# Vector fields in (axion) inflation

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Three class of models:

• Mass term

$$-\frac{1}{4}F^2 + \frac{1}{2}m^2A^2$$

• Nonstandard kinetic term

 $-\frac{1}{4}f^{2}\left(\phi\right)F^{2}$ 

• Pseudo-scalar interaction

$$-rac{1}{4}F^2-rac{\phi}{4f}F\, ilde{F}$$

## "Mass term"



• Potential term  $V(A_{\mu}A^{\mu})$ 

• Fixed norm 
$$\lambda \left( A_{\mu}A^{\mu} - v^2 \right)$$

• Nonminimal coupling  $A_{\mu}A^{\mu}R$ 

Ford '89

Ackerman, Carroll, Wise '07

Golovnev, Mukhanov, Vanchurin '08 Kanno, Kirma, Soda, Yokoyama '08 Chiba '08 Kovisto, Mota '08

All these vector models have ghost instabilities

Himmetoglu, Contaldi, MP '09

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{F^2}{4} + \frac{\xi}{2} R A^2 \right]$$

Positive  $\xi \equiv$  wrong sign mass  $\Rightarrow$  longitudinal vector is a ghost

(ghost = field with negative energy, not simply a tachyon)

Computation in Himmetoglu et al '09. Here, Stückelberg procedure:

$$A_{\mu} \to A_{\mu}^{T} + \frac{1}{M} \partial_{\mu} \phi$$

so that  $+ M^2 A^2 \rightarrow M^2 A^T_{\mu} A^{\mu T} + \partial_{\mu} \phi \partial^{\mu} \phi$  (signature - + + +)

Moral: All the above above models introduce an "effective mass term" with "wrong sign", to sustain the vector field during inflation (anisotropic inflation, magnetogenesis). U(1) invariance due to the V(A) term generates a longitudinal mode, which, for the wrong sign, is a ghost.

Cure: search for models with unbroken U(1)

### Non-standard kinetic term

 $\mathcal{L} = -\frac{1}{A}I^2(\phi) F^2 \qquad \text{electric and magnetic fields } E_i = -\frac{\langle I \rangle}{a^2} A'_i \ , \ B_i = \frac{\langle I \rangle}{a^2} \epsilon_{ijk} \partial_j A_k$ 

• electric  $\leftrightarrow$  magnetic duality for  $\langle I \rangle \leftrightarrow \frac{1}{\langle I \rangle}$ 

Assume  $I(\phi)$  and  $\phi(t)$  combine to give  $\langle I \rangle \propto a^n$ 



• EM coupling 
$$e_{\text{physical}} = e I^{-1}(\phi) \propto a^{-n}$$

Strong coupling problem during inflation for n > 0 Demozzi, Mukhanov, Rubinstein '09

Functional form 
$$I(\phi) = \exp\left[-\frac{n}{M_p}\int \frac{d\phi}{\sqrt{2\epsilon(\phi)}}\right] \rightarrow \langle I \rangle \propto a^n$$
  
 $\epsilon = \frac{M_p^2}{2} \left(\frac{1}{V}\frac{dV}{d\phi}\right)^2 \ll 1$  Martin, Yokoyama '07

#### Anisotropic inflation

Watanabe, Kanno, Soda '09

n = -2 (no backreaction, nor strong coupling)  $\rightarrow$  constant electric field at large scales  $\equiv$  allows for uniform and constant  $\langle \vec{E}^{(0)} \rangle$  across the universe

Accounting for slow roll corrections to dS

$$rac{\Delta H}{H} \simeq rac{2
ho_{E^{(0)}}}{V\left(\phi
ight)} \simeq \left(\left|n
ight|-2
ight) \epsilon$$



### Phenomenology

• Power spectrum  $\left< \delta \phi^2 \right>$ 

Dulaney, Gresham '10 Gumrucuoglu, Himmetoglu, MP '10 Watanabe, Kanno, Soda '10

$$P\left(\vec{k}\right) = P\left(k\right) \left[1 + g_*\left(\hat{k} \cdot \hat{v}\right)^2\right]$$

$$g_* \simeq -rac{96 N_{\mathsf{CMB}}^2}{\epsilon} rac{
ho_{E^{(0)}}}{V(\phi)} \simeq 10^7 rac{
ho_{E^{(0)}}}{V(\phi)}$$

• Bispectrum  $\left< \delta \phi^3 \right>$ 

 $k_2$ 

 $k_1$ 

Barnaby, Namba, MP '12

Non-trivial dependence on  $\alpha$  in the squeezed limit  $k_3 \ll k_{1,2}$ . " .... This therefore appears as a signature of non-gaussianity from higher spin fields, and it may be an important distinguishing feature when the model is confronted with observations.... "

Barnaby, Namba, MP '12

Anisotropic inflation (n = -2):  $g_* = O(10^7) \times \frac{\Delta H}{H}$ 

Electric  $\leftrightarrow$  magnetic duality?? What does mean? Bartolo, Matarrese, MP, Ricciardone '12

Magnetogenesis (n = +2):

 $\rho_B \simeq H^4 N , \ g_* = 0$ 

- $n = \pm 2$  produces scale-invariant and constant perturbations. Each mode leaves the horizon with energy  $H^4$ , and then frozen.
- "Random walk" addition of the modes that have left the horizon
   They add up to a classical homogeneous background. For vector
   fields, the background is a vector that points somewhere in space

In any realization, CMB modes see a 
$$\vec{V}_{\text{classical}}$$
 drawn by a statistical distribution of variance  $\sim H^4 (N_{\text{tot}} - N_{\text{CMB}})$ 

In  $I^2F^2$  case,  $N_{\rm tot} - N_{\rm CMB} \simeq 5$  lead to a  $g_* \sim 0.01$ , ruled out

### Axion inflation

 $\Delta\phi > M_p$  not expected in a generic low-energy effective QFT

$$V = V_{\text{renormalizable}}(\phi) + \phi^{4} \sum_{n=1}^{\infty} c_{n} \frac{\phi^{n}}{M^{n}}$$
  
Hard to achieve flatness  $\frac{M_{p}V'}{V}, \frac{M_{p}^{2}V''}{V} \ll 1$  unless  $M \gg M_{p}$   
UV sensitivity of i  
Shift symmetry  $\phi \bigcup \forall + 5$  (rest faxion (refutat)) inflation  
UV sensitivity of inflation  
 $\mathcal{L} = \frac{1}{2} \left( \partial_{\mu} \phi \bigcup \phi + 5 \phi \right) \int \phi^{\mu} \phi^{$ 



### Aligned natural inflation

$$V = \Lambda_1^4 \left[ 1 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right) \right] + \Lambda_2^4 \left[ 1 - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right) \right]$$
$$f_{\text{eff}} \gg f_i, g_i \quad \text{if} \quad \frac{f_1}{g_1} \simeq \frac{f_2}{g_2}$$

Proposed to produce  $f_{eff} > M_p$  from sub-Planckian  $f_i$ ,  $g_i$ (gravitational instanton corrections may still be a problem)



#### • In agreement with CMB

denote by  $\phi(\psi)$  the light (heavy) eigenstate Fields rescaled  $\simeq$  curvature in 2 directions inflation on valleys with  $\frac{\partial V}{\partial \psi} = 0$ ,  $\frac{\partial^2 V}{\partial \psi^2} > 0$ 

 $r_f = 1.5; r_\Lambda = 0.4144; r_g = 1; \alpha = 0.01; f_{\phi} = 5.5 M_p$ 

MP, Unal '15

For some parameters inflationary trajectories ending because (1) reach a minimum or (2) become unstable in heavy direction



All above natural inflation

Target for CMB-S4, LiteBIRD, ...

Solved analytically:  $n_{s-1}$  and r given as analytic functions of the model parameters and number of e-folds in Greco, MP '24

Also shown that, after leaving the metastable inflationary valley, system often reaches a stable inflationary valley, so two-stage inflation



Fast oscillations  $\rightarrow$  enhanced power at intermediate scales. Potential signature of the model, Greco, MP 2507.xxxxx



Barnaby, MP '10



- Sourced GW are chiral Sorbo '11
- Sourced scalars non-Gaussian  $ightarrow f/c_A \gtrsim 10^{16}\,{
  m GeV}$  Barnaby, MP '10
- GW less produced  $(1/M_p \text{ vs } c_A/f)$  and not obsservable

General lesson: Several mechanisms for additional GW, result in a decrease of ronce also extra density perturbations are accounted for



#### How robust ? Cost for evading it ?

- No direct coupling with inflaton (Source gravitationally coupled to both GW and inflaton)
- Relativistic source (GW are produced by quadrupole moment;  $\zeta$  by energy density)
- Source active only for limited time (GW observed only on a small window;

 $\zeta$  provides constrains on many more scales)

These 3 ingredients present in Namba, MP, Shiraishi, Sorbo, Unal '15



• Gives visible r at arbitrarily small  $r_{vacuum}$  / scale of inflation

Three examples with  $\epsilon_{\phi} = 10^{-5}$  (so that  $r_{\text{vacuum}} = 16 \epsilon$  is unobservable):



• Distinguishable from vacuum GW by tensor running

Moral: Hard, but not impossible, to violate  $V \leftrightarrow r$  relation. Limits from scalar non-G  $\rightarrow$  specific conditions  $\rightarrow$  distinguish from vacuum GW

#### Naturally blue signals in axion inflation

Back to production from inflaton Recall  $A_+ \propto e^{\pi\xi}$ ,  $\xi = \frac{\phi}{2fH} \propto \sqrt{\epsilon}$ 

Inflaton speeds up  $\rightarrow$  signals naturally increase at smaller scales

- GW at interferometers
   Cook, Sorbo '12; Barnaby, Pajer, MP '12
   Domcke, Pieroni, Binétruy '16
- PBH Linde, Mooij, Pajer '12; Bugaev, Klimai '13
   Garcia-Bellido, MP, Unal '16, '17



CMB signatures  $\rightarrow \xi \lesssim 2.5$ . In this regime, amplified  $\vec{A}$  have negligible backreaction on the background dynamics. No longer the case for  $\xi \gtrsim 5$ 

MP, Sorbo, Unal '16



$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = \frac{c_A}{f}\vec{E}\cdot\vec{B}$$

$$H^{2} = \frac{1}{3M_{p}^{2}} \left[ \frac{1}{2} \dot{\phi}^{2} + V + \frac{\vec{E}^{2} + \vec{B}^{2}}{2} \right]$$

(1) Steady state backreaction Analytic computations for  $\xi \simeq \text{const}$ 

(2) Homogeneous backreaction  $\{\phi($ 

$$\left\{ \phi\left(t
ight) \;,\; ec{A}\left(t,\,ec{x}
ight) 
ight\}$$

(3) Inhomogeneous backreaction Full system

SU(2) case in Iarygina, Sfakianakis, Sharma, Brandenburg'24

#### Steady state backreaction

- Strong backreaction regime studied by Anber, Sorbo '09
- Old idea of warm inflation φ + 3Hφ + V' = -Γφ Berera '95
   Dissipation reduces the inflaton motion. Can allow inflation in steep potentials (large V') and for reduced field excursion (small Δφ)
   Both aspects might be beneficial in the recent swampland program
- Anber-Sorbo mechanism simple & well defined QFT realization

 $\ddot{\phi} + 3H\dot{\phi} + V' = \frac{c_A}{f} F \tilde{F} \left[ \dot{\phi} \right]$   $\swarrow \checkmark \checkmark$ Perfect balance assumed at all times  $\rightarrow$  Steady state evolution

$$\Rightarrow V' \simeq -2.4 \cdot 10^{-4} \frac{c_A H^4}{f} \frac{e^{2\pi\xi}}{\xi^4} \quad , \quad \xi \equiv \frac{c_A \dot{\phi}}{2fH}$$

Anber, Sorbo '09

### Homogeneous backreaction (homogeneous $\phi$ )

#### Oscillations about AS from numerical integration of simplified system

Cheng, Dall'Agata, Domcke, Durrer, González-Martín, Garg, Gorbar, Guidetti, Lee, Momot, Ng, Notari, Papageorgiou, MP, Rudenok, Schmitz, Sobol, Sorbo, von Eckardstein, Tywoniuk, Vilchinskii, Welling, Westphal, ...



• Interpreted as delayed effect between the moment the gauge quanta are produced and the moment they backreact on  $\phi(t)$ 

Domcke, Guidetti, Welling, Westphal '20

Analytical study: 
$$\phi(t) = \bar{\phi}(t) + \delta\phi(t)$$
,  $A^{\mu}(t, \vec{k}) = \bar{A}^{\mu}(t, \vec{k}) + \delta A^{\mu}(t, \vec{k})$   
of the homogeneous inflaton & gauge modes around the AS solution  
MP, Sorbo '22

$$\delta\phi'' + 2aH\delta\phi' + a^2V''\delta\phi = -\frac{c_A}{fa^2} \int \frac{d^3k}{(2\pi)^3} \frac{k}{2} \frac{\partial}{\partial\tau} \left[\bar{A}\,\delta A^* + \bar{A}^*\,\delta A\right]$$
$$\delta A'' + \left(k^2 - \frac{k\,\bar{\phi}'}{f}\right)\delta A = \frac{c_A\,\bar{A}}{f}\,\delta\phi'$$

• Formally solve 2nd eq for  $\delta A$  as a function of  $\delta \phi'$ 

$$\delta A(\tau, k) = \frac{c_A k}{f} \int^{\tau} d\tau' G_k(\tau, \tau') \bar{A}(\tau', k) \delta \phi'(\tau')$$

• Insert solution in 1st eq  $\rightarrow$  integro-differential eq for  $\delta\phi$ 

 $\delta\phi''(\tau) + 2aH\delta\phi'(\tau) + a^2V''\delta\phi(\tau) \simeq$ 

$$\frac{c_A^2}{f^2 a^2} \frac{\mathrm{e}^{2\pi\xi}}{2^8 \pi^2 \xi^5} \int^{\tau} \frac{d\tau'}{\left(-\tau'\right)^4} \delta\phi'\left(\tau'\right) \frac{\partial}{\partial\tau} \int_0^{4\xi_{\gamma}^2} dy \, y^3 \sqrt{\tau\tau'} \left[\mathrm{e}^{-4\sqrt{y}} - \mathrm{e}^{-4\sqrt{y}} \sqrt{\frac{-\tau}{-\tau'}}\right]$$

• Look for  $\delta\phi \propto (-\tau)^{-\beta} \equiv a^{\operatorname{Re}\beta} \cos(\operatorname{Im}\beta \times N + \operatorname{phase})$ 

Inserting this and doing the integrals  $\rightarrow$  homogeneous eq in time (all terms scale as  $\tau^{-\beta-2}$ ). Therefore left will an algebraic equation for complex  $\beta$ .

$$\frac{\xi f V''}{c_A (-V')} \simeq \frac{4\beta \left(\beta + 3\right)}{\left(2\beta - 1\right) \left(2\beta + 7\right)} \left[\frac{1}{\left(8\xi_{\gamma}\right)^{2\beta - 1}} \frac{\Gamma \left(2\beta + 8\right)}{\Gamma \left(9\right)} - 1\right] \equiv \mathcal{F}\left[\beta, \, \xi_{\gamma}\right]$$



$$\delta\phi \propto (-\tau)^{-\beta} \equiv a^{\operatorname{Re}\beta} \operatorname{cos}\left(\operatorname{Im}\beta \times N + \operatorname{phase}\right)$$
$$\frac{\xi f V''}{c_A (-V')} \simeq \frac{4\beta (\beta+3)}{(2\beta-1)(2\beta+7)} \left[\frac{1}{(8\xi_{\gamma})^{2\beta-1}} \frac{\Gamma (2\beta+8)}{\Gamma (9)} - 1\right] \equiv \mathcal{F}[\zeta, \xi_{\gamma}]$$
$$\frac{\rho_A}{\rho_{\phi}} \sim \left|\frac{\xi f V''}{c_A (-V')}\right| \ll 1 \text{ in AS}$$



Look for most unstable

20

 $\delta\phi(t)$  linear combination of these modes



solution (greatest  $\beta_R$ )

with  $\mathcal{F} = 0$ 



#### Bursts of GW production

• Expect gauge field amplification and related phenomenology enhanced at scales O(H) when  $\dot{\phi}$  is maximum  $\rightarrow$  Recurrent peaks in power spectra



CanStockPhoto.com

Correlated peaks across different GW observatories

Pulsar Timing Arrays

Astrometry

Interfrometers

Garcia-Bellido, Papageorgiou, MP, Sorbo '23



• Based on homogeneous backreaction:  $\phi(t) + \vec{A}(t, \vec{x}) + \text{linear } \delta g_{\mu\nu}^{TT}(t, \vec{x})$ Can we trust it? As usual, the problem is sourced GW vs. scalar perturbations. Add  $\delta\phi$  (perturbatively) and see when it is no longer safe to neglect it

Within the Gradient Expansion Formalism

Domcke, Ema, Sandner '24 Durrer, von Eckardstein, Garg, Schmitz, Sobol, Vilchinskii '24

 $\bigstar$  In the  $\left\{\phi(t), \vec{A}_{\vec{k}}(t)\right\} + \left\{\delta\phi, \delta g_{\mu\nu}^{TT}\right\}$  system

Barbon, Ijaz, MP 2507.xxxxx



Inhomogeneous backreaction - weak backreaction regime

- Analytic results for power spectrum  $P \propto \langle \delta \phi^2 \rangle$  and bispectrum  $B \propto \langle \delta \phi^3 \rangle$ in this regime based on several approximations: constant  $\xi$  and H, specific UV regularization Barnaby, MP '10
- Under control (backreaction and perturbativity) for  $\xi \lesssim 5$

Excellent agreement with full lattice simulations

MP, Sorbo, Unal '16

Caravano, Komatsu, Lozanov, Weller '22





#### Inhomogeneous backreaction - strong backreaction regime

Reheating: Adshead, Cuissa, Figueroa, Florio, Giblin, Pieroni, Scully, Sfakianakis, Shaposhnikov, Torrenti, Valkenburg, Weiner, ...

Caravano, Komatsu, Lozanov, Weller '22; Figueroa, Lizarraga, Urio, Urrestilla, '23; Caravano, MP '24;

Sharma, Brandenburg, Subramanian, Vikman '24; Figueroa, Lizarraga, Loayza, Urio, Urrestilla '24



- Extremely challenging (relatively fewer N)
- Reduced oscillatory pattern
- Strong sensitivity to coupling

#### Spectator axion on the lattice

Caravano, MP '24

 $\frac{\sigma}{c}$ 

 $V(\sigma) = \frac{\Lambda^4}{2} \left[ \cos\left(\frac{\sigma}{f}\right) + 1 \right]$ 

 $\frac{V}{\Lambda^4}$ 

0.8 0.6 0.4

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - U(\varphi) - \frac{1}{2}(\partial\sigma)^2 - V(\sigma) - \frac{1}{4}F^2 - \frac{c_A\sigma}{4f}F\tilde{F}$$
inflaton sector
extra sector

Axion rolls for  $\Delta N = \frac{3H^2}{m^2} = O(1) e - folds of inflation$ 

- Gives visible r at small  $r_{vacuum}$  / scale of inflation
- Relevant dynamics covered by the lattice





#### Increased sourced tensor / scalar at small $c_s$

$$\mathcal{L} = K(X) - V(\phi) - \frac{\phi}{4f} F \tilde{F} , \quad X \equiv -\frac{1}{2} (\partial \phi)^2 \qquad \text{Bartolo '25}$$

Kume MP

Canonical mode :  $v = \frac{a\sqrt{K'}}{c_s}\delta\phi$  ,  $c_s^2 \equiv \frac{K'}{K' + \dot{\phi}^2 K''}$  sound speed

Production parameter : 
$$\xi \equiv rac{\dot{\phi}}{2Hf} = \sqrt{rac{\epsilon}{2\,K'}} rac{M_p}{f}$$

$$\Rightarrow \text{ coupling } \frac{\delta\phi}{4f}F\,\tilde{F} = \frac{1}{4aM_p}\,c_s\sqrt{\frac{2}{\epsilon}}\,\xi\,v\,F\,\tilde{F}$$

Curvature perturbations :  $\zeta = -\frac{H}{\dot{\phi}}\delta\phi = -\frac{c_s}{\sqrt{2\epsilon}}\frac{v}{aM_r}$ 



$$\Rightarrow \zeta_{s} \sim \frac{H^{2}}{M_{p}^{2}} e^{2\pi\xi} \left[ \frac{c_{s}^{2}}{\epsilon} + O(1) \right] \sim c_{s}^{3} P_{\zeta}^{(0)} e^{2\pi\xi} \left[ 1 + O\left(\frac{\epsilon}{c_{s}^{2}}\right) \right]$$
  

$$\uparrow \qquad \uparrow$$
Time integration & coupling  
gauge amplitude

Tensor mode unaffected  $\Rightarrow \delta g_s^{TT} \sim \frac{H^2}{M_p^2} e^{2\pi\xi} \sim c_s \epsilon P_{\zeta}^{(0)} e^{2\pi\xi}$ 



Sourced tensor > vacuum

# Conclusions

- Vector fields during inflation lead to distinct signatures but challenging theoretical realizations beyond pseudo-scalar coupling
- Axion inflation remains an interesting framework for large field inflation. Need to go beyond the simplest model and address  $\Delta \phi > M_p$  in quantum gravity
- Gauge field amplification in axion inflation natural mechanism, with rich pheno (non-Gauss, chiral GW at many scales, PBH...)
- Old papers ( $\sim$  2010) analytical computations. Confirmed in the weak backreaction regime. Lattice simulations have matured to provide novel and interesting results for strong backreaction

## **EXTRA**

## Testing a chiral SGWB

• @ CMB scales (TB & EB correlations)



#### Measurement of GW polarization at LISA / ET

Two GWs related by a mirror symmetry produce the same response in a planar detector. Cannot detect net circular polarization of an isotropic SGWB

Isotropy in any case broken by peculiar motion of the solar system. Assumption,  $v_d \simeq 10^{-3}$  as CMB

$$\mathsf{SNR}_{\mathsf{LISA}} \simeq rac{v_d}{10^{-3}} rac{\Omega_{\mathsf{GW},\mathsf{R}} - \Omega_{\mathsf{GW},\mathsf{L}}}{1.2 \cdot 10^{-11}} \sqrt{rac{T}{3\,\mathsf{years}}}$$

Domcke, García-Bellido, MP, Pieroni Ricciardone, Sorbo, Tasinato '19

(one order of magnitude greater than estimate in Seto '06)

- Do the GW and the CMB dipole coincide ?
- One order of magnitude improvement with LISA-Taiji

Orlando, Pieroni, Ricciardone '20





#### Measurement at ground-based interferometers



### Large-scale SGWB anisotropies



• Treatment as CMB Alba, Maldacena '15; Contaldi '16; Cusin, Pitrou, Uzan '17; Jenkins, Sakellariadou '18; Bartolo, Bertacca, Matarrese, MP, Ricciardone, Riotto, Tasinato '19

$$\begin{split} \Omega_{\rm GW}(f) \ \to \ \Omega_{\rm GW}(f,\,\hat{n}) \equiv \Omega_0 \left(\frac{f}{f_0}\right)^{\alpha} \sum_{\ell m} \delta_{\ell m}^{\rm GW} \, Y_{\ell m}\left(\hat{n}\right) & \text{Bartolo et al '22,} \\ & \text{LISA CosWG} \\ & & \uparrow \\ & \text{Amplitudes relative to the monopole,} \\ & \delta_{00}^{\rm GW} = \frac{1}{Y_{00}} = \sqrt{4\pi} \simeq 3.5 \end{split}$$

#### Mentasti, Contaldi, MP '23



10 years

 $\Omega_{0,fiducial}=2\times 10^{-11}$ 

at 
$$f_0 = 1 \text{ mHz}$$

