Axion String Evolution in Terasite Simulations

Based on **arXiv:2410.18064, PhysRevD 111 063532**
Computational resources from CSC Finland: 462000084

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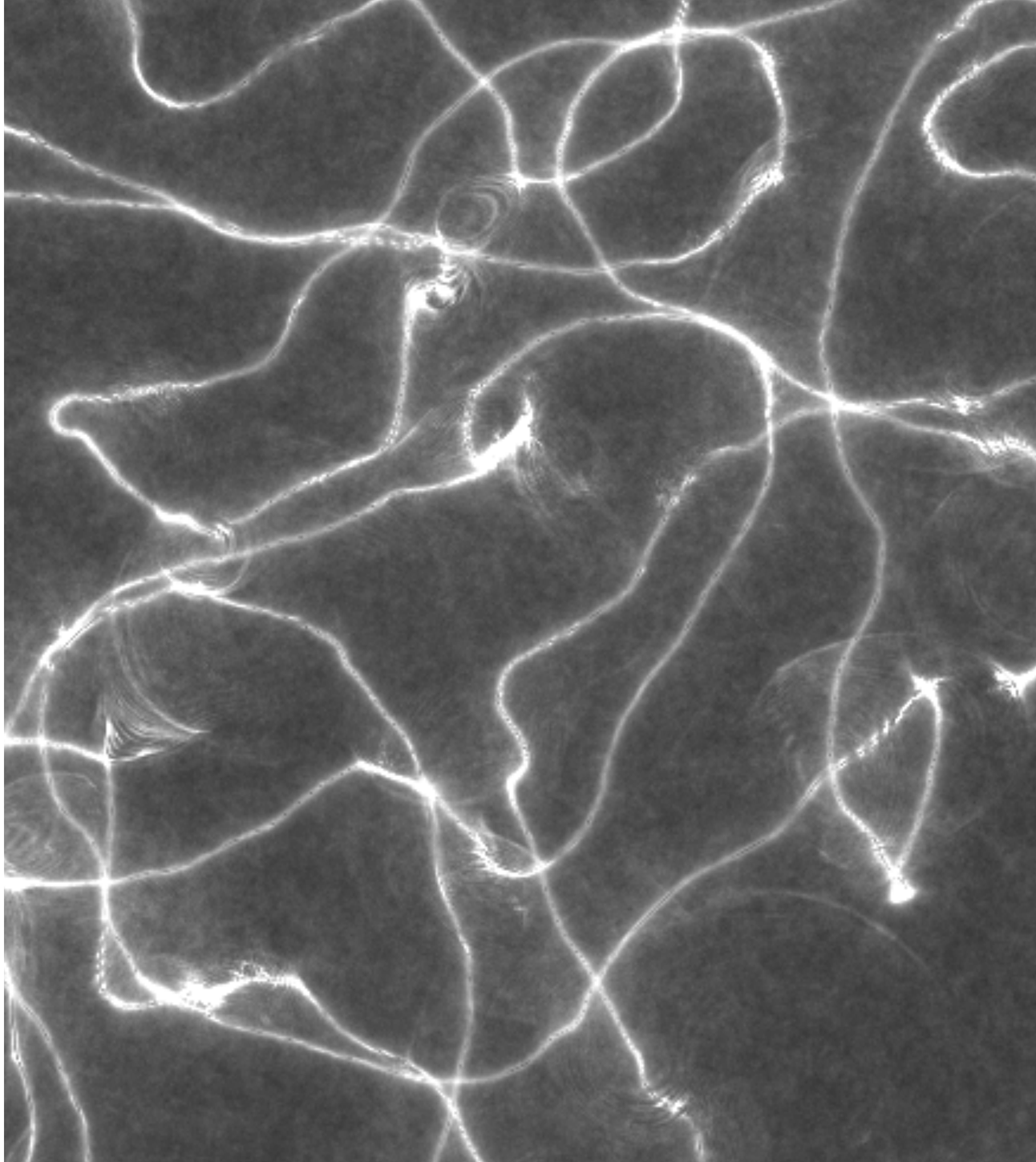
Introduction

- Introduction
- Scaling density of Axion Strings in Terasite Simulations
- Long Term Growth Analysis
- Conclusions



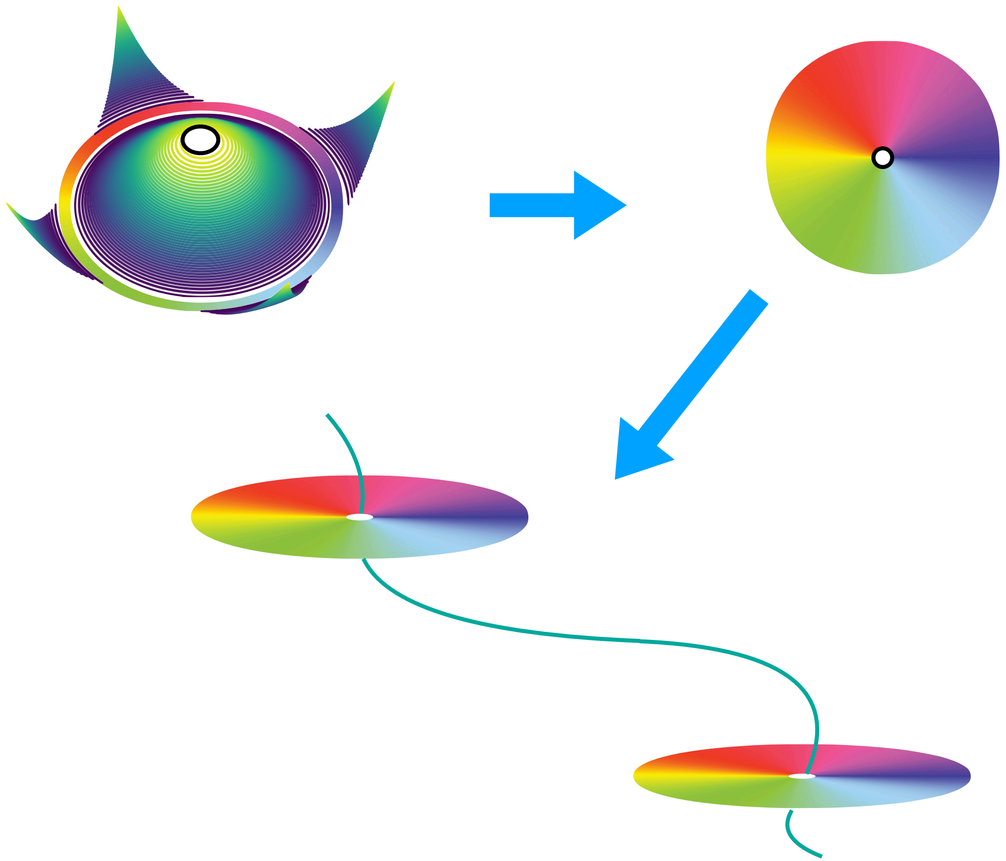
Global strings

- Consider a singlet **scalar field** invariant under **U(1) Global** symmetry, (ϕ is a complex scalar field), which spontaneously breaks

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi - \frac{1}{4}\lambda(\phi^*\phi - \eta^2)^2\right)$$

  - **String solutions** can then exist →

- **Global strings** have different properties from “conventional” gauged/local strings:
  - **Energy not** entirely **localised** to the string world sheet
  - Existence of **long-range interactions**
  - **Massless radiation** expected to be emitted



[Kibble, 1976]

Axions in the post-inflationary scenario I

- Peccei-Quinn mechanism [Peccei & Quinn, 1977] was proposed to solve the Strong CP problem…
- Extra field is added to Standard Model, and the angular excitations of this field are known as **Axions**
  - Solid dishwashing product, but also happens to be a dark matter candidate.
- If the **PQ phase transition** is **after inflation**, one will form **global strings** [Davis & Shellard, 1986]
  - They survive until the QCD phase transition (for QCD axions), roughly 70 e-folds later
  - And the **string network** will emit **axion waves** continuously throughout this time
- At QCD PT, a system of domain walls connecting strings will form and the whole system will collapse ($N_{DW}=1$)

String Formation

String + Domain Wall Collapse

PQ — **QCD**

$\sim 10^{10} - 10^{12} GeV$

SIMULATIONS — **EXTRAPOLATION**

$\sim 100 MeV$

Axions in the post-inflationary scenario II

- Multiple factors which can greatly impact the predicted dark matter abundance, we highlight two:

  - The **number of strings per Hubble** volume at the QCD phase transition ζ

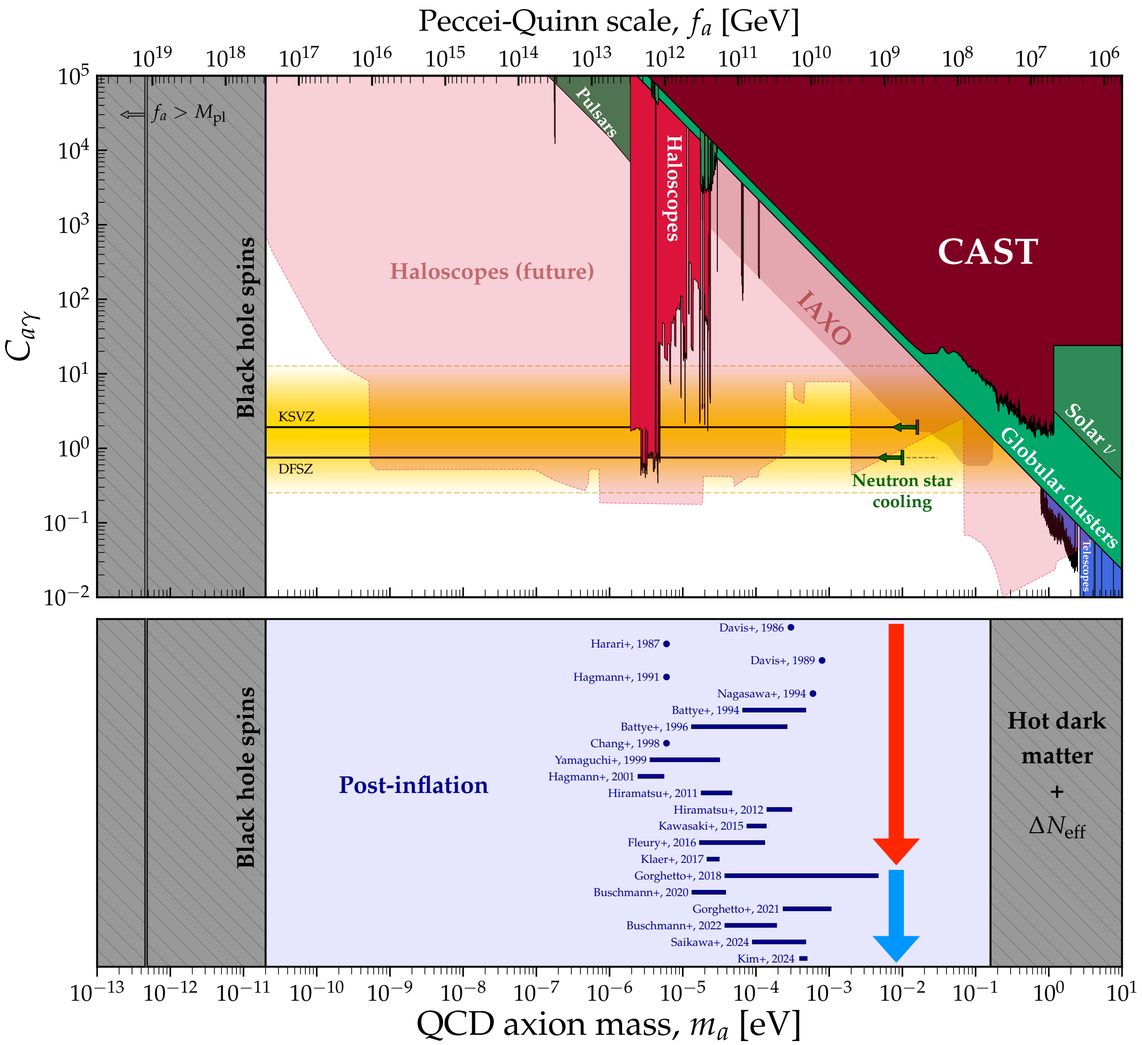
    $$\zeta \approx \mathcal{O}(1) \approx const\,. \text{ or } \zeta \propto log(m_s t)$$

  - The shape of the axion emission spectra. **Spectral index:**

    $$q \approx 1 \quad \text{or} \quad q > 1\,?$$

    [Hariri & Sikivie, 1987] [Davis & Shellard, 1986]

Summarised on the right are mass predictions of multiple simulation groups: rather active research area! Important for axion detection.



[Cajoharo et al, 2024]

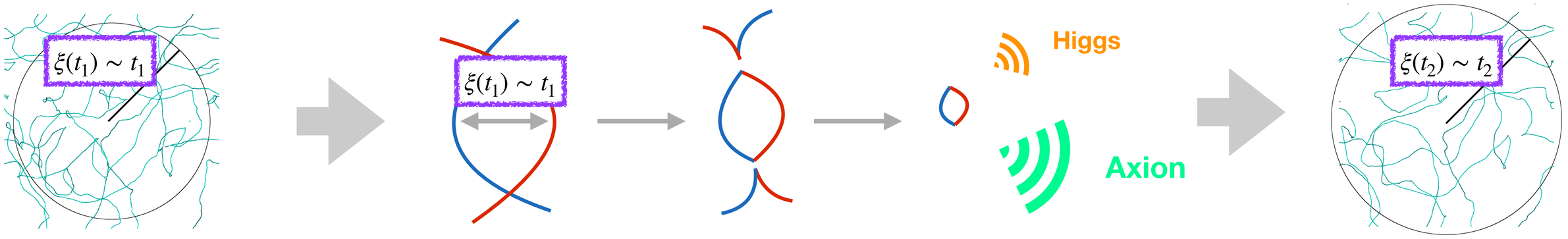
Standard Scaling

- Defect networks are expected (deep in eras where $a \propto t^{\lambda}$) to evolve according to the **scaling regime / scale-invariant regime** [Kibble, 1976], [Zeldovich, 1980], [Villenkin, 1981 & 1982] where,
  - Mean String **separation**, $\xi \propto t$ and the string **velocity** $v \propto const$ and therefore $\zeta = const$
  - Scaling given $t \gg t_c; \xi, \bar{\xi} \gg w_s$ and results in:

$$\xi \sim \bar{\xi} \sim d_H$$

(w_s - string width, $\bar{\xi}$ - mean string curvature)

  - Self-similar evolution sustained via energy loss mechanisms (loop chopping, expansion, emission of radiation)

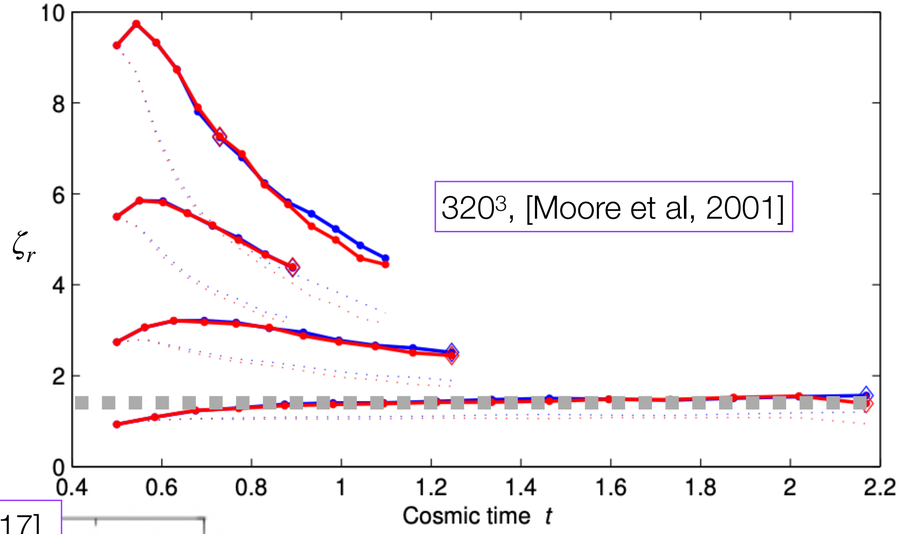


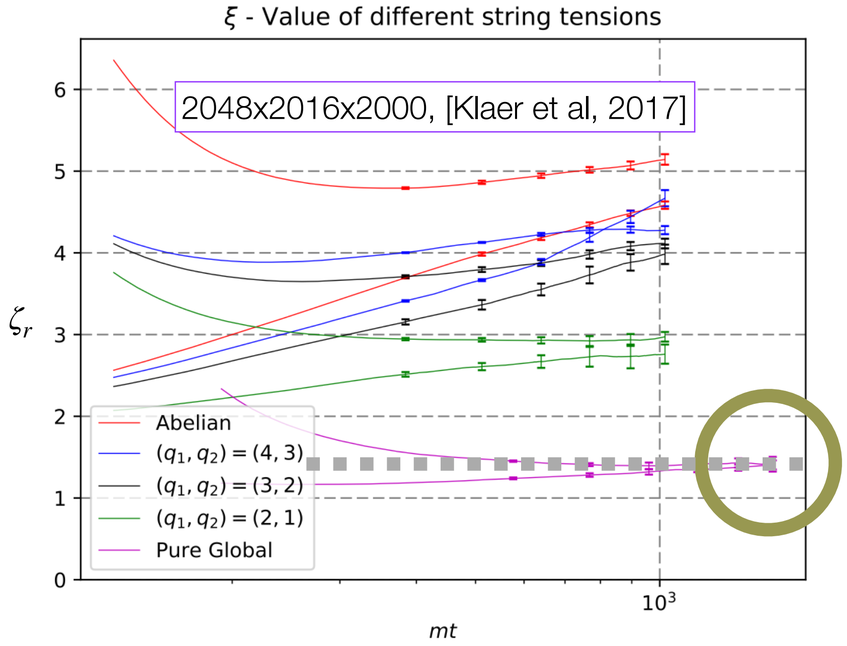
Axion strings - Standard Scaling (pre-2018)

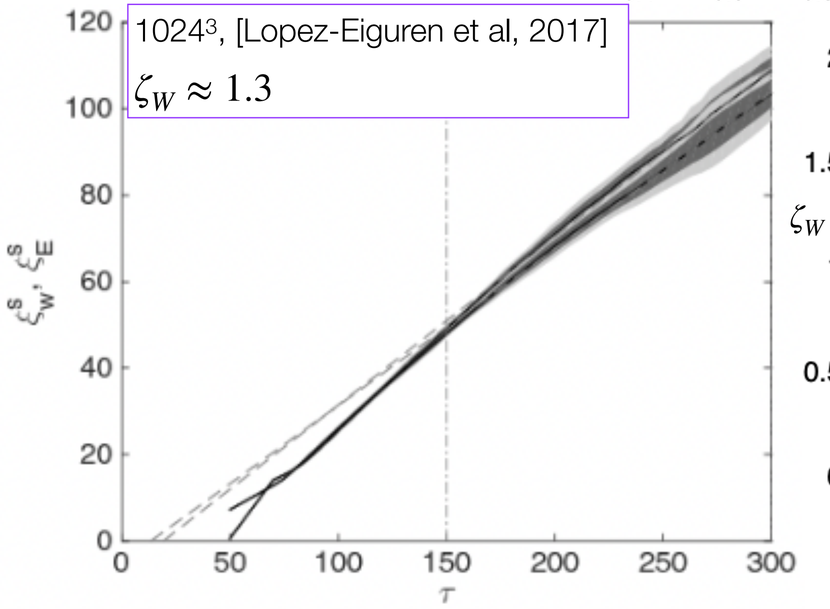
- So far, all consistent with string density parameter / number of strings per Hubble patch $\zeta \approx 1$ (grey dashed line, foreshadows our latest result)
  - Some definitions for **rest-frame** and **Universe-frame**:

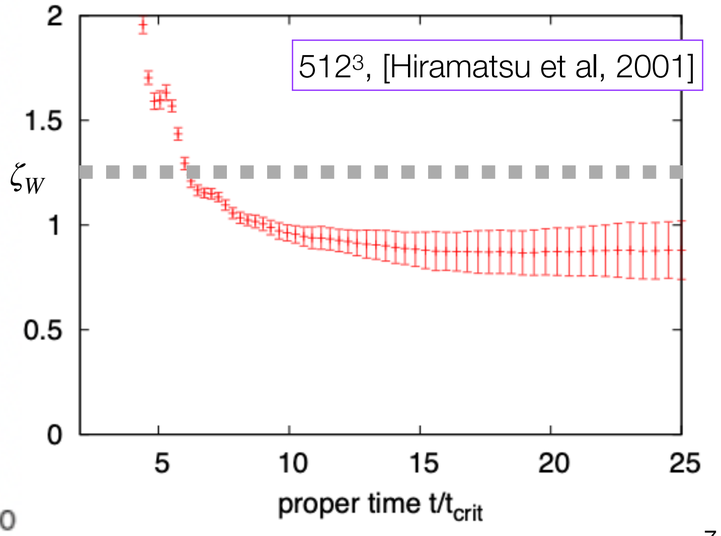
$$\zeta_r = \gamma \zeta_W$$

- Root-mean-square velocity $v \approx 0.6;\ \gamma \approx 1.3$ [Klaer et al, 2017]





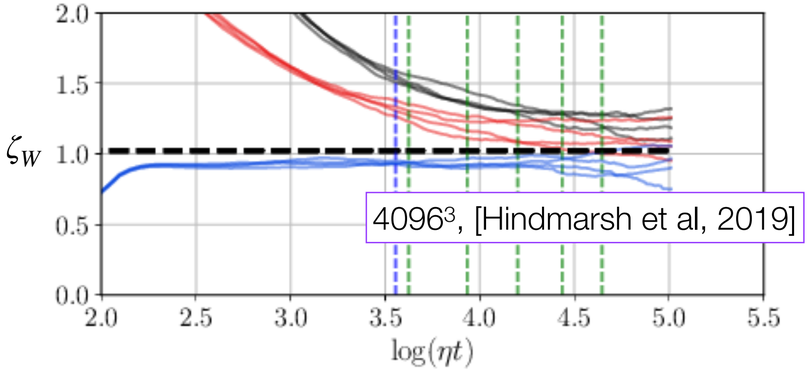


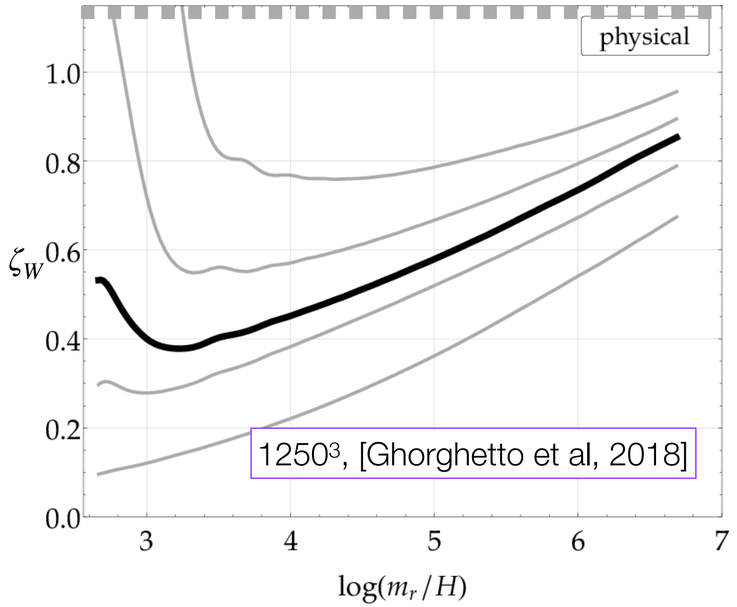


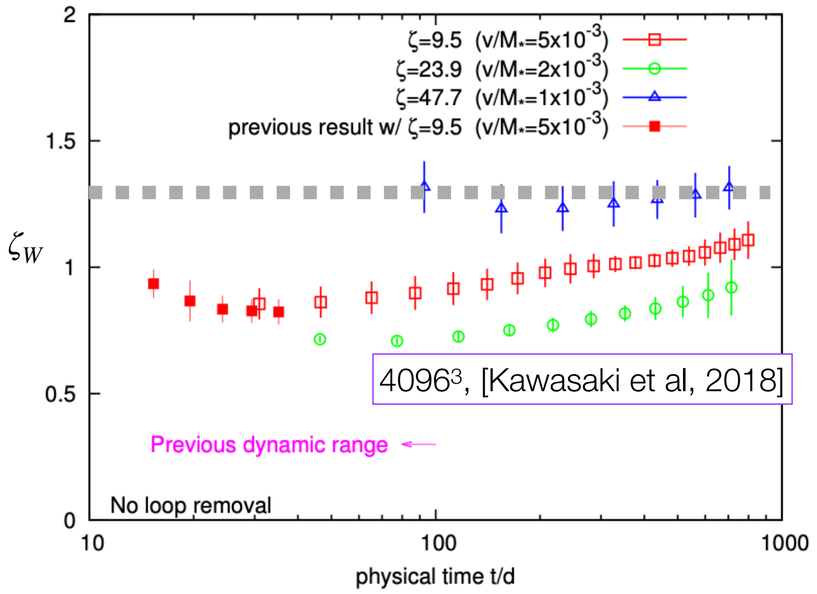
Axion strings - Scaling Violation (2018/2019)

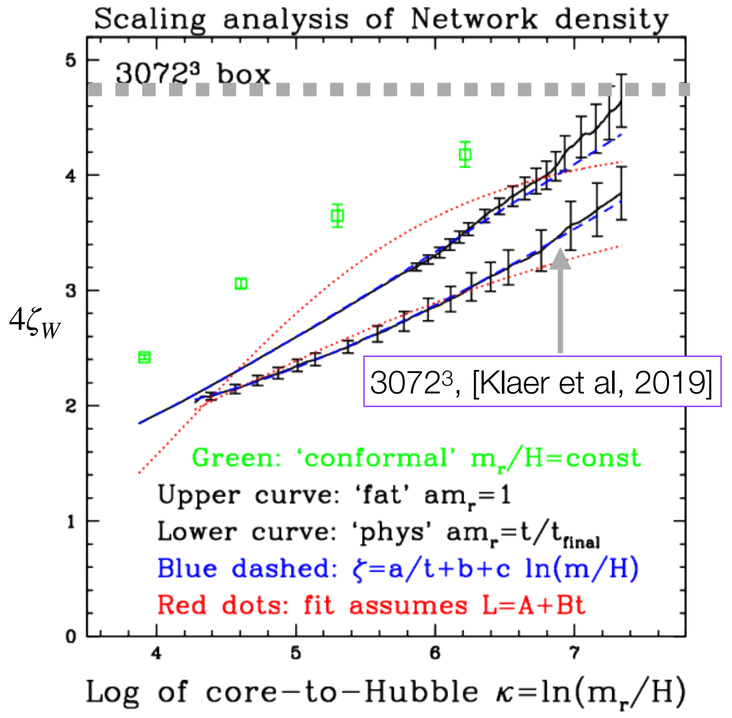
- In [Ghorghetto et al, 2018] it is proposed that the density parameter should grow logarithmically with time due to the time-dependent string tension $\mu(t) \propto \log(t)$. **NOTE:** this is **not scaling** in the original sense. Could this be a low-density transient? (See upper right figure)

- Here fits of the form $c_1 log(t) + c_0$ to $\zeta_W = \frac{t^2 l}{a^2 V}$ are used to argue that this is the true long-term behaviour of the network. However, final values $\zeta_W \sim 1$.









Axion strings - Scaling Violation (2020s)

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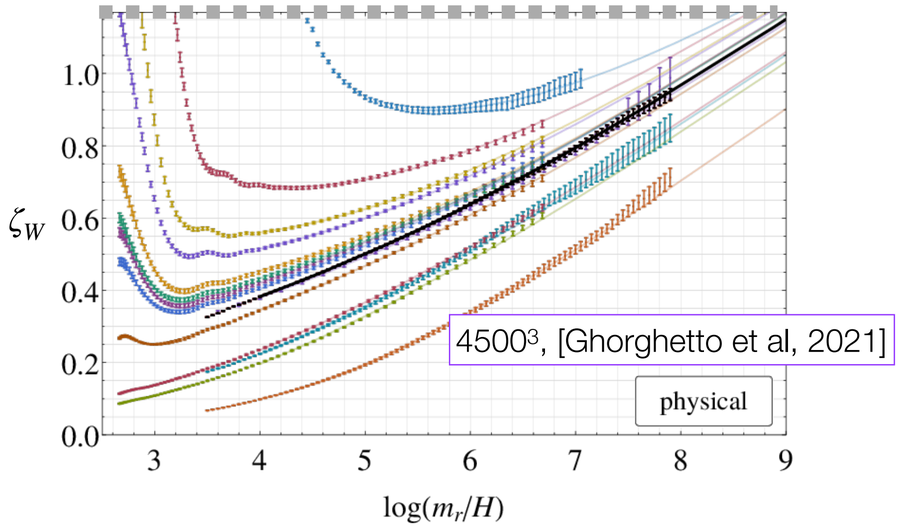
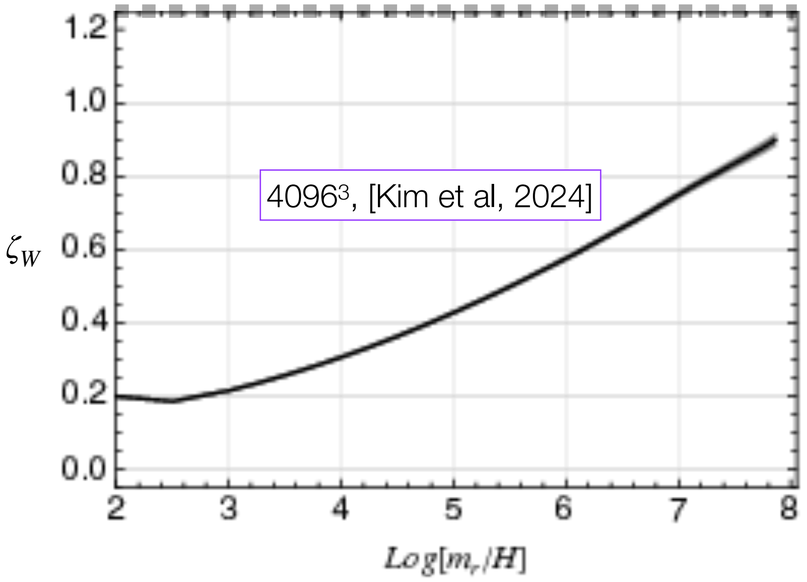
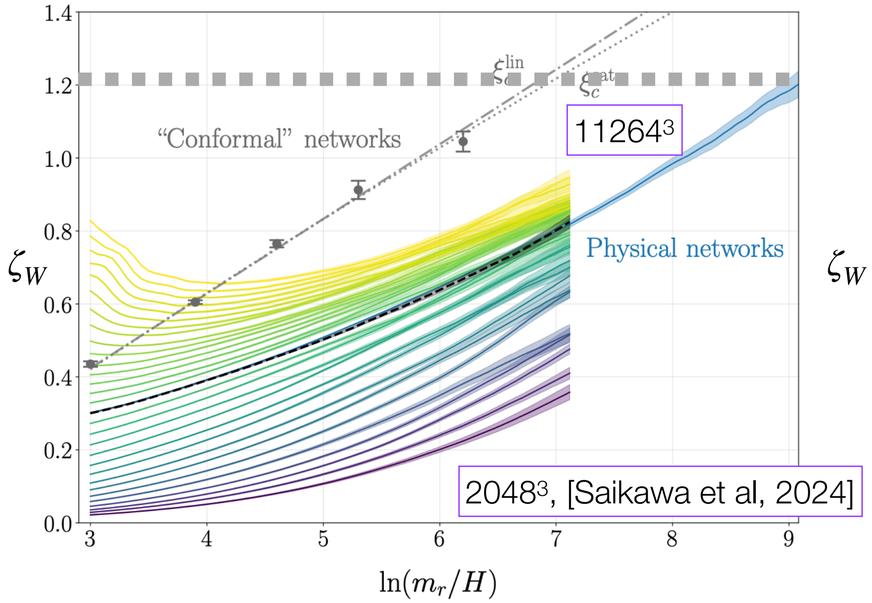
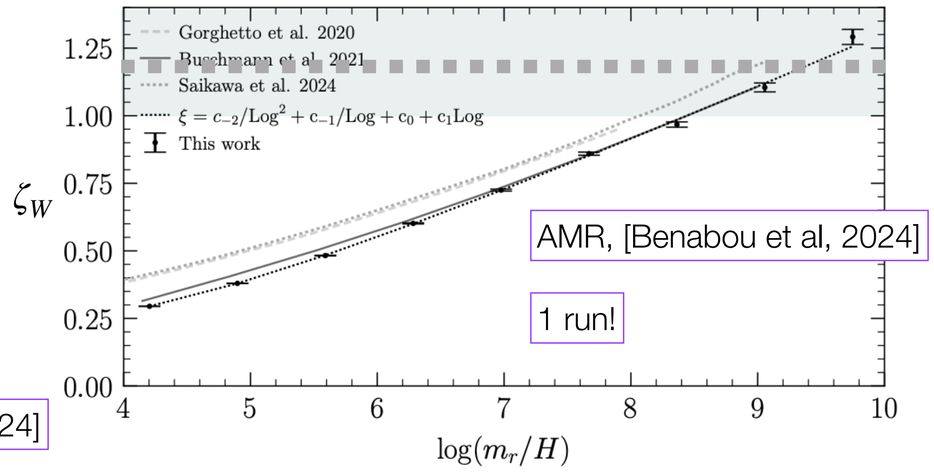








String Evolution as an Autonomous System

- Take the action for an ideal axion string [Kalb et al, 1974; Lund et al. 1976; Dabholkar et al, 1990]. Here we will work with world sheet coordinates $X^\mu = X^\mu(\sigma^0, \sigma^1)$, which represent 1+1 world sheet in space-time:

$$S = -\mu \int \sqrt{-\gamma} d^2\sigma - \frac{1}{6} \int H^2 d^4x - 2\pi f_a \int B_{\mu\nu} d\sigma^{\mu\nu}$$

$\gamma = det(\gamma_{ab})$ is the determinant of the induced metric, $\gamma_{ab} = g_{\mu,\nu} x^\mu_{,a} x^\nu_{,b}$. Let us neglect interactions (& axion) for now.

- Assume transverse-temporal gauge: $\sigma_0 = \tau$ (or, $X^\mu = (\tau, x^i)$) and $\dot{X} \cdot X' = 0$ and the equations of motion in an FLRW background are:

$$\ddot{x} + 2\frac{\dot{a}}{a}\left(1 - \dot{x}\right)\dot{x} = \epsilon^{-1}\left(\epsilon^{-1} x'\right) \qquad \text{and} \qquad \dot{\epsilon} = -2\frac{\dot{a}}{a}\dot{x}^2\epsilon$$

- Where $\epsilon = \sqrt{\frac{x'^2}{1 - \dot{x}^2}}$. Apply an averaging procedure on **rest-frame length** and **r.m.s velocity**:

. $l_r = \int \epsilon d\sigma = \frac{a^2 V}{\xi_r^2}$ and $v^2 = \int \dot{x}^2 \epsilon d\sigma / l_r$

Assume One-Scale approximation:
$\xi \sim \bar{\xi} \sim L$ (correlation length)

Add Loop chopping:
$$\frac{d\rho}{dt}_{loops} = cv\frac{\rho}{L}$$

String Evolution as an Autonomous System

Result: **Velocity-dependant One-Scale** (VOS) [Martins & Shellard, 1996]. For ideal Nambu-Goto strings:

$$2\frac{d\xi}{dt} = 2H\xi(1+v^2) + cv$$

$$\frac{dv}{dt} = \left(1-v^2\right)\left(\frac{k(v)}{\xi} - 2Hv\right)$$

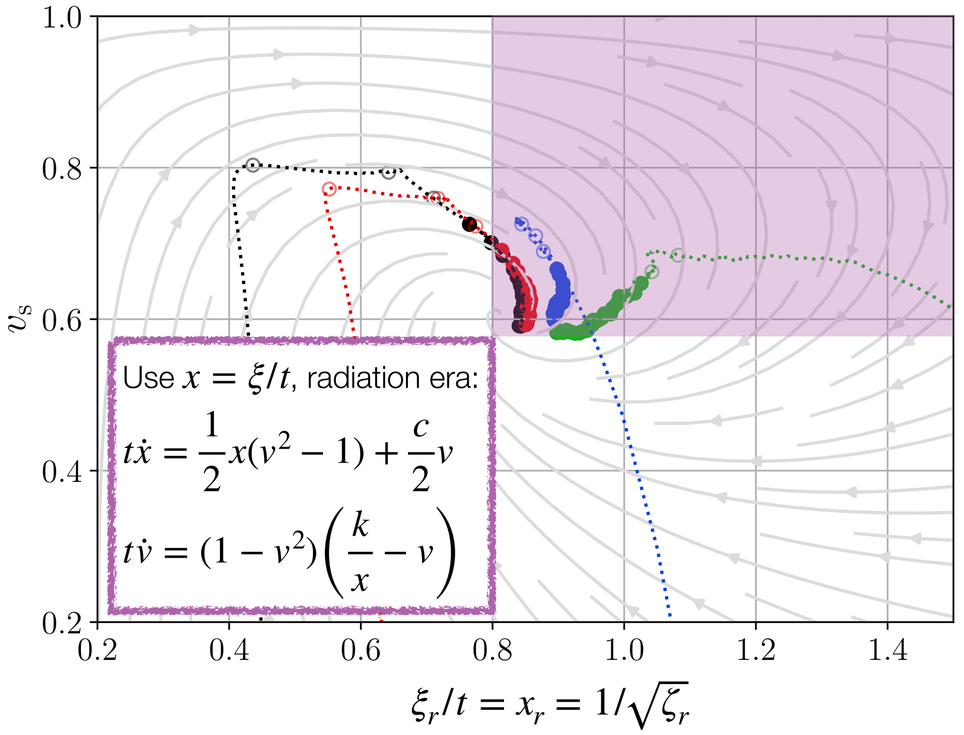
c = loop chopping efficiency

k = curvature parameter

Scaling appears naturally as an **attractor fixed point**.

- Energy conservation + Newton's second law (relates acceleration with curvature)

- In [Hindmarsh et al, 2021] a simplified version of this model is used for Global Strings…
  - …where the curvature parameter is taken as constant (string interaction force absorbed)



[Hindmarsh et al, 2021] used this simple model to estimate a axion strings FIXED POINT AT:

$$x_* = 0.814 \pm 0.037 \qquad v_* = 0.609 \pm 0.014$$

$$\zeta_* = 1.20 \pm 0.09$$

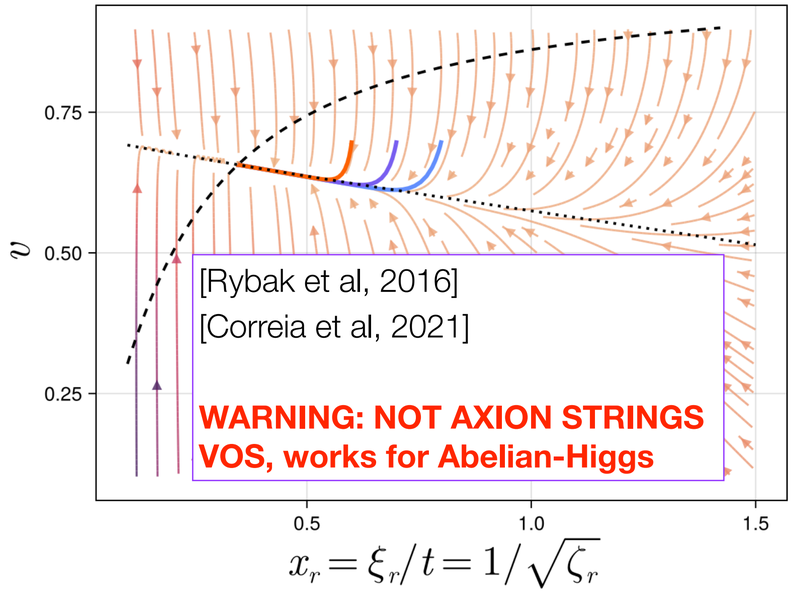
Scaling Violation or Slow approach to fixed point?

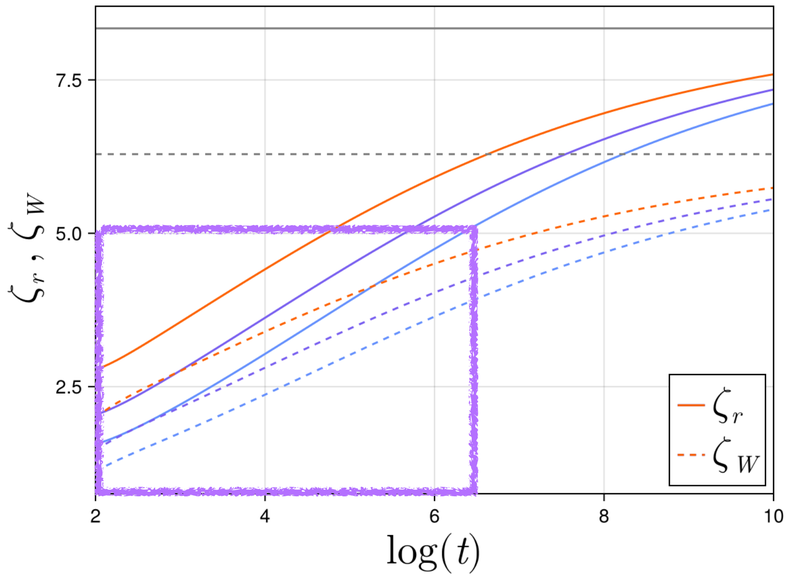
- The previous VOS dynamical system can always be recast into the following form (autonomous system, no dependence on t $\rightarrow$ follows from conservation laws),

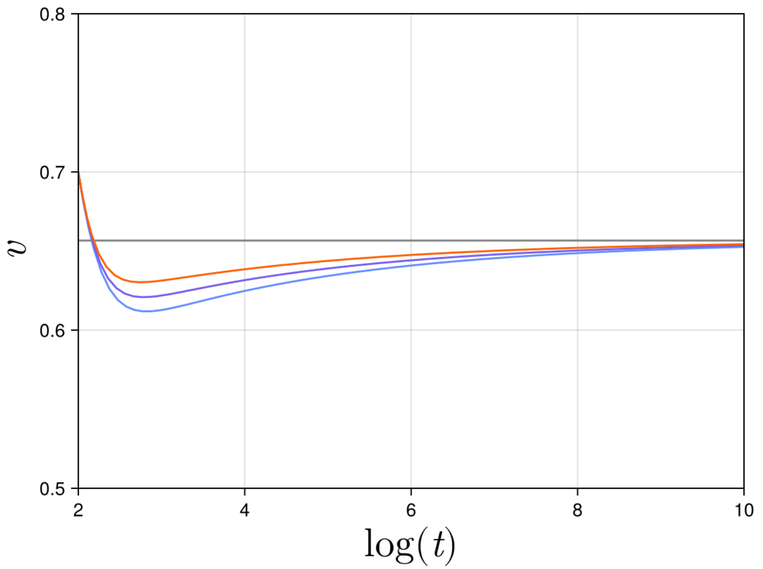
$$\frac{dx}{d\Lambda} = f_x(x, v) \qquad \frac{dv}{d\Lambda} = f_v(x, v)$$

Where we have introduced $\Lambda = log(t/t_0)$. This implies **no drift** of the fixed point.

- Non-spiral approach to a fixed point at **low** ζ near $dv/dt = 0$ null-cline can lead to similar behaviour to scaling violation.

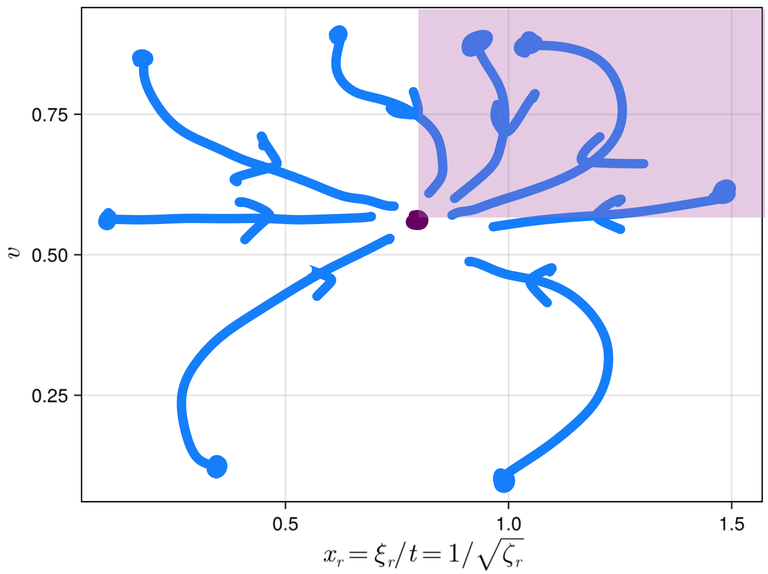




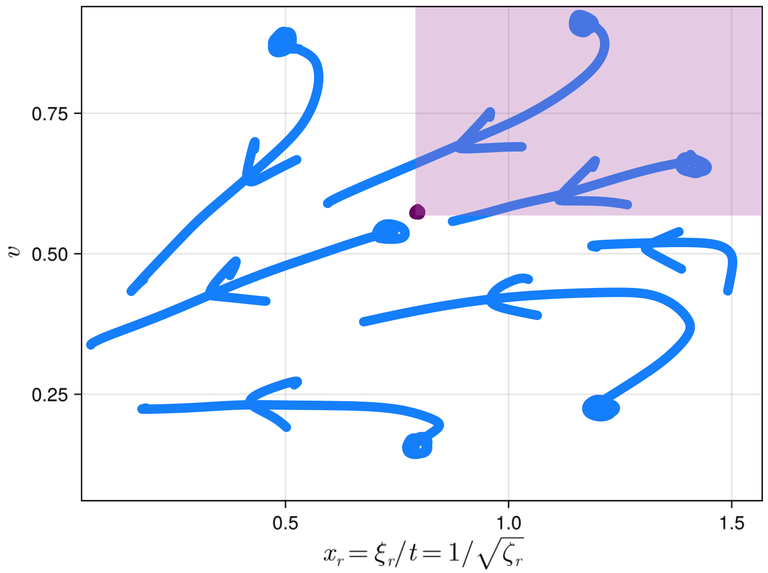


Scaling Violation or Slow approach to fixed point?

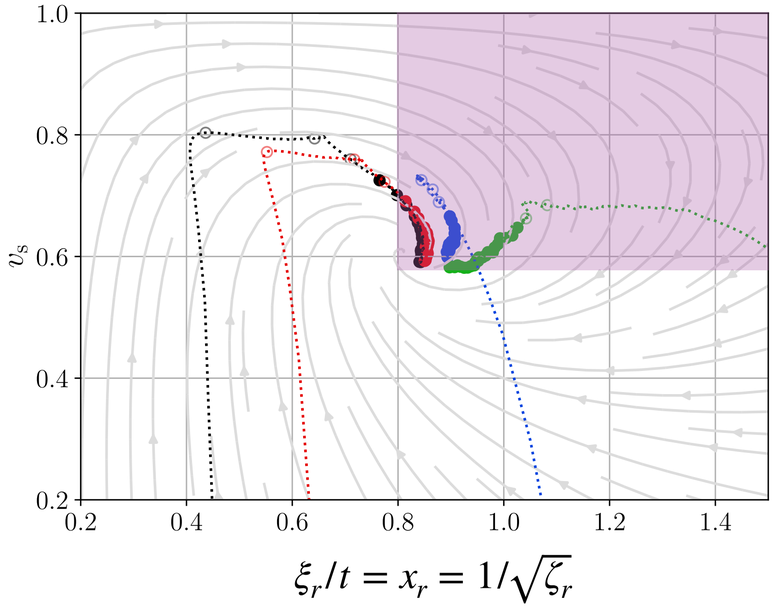
- Can we somehow fill the phase space to **distinguish** between a **fixed point** and a **scaling violation**?
- We should start with initial conditions in all four “quadrants” of the phase space (**high/low density** and **velocity**) and not only the purple part (high velocity, low density). Some work is ongoing on this.



Non-spiral approach to fixed point, sketched



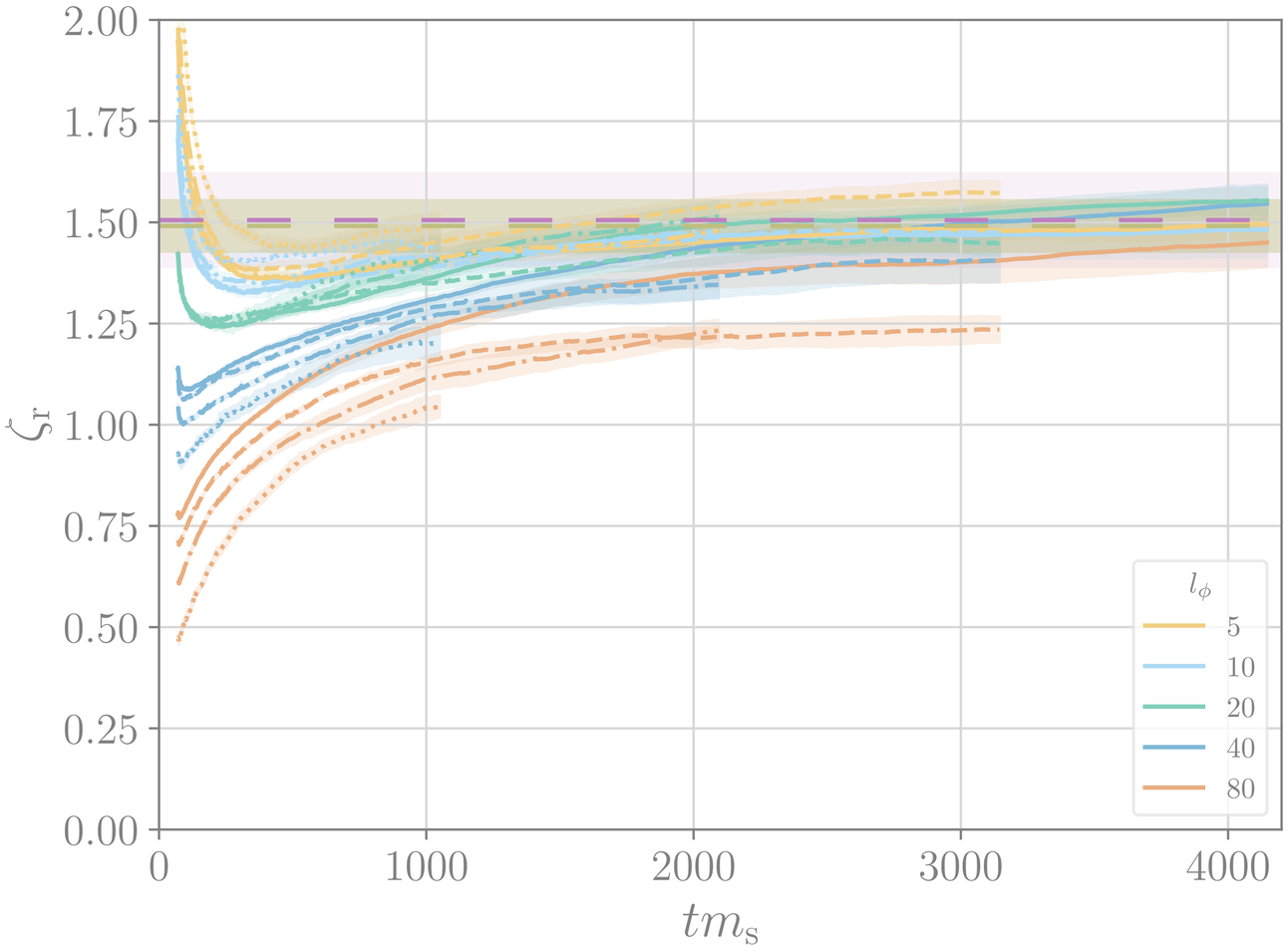
Scaling Violation, sketched



Simulation data [Hindmarsh, 2021]

The Scaling Density of Axion Strings in Terasite Simulations

arXiv:2410.18064, PhysRevD 111 063532
CSC project: 462000084



Simulations setup: initial conditions

- We report on **16384^3** fixed grid simulations performed using HILA (https://github.com/CFT-HY/HILA), ran at **LUMI** (8th at TOP500),

$$\ddot{\phi} + 2\frac{\dot{a}}{a}\phi - \nabla^2\phi = -a^2\lambda(\phi^T\phi - \eta^2)\phi$$

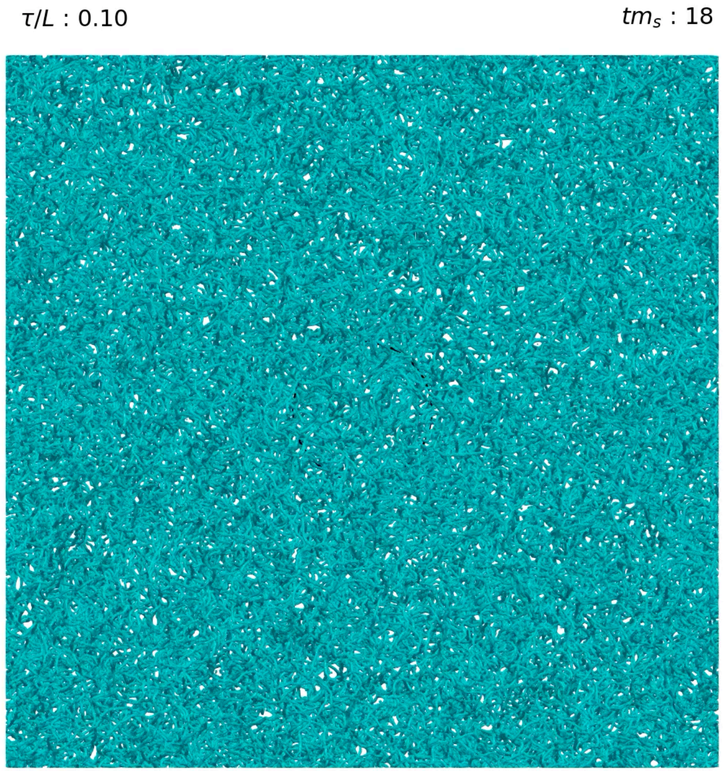
- **“Initial conditions don’t matter** - string networks will be driven towards an attractor”. Yes, but **dynamic range is not infinite**. It is always possible one never quite reaches this attractor.

- Gaussian random field with different initial correlation l_ϕ lengths at conformal time $\tau = 50$, diffusion until $\tau_{diff} = 70$
- “core growth” (also known as conformal strings) until τ_{cg}, after that radiation epoch evolution until half-a-light crossing time

Box size	Seeds	Runs	l_ϕ
4k	9	45	5,10,20,40,80
8k	9	45	5,10,20,40,80
12k	6	30	5,10,20,40,80
16k	8	56	5,10,20,40,80,160,320

Simulations setup: initial conditions

- Visually it is obvious that low initial density can lead to a state where the number of strings per horizon is lower than the fixed point value $\zeta_W \leq 1.2$ (comoving version $\zeta_c \approx 4$).
- Compare $l_\phi = 5$ on the left-hand-side and $l_\phi = 80$ on the right-hand-side



- $|\phi| = 0.6$ isosurfaces
- Circle proportional to conformal time τ
- Order four strings per sphere of comoving Hubble radius ($\zeta_c \approx 4$)

Simulations setup: observables

- Understanding and interpreting the evolution of this dynamical system, requires measuring both **string length** and **velocity**

- Measure string length, in both **Universe frame** (left, other groups do this only) and **string rest-frame** (right, related to **energy density, what VOS predicts**):

$$\ell_{\mathrm{w}} = \frac{2}{3} n_{\mathrm{p}} \Delta x$$

$n_p \rightarrow$ number of plaquettes pierced by strings

$$\ell_{\mathrm{w}} = \ell_{\mathrm{r}} \langle \gamma^{-1} \rangle$$

$$\ell_{\mathrm{r}} = \frac{E + f_{V,\mathcal{W}} L}{\mu_{\mathcal{W}} (1 - f_{V,\mathcal{W}})}$$

$\mu_{\mathcal{W}} \rightarrow$ wighted energy per unit length

$f_{V,\mathcal{W}} \rightarrow$ fraction contributed by the weighted potential energy density

$E \rightarrow$ Energy

$L \rightarrow$ Lagrangian

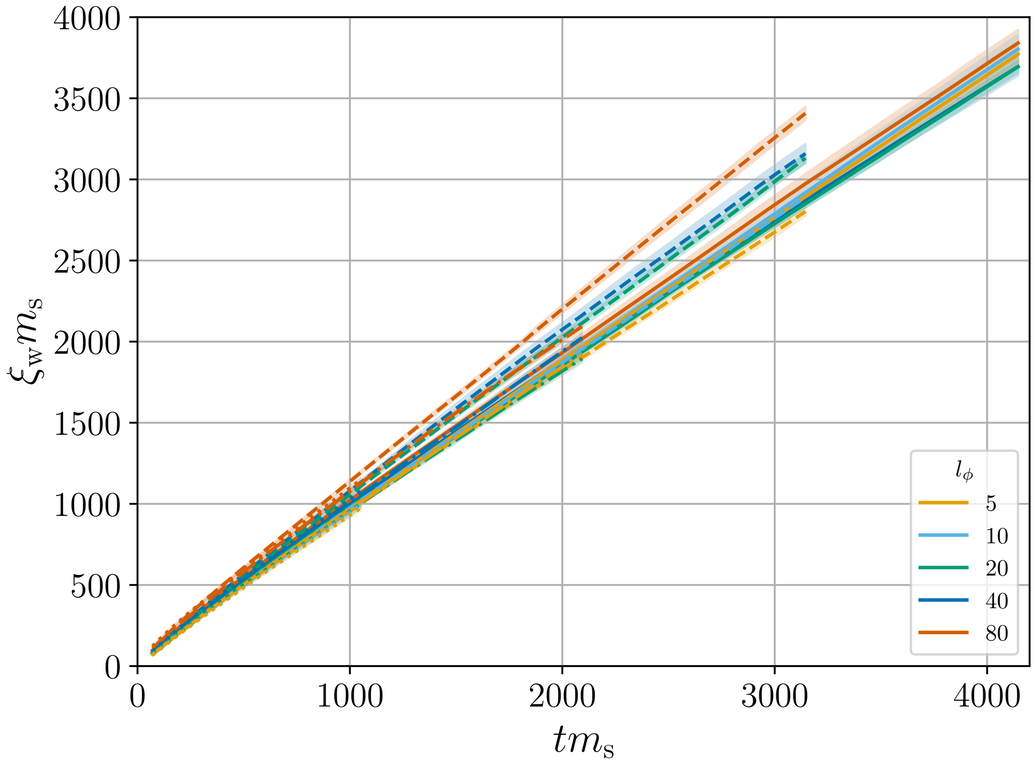
- String separation $\xi = \sqrt{\frac{V}{l}}$ and the related $x = \xi / t$ String density parameter $\zeta = \frac{l t^2}{a^2 V} = \frac{1}{x^2}$.

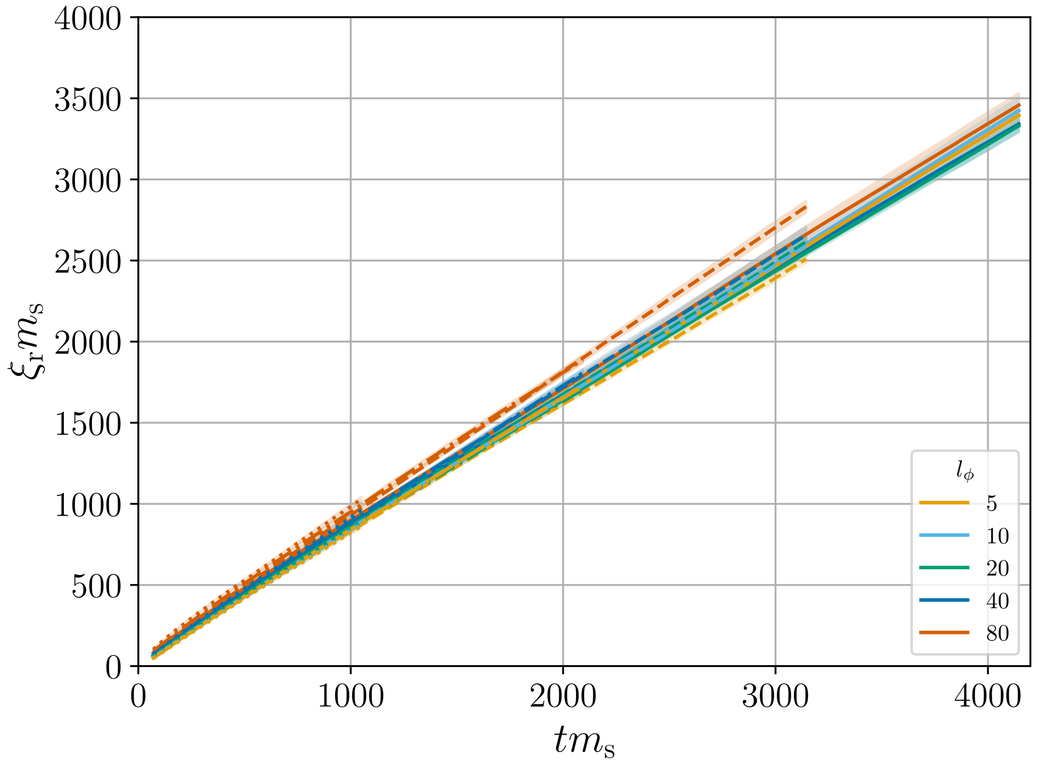
- Some groups measure **velocity**, although not all: $v_L^2 = \frac{E - L}{E + f_{V,V} L}$

Important **consistency check:** $\zeta_r \gamma^{-1} \approx \zeta_W$

Standard Scaling Analysis: Time Series Data

- From looking at string separation, one does **not** see any **evidence** of **deviations** from **standard scaling**
- Here it looks like using either **Universe frame** or **rest-frame** measures of length is 100% **inconsequential**…

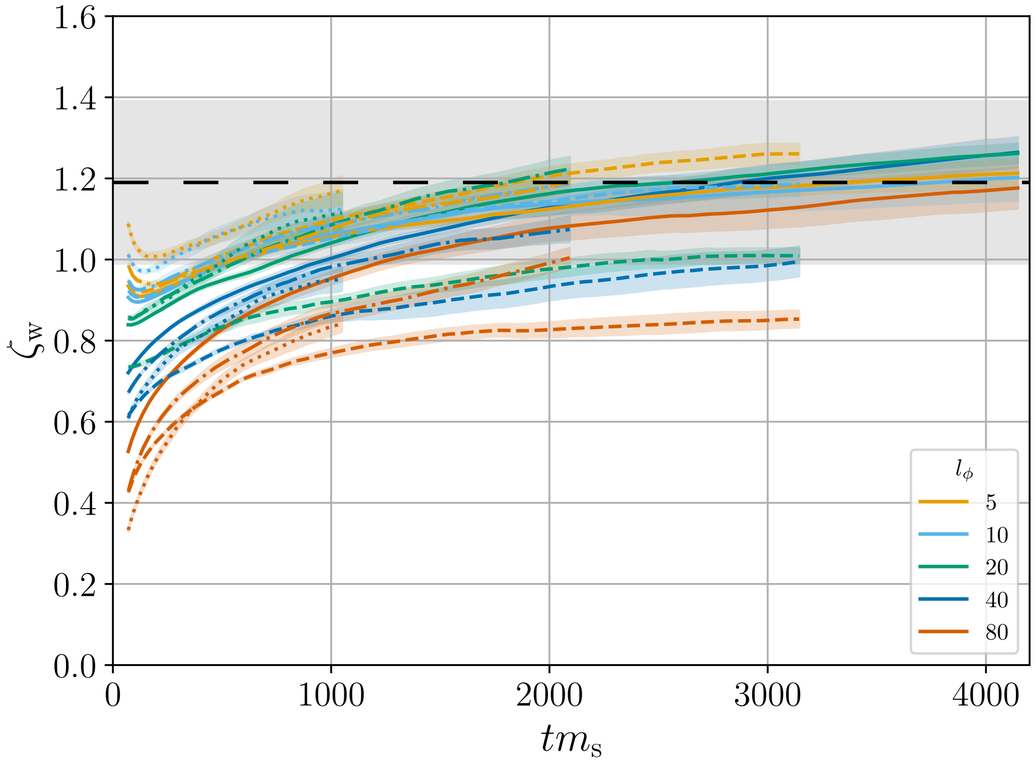


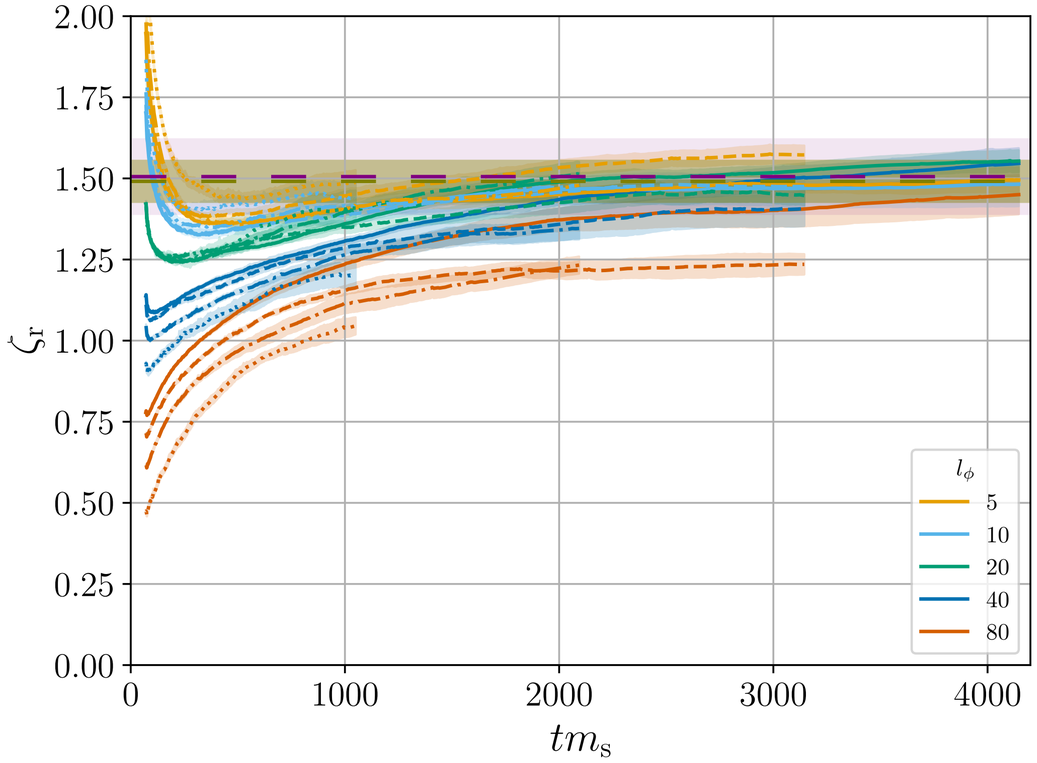


Standard Scaling Analysis: Time Series Data

- We remind the audience that other groups only measure ζ_W, but not ζ_r (and most of the times not v)

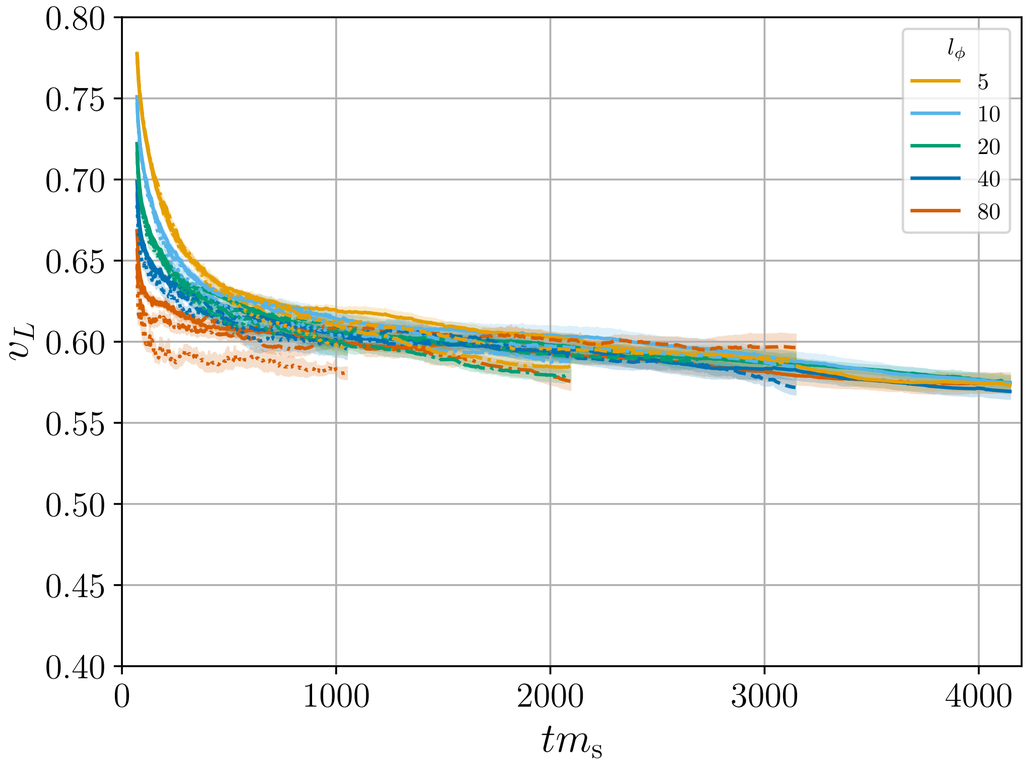
- It seems there is a difference between **Universe frame** and **rest-frame** string density parameters. Now we must look at **velocities. Rest-frame converges** to constant - depending on initial condition and dynamic range.

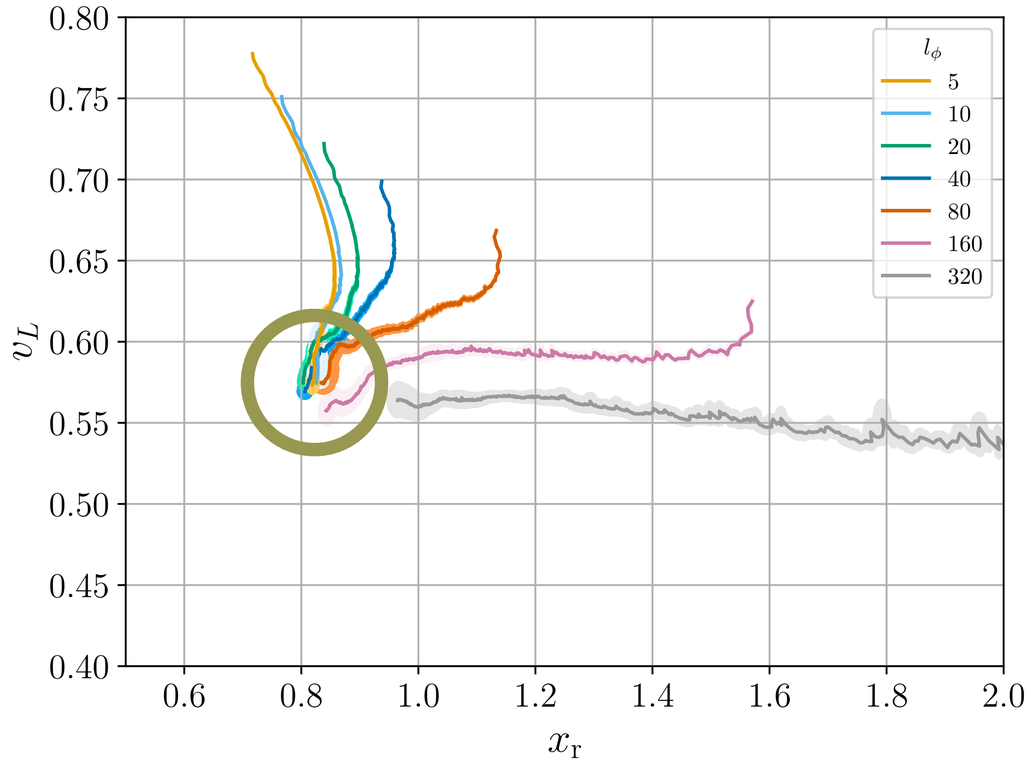




Standard Scaling Analysis: Time Series Data

- **Velocity** is still **evolving**. Although **timid**, it is sufficient to take make the length contracted $\zeta_W = \gamma^{-1}\zeta_r$ evolve as well
- The phase space plot (showing only 16k) is quite informative too:
  - **Null-clines** at $dv/dt = 0$ and $dx_r/dt = 0$ explain how to interpret evolution and their crossing leads to a fixed point

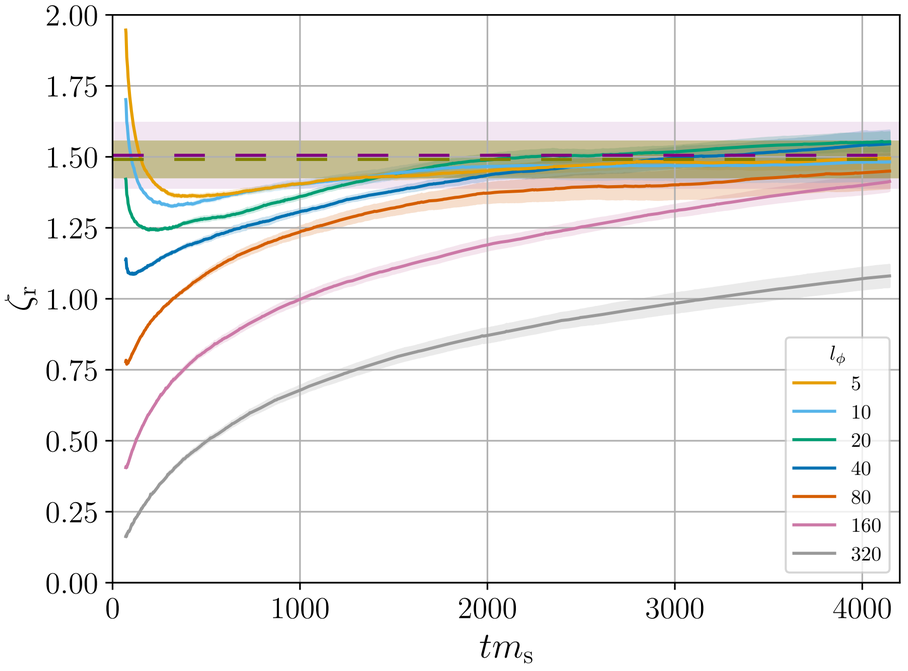


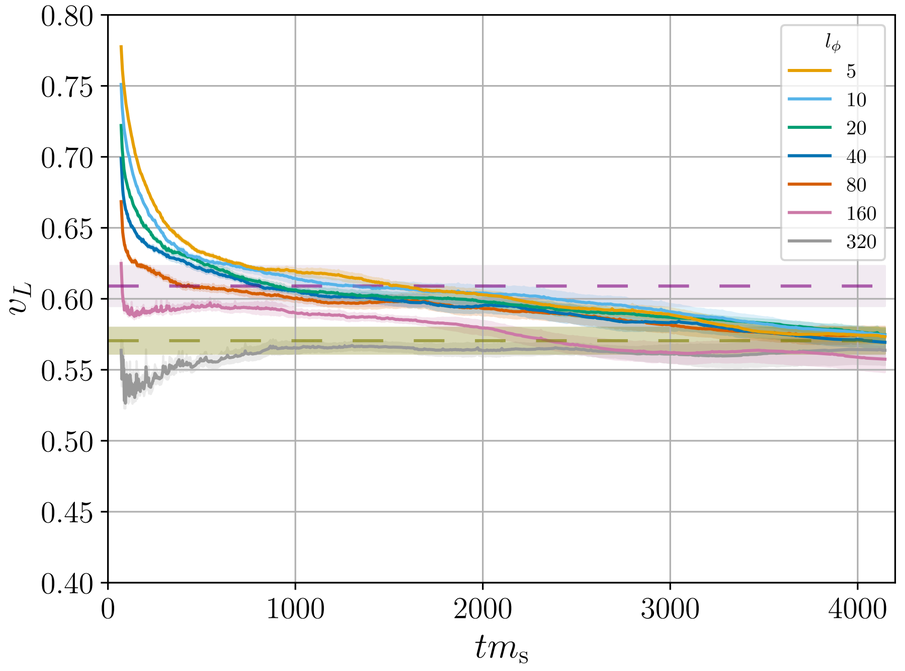


Standard Scaling Analysis: Main result

- We observe **convergence** of **rest-frame** string density parameter**,** ζ_r. **Well-predicted** by the naive (c, k) VOS!
- **Velocity** is **under predicted** by the simple VOS roughly by 5% → we need a more **complete model**.
  - We therefore find it justified to extrapolate from standard scaling: $\hat{\xi}_{r,*} = 1.491(93)$ and $\hat{v}_{L,*} = 0.5705(93)$

$$\hat{\xi}_{w,*} = 1.220(57) \qquad max(\zeta_W) = 1.46$$





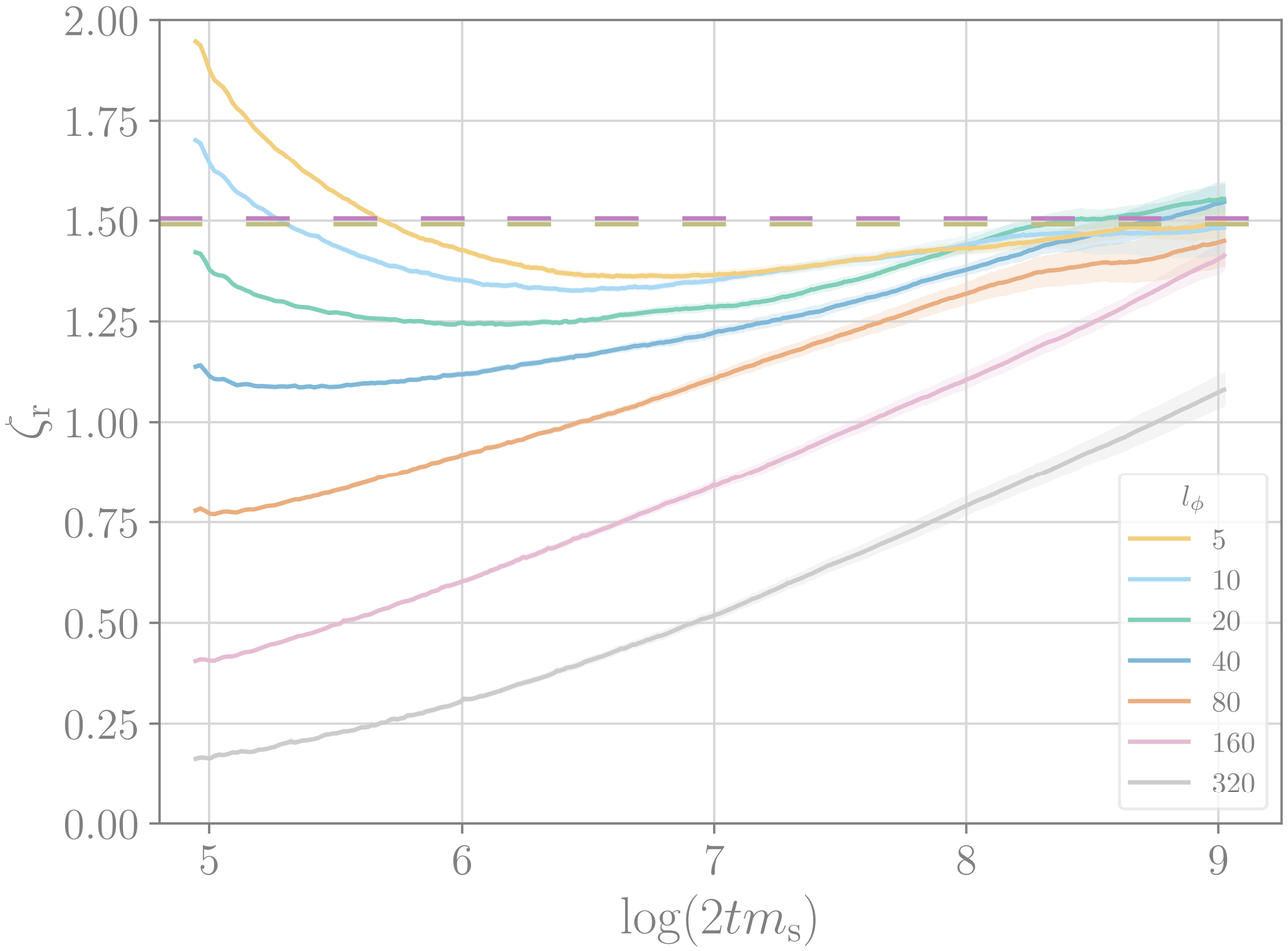
NOTE:

$\hat{\xi}_w \gamma(\hat{v}_L)$ **coincides** with $\hat{\xi}_r$.
Convergence after boost!

l_ϕ	$\hat{\zeta}_w$	$\hat{\zeta}_r \gamma^{-1}(\hat{v}_L)$
5	1.211(34)	1.214(24)
10	1.200(56)	1.202(46)
20	1.258(24)	1.259(21)
40	1.259(37)	1.257(30)
80	1.172(51)	1.174(42)
160	1.156(34)	1.156(24)
320	0.882(32)	0.880(34)

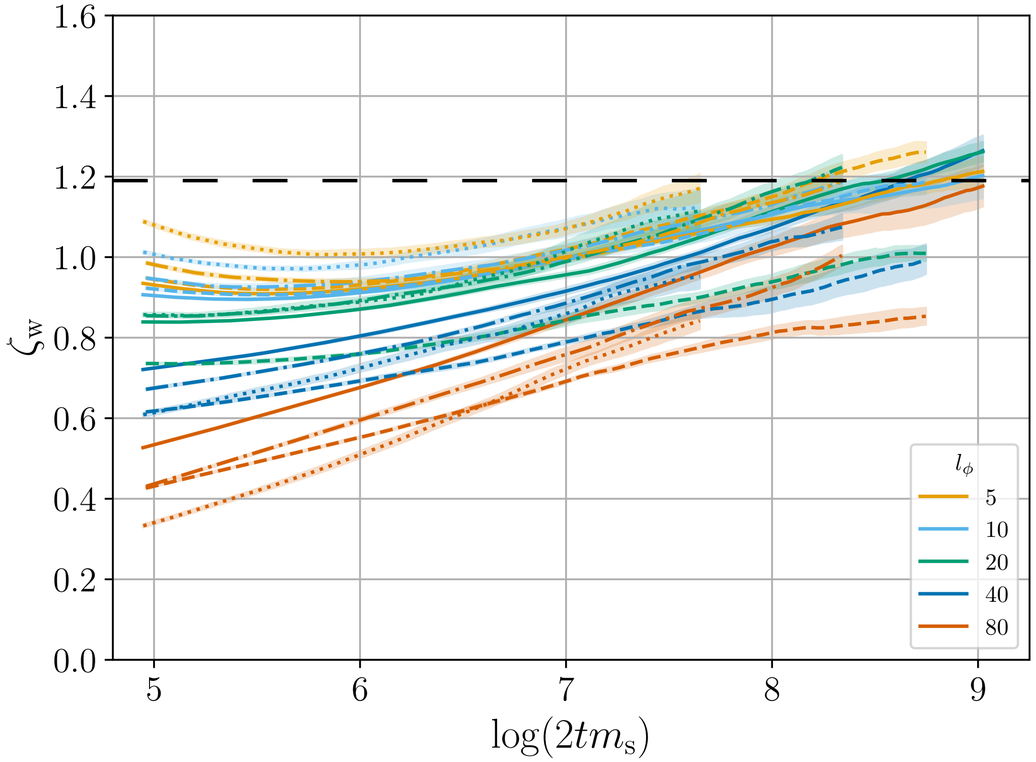
Long Term Growth Analysis

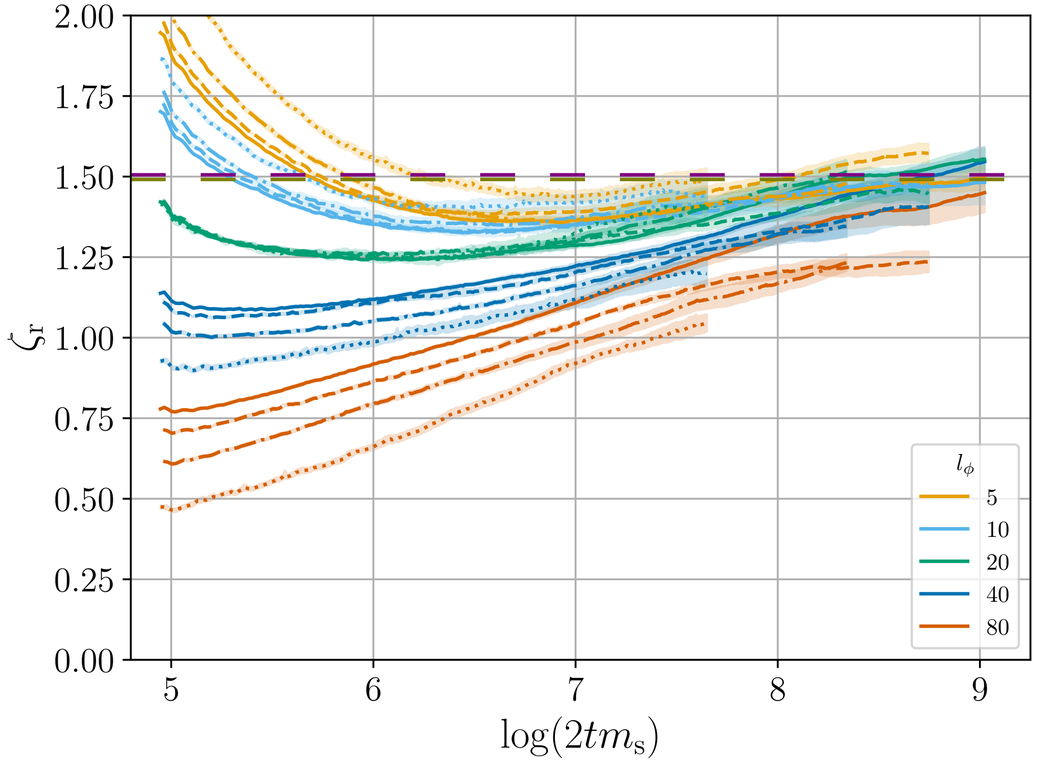
arXiv:2410.18064, PhysRevD 111 063532
CSC project: 462000084



Long Term Growth Analysis

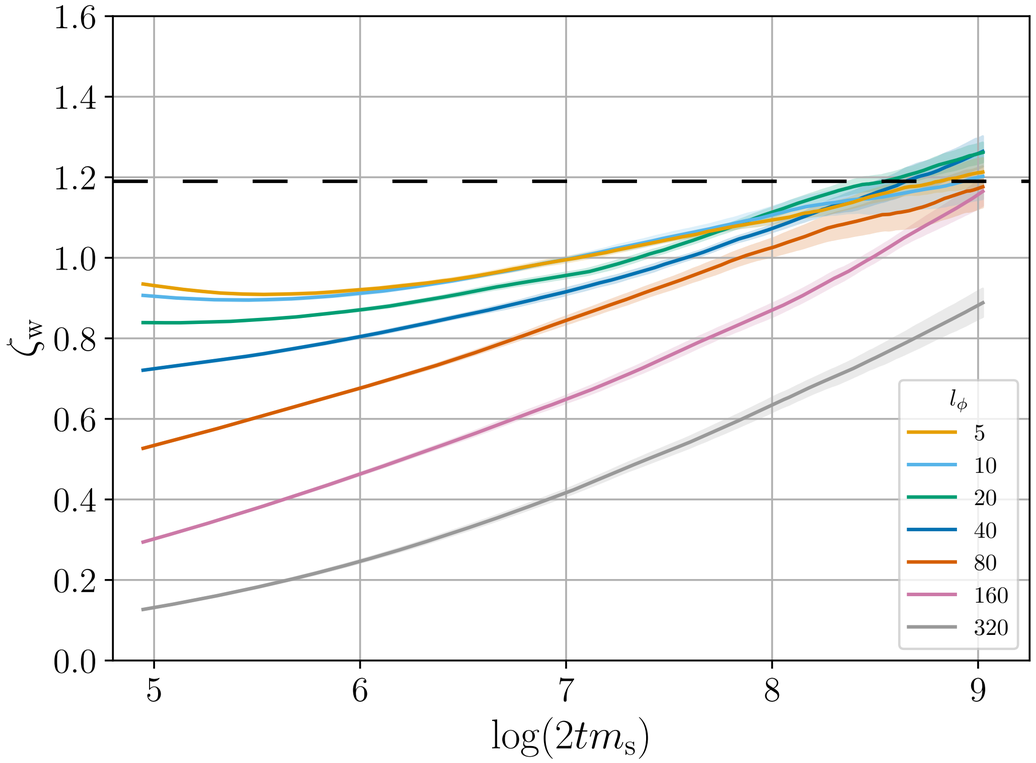
- Although one can seemingly presume that these ζ_W curves imply long-term growth (at least from a cursory look at all resolutions / dynamics ranges altogether)…
- Finding the scaling violation at low resolution and **extrapolating forwards** would **significantly exceed the fixed point** values. **This does not occur.** Again we note most groups choose ICs with $\zeta_W \approx 0.5$ at $log(2tm_s) = 5$

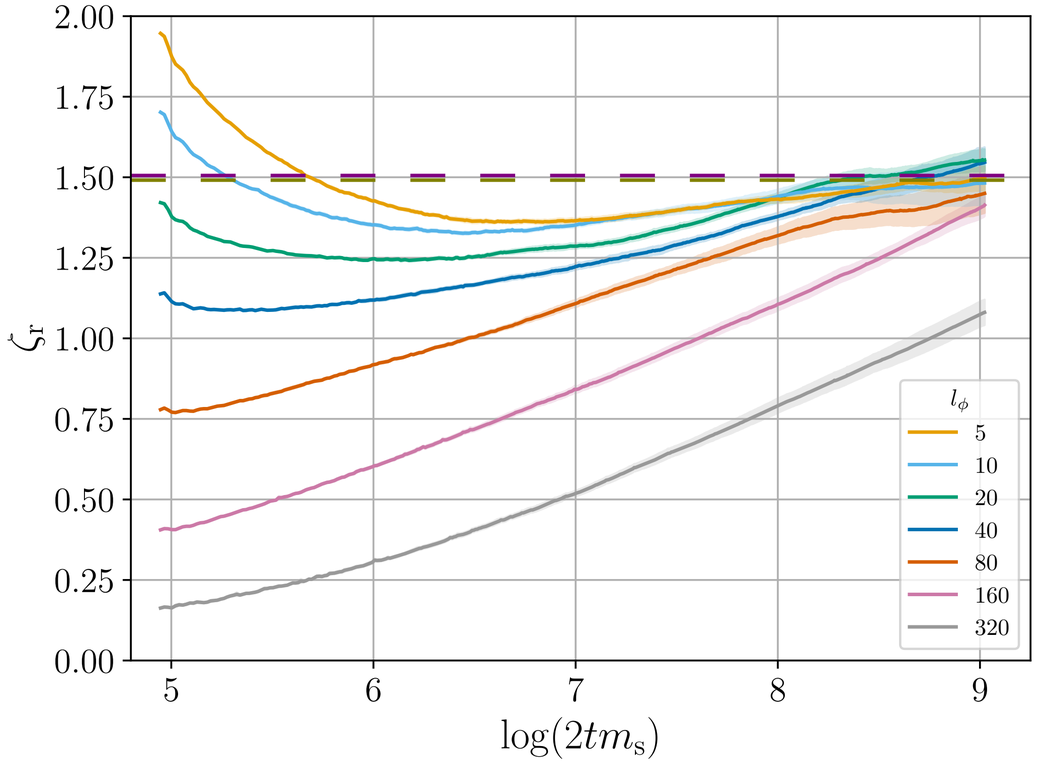




Long Term Growth Analysis

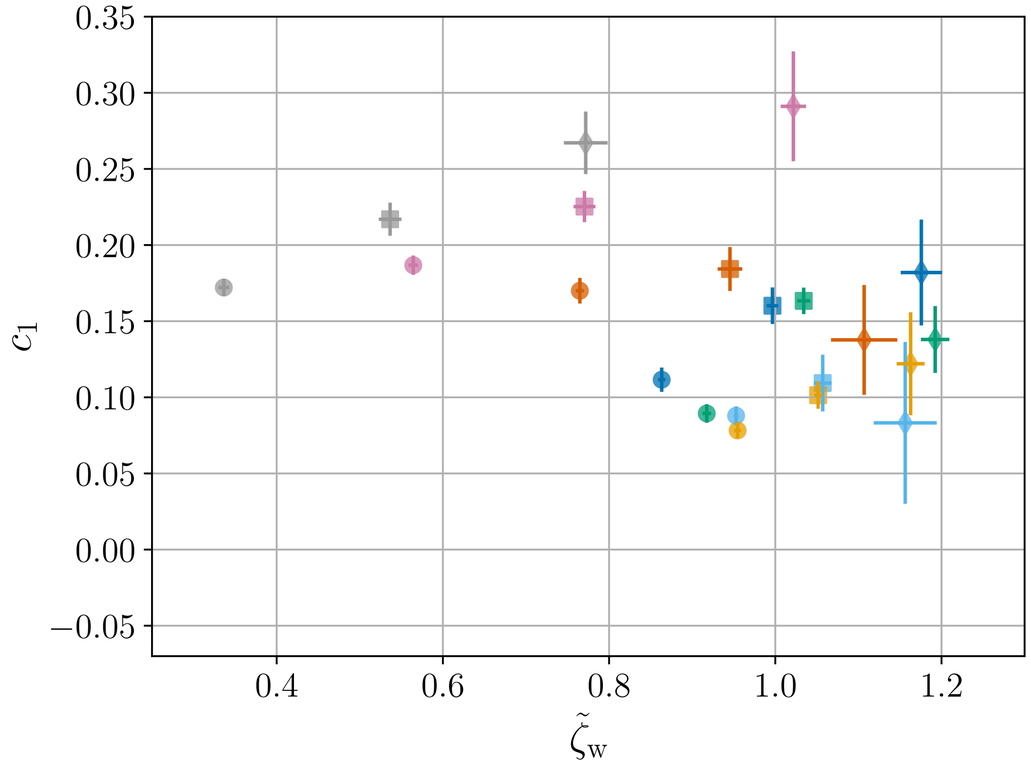
- Flattening out of curves can be seen more easily if we pick out a specific resolution / dynamic range (16k)
- This choice of variables will highlight the **initial conformal time range**, where evolution is most **sensitive** to **initial conditions**. **Scaling** “**compressed**” in the **last e-fold.** Again we repeat **velocity masks convergence of** ζ_W.

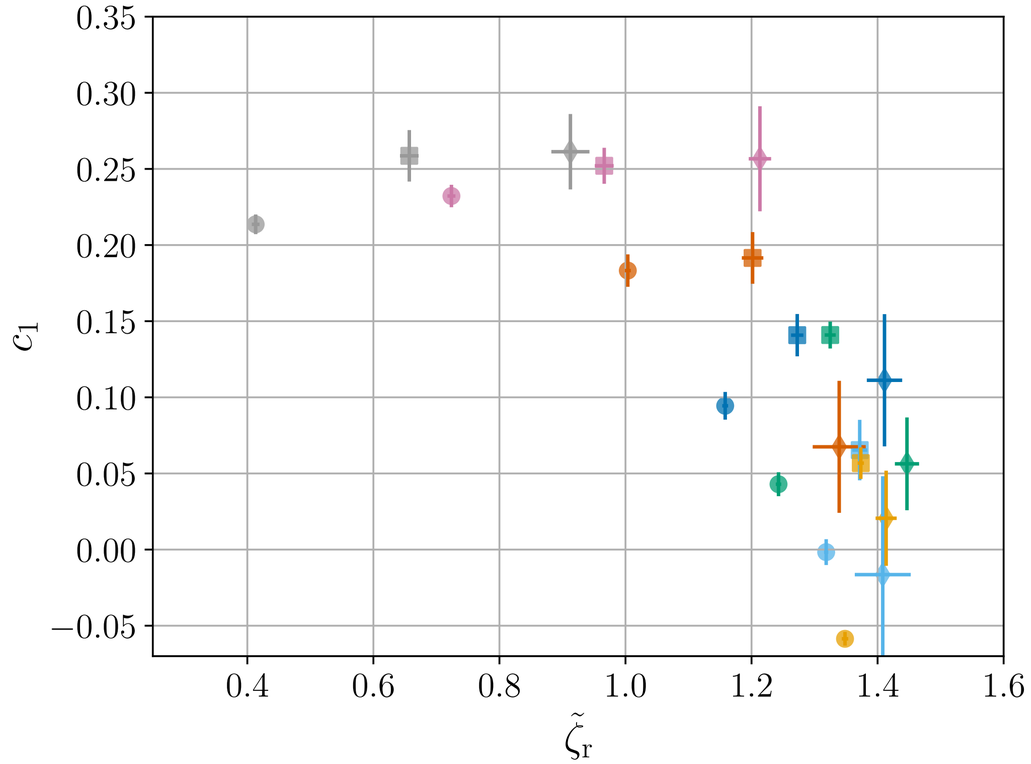




Long Term Growth Analysis

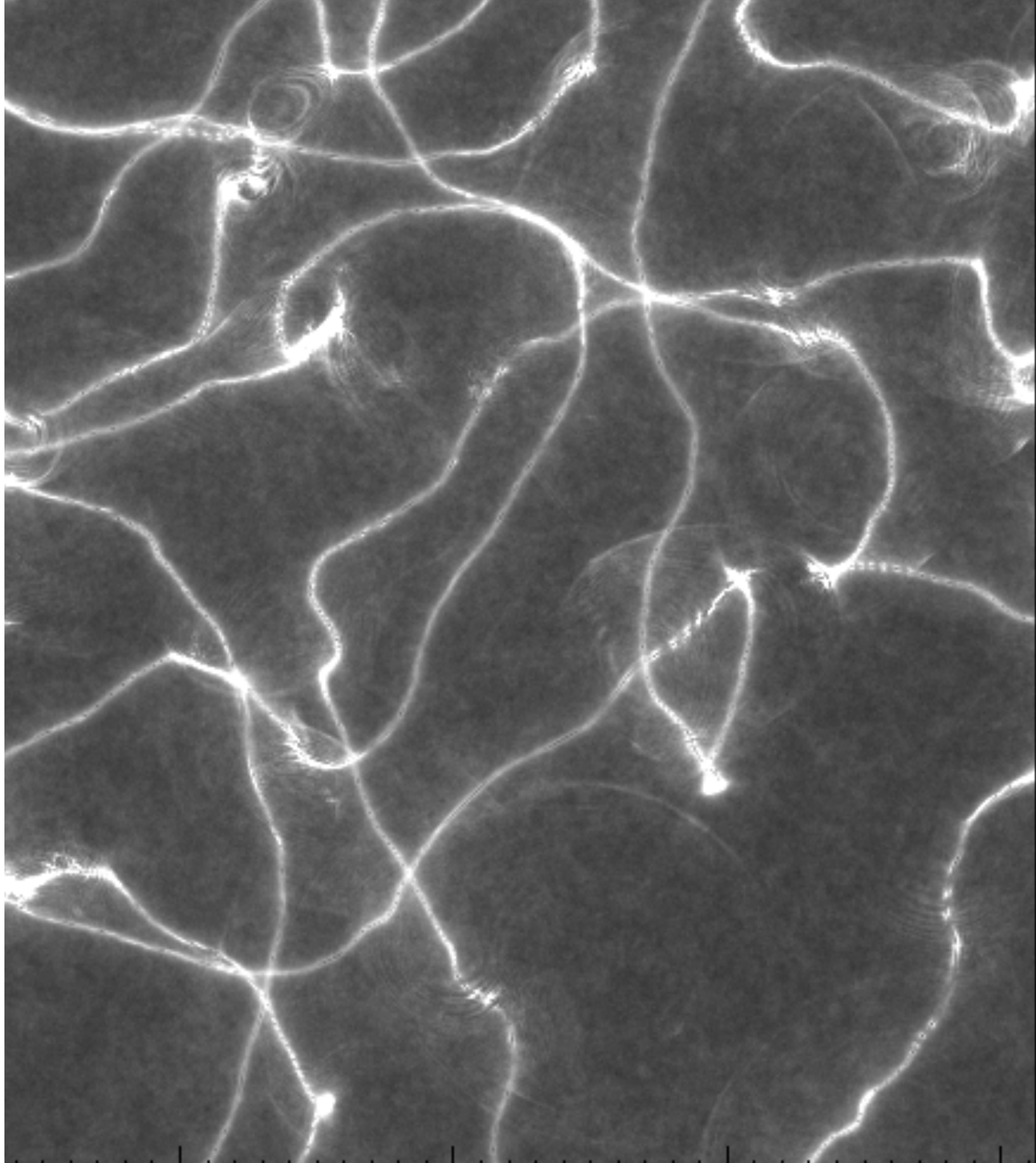
- We will attempt fits of the form $c_1 log(t) + c_0$, in different intervals of logarithm of cosmic time [6, 7], [7, 8] and [8, 9] for both $\zeta_{r,W}$
- We can observe **strict convergence** in ζ_r in $5 \leq l_\phi \leq 20$; although ζ_W **convergence masked** by **velocity** evolution
    - If the **logarithmic growth** were the correct late time behaviour, slopes would **increase with time**. **This is not observed**.





Conclusions

"The report of my death was an exaggeration”
- ~~Mark Twain~~ / Standard scaling



Conclusions

- We have, for the first time, observed **convergence of** ζ_r for simulations with $5 \leq l_\phi \leq 20$
- **RMS velocity convergence** observed in $l_\phi = 320$
- Our results support **standard scaling** (5% accuracy) with,

$$\zeta_{r,*} = 1.491(93) \qquad \hat{\zeta}_w = 1.220(57) \qquad v_{L,*} = 0.5705(93)$$

- Evolution in ζ_W due to two factors (**two null-clines**):

x_r at low initial string densities $\qquad$ v_L at higher initial densities

- ζ_r and $\zeta_W \gamma$ values **agree** at the end of the simulations:

  suggesting that the **universe length** is **close** to **converging** too

  Implied **scaling violation** of $\zeta_W \approx 0.24 log(m_s t)$ can be linked to **low initial density** and a symptom of an approach to a fixed point.

Conclusions

- Extrapolation from 4k simulations, **agree** with 16k simulations for ζ_r. 16k **velocity** is roughly 5% **lower.**
- A **VOS** model for global strings **needs** to be **developed and calibrated**. Attempts are scattered throughout the literature:
    - [Klaer & Moore, 2018], [Martins, 2018], [Revello & Villa, 2024] are some examples either with axion emission terms or time-varying tensions (the latter reparametrization invariant).
    - Testing corrections to the fixed point must be done via full model calibration, taking approach into account.
- Our methods of generating initial conditions either end in a constant rest-frame string density parameter, **or** a constant velocity → refining methods of generating ICs would benefit all groups.
    - Some ongoing work on modifying ICs to start physical evolution in different parts of phase space;
    - Will eventually allow us to start as close as possible to the fixed point = maximises useful dynamic range.
- Axion spectra - see **Mark Hindmarsh**'s talk for preliminary results.

Tack så mycket! /
Thank you! /
Obrigado!

PS: Fooseball anyone?