李政道研究所
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Cosmology of preferred axion models

AXIONS IN STOCKHOLM, WORKSHOP (WEEK 1), JUNE 2025

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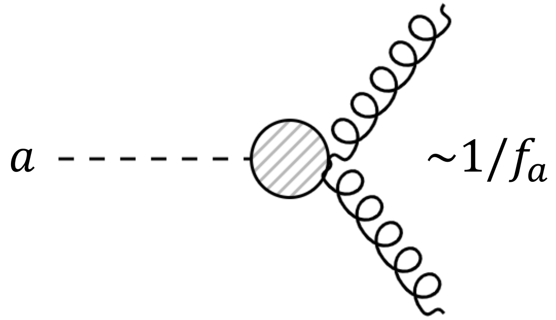
Based on *JCAP* 03 (2024) 061, *JCAP* 03 (2025) 014 and arXiv:2505.04614
With J. Osinski, L. Roszkowski, U. Min, A. Ghoshal and D. Paul

Axion dark matter: a simple solution

- The **QCD** axion provides an elegant solution to two big problems in fundamental physics.
- It dynamically explains why there is no CP violation observed in the strong sector.
- It has two potential mechanisms for producing a dark matter relic.
- All comes from the anomaly term

$$\mathcal{L}^{a}_{\text{eff}} = \frac{a\,(t,x)}{f_a}\frac{\alpha_s}{8\pi}G^{a}_{\mu\nu}\tilde{G}^{\mu\nu}_{a}$$



- Quantum chromodynamics and cosmology do the rest.

Misalignment mechanism

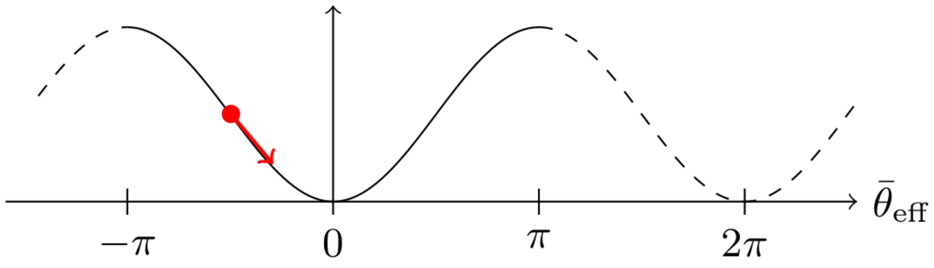
A scalar field such as the axion has the following equation of motion in FLRW

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} + 3H(t)\frac{\mathrm{d}}{\mathrm{d}t}\right)\theta(t) + \tilde{m}_a^2(t)\sin(\theta(t)) = 0$$

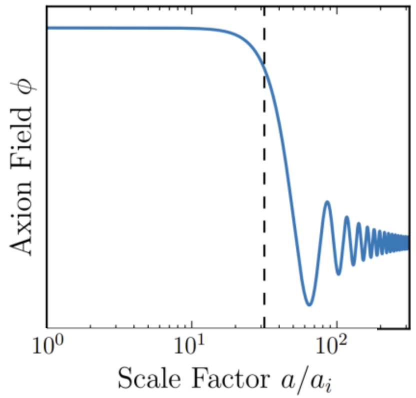
$$\theta(t) = a(t)/f_a$$

$$\tilde{m}_a(T) \simeq m_a \times \begin{cases} 1 & \text{for } T \leq T_{\mathrm{QCD}}\,, \\ \left(\dfrac{T}{T_{\mathrm{QCD}}}\right)^{-4} & \text{for } T \geq T_{\mathrm{QCD}}\,, \end{cases}$$

The initial field value is random



[D. Marsh, axion cosmology]



Axion field redshifts like a matter field at late times

$$\rho_{\mathrm{axion}} \propto a^{-3}$$

QCD axion as dark matter

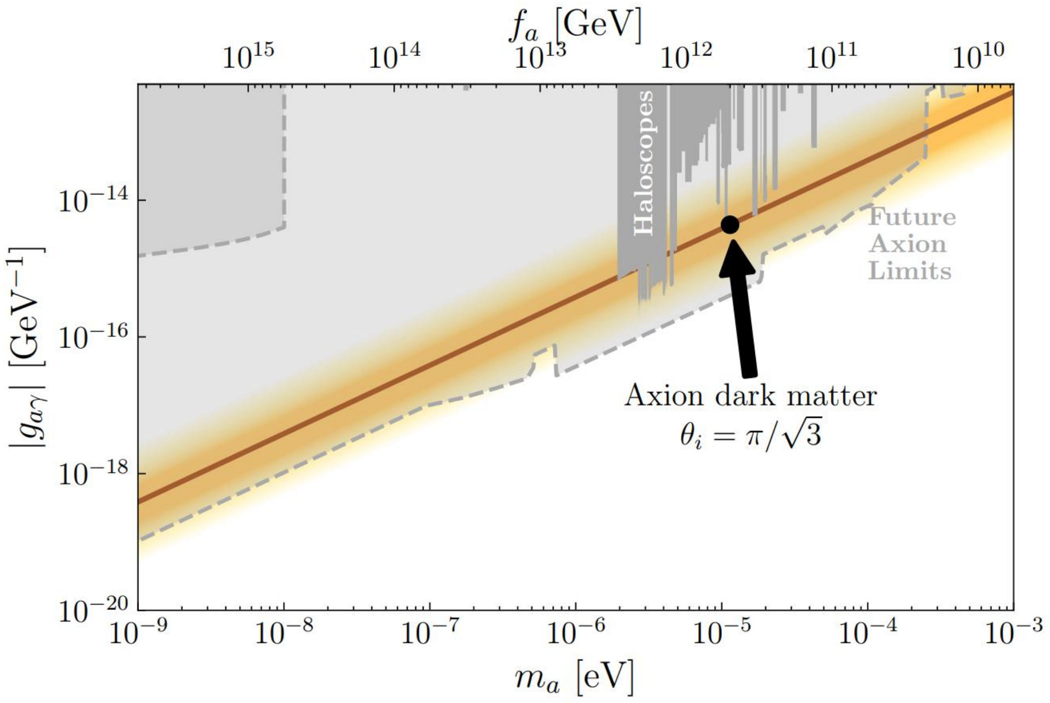
Misalignment production in standard cosmology

$$\Omega_a h^2 \approx 0.12 \left(\frac{\theta_{\rm i}}{2.15}\right)^2 \left(\frac{28\ \mu\text{eV}}{m_a}\right)^{7/6}$$

In the post-inflationary breaking you expect random θ_i in range $[-\pi, \pi)$.

Take random values in each Hubble patch

$$\theta_i \equiv \sqrt{\langle \theta_i^2 \rangle} = \frac{\pi}{\sqrt{3}} \simeq 1.81 \xrightarrow{\text{Anharmonic corrections}} \approx 2.15$$

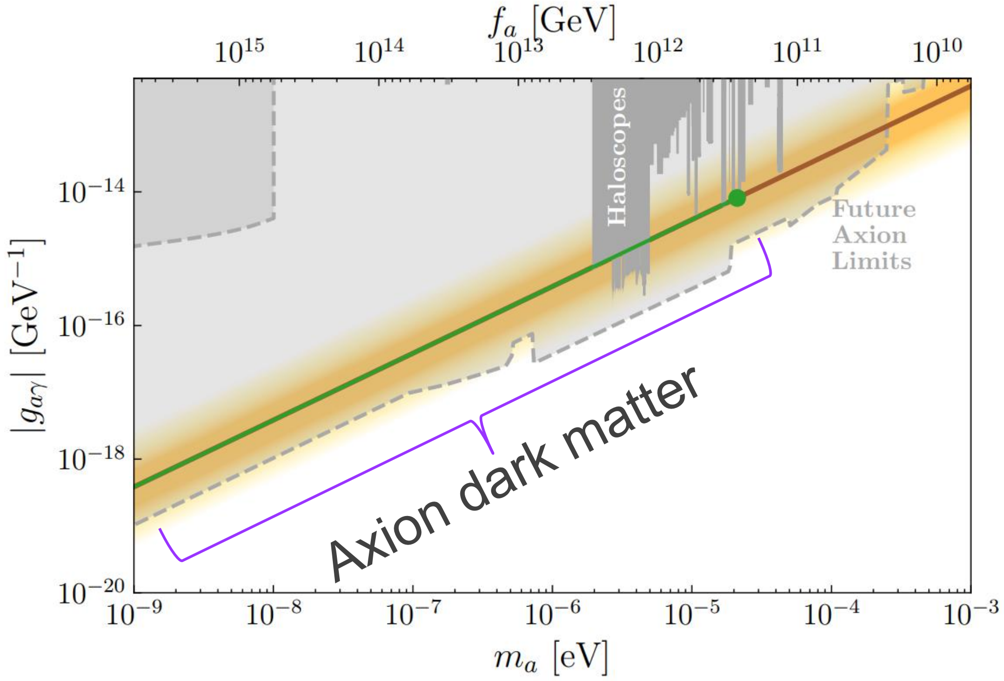


$$m_a = 5.7 \times 10^{-5}\,\text{eV} \left(\frac{10^{11}\,\text{GeV}}{f_a}\right)$$

PQ breaking and inflation

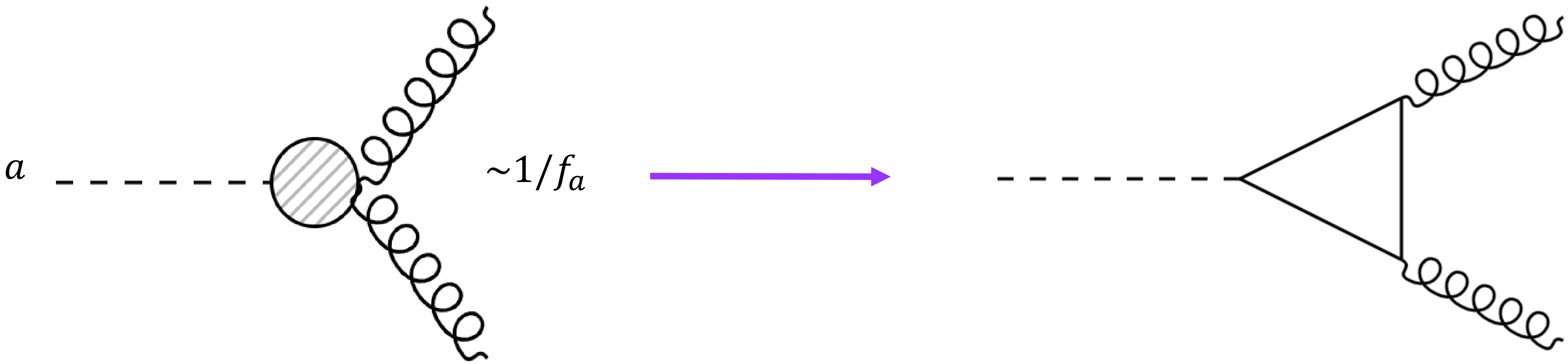
- If PQ symmetry is broken before inflation, the whole observable universe has the same initial angle θ_i, QCD axion could be much lighter.

$$\Omega_a h^2 \approx 0.12 \left(\frac{\theta_{\rm i}}{2.15}\right)^2 \left(\frac{28\ \mu\text{eV}}{m_a}\right)^{7/6}$$



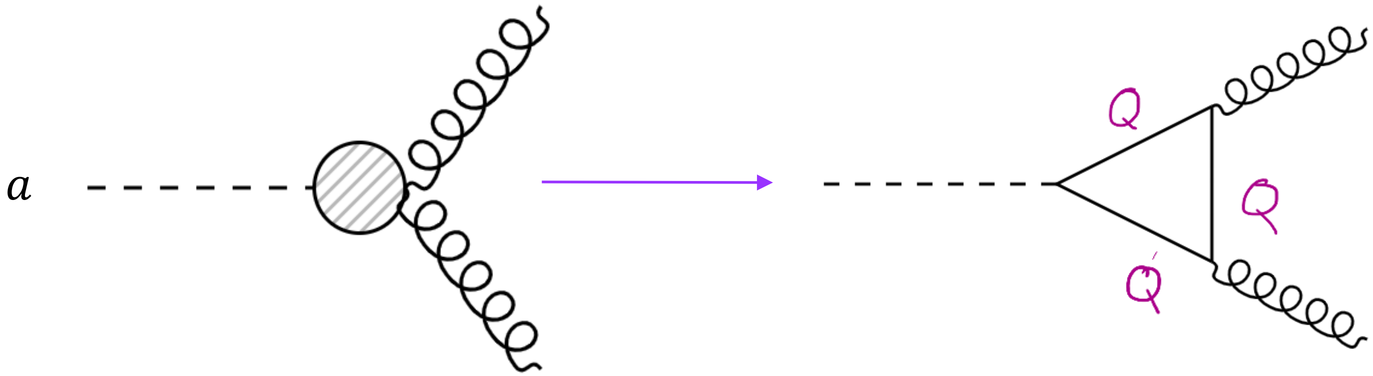
- The discovery of a light axion would be an indication of pre-inflationary PQ breaking.
- Other phenomenological considerations,
  - Thermal axion contributions to dark radiation.
  - Isocurvature bounds on scale of inflation.

Axion dark matter: not so simple



Complicated by completions

$$\mathcal{L}^a_{\text{eff}} = \frac{a\,(t,x)}{f_a}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$



UV completions must involve strongly coupled particles.

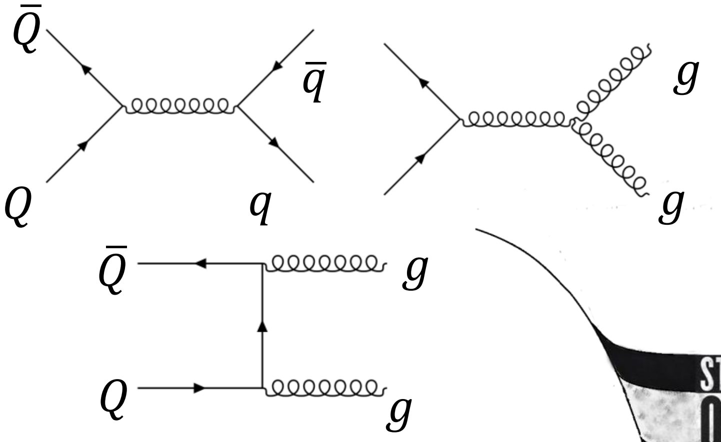
KSVZ: SM fields are $U(1)_{\text{PQ}}$ neutral → Introduce heavy quarks which need to decay

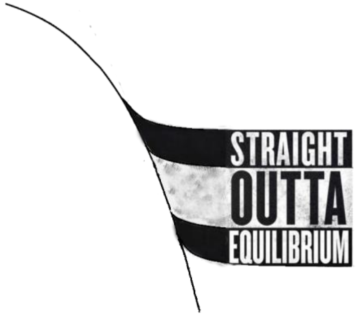
DFSZ: SM fields are charged under $U(1)_{\text{PQ}}$ → suffer from a domain wall problem

These problems can be avoided with pre-inflationary PQ breaking

Heavy quark disaster

These new strongly interacting massive particles undergo thermal freeze-out in a similar way to weak-scale dark matter, but now they are more massive and overproduced!





Assuming stable Q,

$$Y_Q^\infty \approx \frac{x_f}{\lambda} \approx \frac{10\, H(m_Q)}{m_Q^3 \langle \sigma v \rangle}$$

leads to

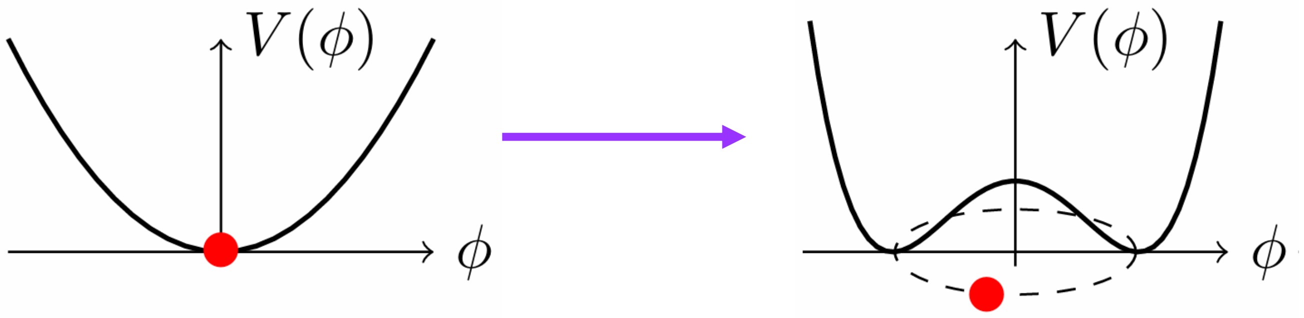
$$\frac{\rho_Q}{\rho_{\rm R}^{\rm SM}} \sim 10^{10} \left(\frac{m_Q}{10^{12}\,{\rm GeV}} \right)^2 \left(\frac{1\,{\rm MeV}}{T} \right)$$

$m_Q \geq 10^7$ GeV, quarks dominate before BBN

How heavy is the quark?

The **KSVZ** quark gets its mass from $U(1)_{\mathrm{PQ}}$ symmetry breaking

$$\mathcal{L}_{\mathrm{PQ}} = |\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q \; - (y_Q \overline{Q}_L Q_R \Phi + \mathrm{H.c})$$



$\Phi \to (v_\varphi/\sqrt{2}) e^{ia/f_a}$ so $m_Q = y_Q f_a/\sqrt{2}$, typical choice is $y_Q \approx 1$.

So $m_Q \sim f_a$

Heavy quarks must decay

- If such heavy quarks will be overabundantly produced via freeze-out they must decay,

$$\mathcal{L}_{\rm PQ} = \underbrace{|\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q}\; - (y_Q \underbrace{\overline{Q}_L Q_R} \Phi + \text{H.c})$$

$$U(1)^3 \equiv U(1)_{Q_L} \times U(1)_{Q_R} \times U(1)_\phi \to U(1)_{\rm PQ} \times U(1)_Q$$

- Must introduce Q-breaking term
- This is only possible for some charge assignments
- For example,

$$R_Q\colon \left(\underset{SU(3)_c}{3}, \underset{SU(2)_W}{1}, \underset{U(1)_Y}{-\frac{1}{3}}\right) \quad \text{and} \quad \overbrace{(\chi_L, \chi_R)}^{\text{PQ charge}} = \left(\frac{1}{2}, -\frac{1}{2}\right) \longrightarrow$$

In this case, decay impossible!

Not many choices for SM charges

Sticking with only renormalizable terms is already quite restrictive, especially if $N_{\mathrm{DW}} = 1$

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{LP}$[GeV]	E/N	N_{DW}
R_1: $(3, 1, -\frac{1}{3})$	$\overline{Q}_L d_R$	$9.3 \cdot 10^{38}(g_1)$	$2/3$	1
R_2: $(3, 1, +\frac{2}{3})$	$\overline{Q}_L u_R$	$5.4 \cdot 10^{34}(g_1)$	$8/3$	1
R_3: $(3, 2, +\frac{1}{6})$	$\overline{Q}_R q_L$	$6.5 \cdot 10^{39}(g_1)$	$5/3$	2
R_4: $(3, 2, -\frac{5}{6})$	$\overline{Q}_L d_R H^\dagger$	$4.3 \cdot 10^{27}(g_1)$	$17/3$	2

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{LP}$[GeV]	E/N	N_{DW}
R_5: $(3, 2, +\frac{7}{6})$	$\overline{Q}_L u_R H$	$5.6 \cdot 10^{22}(g_1)$	$29/3$	2
R_6: $(3, 3, -\frac{1}{3})$	$\overline{Q}_R q_L H^\dagger$	$5.1 \cdot 10^{30}(g_2)$	$14/3$	3
R_7: $(3, 3, +\frac{2}{3})$	$\overline{Q}_R q_L H$	$6.6 \cdot 10^{27}(g_2)$	$20/3$	3

L. Di Luzio et. al.
[arXiv:1610.07593]

From here, can determine distinct models from PQ charges

$$
\begin{aligned}
\mathcal{O}_4^M &= M_d \overline{Q}_L d_R, && \text{for } (\chi_L, \chi_R) = (0, -1), && \text{Model A,} \\
\mathcal{O}_4^H &= y_{1,q} H \overline{q}_L Q_R, && \text{for } (\chi_L, \chi_R) = (1, 0), && \text{Model B,} \\
\mathcal{O}_4^\Phi &= y_{2,d} \Phi \overline{Q}_L d_R, && \text{for } (\chi_L, \chi_R) = (1, 0), && \text{Model B,} \\
\mathcal{O}_4^{\Phi^\dagger} &= y_{3,d} \Phi^\dagger \overline{Q}_L d_R, && \text{for } (\chi_L, \chi_R) = (-1, -2), && \text{Model C.}
\end{aligned}
$$

Not many choices for SM charges

Sticking with only renormalizable terms is already quite restrictive, especially if $N_{\mathrm{DW}} = 1$

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{LP}$[GeV]	E/N	N_{DW}
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$R_3: (3, 2, +\frac{1}{6})$	$\overline{Q}_R q_L$	$6.5 \cdot 10^{39}(g_1)$	$5/3$	2
$R_4: (3, 2, -\frac{5}{6})$	$\overline{Q}_L d_R H^\dagger$	$4.3 \cdot 10^{27}(g_1)$	$17/3$	2

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{LP}$[GeV]	E/N	N_{DW}
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$R_7: (3, 3, +\frac{2}{3})$	$\overline{Q}_R q_L H$	$6.6 \cdot 10^{27}(g_2)$	$20/3$	3

L. Di Luzio et. al. (PRL) [arXiv:1610.07593]

Can also go to the non-renormalizable level to determine the limit?

$$\mathcal{L}_{Qq} = \mathcal{L}^{d\leq 4}_{Qq} + \mathcal{L}^{d>4}_{Qq} = \mathcal{L}^{d\leq 4}_{Qq} + \frac{1}{\Lambda^{(d-4)}}\mathcal{O}^{d>4} + \mathrm{h.c.}$$

This leads to decays which are suppressed by powers of $\Lambda \neq f_a$

$$\Gamma_{d,n_f} = \frac{m_Q}{4\,(4\pi)^{2n_f-3}\,(n_f-1)!\,(n_f-2)!}\left(\frac{m_Q^2}{\Lambda^2}\right)^{d-4}$$

Use Standard Cosmology

Decay terms are limited by

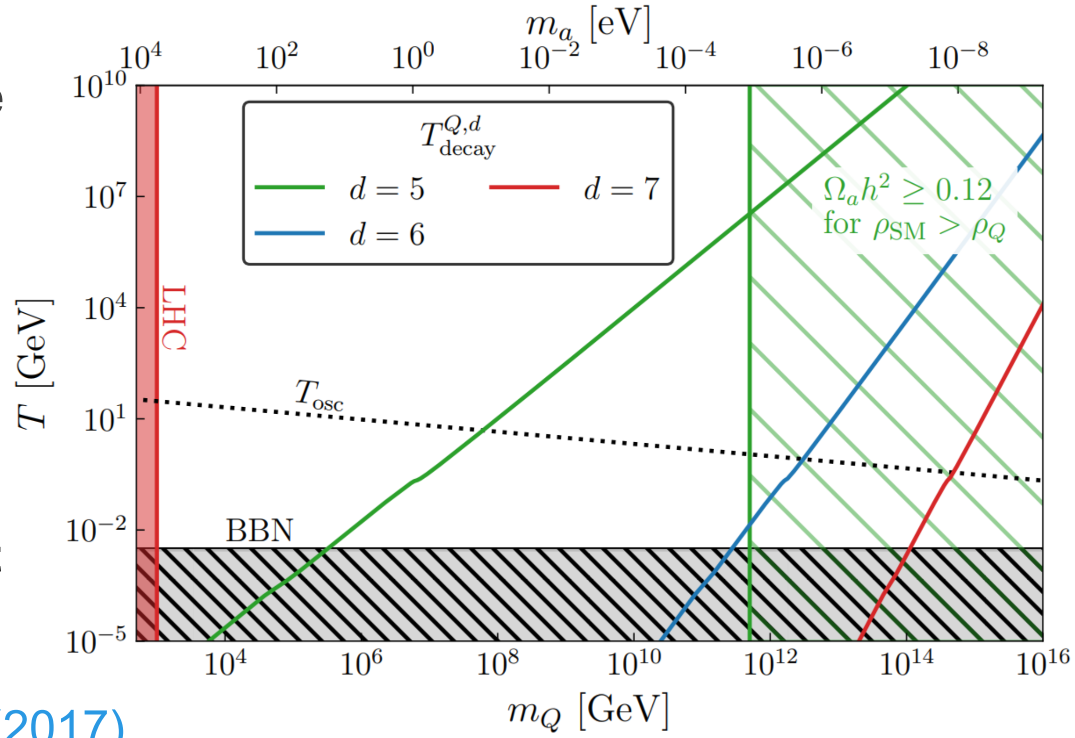
1) Ensuring misalignment doesn't overproduce axions
2) Ensuring Q decay occurs before BBN

approximately

$$\tau \lesssim 0.01\,\mathrm{s}$$

Preferred axion models **decay via dimension 5 at most!**

Put forward by Luzio, Mescia and Nardi in [*PRL* 118 (2017) 3, 031801] and [*PRD* 96 (2017) 7, 075003].



Can explore *preferred* models

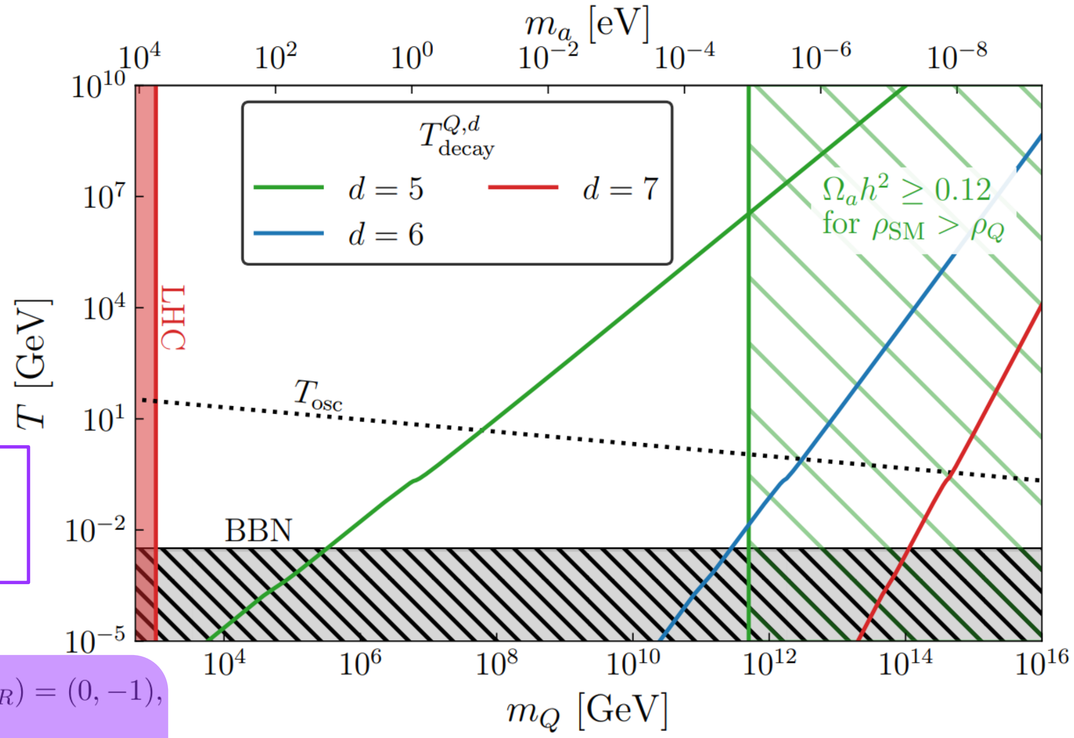
Preferred axion models **decay via dimension 5 at most!**

$$\text{KSVZ–I} : (3,\ 1,\ -1/3), \text{ or}$$
$$\text{KSVZ–II} : (3,\ 1,\ +2/3).$$

$$\mathcal{O}_4^M = M_d \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$
$$\mathcal{O}_4^H = y_{1,d} H \overline{d}_L Q_R, \quad \text{for } (\chi_L, \chi_R) = (1, 0),$$
$$\mathcal{O}_4^\Phi = y_{2,d} \Phi \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (1, 0),$$
$$\mathcal{O}_4^{\Phi^\dagger} = y_{3,d} \Phi^\dagger \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (-1, -2),$$
$$\mathcal{O}_5^\Phi = \frac{\lambda_{2,d}}{\Lambda} \Phi^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (2, 1),$$
$$\mathcal{O}_5^{\Phi H} = \frac{\lambda'_{2,d}}{\Lambda} \overline{Q}_R q_L H^\dagger \Phi, \quad \text{for } (\chi_L, \chi_R) = (2, 1),$$
$$\mathcal{O}_5^{\Phi^\dagger} = \frac{\lambda_{3,d}}{\Lambda} (\Phi^\dagger)^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (-2, -3),$$

5 models for each KSVZ model type

$$\mathcal{O}_5^{|H|^2} = \frac{\lambda_d}{\Lambda} |H|^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$
$$\mathcal{O}_5^{|\Phi|^2} = \frac{\lambda'_d}{\Lambda} |\Phi|^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$
$$\mathcal{O}_5^H = \frac{\lambda_{1,d}}{\Lambda} \Phi H \overline{d}_L Q_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$



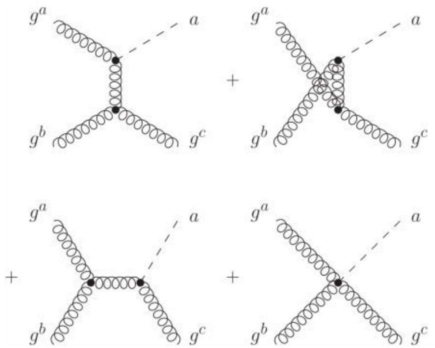
[arXiv:2411.17320]
AC + Ui Min

Decay of heavy quarks

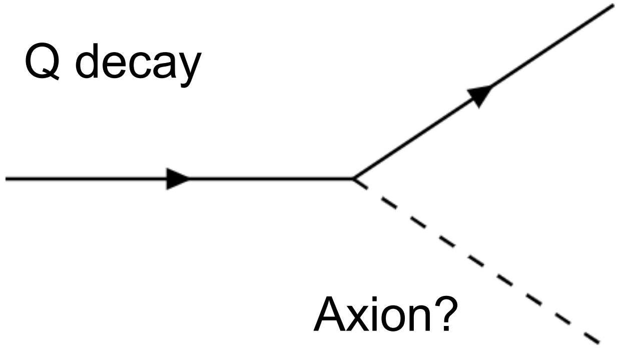
- Heavy quark decay products may leave a trace in the early universe.

$$\frac{\mathrm{d}\rho_a}{\mathrm{d}t} = -4H\rho_a + \mathrm{BR}_{\mathrm{axion}}\Gamma_Q\rho_Q + \langle E^{\mathrm{axion}}_{\mathrm{scat}}\rangle\gamma_a\left(1 - \frac{n_a}{n_a^{\mathrm{eq}}}\right),$$

$$\frac{\mathrm{d}n_Q}{\mathrm{d}t} = -3Hn_Q - \Gamma_Q n_Q - \langle\sigma\, v\rangle\left[n_Q^2 - (n_Q^{\mathrm{eq}})^2\right].$$



Q decay



Axion?
Dark radiation?

$T^{\mathrm{decoupling}}_{\mathrm{axion}} < T^{\mathrm{decay}}_{Q}$?

P. Graf et. al. (PRD) [arXiv:1008.4528] + more

Light remnants of heavy quarks

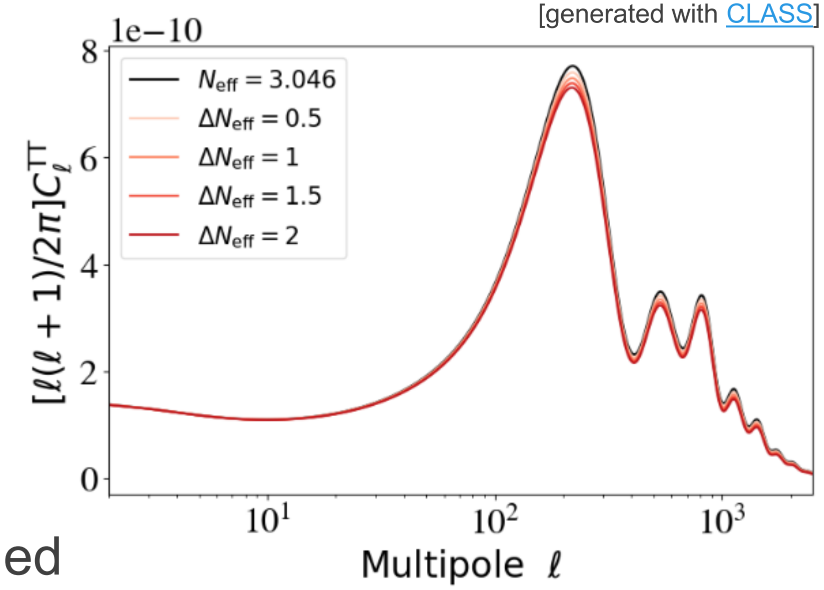
In the standard picture of the Big Bang, we have two particles species that remain relativistic until recombination.

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu$$

$$\rho_{\rm rad} = \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{\rm eff}\right]$$

$$(N_{\rm eff})_{\rm P18} = 2.88^{+0.44}_{-0.42}$$

[generated with CLASS]



Consistent with 3 light neutrinos, additions constrained

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu + \rho_{\rm DR} \longrightarrow \Delta N_{\rm eff} \equiv \left\{\frac{8}{7}\left(\frac{4}{11}\right)^{-\frac{4}{3}} + N_{\rm eff}^{\rm SM}\right\}\frac{\rho_a}{\rho_{\rm R}^{\rm SM}}$$

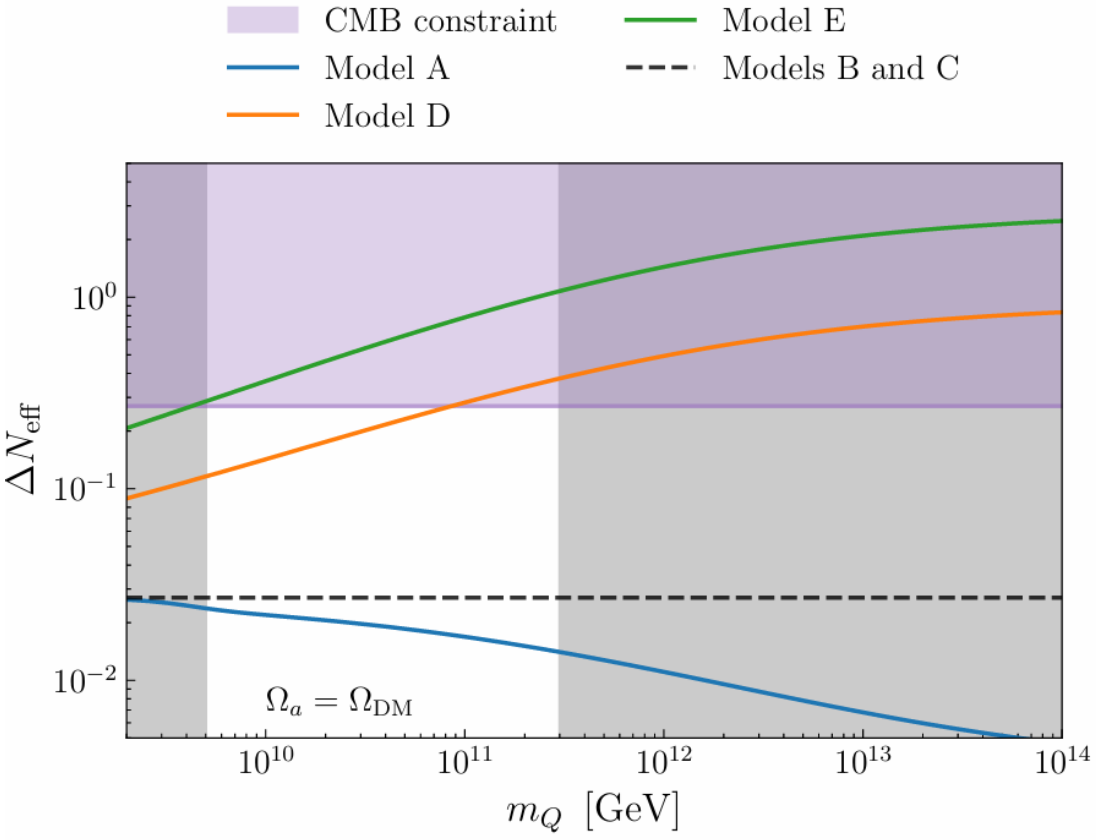
$$\Delta N_{\rm eff} \leq 0.276$$

CMB disfavors some models

Decay	Model A	Model B	Model C	Model D	Model E
$\Gamma(Q \to a\, d) = \frac{3}{32\pi}\mathcal{C}^2 m_Q$	—	$\frac{y_{2,d}}{\sqrt{2}}$	$\frac{y_{3,d}}{\sqrt{2}}$	$\frac{\lambda_{2,d} f_a}{\Lambda}$	$\frac{\lambda_{3,d} f_a}{\Lambda}$
$\Gamma(Q \to H\, q_L) = \frac{3}{32\pi}\mathcal{C}^2 m_Q$	$\frac{\lambda_{1,q} f_a}{\Lambda}$	$y_{1,q}$	—	$\frac{\lambda_{2,q} f_a}{\Lambda}$	—
$\Gamma(Q \to a\, a\, d) = \frac{1}{256\pi^3}\mathcal{C}^2 m_Q^3$	—	$\frac{y_{2,d}}{2 f_a}$	$\frac{y_{3,d}}{2 f_a}$	—	$\frac{\lambda_{3,d}}{\Lambda}$
$\Gamma(Q \to H\, H\, d) = \frac{1}{256\pi^3}\mathcal{C}^2 m_Q^3$	$\frac{\lambda_d}{\Lambda}$	—	—	—	—
$\Gamma(Q \to H\, q_L\, a) = \frac{1}{512\pi^3}\mathcal{C}^2 m_Q^3$	$\frac{\lambda_{1,q}}{\Lambda}$	—	—	$\frac{\lambda_{2,q}}{\Lambda}$	—
M_d	M_d	$\frac{y_{2,d} f_a}{\sqrt{2}}$	$\frac{y_{3,d} f_a}{\sqrt{2}}$	$\frac{\lambda_{2,d} f_a^2}{2\Lambda}$	$\frac{\lambda_{3,d} f_a^2}{2\Lambda}$

[arXiv:2411.17320]

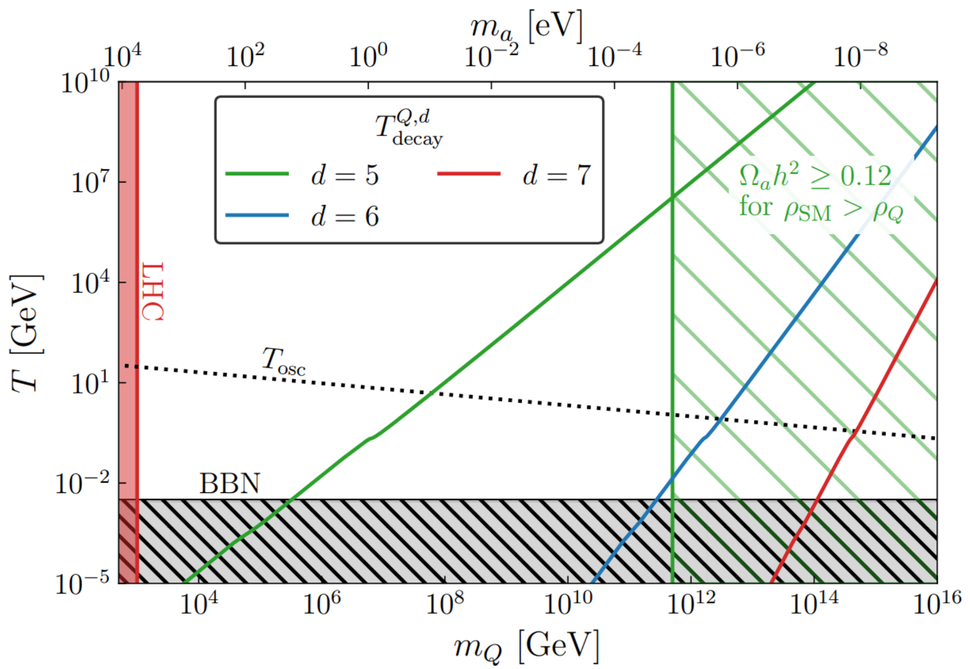
AC + Ui Min (JCAP)

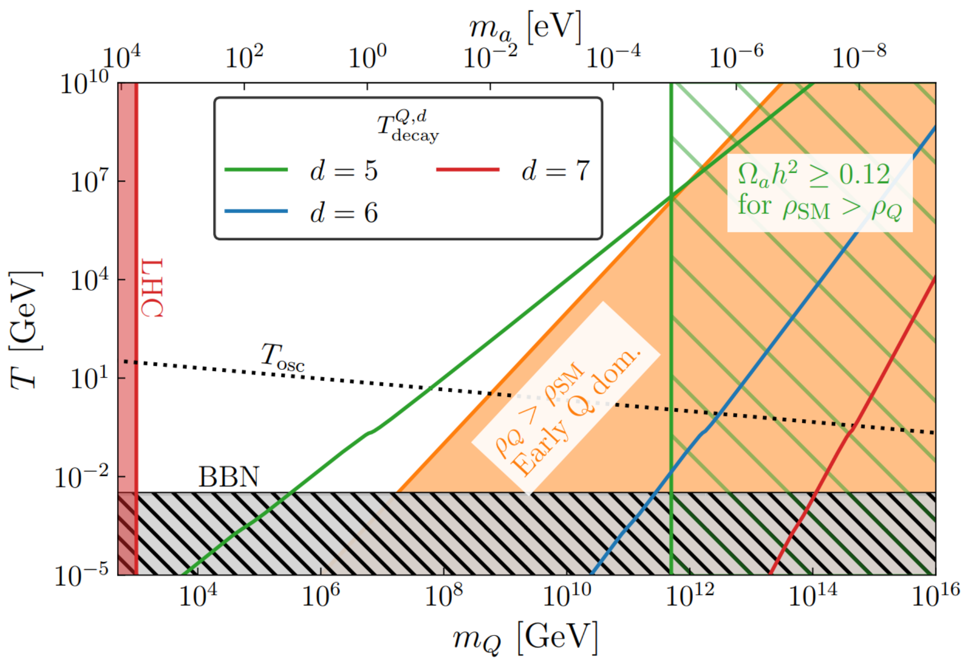


Preferred axion models too restrictive

- Constraints on heavy quark decay terms assumed standard cosmology and ignored impact of heavy quarks themselves on the cosmology

$$\frac{\rho_Q}{\rho_{\rm R}^{\rm SM}} \sim 10^{10} \left(\frac{m_Q}{10^{12}\,{\rm GeV}}\right)^2 \left(\frac{1\,{\rm MeV}}{T}\right)$$

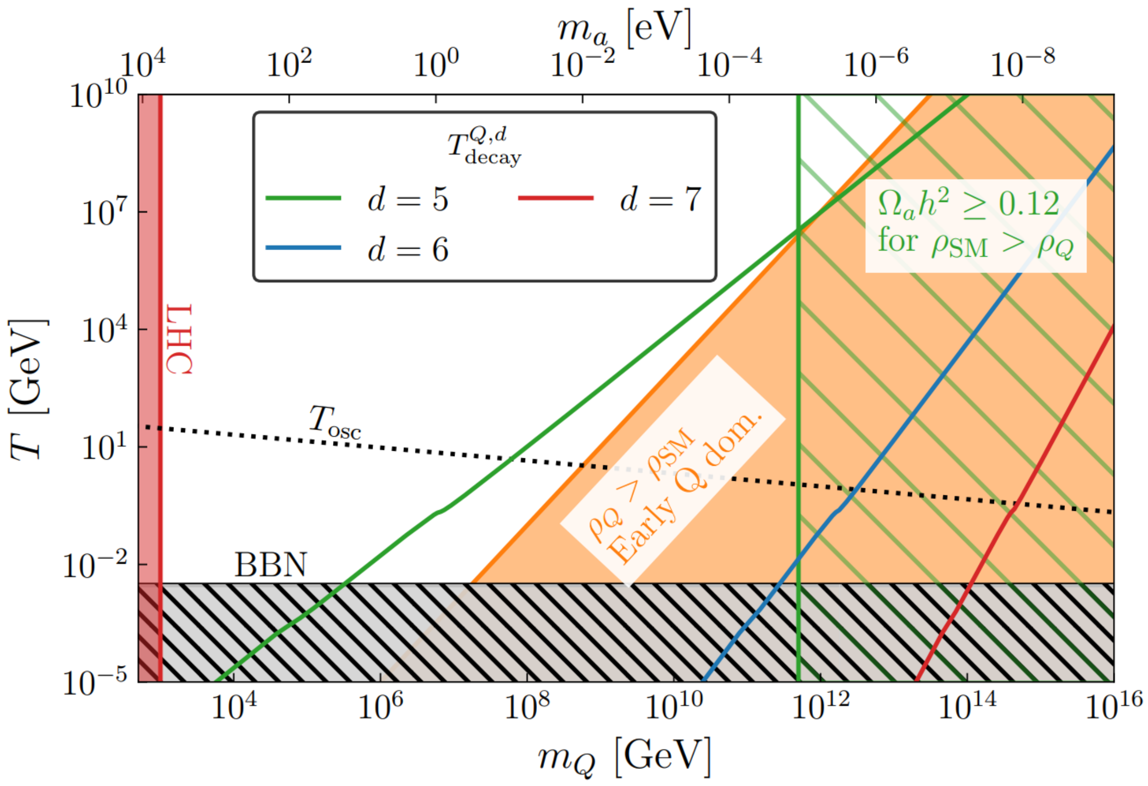




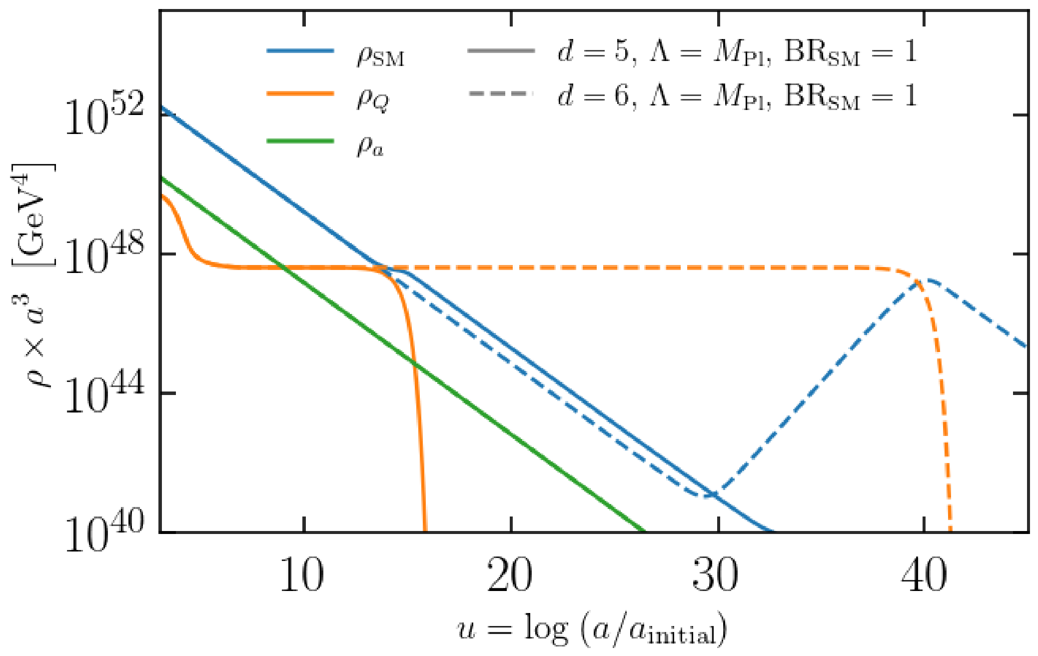
Heavy quark domination

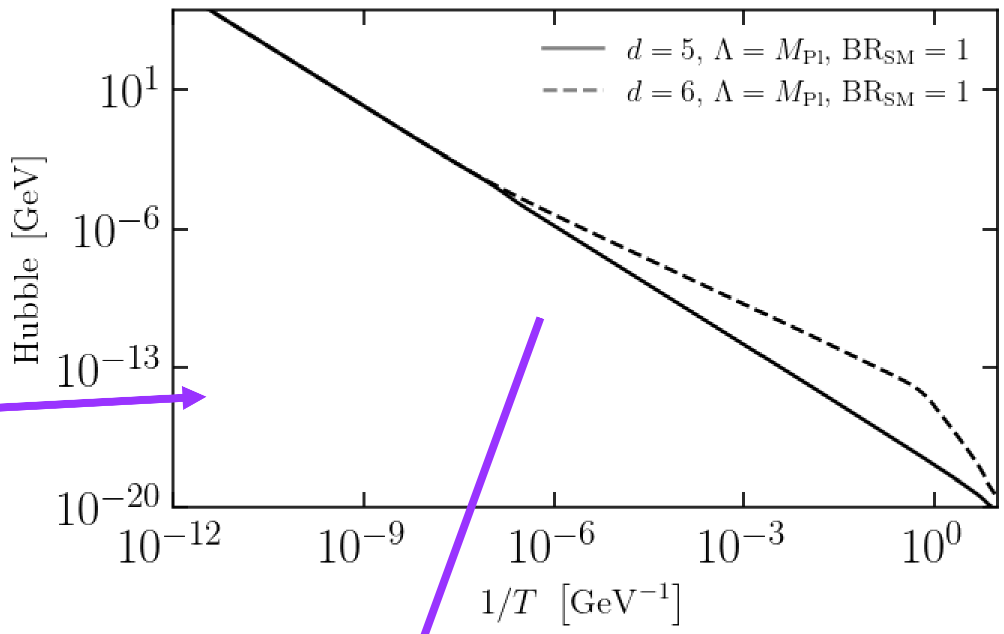
- For these higher dimensional Q decay models, the heavy quarks will dominate the early universe.
- This alters the misalignment mechanism, has been known for decades *Steinhart* et. al. (1984) + *Lazarides* et. al. (1990)
- We show T_{osc}, temperature when axion field oscillations begin

$$3H\left(T_{\mathrm{osc}}\right) = \tilde{m}_a\left(T_{\mathrm{osc}}\right)$$



First you approximate, then you solve





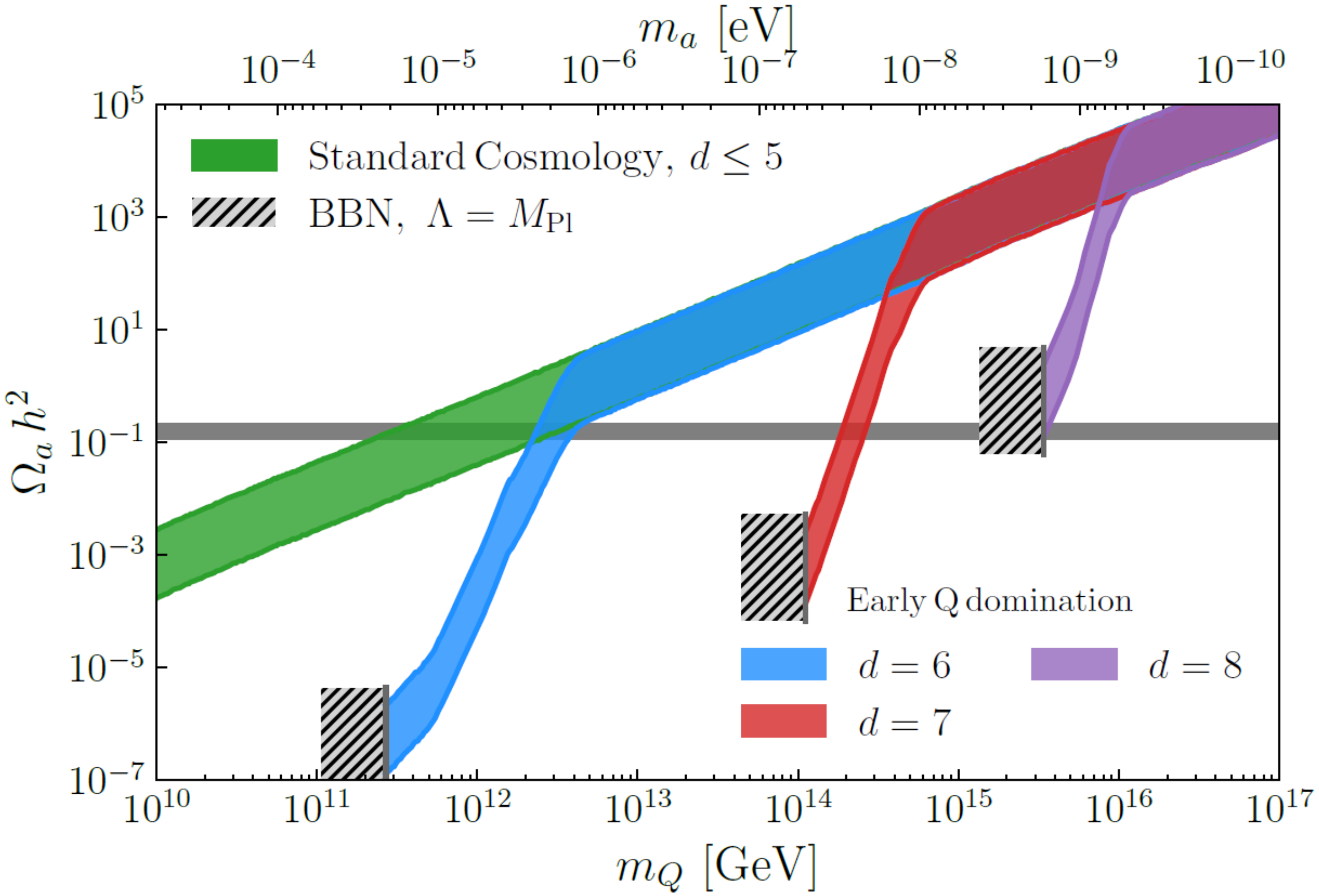
Scale factor

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}+3H(t)\frac{\mathrm{d}}{\mathrm{d}t}\right)\theta(t)+\tilde{m}_a^2(t)\sin(\theta(t))=0$$

[MiMes] misalignment solver

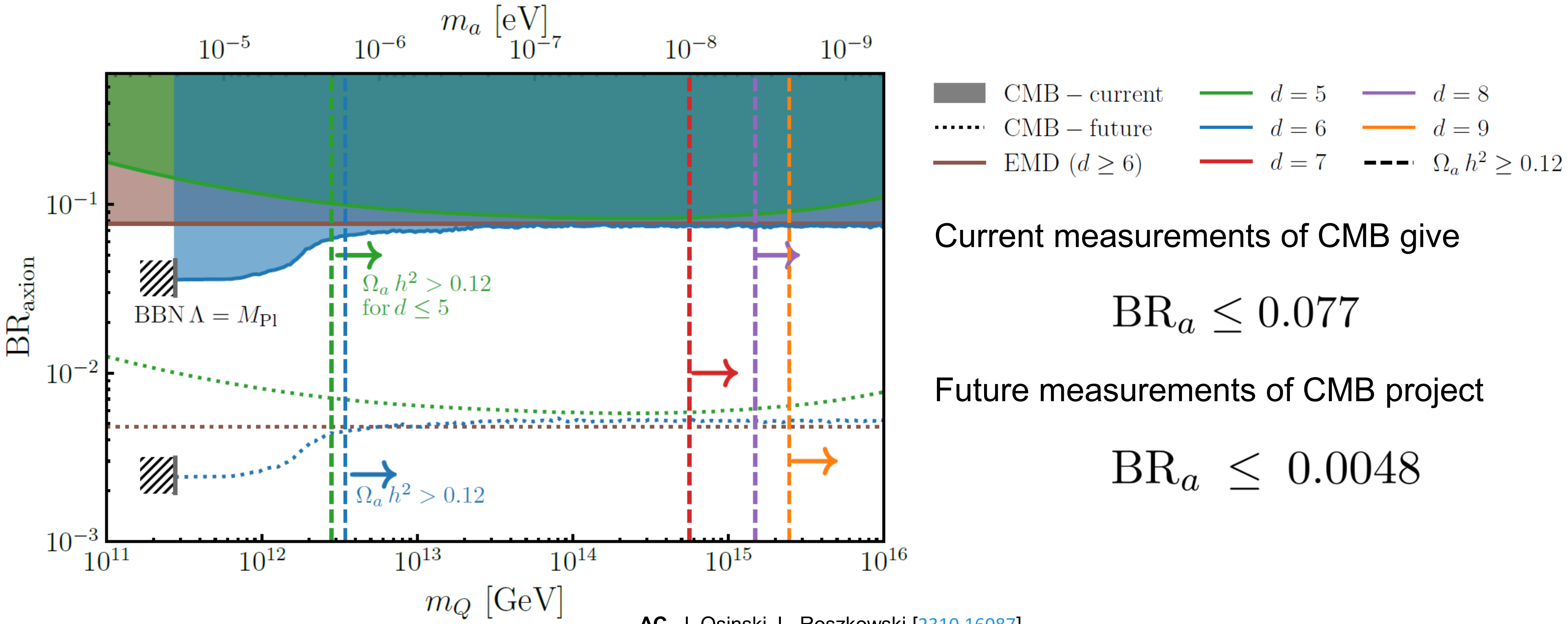
Heavy quark domination dilutes Ω_a

- We were the first to point out the axion models themselves could provide this phase.
- Plotting band of different initial angles $\theta_i \in \left[\frac{1}{2}, \frac{\pi}{\sqrt{3}}\right]$
- Dimension 6-7-8 now are viable.
- More axion models available and parameter space.



AC, J. Osinski, L. Roszkowski [2310.16087]

Constraints from dark radiation



Current measurements of CMB give

$$\mathrm{BR}_a \leq 0.077$$

Future measurements of CMB project

$$\mathrm{BR}_a \leq 0.0048$$

AC, J. Osinski, L. Roszkowski [2310.16087]

More models without domain walls

Recently, Di Luzio et. al. confirmed my findings and catalogued higher dimensional models

L. Di Luzio et. al. [arXiv:2412.17896]

Rep. $(\mathcal{C},\mathcal{I},6\mathcal{Y})$			E/N	$N_{\rm DW}$	Min. d	Example operator	LP [GeV]
3	1	−2	2/3	1	3	$\bar{\mathcal{Q}}_L\, d_R$	2.0×10^{39}
3	1	4	8/3	1	3	$\bar{\mathcal{Q}}_L\, u_R$	6.8×10^{35}
3	1	−14	98/3	1	6	$\bar{\mathcal{Q}}_L\, d_R\,(\bar{e}^c_R\, e_R)$	2.2×10^{22}
$\overline{3}$	1	8	32/3	1	6	$\bar{u}_R\,\gamma_\mu\, e_R\,\bar{d}_R\,\gamma^\mu\,\mathcal{Q}_R$	3.0×10^{28}
$\overline{3}$	1	−10	50/3	1	6	$(\bar{d}_R\, d^c_R)\,\bar{e}_R\,\mathcal{Q}_L$	6.4×10^{25}
3	1	16	128/3	1	6	$\bar{\mathcal{Q}}_L\, u_R\,(\bar{e}_R\, e^c_R)$	1.8×10^{21}
$\overline{3}$	1	20	200/3	1	9	$(\bar{d}^c_R\, d_R)\,(\bar{e}^c_R\, e_R)\,\bar{u}_R\,\mathcal{Q}_L$	6.2×10^{19}
3	1	22	242/3	1	9	$\bar{\mathcal{Q}}_L\, u_R\,(\bar{\ell}_L\,\ell^c_L)\,(\bar{e}_R\, e^c_R)$	2.0×10^{19}

$\mathrm{Br}_{\mathrm{SM}} \approx 1$

$\therefore\ \Delta N_{\mathrm{eff}} \ll 0.027$

$$g_{a\gamma} \equiv \frac{\alpha}{2\pi}\frac{1}{f_a}\left(\frac{E}{N} - 1.92(4)\right)$$

GUT-scale PQ breaking & $N_{\mathrm{DW}} = 1$

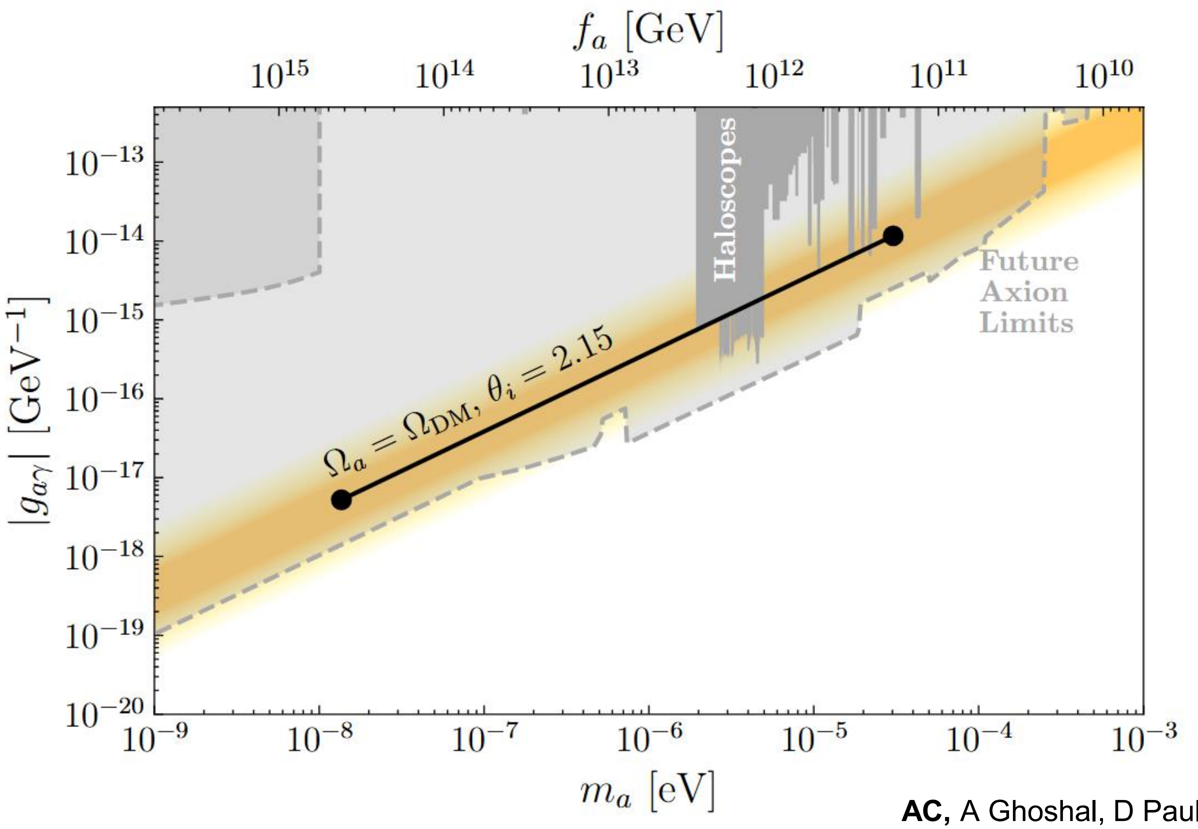
Previously taken $m_Q = f_a$ and $\Lambda = M_{\mathrm{pl}}$

Can relax this and obtain order of magnitude lower mass.

The plot shows dimension 6 models

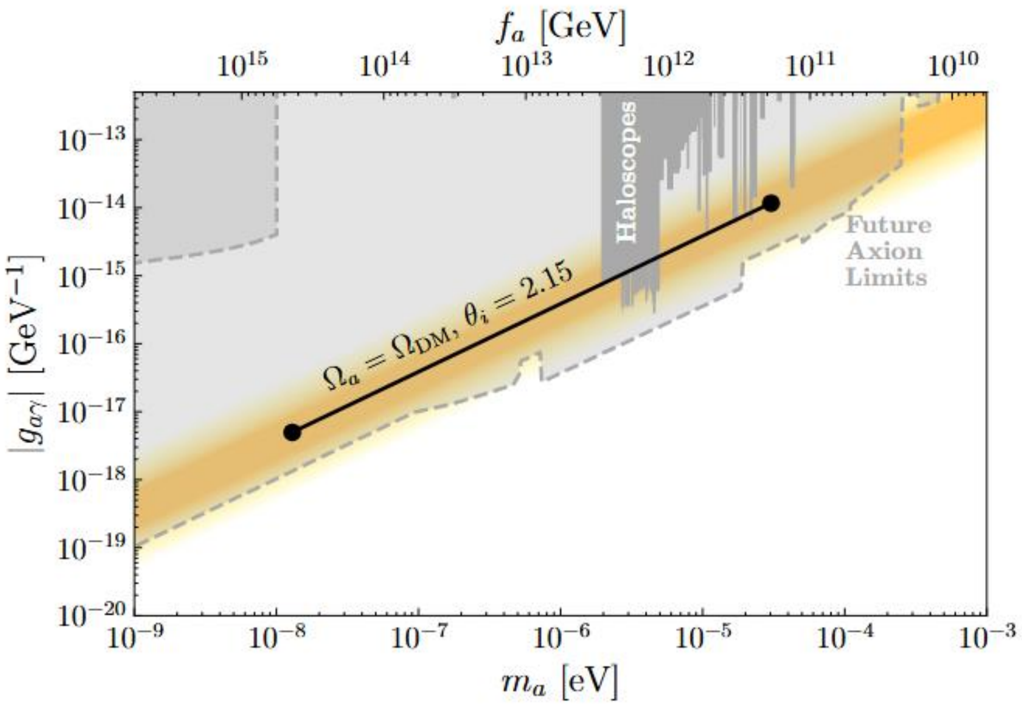
The point with smallest m_a corresponds to:

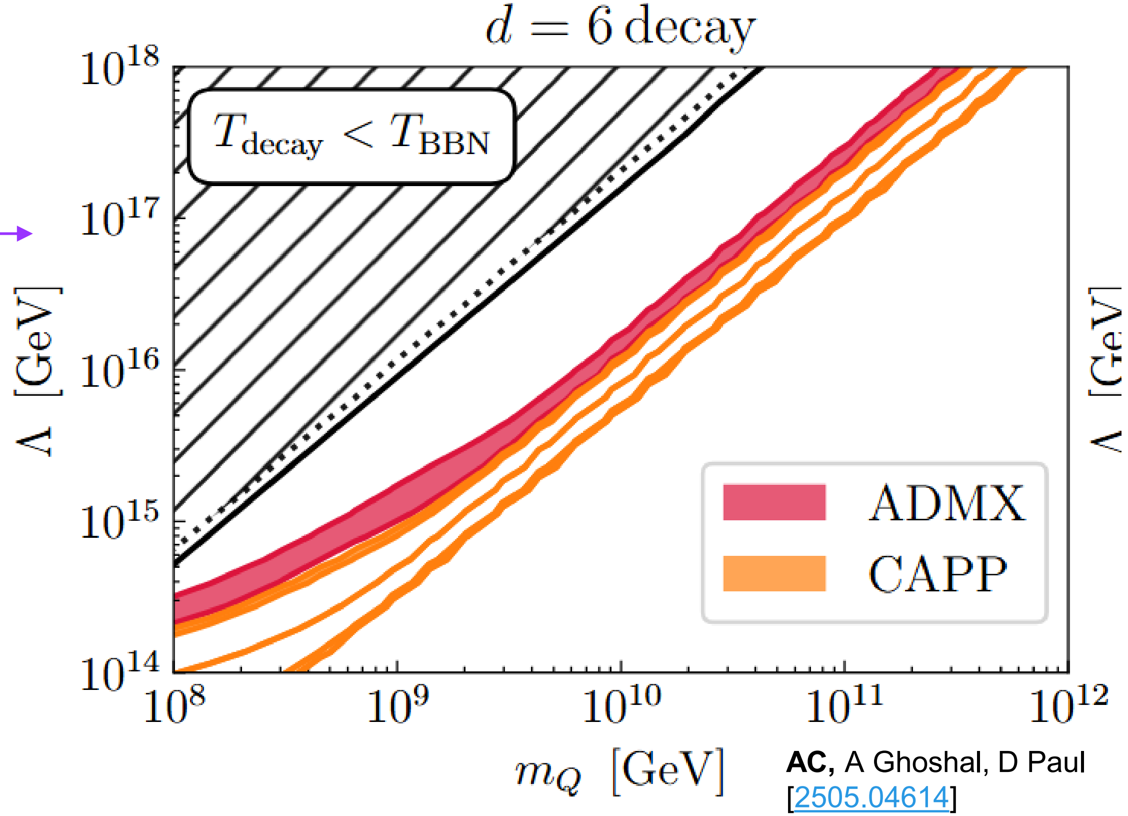
$$f_a = 4 \times 10^{14}\ \mathrm{GeV}$$
$$m_Q = 4 \times 10^{11}\ \mathrm{GeV}$$
$$\Lambda = 4 \times 10^{18}\ \mathrm{GeV}$$



AC, A Ghoshal, D Paul
[2505.04614]

GUT-scale PQ breaking & $N_{\mathrm{DW}} = 1$





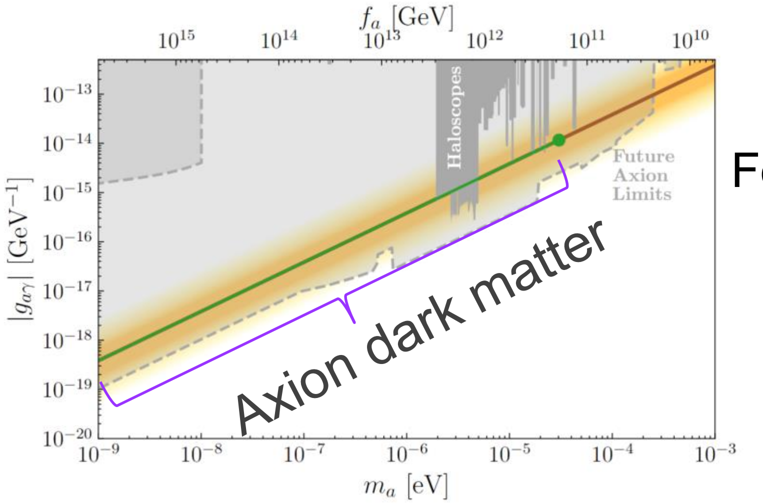
AC, A Ghoshal, D Paul [2505.04614]

When does PQ break?

BEFORE INFLATION

Can have $m_a \leq 10\ \mu\text{eV}$

No detectable dark radiation component from axion.



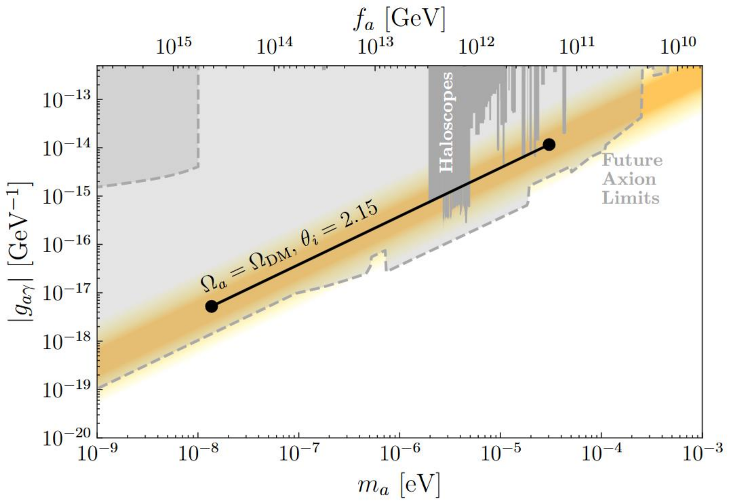
For $m_a = 10^{-8}$ eV

$\theta_i = 10^{-2}$

AFTER INFLATION

Now can have $m_a \leq 10\ \mu\text{eV}$ with HQD

The only models that survive with have no detectable dark radiation component from axion.



Both scenarios have the same phenomenological output.

Primordial GWs as a tracer of HQD

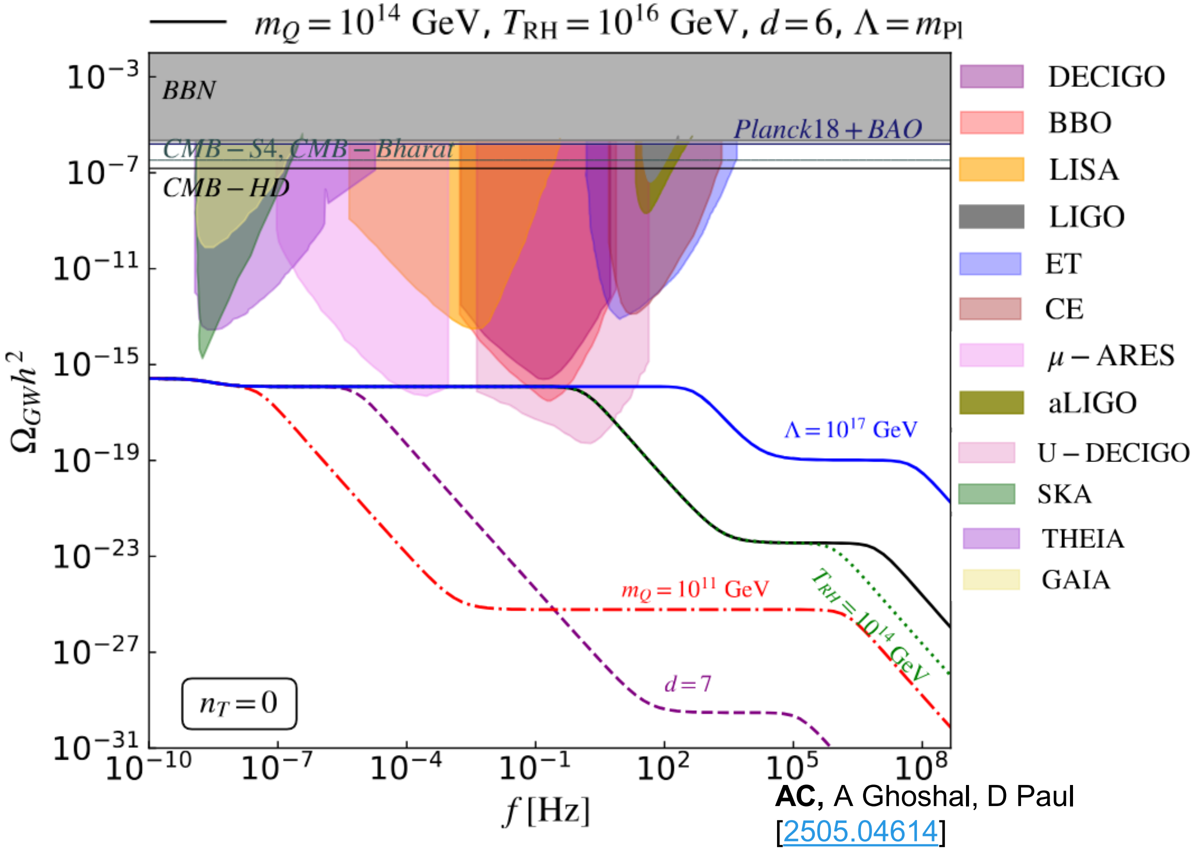
- These are tensor perturbations from inflation, its power spectrum parameterized by

$$P_T^{\text{prim.}}(k) = A_T(k)\left(\frac{k}{k_*}\right)^{n_T}$$

- Current constraints on tensor to scalar ratio

$$r = \frac{A_T}{A_S} < 0.036$$

- The tilt n_T is determined in vanilla slow-roll inflation to be $n_T = -\frac{r}{8}$.



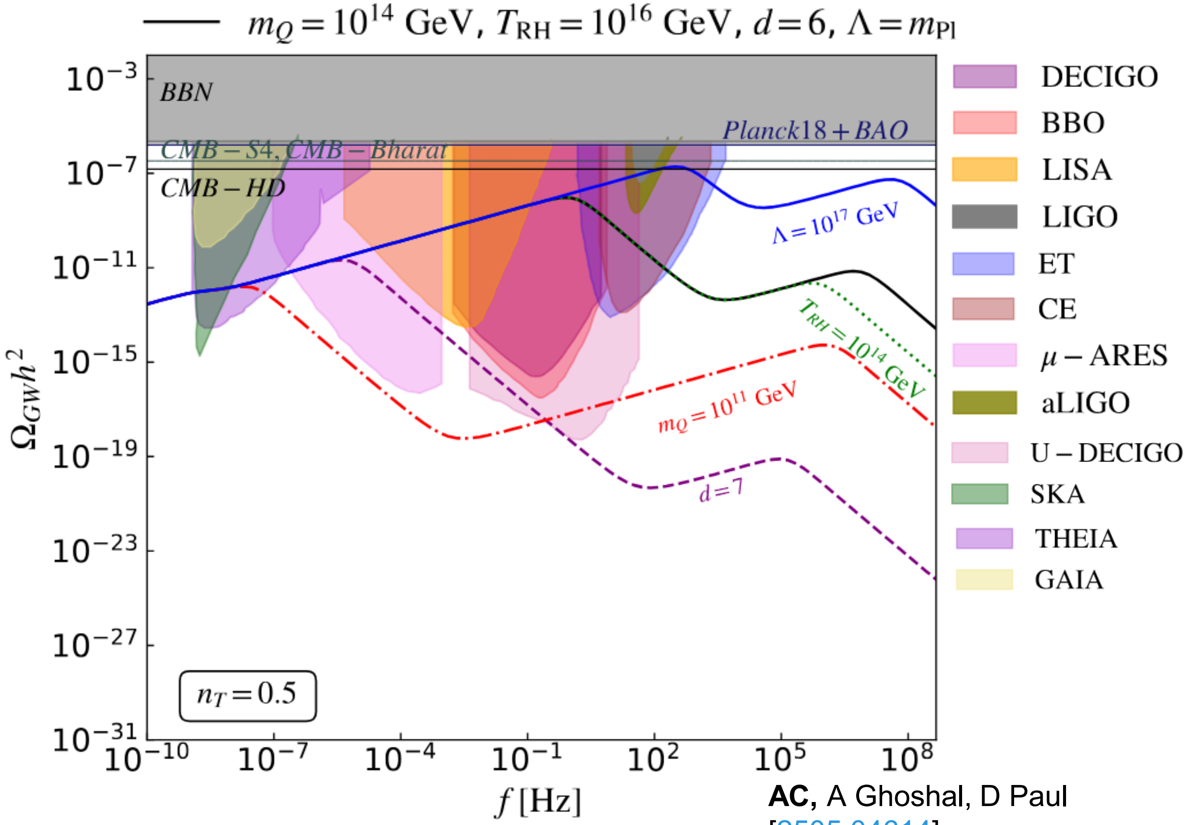
AC, A Ghoshal, D Paul [2505.04614]

Maximum (blue-)tilt

- We take an optimistically blue-tilted power spectrum to assess the maximum sensitivity of GW experiments.
- The sensitivity plots shown on the right are just illustrative (from GWplotter for example)
- We perform our forecasts using the signal-to-noise ratio for each detector

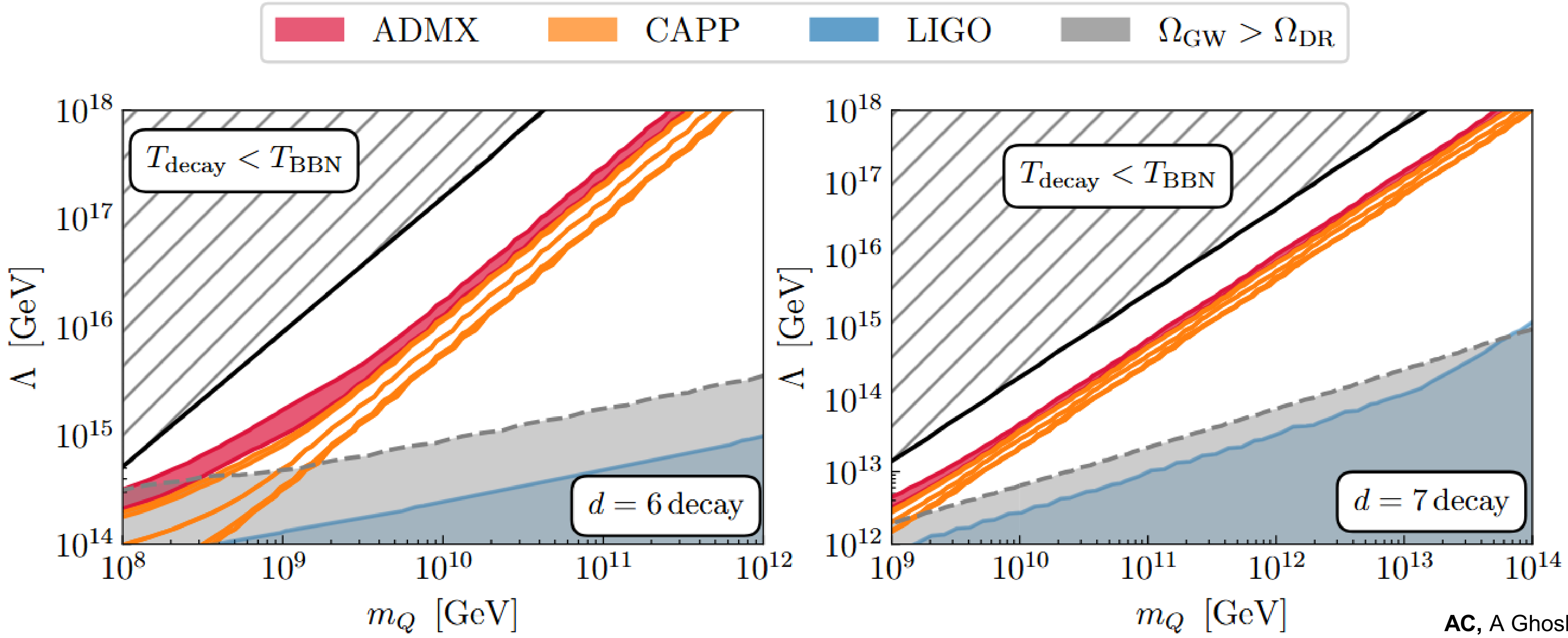
$$\mathrm{SNR} \equiv \sqrt{\tau_{\mathrm{obs}} \int_{f_{\min}}^{f_{\max}} \mathrm{d}f \left(\frac{\Omega_{\mathrm{GW}}(f, \{\theta\})h^2}{\Omega_{\mathrm{GW}}^{\mathrm{noise}}(f)h^2} \right)^2}$$

Detectors	Frequency range	τ_{obs}
SKA	$[10^{-9} - 4 \times 10^{-7}]$ Hz	15 years
μ-ARES	$[10^{-7} - 1]$ Hz	4 years
LISA	$[10^{-4} - 1]$ Hz	4 years
BBO	$[10^{-3} - 7]$ Hz	4 years
ET	$[1 - 10^{3}]$ Hz	5 years



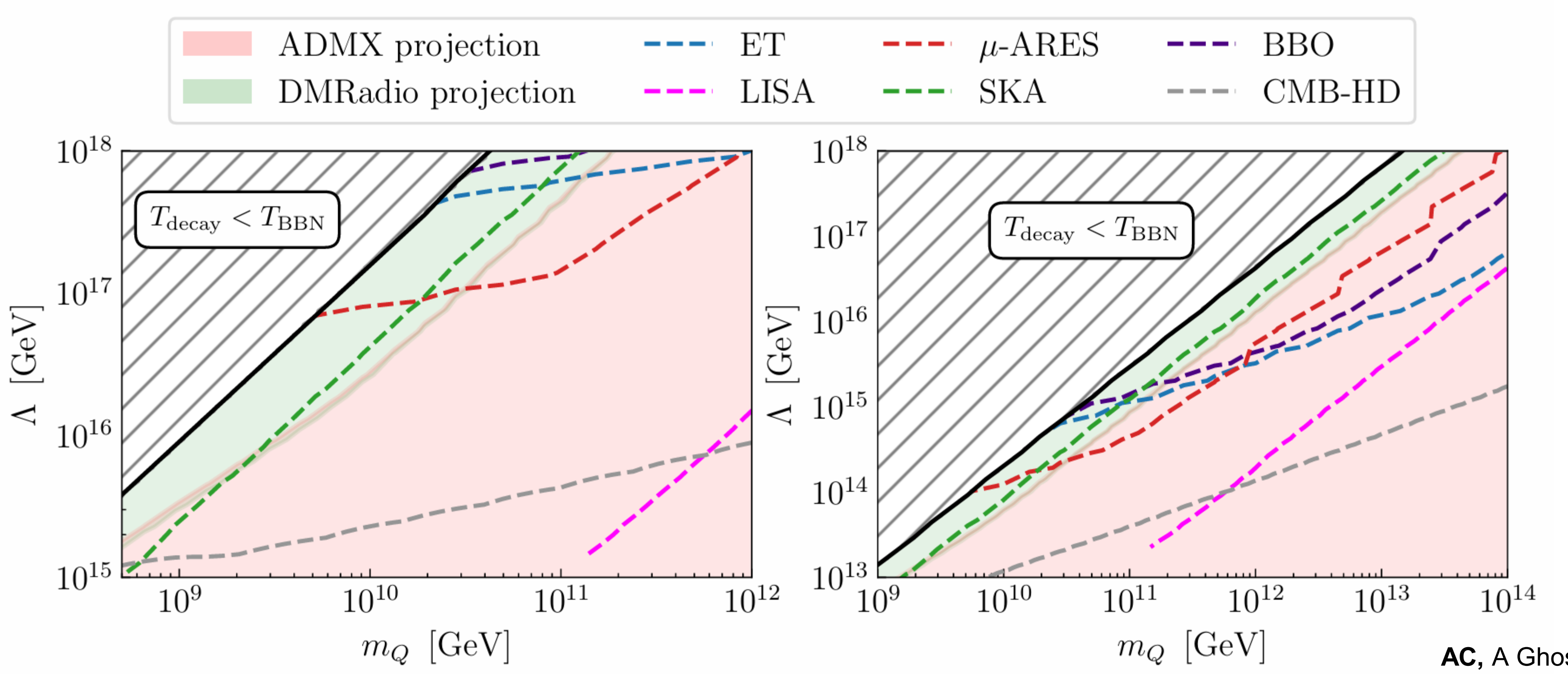
AC, A Ghoshal, D Paul [2505.04614]

Current GW-axion landscape



AC, A Ghoshal, D Paul
[2505.04614]

Future prospects



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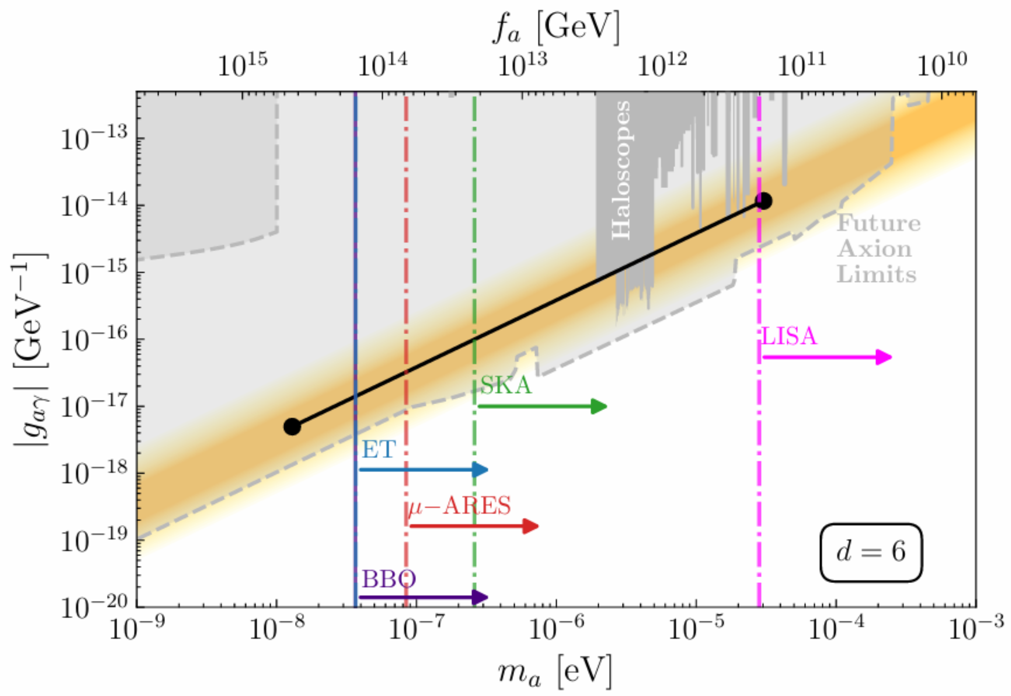
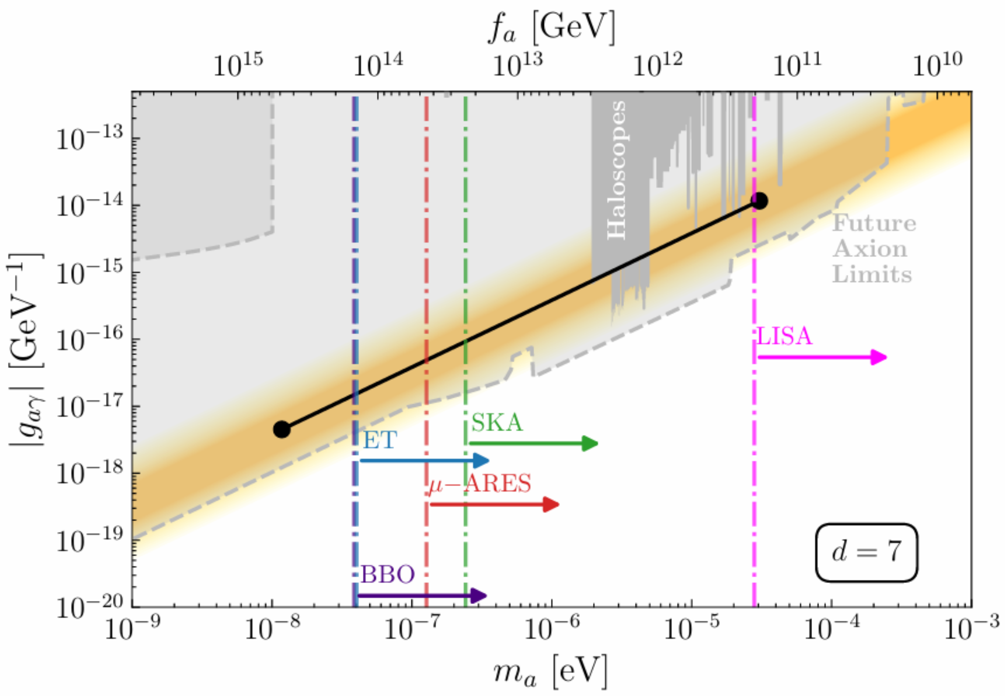
Where GWs will be able to probe

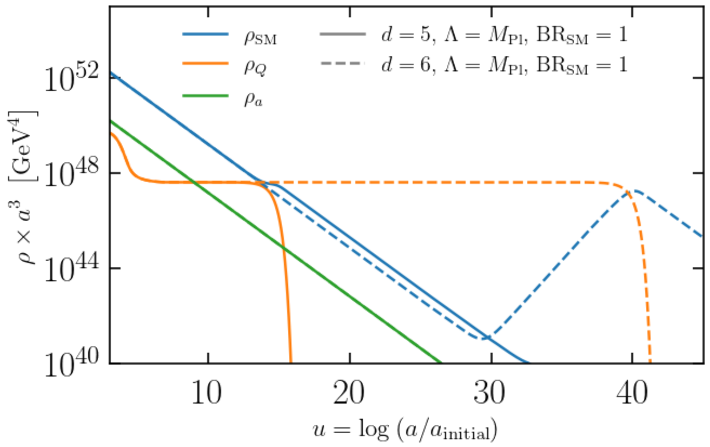


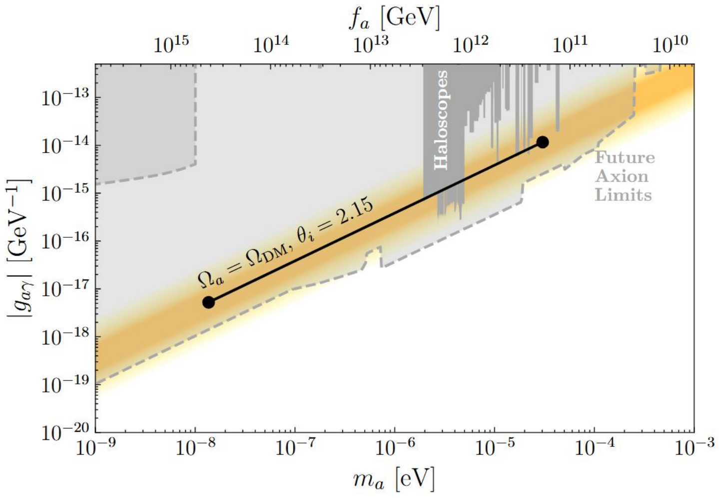


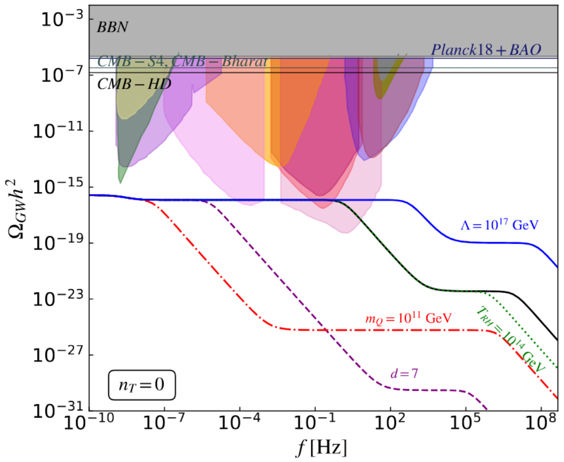
AC, A Ghoshal, D Paul
[2505.04614]

Conclusions

- High energy axion models have phenomenological consequences.
- Heavy quarks domination allows for a greater number of models to be viable. Including a number without a domain wall problem.
- Axion dark matter as light as $m_a = 10^{-8}$ eV can be achieved in post-inflationary breaking.
- We propose using inflationary GWs to learn about heavy quark domination.







Back-up slides

谁让你非要问的！

Thermal freeze-out

Weak-scale particles are in thermal equilibrium in the early universe

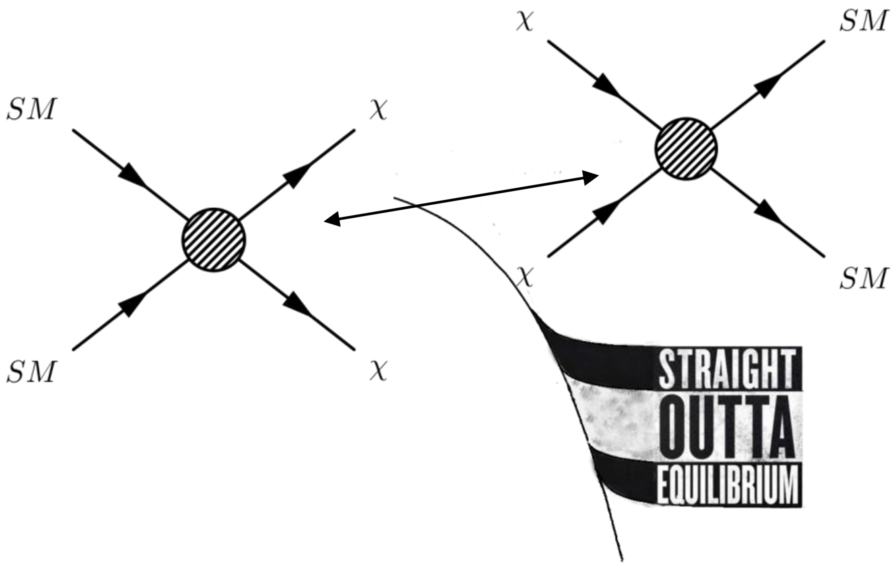
- Rate of pair annihilation and creation is **balanced**

$$\dot{n}_\chi + 3Hn_\chi = \langle \sigma\, v \rangle \left((n_{\rm SM}^{\rm eq})^2 - n_\chi^2 \right)$$

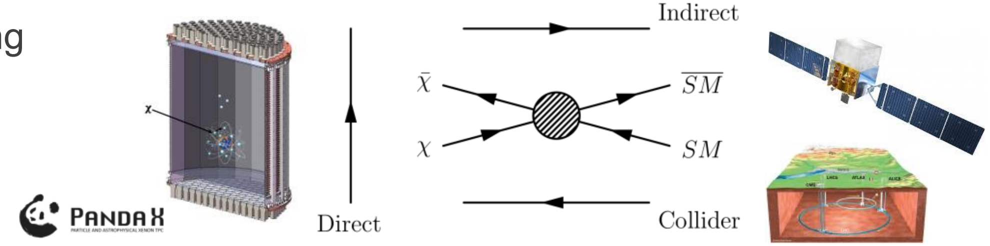
Hubble parameter

cross-section

number density

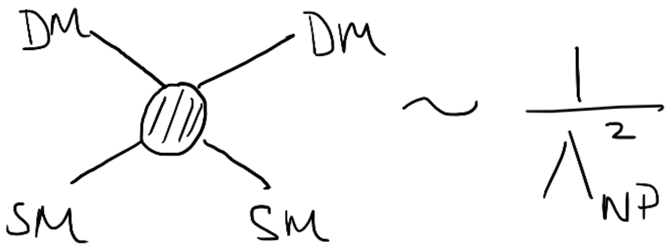


- This possibility is appealing because
  - It's insensitive to initial conditions of the Big Bang
  - Testable through multiple techniques



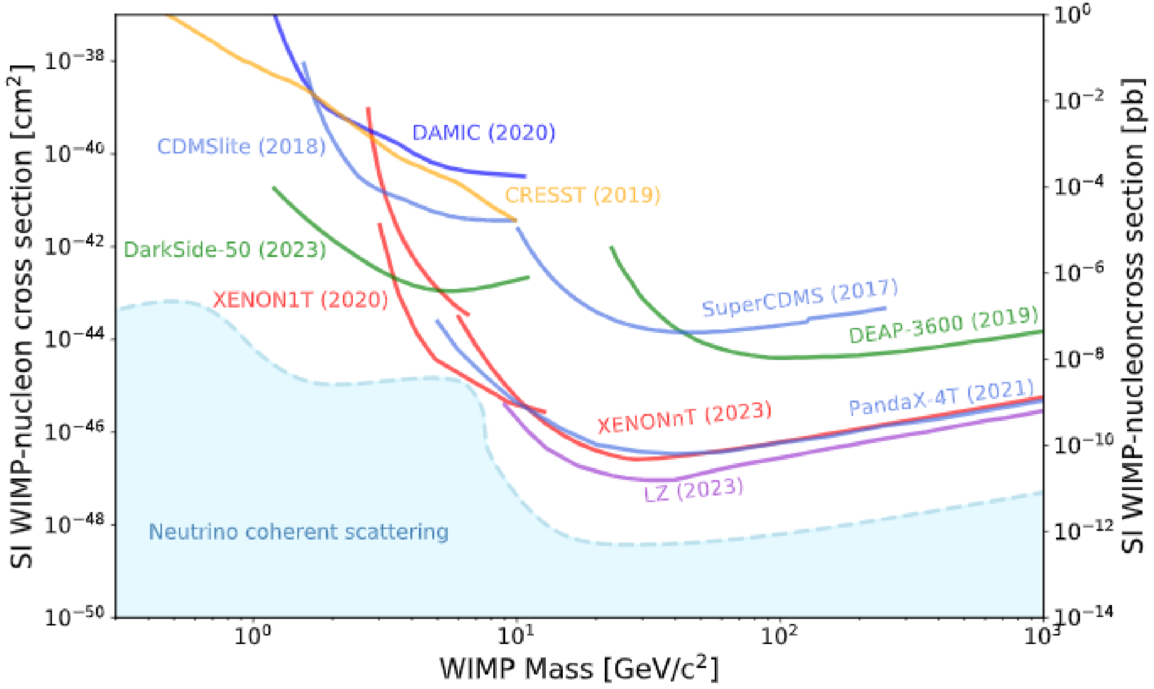
No sign of weak-scale dark matter

The freeze-out dark matter scenario is being tested… so far… no signal



It is still possible… but suggest $\Lambda_{\text{NP}} \geq 10\ \text{TeV}$

Other mechanisms gaining in popularity, many invoke new physics at an even higher scale and could be related to other **problems with the SM.**

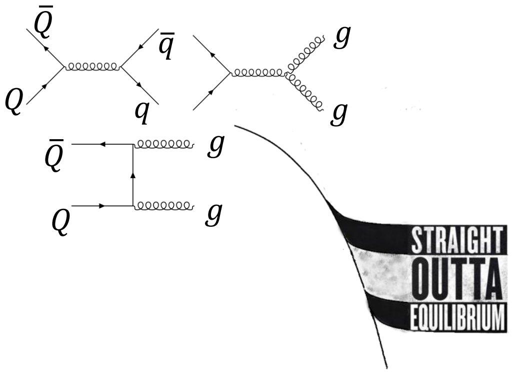


Heavy Quark Abundance

$$\mathcal{L}_{\rm PQ} = |\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q \; - (y_Q \overline{Q}_L Q_R \Phi + {\rm H.c})$$

$\Phi \to (v_\varphi/\sqrt{2}) e^{ia/f_a}$ so $m_Q = y_Q f_a/\sqrt{2}$ where f_a is constrained to be $\geq 10^8$ GeV

These strongly coupled Q particles will **freeze-out like dark matter**



$$Y_Q^\infty \approx \frac{x_f}{\lambda} \approx \frac{10\, H(m_Q)}{m_Q^3 \langle \sigma v \rangle}$$

$$H(m_Q) \sim m_Q^2/M_{\rm pl}$$

$$\langle \sigma v \rangle \sim 1/m_Q^2$$

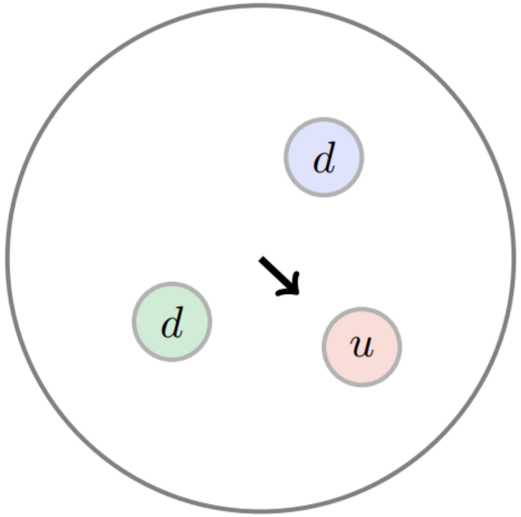
$$Y_Q^\infty \sim 10 \frac{m_Q}{M_{\rm pl}}$$

$\rho_Q = m_Q n_Q \Rightarrow \rho_Q \propto m_Q^2 T^3$ and $\rho_{\rm SM} \propto T^4$ leads to $\frac{\rho_Q}{\rho_{\rm R}^{\rm SM}} \sim 10^{10} \left(\frac{m_Q}{10^{12}\,{\rm GeV}}\right)^2 \left(\frac{1\,{\rm MeV}}{T}\right)$

The strong charge-parity problem

The strong CP problem asks why there is no charge-parity violation in strong interactions when the SM gauge-group appears to allow it

$$\mathcal{L}_{\cancel{CP}} = \theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$$



Predicts an electric dipole moment of the neutron (and others)

$$d_n = (2.4 \pm 1.0)\,\theta\, \times 10^{-3}\,\mathrm{e\,fm}$$

Observations put this down to $|\theta| < 0.8 \times 10^{-10}$

In fact, the observable is

$$\bar{\theta} = \theta + \arg\left(\det M_q\right)$$

Two independent phases cancel out precisely, $\bar{\theta} \lesssim 10^{-10}$

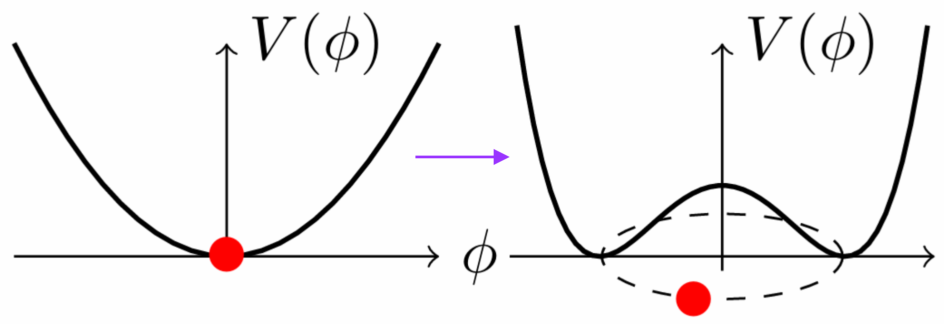
The Peccei-Quinn Mechanism

A new global spontaneously broken $U(1)$ symmetry proposed by R. D. Peccei and H. R. Quinn (1977). Referred to as $U(1)_{\mathrm{PQ}}$, breaks at some scale $\sim f_a$.

Complex scalar field

$$\mathcal{L} = (\partial_\mu \Phi^*)(\partial^\mu \Phi) + m^2 \Phi^* \Phi - \frac{\lambda}{4} \Phi^2 \Phi^{*2}$$

Symmetry breaking leads to a non-zero vacuum expectation value



[Julia Stadler]

$$\langle \Omega_\theta | \Phi | \Omega_\theta \rangle = \sqrt{\frac{2m^2}{\lambda}} e^{i\theta}$$

Expand around new minimum

$$\Phi \to \frac{1}{\sqrt{2}} (f_a + \phi)\ e^{ia/f_a}$$

Weinberg and Wilczek independently realized that this implied the existence of a new boson, the axion (1978).

The Peccei-Quinn Mechanism

The CP violating term now has a dynamical θ parameter

$$\mathcal{L}_{\text{eff}}^{a} = \frac{a\,(t,x)}{f_a}\frac{\alpha_s}{8\pi}G^{a}_{\mu\nu}\tilde{G}^{\mu\nu}_{a} = \theta\,(t,x)\,\frac{\alpha_s}{8\pi}G^{a}_{\mu\nu}\tilde{G}^{\mu\nu}_{a}$$

QCD dynamics generate a potential for the axion field.

$$V_{\text{QCD}}(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2}\sin^2\frac{N\theta}{2}}$$

This potential dynamically drives the field to cancel the CP violation term!

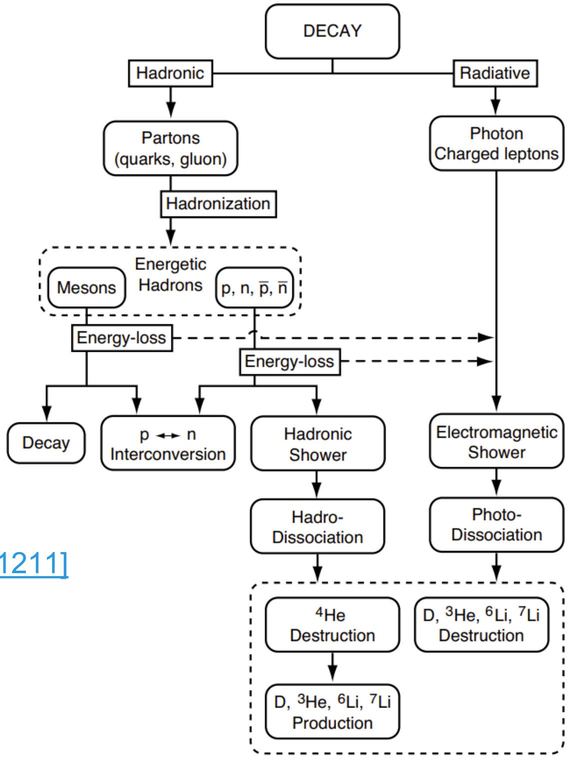
$$\bar{\theta} \to 0$$

Also generate the mass of axion

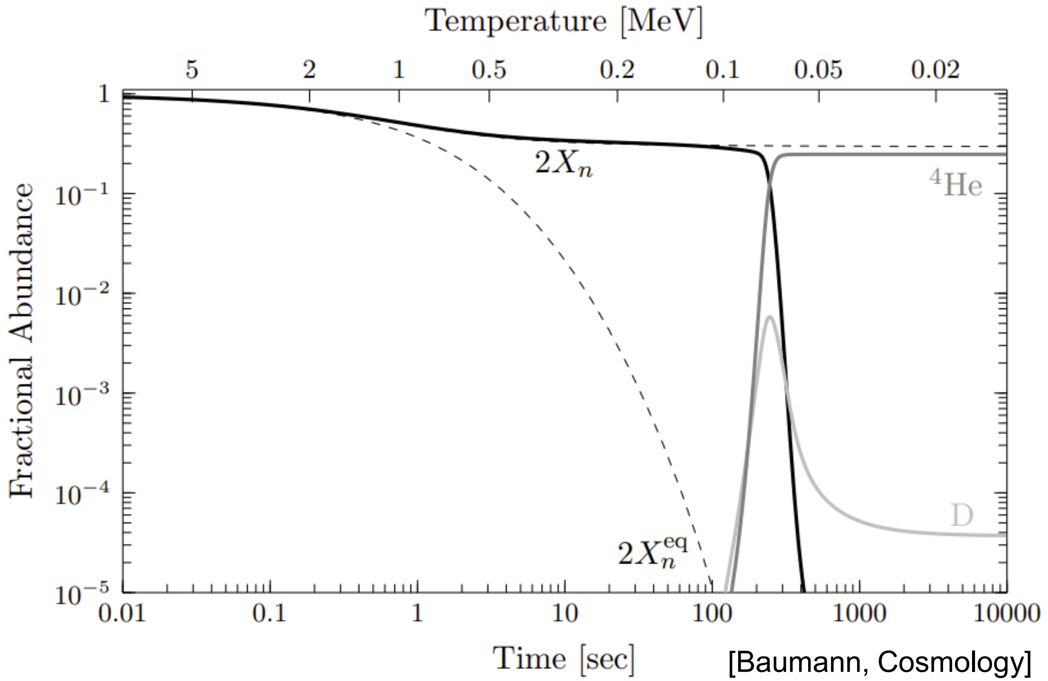
$$m_a = \frac{m_\pi f_\pi}{f/N}\sqrt{\frac{m_u m_d}{2(m_u + m_d)^2}} \approx 6\,\mu\text{eV}\,\frac{10^{12}\,\text{GeV}}{f/N}.$$

Big Bang nucleosynthesis (BBN)

Decays of heavy particles can disrupt the delicate balance of BBN.



[arXiv:1709.01211]



[Baumann, Cosmology]

Limits typically

$\tau \lesssim 0.01\,\mathrm{s}$ or $T_{\mathrm{decay}}^{Q} \gtrsim T_{\mathrm{BBN}} \approx 3\,\mathrm{MeV}$

We perform a full numerical treatment

To assess the effect, we solve the coupled Friedmann-Boltzmann equations

$$\frac{3H^2 M_{\rm Pl}^2}{8\pi} = \rho_{\rm R}^{\rm SM} + \rho_{\rm DR}^{\rm axion} + \rho_Q$$

Entropy density evolution:

$$\frac{\mathrm{d}s_{\rm R}^{\rm SM}}{\mathrm{d}t} = -3Hs_{\rm R}^{\rm SM} + \frac{\mathrm{BR}_{\rm SM}\Gamma_Q}{T}\rho_Q\,,$$

Thermal axion energy density evolution:

$$\frac{\mathrm{d}\rho_a}{\mathrm{d}t} = -4H\rho_a + \mathrm{BR}_{\rm axion}\Gamma_Q\rho_Q + \langle E_{\rm scat}^{\rm axion}\rangle\gamma_a\left(1 - \frac{n_a}{n_a^{\rm eq}}\right)\,,$$

Heavy quark energy density evolution:

$$\frac{\mathrm{d}n_Q}{\mathrm{d}t} = -3Hn_Q - \Gamma_Q n_Q - \langle\sigma\, v\rangle\left[n_Q^2 - (n_Q^{\rm eq})^2\right]\,.$$

We perform a full numerical treatment

To fully assess the effect we solve the coupled Friedmann-Boltzmann equations

$$\frac{3H^2 M_{\mathrm{Pl}}^2}{8\pi} = \rho_{\mathrm{R}}^{\mathrm{SM}} + \rho_{\mathrm{DR}}^{\mathrm{axion}} + \rho_Q$$

“Simplified” branching ratio
If only $Q \to a + q$ the only decay channel, $\mathrm{BR}_{\mathrm{SM}} = \mathrm{BR}_{\mathrm{axion}} = 1/2$

Entropy density evolution: $$\frac{\mathrm{d}s_{\mathrm{R}}^{\mathrm{SM}}}{\mathrm{d}t} = -3H s_{\mathrm{R}}^{\mathrm{SM}} + \frac{\boxed{\mathrm{BR}_{\mathrm{SM}}}\Gamma_Q}{T}\rho_Q \, ,$$

Thermal axion energy density evolution: $$\frac{\mathrm{d}\rho_a}{\mathrm{d}t} = -4H\rho_a + \boxed{\mathrm{BR}_{\mathrm{axion}}}\Gamma_Q\rho_Q + \langle E_{\mathrm{scat}}^{\mathrm{axion}}\rangle\gamma_a\left(1 - \frac{n_a}{n_a^{\mathrm{eq}}}\right) \, ,$$

Heavy quark energy density evolution: $$\frac{\mathrm{d}n_Q}{\mathrm{d}t} = -3Hn_Q - \Gamma_Q n_Q - \langle\sigma\, v\rangle\left[n_Q^2 - (n_Q^{\mathrm{eq}})^2\right] \, .$$

We perform a full numerical treatment

To fully assess the effect we solve the coupled Friedmann-Boltzmann equations

$$\frac{3H^2 M_{\rm Pl}^2}{8\pi} = \rho_{\rm R}^{\rm SM} + \rho_{\rm DR}^{\rm axion} + \rho_Q$$

γ_a: Thermal axion production rate more on this later.

$$\left\langle E_{\rm scat}^{\rm axion} \right\rangle \sim 3T_{\rm SM}$$

Entropy density evolution:

$$\frac{{\rm d}s_{\rm R}^{\rm SM}}{{\rm d}t} = -3H s_{\rm R}^{\rm SM} + \frac{{\rm BR}_{\rm SM}\Gamma_Q}{T}\rho_Q \,,$$

Thermal axion energy density evolution:

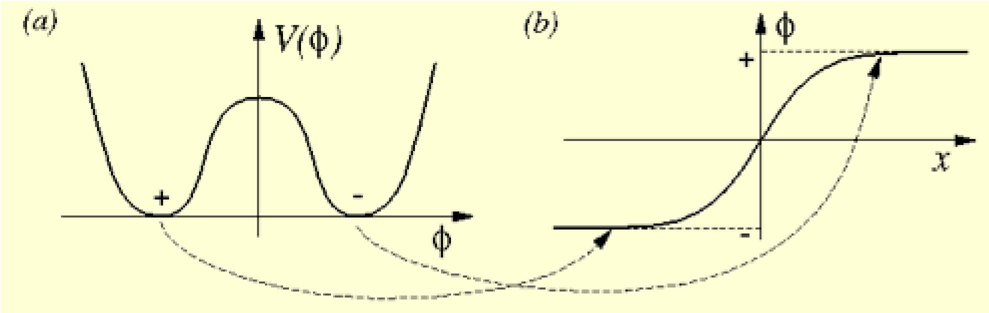
$$\frac{{\rm d}\rho_a}{{\rm d}t} = -4H\rho_a + {\rm BR}_{\rm axion}\Gamma_Q\rho_Q + \boxed{\langle E_{\rm scat}^{\rm axion}\rangle\gamma_a}\left(1 - \frac{n_a}{n_a^{\rm eq}}\right) \,,$$

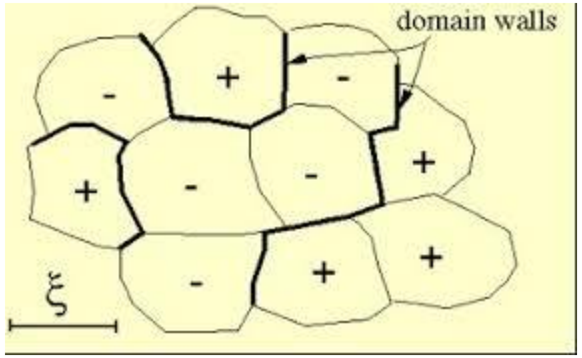
Heavy quark energy density evolution:

$$\frac{{\rm d}n_Q}{{\rm d}t} = -3Hn_Q - \Gamma_Q n_Q - \langle\sigma v\rangle\left[n_Q^2 - (n_Q^{\rm eq})^2\right] \,.$$

Domain walls

- Topological defect where two distinct vacua are separated by a potential





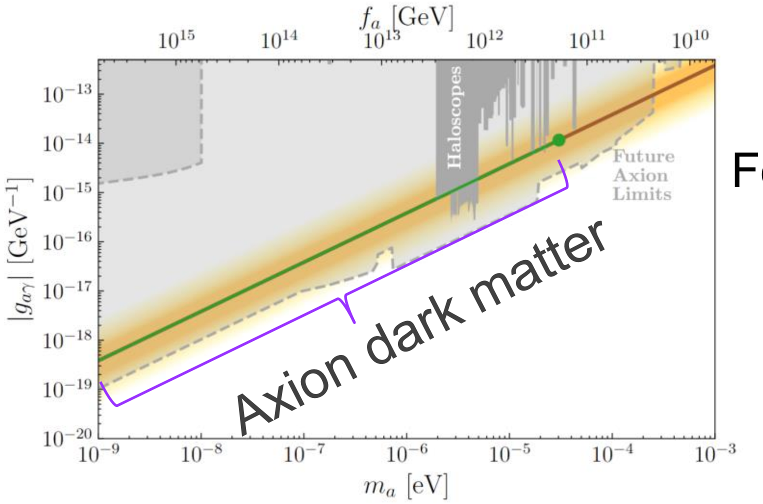
- Stable domain walls scale like a^{-2} so can quickly dominate.
- Much axion model building effort has gone into getting these things to decay or be destroyed.
- I think its interesting to first explore models where this isn't a problem.

When does PQ break?

BEFORE INFLATION

Can have $m_a \leq 10\ \mu\text{eV}$

No detectable dark radiation component from axion.



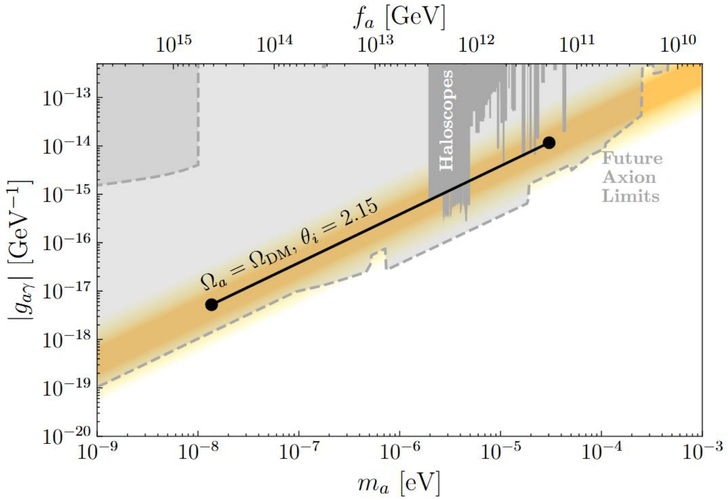
For $m_a = 10^{-8}$ eV

$\theta_i = 10^{-2}$

AFTER INFLATION

Now can have $m_a \leq 10\ \mu\text{eV}$ with HQD

The only models that survive with have no detectable dark radiation component from axion.



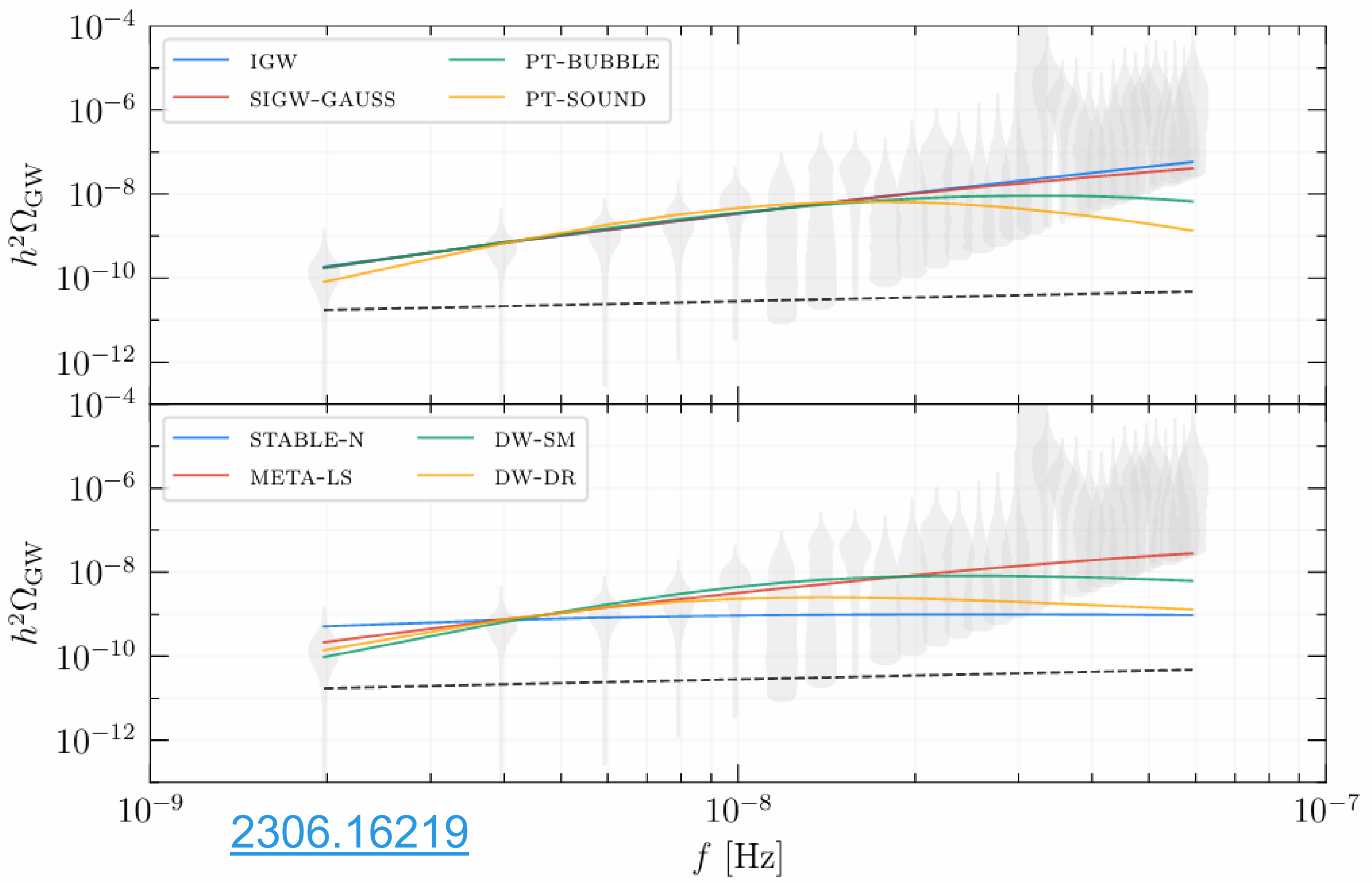
Both scenarios have the same phenomenological output.

Primordial Gravitational waves

LIGO discovering the first gravitational waves in 2015 represents a new way of learning about the Universe.

More recently (2023) Pulsar timing arrays (PTAs) have confirmed what looks like a "stochastic" gravitational wave signal.

Most likely a signal from supermassive black holes, but could be something more exotic



[2306.16219](2306.16219)

Inflationary gravitational waves

For astrophysical gravitational waves, one works in linearized gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

In cosmological settings, the background is expanding

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

The resulting wave equation is (problem 6.4 Baumann's book)

$$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = 0$$

When treating the quantum fluctuations of tensor perturbations during inflation, one obtains the following power spectrum

$$P_T^{\text{prim.}}(k) = A_T(k)\left(\frac{k}{k_*}\right)^{n_T}$$

Current constraints on IGW

The precise form of the power spectrum depends on the specifics of inflation.

$$P_T^{\text{prim.}}(k) = A_T(k)\left(\frac{k}{k_*}\right)^{n_T}$$

Tensor perturbations (GWs) alter the polarization of CMB photons

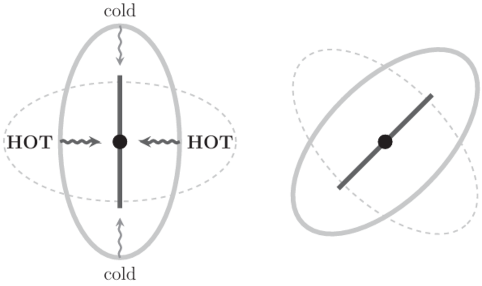


Fig. 7.35 Cartoon illustrating that the anisotropic stretching and compressing of space by a gravitational wave creates a temperature quadrupole and hence leads to CMB polarization. The two polarizations of the gravitational wave produce polarization of the CMB photons with a relative angle of 45°. This is why gravitational waves produce both E and B-modes, while density perturbations create only E-modes.

Current constraints on measurements of B-modes in the CMB constrain the ratio

$$r = \frac{A_T}{A_S} < 0.036$$

The tilt n_T is determined in vanilla slow-roll inflation to be $n_T = -\frac{r}{8}$, but there are numerous alternatives to this relation, string inflation… ekpyrotic… particle production at reheating

NANOGrav signal an IGW?

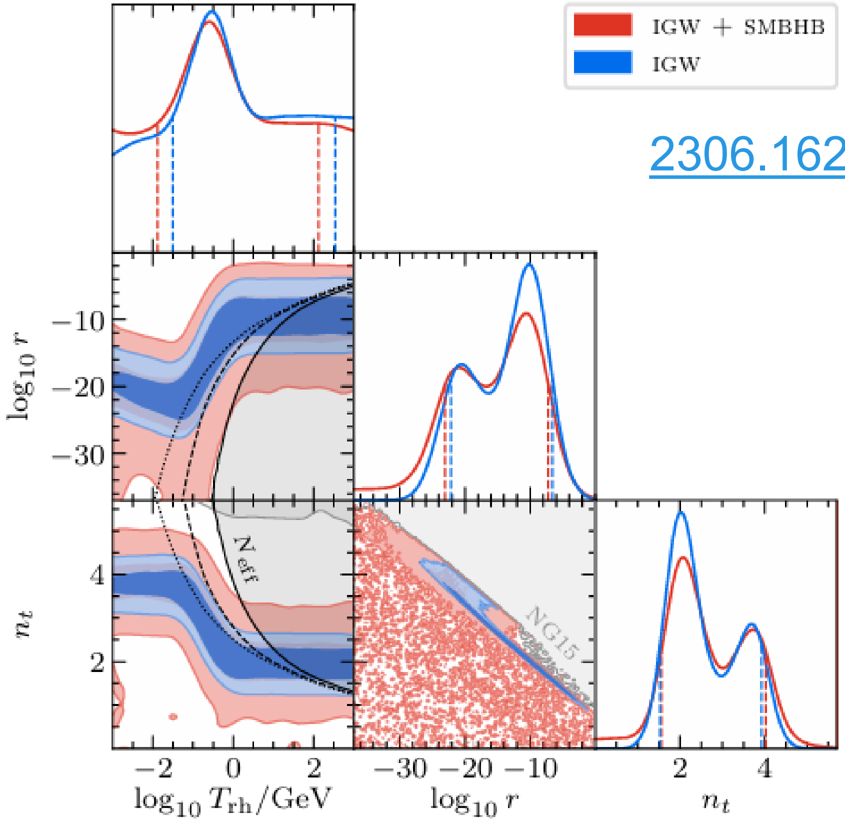
With the 15-year dataset NANOGrav fit for

$$P_T^{\text{prim.}}(k) = A_T(k)\left(\frac{k}{k_*}\right)^{n_T}$$

The collaboration report a good fit with a very small scalar to tensor ratio but a very “blue-tilted” spectrum.

They also include the reheat temperature because that controls the maximum frequency possible

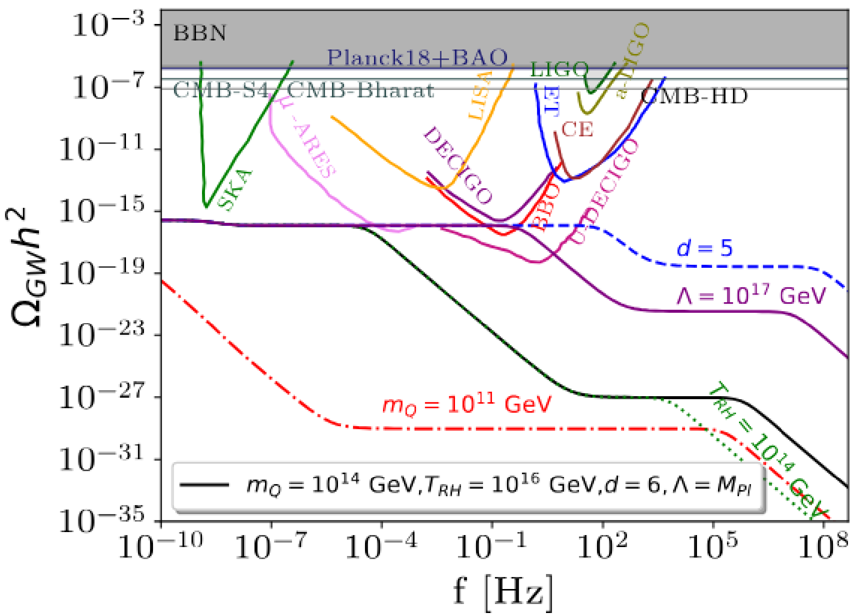
$$\Omega_{\text{GW}}(k) = \frac{1}{12}\left(\frac{k}{a_0 H_0}\right)^2 T_T^2(k)\, P_T^{\text{prim}}(k),$$



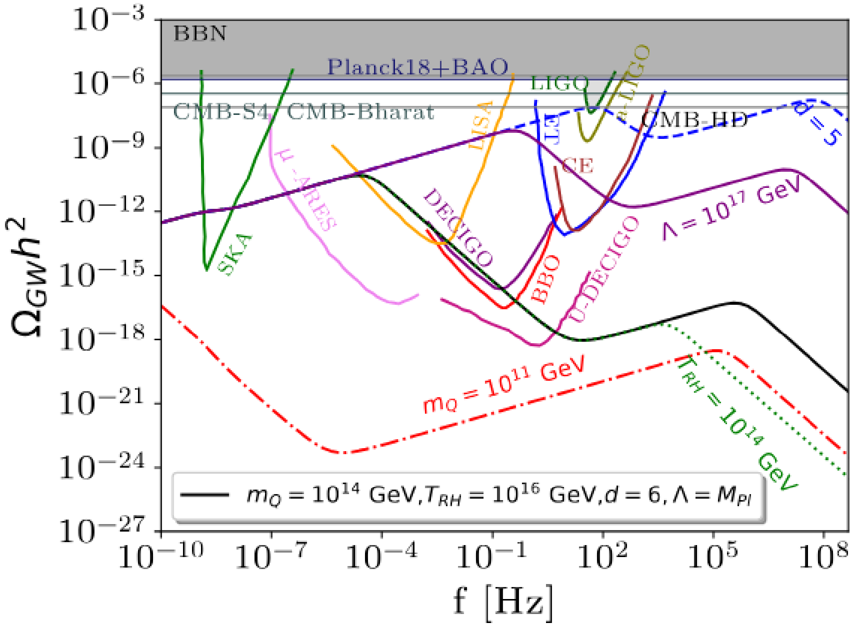
[2306.16219](2306.16219)

Forecasting IGW signal to learn about axion models

We take blue-tilted spectra and project the sensitivities of future experiments



(a) $n_T = 0$



(b) $n_T = 0.5$

Period of Early Matter Domination

We use the Transfer functions to model the GW evolution

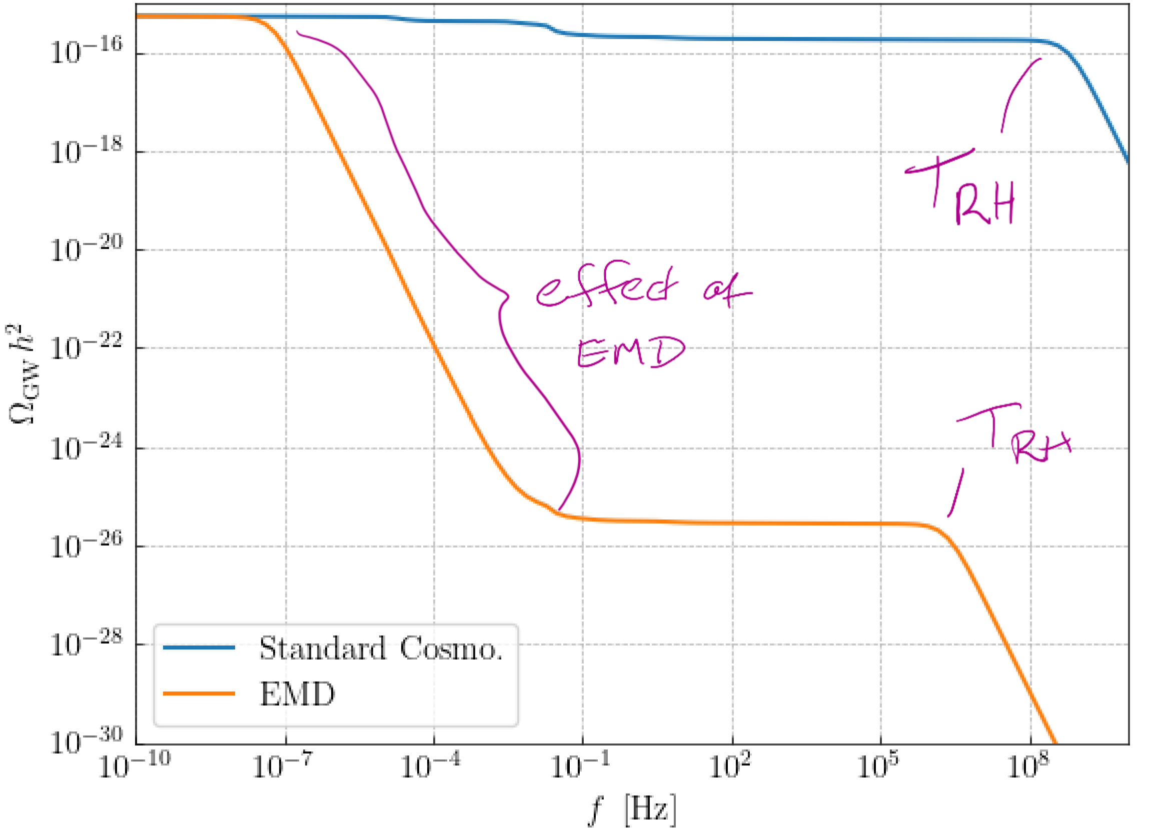
$$\Omega_{\rm GW}(k) = \frac{1}{12}\left(\frac{k}{a_0 H_0}\right)^2 T_T^2(k)\, P_T^{\rm prim}(k),$$

$$T_T^2(k) \equiv \Omega_m^2 \left(\frac{g_*(T_{\rm in})}{g_*^0}\right)\left(\frac{g_{*S}^0}{g_{*S}(T_{\rm in})}\right)^{\frac{4}{3}}\left(\frac{3j_1(z_k)}{z_k}\right)^2 F(k)$$

$$F(k)\Big|_{\rm standard} \equiv T_1^2\left(\frac{k}{k_{\rm eq}}\right) T_2^2\left(\frac{k}{k_{\rm RH}}\right)$$

These functions are determined by solving the wave equation in an evolving cosmology

$$F(k)\Big|_{\rm HQD} \equiv T_1^2\left(\frac{k}{k_{\rm eq}}\right) T_2^2\left(\frac{k}{k_{\rm dec}}\right) T_3^2\left(\frac{k}{k_{\rm dec,\,S}}\right) T_2^2\left(\frac{k}{k_{\rm RH,\,S}}\right)$$



1407.4785 and 0804.1827

Period of Early Matter Domination

One intuitive way to understand the suppression of the GWs under EMD as with respect to radiation domination.

The expansion rate is faster in EMD, so if a mode has spent more time in the Hubble horizon, it has red-shifted more.

Also when a mode enters horizon changes

$k = a_k H_k$ defines the horizon crossing.

$a_k \propto k^{-1}$ in radiation domination

$a_k \propto k^{-2}$ in matter domination

Therefore a_k / a_{RH} is more suppressed in matter domination