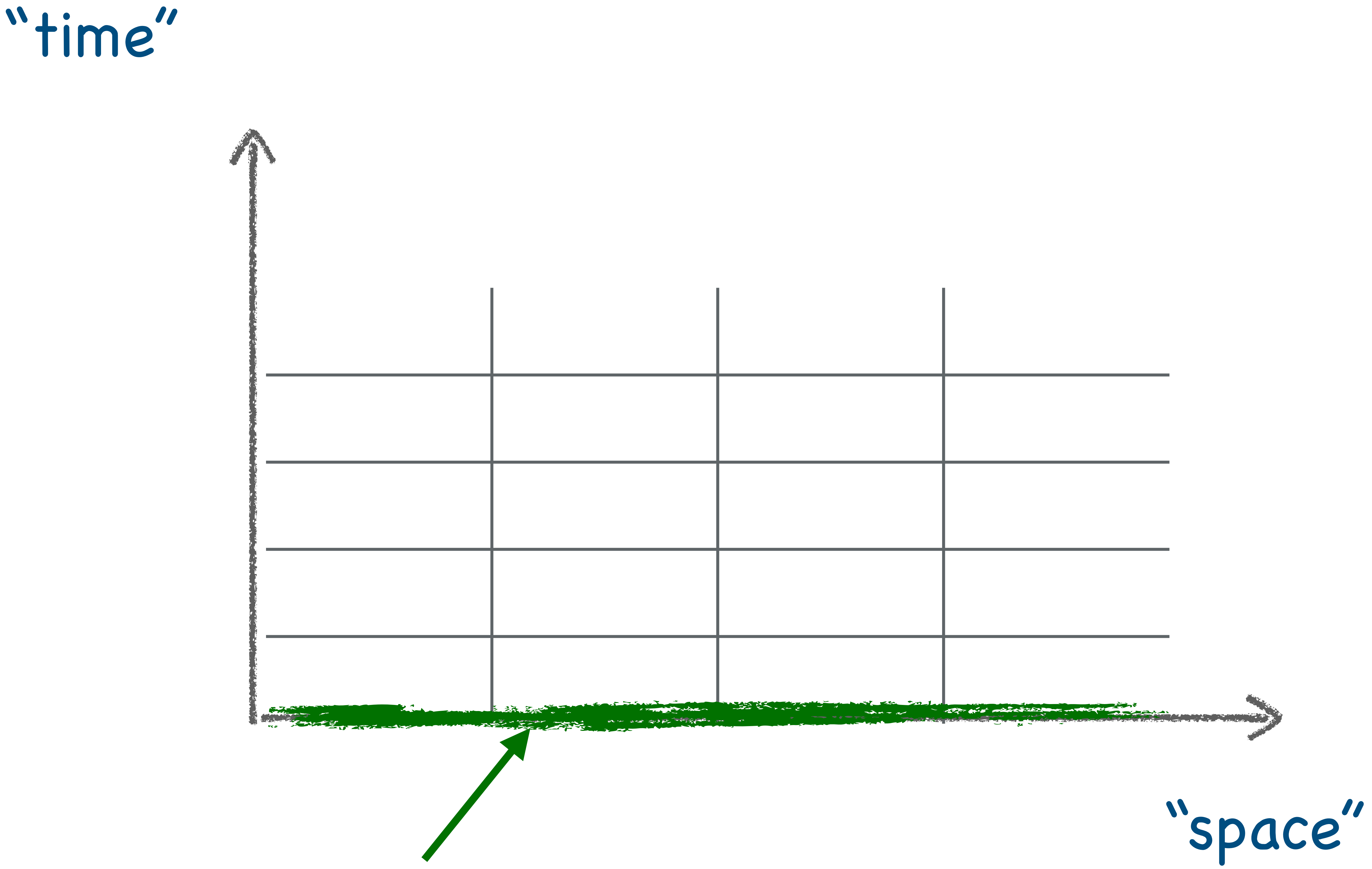
LARGE GRAVITATIONAL BACKREACTION
(STRONG GRAVITY, RELATIVISTIC MOTION)

When do we need numerical relativity for cosmology?



Challenges

Constructing initial data



initial data $(\partial_t g_{\mu\nu},\ g_{\mu\nu},\ T_{\mu\nu})$

Constraints

Goals:

1. Solve the constraints

2. Have the "right" physical system

Notice the fundamental contradiction

I don’t know the “correct” 4D solution
(that is why I’m solving it numerically)

I want to start with a 3D slice of the “correct” solution

The goal of constructing initial data

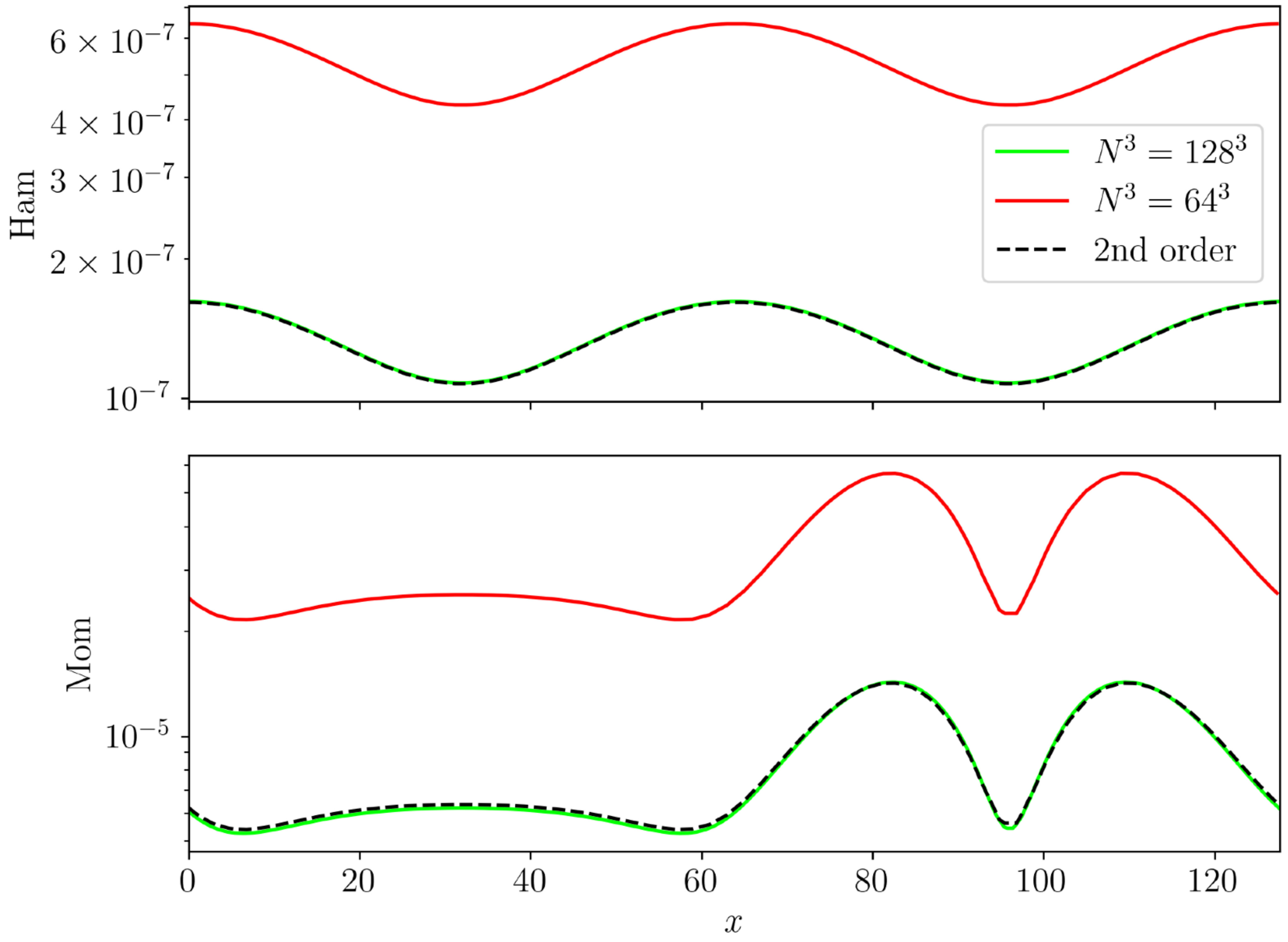
1. Solve the constraints

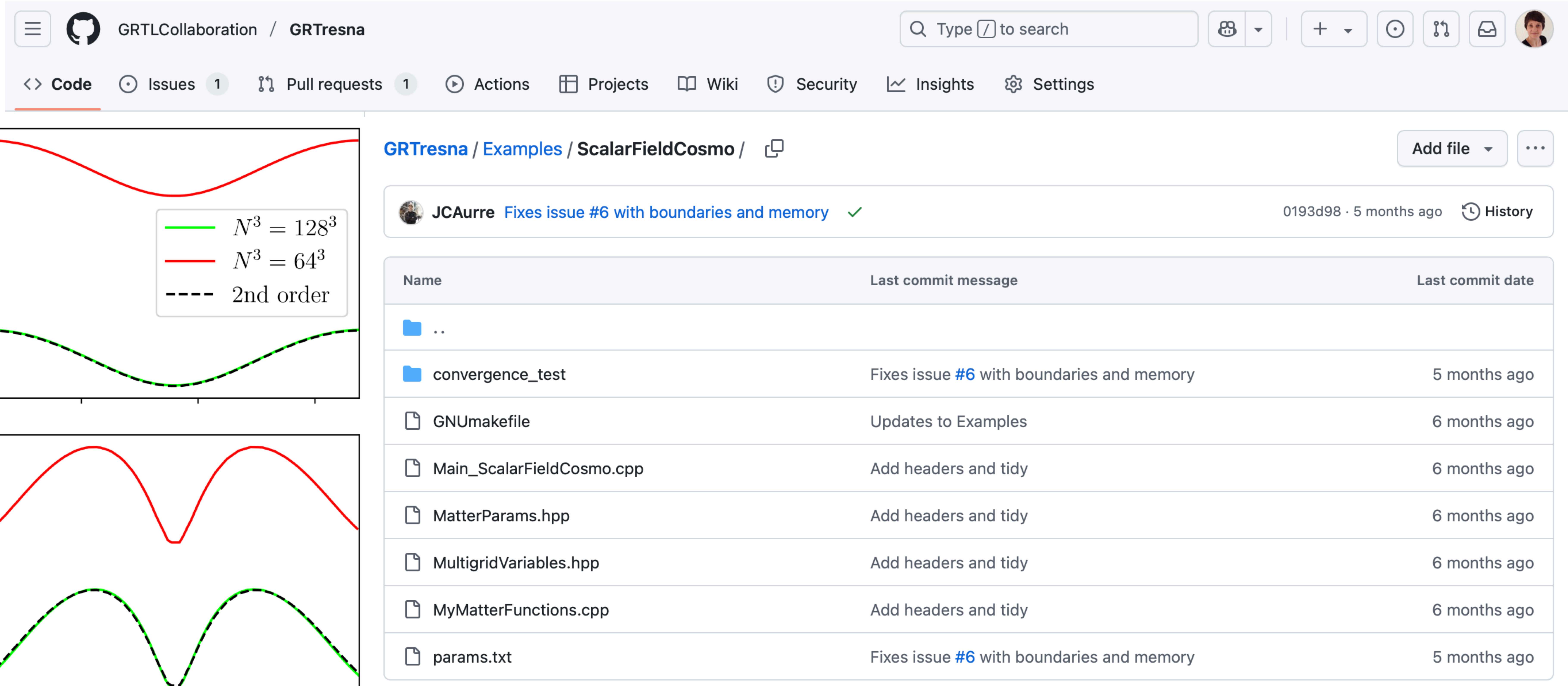
2. Have the “right” physical system

(1) is technically difficult beyond spherical symmetry (especially in periodic spacetimes), but usually doable

(2) is not a completely solved problem, due to the difficulties in separating gauge and physical degrees of freedom

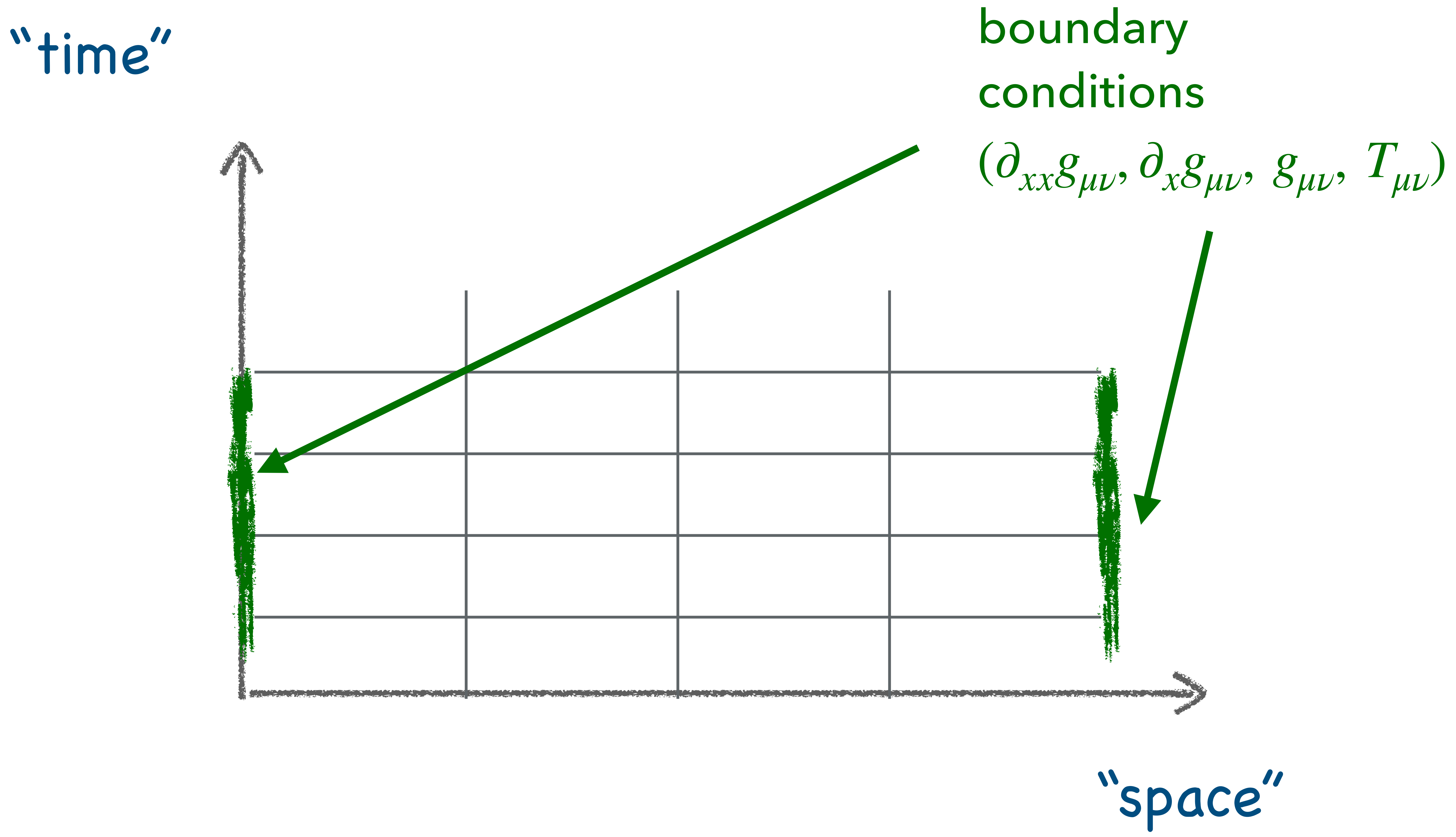
Solving the constraints





https://github.com/GRTLCollaboration/GRTresna

Setting boundary conditions



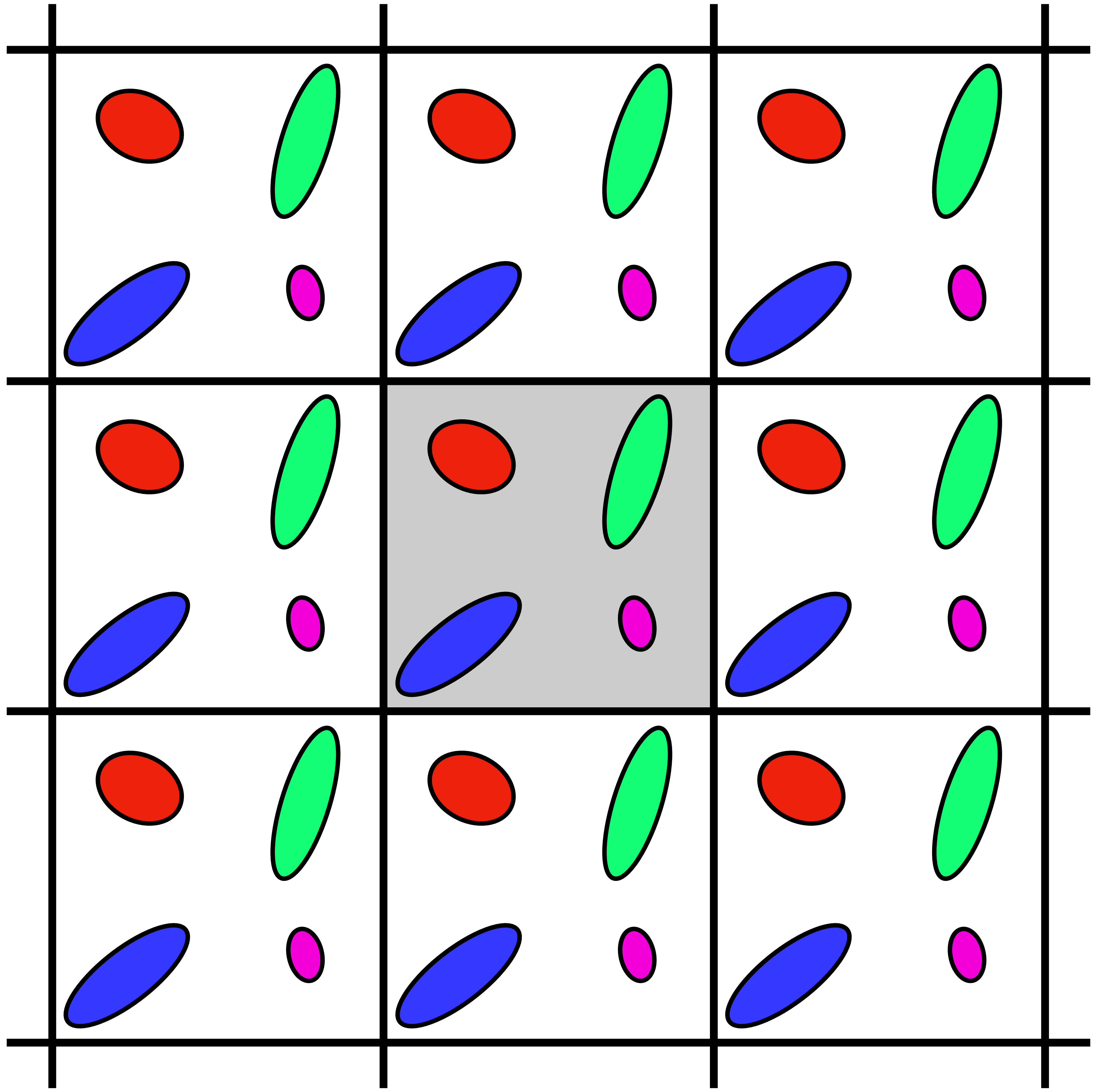
Goals:

1. Have the "right" physical boundary

or

2. Do something that is largely insensitive to our ignorance about the global structure of our universe

Boundary conditions



Impact on results?

Assumption of homogeneity on some scale?

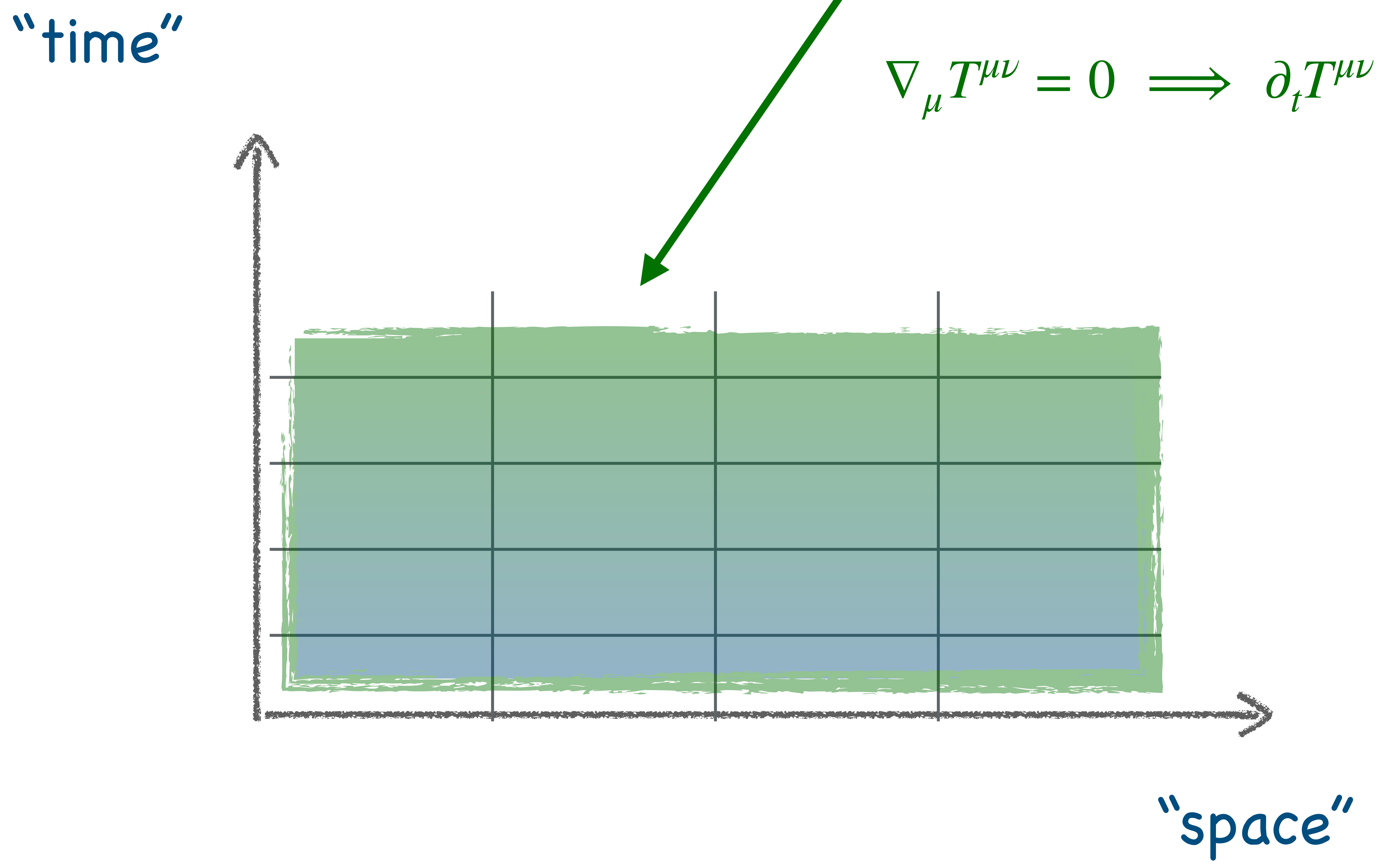
How to extract GW data in non asymptotically flat spacetimes?

Choosing gauge conditions

Evolution

Fill using Einstein equation $\partial_{tt} g_{\mu\nu} = f(\partial_t g_{\mu\nu}, g_{\mu\nu}, T_{\mu\nu})$

$\nabla_\mu T^{\mu\nu} = 0 \implies \partial_t T^{\mu\nu} = f(T_{\mu\nu}, g_{\mu\nu})$

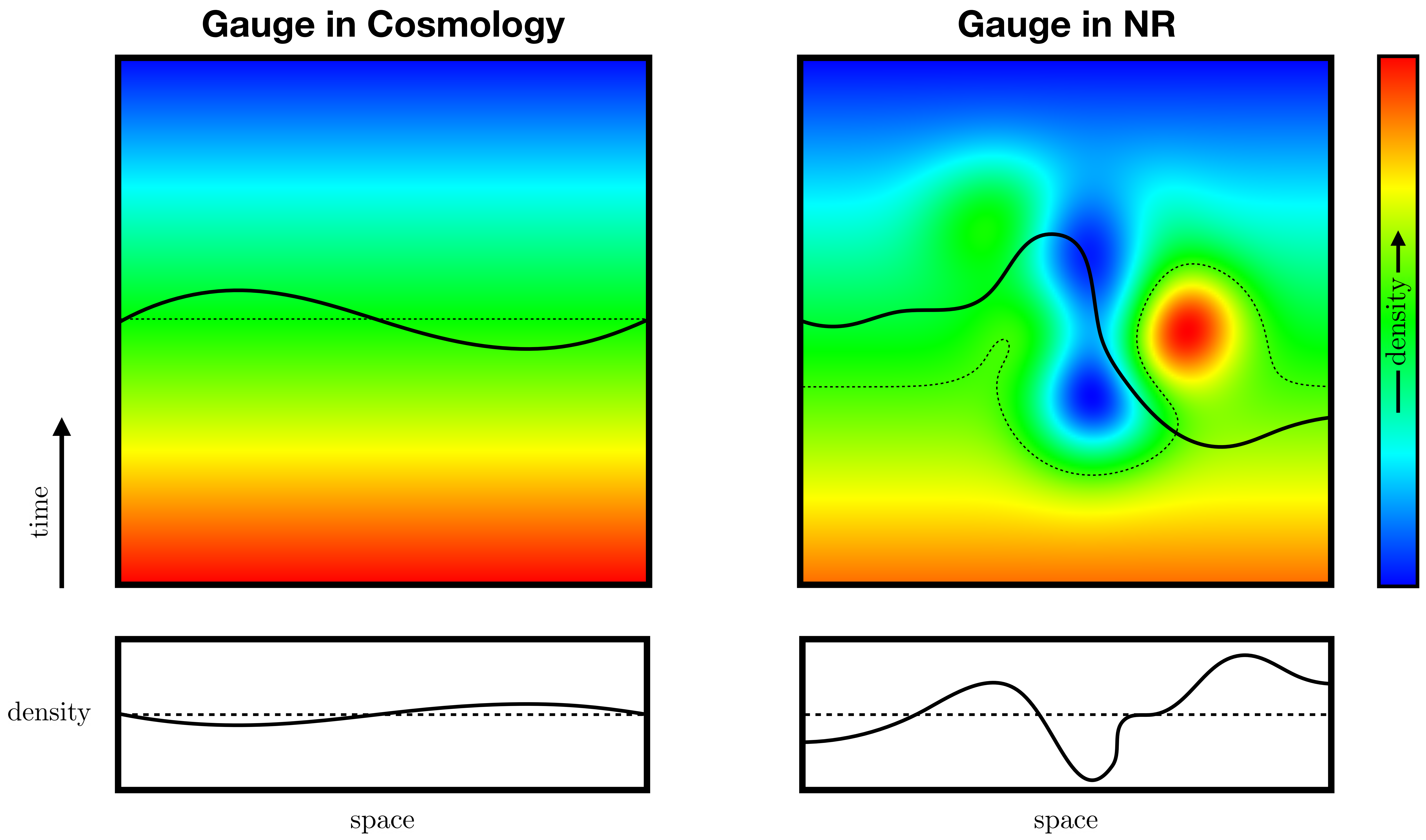


Goals:

1. Stability

2. Sample spacetime efficiently

Gauge conditions



Most gauges better adapted to BH spacetimes

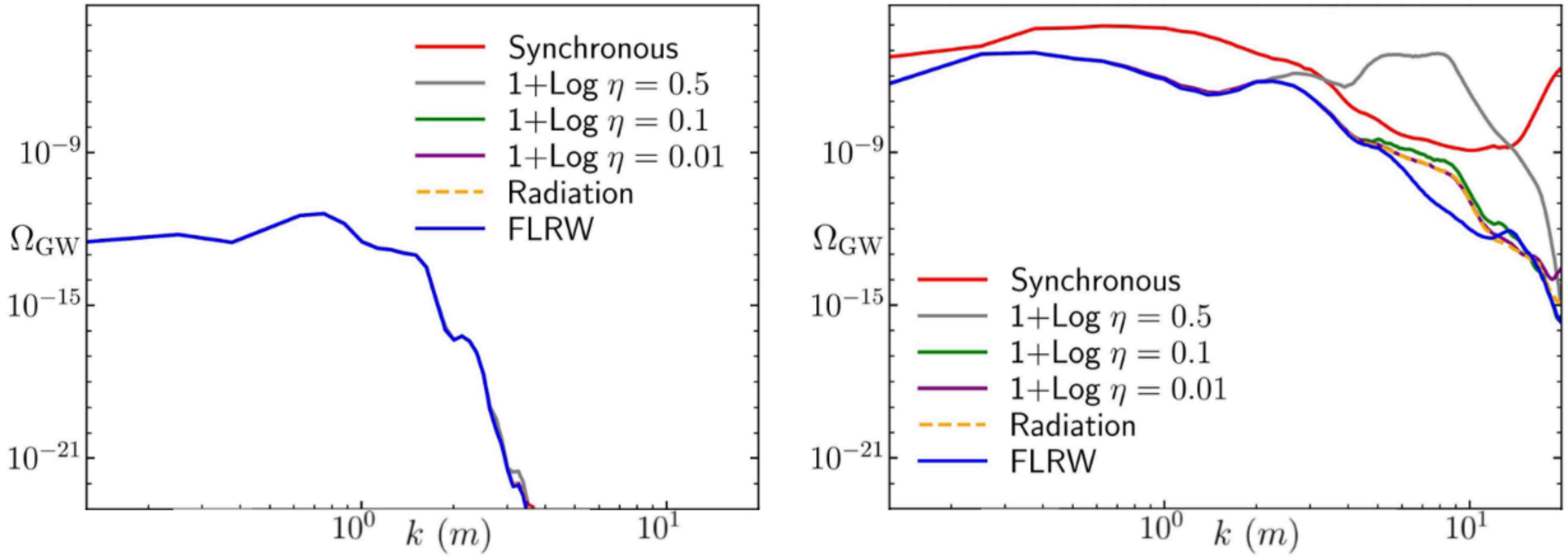
How to interpret global results?

Am I seeing gauge instabilities or physical BHs?

Interpretation

Early time

Late time



Easy question:

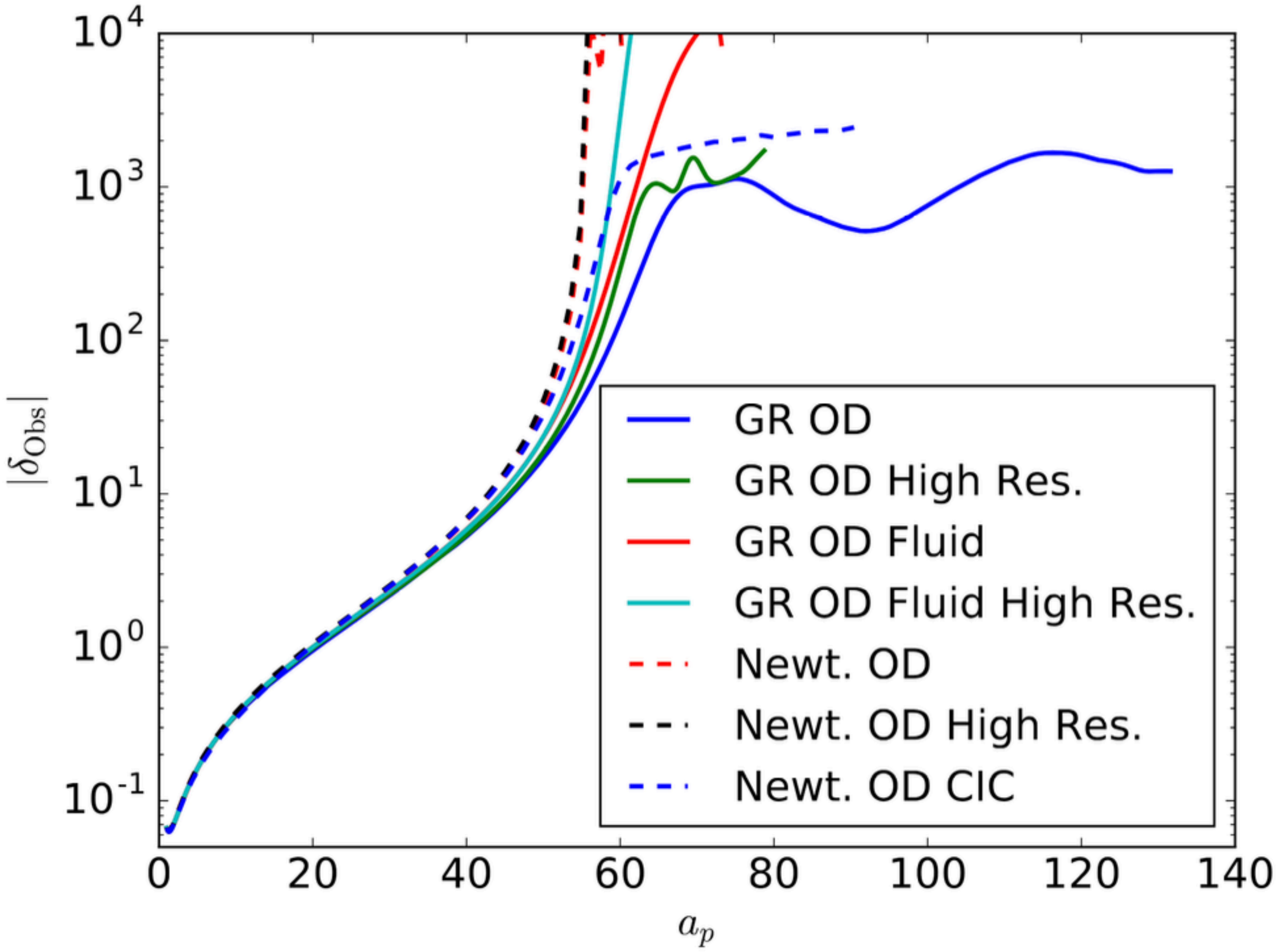
What is the average energy density of GWs on this slice of spacetime?

X Kou., JB Mertens., C Tian, SY Zhou
Gravitational waves from fully general relativistic oscillon preheating.
Phys. Rev. D 105(12), 123505 (2022)

Interpretation

Better question:

What does a specific geodesic observer see as they move through the spacetime?



WE East, R Wojtak, F Pretorius
Einstein-Vlasov Calculations of Structure Formation
Phys. Rev. D 100(10), 103533 (2019)

Interpretation

Best question:

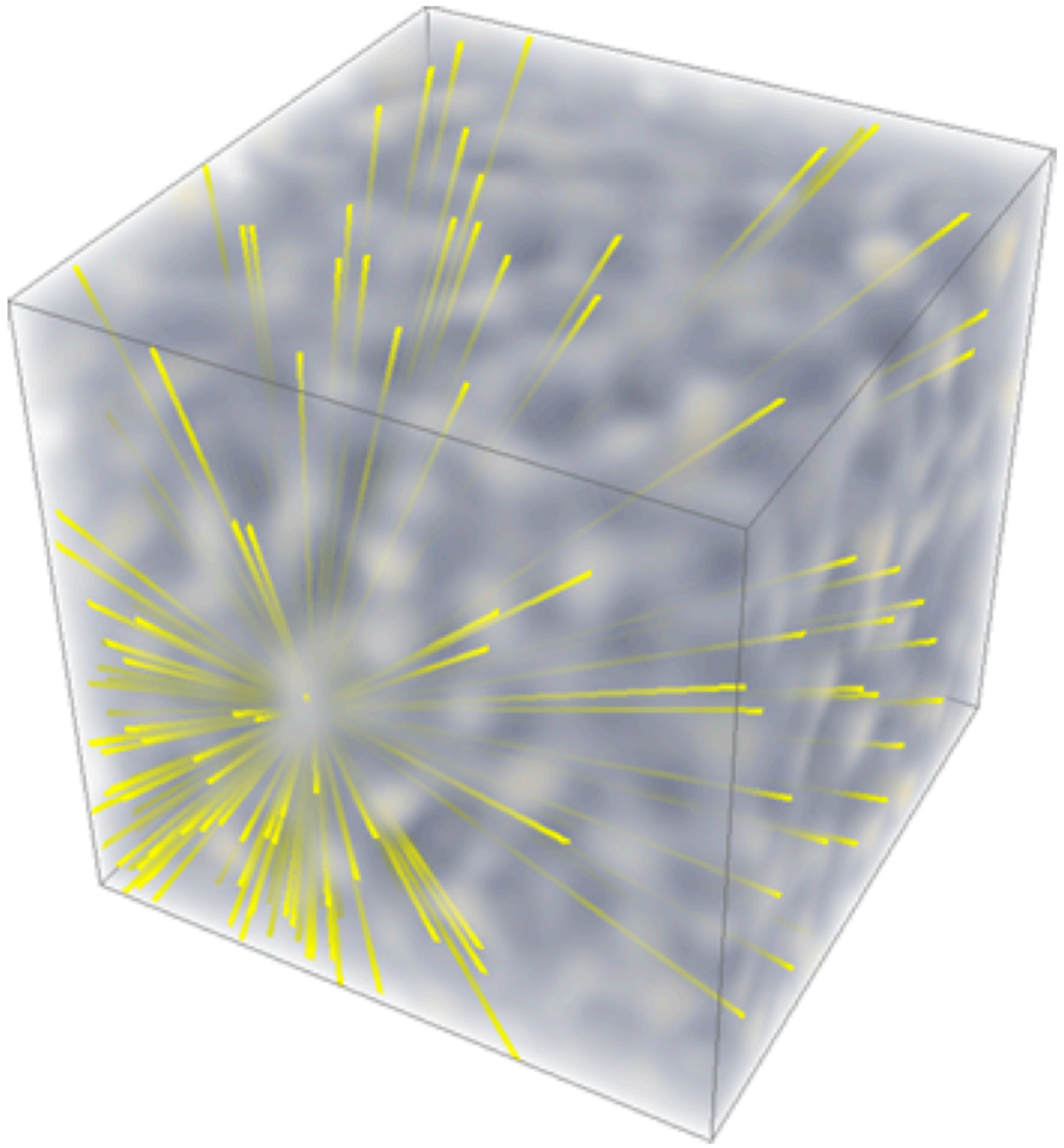
What is the statistical distribution of measured values on the past lightcones of observers at z=0?

Said (almost) no numerical relativist ever

Interpretation

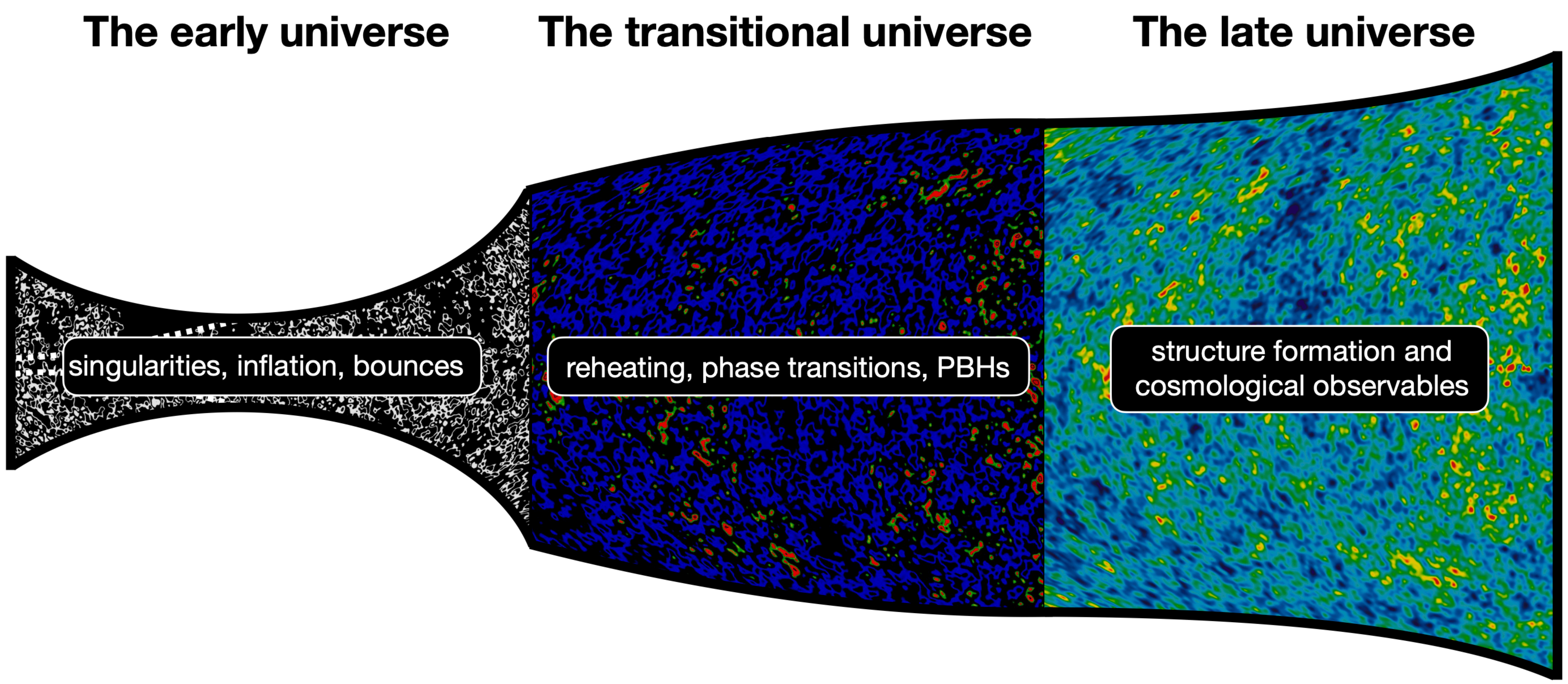
Best question:

What is the statistical distribution of measured values on the past lightcones of observers at z=0?



John T. Giblin, Jr, James B. Mertens, and Glenn D. Starkman
Observable Deviations from Homogeneity in an Inhomogeneous Universe
The Astrophysical Journal, Volume 833, Number 2

A very brief tour of NR for cosmology + axions (scalar fields)



The early universe

Can inflation get started from inhomogeneous initial data?

What is the impact of higher derivative corrections?

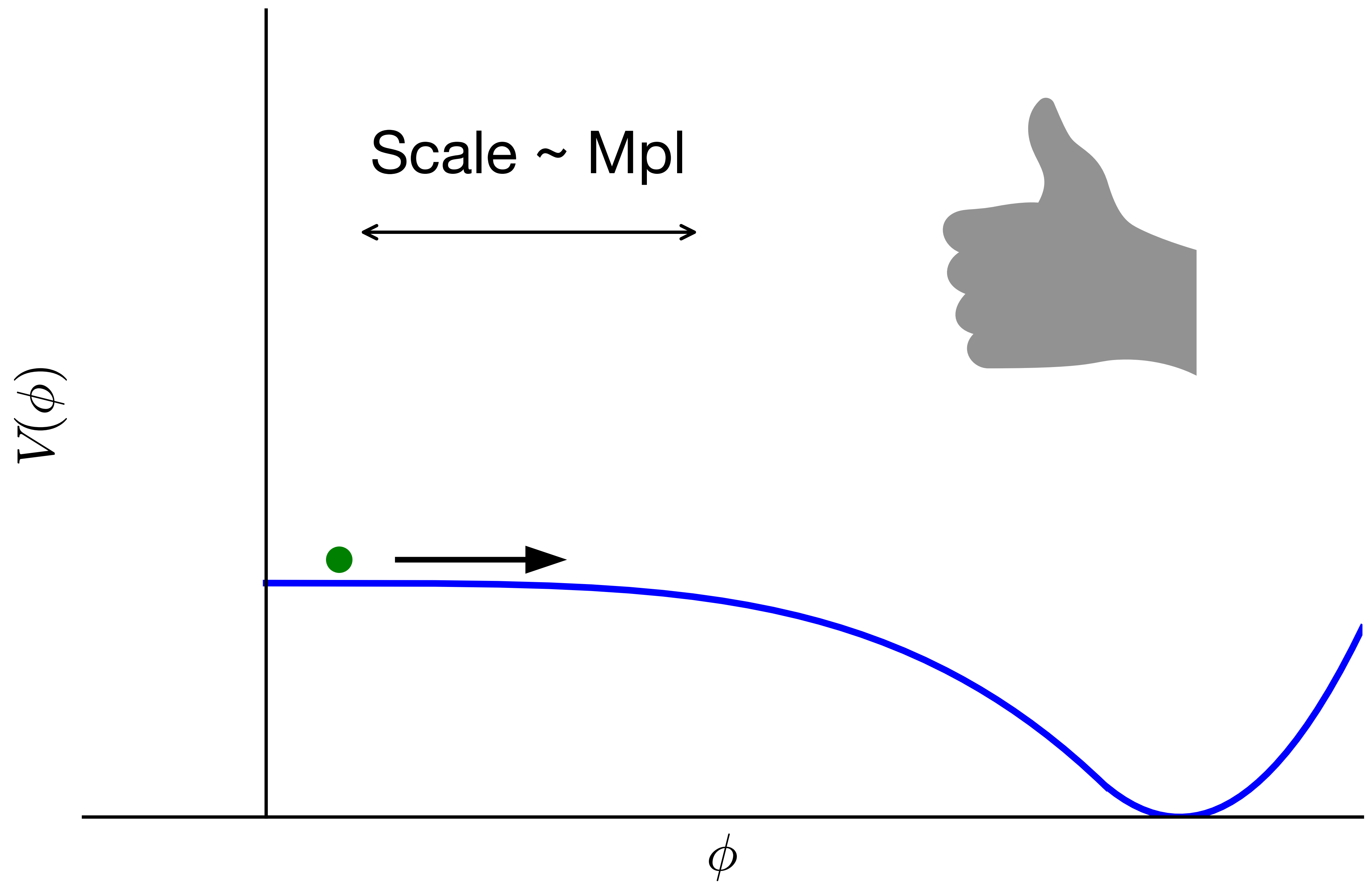
Are there signatures from early inhomogeneities?

Can inflation get started when there are inhomogeneities in the field and metric?

W. E. East, M. Kleban, A. Linde and L. Senatore
Beginning Inflation in an inhomogeneous universe
JCAP 1609 (2016) no.09, 010

KC, E. A. Lim, B. S. DiNunno, W. Fischler, R. Flauger, S. Paban
Robustness of Inflation to Inhomogeneous Initial Conditions
JCAP 1709 (2017) no.09, 025

See also review references
https://arxiv.org/pdf/2409.01939



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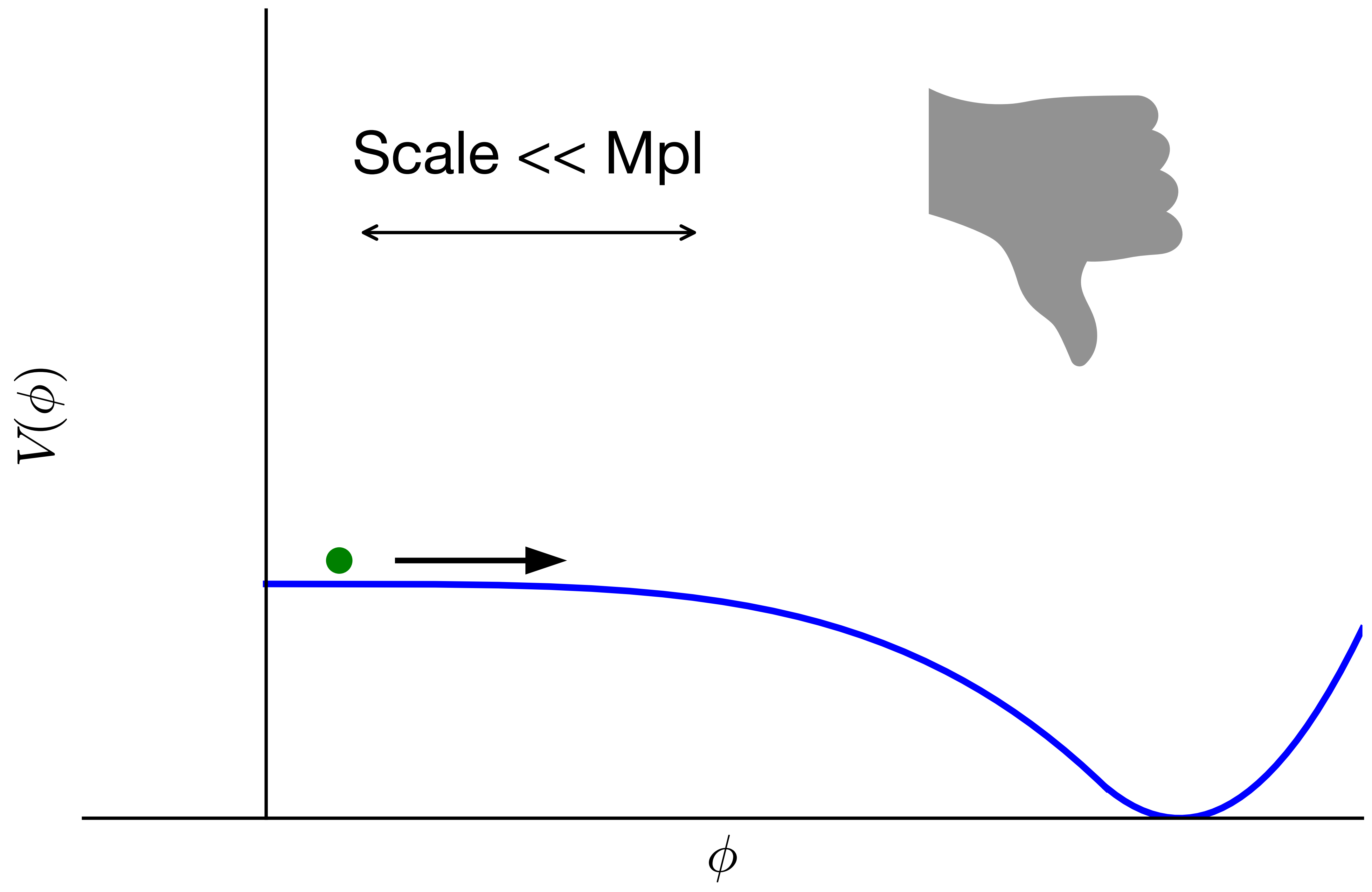
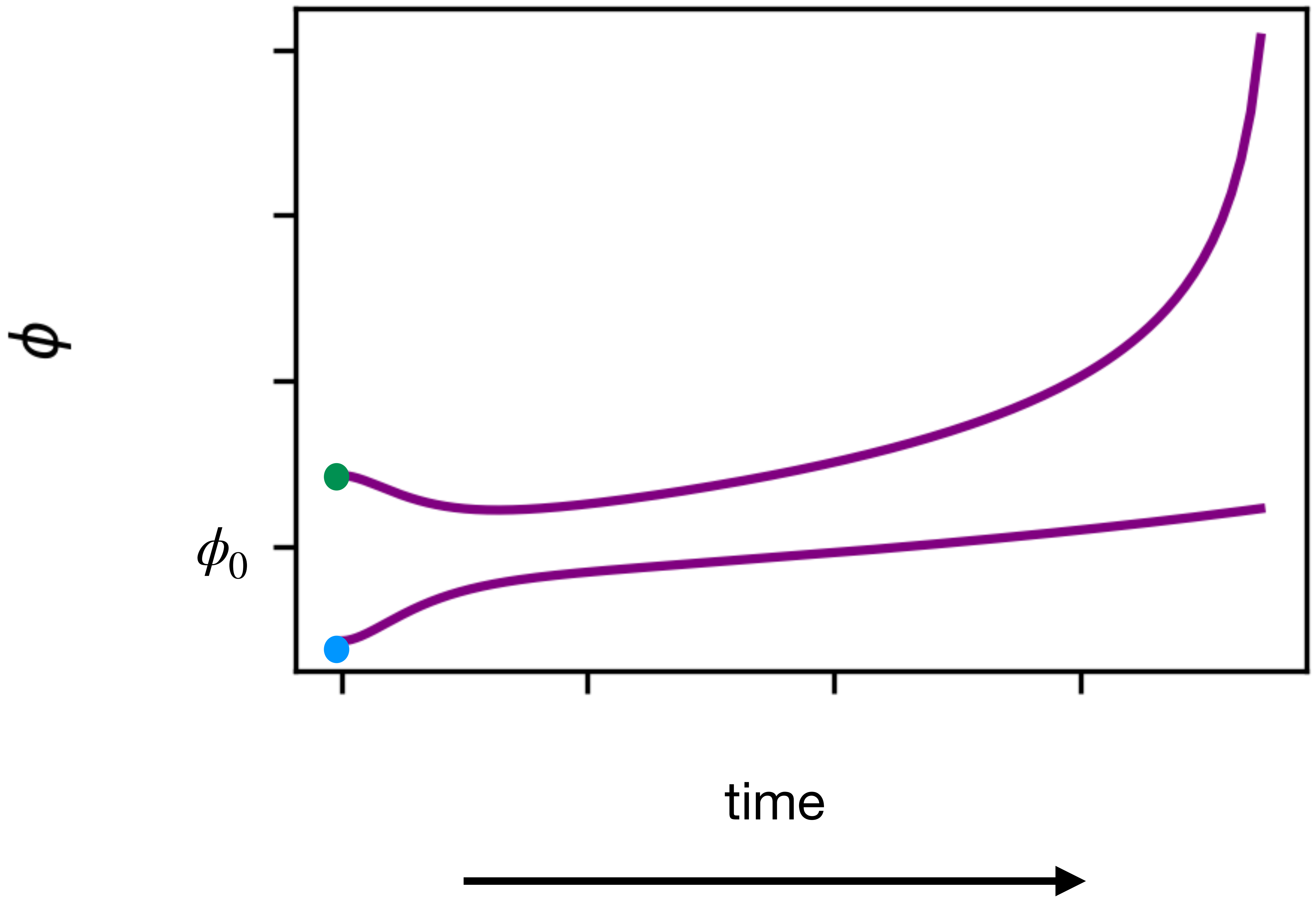
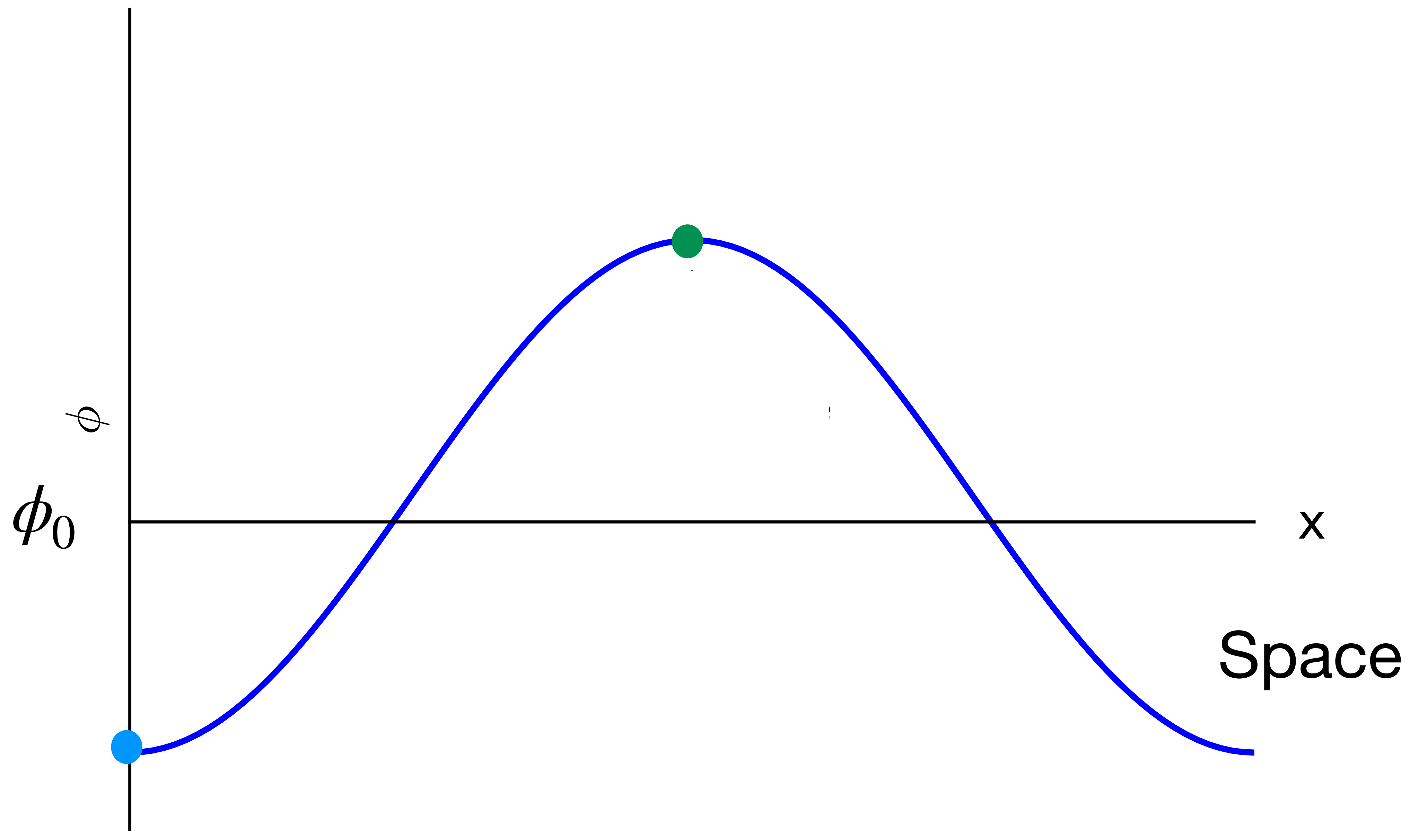


Inhomogeneous inflation

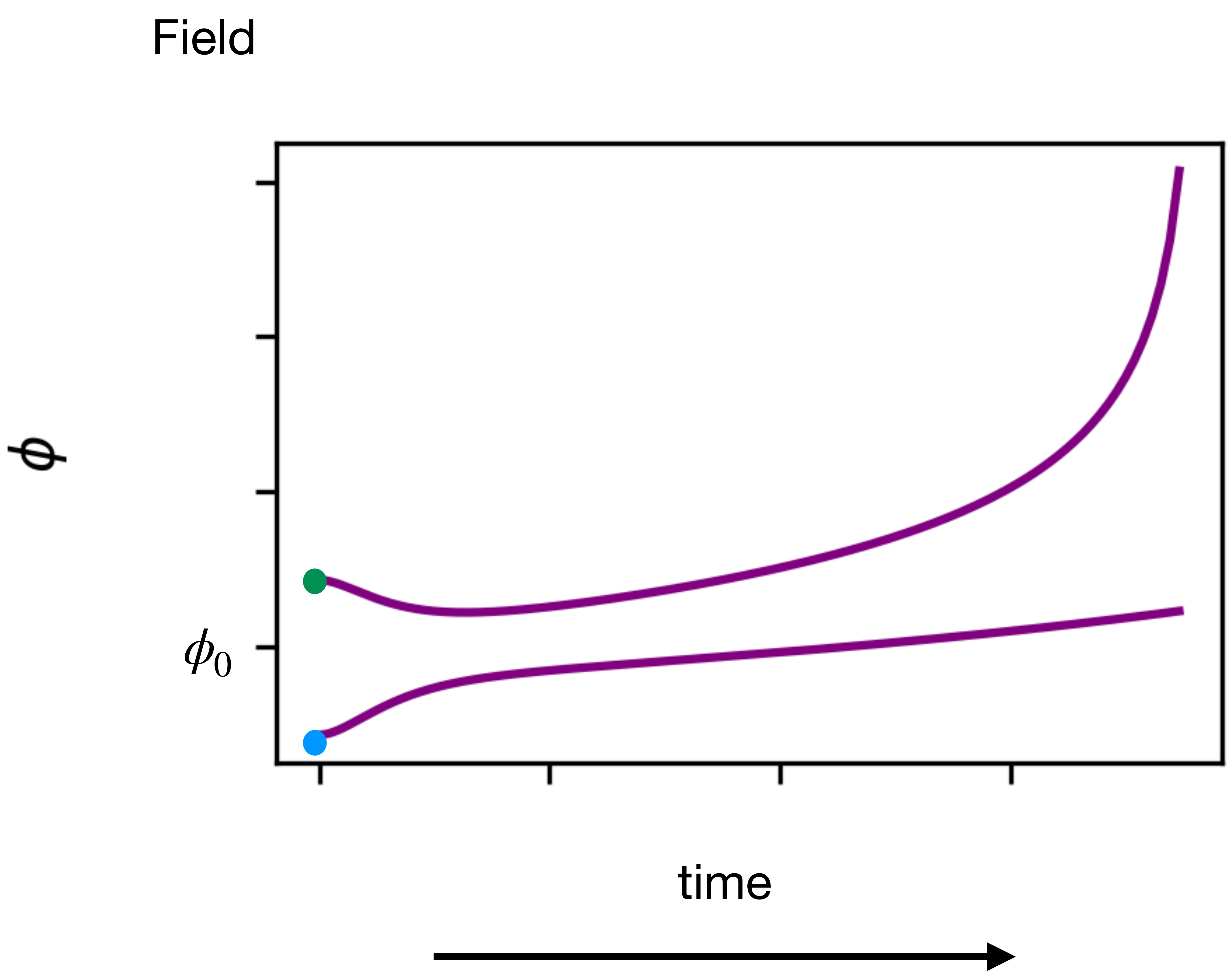
Field

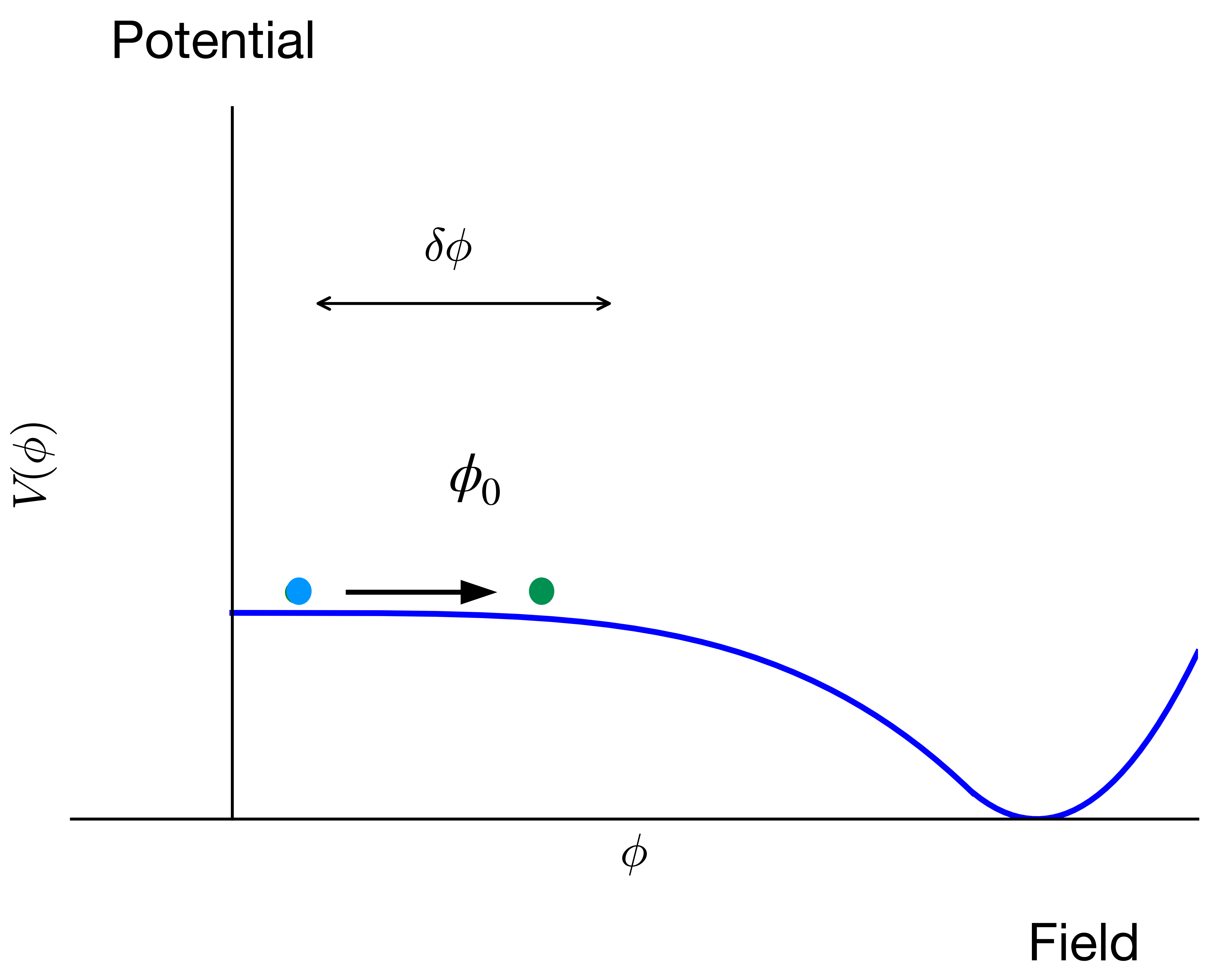


Field

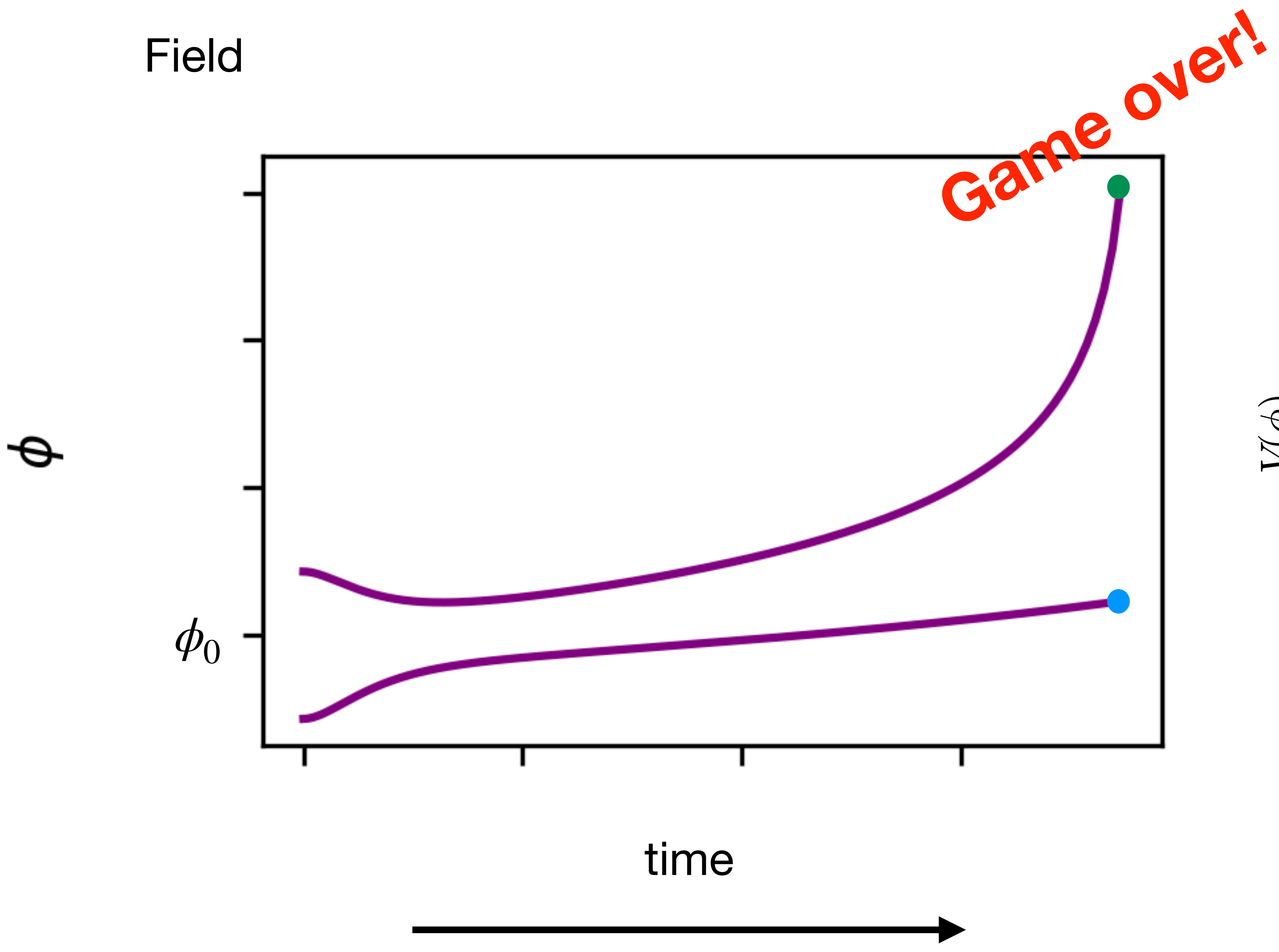


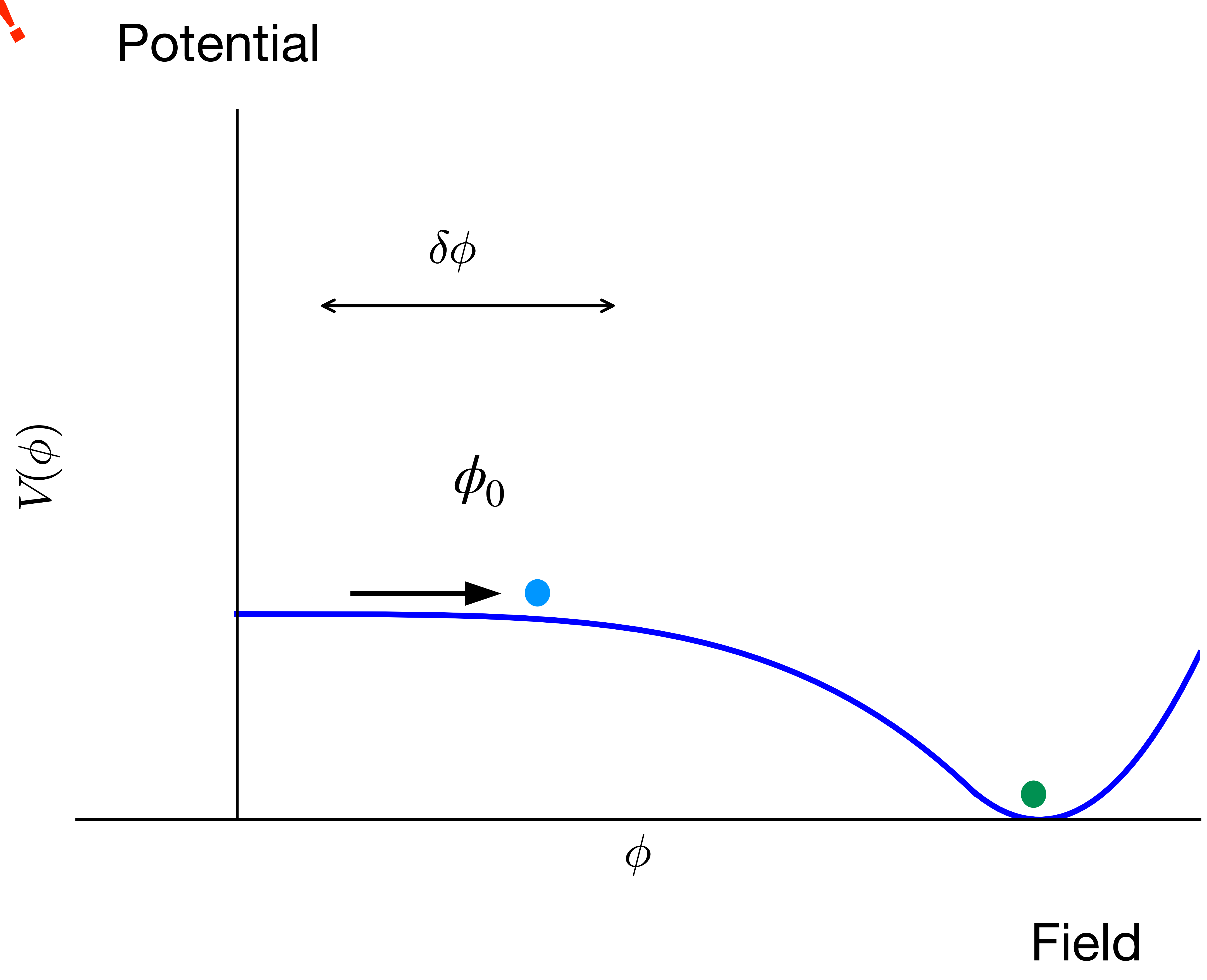
Inhomogeneous inflation



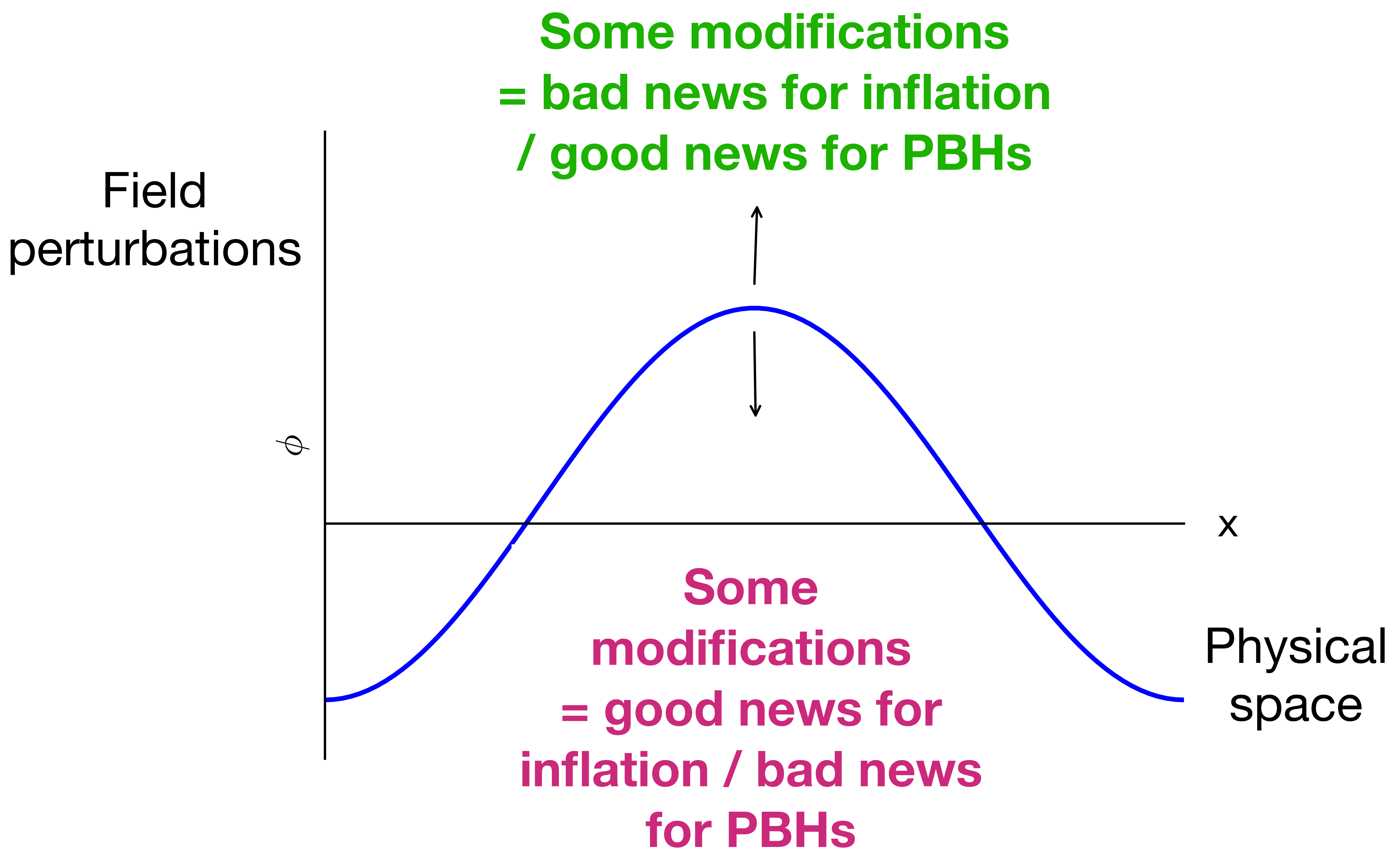


Inhomogeneous inflation





Do higher order modifications to GR change the dynamics of strongly inhomogeneous cosmologies?



e.g. 4dST ~ Einstein scalar Gauss Bonnet

Most general parity-invariant scalar-tensor theory of gravity up to (derivatives)^4:

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(R - X + g_2(\phi) X^2 - V(\phi) + \lambda(\phi) \mathscr{L}_{GB} \right)$$

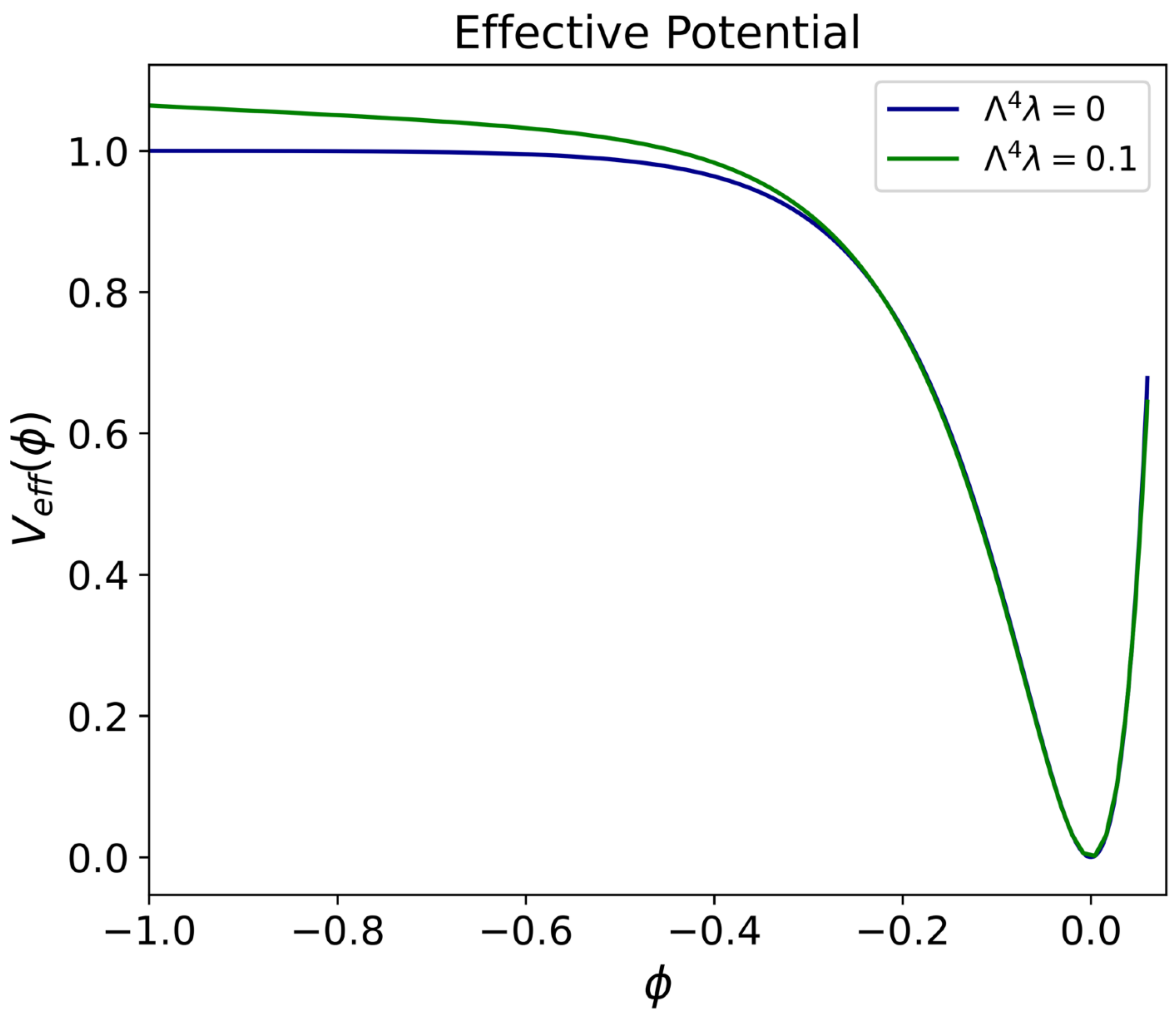
where $X = \nabla^\mu \phi \, \nabla_\mu \phi$

$$\mathscr{L}_{GB} = R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$$

The additional terms result (at zeroth order) in an additional tilt in the effective potential

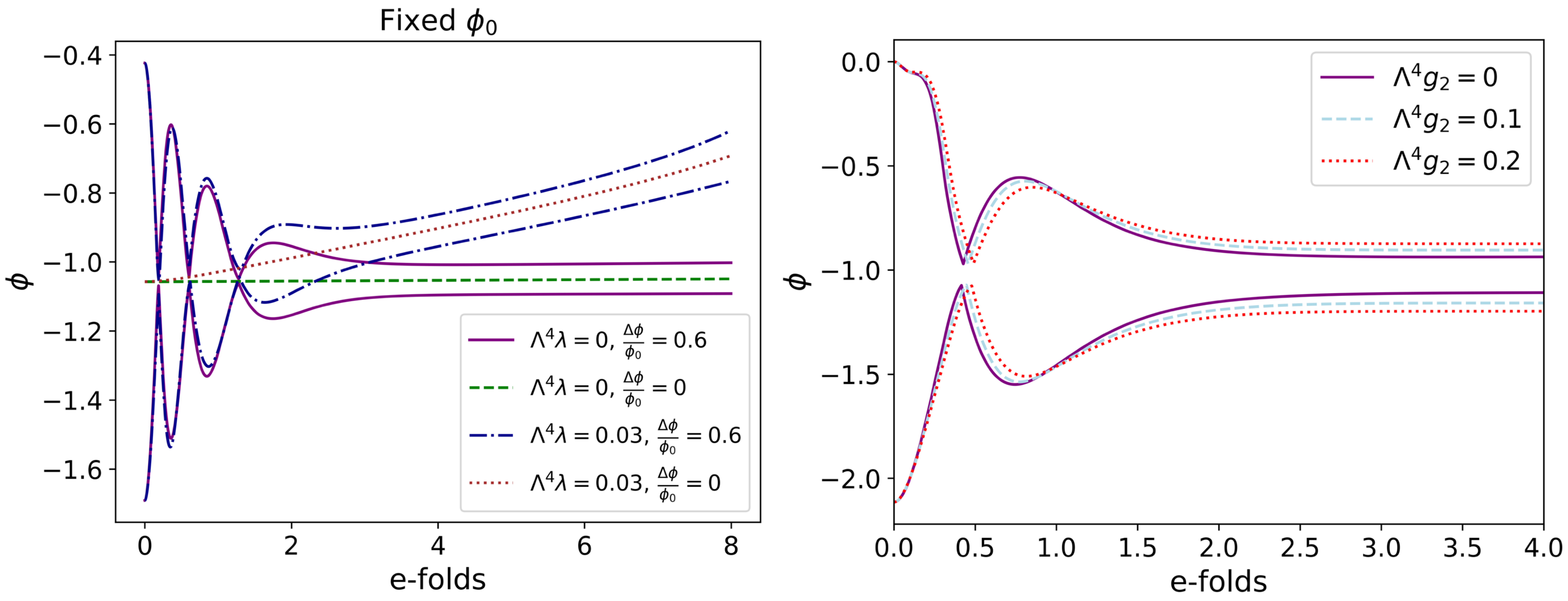
$$V_{eff}(\phi) = V(\phi) + V_{GB}(\phi)$$

$$V_{GB}(\phi) \sim -\lambda\phi(\dot{H}H^2 + H^4)$$



See e.g. S Nojiri, and SD Odintsov and M Sasaki
Gauss-Bonnet dark energy
Phys. Rev. D 71, 123509 (2005)

This tilt gives the dominant effect, effect on perturbations small



S Brady, KC, P Figueras, A Kovacs
Inflaton Dynamics in Higher Derivative Scalar-Tensor Theories of Gravity
gr-qc arXiv:2505.17986

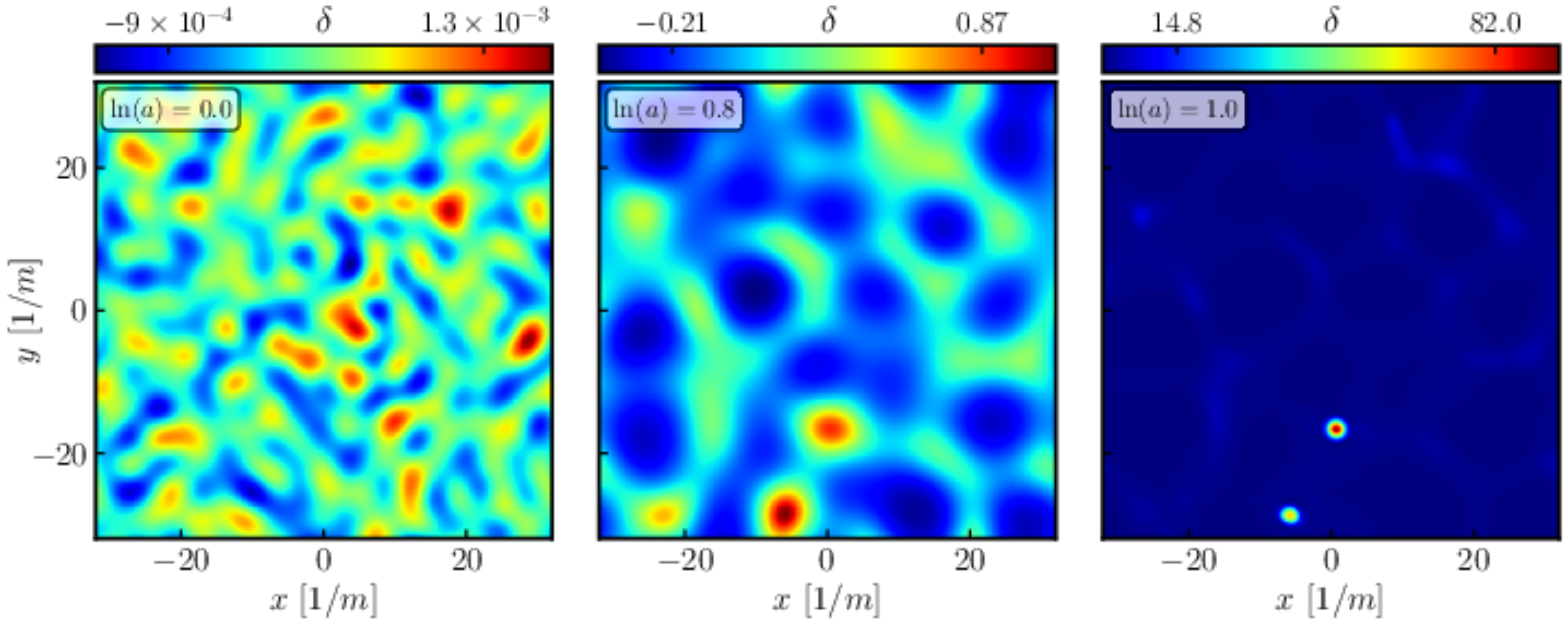
The transitional universe

Can reheating form dense structures/PBHs?

Does PBH formation change beyond spherical symmetry?

What are the signatures of phase transitions if we include back reaction?

Can we form PBHs from small initial perturbations from preheating-like mechanism?

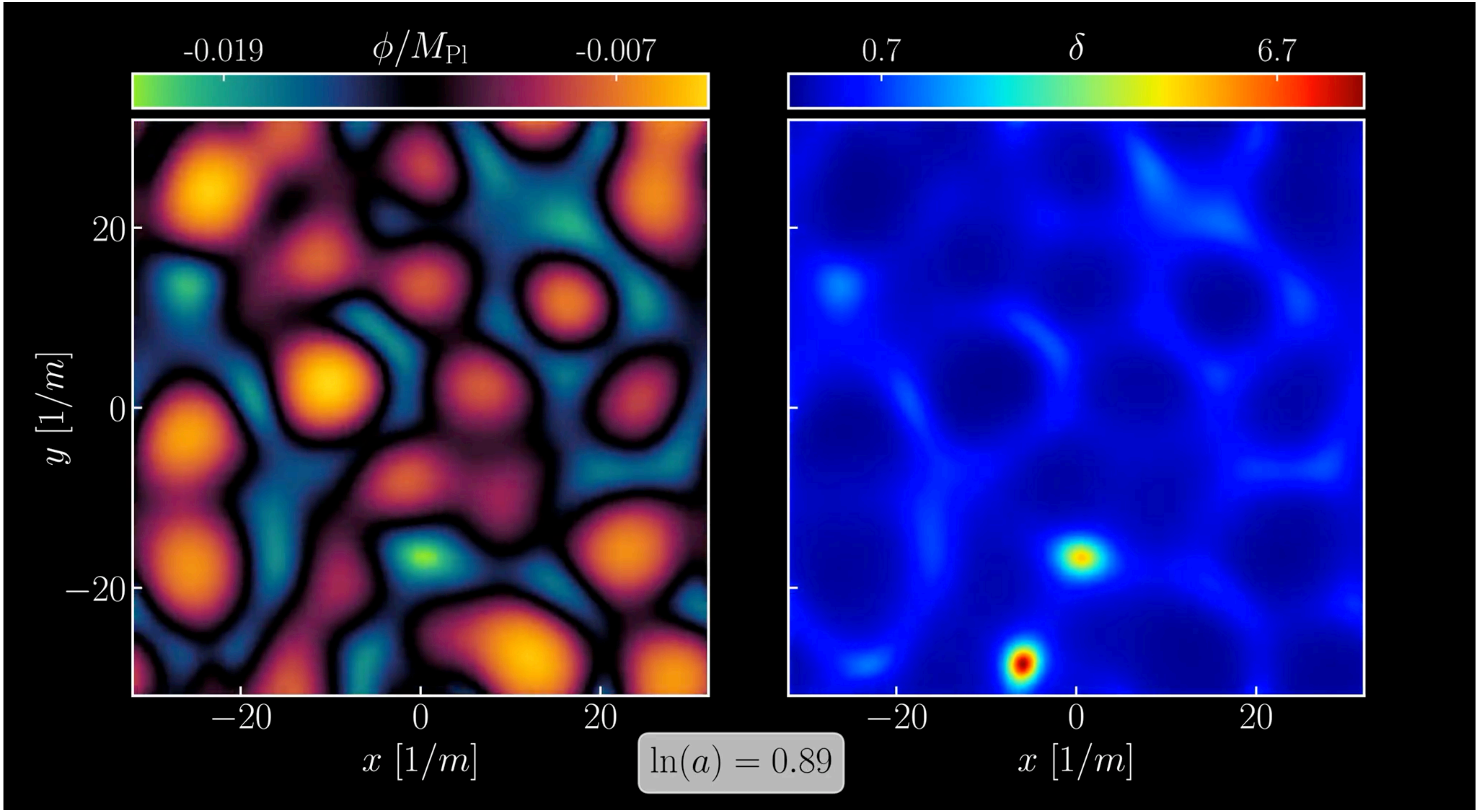


Density contrast

J. Aurrekoetxea, KC, F Muia
Oscillon formation during inflationary preheating with general relativity
Phys.Rev.D 108 (2023) 2, 023501

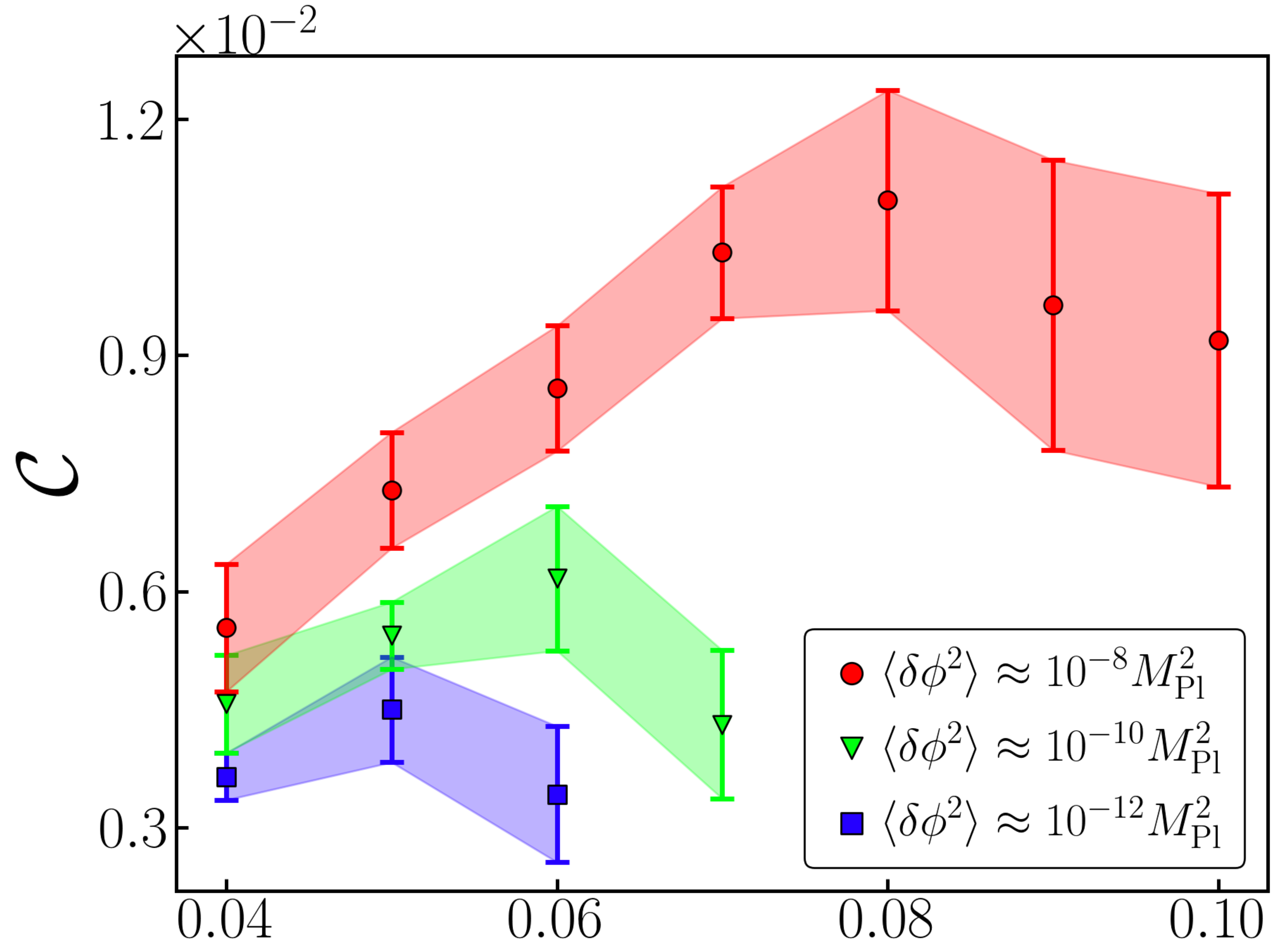
Scalar field

Density perturbations



Oscillons have a maximum compactness

Higher
Compactness



J. Aurrekoetxea, KC, F Muia
Oscillon formation during
inflationary preheating with
general relativity
Phys.Rev.D 108 (2023) 2, 023501

More large field alpha attractor model

The late universe

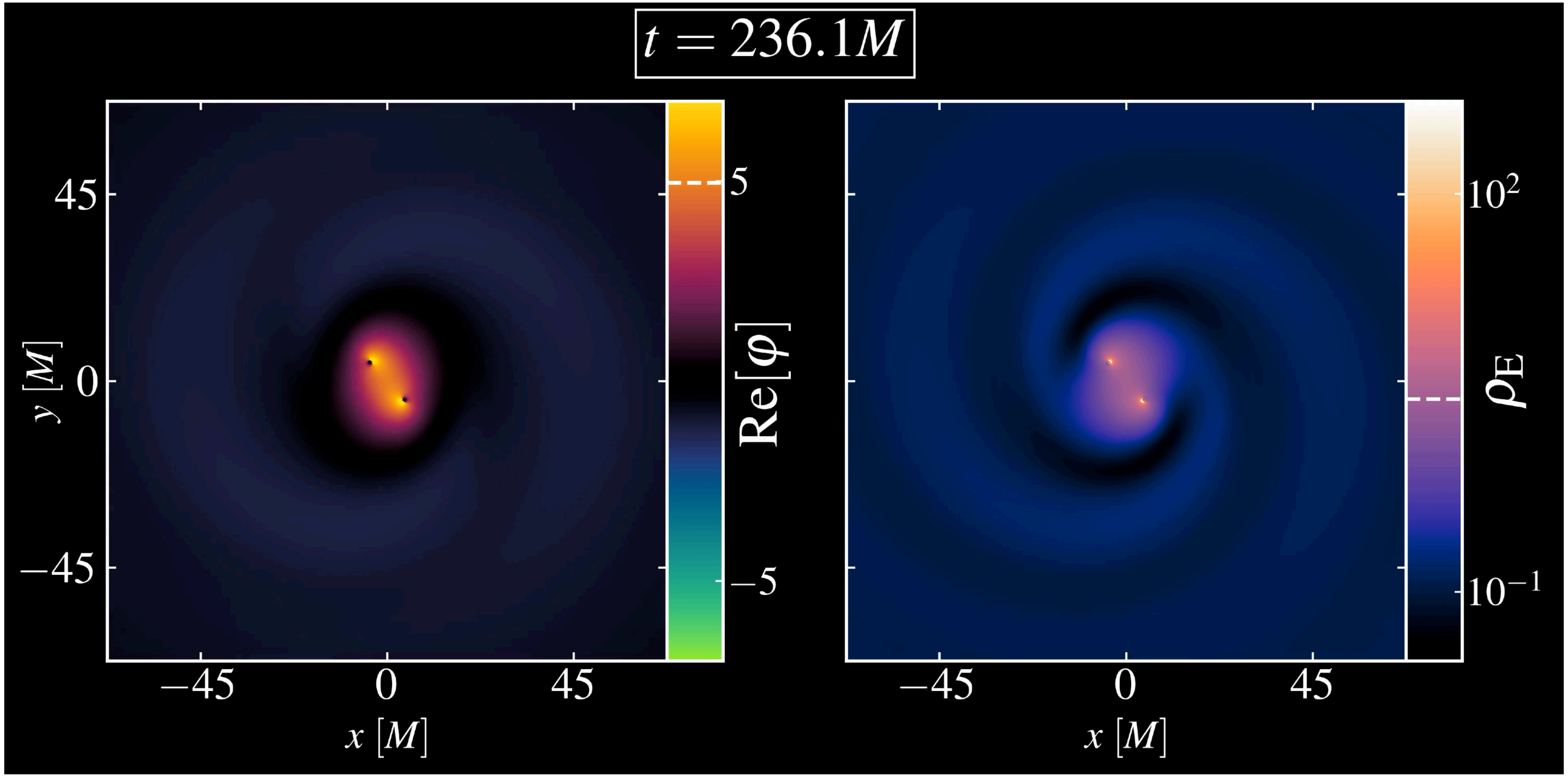
Are important non linear effects in GR neglected in standard FLRW + Newtonian particle treatment?

Is dark matter axions/wave like?

Are there observable effects in the non linear regime?

Axion-like DM can cluster around binaries

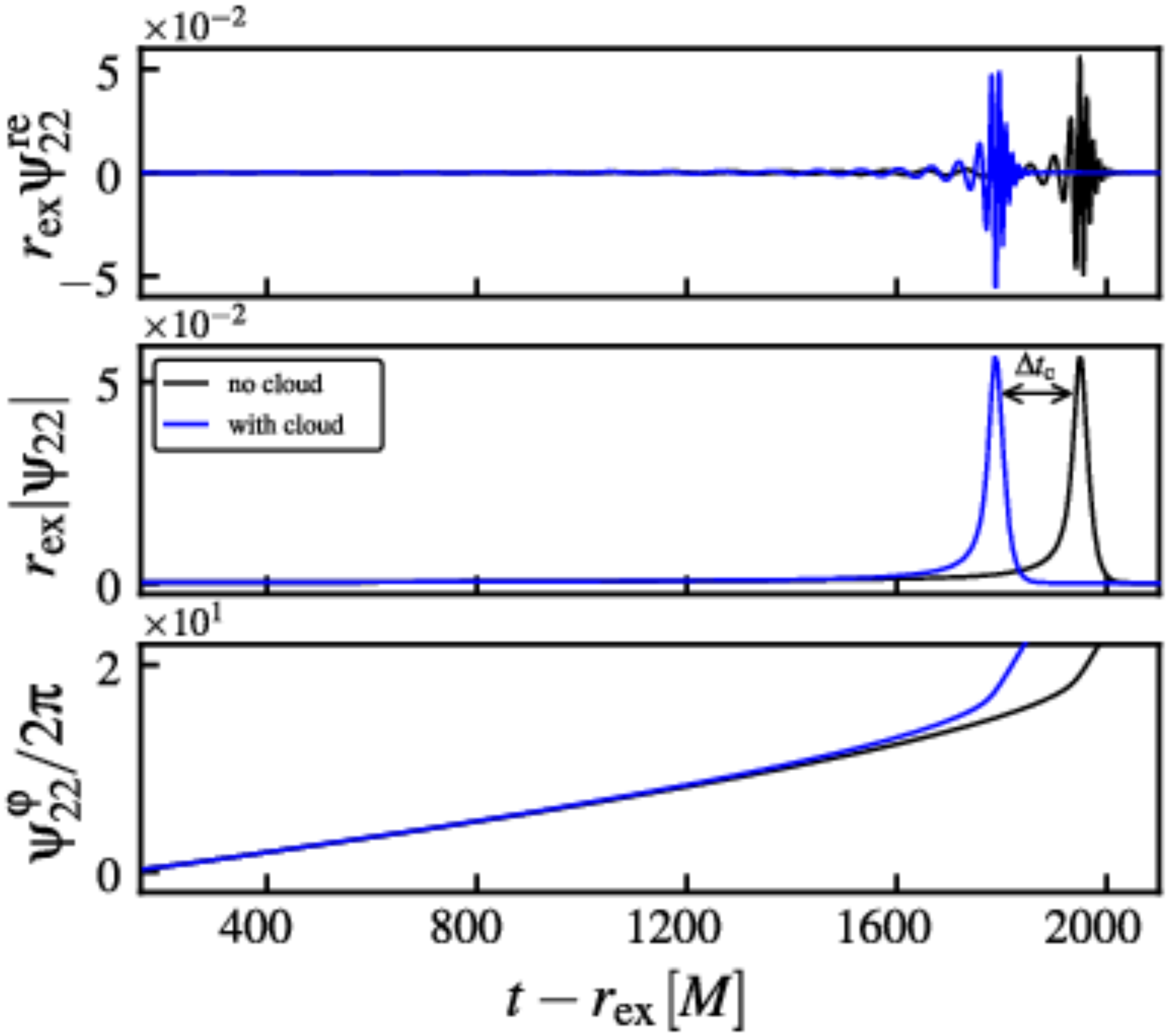
Field



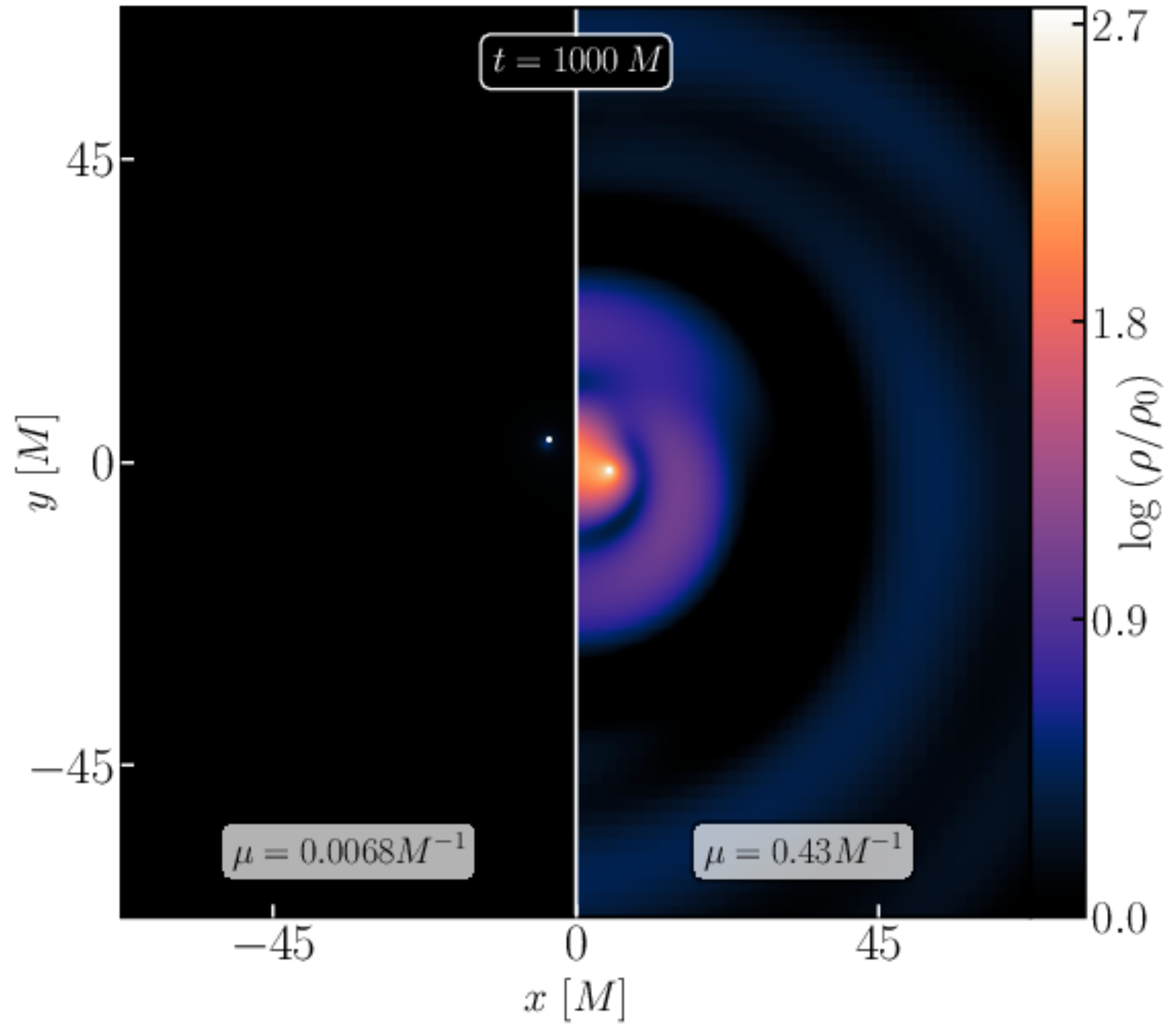
energy density

J Bamber, JC Aurrekoetxea, KC, P Ferreira 2023
Phys Rev D 107 2, 024035

This leads to a small dephasing of GWs



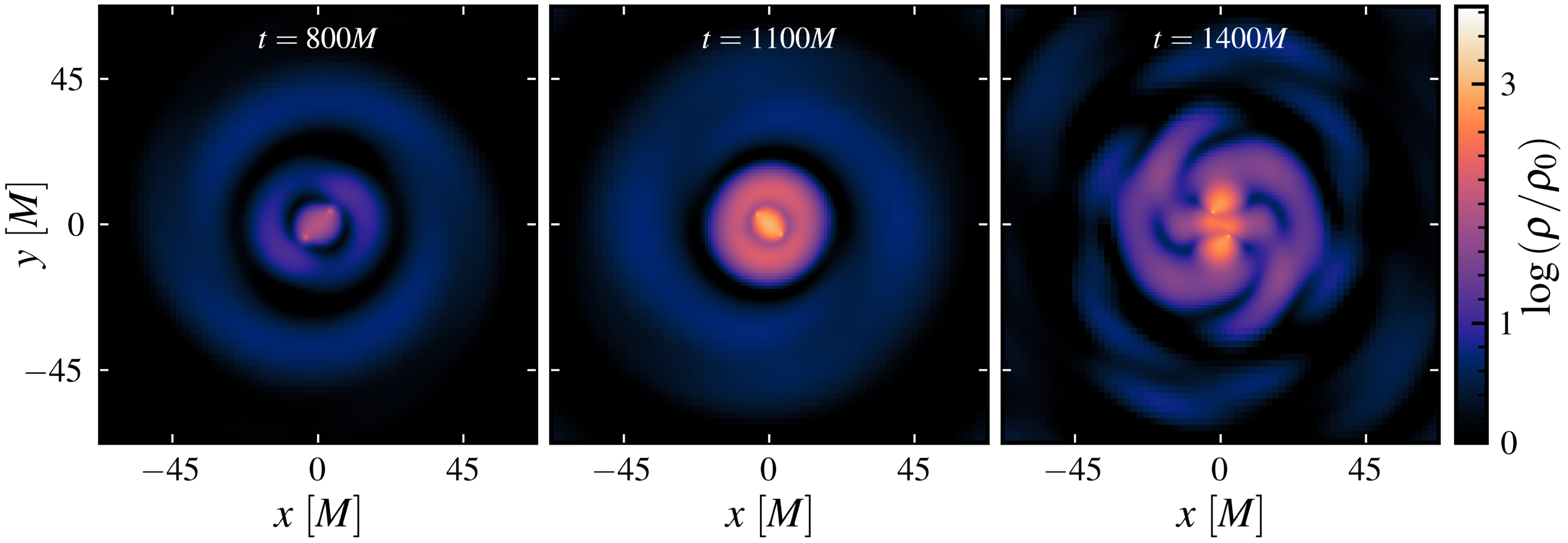
density



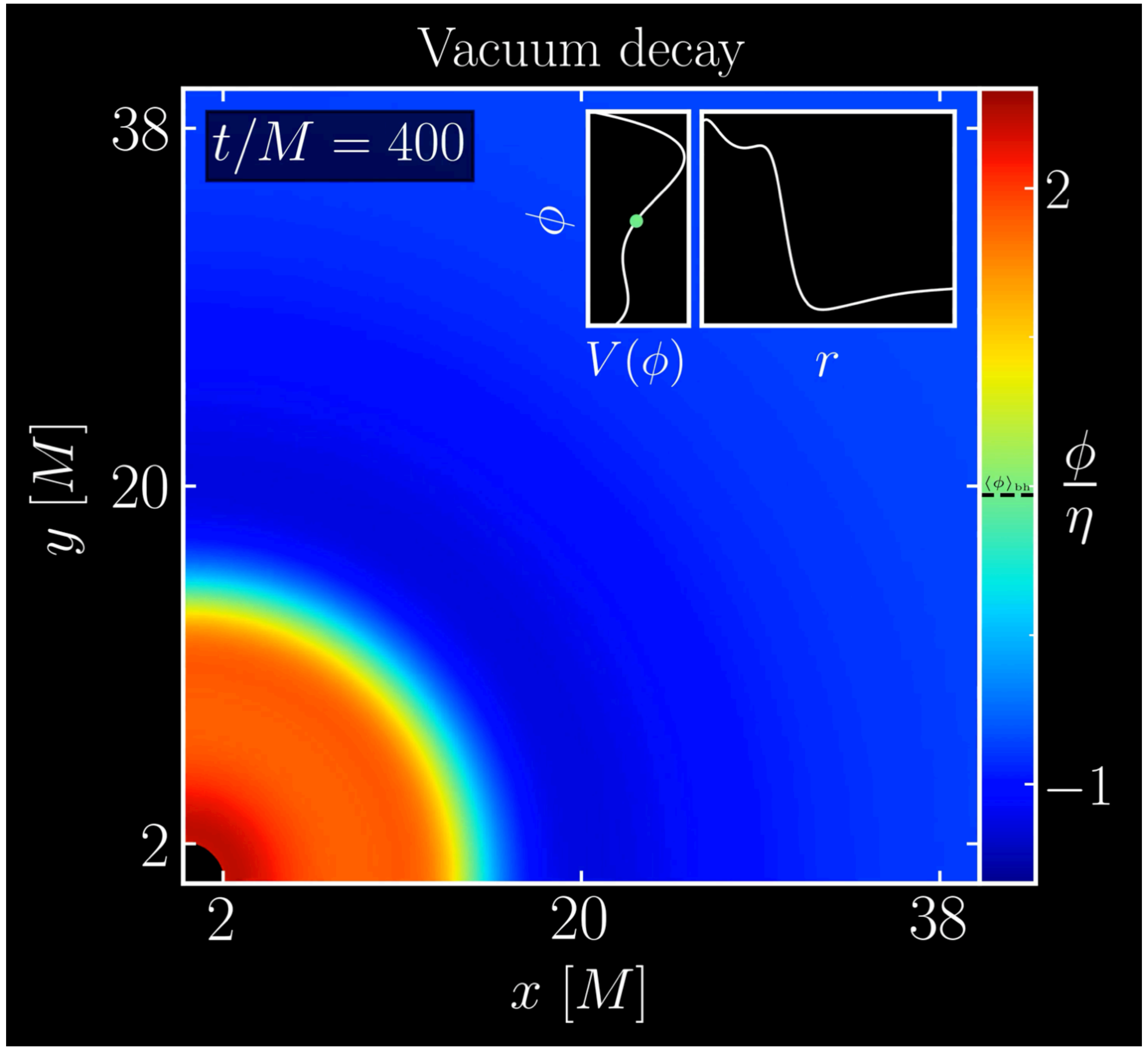
JC Aurrekoetxea, KC, J Bamber, P Ferreira 2023
Phys.Rev.Lett. 132 (2024) 21, 211401

Attractive self interactions result in "bosenova" like bursts

JC Aurrekoetxea, J Marsden, KC, P Ferreira 2023
Phys.Rev.Lett. 132 (2024) 21, 211401



Overdensities from accretion around black holes may lead to phase transitions



J Marsden, J. Aurrekoetxea, KC, P Ferreira 2024
Symmetry restoration and vacuum decay from accretion around black holes
Phys.Rev.D 111 (2025) 4, L041501

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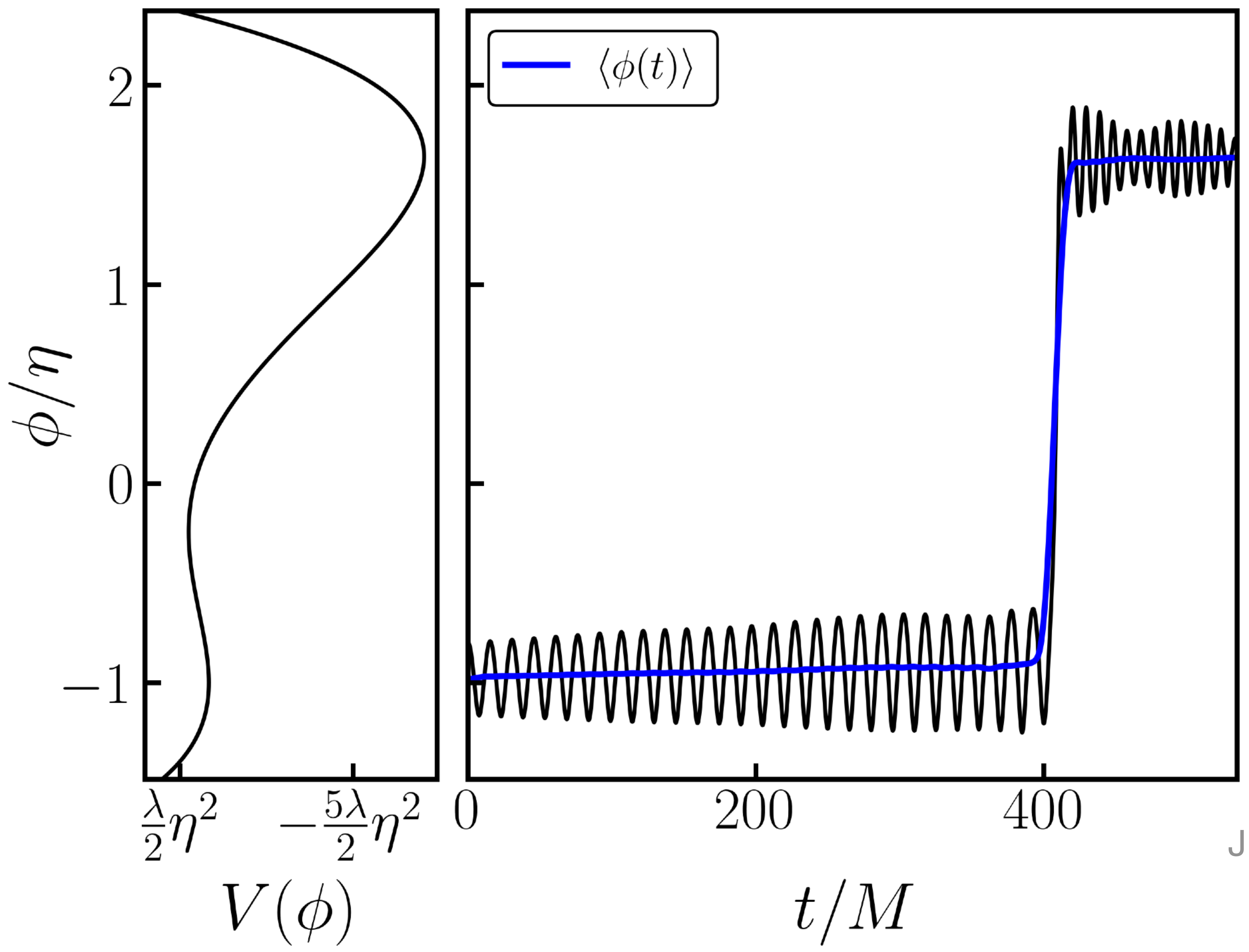

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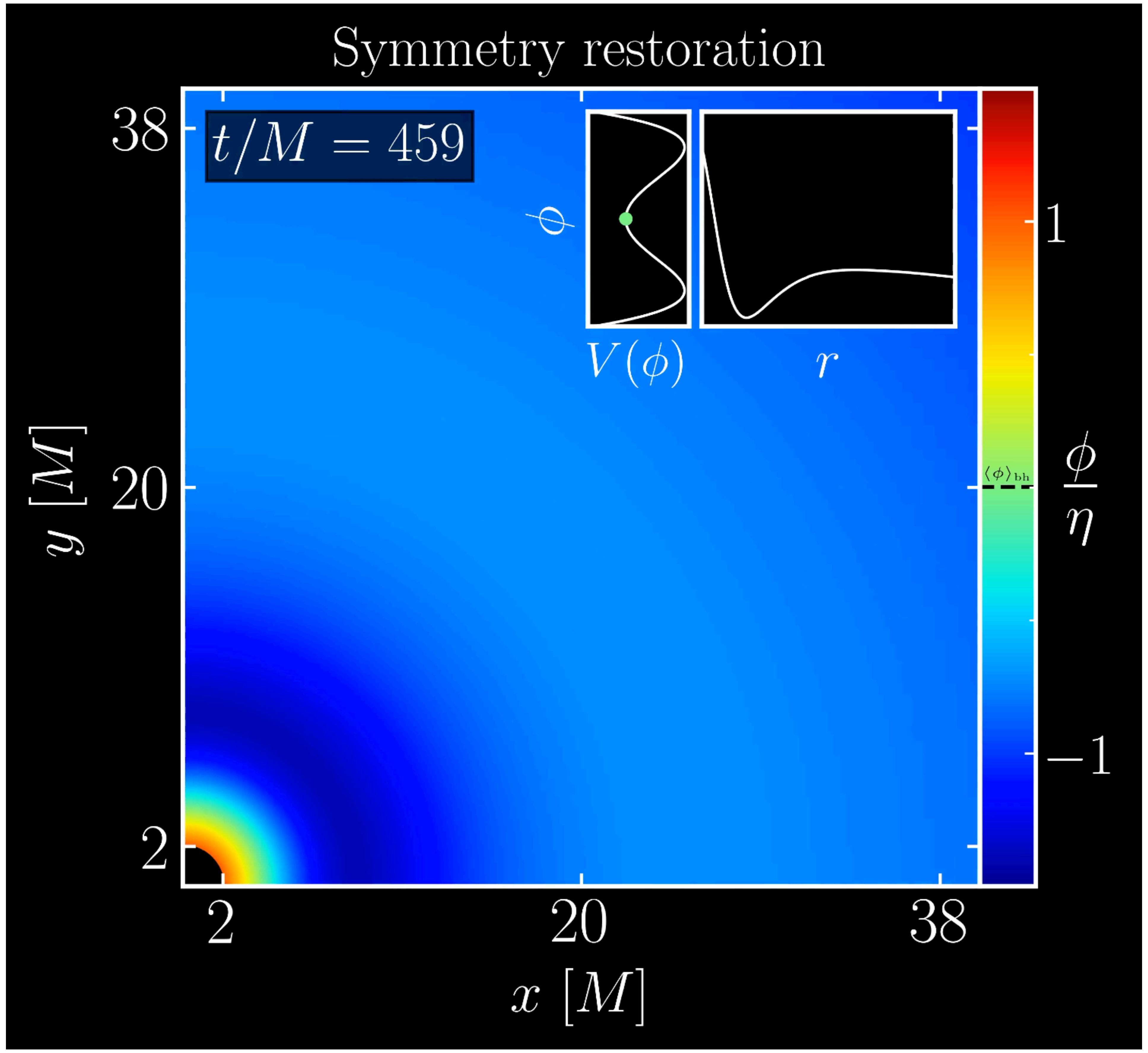

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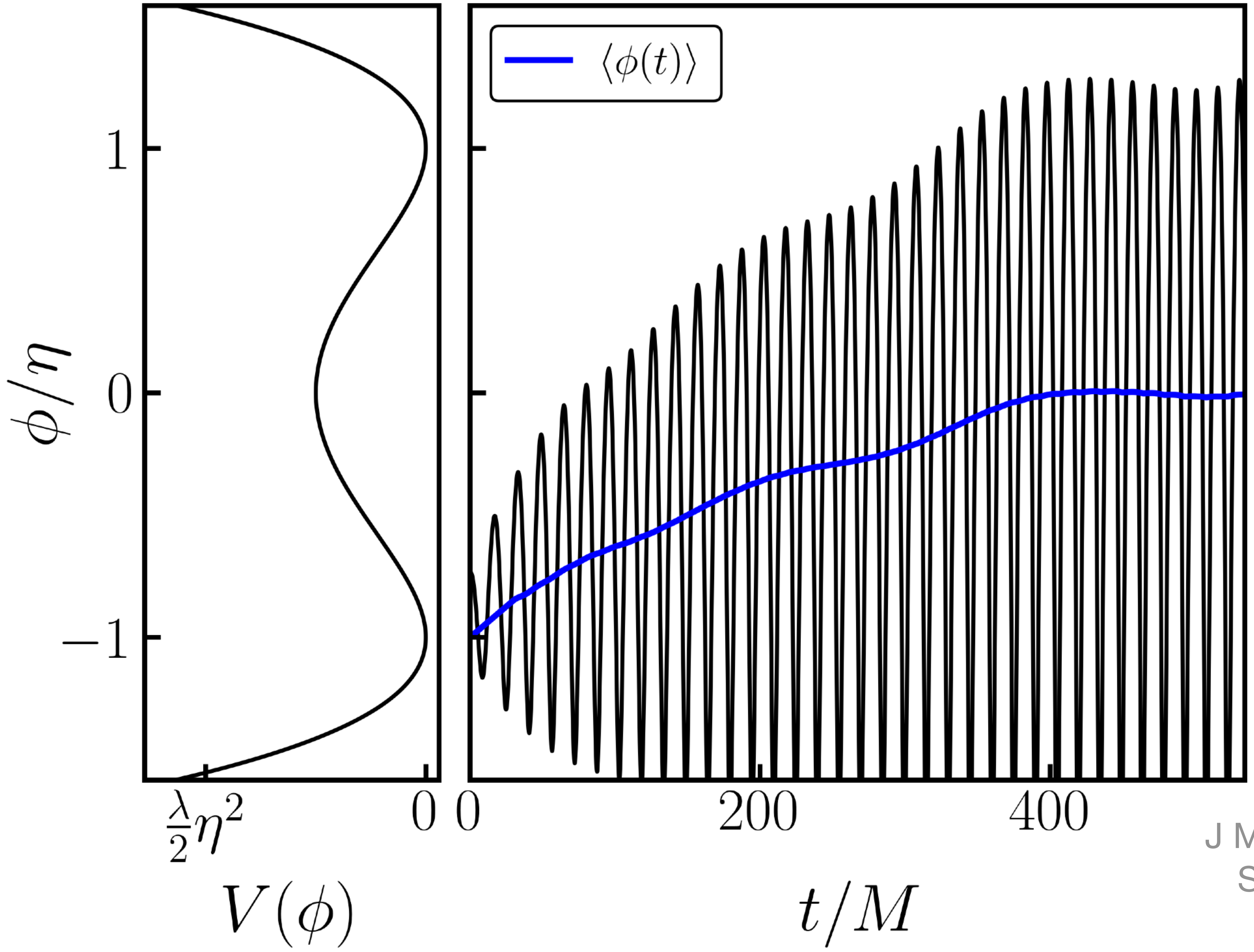

J Marsden, J. Aurrekoetxea, KC, P Ferreira 2024
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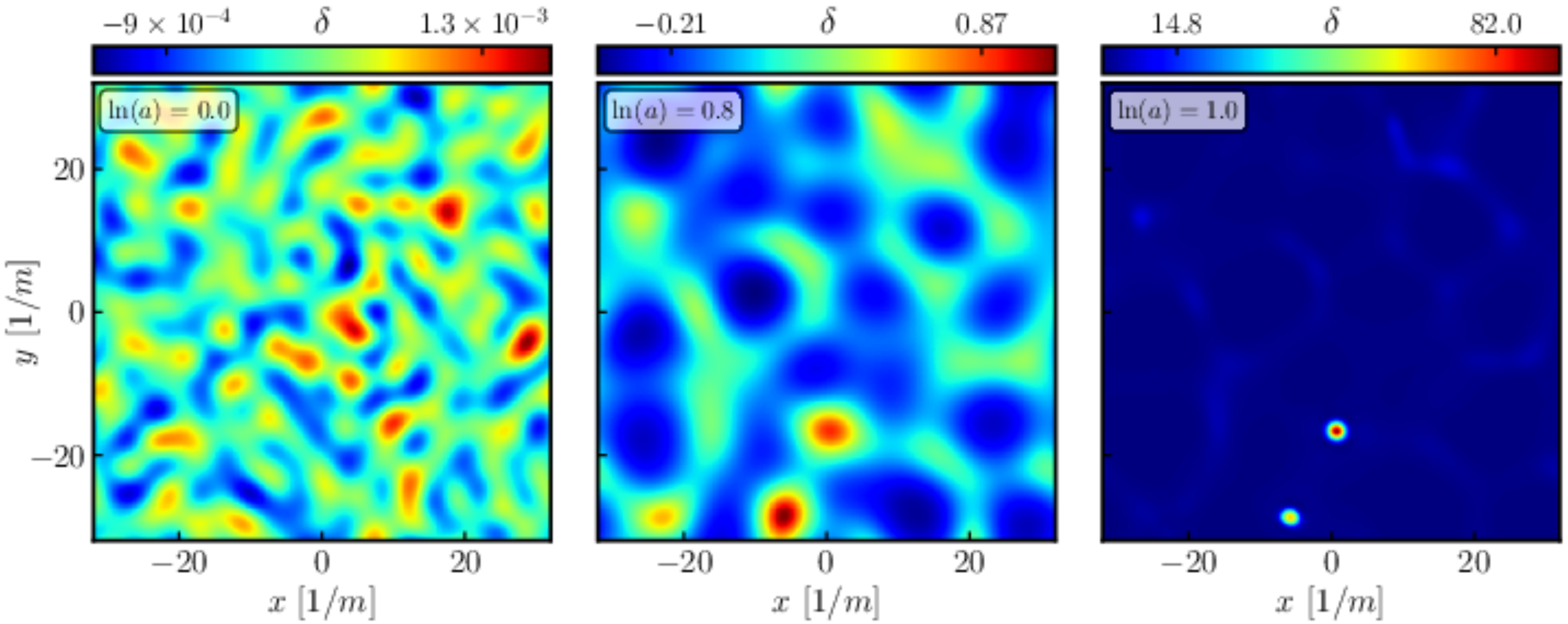
Summary

NR is not always necessary but can be an important tool for non linear, strong field gravity regimes, especially in the early universe



Lots more to do!

7 Summary table and open questions

A summary table of the work that has been completed in the various areas covered by the review is provided in Tab. 1 for reference. We also provide here an (incomplete) list of open questions and outstanding directions that have been highlighted by the review:

NR techniques for cosmology

1. What are the optimum boundary and gauge conditions for cosmological simulations, and how do they depend on the physics being studied?
2. How can we access a wider range of physical scales or make approximations to allow us to study GR effects across scales, e.g. in string networks?
3. What are the best diagnostics for inhomogeneous spacetimes? Can spatial averages be trusted or do we always need to use geodesic observers? If so, how should they be positioned to sample the spacetime?
4. How can we accurately quantify GWs or other stochastic background observables in cosmological spacetimes?
5. How can we construct initial data that is physically "correct", as well as constraint satsfying? To what extent do arbitrary choices like conformal flatness limit the physics we can study? Can we go beyond these?
6. Can we develop open source tools that are general purpose for different cosmological spacetimes, e.g. a general initial condition solver for 3+1D periodic spacetimes?

Genesis: the early universe

1. Does the BKL conjecture continue to hold for asymptotically flat spacetimes?
2. What is the role of periodic boundary conditions in findings about the robustness of inflation simulations? Do they provide a bias towards inflation?
3. What is the impact of considering multi-field models on the robustness of inflation?
4. How can the smooth contraction phase connect to the bounce in Ekpyrosis scenarios?
5. What are the observable consequences of either inflation or bouncing scenarios in the CMB or other observations like GW backgrounds?
6. Do modifications to gravity at higher energies change these behaviours? Can we connect to quantum gravity/string theory predictions?

Exodus: the transitional universe

1. How does gravity and AMR impact the evolution of a network of early universe relics? e.g. oscillons, topological defects.
2. Are there any (p)reheating scenarios that generate significant amounts of PBHs with small amplitude fluctuations?

	Dimension		Boundaries		Matter type		GR formulation		
	No 3+1D	3+1D	Periodic	FLRW	Fluids	Fund. fields	BSSN/CCZ4	GHC	Other
Singularities	[89, 108–119, 121–124, 126–128]	[134] (fully 3D), [135] (2D using 3D code)	[108–114, 114–119]	[119] (AP)	– –	[89, 119, 126–128]	– –	[134, 135]	[89, 108–119, 121–124, 126–128]
Inflation	[49, 81, 141, 142, 146, 147, 153]	[72, 75, 88, 90–93, 143, 144, 146, 149] (fully 3D), [84] (axisymmetric)	[49, 72, 75, 80, 90–93, 143, 144, 146–149, 153]	[54, 81, 141, 142]	[142]	[49, 72, 75, 80, 90–93, 141–144, 146–149, 153]	[75, 80, 91–93, 148, 149]	[72, 90]	[49, 147, 153]
Bounces	[160, 161, 163, 165, 168, 169]	[165] (fully 3D),[53, 164] (axisymmetric)	[160, 161, 163–165, 168, 169]	[53]	– –	[53, 160, 161, 163–165, 168, 169]	– –	[53, 169]	[160, 161, 163–165, 168]
Bubble collisions	[187, 193–196, 199, 200]	– –	– –	[187, 193–196, 199, 200]	– –	[187, 193–196, 199, 200]	– –	– –	[187, 193–196, 199, 200]
Reheating	[82, 201, 203]	[73, 83, 97, 98, 102, 205, 207]	[73, 82, 83, 98, 102, 201, 205, 207]	[203] (extrapolating)	– –	[73, 82, 83, 97, 98, 102, 201, 203, 205, 207]	[73, 97, 98, 102, 207]	– –	[82, 83, 201, 203, 205]
Phase transitions	[212, 213]	[214–217]	[214–217]	[212, 213]	[213]	[212–217]	[214–217]	– –	[212, 213]
Primordial BH	[51, 52, 231–233, 236, 239, 243–247, 249–258]	[74, 266–268]	[74, 266, 268], [267] (collective)	[51, 52, 231–233, 239, 243–247, 249–258]	[51, 52, 231–233, 236, 239, 243–247, 249–258, 266, 267]	[74, 268]	[52, 74, 266–268]	– –	[51, 231–233, 236, 239, 243–247, 249–258]
Backreaction		[79, 84, 208, 285, 286, 288–296, 298, 300, 301, 304–310, 312]	[79, 84, 208, 285, 286, 288–296, 298, 300, 301, 304–310, 312]	– –	[79, 84, 208, 285, 286, 288–296] (perfect fluids), [298, 300, 301] (particles), [304–310] (BH lattice), [312] (GWs)	– –	[79, 84, 208, 285, 286, 288–294, 300, 301, 312]	[286, 298]	[285]
Observables	[283, 313, 319–325]	– –	[283, 313, 319–325]	– –	[283, 313, 319–325]	– –	[283, 313, 319–325]	– –	– –

Table 1: Summary of the work in this review.

3. How can we best quantify the stochastic GW background produced from early universe phenomena like reheating?
4. What are the effects of non-spherical perturbations (i.e. going to 3+1D simulations) in the formation of PBHs?
5. Can we make quantitative predictions that connect observational data to parameters of the fundamental theory?

Revelation: The late universe

1. What is the impact of including GR effects on systematics for next generation cosmological surveys?
2. Does the assumption of periodicity that is used in most simulations impact on the (lack of) development of non-FLRW features?
3. Are there other GR effects that could explain cosmological tensions?
4. What are the signatures of modified gravity effects in the large scale structure statistics, e.g. non-linear screening?

https://link.springer.com/article/10.1007/s41114-025-00058-z

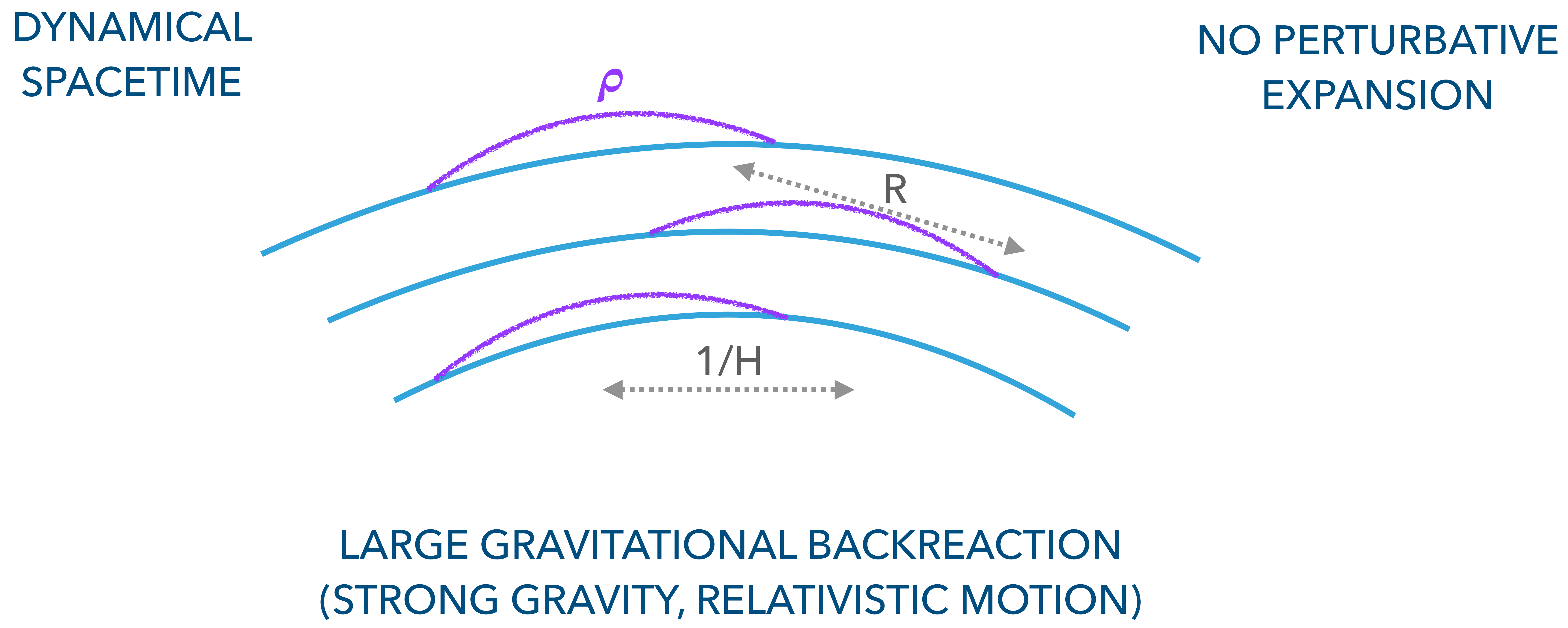
For discussion

1. What are the open questions for which NR could be useful?

2. What are the observables that we need to extract from the simulations, and how best to measure them?

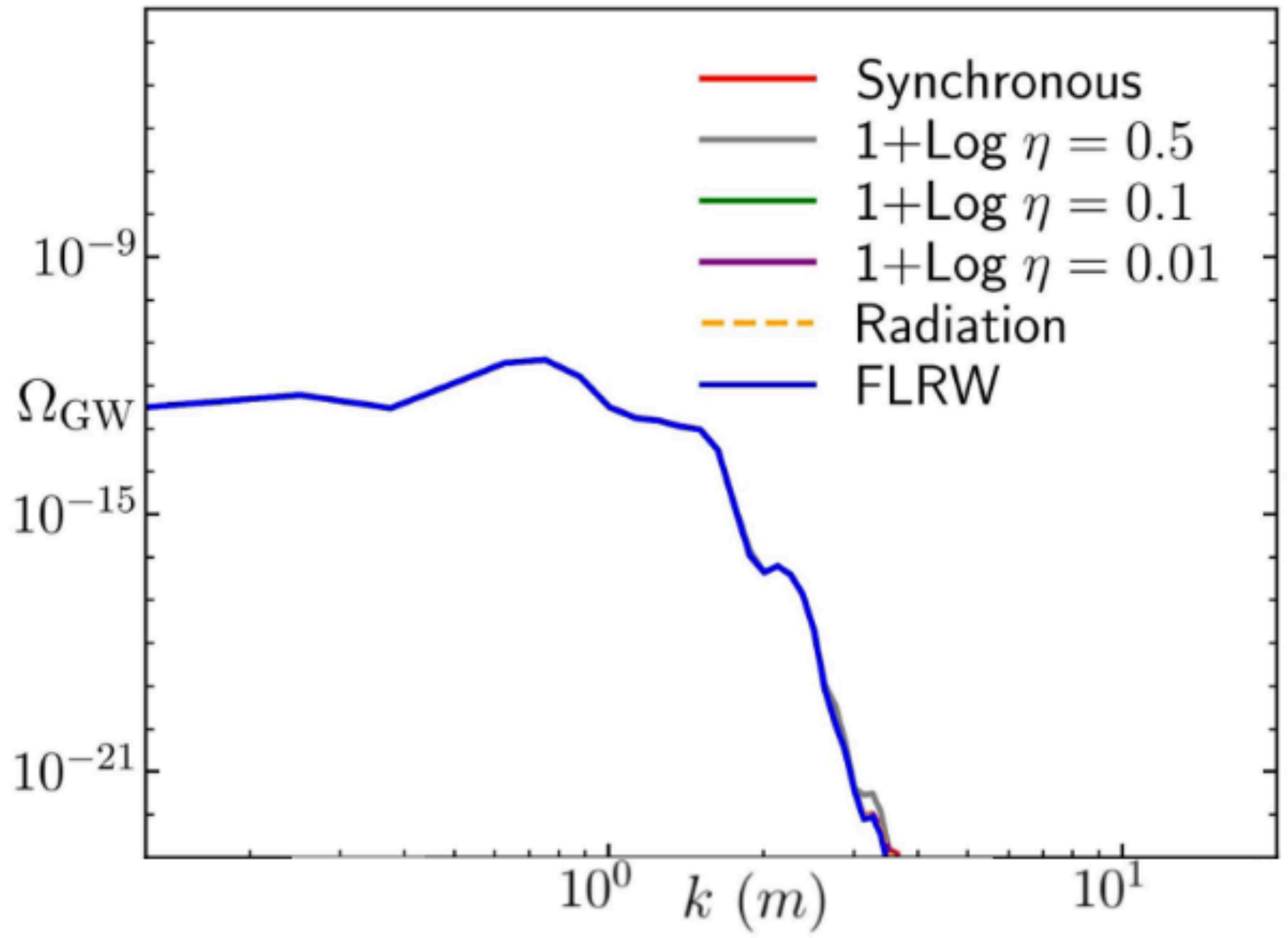
3. Can we connect observables to the theory space or is there significant degeneracy?

When do we need numerical relativity for cosmology?

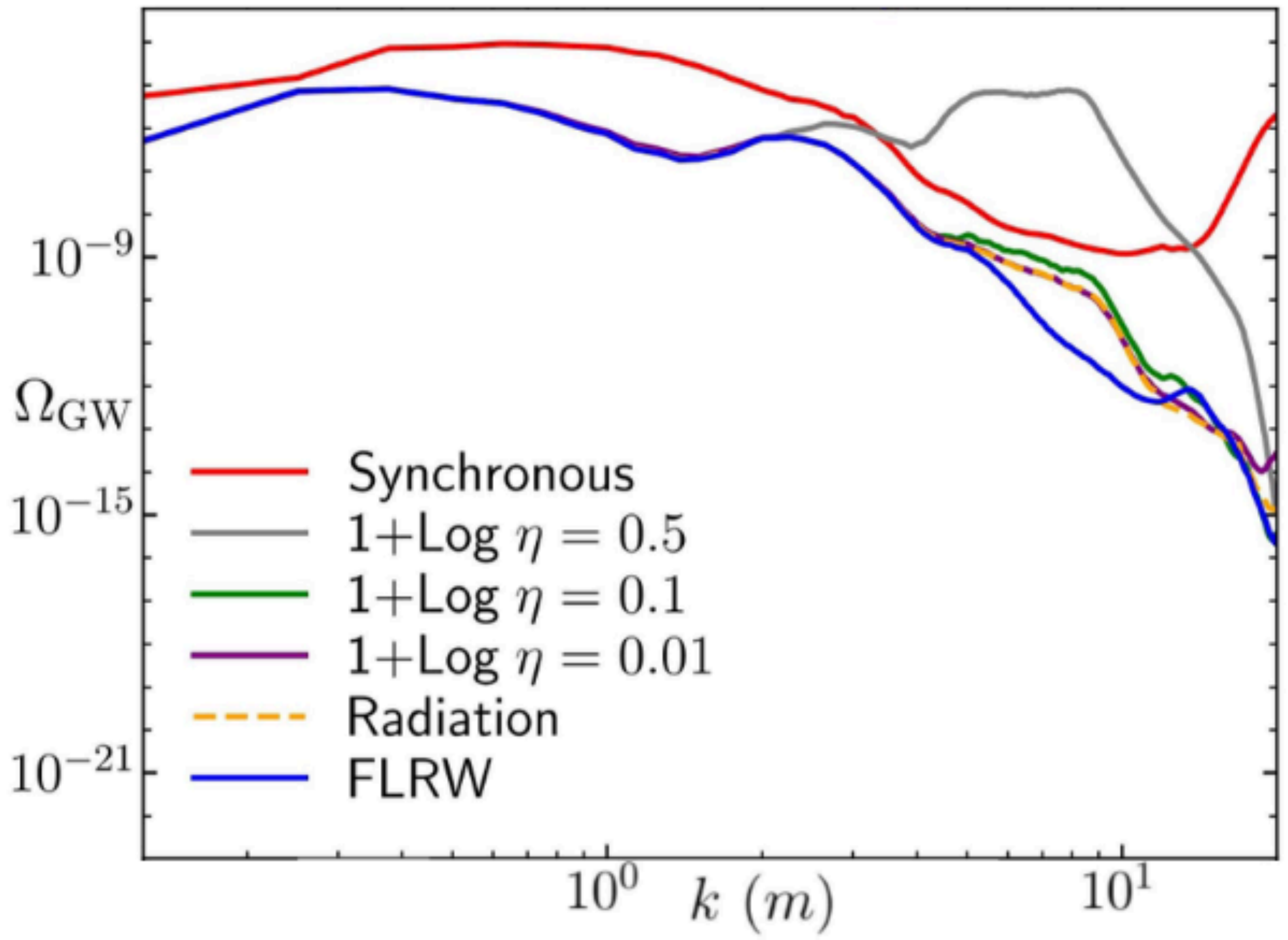


Observables

Early time



Late time

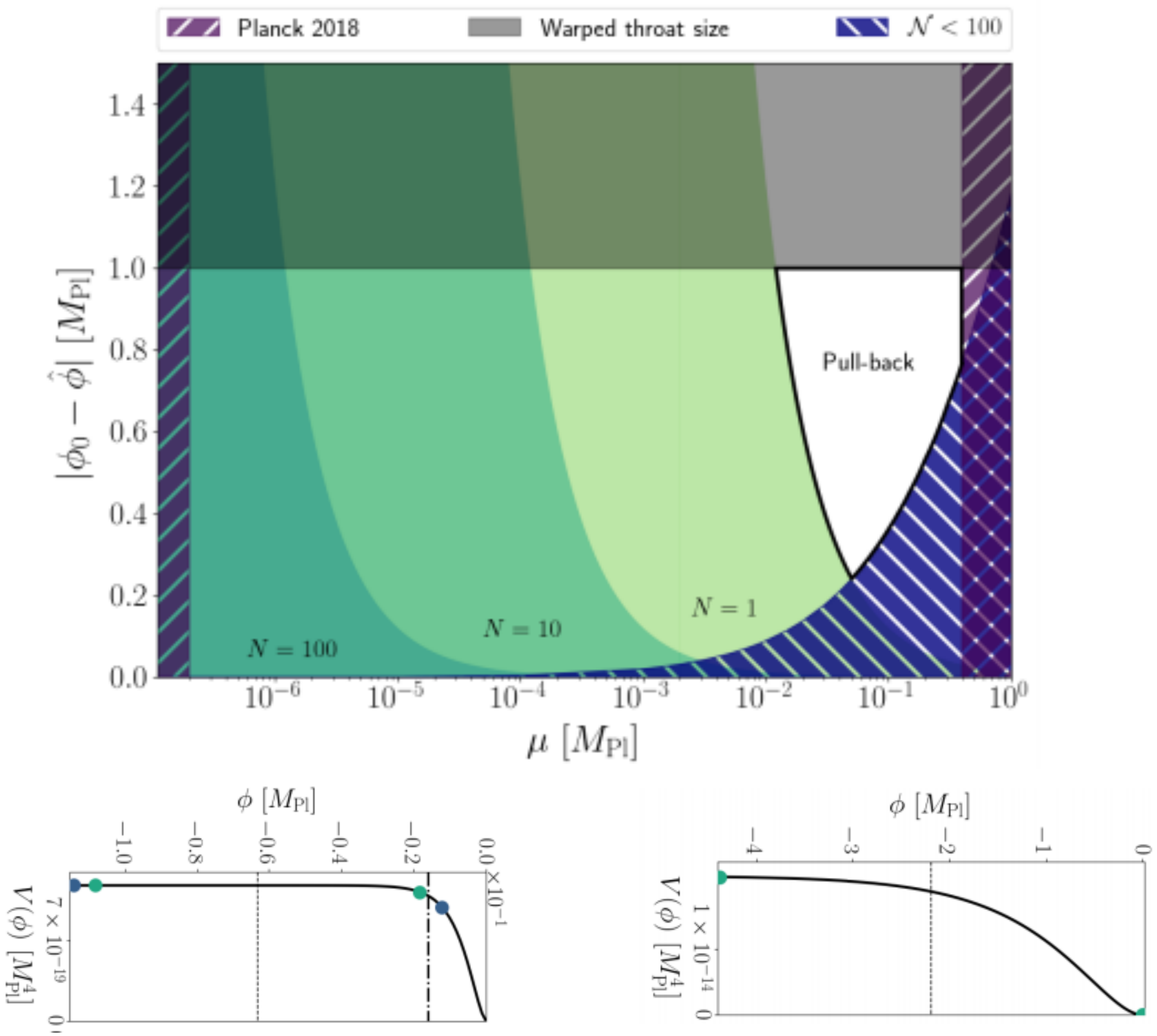


"What is the average energy density of GWs on this slice of spacetime?" is not the best question in NR

X Kou., JB Mertens., C Tian, SY Zhou
Gravitational waves from fully general relativistic oscillon preheating.
Phys. Rev. D 105(12), 123505 (2022)

Connection to theory space

Distance of average field value from minimum



small field **large field**

J. C. Aurrekoetxea, K. Clough, E. A. Lim, R. Flauger
The Effects of Potential Shape on Inhomogeneous Inflation
JCAP 05 (2020) 030