Simulating electric currents at the end of axion inflation

NORDITA-2025-030

Schwinger effect in axion inflation on a lattice

Oksana Iarygina,[1,2,*] Evangelos I. Sfakianakis,[3,4,†] and Axel Brandenburg[1,2,5,6,‡]

arXiv:2506.20538v1 [astro-ph.CO] 25 Jun 2025

entropy MDPI

Article

Electromagnetic Conversion into Kinetic and Thermal Energies

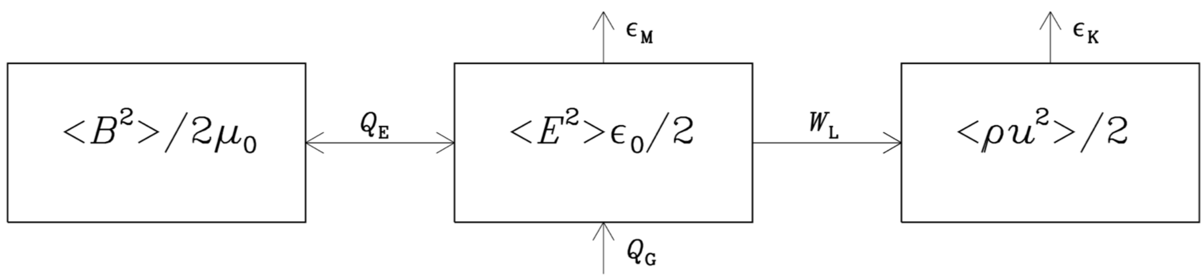
Axel Brandenburg [1,2,3,4,*] and Nousaba Nasrin Protiti [1,2]

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\left\langle \boldsymbol{B}^2/2\mu_0 \right\rangle + \left\langle \epsilon_0 \boldsymbol{E}^2/2 \right\rangle\right) = -\langle \boldsymbol{J}\cdot\boldsymbol{E}\rangle$$

$$\boldsymbol{J} = \sigma\left(\boldsymbol{E} + \boldsymbol{u}\times\boldsymbol{B}\right)$$

$$\boldsymbol{E} = \boldsymbol{J}/\sigma - \boldsymbol{u}\times\boldsymbol{B}$$

$$\langle \boldsymbol{J}\cdot\boldsymbol{E}\rangle = \langle \boldsymbol{J}^2/\sigma\rangle + \langle \boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B})\rangle$$

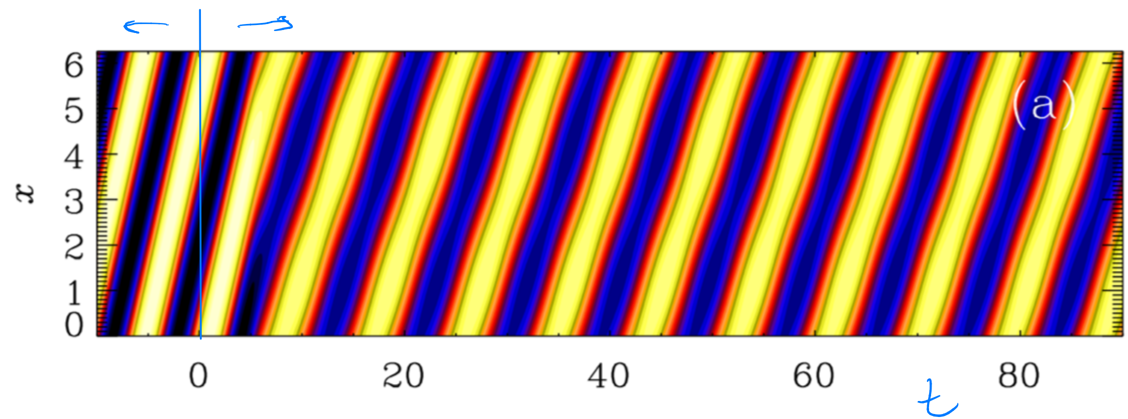


Work by Lorentz force

Limit of "ideal" MHD

Perfect conductivity
—> No dissipation

$$\langle \boldsymbol{J}\cdot\boldsymbol{E}\rangle = \cancel{\langle J^2/\sigma\rangle} + \langle \boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B})\rangle$$



Vacuum, zero conductivity
—> also no dissipation

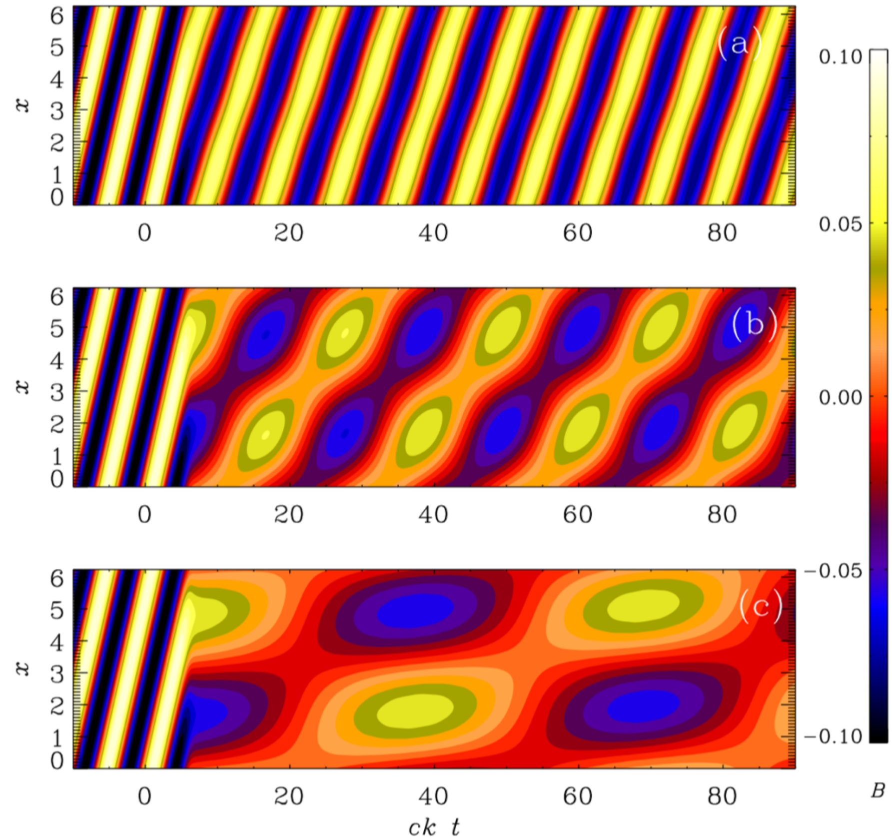
t<0 electromagnetic waves
t>0 Alfven waves

Alfven speed depends on background magnetic field strength

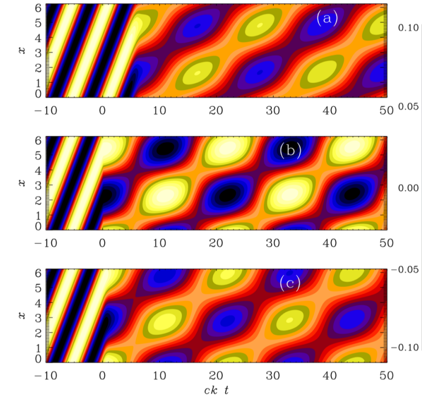
1

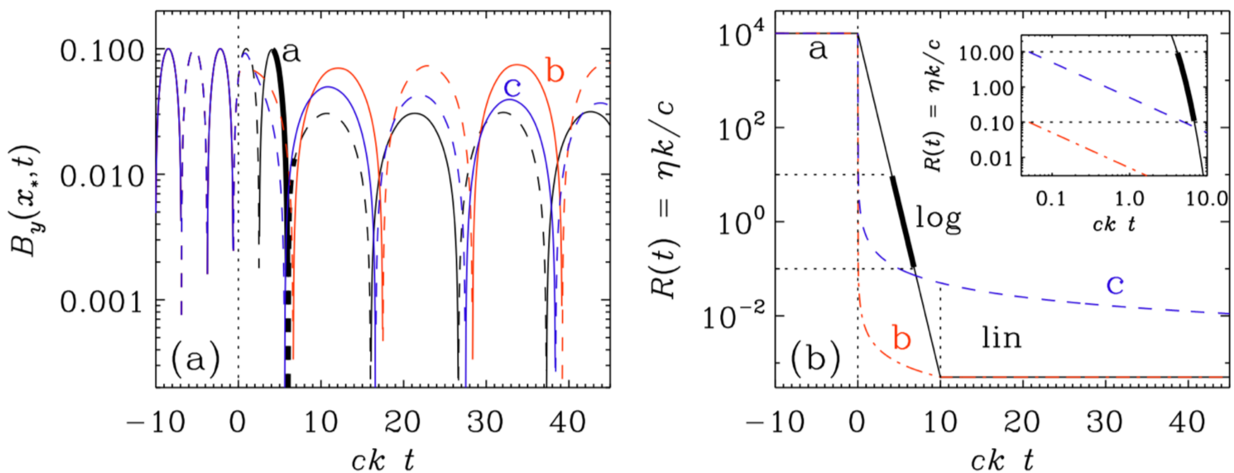
0.3

0.1



Intermediate conductivity:
—> dissipative losses,
—> depend on duration of transition





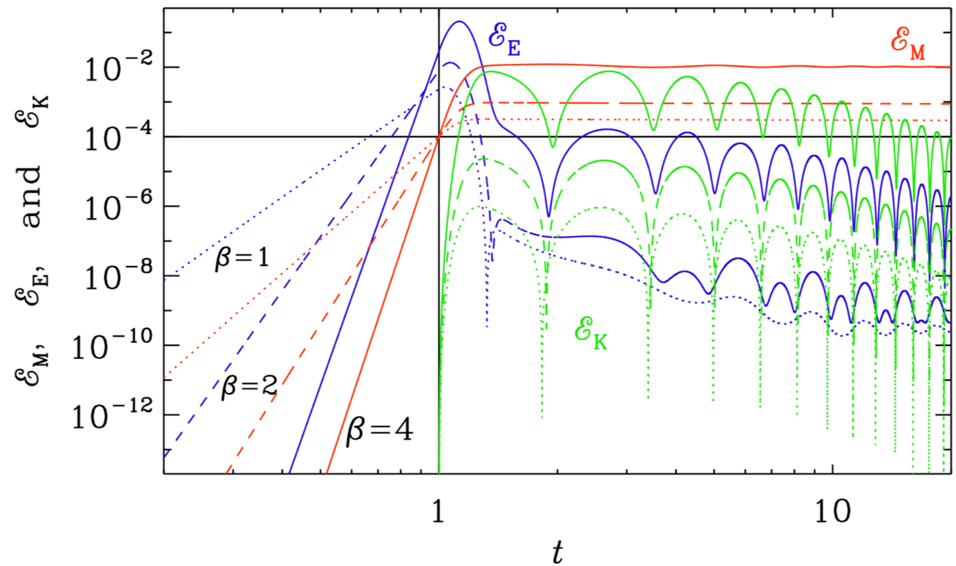
resistivity η
$= 1/\sigma$

MHD after inflation

$$f^2 F^{\mu\nu} F_{\mu\nu}$$

$$f \propto a^{-\beta}$$

Sharma+17



$$\mathcal{E}_\mathrm{M} \equiv \langle \boldsymbol{B}^2/2\mu_0 \rangle, \quad \mathcal{E}_\mathrm{E} \equiv \langle \epsilon_0 \boldsymbol{E}^2/2 \rangle, \quad \text{and} \quad \mathcal{E}_\mathrm{K} \equiv \langle \rho \boldsymbol{u}^2/2$$

Loss of electric energy —> Gain of kinetic and magnetic energy

The Journal of Open Source Software

The Pencil Code, a modular MPI code for partial differential equations and particles: multipurpose and multiuser-maintained

The Pencil Code Collaboration[1], **Axel Brandenburg**[1, 2, 3], **Anders Johansen**[4], **Philippe A. Bourdin**[5, 6], **Wolfgang Dobler**[7], **Wladimir Lyra**[8], **Matthias Rheinhardt**[9], **Sven Bingert**[10], **Nils Erland L. Haugen**[11, 12, 1], **Antony Mee**[13], **Frederick Gent**[9, 14], **Natalia Babkovskaia**[15], **Chao-Chin Yang**[16], **Tobias Heinemann**[17], **Boris Dintrans**[18], **Dhrubaditya Mitra**[1], **Simon Candelaresi**[19], **Jörn Warnecke**[20], **Petri J. Käpylä**[21], **Andreas Schreiber**[15], **Piyali Chatterjee**[22], **Maarit J. Käpylä**[9, 20], **Xiang-Yu Li**[1], **Jonas Krüger**[11, 12], **Jørgen R. Aarnes**[12], **Graeme R. Sarson**[14], **Jeffrey S. Oishi**[23], **Jennifer Schober**[24], **Raphaël Plasson**[25], **Christer Sandin**[1], **Ewa Karchniwy**[12, 26], **Luiz Felipe S. Rodrigues**[14, 27], **Alexander Hubbard**[28], **Gustavo Guerrero**[29], **Andrew Snodin**[14], **Illa R. Losada**[1], **Johannes Pekkilä**[9], and **Chengeng Qian**[30]

DOI: 10.21105/joss.02807

Software

- Review
- Repository
- Archive

Editor: Arfon Smith

Reviewers:

- @zingale
- @rtfisher

multipurpose

2 Papers by topic

The Pencil Code has been used for the following research topics

1. **Interstellar and intercluster medium as well as early Universe**

   (a) *Interstellar and intercluster medium* (Korpi-Lagg *et al.*, 2024; Elias-López *et al.*, 2024, 2023; Pavaskar *et al.*, 2023; Gent *et al.*, 2023; Brandenburg and Ntormousi, 2022; Maiti *et al.*, 2021; Gent *et al.*, 2021; Li and Mattsson, 2021; Candelaresi and Del Sordo, 2021, 2020; Li and Mattsson, 2020; Brandenburg and Furuya, 2020; Brandenburg and Brüggen, 2020; Gent *et al.*, 2020; Evirgen and Gent, 2019; Evirgen *et al.*, 2019; Seta and Beck, 2019; Rodrigues *et al.*, 2019; Brandenburg, 2019a; Väisälä *et al.*, 2018; Zhang

   (b) *Small-scale dynamos and reconnection* (Kishore and Singh, 2025a; Brandenburg and Ntormousi, 2025; Warnecke *et al.*, 2025; Gent *et al.*, 2024; Zhou and Jingade, 2024; Qazi *et al.*, 2025, 2024; Brandenburg and Larsson, 2023; Warnecke *et al.*, 2023; Gent *et al.*, 2022; Brandenburg *et al.*, 2023c; Zhou *et al.*, 2022; Bhat, 2021; Park and Cheoun, 2021; Santos-Lima *et al.*, 2021; Park, 2020; Pusztai *et al.*, 2020; Rüdiger *et al.*, 2020; Seta *et al.*, 2020; Käpylä, 2019; Bhat *et al.*, 2019; Brandenburg and Rempel, 2019; Brandenburg *et al.*, 2018a; Käpylä *et al.*, 2018; Bhat *et al.*, 2016b; Bhat and Subramanian, 2013; Brandenburg, 2011c; Baggaley *et al.*, 2009, 2010; Schekochihin *et al.*, 2005, 2007; Haugen and Brandenburg, 2004b; Haugen *et al.*, 2003, 2004a,c; Dobler *et al.*, 2003).

   (c) *Primordial magnetic fields and decaying turbulence* (Dehman and Brandenburg, 2025; Vachaspati and Brandenburg, 2024; Brandenburg and Banerjee, 2024; Dwivedi *et al.*, 2024; Brandenburg *et al.*, 2024a, 2023d; Mtchedlidze *et al.*, 2024, 2023, 2022; Bhat *et al.*, 2021; Brandenburg, 2023a, 2020a; Brandenburg *et al.*, 2020b, 2019b; Kahniashvili *et al.*, 2020; Brandenburg *et al.*, 2018b; Trivedi *et al.*, 2018; Brandenburg *et al.*, 2017d; Brandenburg and Kahniashvili, 2017; Kahniashvili *et al.*, 2017; Reppin and Banerjee, 2017; Park, 2017; Osano and Adams, 2017; Adams and Osano, 2016; Osano and Adams, 2016b,a; Kahniashvili *et al.*, 2016; Brandenburg *et al.*, 2015; Adams and Osano, 2014; Kahniashvili *et al.*, 2012, 2013; Tevzadze *et al.*, 2012; Candelaresi and Brandenburg, 2011a; Kahniashvili *et al.*, 2010; Del Sordo *et al.*, 2010; Christensson *et al.*, 2005; Yousef *et al.*, 2004).

   (d) *Relic gravitational waves & axions* (Sharma *et al.*, 2025c; Brandenburg *et al.*, 2024b,c; Iarygina *et al.*, 2024; Sharma *et al.*, 2023; He *et al.*, 2023; Roper Pol, 2022; Sharma and Brandenburg, 2022; AlbertoRoper, 2022; Kahniashvili *et al.*, 2022; Roper Pol, 2021; Roper Pol *et al.*, 2022b; He *et al.*, 2021b,a; Brandenburg *et al.*, 2021b,d; Brandenburg and Sharma, 2021; Brandenburg *et al.*, 2021a,c; Kahniashvili *et al.*, 2021; Roper Pol *et al.*, 2020b,a).

2. **Planet formation and inertial particles**

   (a) *Planet formation* (Rice *et al.*, 2025; Eriksson *et al.*, 2025; Shi *et al.*, 2025; Baehr *et al.*, 2022; Yang and Zhu, 2021; Raettig *et al.*, 2021; Baehr and Zhu, 2021b,a; Zhu and Yang, 2021; Klahr and Schreiber, 2021, 2020; Yang and Zhu, 2020; Eriksson *et al.*, 2020; Gerbig *et al.*, 2020; Castrejon *et al.*, 2019; Baehr and Klahr, 2019; McNally *et al.*, 2018; Schreiber and Klahr, 2018; Hernandez *et al.*, 2019; Manser *et al.*, 2019; Yang *et al.*, 2018; Rice and Nayakshin, 2018; Richert *et al.*, 2018; Kuchner *et al.*, 2018;

3. **Accretion discs and shear flows**

   (a) *Accretion discs and shear flows* (Meftah, 2025; Lyra *et al.*, 2024; Sengupta *et al.*, 2024; Cañas *et al.*, 2024; Zhou, 2024; Mondal and Bhat, 2023; Meftah, 2023; Tharakkal *et al.*,

   (a) *Coronal heating and coronal mass ejections* (Kishore and Singh, 2025a,b; Singh *et al.*, 2025; Vemareddy, 2024; Kesri *et al.*, 2024; Maity *et al.*, 2024b; Dey *et al.*, 2024; Vemareddy *et al.*, 2024; Zhang *et al.*, 2023; Dey *et al.*, 2022; Chatterjee and Dey, 2022; Jakab and Brandenburg, 2021; Zhuleku *et al.*, 2021; Adrover-González and Terradas,

   (b) *Large-scale dynamos, helical turbulence, and catastrophic quenching* (Brandenburg *et al.*, 2025b; Brandenburg and Vishniac, 2025; Rogachevskii *et al.*, 2025; Brandenburg *et al.*, 2025a; Mondal *et al.*, 2025; Shchutskyi *et al.*, 2025; Hidalgo *et al.*, 2025; Zhou and Lai,

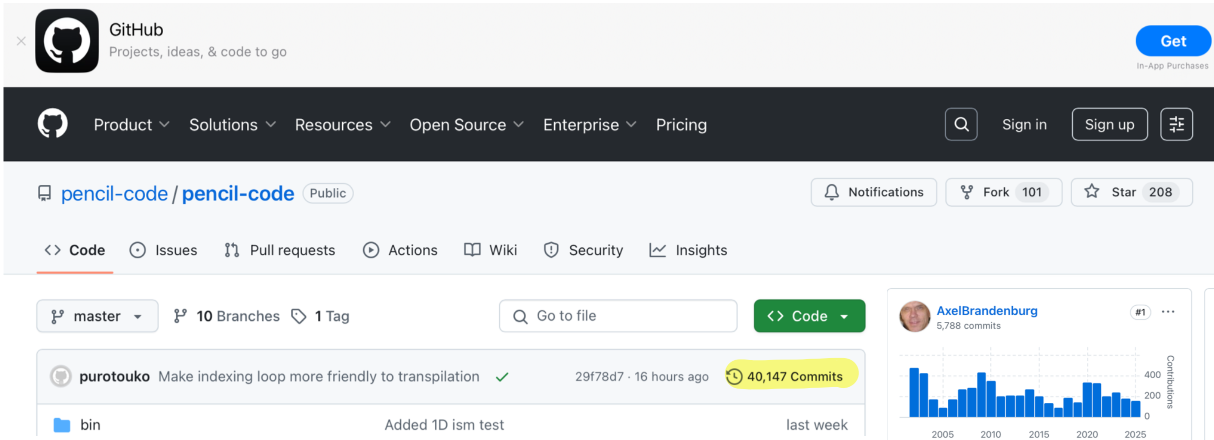
   (b) *Hydrodynamic and MHD instabilities* (Oliveira *et al.*, 2021; Del Sordo *et al.*, 2012; Chatterjee *et al.*, 2011b,c; Bejarano *et al.*, 2011; Brandenburg and Rüdiger, 2005; Brandenburg *et al.*, 2004c; Brandenburg, 2003).

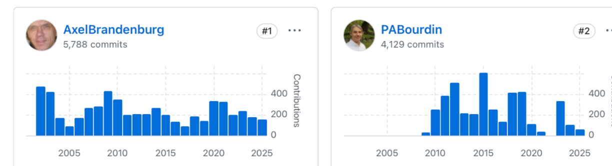
   (c) *Chiral MHD* (Schober *et al.*, 2024b,a; Brandenburg *et al.*, 2023a,b; Schober *et al.*, 2022a,b, 2020a,b, 2019, 2018; Brandenburg *et al.*, 2017e).

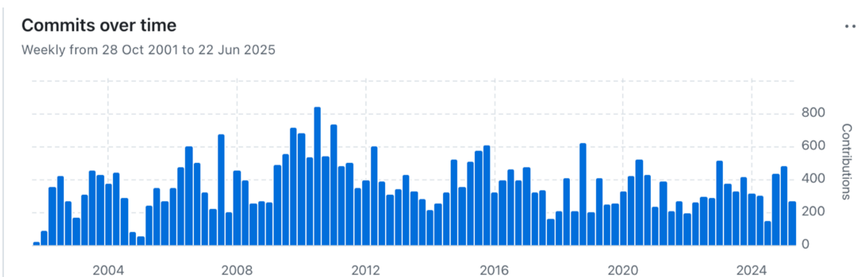
   (d) *Hydrodynamic and MHD turbulence* (Brandenburg and Scannapieco, 2025; Park, 2025; Roper Pol and Salvino Midiri, 2025; Brandenburg *et al.*, 2025c, 2023e; Brandenburg and Boldyrev, 2020; Aiyer *et al.*, 2017; Yokoi and Brandenburg, 2016; Brandenburg and Petrosyan, 2012; Del Sordo and Brandenburg, 2011a,b; Brandenburg and Nordlund, 2011; Haugen and Brandenburg, 2004a, 2006; Brandenburg *et al.*, 2005c; Pearson *et al.*, 2004).

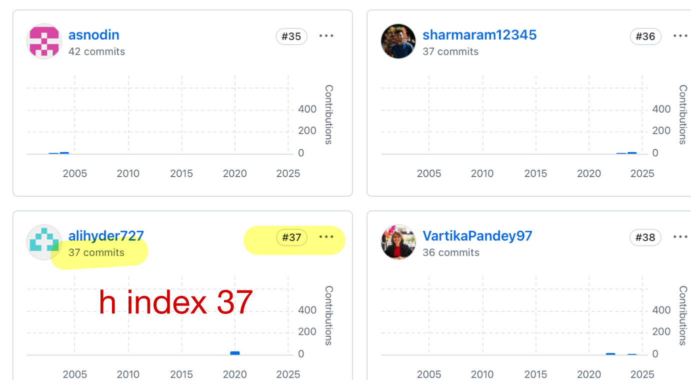
   (e) *Turbulent combustion, front propagation, radiation & ionization* (Yuvraj *et al.*, 2025; Lipatnikov, 2024b; Wang *et al.*, 2024; Ganti *et al.*, 2023; Yuvraj *et al.*, 2023; Lipatnikov and Sabelnikov, 2022, 2023; Karchniwy *et al.*, 2022; Bhatia and De, 2021; Zhang

multiuser-maintained









Continuous commits (>40,000) since 2001

Automatic testing: lowers threshold for newcomers

Commits on Jun 23, 2025

- **MR: declaration missing**
  Matthias Rheinhardt committed 3 days ago · ✓ 1 / 1
- **MR: k1_ff -> k1_ff_mag, but k1_ff still in run_pars**
  Matthias Rheinhardt committed 3 days ago · ✗ 0 / 1
- **increase n_pars in gravitational_waves_hTXk**
  ToxPuro committed 4 days ago · ✓ 1 / 1
- **non-rhs kernels of for gravitational_waves_hTXk.f90**
  ToxPuro committed 4 days ago · ✓ 1 / 1

Commits on Jun 21, 2025

- **magnetic.f90: Initialize current to zero, if lohm_evolve=T.** ···
  AxelBrandenburg committed last week · ✓ 1 / 1
- **initialize lohm_evolve=.false.**
  AxelBrandenburg committed last week · ✓ 1 / 1

Commits on Jun 19, 2025

- **MR: removed uneeded; improved cleaning; comments**
  mrheinhardt committed last week · ✓ 1 / 1
- **MR: back to underscoring: doesn't matter for Linux, is needed for MacOS**
  mrheinhardt committed last week · ✓ 1 / 1

Automatic tests

To ensure reproducability, the Pencil Code is tested for a number of sample applications. This is important for us in order to make sure certain improvements in some parts of the code do not affect the functionality of other parts. For other users who suspect that a new problem has emerged it could be useful to first see whether this problem also shows up in our own tests. The latest test results for a can be seen online:

level	name	description	time	OK	runs	host	compiler	maintainer
0	minimal	no- & most-modules	minutely	✓	latest (previous)	pencil-code.org	GNU 9.4	Philippe
0+1	basic	same as Travis	minutely	✓	latest (previous)	Norlx51	GNU 13.3	Axel/Philippe
2	normal	without basic	*/*:15	✗	latest (previous)	Norlx51	GNU 13.3	Axel/Philippe
0-2	default	basic + normal	*/2:03	✗	latest (previous)	Norlx65	GNU 13.3	Philippe/Axel
3	extended	without default	*/*:55	✓	latest (previous)	Norlx51	GNU 13.3	Axel/Philippe
0-3	full test	default + extended	*/6:31	⌛	latest (previous)	pencil-code.org	GNU 9.4	Philippe
4	fixme	succeeded before	*/6:45	⌛	latest (previous)	Norlx51	GNU 13.3	Axel/Philippe
5	overlong	runs less often	15:31	⏲	latest (previous)	Norlx65	GNU 13.3	Philippe/Axel
6-9	defective	known to fail	03:31	⏲	latest (previous)	Norlx65	GNU 13.3	Philippe/Axel

Legend: */* means every hour, */6:31 means 31 minutes after full hours divisible by 6.
Status of auto-tests: ⏲ scheduled; ⌛ running; ✗ failed; ✓ succeeded.
Tests are triggered only if there are new updates to the code.

Record for each run

```
fre  6 jun 2025 21:55:19 CEST
Submitted batch job 10468169
10468169  # RUN STARTED on nid001108 fre 6 jun 2025 23:05:50 CEST (SVN Revision: 40552, date of run.x: 2025-06-04 09:44)
10468169  # RUN FINISHED on nid001108 lör 7 jun 2025 22:21:57 CEST (SVN Revision: 40552, date of run.x: 2025-06-04 09:44)

sön  8 jun 2025 23:05:57 CEST
Submitted batch job 10497869
10497869  # RUN STARTED on nid001070 sön 8 jun 2025 23:14:30 CEST (SVN Revision: 40595, date of run.x: 2025-06-07 07:52)
10497869  # RUN FINISHED on nid001070 mån 9 jun 2025 20:49:06 CEST (SVN Revision: 40595, date of run.x: 2025-06-07 07:52)
```

Pencil Code school and user meeting

20–31 Oct 2025
CERN
Europe/Zurich timezone

Enter your search term

1st Pencil Code school on early Universe physics and gravitational waves (Oct 20-24)

The Pencil Code school on early Universe physics and gravitational waves will take place on October 20-24 as part of a two-week CERN TH institute.

The school targets early-career and senior researchers that are interested in learning and developing numerical skills applied to early Universe physics using Pencil Code.

The lectures will cover numerical aspects:

- *Introduction to Pencil Code*
- *Finite-difference schemes for partial differential equations*
- *Post-processing of data with IDL and Python*
- *GPU acceleration of Pencil Code*

as well as applications to particular physics cases with hands-on exercises on:

- *Magnetohydrodynamics of the early Universe*
- *Generation and evolution of primordial magnetic fields*
- *Chiral magnetohydrodynamics*
- *First-order phase transitions*
- *Gravitational wave production*
- *Axion inflation*

Registration is open and will close on ***July 31st***. The school is limited to a maximum of 30 participants.

Participants of the school are encouraged to also participate in the user meeting (Oct 27-31) and need to register separately.

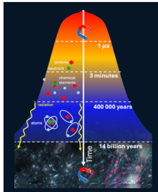
Numerical Experiments

Numerical Experiments, School on Astrophysical Turbulence and Dynamos, ICTP Trieste, 20-30 April 2009.

- LCD workshop2016 (Boulder, 10-12 May 2016)
- MHD course (Stockholm, January 2012)
- Evry Schatzman school'09 in Aussois,
- Solar Physics and MHD course (Stockholm, May 2009)
- Schedule for Trieste, April 2009

September 2009 (PowerPoint Presentation)

Pencil Code home page, Manual, Manual-II, PowerPoint Presentation, https://github.com/pencil-code



Nordita Winter School 2026 - Cosmological Magnetic Fields: Generation, Observation, and Modeling

12-23 Jan 2026 — Albano Building 3

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- Application
- Timetable
- Slides From Lectures
- Practical Information
  - What is Nordita?
  - Directions to Nordita
  - Directions to BizApartment Hotel

Venue

Registration, 12 Jan. 09:15: Albano Campus, House 3, floor 6 (Nordita building)

Lectures: Room 4205, Conference Center, Albano Campus, House 3, floor 4 (Nordita building)

Workspaces: Use the open desks throughout floor 6 and floor 5 (east).

Coffee: Help yourself to free coffee in Nordita's kitchens on floor 6 and floor 5 (east).

Scope

Magnetic fields are omnipresent in the Universe, we find them in galaxies and galaxy clusers, in filaments and voids of the Large Scale Structure. The presence of magnetic fields in voids hints to the possibility that the initial fields have been generated in the early Universe, within the first

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Schwinger effect in axion inflation on a lattice

Oksana Iarygina,[1,2,*] Evangelos I. Sfakianakis,[3,4,†] and Axel Brandenburg[1,2,5,6,‡]

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \right.$$
$$\left. -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} + \mathcal{L}_{ch} \right]$$

$\mathcal{L}_{ch} = \mathcal{L}_{ch}(A_\mu^{\mathrm{ph}}, \chi)$ describes all charged fields.

$$J^\mu = -\partial\mathcal{L}_{ch}/\partial A_\mu = \left(\rho_{\mathrm{ch}}, \boldsymbol{J}^{\mathrm{ph}}/a\right)$$

$$\partial_\tau^2\phi + 2\mathcal{H}\partial_\tau\phi - \boldsymbol{\nabla}^2\phi + a^2\frac{dV}{d\phi} = \frac{\alpha}{a^2 f}\boldsymbol{E}\cdot\boldsymbol{B},$$

$$\partial_\tau\boldsymbol{E} - \mathrm{rot}\,\boldsymbol{B} + \frac{\alpha}{f}\left(\partial_\tau\phi\boldsymbol{B} + \boldsymbol{\nabla}\phi\times\boldsymbol{E}\right) + \boldsymbol{J} = 0,$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = -\frac{\alpha}{f}\boldsymbol{\nabla}\phi\cdot\boldsymbol{B}, \quad \boldsymbol{\nabla}\cdot\boldsymbol{B} = 0,$$

$$\partial_\tau\boldsymbol{B} + \mathrm{rot}\,\boldsymbol{E} = 0,$$

$$\mathcal{H}^2 = \frac{8\pi}{3m_{\mathrm{Pl}}^2}a^2\left(\rho_\phi + \rho_E + \rho_B + \rho_\chi\right),$$

EoM

$$\rho_\phi = \langle(\partial_\tau\phi)^2/2a^2 + (\boldsymbol{\nabla}\phi)^2/2a^2 + V\rangle$$

$$V(\phi) = \frac{1}{2}m^2\phi^2 \qquad m = 1.04\times 10^{-6}\, m_{\mathrm{Pl}}.$$

$$\partial_\tau\rho_\chi + 4\mathcal{H}\rho_\chi = \frac{1}{a^3}\left(\langle\sigma_E\rangle\langle\boldsymbol{E}^2\rangle + \langle\sigma_B\rangle\langle\boldsymbol{E}\cdot\boldsymbol{B}\rangle\right)$$

Ambiguity in choices

Electric, magnetic, mixed pictures

$$\boldsymbol{J} = \sigma_E\boldsymbol{E}, \quad \sigma_E = \frac{(e|Q|)^3}{6\pi^2\mathcal{H}}|B|\coth\left(\frac{\pi B}{E}\right), \tag{8}$$

$$\boldsymbol{J} = \sigma_B\boldsymbol{B}, \quad \sigma_B = \frac{(e|Q|)^3}{6\pi^2\mathcal{H}}\mathrm{sign}(\boldsymbol{E}\cdot\boldsymbol{B})\,E\coth\left(\frac{\pi B}{E}\right), \tag{9}$$

$$\boldsymbol{J} = \sigma_E\boldsymbol{E} + \sigma_B\boldsymbol{B}, \tag{10}$$

All give the same current

$$J = \frac{(e|Q|)^3}{6\pi^2\mathcal{H}}E|B|\coth\left(\frac{\pi|B|}{E}\right)e^{-\frac{\pi m^2 a^2}{e|Q|E}}$$

Non-collinear case

R. von Eckardstein, K. Schmitz, and O. Sobol, JHEP **02**, 096 (2025), arXiv:2408.16538 [hep-ph].

$$\sigma_E = \frac{|\boldsymbol{J}'|E'}{\gamma\, I^2}\,, \quad \sigma_B = \frac{|\boldsymbol{J}'|}{E'\gamma\, I^2}(\boldsymbol{E}\cdot\boldsymbol{B}),$$

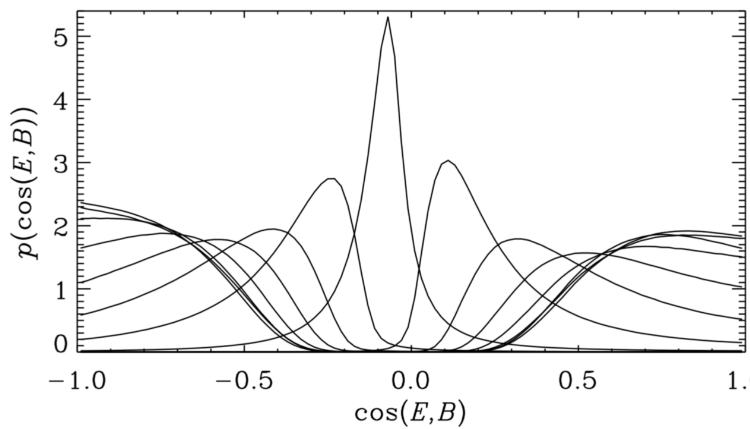
$$I^2 \equiv \sqrt{(\boldsymbol{E}^2 - \boldsymbol{B}^2)^2 + 4(\boldsymbol{E}\cdot\boldsymbol{B})^2}$$

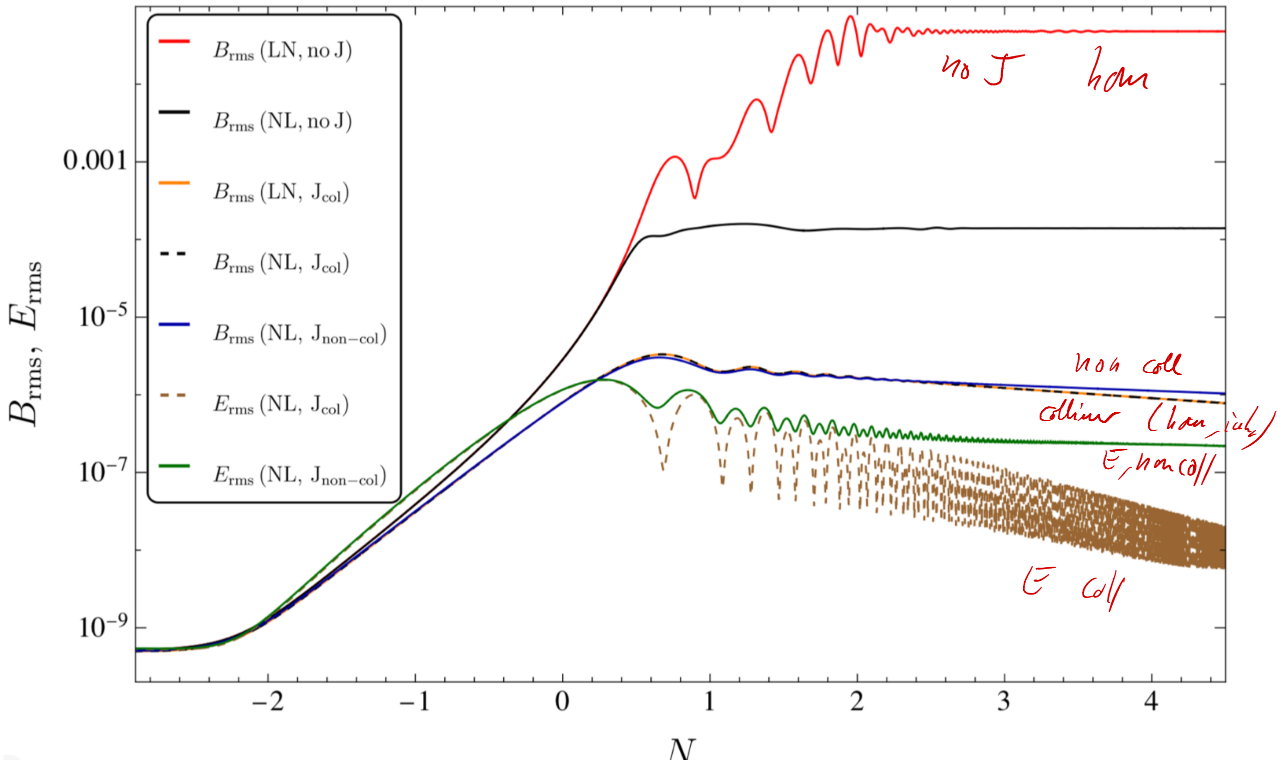
$$J' = \frac{(e|Q|)^3}{6\pi^2\mathcal{H}} E'|B'| \coth\left(\frac{\pi|B'|}{E'}\right),$$

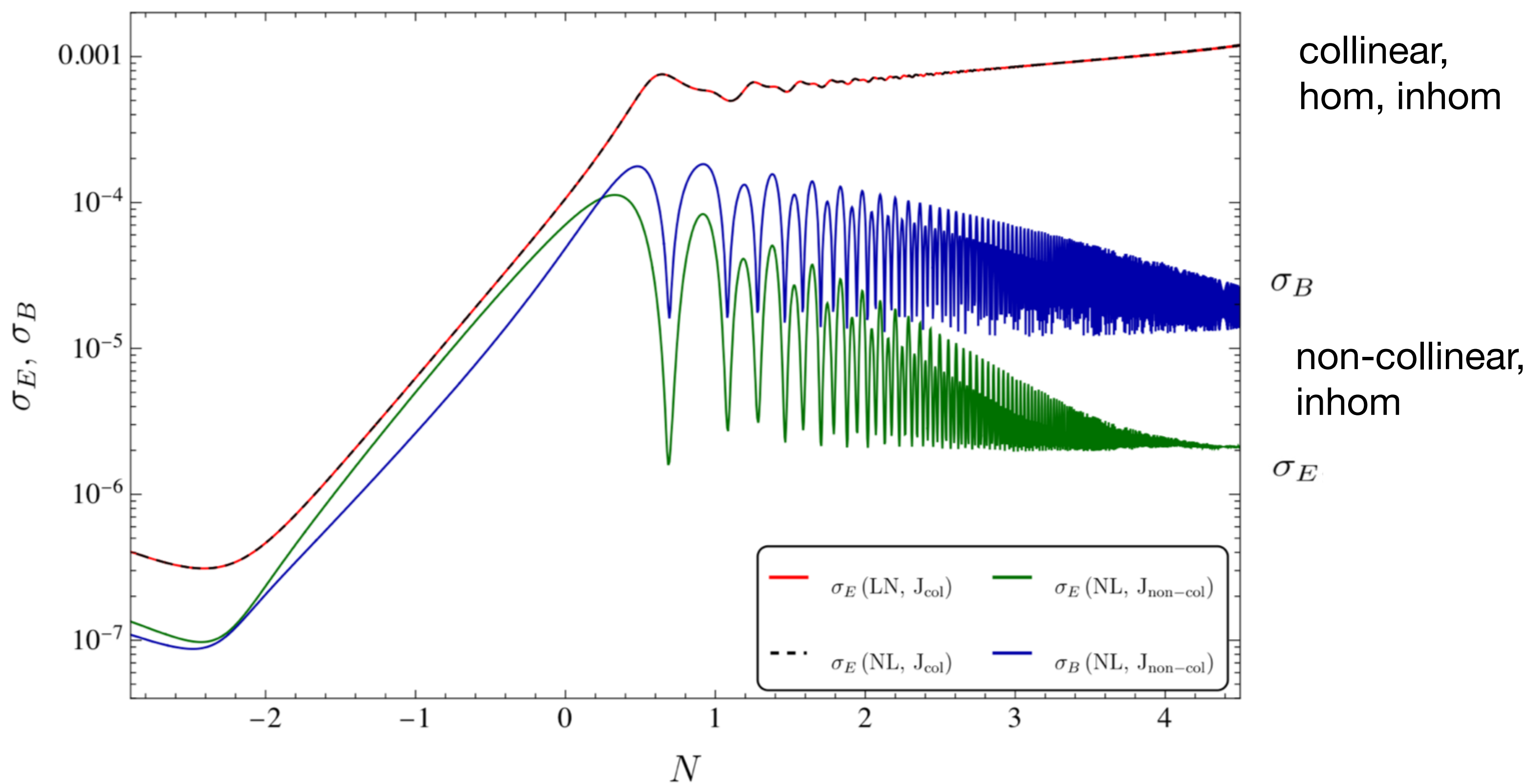
$$E' = \frac{1}{\sqrt{2}}\left[\boldsymbol{E}^2 - \boldsymbol{B}^2 + I^2\right]^{1/2},$$

$$B' = \frac{\mathrm{sign}(\boldsymbol{E}\cdot\boldsymbol{B})}{\sqrt{2}}\left[\boldsymbol{B}^2 - \boldsymbol{E}^2 + I^2\right]^{1/2},$$

$$\gamma = \frac{1}{\sqrt{2}}\left[1 + \frac{\boldsymbol{E}^2 + \boldsymbol{B}^2}{I^2}\right]^{1/2}.$$




$B_{\rm rms}$ (LN, no J)
$B_{\rm rms}$ (NL, no J)
$B_{\rm rms}$ (LN, $J_{\rm col}$)
$B_{\rm rms}$ (NL, $J_{\rm col}$)
$B_{\rm rms}$ (NL, $J_{\rm non-col}$)
$E_{\rm rms}$ (NL, $J_{\rm col}$)
$E_{\rm rms}$ (NL, $J_{\rm non-col}$)
$B_{\rm rms}, E_{\rm rms}$
0.001
10^{-5}
10^{-7}
10^{-9}
−2
−1
0
1
2
3
4
N
no J
hom
non coll
E, non coll
E coll



0.001
10^{-4}
10^{-5}
10^{-6}
10^{-7}
-2
-1
0
1
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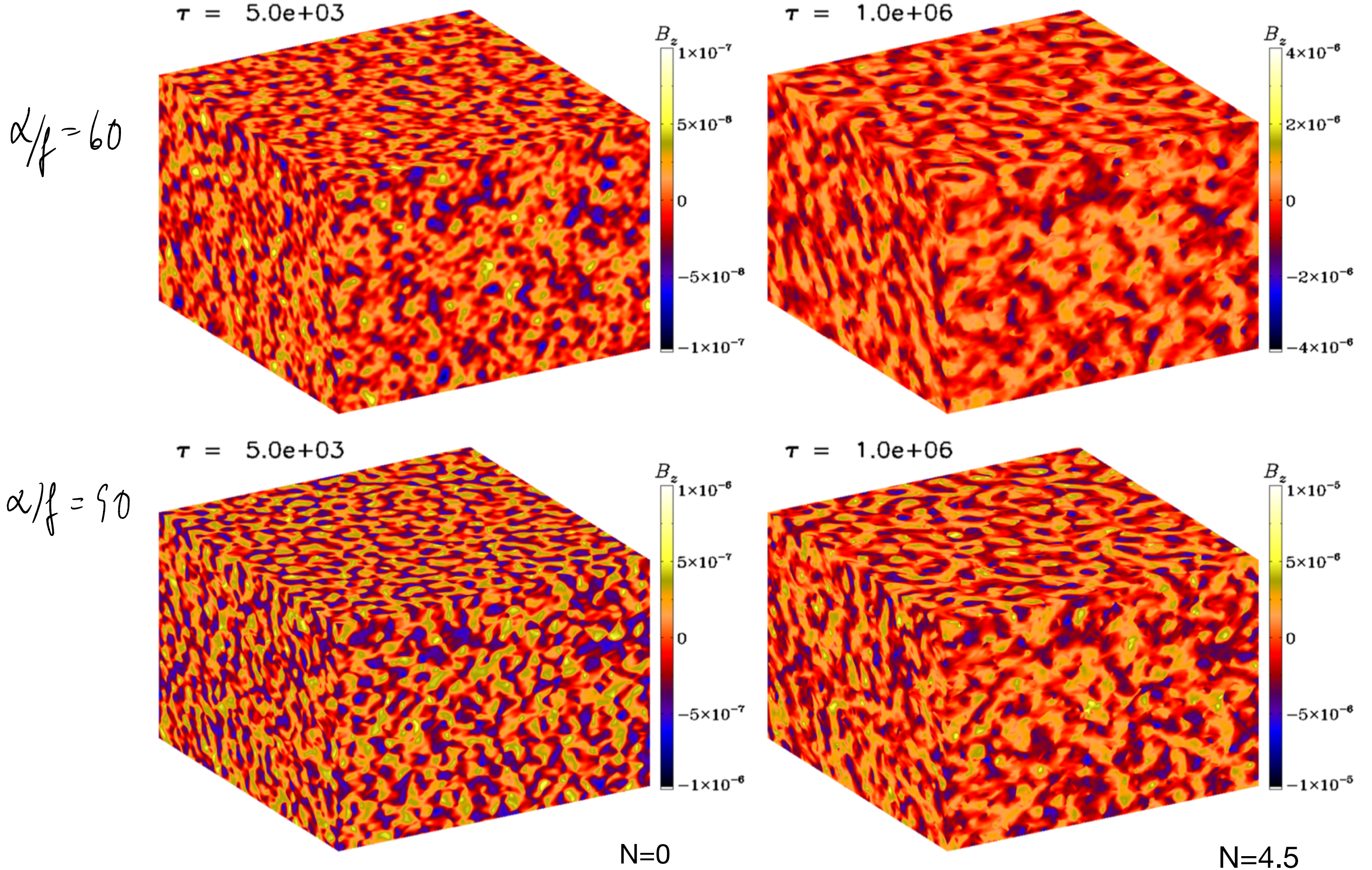
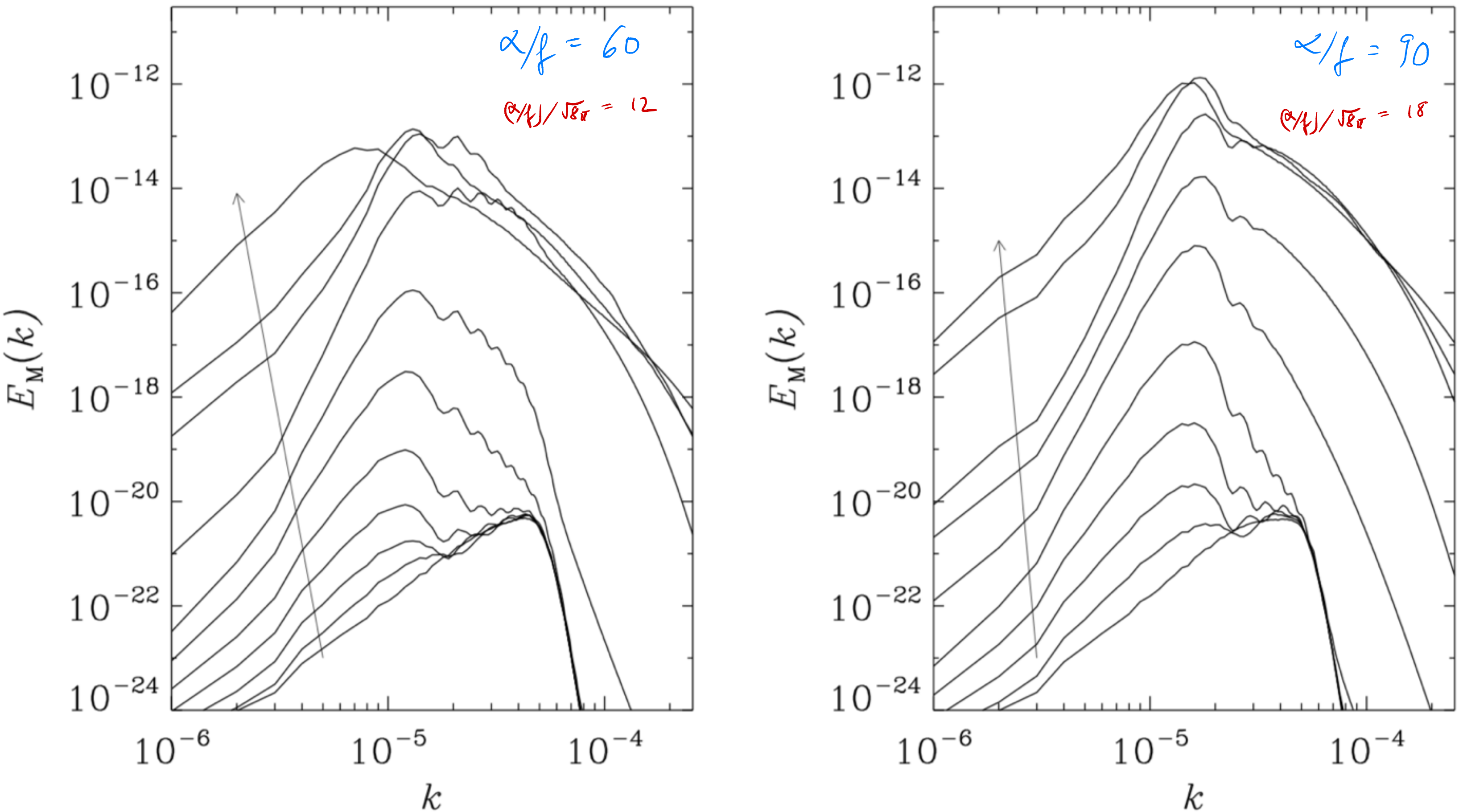
N
σ_E, σ_B
σ_E (LN, $\mathrm{J_{col}}$)
σ_E (NL, $\mathrm{J_{col}}$)
σ_E (NL, $\mathrm{J_{non-col}}$)
σ_B (NL, $\mathrm{J_{non-col}}$)
collinear, hom, inhom
σ_B
non-collinear, inhom
σ_E

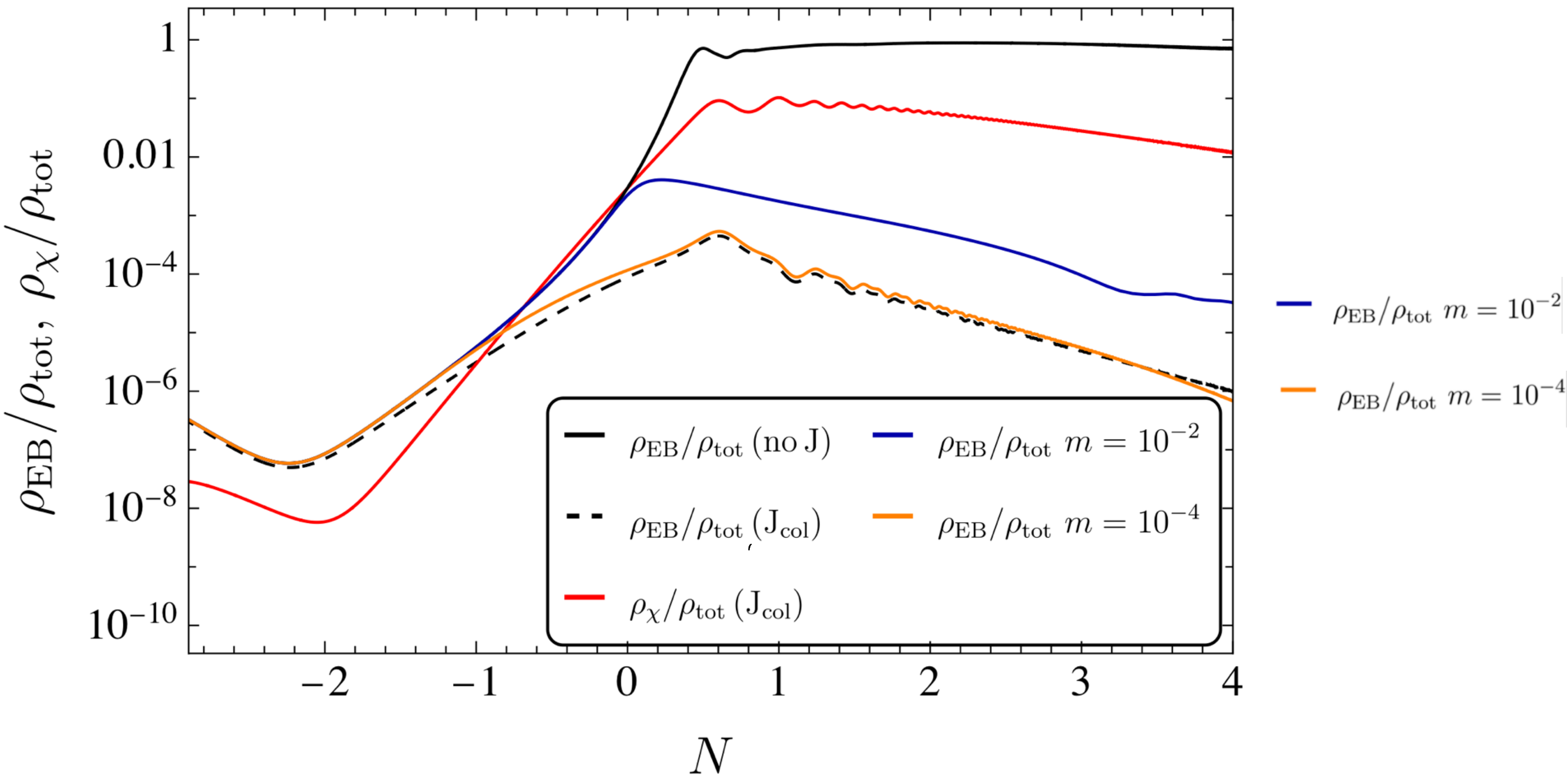


τ = 5.0e+03
τ = 1.0e+06
$\alpha/f = 60$
$\alpha/f = 90$
B_z
1×10^{-7}
5×10^{-8}
0
-5×10^{-8}
-1×10^{-7}
4×10^{-6}
2×10^{-6}
-2×10^{-6}
-4×10^{-6}
1×10^{-6}
5×10^{-7}
-5×10^{-7}
-1×10^{-6}
1×10^{-5}
5×10^{-6}
-5×10^{-6}
-1×10^{-5}
N=0
N=4.5

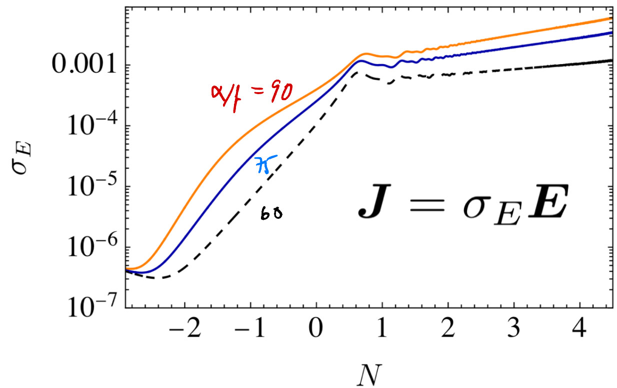


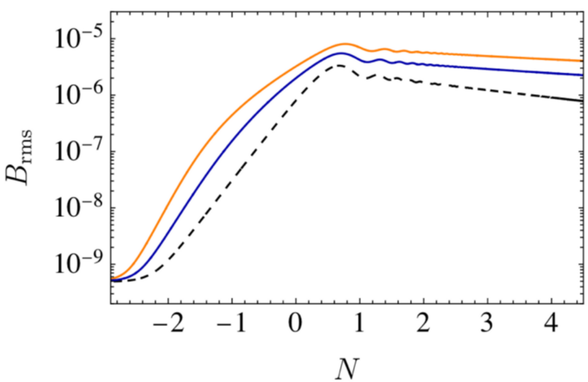
$\alpha/f = 60$
$(\alpha/f)/\sqrt{8\pi} = 12$
$\alpha/f = 90$
$(\alpha/f)/\sqrt{8\pi} = 18$
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
$E_M(k)$
10^{-6}
10^{-5}
10^{-4}
k

$$J = \frac{(e|Q|)^3}{6\pi^2\mathcal{H}} E|B| \coth\left(\frac{\pi|B|}{E}\right) e^{-\frac{\pi m^2 a^2}{e|Q|E}}$$



Universality!





$$E \approx B$$

$$\partial_\tau \boldsymbol{E} - \mathrm{rot}\, \boldsymbol{B} + \frac{\alpha}{f}\left(\partial_\tau \phi \boldsymbol{B} + \boldsymbol{\nabla}\phi \times \boldsymbol{E}\right) + \boldsymbol{J} = 0,$$

$$\partial_\tau \phi = m_{\rm Pl} \mathcal{H} \sqrt{\epsilon / 4\pi}$$

$$\sigma_E \approx (\alpha/f)\partial_\tau \phi$$

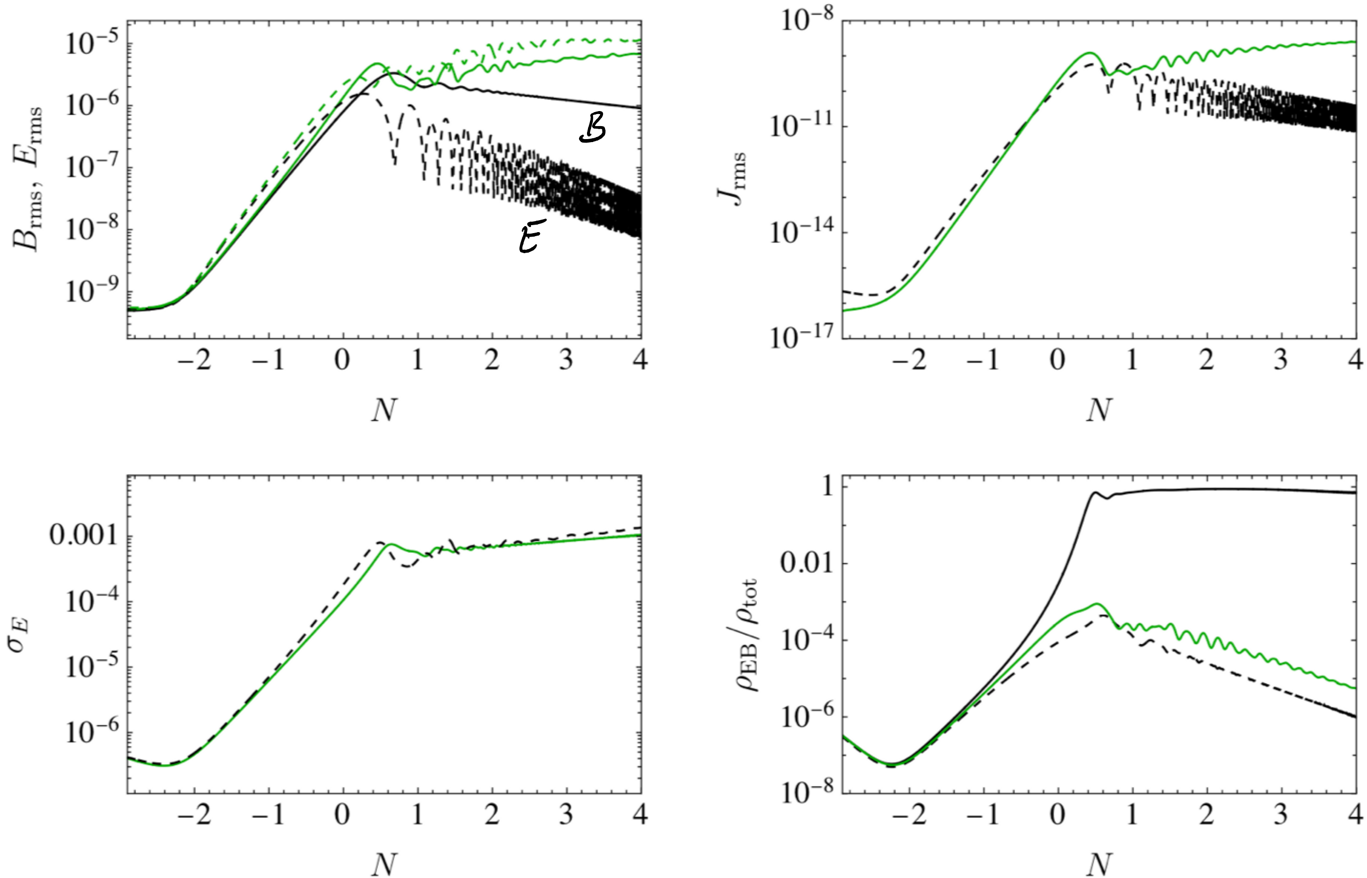
$$(\alpha/f)\partial_\tau \phi \sim \mathcal{O}(100)\mathcal{H}$$

$$\mathcal{H} \sim 10^{-5} m_{\rm Pl}$$

$$\sigma_E \sim 10^{-3} m_{\rm Pl}$$

$$J = \frac{(e|Q|)^3}{6\pi^2\mathcal{H}} E|B| \coth\left(\frac{\pi|B|}{E}\right) \longrightarrow \partial_\tau J = \frac{(e|Q|)^3}{2\pi^2} E|B| \coth\left(\frac{\pi|B|}{E}\right)$$

dynamical current



dyn.

non-dyn.

Conclusions

Backreaction quenches gauge field amplification
Homogeneous & inhomogeneous cases now similar
Universal critical conductivity & field strength
Axion inflation for intergalactic magnetogenesis ruled out
? Large fermion masses?
Dynamical current to be studied further