Ultralight Dark Matter:
New Opportunities from Stochastic Fluctuations

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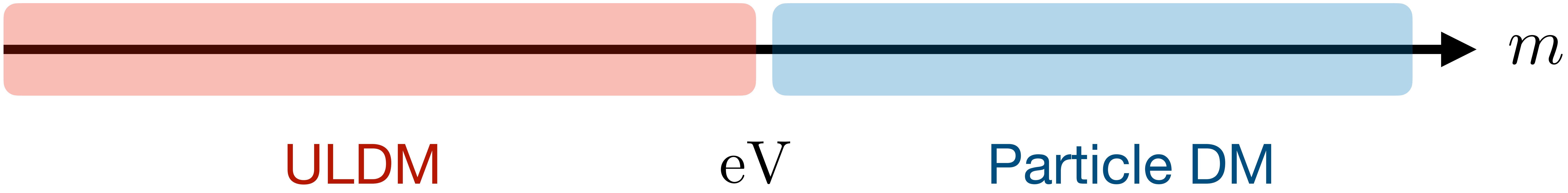
New Opportunities from Stochastic Fluctuations

Ultralight Dark Matter

Ultralight dark matter (ULDM)

as bosonic DM candidates with $\mathrm{m} < \mathrm{eV}$



$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 + h(\phi) \mathcal{O}_{\mathrm{SM}} \right]$$

$$ds^2 = (1 + 2\Phi) dt^2 - (1 - 2\Psi) dx^2$$

Ultralight dark matter (ULDM)
as bosonic DM candidates with $\mathrm{m} < \mathrm{eV}$

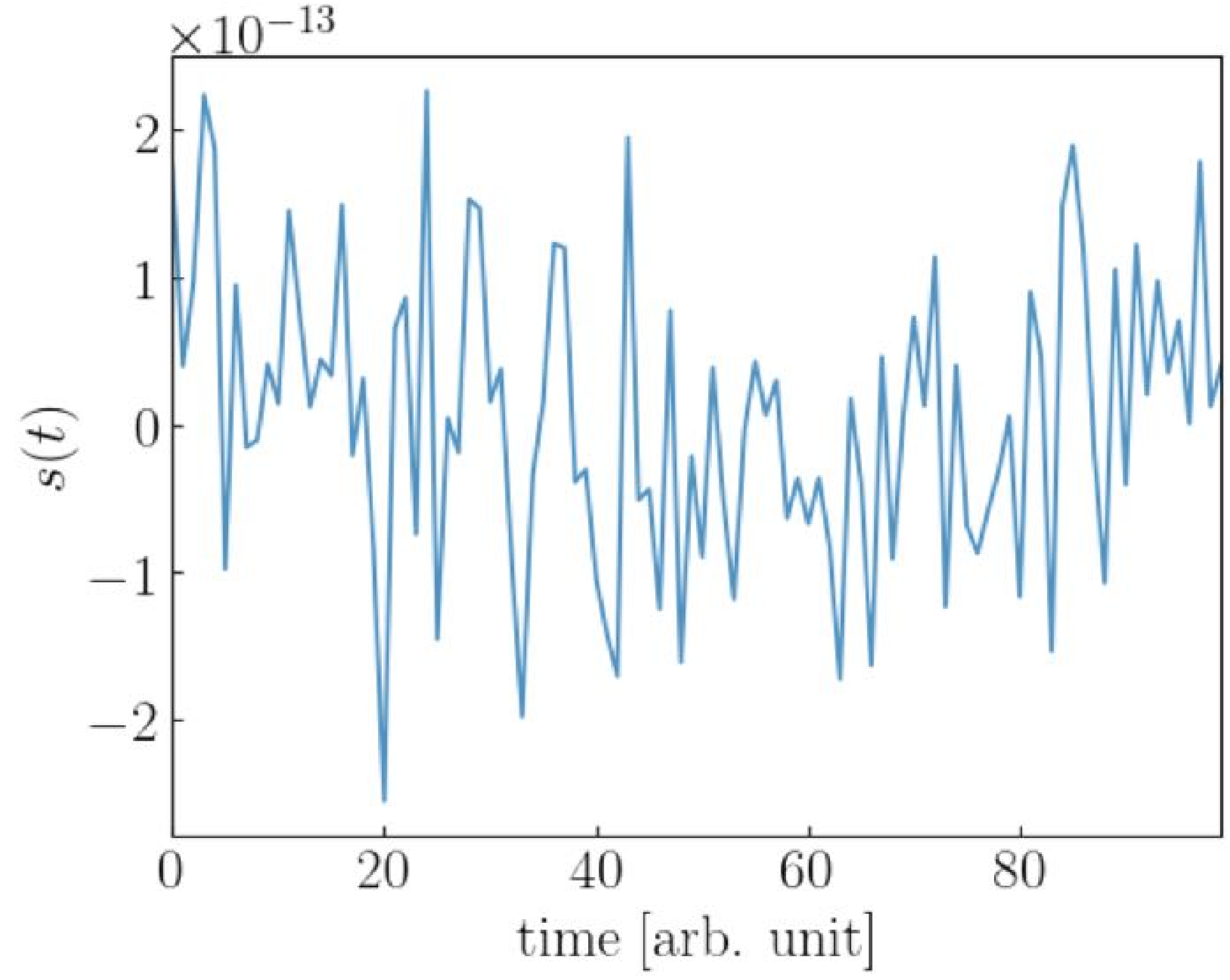
$$m \lesssim 10\,\mathrm{eV}$$

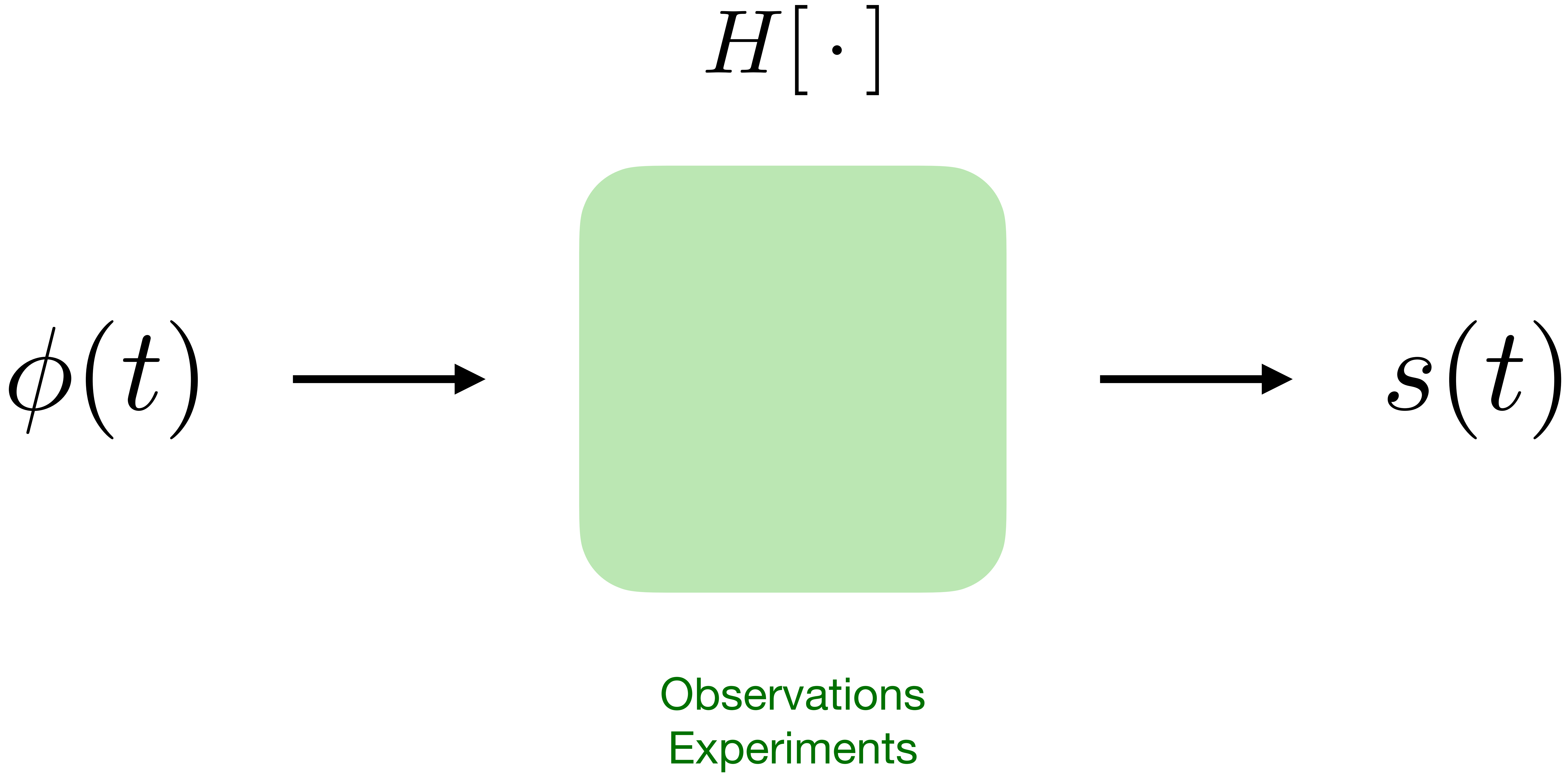
$$N_{\mathrm{occ}} \sim n_{\mathrm{dm}} \lambda^3 \sim \left(\frac{10\,\mathrm{eV}}{m}\right)^4$$

What signal should we expect?

$$s(t) = H[\phi(t)]$$

How might we find them?





$$s(t) = H[\phi(t)]$$

response of system depends on
specifics of the system / types of ULDM interaction

$$s(t) = H[\phi(t)]$$

$\propto \phi$ linear

ϕ^2 quadratic (nonlinear)

Example 1

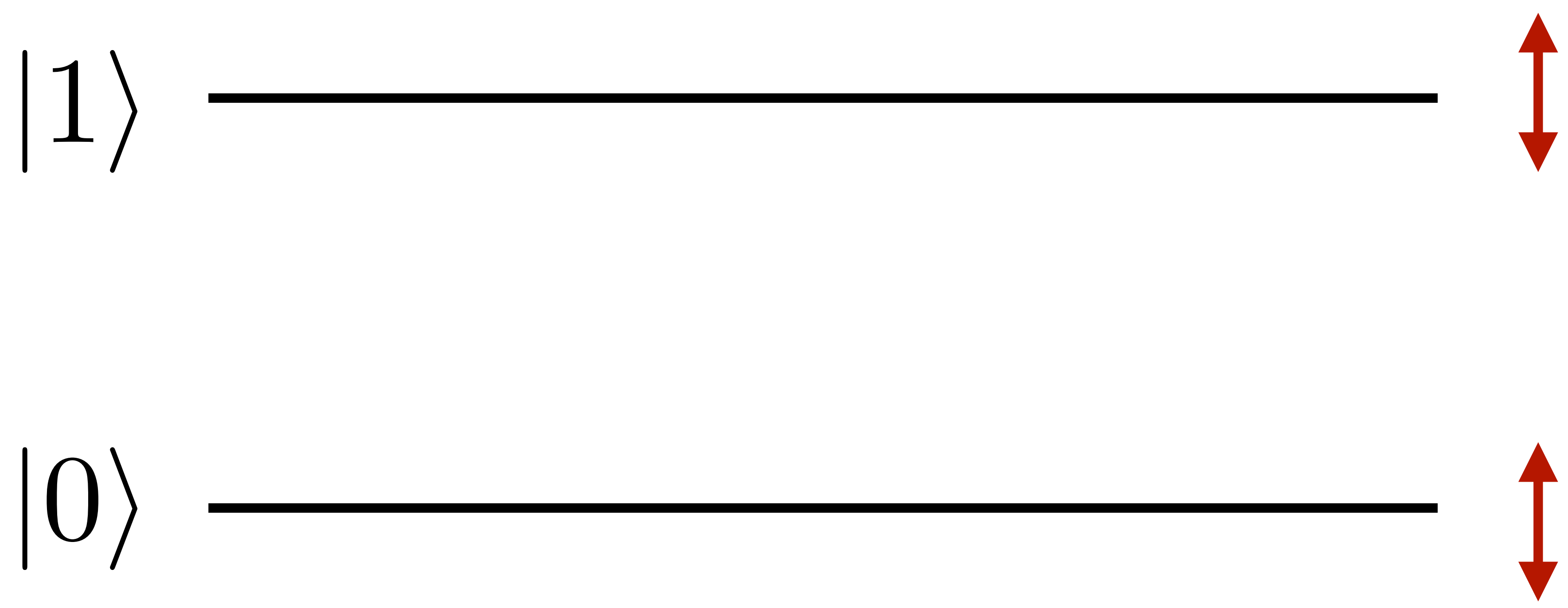
(non-gravitationally interacting ULDM)

the response of atomic system

$$\mathcal{L} = -\frac{1}{4\alpha}(1 - g\phi)FF$$

$$\frac{\delta\alpha}{\alpha} = g\phi$$

Example 1
(non-gravitationally interacting ULDM)
the response of atomic system



$$s(t) = \frac{\delta E}{E}(t) = c\frac{\delta\alpha}{\alpha}(t) = cg\phi(t)$$

Example 1
(non-gravitationally interacting ULDM)
the response of atomic system

An example of **linear** signal

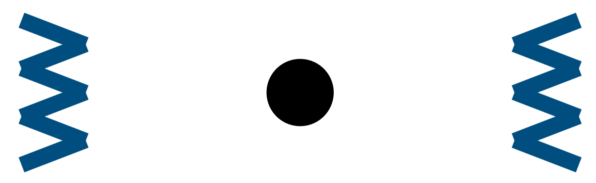
$$s(t) = H[\phi(t)] \propto \phi$$

$$\frac{\delta E}{E} = c\frac{\delta \alpha}{\alpha} = cg\phi$$

Example 2

(gravitationally interacting ULDM)

the response of a point-like particle

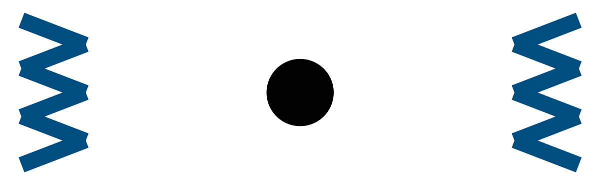


$$\ddot{x} = -\nabla\Phi$$

$$\nabla^2\Phi = 4\pi G\delta\rho$$

$$\rho = \frac{1}{2}\left[\dot{\phi}^2 + (\nabla\phi)^2 + m^2\phi^2\right]$$

Example 2
(gravitationally interacting ULDM)
the response of a point-like particle



An example of **quadratic** signal

$$s(t) = H[\phi(t)] \propto \phi^2$$

whether it is linear or quadratic

the signal will be a function of underlying ULDM field

$$s(t) = H[\phi(t)]$$

the signal will inherit
temporal/spectral characteristics
of underlying ULDM field

$$s(t) = H[\phi(t)]$$

to understand what signal one should expect
we first need to understand

$$\phi(t)$$

how do we describe ULDM field?

A ‘spherical cow’ level description of ULDM

$$\phi(t, \boldsymbol{x}) = \frac{\sqrt{2\rho_0}}{m} \cos(mt + \boldsymbol{k} \cdot \boldsymbol{x})$$

coherently oscillating at

$$\omega = m$$

A ‘spherical cow’ level description of ULDM

$$\phi(t, \boldsymbol{x}) = \frac{\sqrt{2\rho_0}}{m} \cos(mt + \boldsymbol{k} \cdot \boldsymbol{x})$$

amplitude determined by

$$\rho_0 = \frac{1}{2}(\dot{\phi}^2 + m^2\phi^2)$$

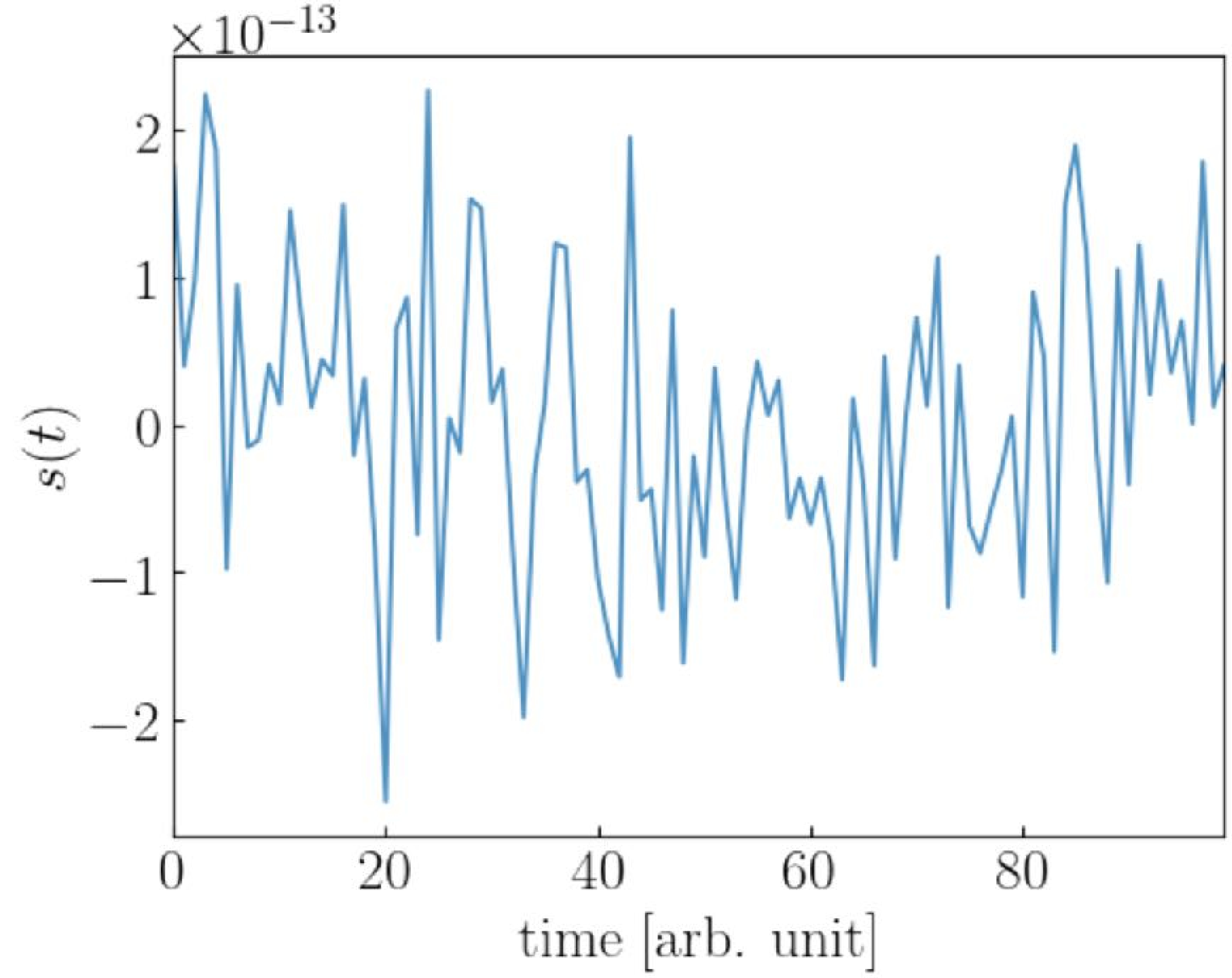
with this simplified description

expected ULDM signals become clear

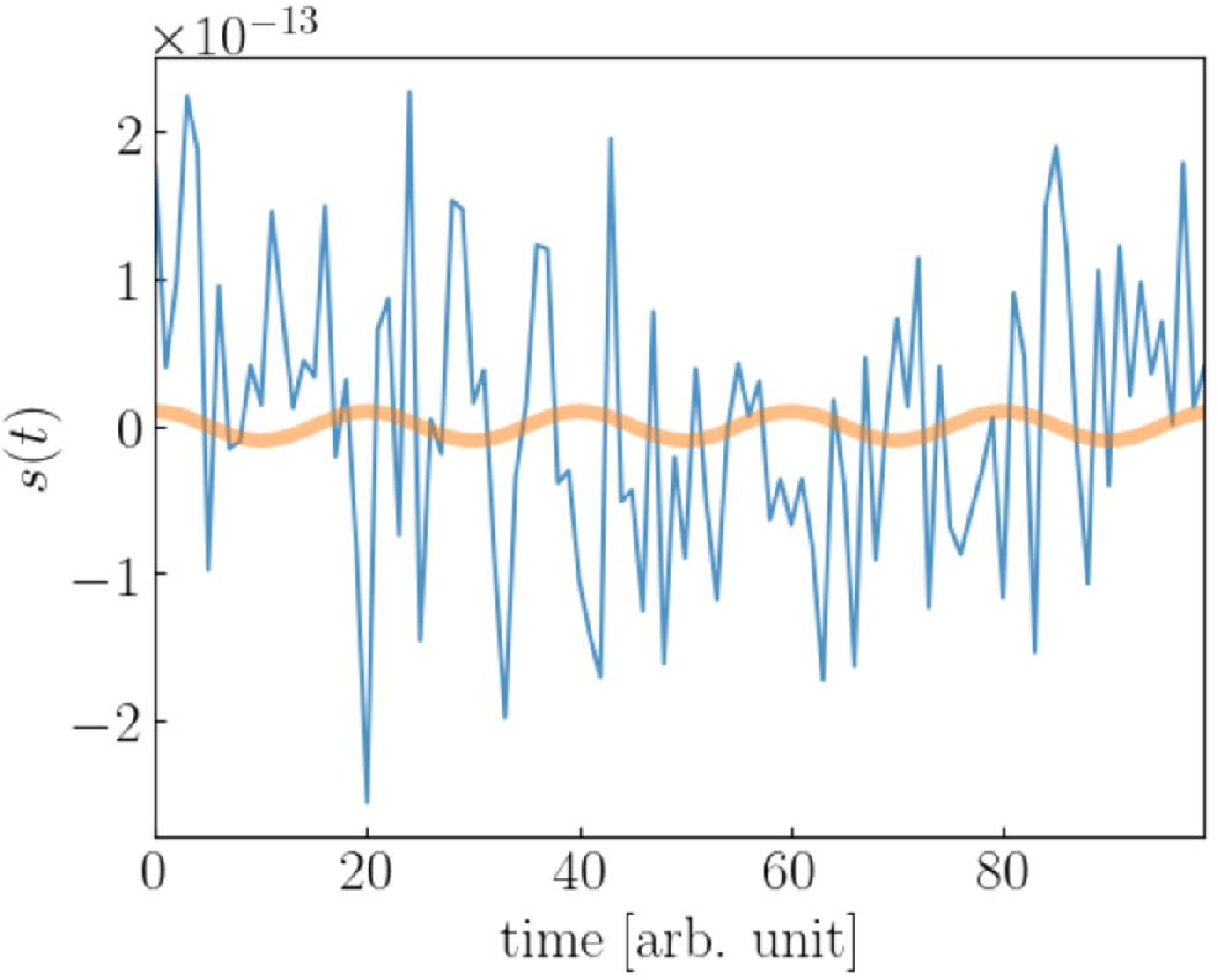
$$s(t) \propto \phi(t) = \phi_0 \cos(mt)$$

$$\propto \phi^2(t) = \phi_0^2 \cos^2(mt) \sim \phi_0^2 \cos(2mt)$$

How might we find them?



we look for coherent signals !

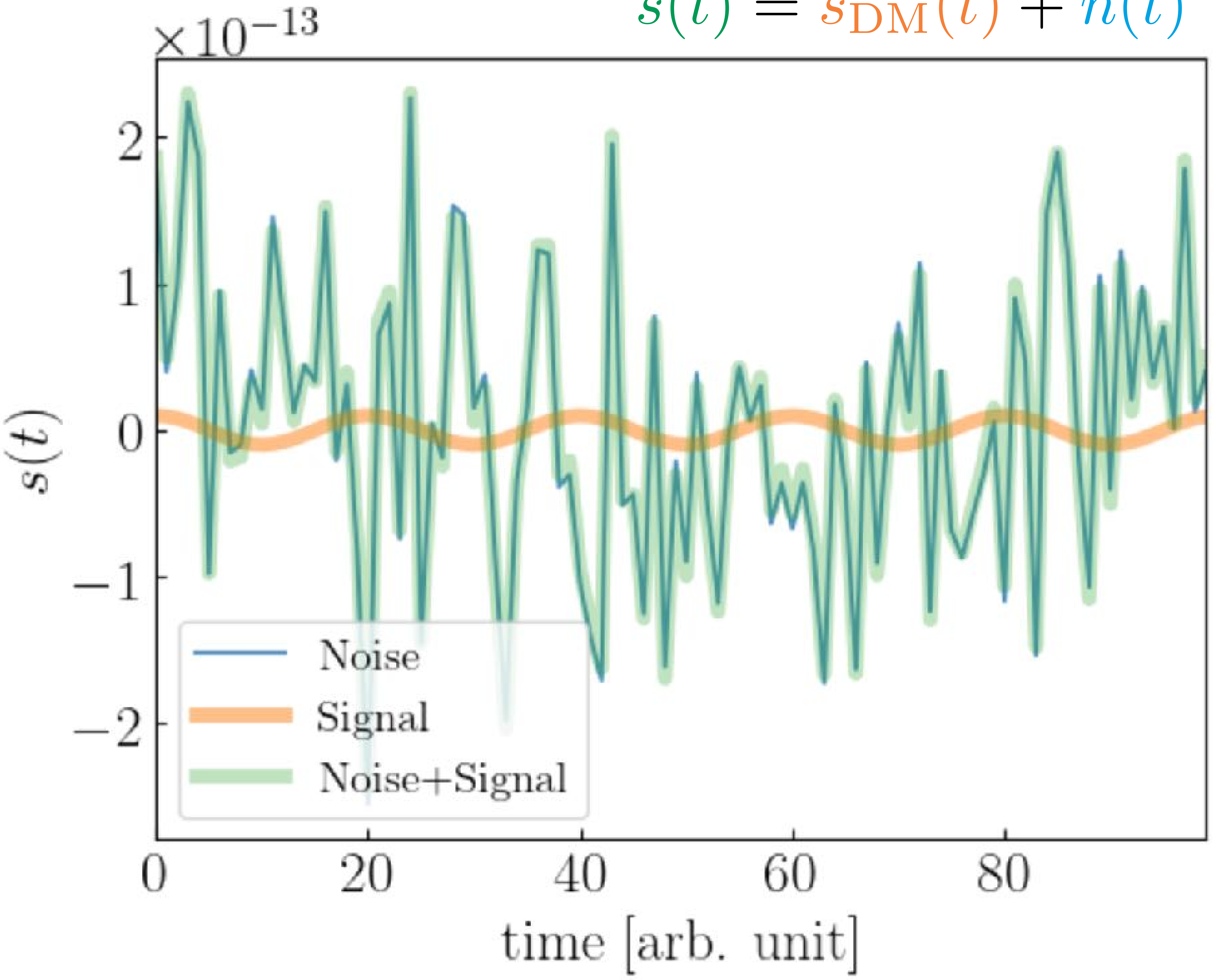


$\cos(mt)$ linear

$\cos(2mt)$ quadratic

how do we find such signals practically?

$$s(t) = s_{\rm DM}(t) + n(t)$$



$$\propto T \qquad \propto \sqrt{T}$$

$$Y = \int dt\, s(t) K_\omega(t) = \int dt \left[s_{\rm DM}(t) K_\omega(t) + n(t) K_\omega(t) \right]$$

For coherent signal

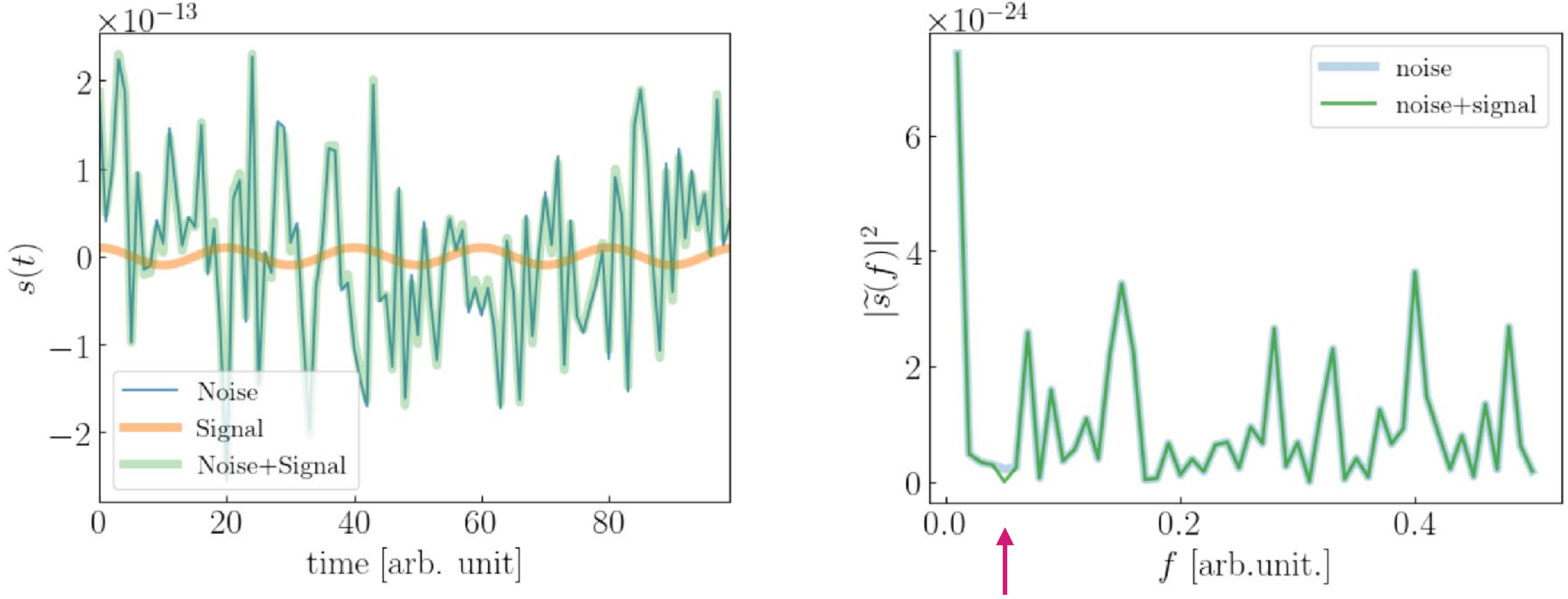
this quantity is nothing but Fourier transformation

$$\cos(mt)$$

$$Y = \int dt\, s(t) K_\omega(t) \ \sim\ \tilde{s}(\omega)$$

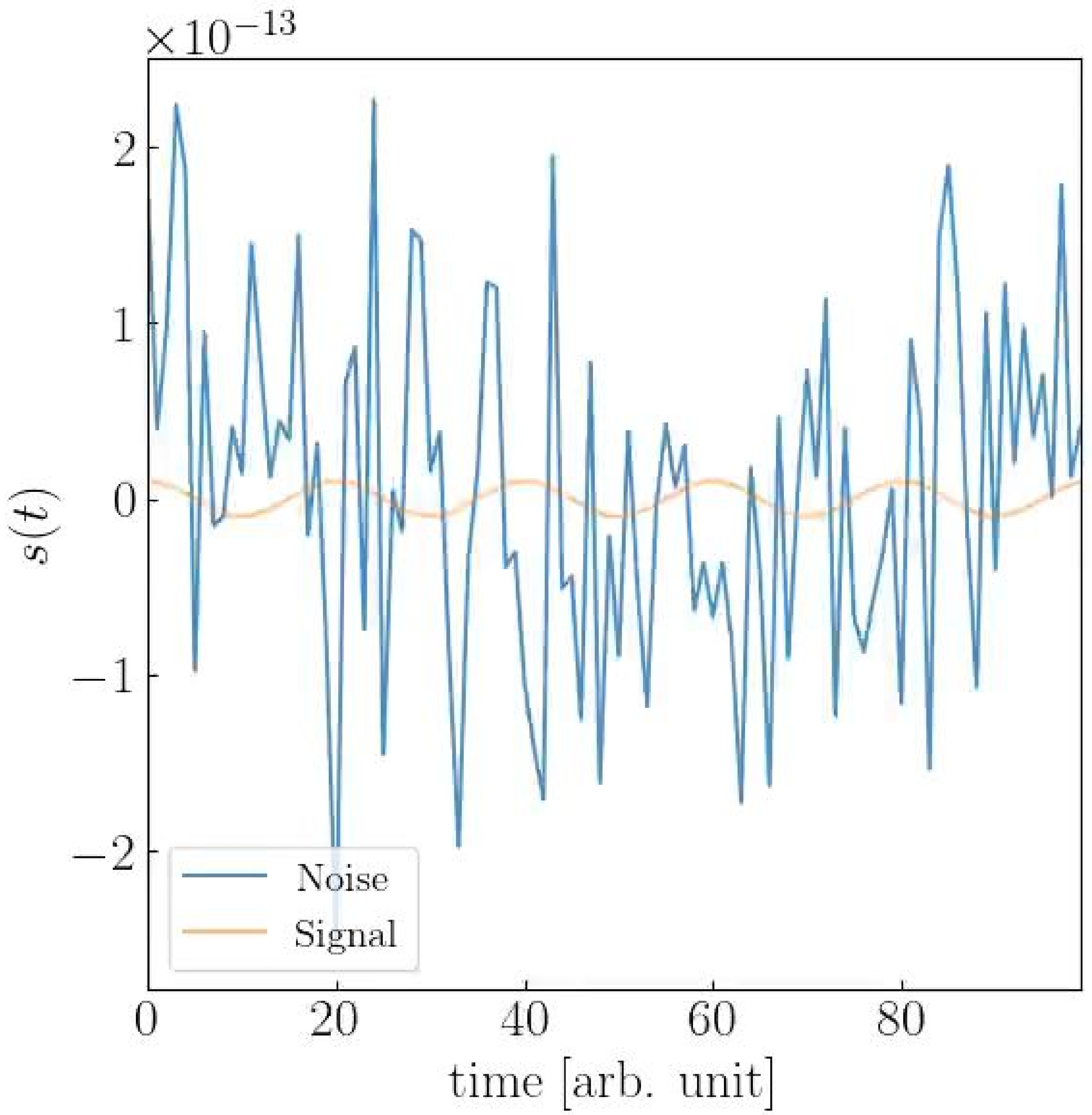
$$\cos(\omega t)$$

$$\sim \delta_T(\omega - m)$$

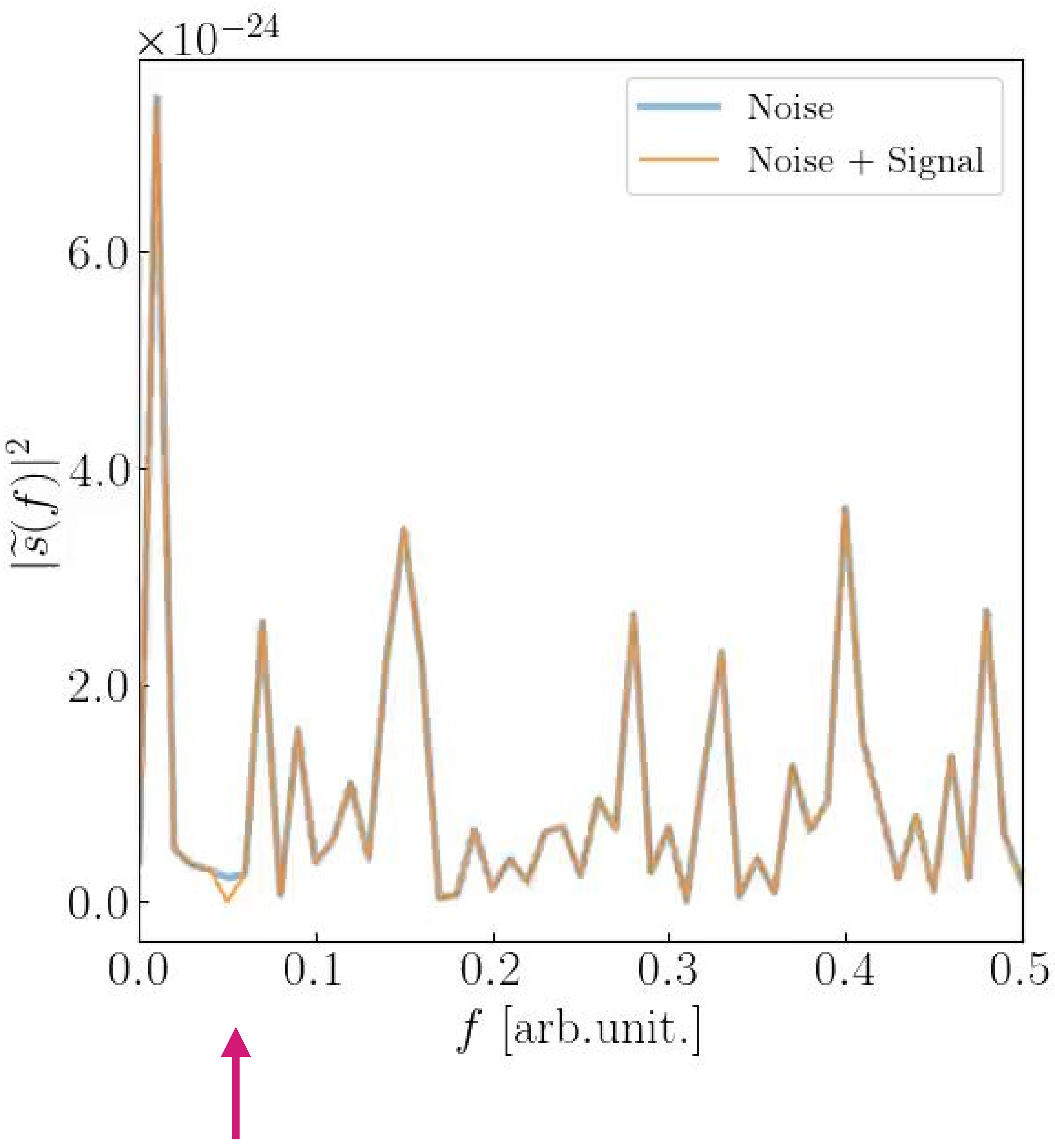


Coherent signal expected to appear here!

$\omega = m$ or $2m$



$\times 10^{-13}$
2
1
0
−1
−2
$s(t)$
Noise
Signal
0
20
40
60
80
time [arb. unit]



$\times 10^{-24}$
Noise
Noise + Signal
6.0
4.0
2.0
0.0
$|\tilde{s}(f)|^2$
0.0
0.1
0.2
0.3
0.4
0.5
f [arb.unit.]

this simplified picture

cannot capture ‘stochastic signal’

$$\alpha_{\boldsymbol{k}_i} = r_{\boldsymbol{k}_i} e^{i\theta_{\boldsymbol{k}_i}}$$

beyond 'spherical cow' description

random variables

$$\phi(t, \boldsymbol{x}) = \sum_{\boldsymbol{k}_i} \frac{1}{\sqrt{2mV}} \left[\alpha_{\boldsymbol{k}_i} e^{-ik_i \cdot x} + \alpha^*_{\boldsymbol{k}_i} e^{ik_i \cdot x} \right]$$

$$= \sum_{\boldsymbol{k}_i} \sqrt{\frac{2}{mV}} r_{\boldsymbol{k}_i} \cos(\omega_{\boldsymbol{k}_i} t - \boldsymbol{k}_i \cdot \boldsymbol{x} + \theta_{\boldsymbol{k}_i})$$

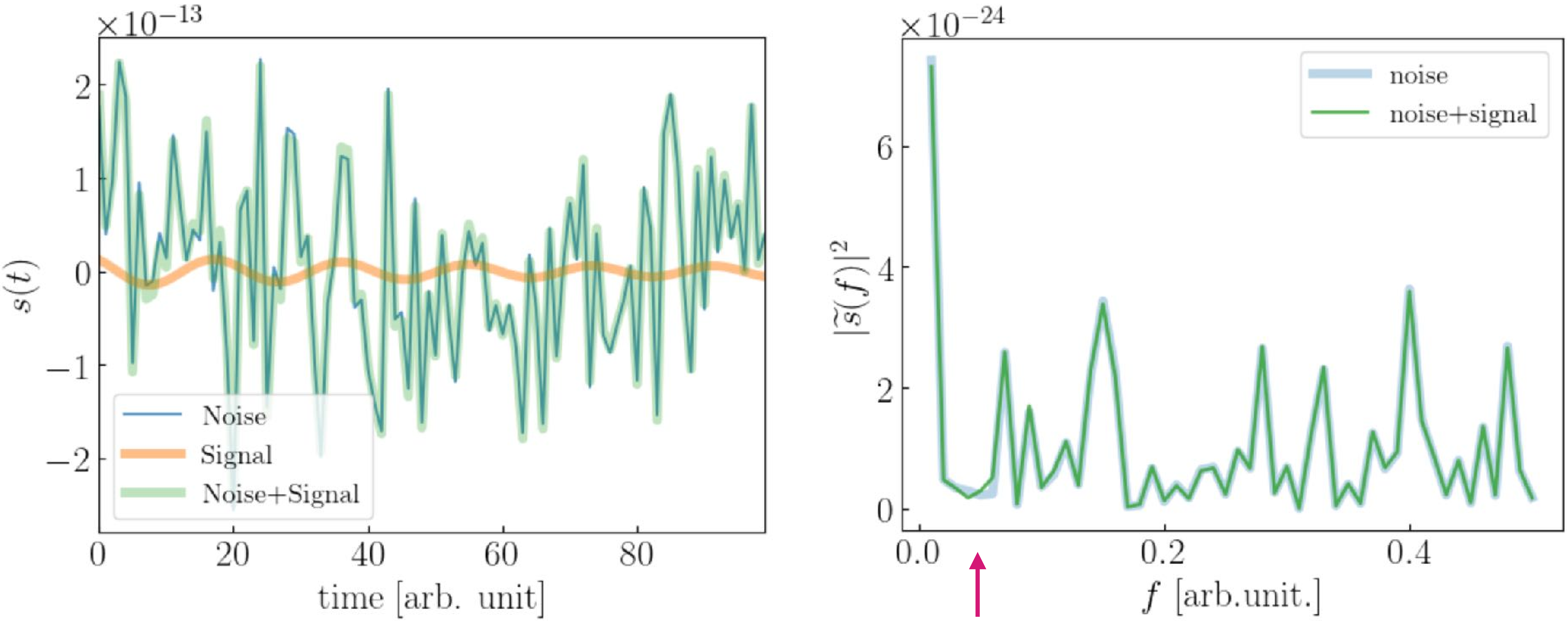
this expression is nothing but sum of cosine waves what signals should we expect?

for **linear signals**

it is qualitatively the same as before

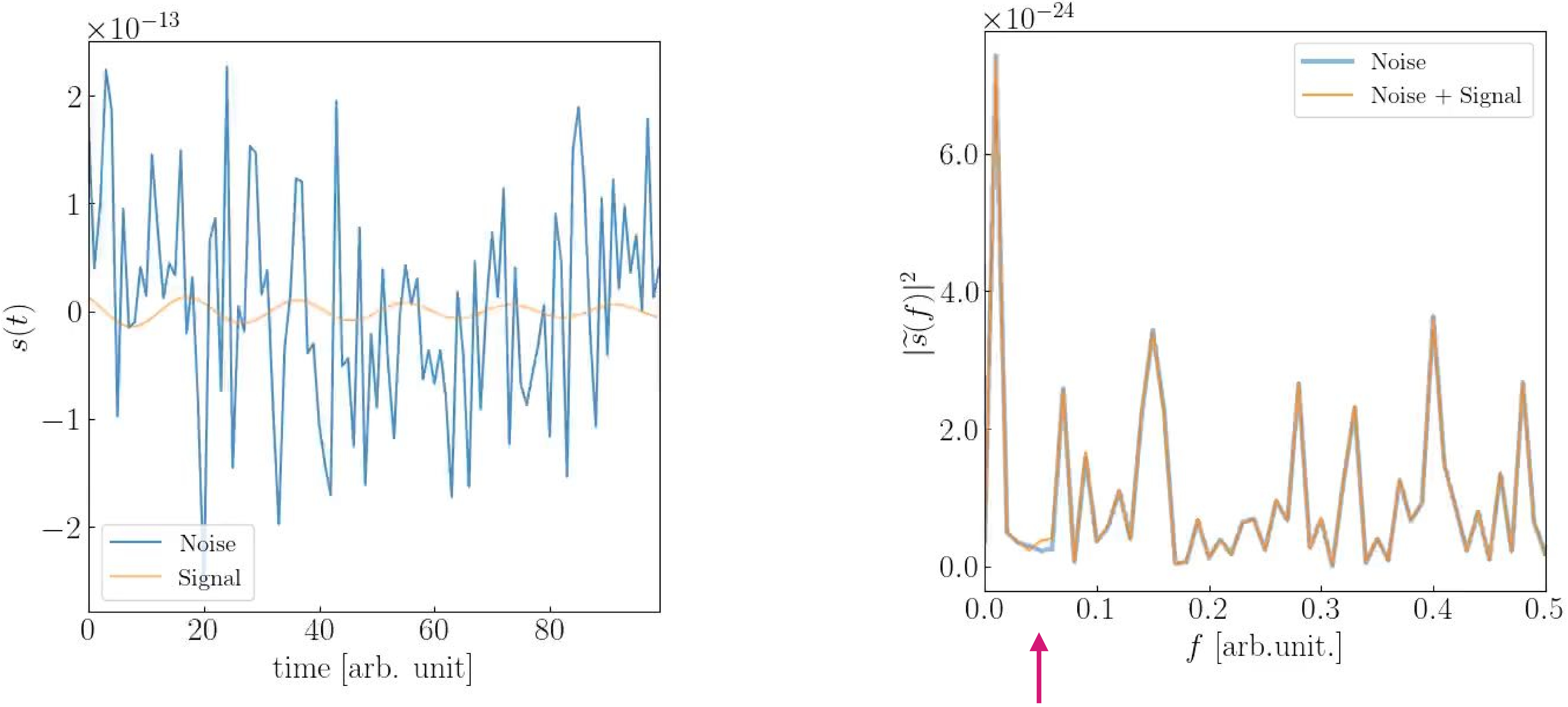
$$s(t) \propto \phi(t) \propto \sum_{\boldsymbol{k}_i} r_{\boldsymbol{k}_i} \cos(mt + \theta_{\boldsymbol{k}_i})$$

we look for coherent oscillations



Coherent signal expected to appear here

$\omega = m$



×10^−13
2
1
0
−1
−2
s(t)
Noise
Signal
0
20
40
60
80
time [arb. unit]
×10^−24
Noise
Noise + Signal
6.0
4.0
2.0
0.0
|s̃(f)|^2
0.0
0.1
0.2
0.3
0.4
0.5
f [arb.unit.]

for **quadratic signals**

stochastic fluctuation emerges

$$s(t) \propto \phi^2(t)$$

$$\propto \sum_{\boldsymbol{k}_i, \boldsymbol{k}_j} r_{\boldsymbol{k}_i} r_{\boldsymbol{k}_j} \cos(\omega_{\boldsymbol{k}_i} t + \theta_{\boldsymbol{k}_i}) \cos(\omega_{\boldsymbol{k}_j} t + \theta_{\boldsymbol{k}_j})$$

$$\propto \sum_{\boldsymbol{k}_i, \boldsymbol{k}_j} r_{\boldsymbol{k}_i} r_{\boldsymbol{k}_j} \left[\cos\left[(\omega_{\boldsymbol{k}_i} - \omega_{\boldsymbol{k}_j}) t + (\theta_{\boldsymbol{k}_i} - \theta_{\boldsymbol{k}_j}) \right] + \cos\left[(\omega_{\boldsymbol{k}_i} + \omega_{\boldsymbol{k}_j}) t + (\theta_{\boldsymbol{k}_i} + \theta_{\boldsymbol{k}_j}) \right] \right]$$

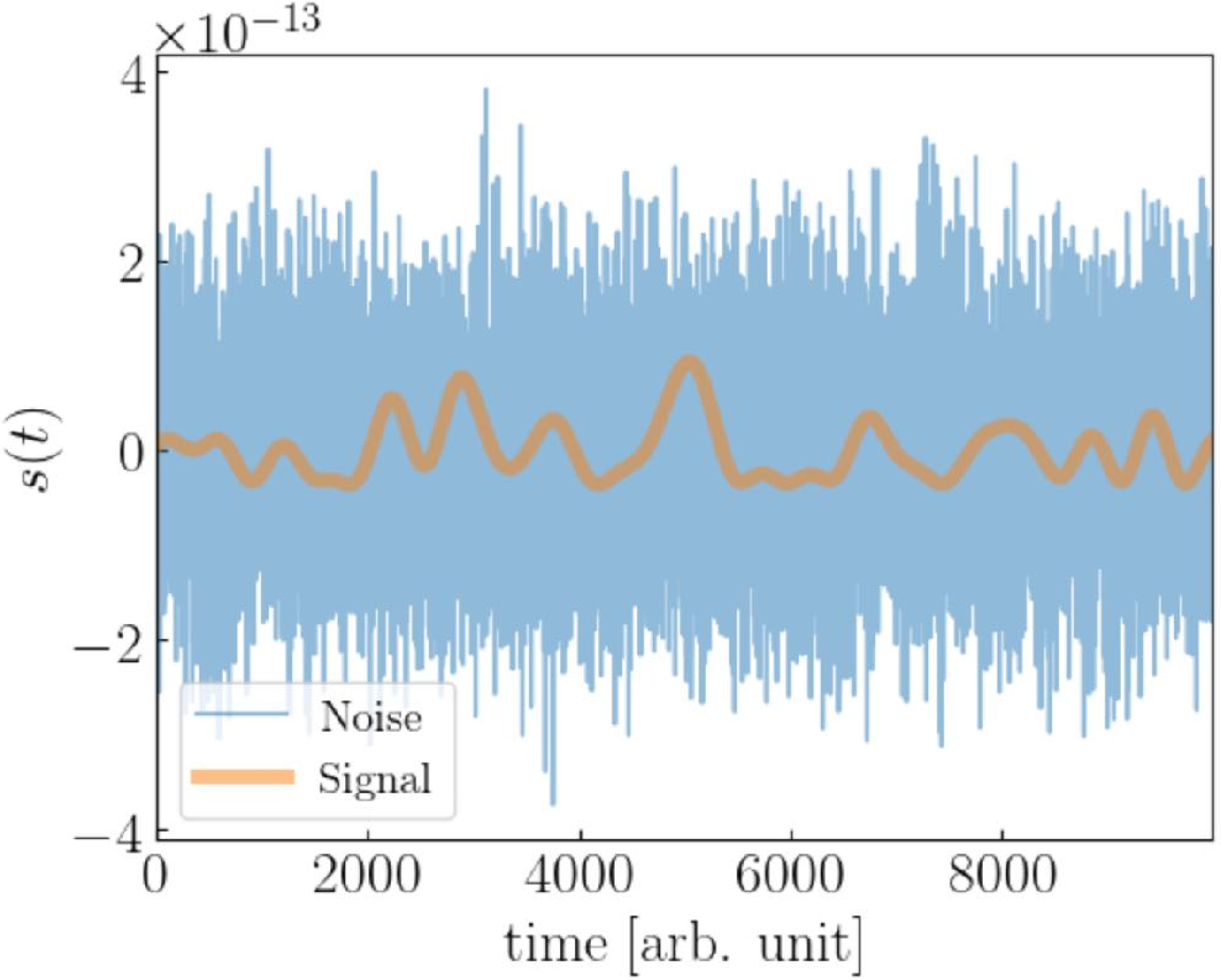
$\omega \lesssim mv^2$ — **stochastic fluctuation**

$\omega \approx 2m$ — coherent fluctuation how would stochastic fluctuations appear in the output?

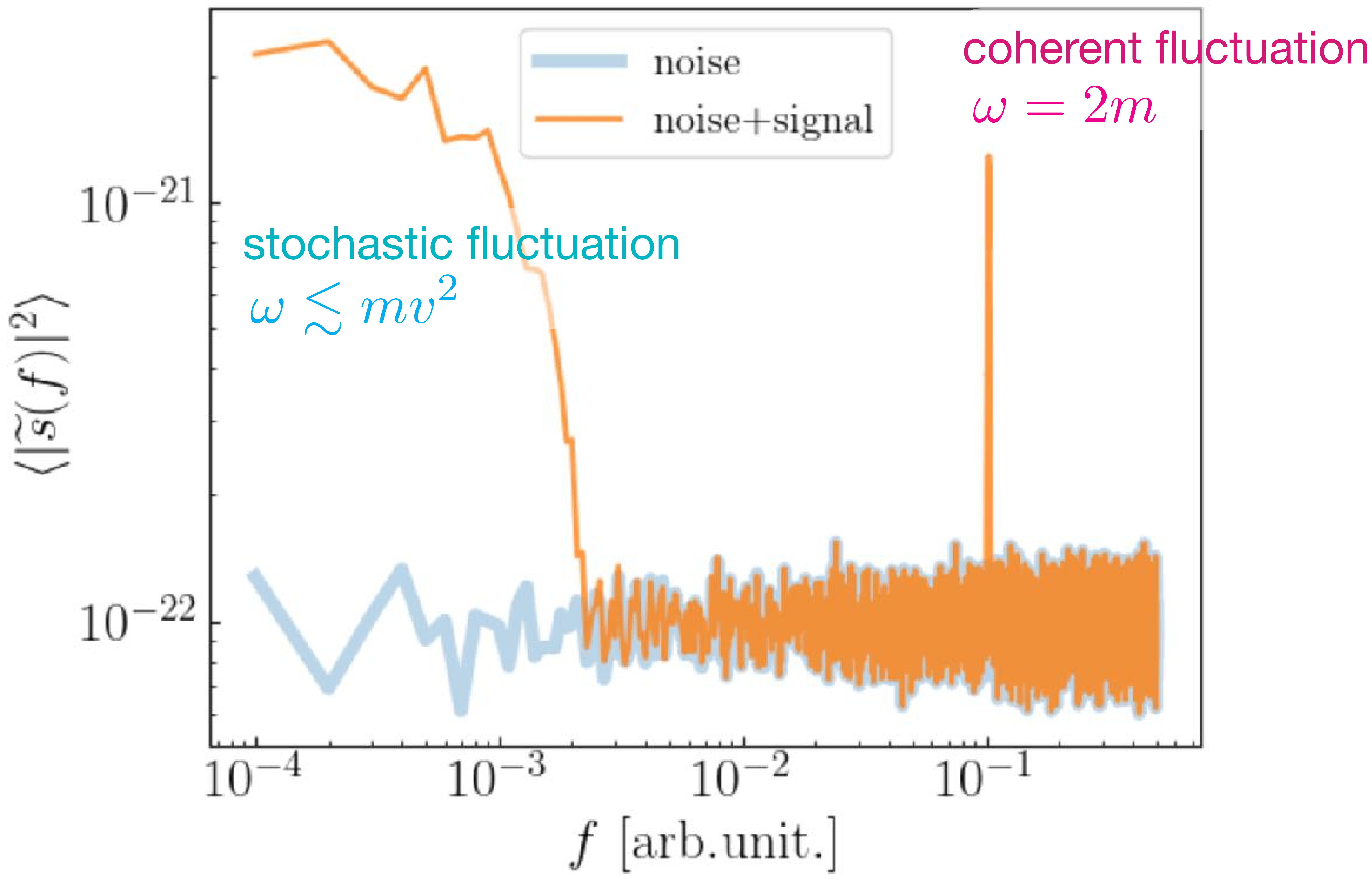
$$s(t) \propto [\phi^2(t)]_{\mathrm{sto}}$$

$$\propto \sum_{\boldsymbol{k}_i, \boldsymbol{k}_j} r_{\boldsymbol{k}_i} r_{\boldsymbol{k}_j} \cos\left[(\omega_{\boldsymbol{k}_i} - \omega_{\boldsymbol{k}_j})t + (\theta_{\boldsymbol{k}_i} - \theta_{\boldsymbol{k}_j})\right]$$

stochastic part of signal will appear



the power spectrum now would look like



suppose that a detector
is sensitive to

$$[f_l,\ f_u]$$

for coherent signals

$$f_{\rm signal} = \frac{m}{2\pi} \in [f_l,\ f_u]$$

for stochastic signals

$$f_{\rm signal} \sim \frac{mv^2}{2\pi} \in [f_l,\ f_u]$$

for coherent signals

$$f_{\rm signal} = \frac{m}{2\pi} \in [f_l,\ f_u]$$

for stochastic signals

$$f_{\rm signal} \sim \frac{mv^2}{2\pi} \in [f_l,\ f_u]$$

for a single given detector

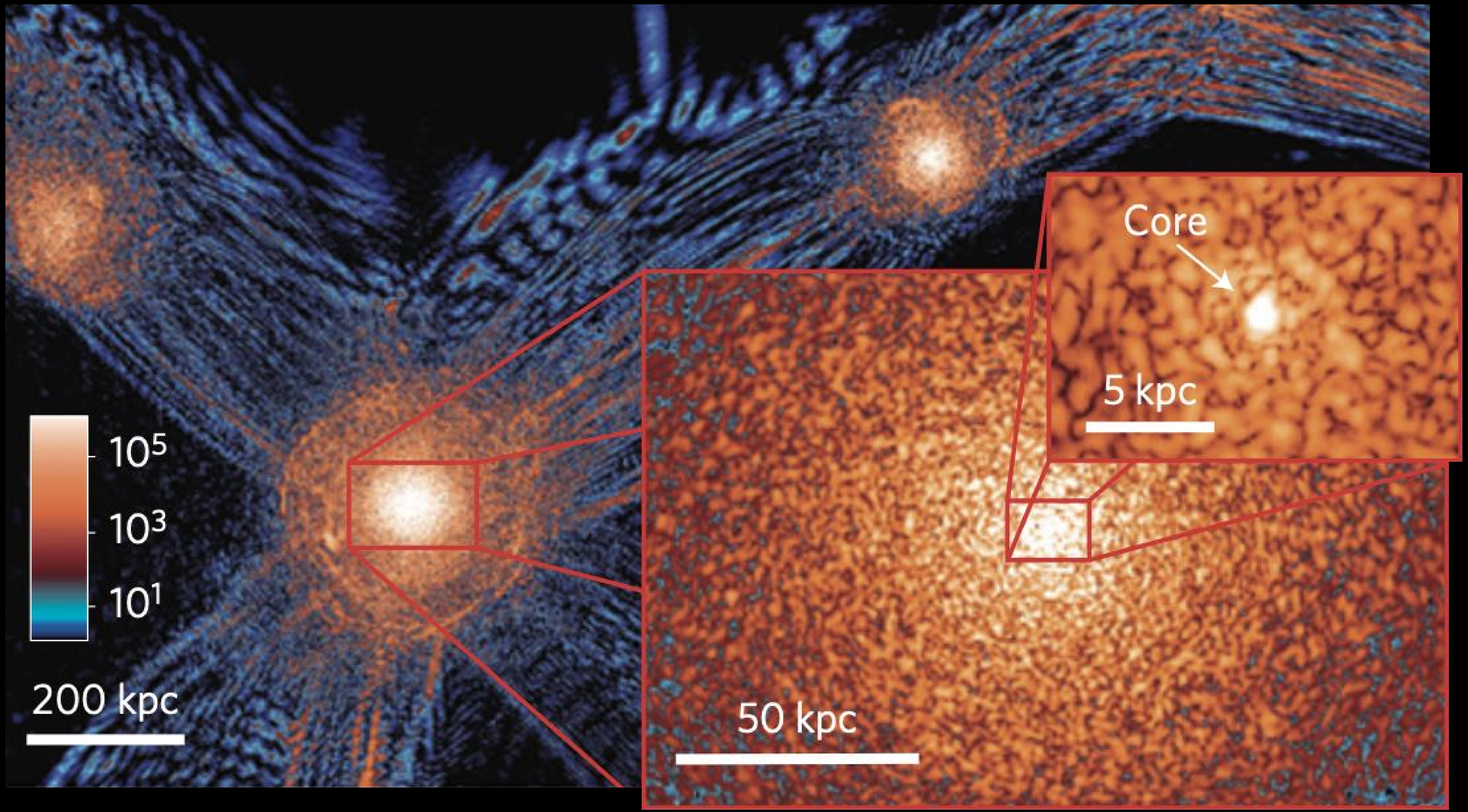
we probe a widely separated mass range simultaneously!

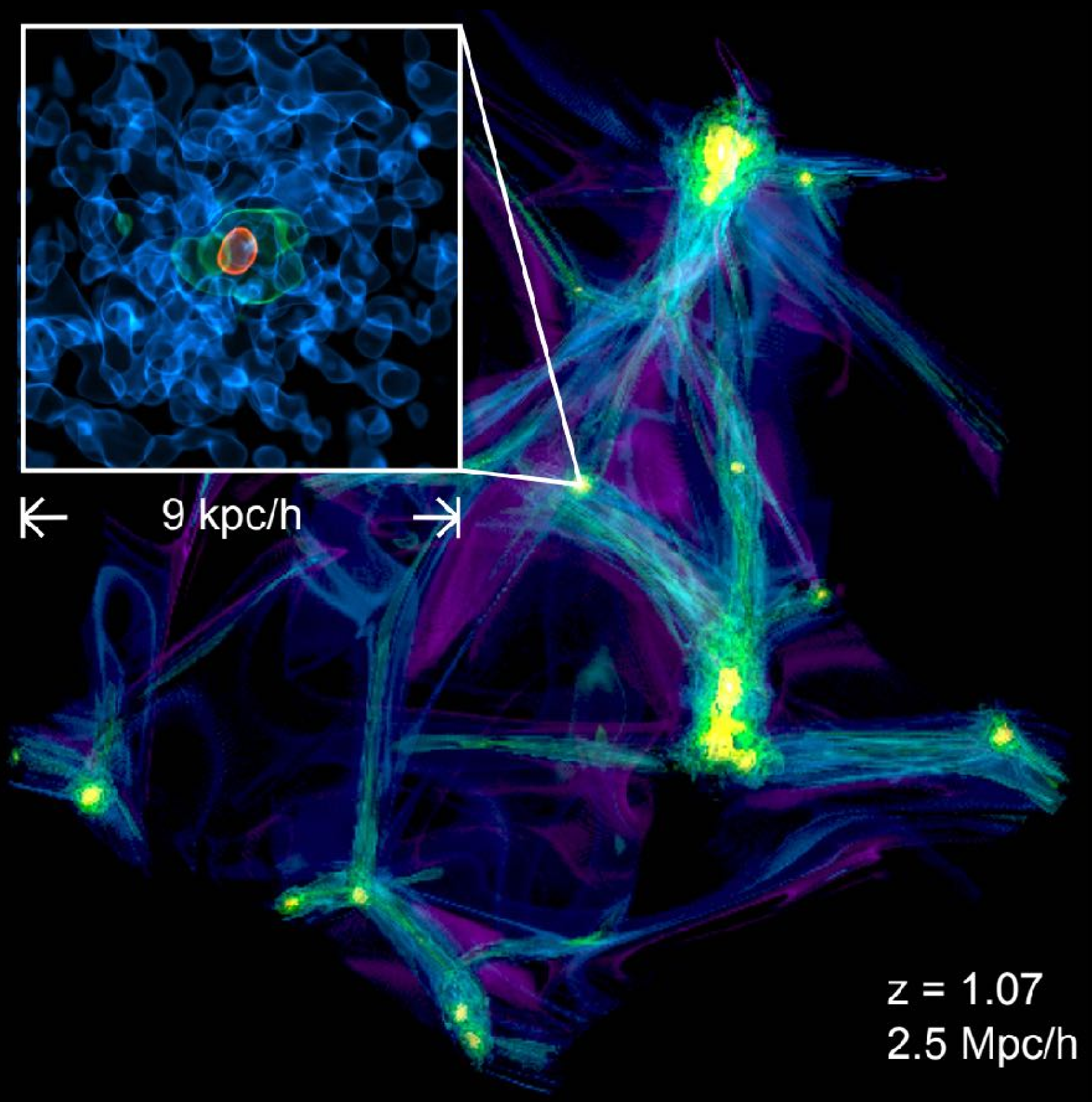
Applications

focusing on gravitational interaction

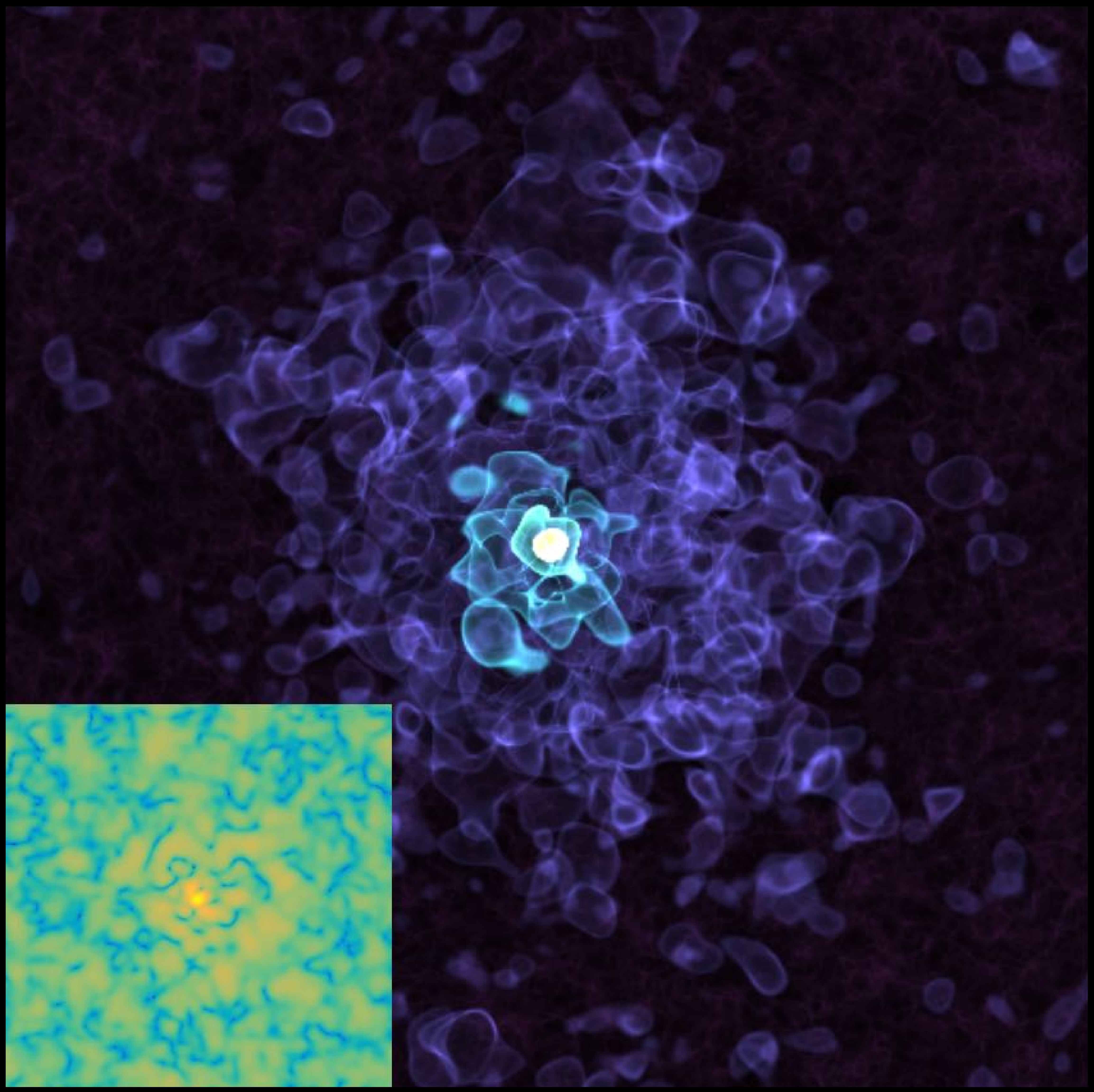
$$g_{\mu\nu} \propto T_{\mu\nu} \propto \phi^2$$

Schive, Chiueh, Broadhurst (14)



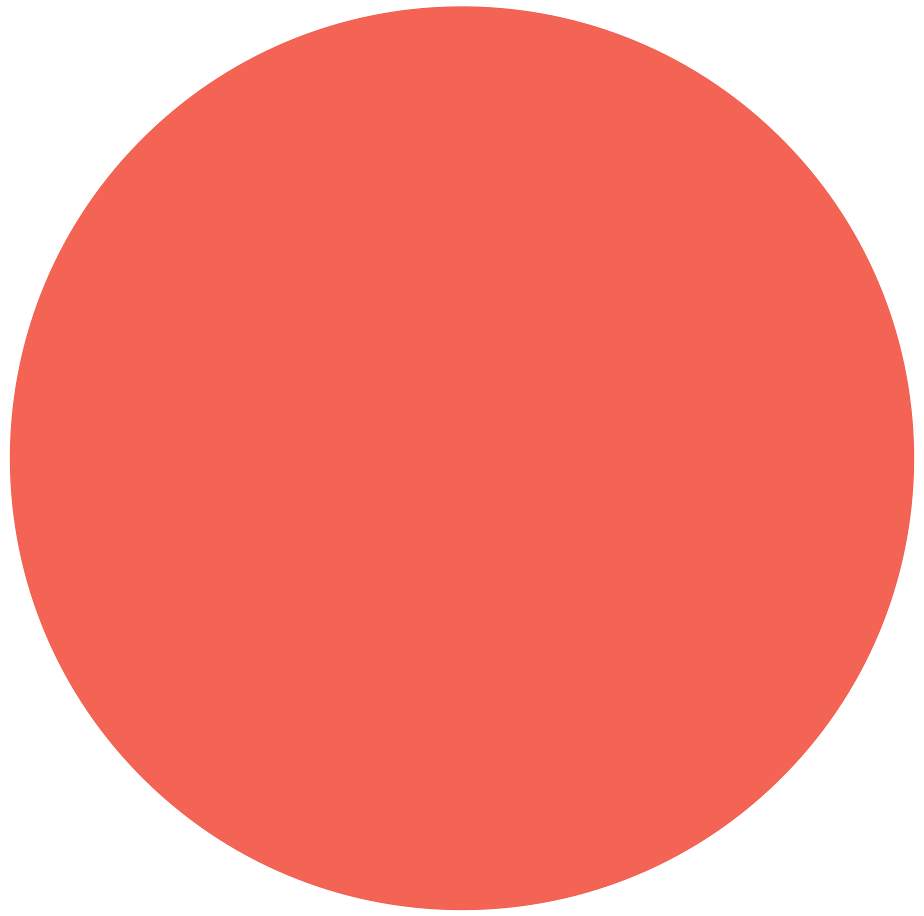


Veltmaat, Niemeyer, Schwabe (18)



An intuitive understanding of the granule structure:

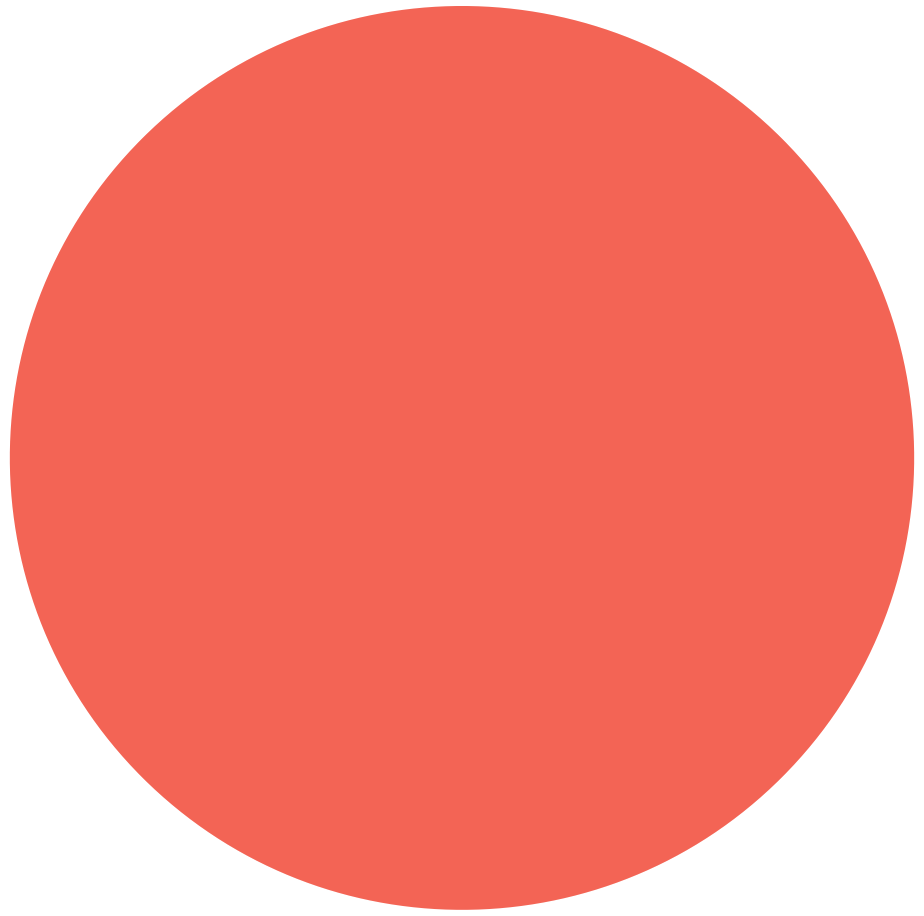
Quasiparticle



$$\ell \sim \lambda = \frac{1}{mv}$$

$$m_{\rm eff} \sim \rho_{\rm DM} \ell^3$$

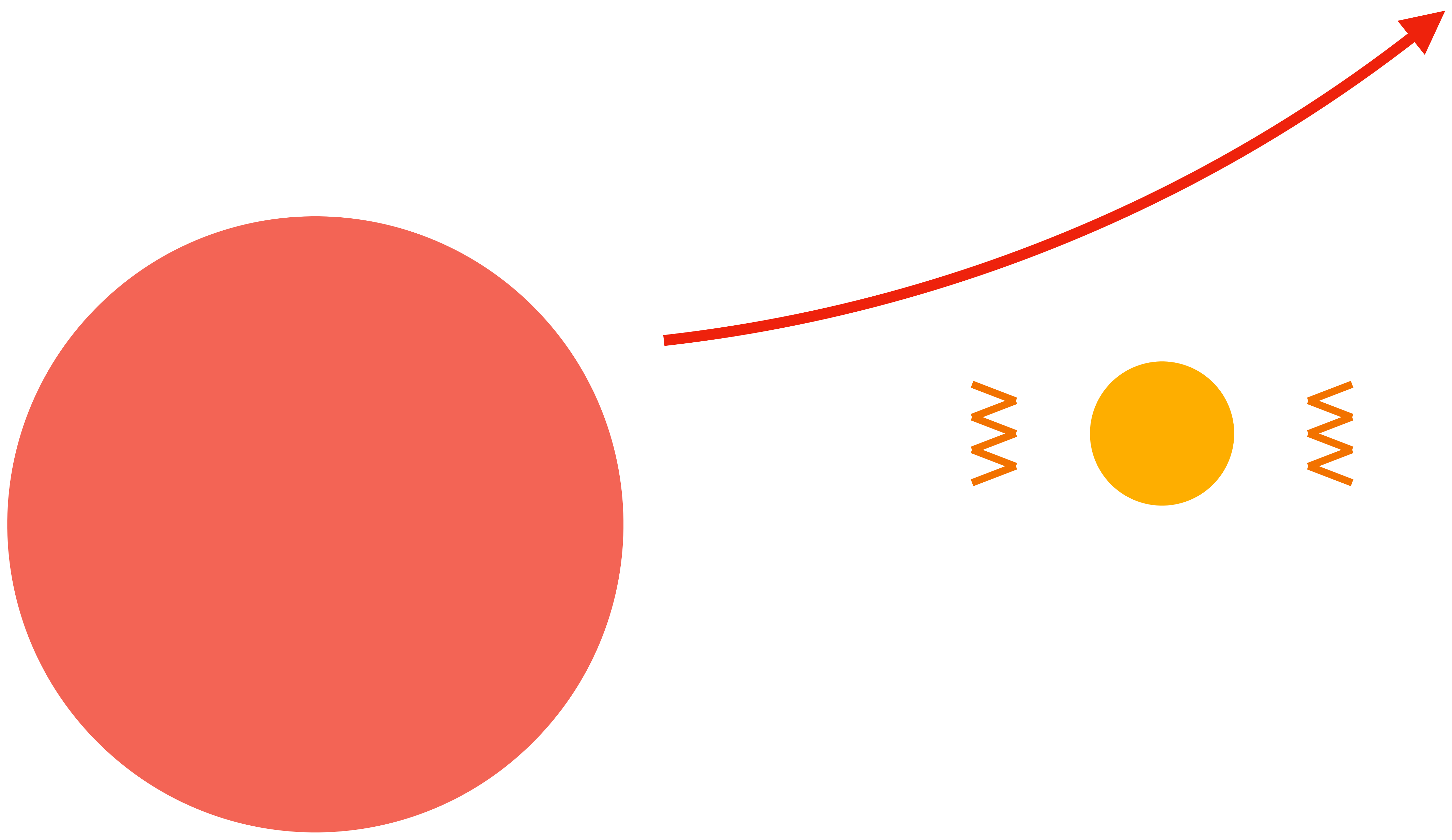
the size and mass of them could be astronomical



$$\ell \sim \lambda = \frac{1}{mv} \sim 10\,\mathrm{AU} \times \left(\frac{10^{-16}\,\mathrm{eV}}{m}\right)$$

$$m_{\mathrm{eff}} \sim \rho_{\mathrm{DM}} \ell^3 \sim 10^{15}\,\mathrm{kg} \times \left(\frac{10^{-16}\,\mathrm{eV}}{m}\right)^3$$

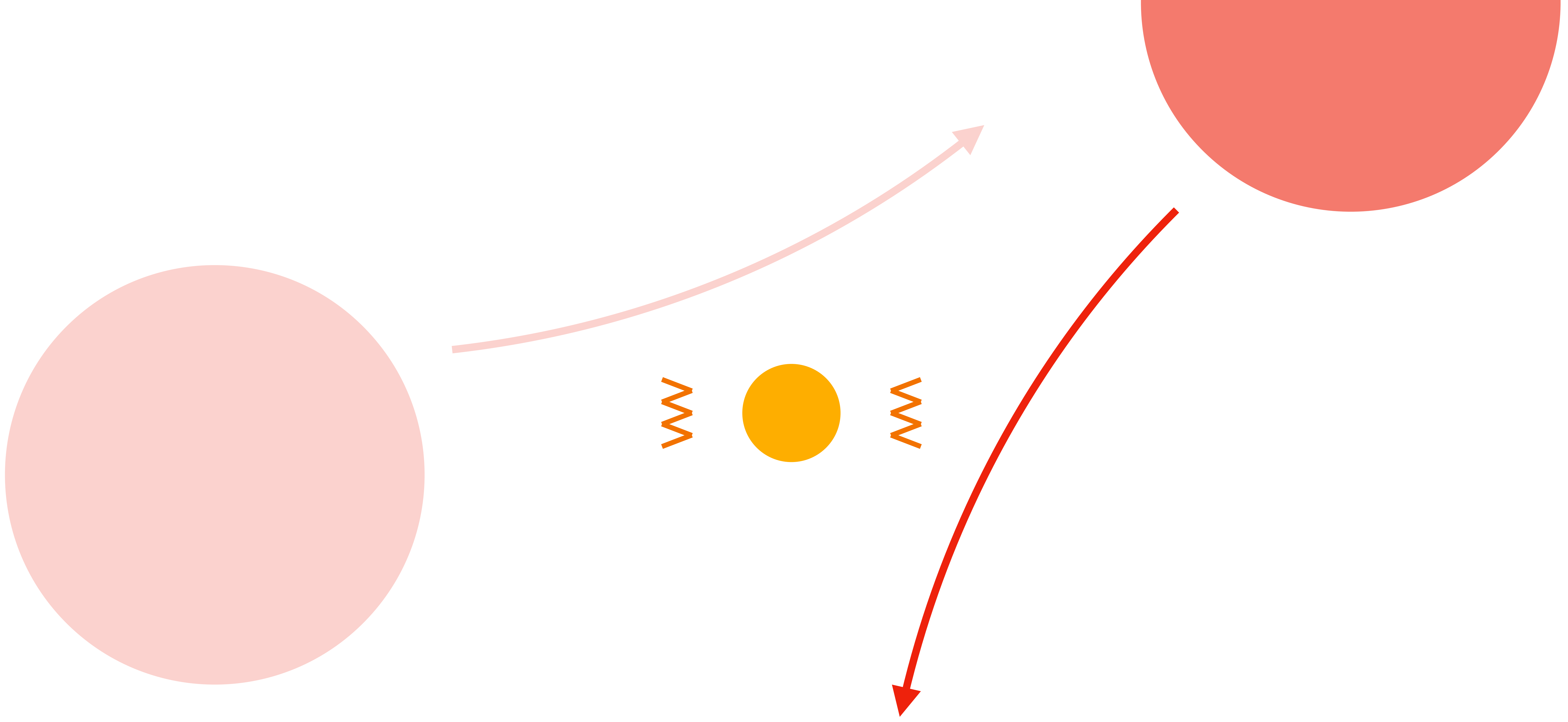
being that massive
it may engage in interaction with stars
and significantly perturb the motion of them

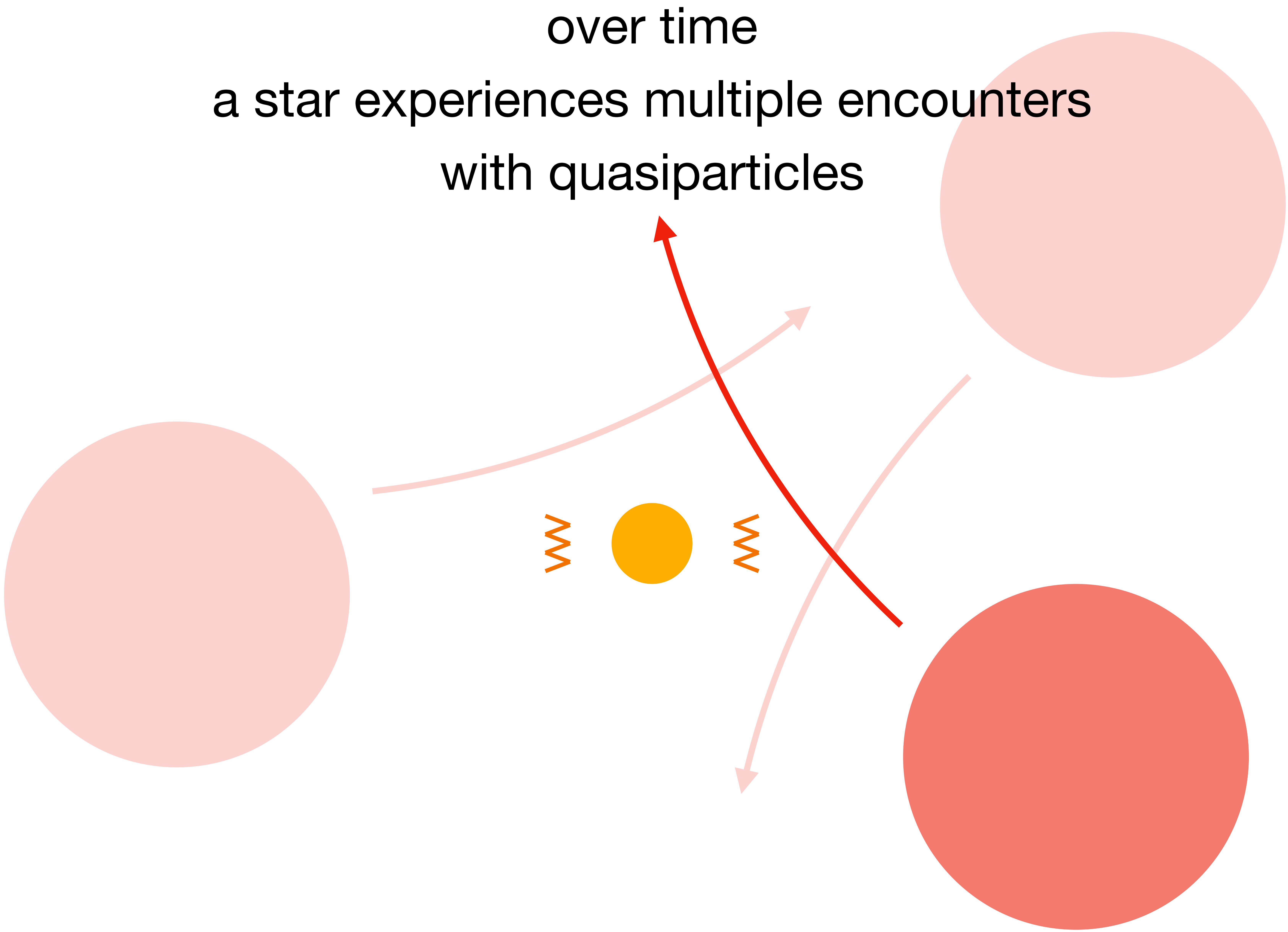


over time

a star experiences multiple encounters

with quasiparticles




over time
a star experiences multiple encounters
with quasiparticles

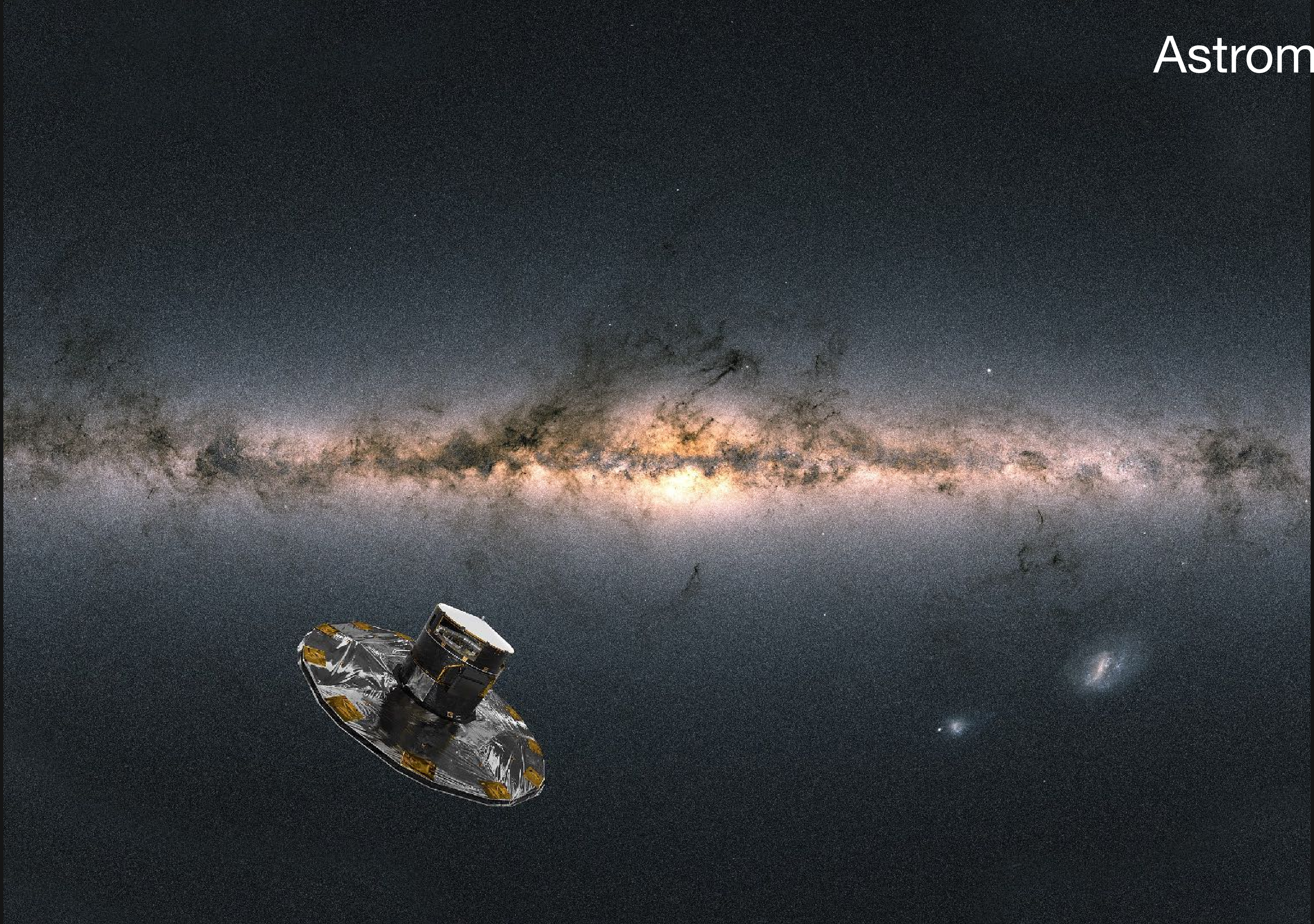
so what?

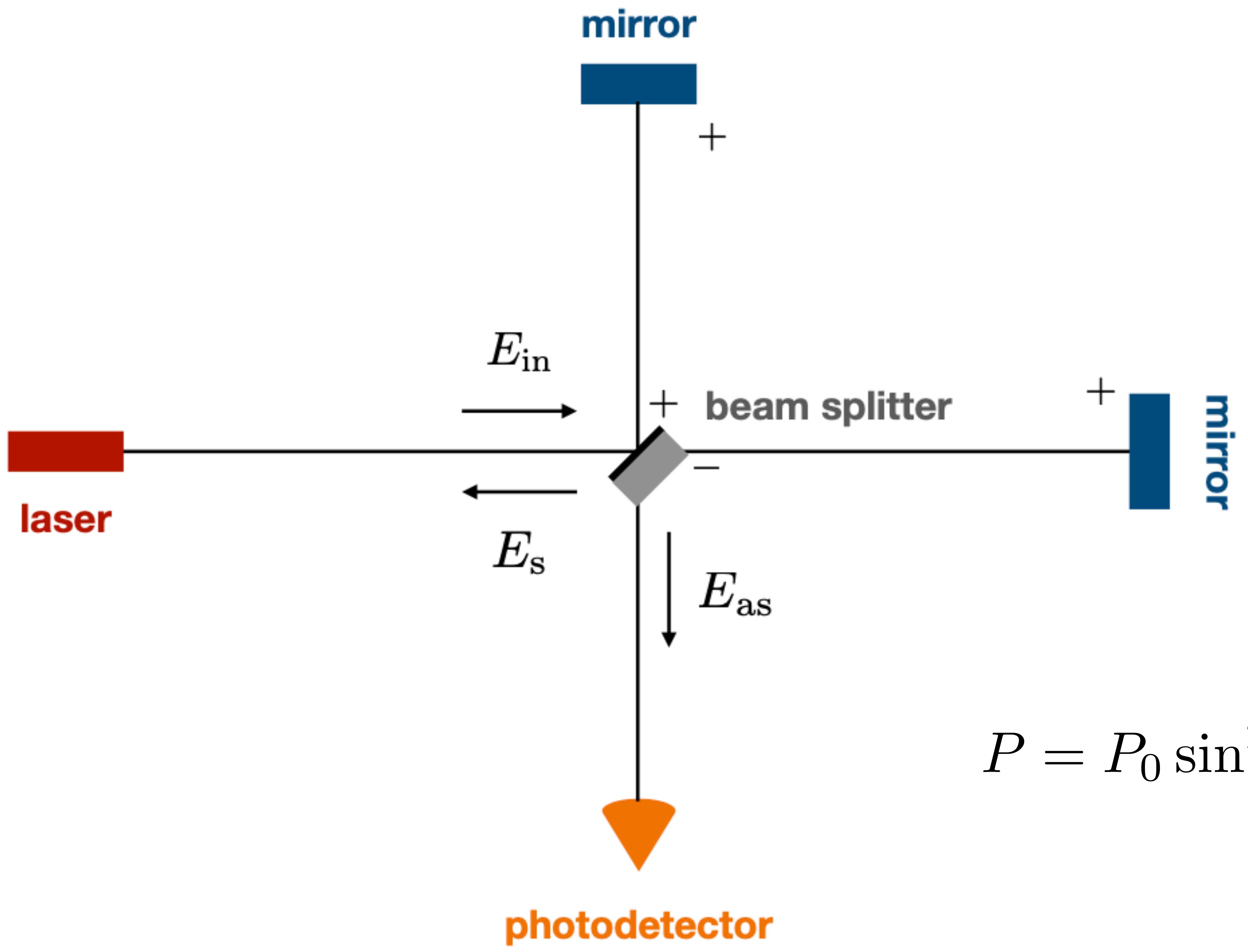
quasiparticles *bombards*
normal matters, leaving *distinctive stochastic signals*
in *gravitational wave detectors*


Interferometry

Pulsar Timing Array

Astrometry

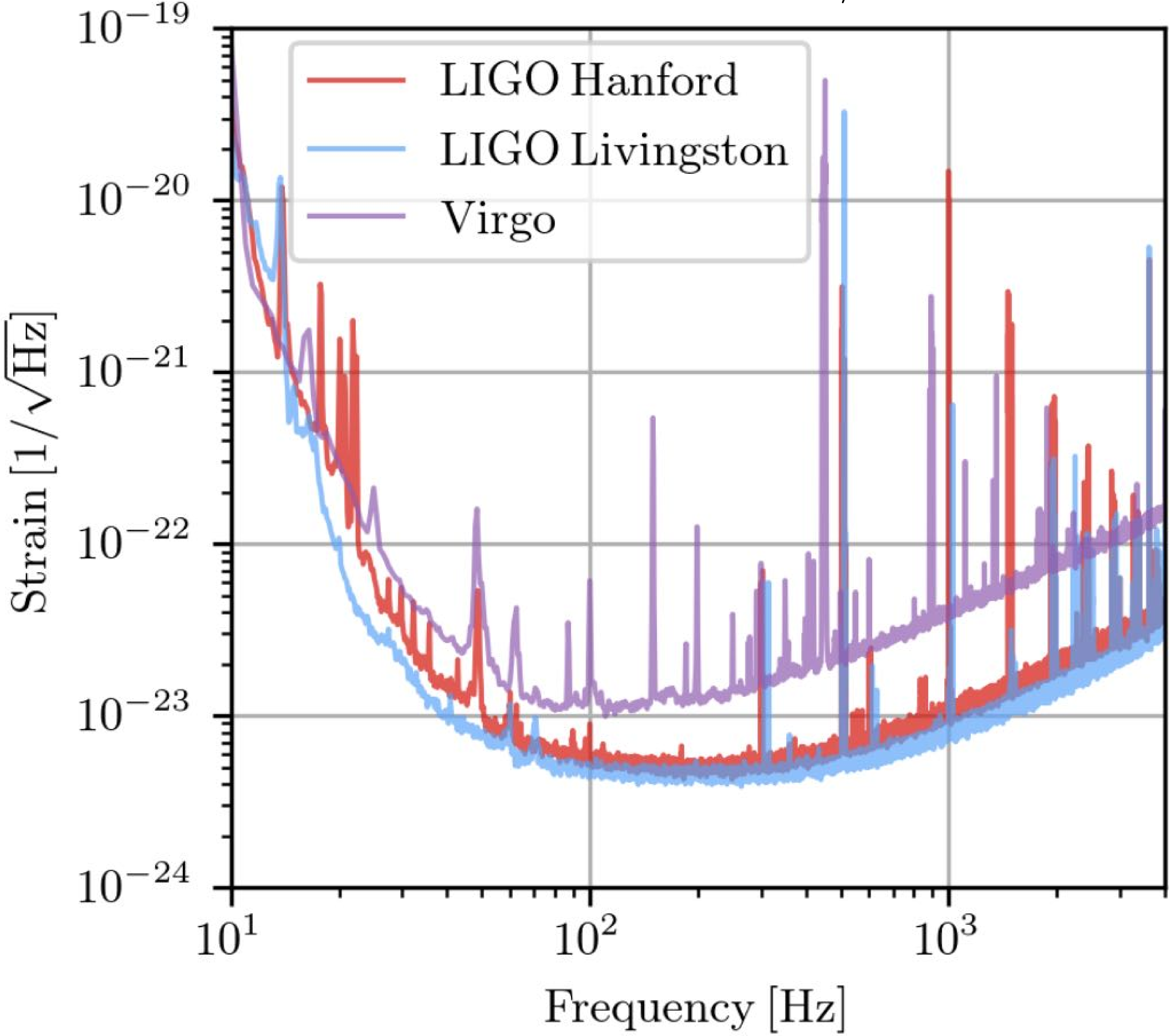


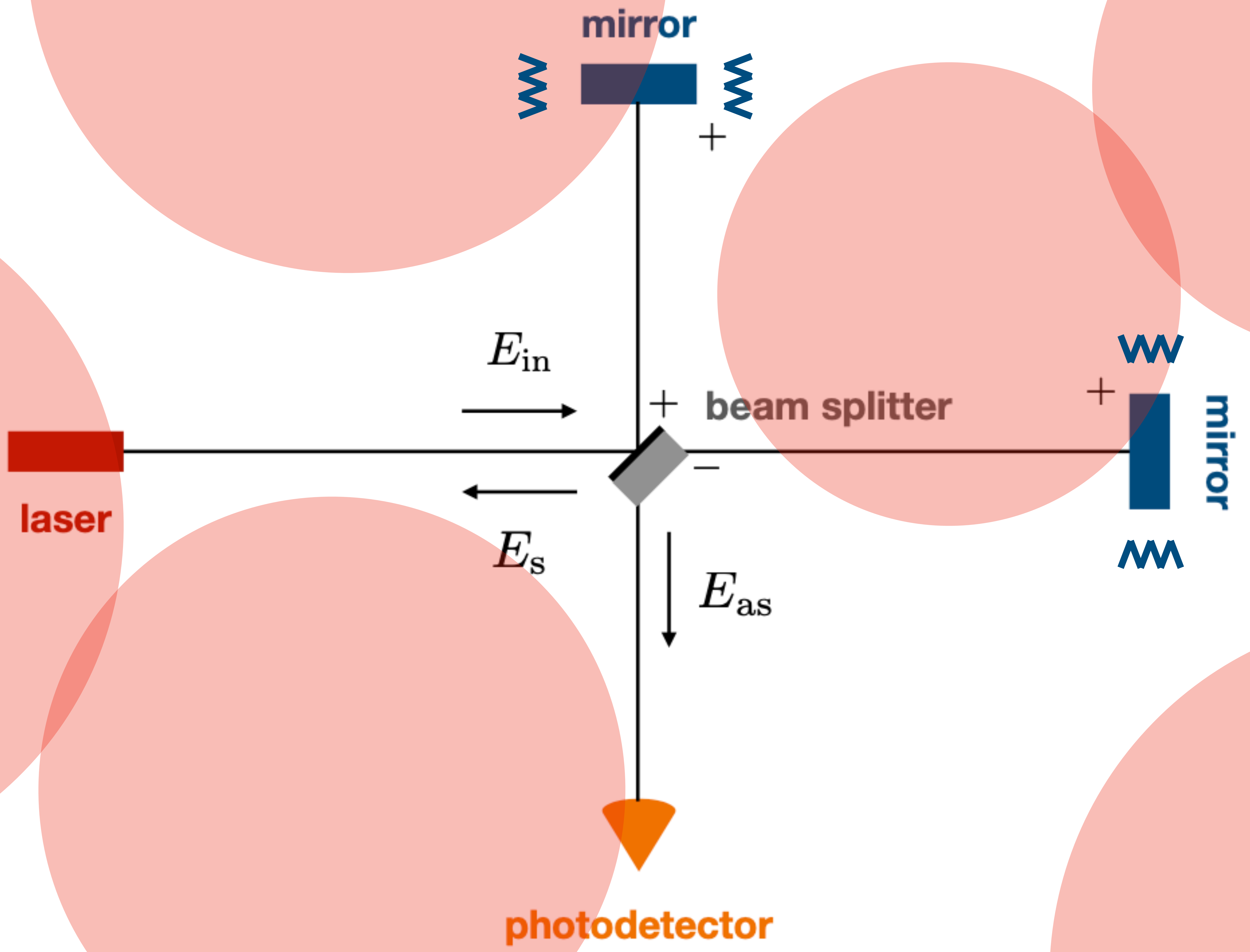


$$P = P_0 \sin^2(k\Delta L)$$

$$S_n^{1/2}(f) \sim S_{\Delta L/L}^{1/2}(f)$$

$$\langle x^2 \rangle = \int df\, S_x(f)$$




mirror
E_{in}
beam splitter
mirror
laser
E_{s}
E_{as}
photodetector

can we actually measure
ULDM signals with GW interferometers?

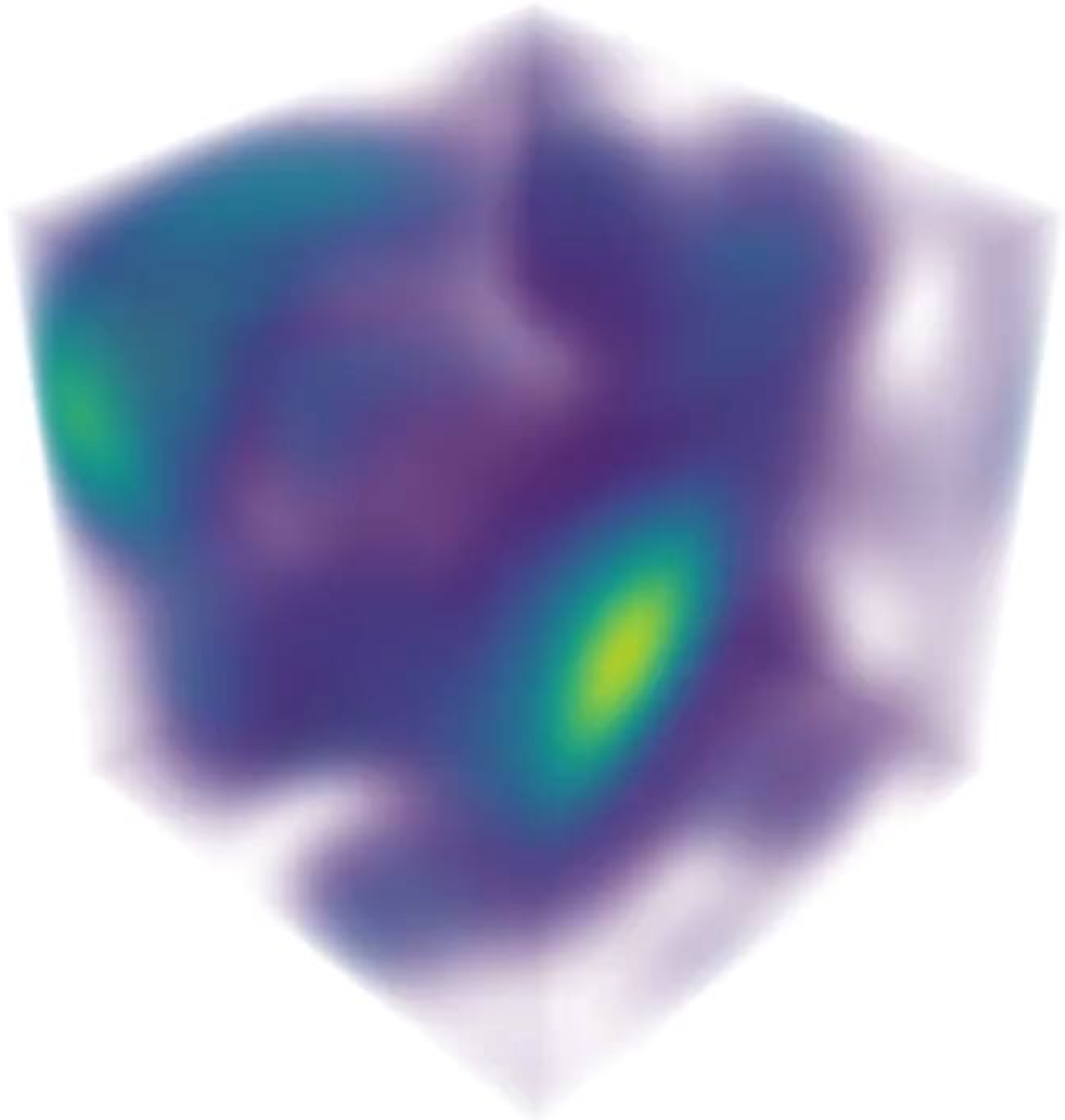
$$\ddot{x} = -\nabla\Phi$$

$$\nabla^2\Phi = 4\pi G\rho$$

what is reflected in *detector observables*

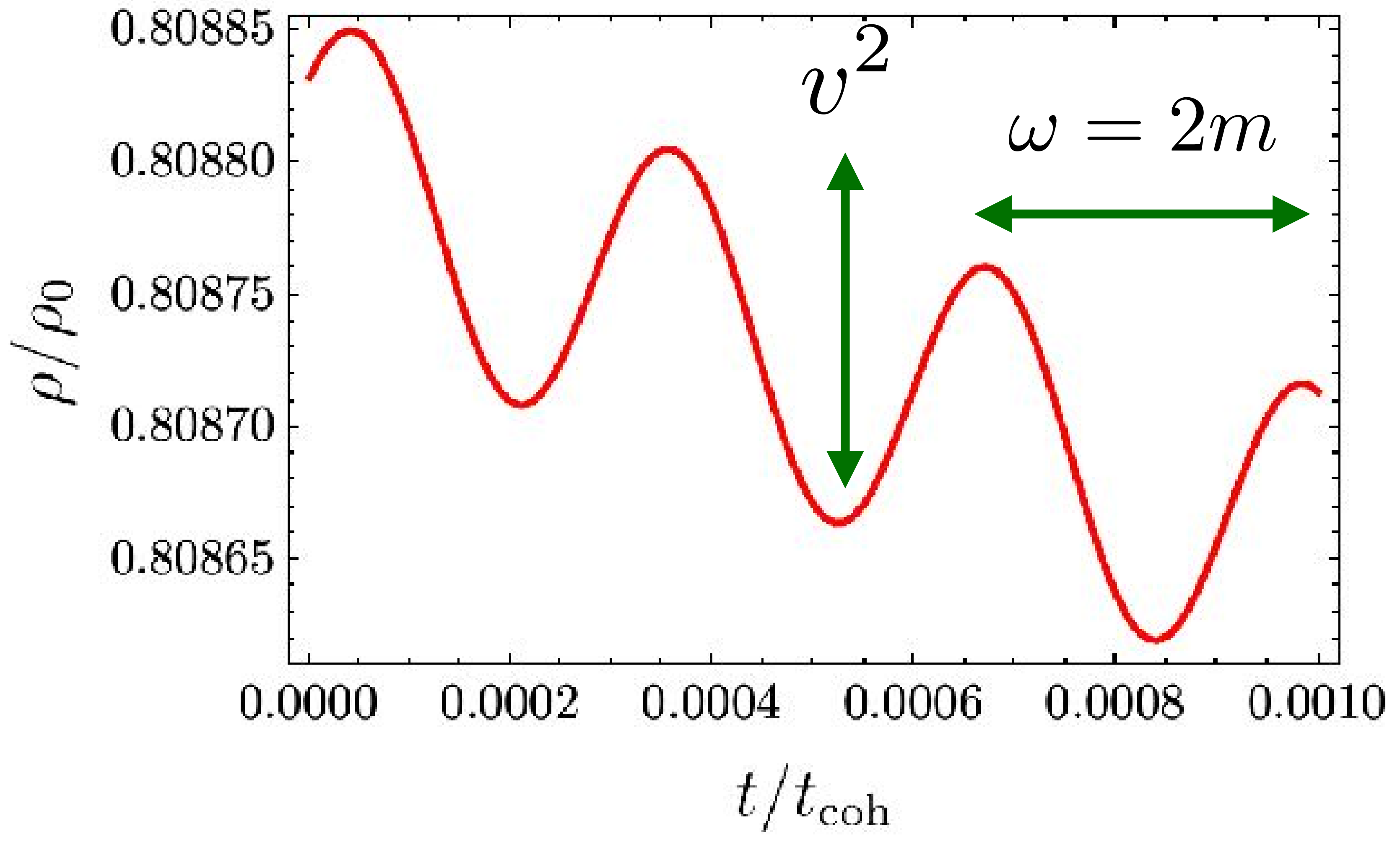
is the *statistical properties* of

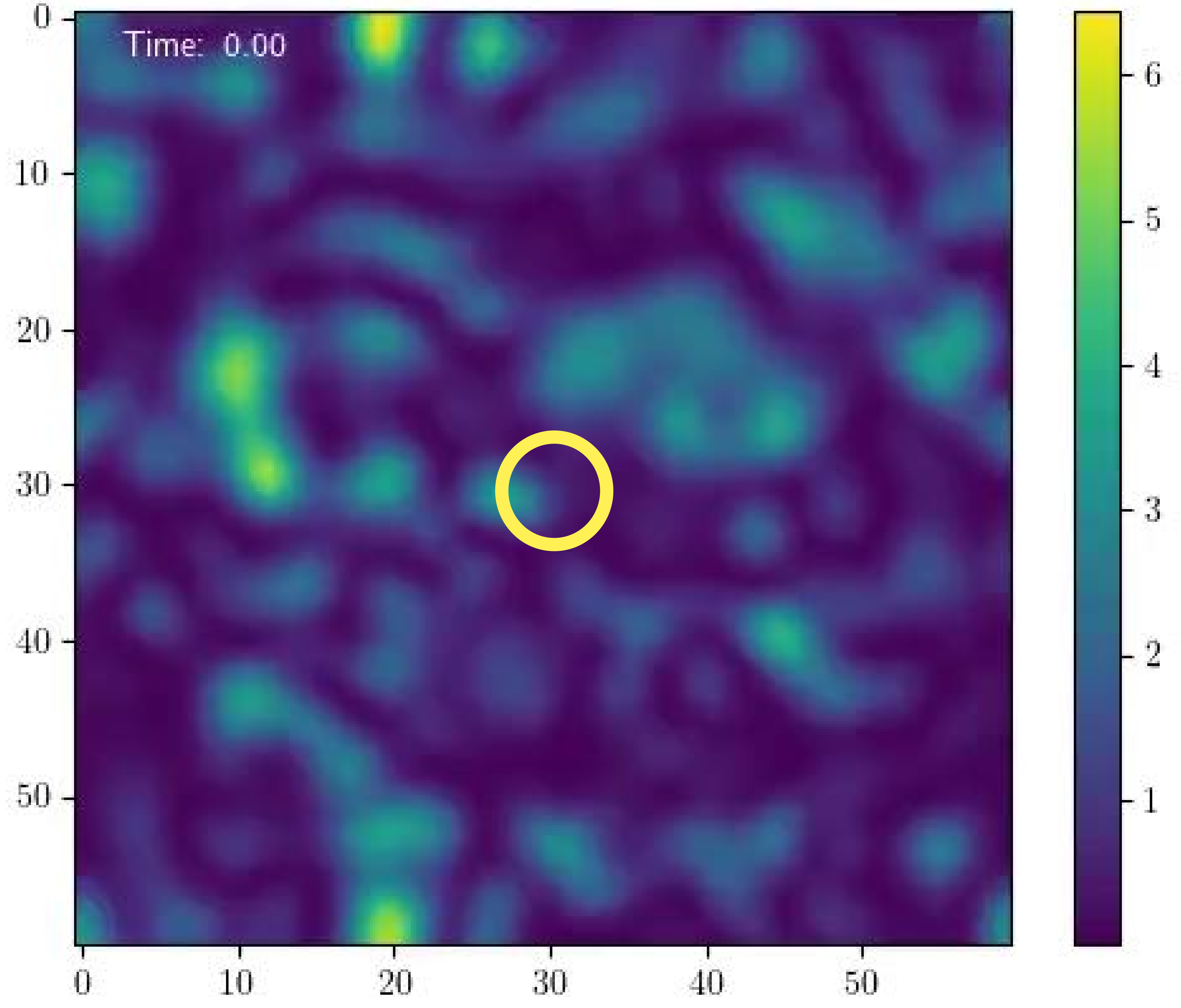
density fluctuations of ULDM

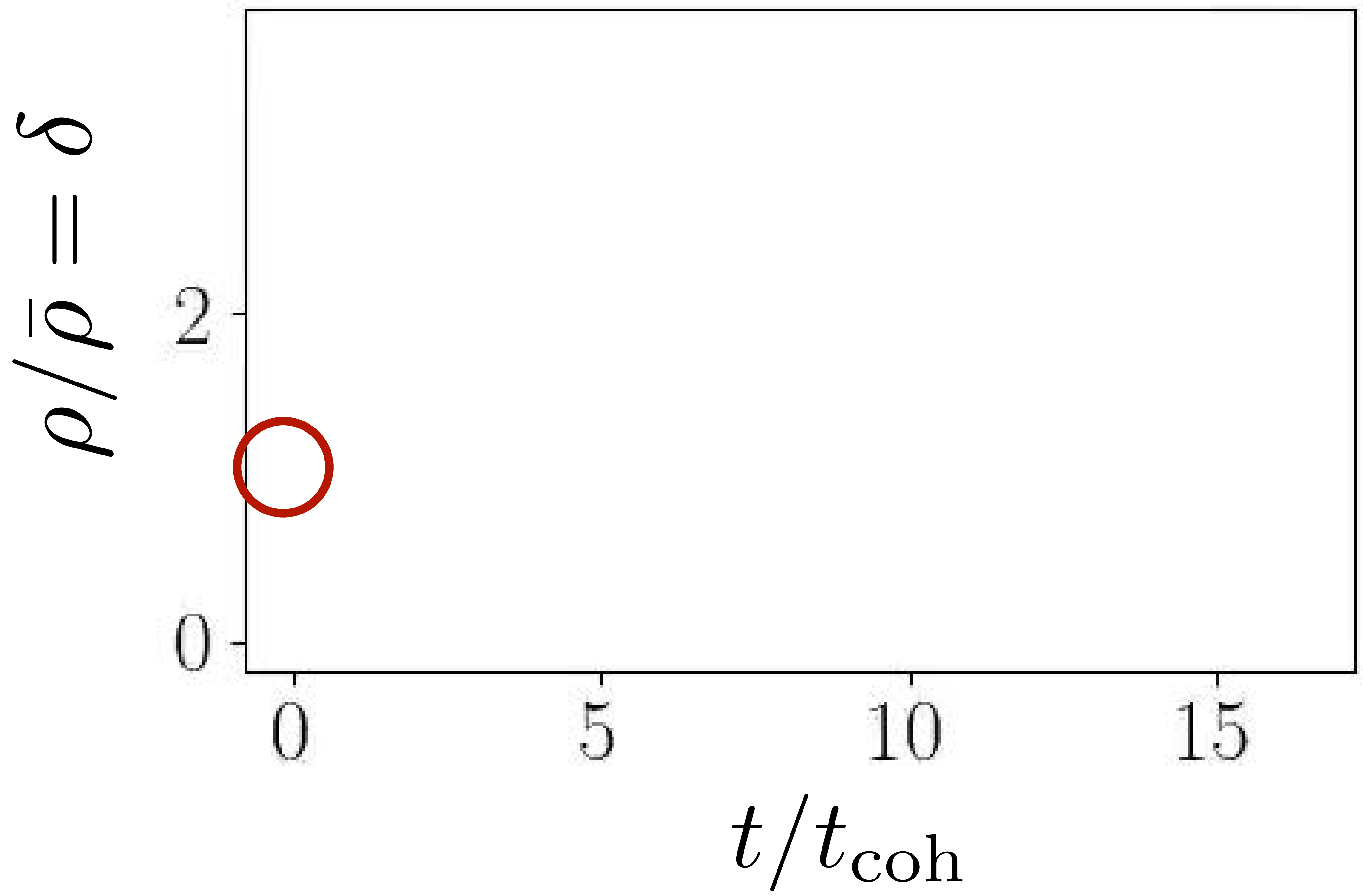


the density-density correlator at the same position is

$$\langle \delta(x)\delta(x)\rangle = \int \frac{d\omega}{2\pi} S_\delta(\omega)$$







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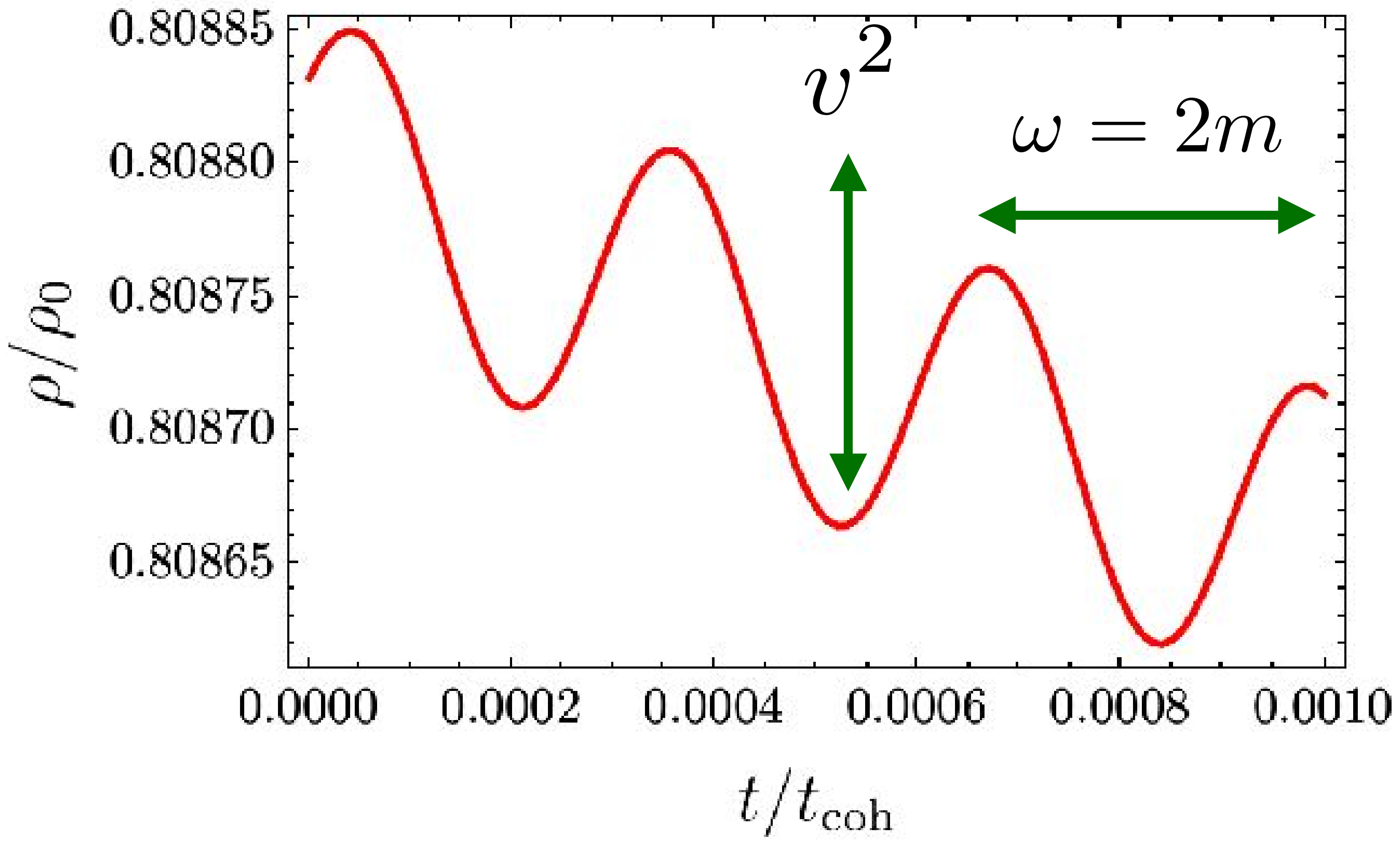
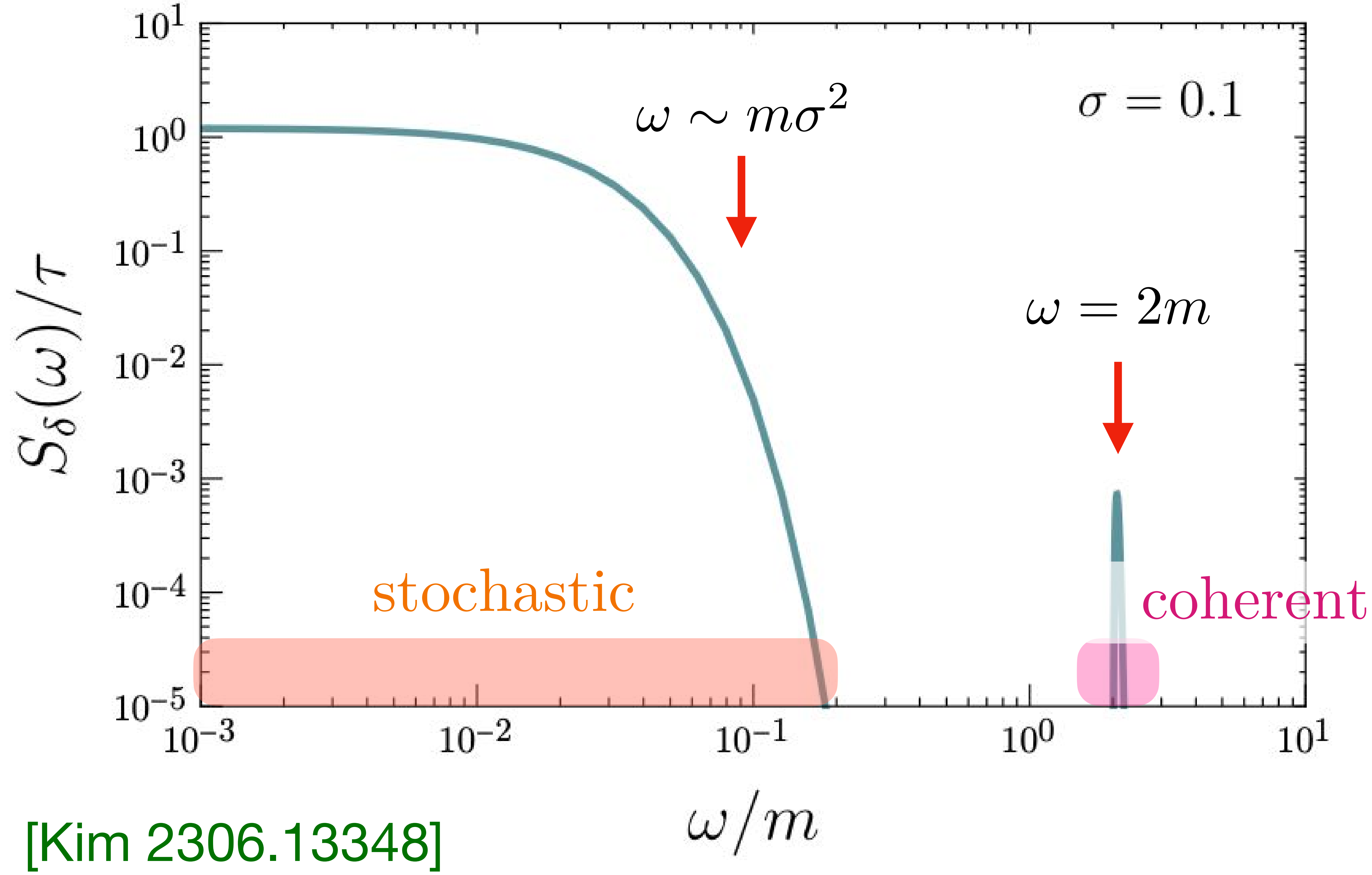
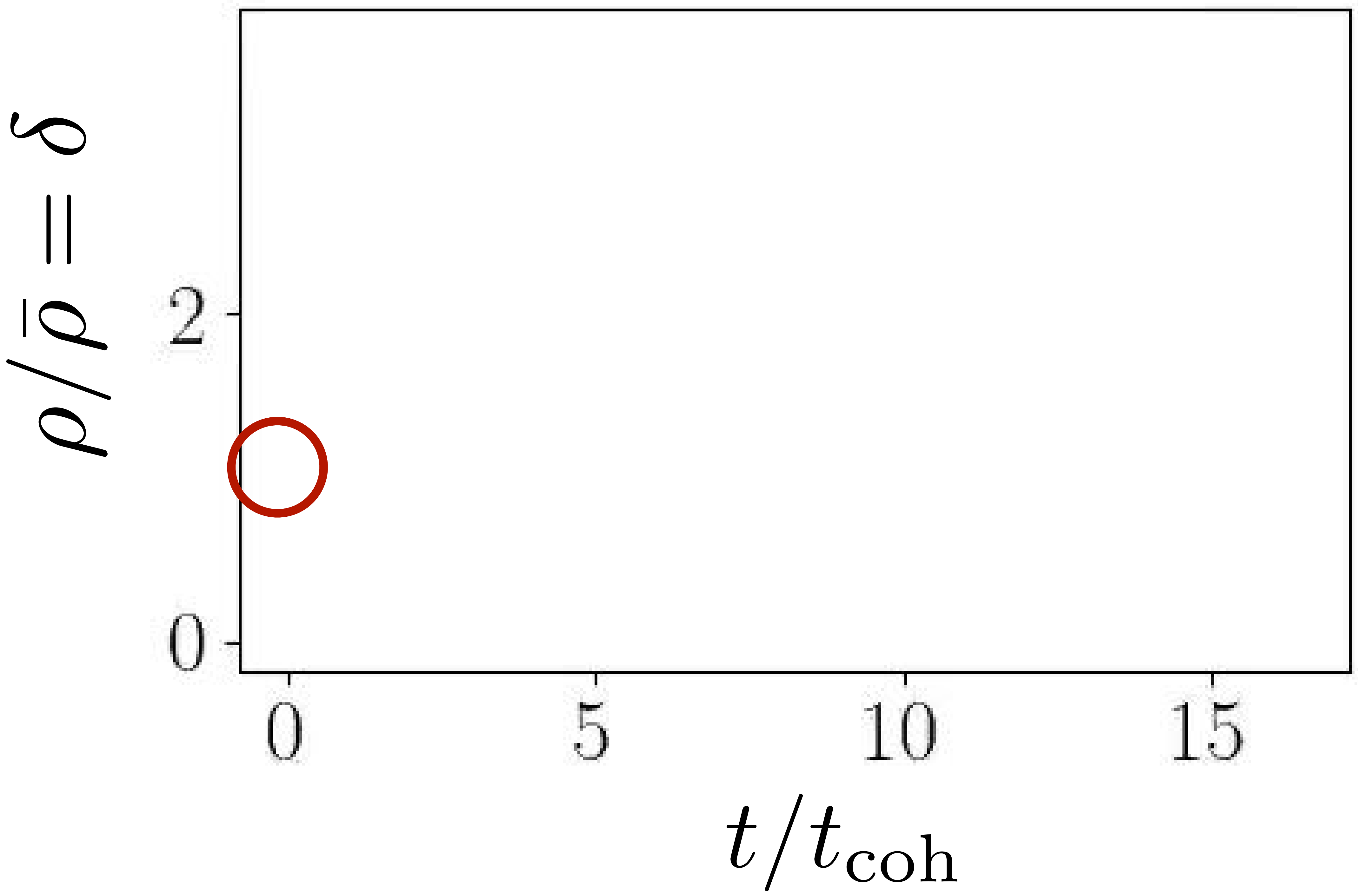


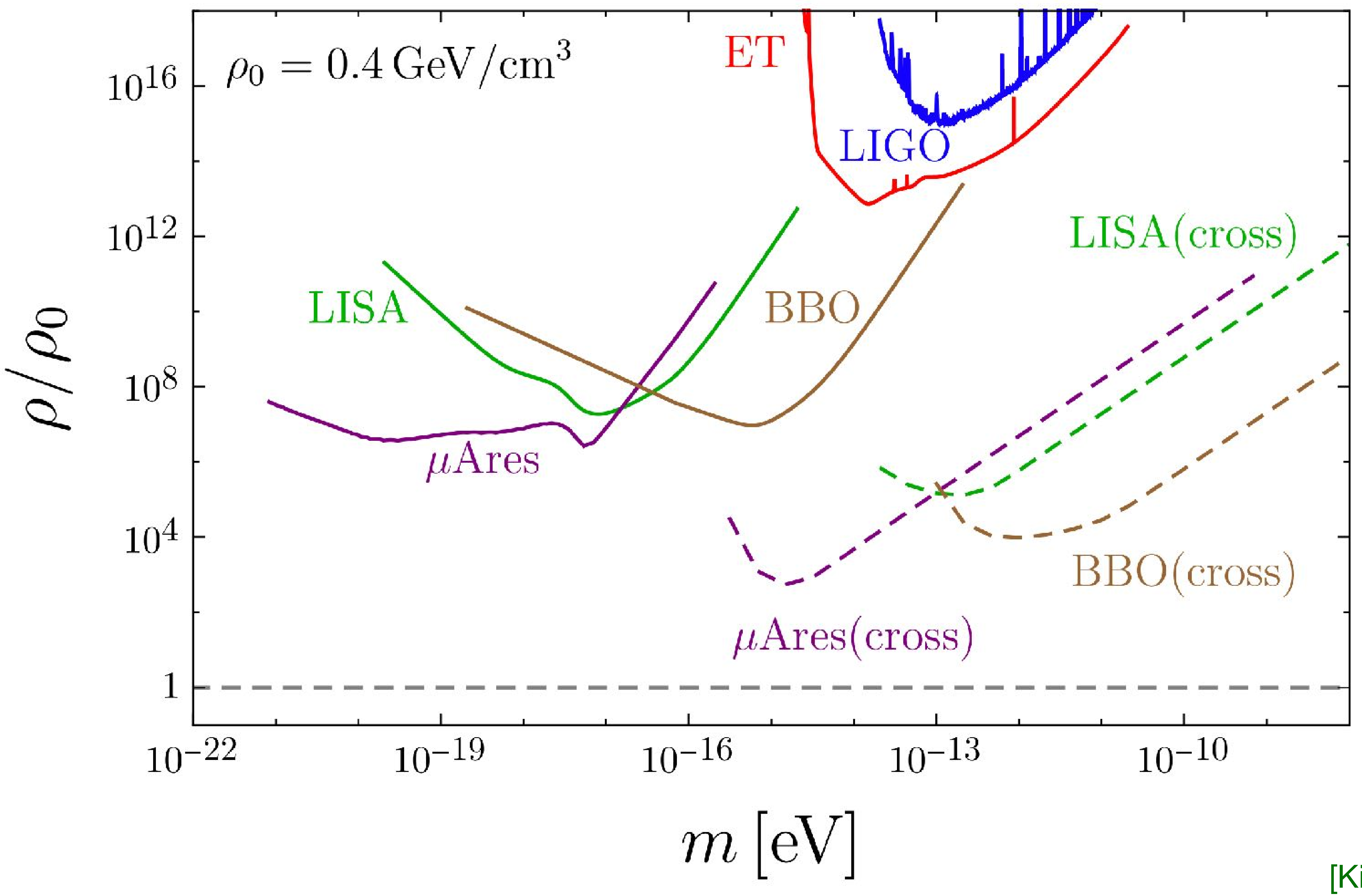
$$S_\delta(\omega) = \tau[\sigma^4 A_\delta(\omega) + B_\delta(\omega)]$$





[Kim 2306.13348]

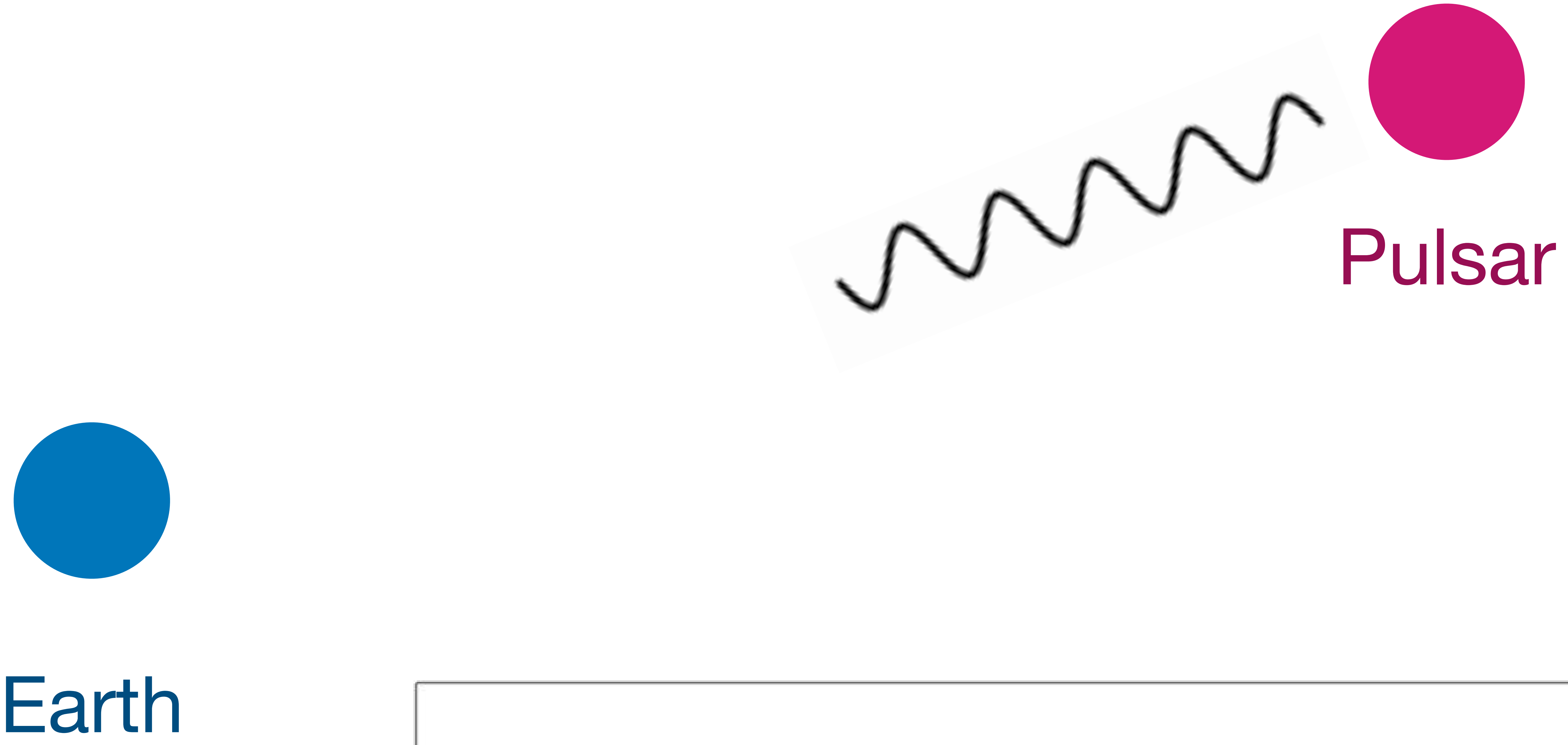
[Kim, Lenoci, Perez, Ratzinger, 2307.14962]

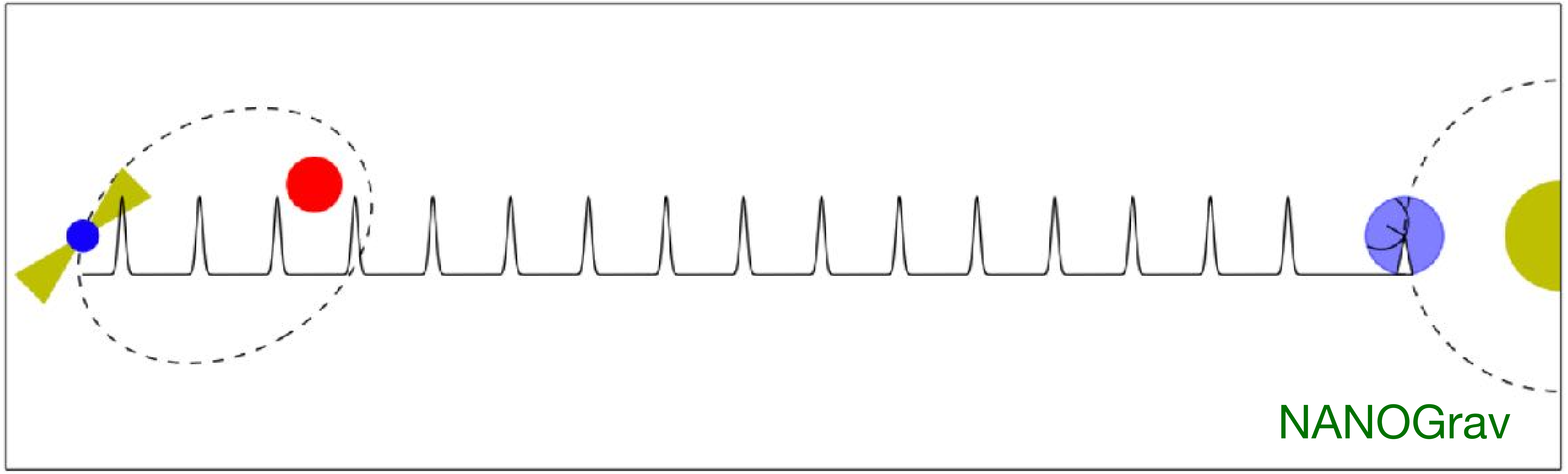


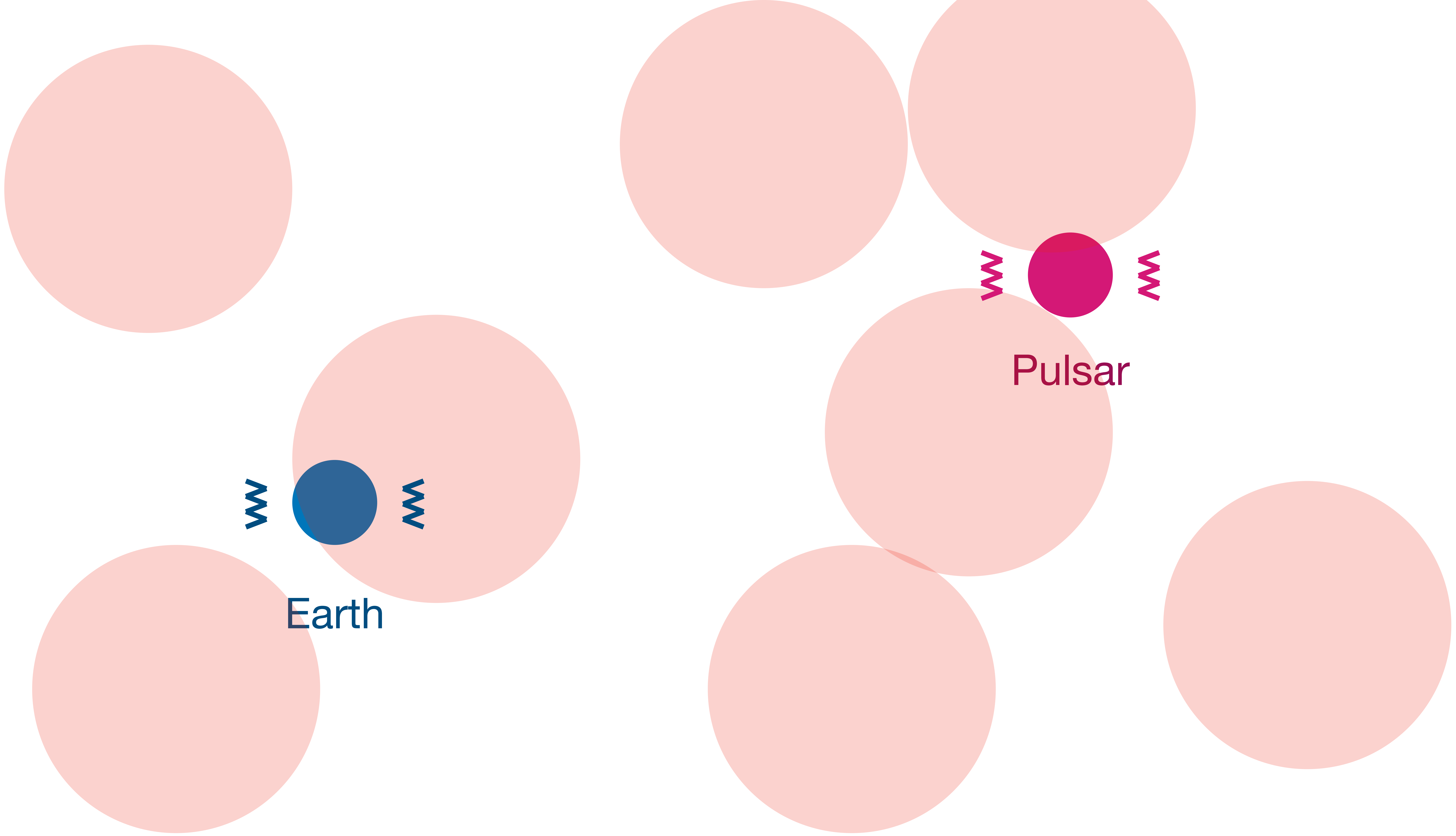
[Kim 2306.13348]

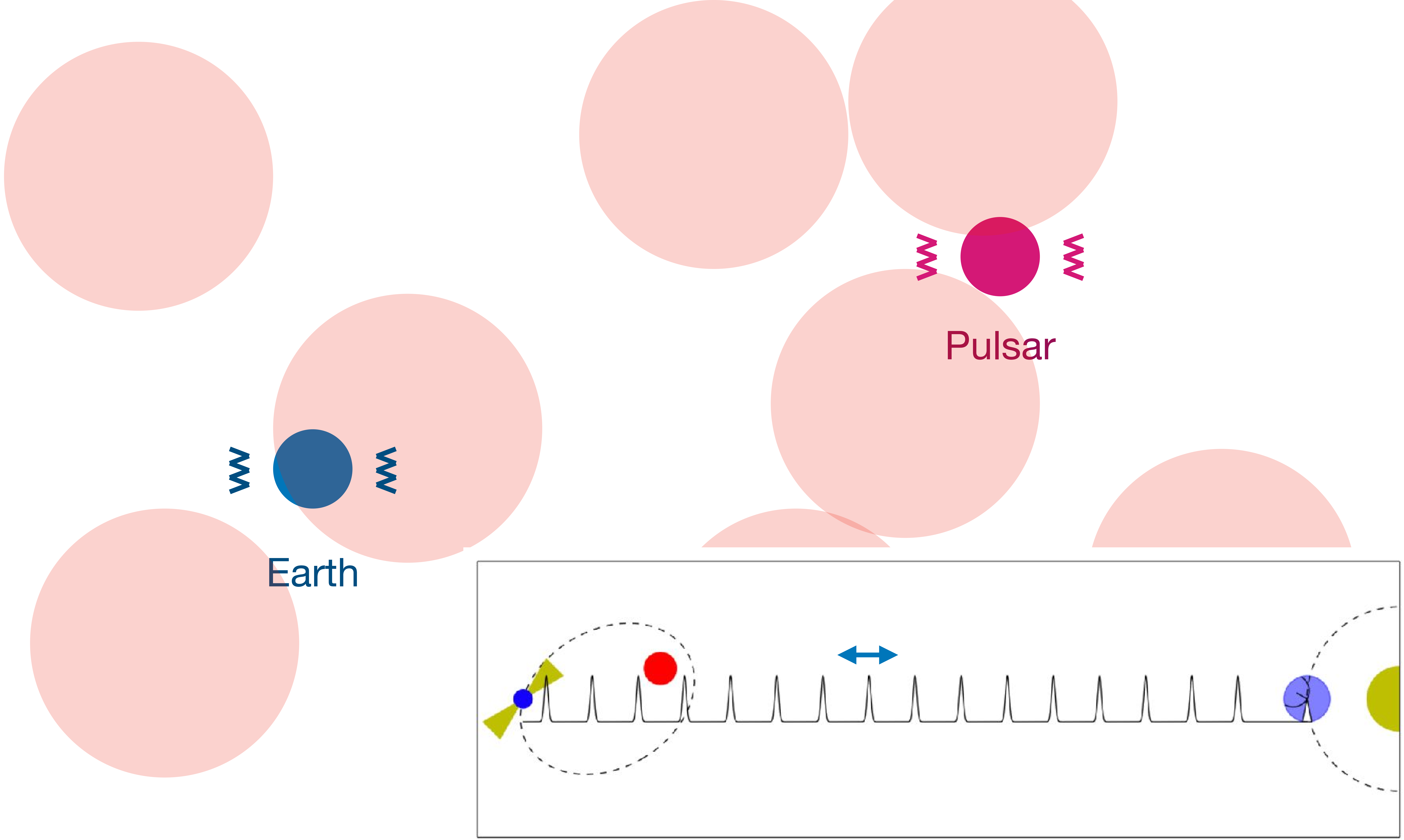
another example:

Pulsar Timing Array


Pulsar
Earth


NANOGrav


Pulsar
Earth



Pulsar
Earth

ultralight dark matter signal is characterised by

spectrum and ***correlation***

$$\langle \delta t_a \delta t_b \rangle = \int df \, \Gamma_{ab}^{\mathrm{ULDM}} S_{\delta t}^{\mathrm{ULDM}}(f)$$

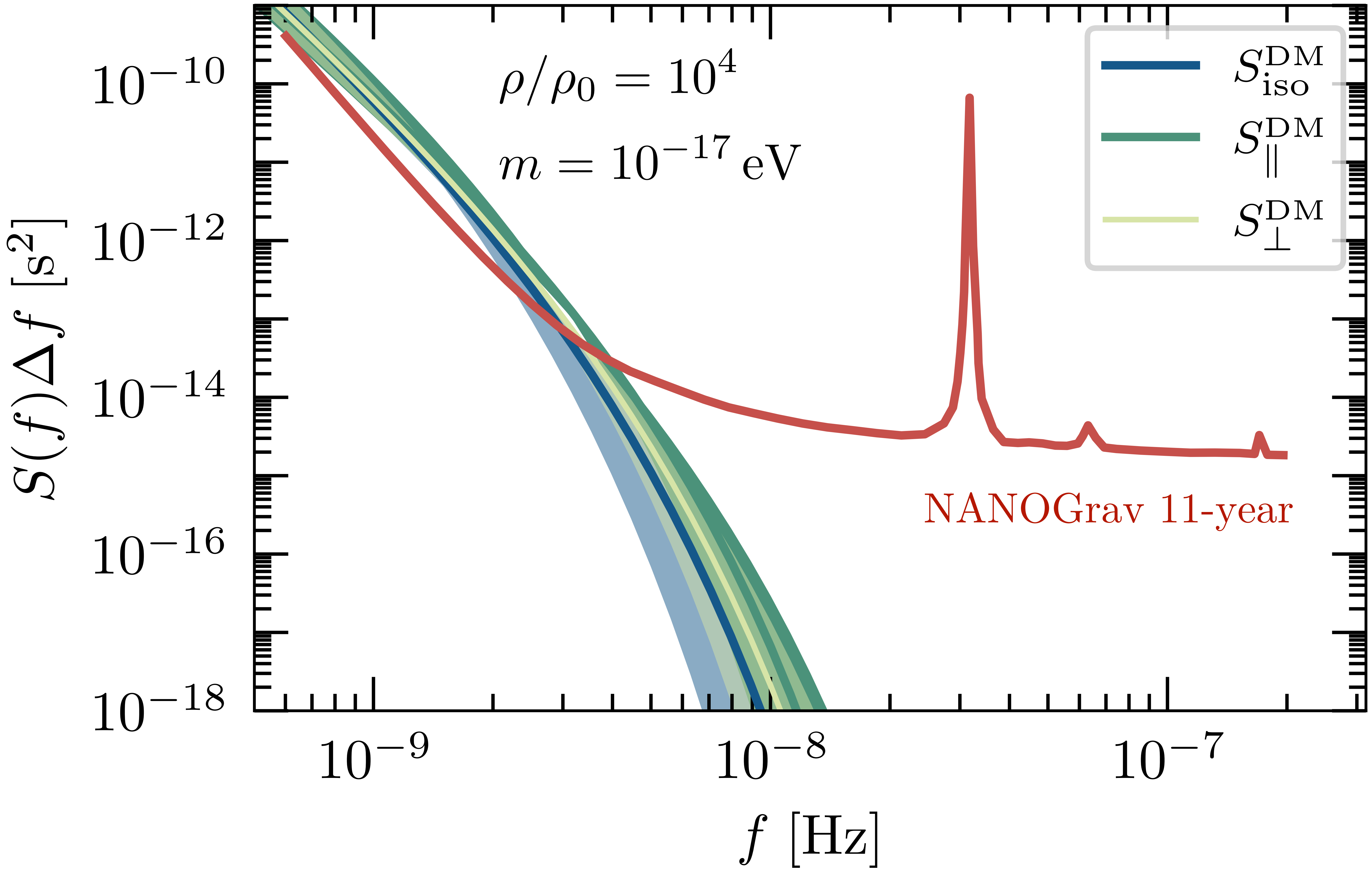
$$\langle \delta t_a \delta t_b \rangle = \int df\, \Gamma_{ab}^{\mathrm{ULDM}} S_{\delta t}^{\mathrm{ULDM}}(f)$$

for isotropic DM distribution

$$\Gamma_{ab} = \frac{1}{2}\left[\delta_{ab} + \hat{n}_a \cdot \hat{n}_b\right]$$

$$S_{\delta t}(f) = \frac{a^2 \tau}{(2\pi f)^4}\left[\frac{64}{3\pi} K_0(\omega/m\sigma^2)\right]$$

[Kim and Mitridate 2312.12225]



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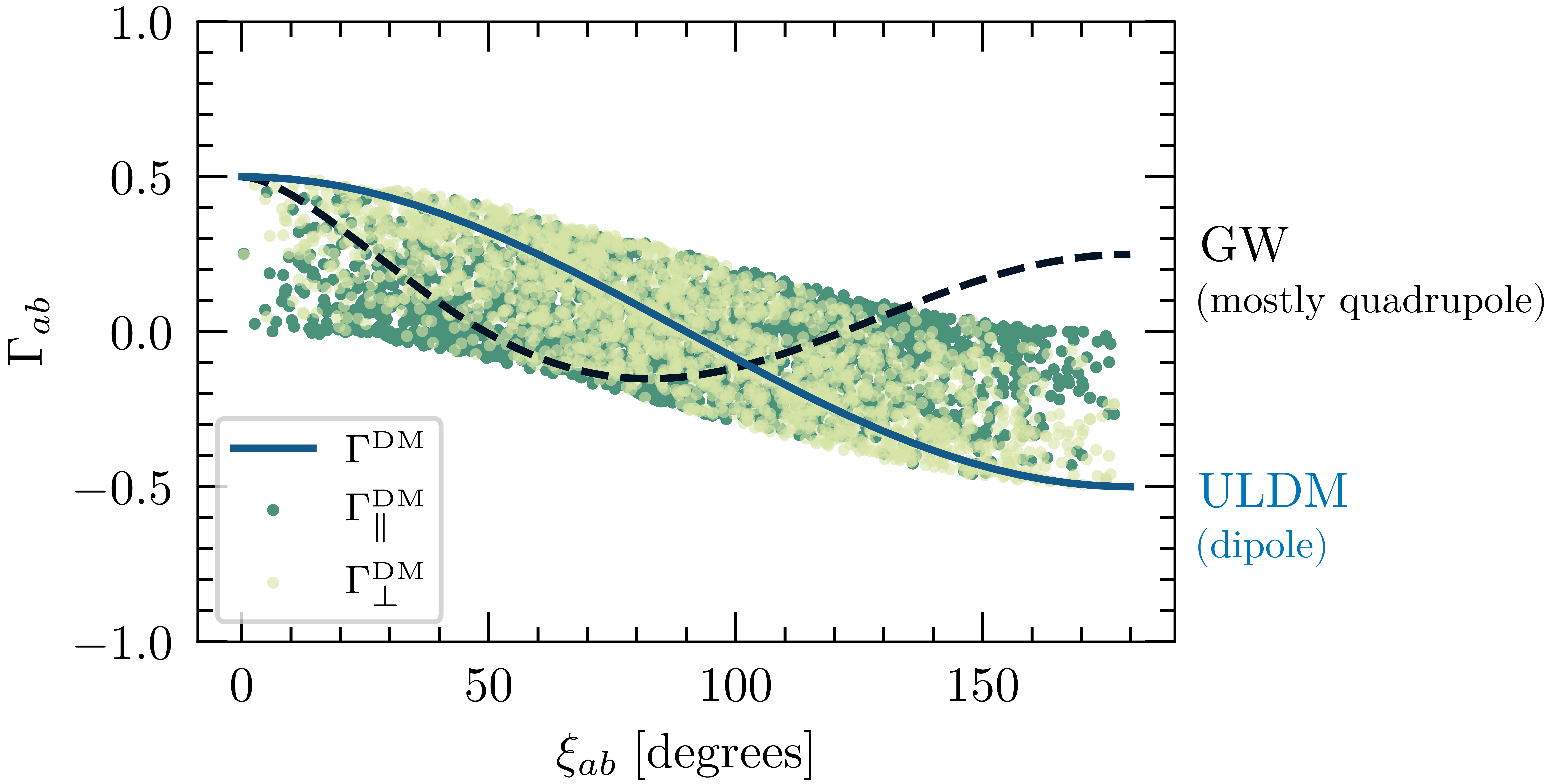
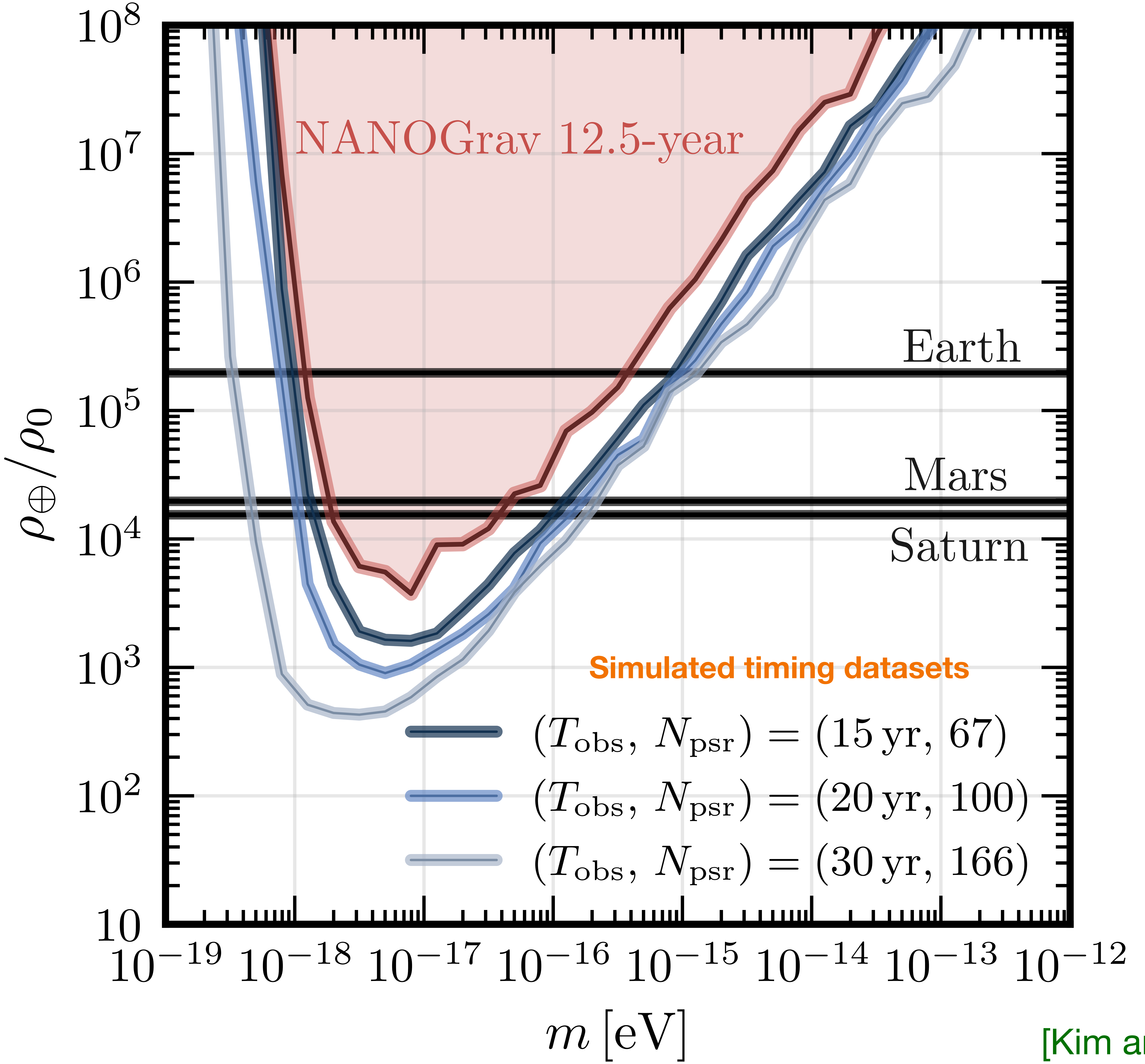


[Kim and Mitridate 2312.12225]



[Kim and Mitridate 2312.12225]

one last example:

Astrometry

astrometry involves
precision measurements of
positions / velocities of stars

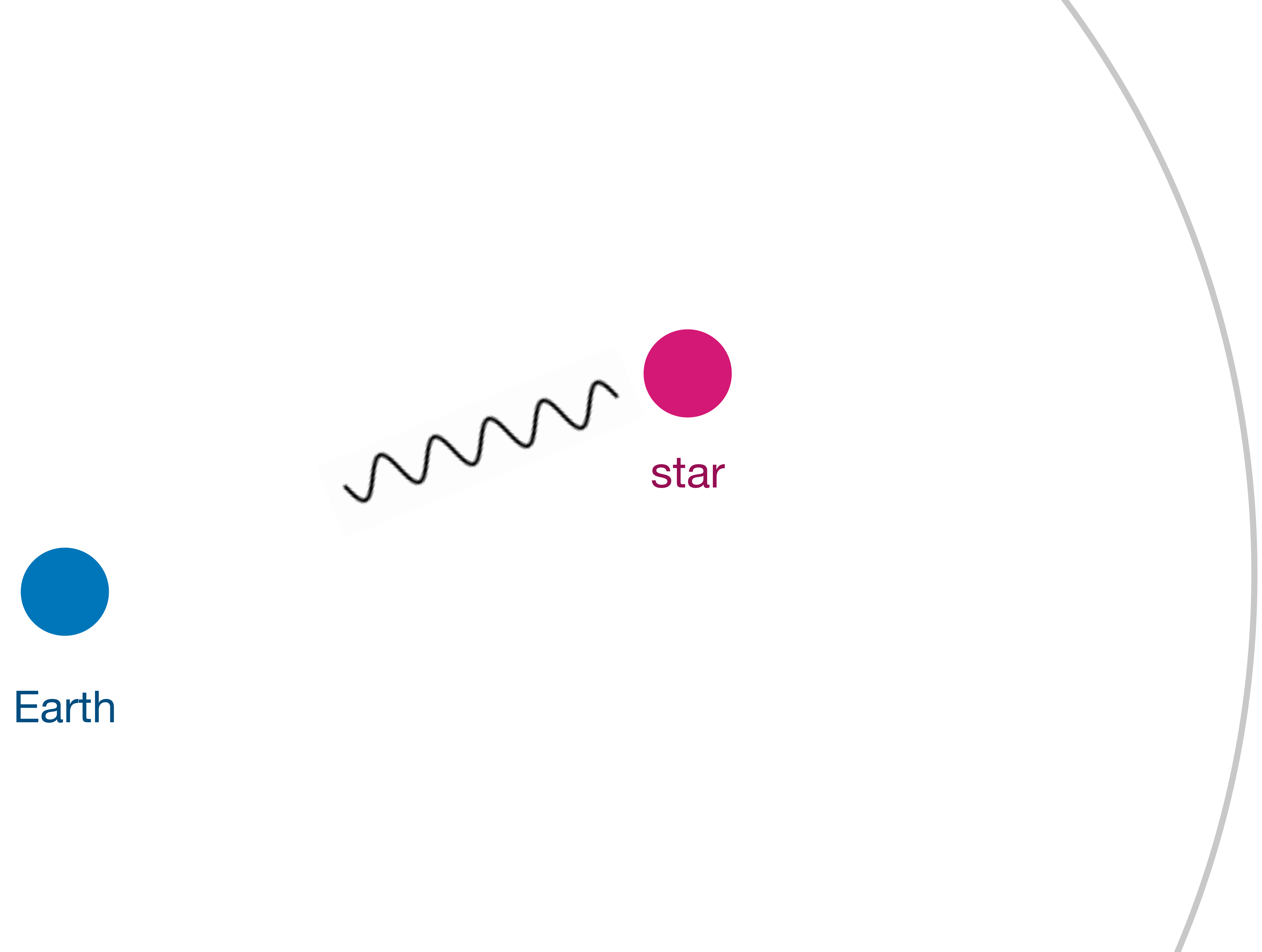
current/future astrometry missions measure

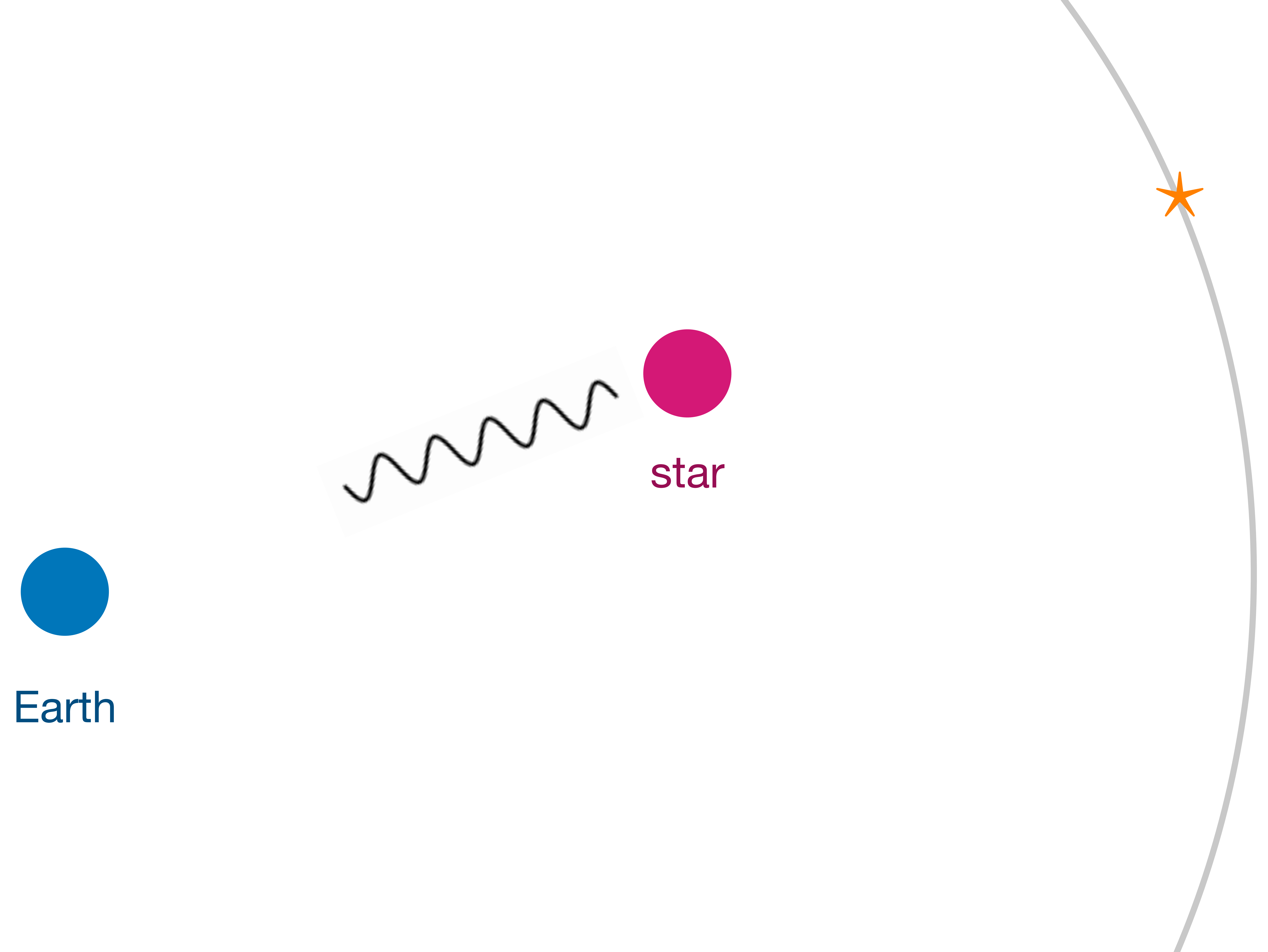
$$N_\star = 10^8 - 10^9$$

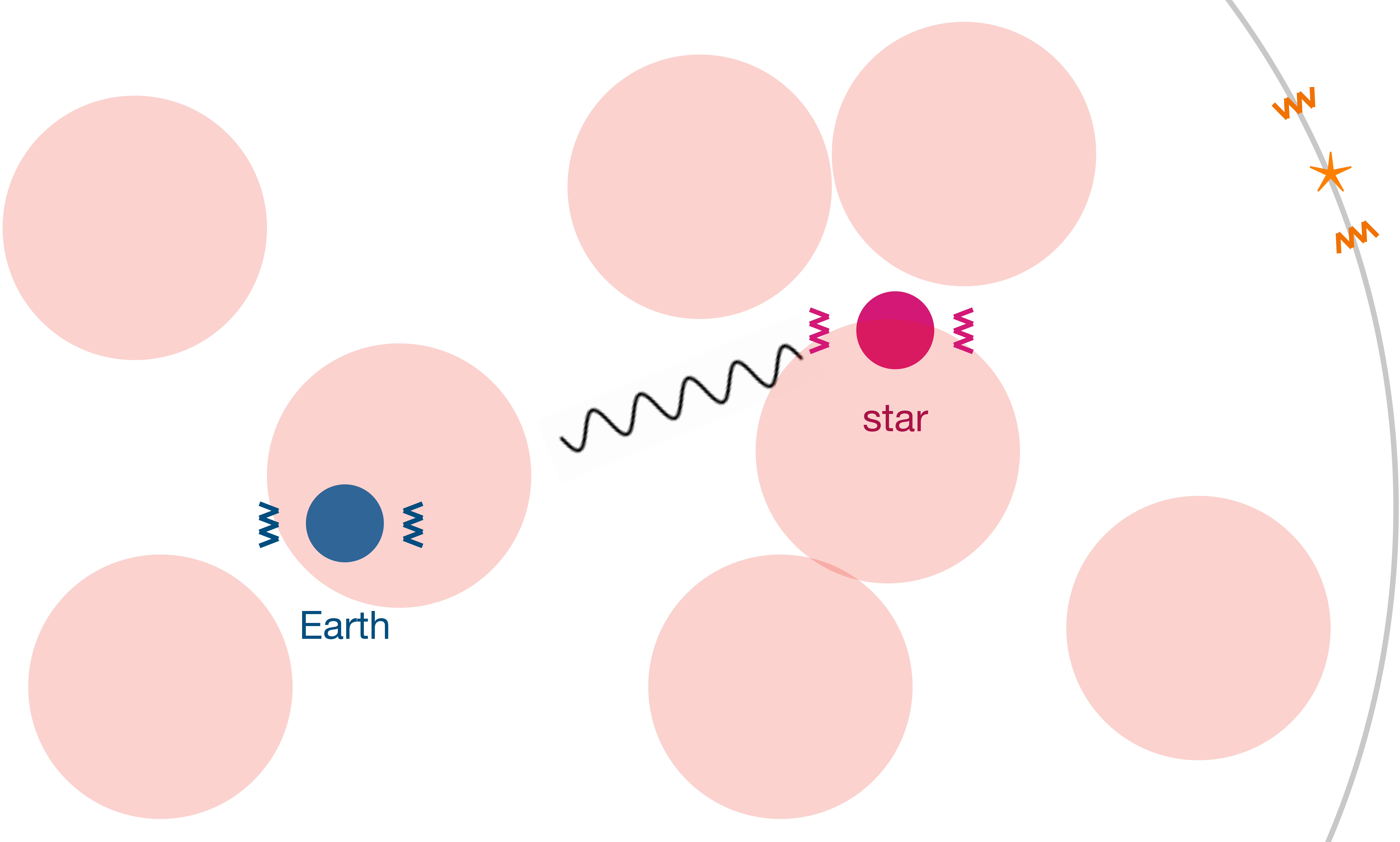
at the precision of

$$\Delta\theta \sim \mathcal{O}(10^2)\mu\mathrm{as}$$

$\mu\mathrm{as} = 5 \times 10^{-12}\,\mathrm{rad}$


star
Earth


star
Earth


star
Earth

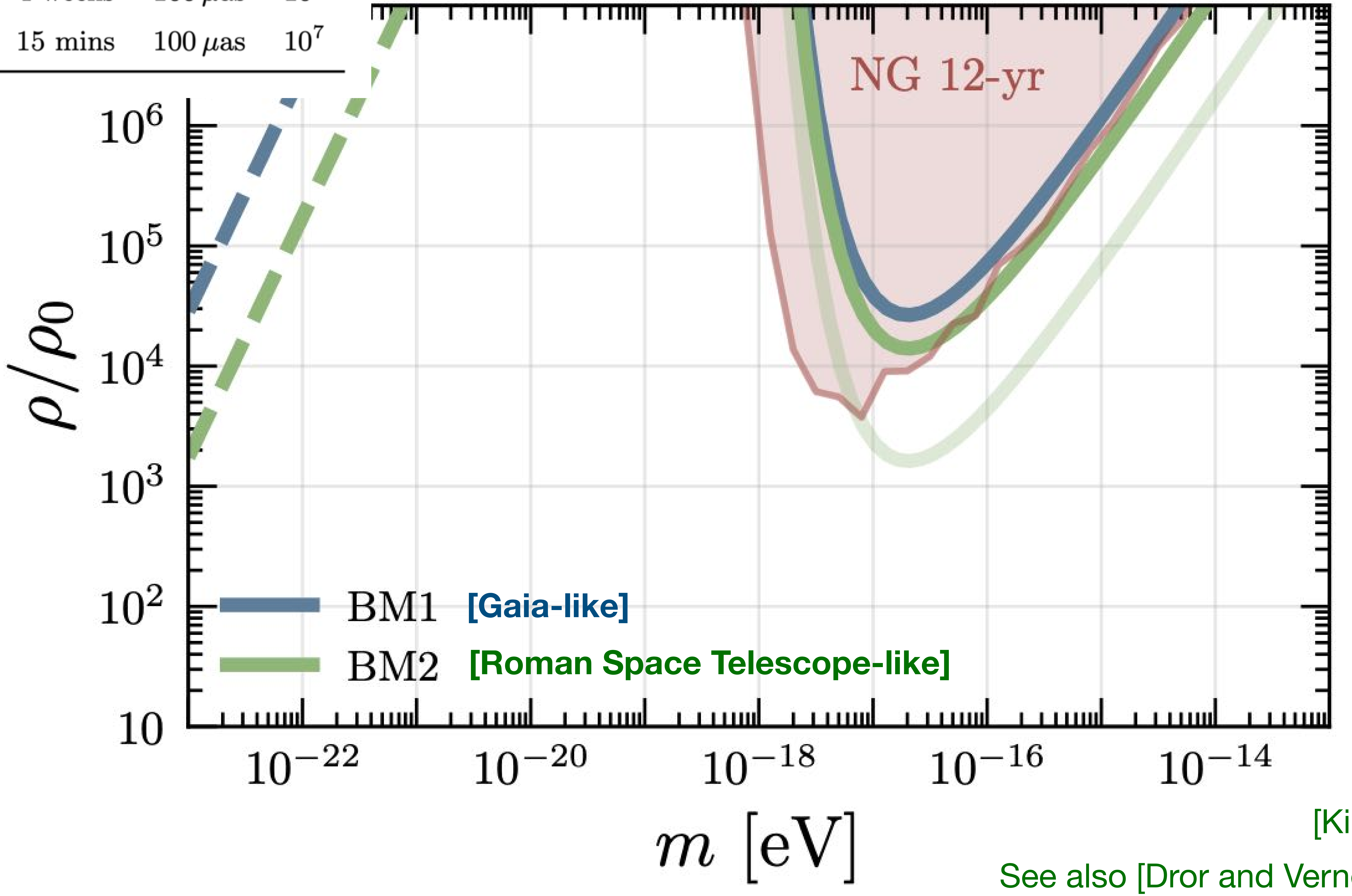
similarly

the signal is characterised by spectrum and correlation

$$\langle \delta n_a^i(t) \delta n_b^j(t') \rangle = \int df\, \Gamma_{ab}^{ij} S(f) \cos[2\pi f(t - t')]$$

[Kim 2406.03539]

	T	Δt	σ_r	$N_\star$
BM1	10-yr	4 weeks	100 μas	10^8
BM2	10-yr	15 mins	100 μas	10^7



[Kim 2406.03539]

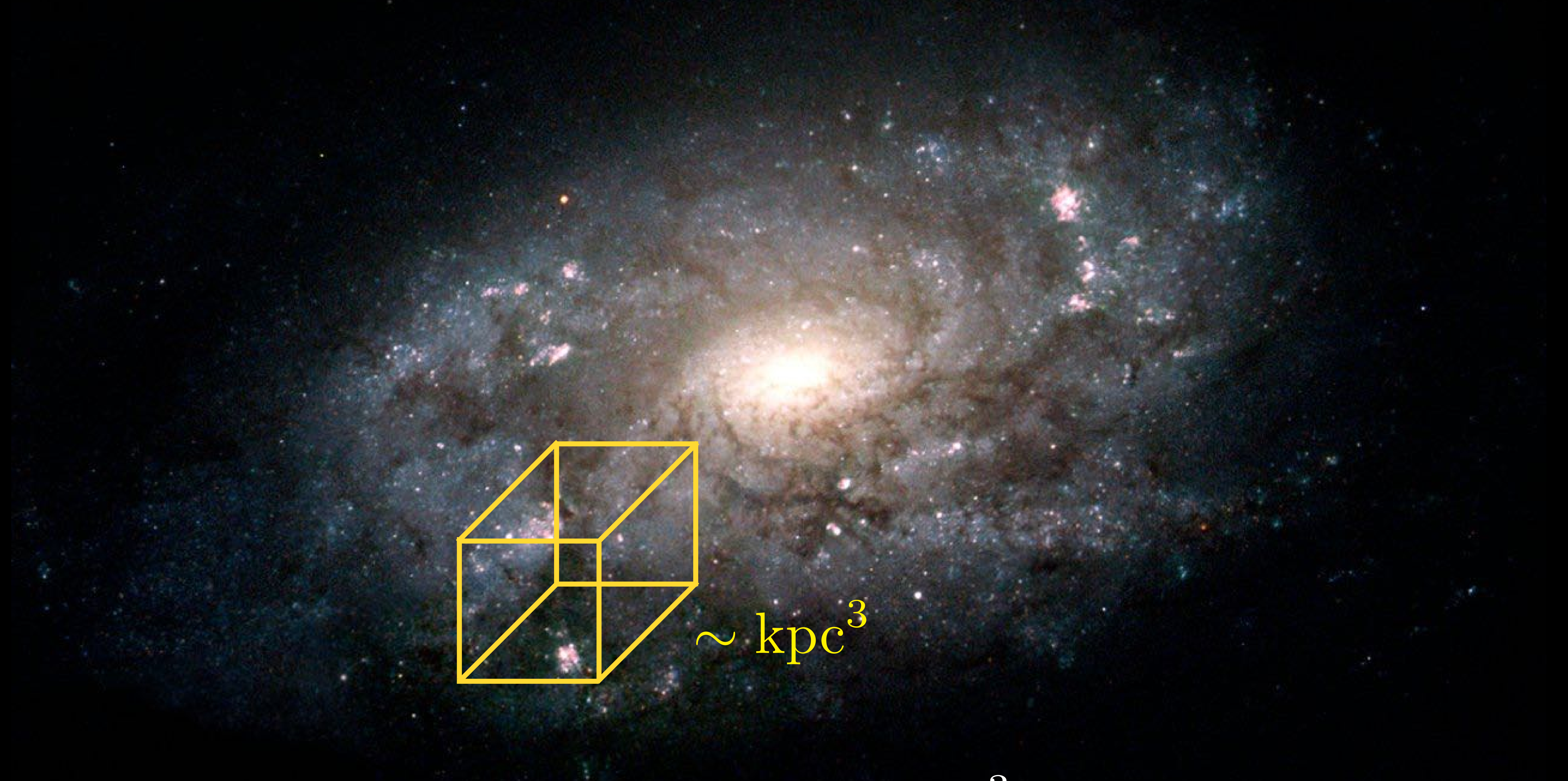
See also [Dror and Verner 2406.03526]

Remark I

all of the results shown here are sensitive to

ULDM density around/within the solar system

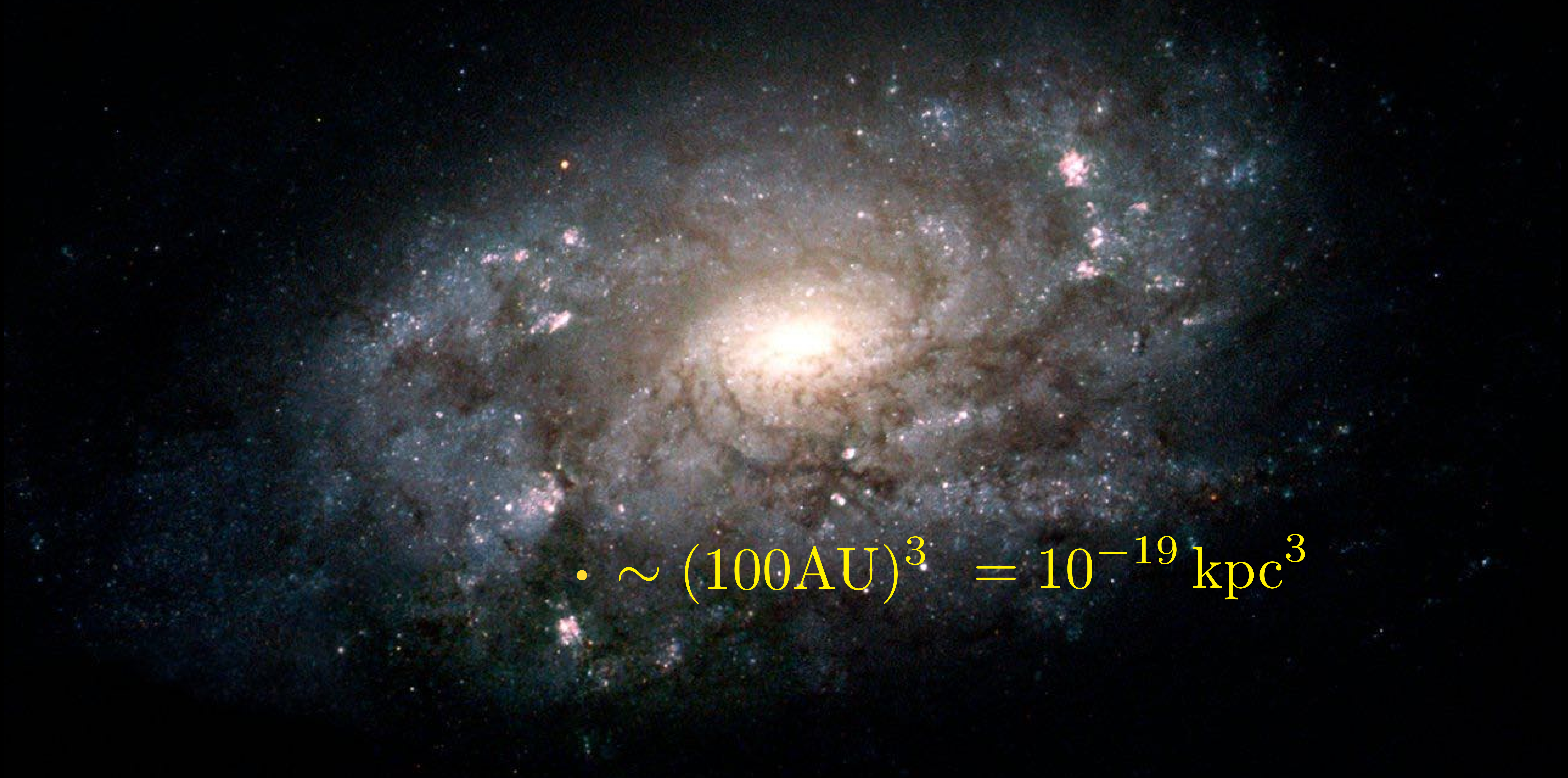
local dark matter density is often derived over kpc scales



$$\rho_0 = 0.4\,\mathrm{GeV/cm^3}$$

is an ***average density over the volume of kpc*** what we are probing is

(or what matters for all terrestrial DM detector is)



$$\bullet \sim (100\mathrm{AU})^3 = 10^{-19}\,\mathrm{kpc}^3$$

currently no measurement on this scale exists only constraints exist

$$\rho/\rho_0 \lesssim 10^{11}$$

From geodetic satellite and LLR [Adler (08)]

$$\rho/\rho_0 \lesssim 6 \times 10^{6}$$

From asteroids in the solar system [Tsai, Eby et al (22)]

$$\rho/\rho_0 \lesssim 2 \times 10^{4}$$

From solar system ephemerides [Pitjev, Pitjeva (13)]

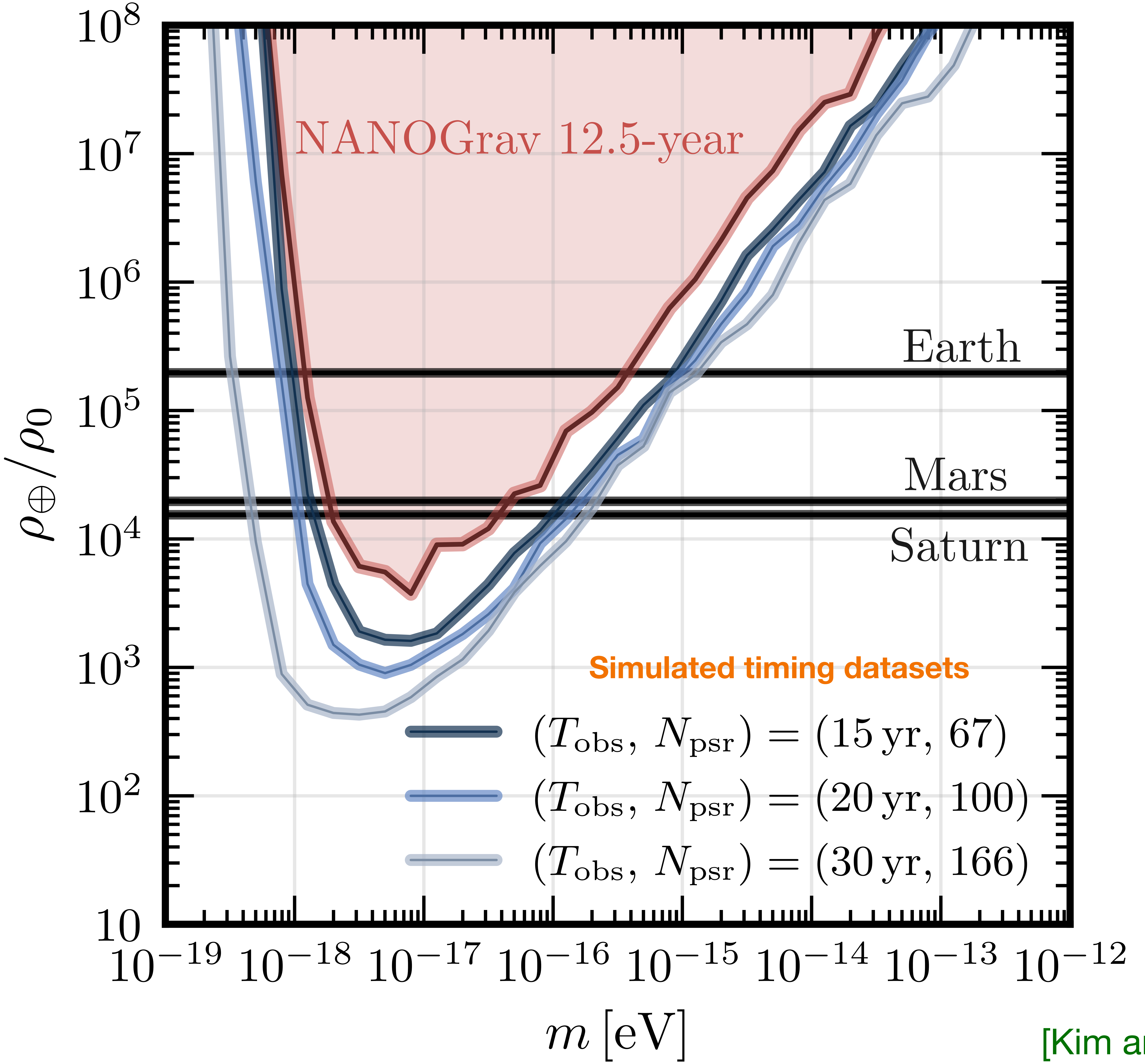
GW detectors will provide

one of the strongest probes of ULDM density

within/around the solar system

Remark II

prospects of pulsar timing array

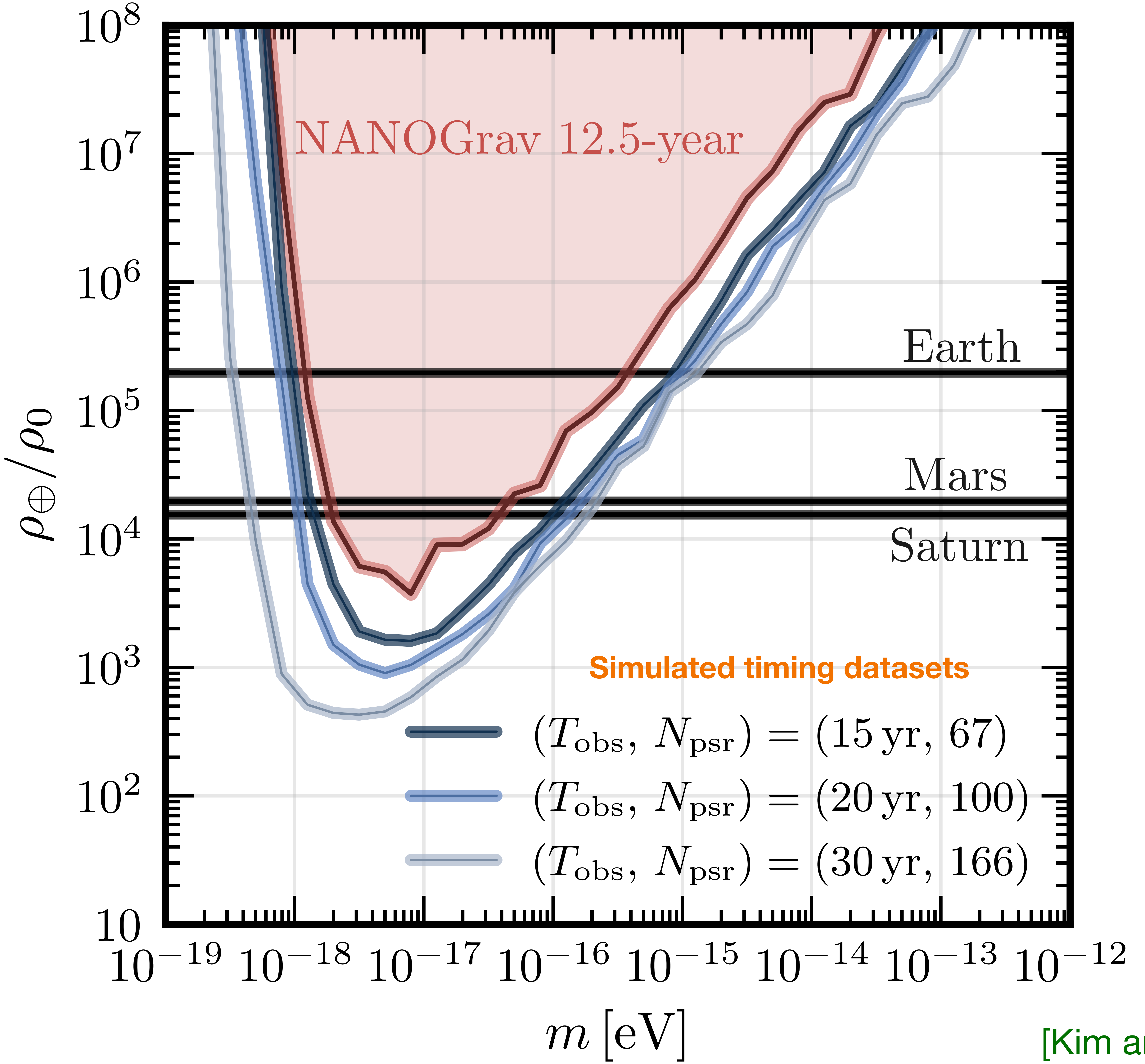


[Kim and Mitridate 2312.12225]

Next International Pulsar Timing Array (IPTA) data release
is expected to include

$$N_{\rm psr} \sim 100$$

$$T_{\rm obs} = 20^{+}\,{\rm yr}$$



[Kim and Mitridate 2312.12225]

with next-gen ratio telescope (e.g. Square Kilometer Array)
an order of magnitude or more improvement might be feasible

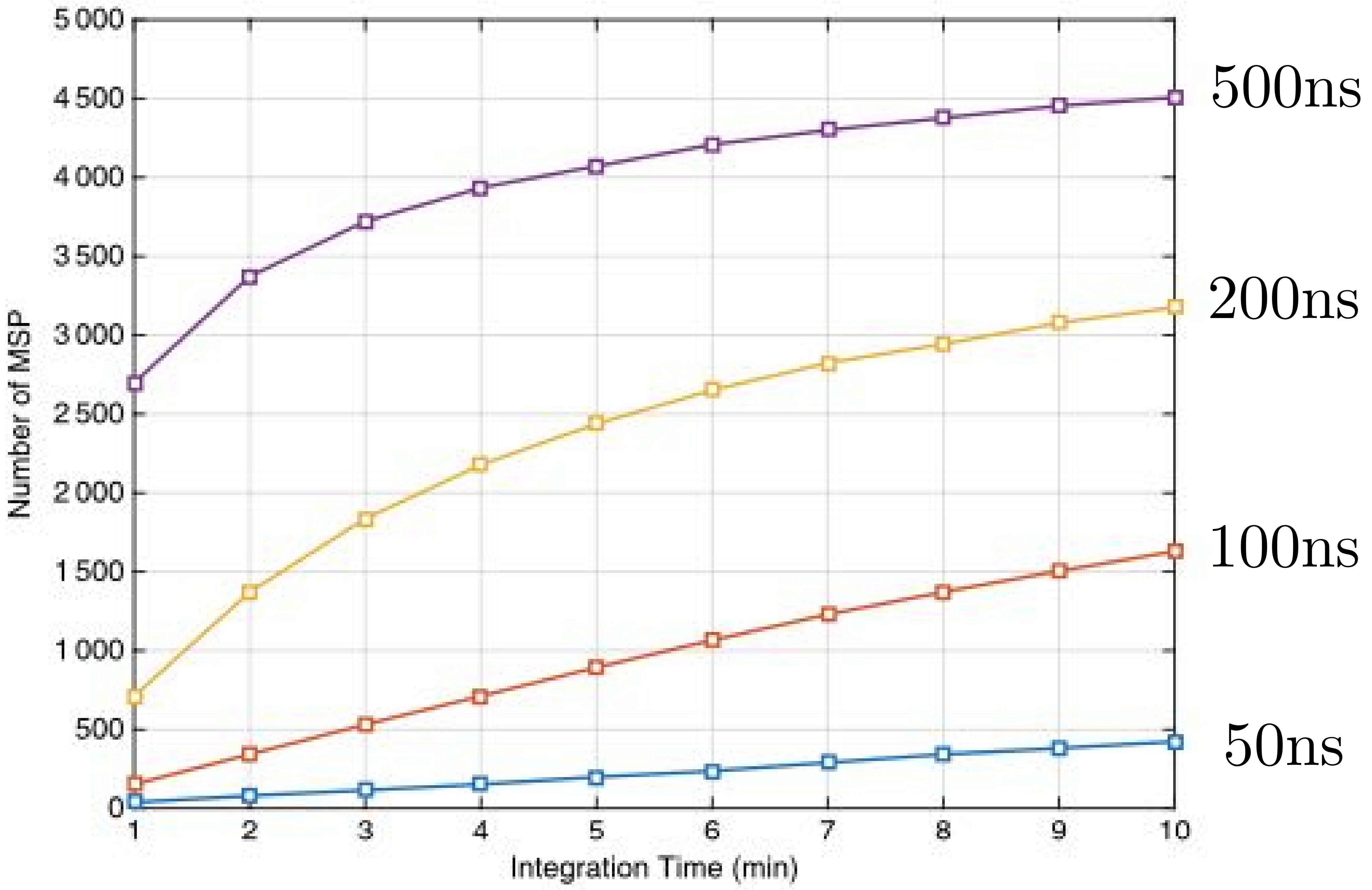


Figure 10. Numbers of MSPs that can archive a certain RMS noise level (or better) with varying integration time. Colour lines indicate different RMS noise levels (from bottom to top): 50 ns (blue), 100 ns (red), 200 ns (yellow), and 500 ns (purple).

[SKA, Weltman et al (20)]