AXION SIGNALS FROM MAGNETOSPHERES

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Based on work published in PRD (ArXiv:2504.04209) and new work in progress

Outline

Part 1

- Short motivation of ultralight particles
- Magnetospheric superradiance
- Some avenues for future work

Part 2

- Resonant axion-photon conversion in magnetars
- Polarization signals
- Effects of axion densities (work in progress)

Ultralight Particles: The Motivation

- Ultralight particles (ULPs) are generic predictions of many beyond standard model (BSM) paradigms such as
  - (Pseudo-)scalar: **QCD axion, axion-like particles (ALPs)** as Goldstone bosons of various broken U(1) symmetries, string theory moduli and dilatons
  - Vector: Dark photons
  - Tensor: Bimetric gravity, massive gravitons
- ULPs are wavelike (10^{-24} eV $\ll m \ll 1$ eV) and couple very weakly to the SM, with couplings generally suppressed by a very high symmetry breaking scales, $g \sim \frac{1}{f_a}$

Superradiance: The Motivation

- Superradiance (SR) $\equiv$ amplification of radiation in a dissipative system
- Three ingredients: Rotation, **dissipation** and bound states
- SR in BHs and stars offers a unique tool to constrain ULPs through their gravitational interaction which
  - is complementary to laboratory searches
  - can constrain ULPs which are too light for laboratory searches
  - can be coupling independent (BHs) or dependent (stars)
- SR in BHs and stars leads to a long list of potentially observable signals: **rapid spindown**, flashes of light, gravitational waves...

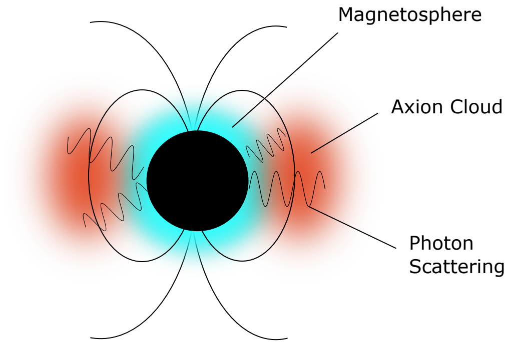
Superradiant growth in a nutshell

- Dissipation: Imagine a wave $\sim e^{-i\omega t}$. Non-hermitian (dissipative) dynamics can lead to a complex frequency and waves that grow exponentially: $\sim e^{i\omega_R t + \mathrm{Im}[\omega] t} \rightarrow \Gamma_{SR} \equiv \mathrm{Im}[\omega]$
- Rotation: A rotating body has $E_{rot} = \frac{1}{2} I\Omega^2, J = I\Omega$. Suppose a wave extracts $\delta E, \delta J = \frac{m}{\omega}\delta E$, causing rotational energy of the body to change by $\delta E_{rot} = \frac{dE_{rot}}{dJ}\delta J = \Omega\delta J = \frac{m\Omega}{\omega}\delta E$. $\delta E_{rot} > \delta E \Rightarrow \omega < m\Omega$, the **superradiance condition**
- Bound states: A gravitational well of size $R \sim \lambda_{db} \sim 1/\mu$ can trap axions of mass μ which bounce and amplify repeatedly

Superradiance in the Magnetosphere

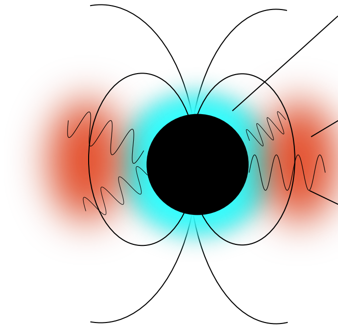
We consider superradiance (SR) in neutron star magnetospheres where the ingredients are:

- Rotation: Magnetosphere rotates with the star
- Dissipation: Finite conductivity plasma → complex frequencies
- Bound states: Approximately Schwarzschild geometry, leading to quasibound axion states



Neutron star SR: step by step

1. An axion perturbs the bound state "axion cloud"
2. The axion converts into a photon via the $g_{a\gamma} aF\tilde{F}$ coupling
3. The photon scatters off of the rotating dissipative plasma, extracting rotational energy
4. The photon stores the energy back into the axion cloud
5. Repeat (until rotational energy is insufficient for SR)$\Rightarrow$ INSTABILITY



Theory

- We now consider axion electrodynamics in presence of a conducting plasma, given by

$$\mathcal{L} \supset \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\mu^2}{2}\phi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g_{a\gamma\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} - A_\mu j^\mu\right], j^\mu \equiv \sigma F^{\mu\nu}u_\nu + \rho u^\mu$$

  where (ρ, σ, u are electric charge, conductivity and fluid velocity)
- One finds the linearized ($F \to F + F_B, \phi \to \phi + \phi_B$) equations of motion

$$\Box\phi + \mu^2\phi = -\frac{g_{a\gamma\gamma}}{2}F_{\mu\nu}\tilde{F}_B^{\mu\nu}$$
$$\partial_\mu F^{\mu\nu} - \sigma F^{\nu\mu}u_\mu = -g_{a\gamma\gamma}\left(\partial_\mu\phi\right)\tilde{F}_B^{\mu\nu}$$

The superradiance rate

The linearized EOMs can be written in matrix form

$$[H_F + V_A + V_{a\gamma\gamma}] \begin{pmatrix} |\phi\rangle \\ |A^0\rangle \\ |\mathbf{A}\rangle \end{pmatrix} = \omega^2 \begin{pmatrix} |\phi\rangle \\ |A^0\rangle \\ |\mathbf{A}\rangle \end{pmatrix}$$

where the free Hamiltonian is $H_F = \mathrm{diag}\left(-d^2/dr_*^2 + U(r), -\nabla^2, -\nabla^2\right)$ and

$$V_{a\gamma\gamma} = ig_{a\gamma\gamma} \begin{pmatrix} 0 & \mathbf{B}(\mathbf{r}) \cdot \hat{p} & -\omega \mathbf{B}(\mathbf{r}) \\ \mathbf{B}(\mathbf{r}) \cdot \hat{p} & 0 & 0 \\ \omega \mathbf{B}(\mathbf{r}) & 0 & 0 \end{pmatrix}$$

$$V_A = i\sigma(\hat{x}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathbf{u}(\mathbf{r}) \cdot \hat{p} & -\omega \mathbf{u}(\mathbf{r}) \\ 0 & \hat{p} & -\omega - \mathbf{u}(\mathbf{r}) \times (\hat{p} \times \cdot) \end{pmatrix}.$$

The superradiance rate

- We then follow Day et al. (arxiv/1904.08341) and compute the SR rate in third-order QM Pert. theory as

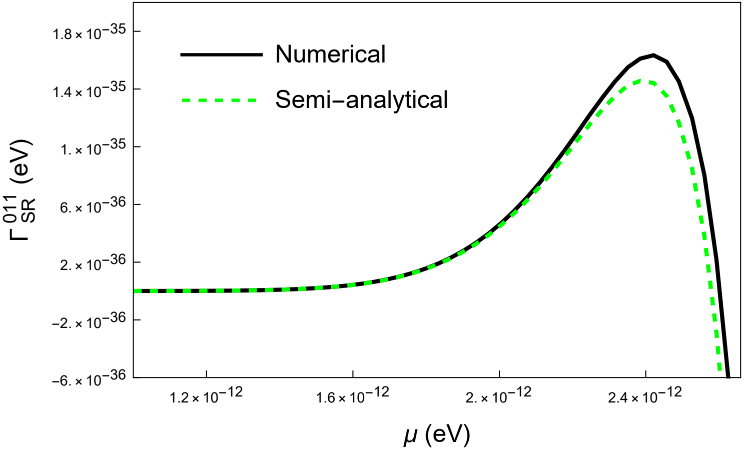
$$\mathrm{Im}[\delta\omega_{\mathrm{nlm}}] = \Gamma_{SR} \sim \langle\phi|V_{a\gamma}|A\rangle\langle A|V_{\sigma}|A\rangle\langle A|V_{a\gamma}|\phi\rangle$$

- Day et al.: proof of principle with $\vec{B} = B\hat{z}$, we use $\mathbf{B}(\mathbf{r}) = \frac{B_0}{r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}})$
- For the $nlm = 011$ state we find in leading order in ωR,

$$\Gamma^{011}_{\mathrm{SR}} = \frac{\pi}{225} g^2_{\mathrm{a}\gamma\gamma} B_0^2 (\mu R)^6 \mathcal{N}_\phi^2 \alpha_{01}^5 \frac{\sigma}{\mu} (\mu R_{\mathrm{LC}})^5 (\Omega - \frac{17}{15}\mu),$$

where $\mathcal{N}_\phi$ is a normalization factor, R_{LC} the light cylinder radius, Ω the NS spin and B the NS magnetic field. Tracks numerical result closely.

Semianalytical vs numerical result



Constraints from Pulsar Timing

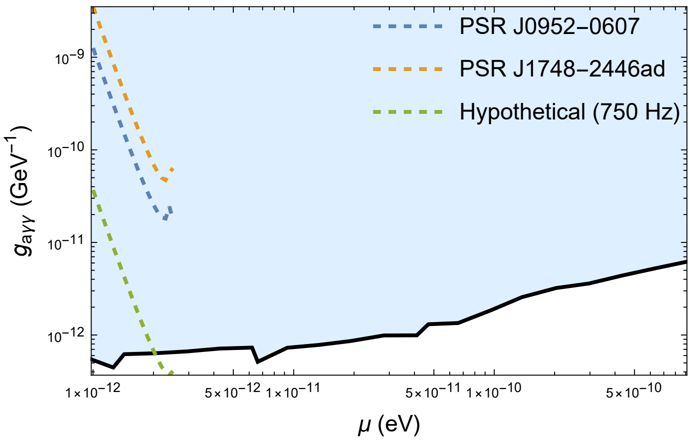
- The characteristic superradiant timescale $\tau_{SR} = \frac{1}{\Gamma_{SR}}$ sets the angular momentum extraction timescale
- Pulsar timing arrays (PTAs) yield very sensitive measurements of NS spindown rates and their derivatives $\rightarrow \tau_{NS} \sim \frac{\Omega}{\dot{\Omega}}$
- We cannot have neutron stars for which $\tau_{SR} \lesssim \tau_{NS}$ as these would have spun out already. Thus those $(\mu, g_{a\gamma\gamma})$ combinations for which this happens are ruled out.

Constraints from Pulsar Timing (*)

J0952-0607: $\Omega = 707$ Hz, $B = 10^8$ G, $M = 2.35\,M_\odot$, $R = 12.5$ km

Note: J1748-2446ad higher spin (716 Hz) but weaker at constraining

Hypothetical: $\Omega = 750$ Hz, $B = 5 \times 10^8$ G, $M = 2.35\,M_\odot$, $R = 13$ km



Magnetospheric modelling

- A non-zero ω_p can decouple the axion and photon sectors $\rightarrow$ how large really is ω_p in MSPs?
- Simulations like in Guèpin, Cerutti & Kotera 2020 (1910.113879) show that for low B the loss of dominant pair production via $\gamma + B \rightarrow e^{+} + e^{-}$ leads to the so called **electrosphere** solution:
  e^{-} stuck at polar caps, no e^{+}, p at equator like leading SR mode cloud

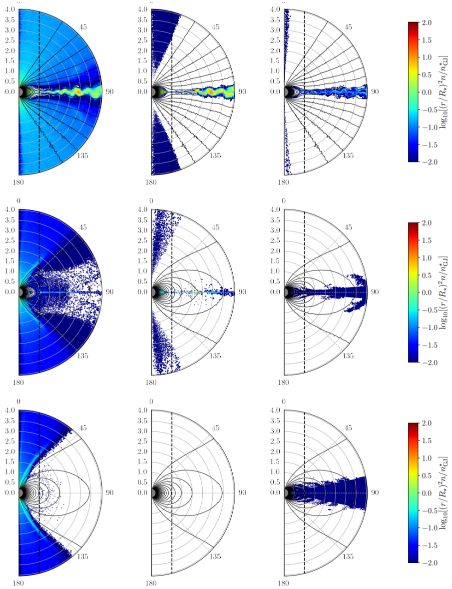


Fig. 1: Density maps for electrons, positrons, and protons (left to right), for $f_{pp} = 0.01$, $f_{pp} = 0.05$ and $f_{pp} = 0.1$ (top to bottom),

Magnetospheric modelling

- Note: Magnetic field lines in electrosphere case remain approximately dipolar. Otherwise one may need multiple field regimes outside light cylinder.
- In reality, the magnetosphere is likely something in between saturated and electrosphere.
- We assume the electrosphere solution.

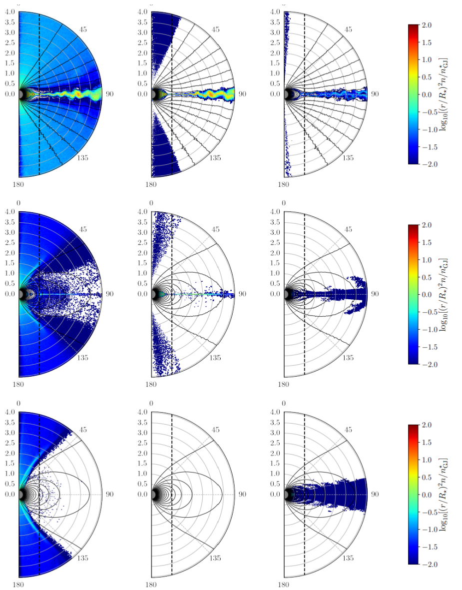


Fig. 1: Density maps for electrons, positrons, and protons (left to right), for $f_{pp} = 0.01$, $f_{pp} = 0.05$ and $f_{pp} = 0.1$ (top to bottom),

Open questions & Future work

- More precise modeling of MSP magnetospheres is required, including plasma and conductivity
- The interaction of the many instabilities possible in stars (in-medium, r-mode, magnetospheric..) could be very strong: SR rates add up, and cloud grows as $e^{\sum_i \Gamma_i t}$
- Binaries (EM, GW, PTA signals)?

Part 2: Axion-photon conversion in magnetar magnetospheres

We will now consider polarization signals from resonant axion-photon conversion in magnetar magnetospheres. The outline is as follows:

- Overview of
  - Resonant axion-photon conversions
  - Polarization signals
- Early work: Axion cloud effects on polarization via quadratic coupling

Theory

We begin by considering axion electrodynamics augmented with the Euler-Heisenberg (EH) term

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\left((\partial_\mu a)^2 - m_a^2 a^2\right) - \frac{g_{a\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\alpha^2}{90 m_e^4}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2\right]$$

- EH Lagrangian is valid at photon energies $\omega < m_e$ as electrons are integrated out.
- obtained by expansion in F/m_e, it fails for large fields. This can be interpreted as the pair creation limit $B_c \sim 4.4 \times 10^{13}$ G.

Resonant conversion

- In non-resonant case the vacuum term suppresses degree linear polarization due to $\Pi_L \sim 1/B^2$ scaling, and the best bounds are actually obtained from white dwarves whose B-field avoid vacuum effects (Dessert et al., 2203.04319)
- What about resonant conversions? Vacuum term becomes a blessing rather than a curse (Song et al., 2402.15144)
- We consider first resonances in axion electrodynamics augmented with EH term (2-state system) for intuition, and later consider the effects of dense axionic clouds

Resonant conversion

It can be shown that the EH lagrangian leads to a Schrödinger equation (2402.15144)

$$\left[i\partial_z + \begin{pmatrix} \Delta_a & \Delta_\mathrm{M} & 0 \\ \Delta_\mathrm{M} & \Delta_\parallel + \Delta_\mathrm{pl} & 0 \\ 0 & 0 & \Delta_\perp \end{pmatrix}\right]\begin{pmatrix} a \\ A_\parallel \\ A_\perp \end{pmatrix} = 0$$

where

$$\Delta_\mathrm{M} = \frac{1}{2} g_{a\gamma} B \sin\Theta, \quad \Delta_a = -\frac{m_a^2}{2\omega}, \quad \Delta_\mathrm{pl} = -\frac{\omega_\mathrm{pl}^2}{2\omega}$$

$$\Delta_\parallel = \frac{7}{2}\omega\xi \sin^2\Theta, \quad \Delta_\perp = \frac{4}{2}\omega\xi \sin^2\Theta$$

Two state system

$$\left[i\partial_z + \begin{pmatrix} \Delta_a & \Delta_\mathrm{M} & 0 \\ \Delta_\mathrm{M} & \Delta_\parallel + \Delta_\mathrm{pl} & 0 \\ 0 & 0 & \Delta_\perp \end{pmatrix} \right] \begin{pmatrix} a \\ A_\parallel \\ A_\perp \end{pmatrix} = 0$$

- a couples only to $A_\parallel \Rightarrow$ Two-state system
- In reality one would solve the problem numerically but we can get some intuition from analytical results

Two state system

In the case of a two state system one can find the mixing angle

$$\tan 2\beta_m = \frac{2\Delta_{\mathrm{M}}}{\Delta_{\|} + \Delta_{\mathrm{pl}} - \Delta_a}$$

which shows the resonance. There are two resonances:

- Axion-plasma: $\Delta_{\|} \ll |\Delta_{\mathrm{pl}}| \sim |\Delta_a|$
- Plasma-vacuum: $|\Delta_a| \ll |\Delta_{\mathrm{pl}}| \sim \Delta_{\|}$

The first dominates far away while the second dominates near star

Adiabaticity

The eigenvalues of the equation of motion are given by

$$\lambda_\pm = \frac{\Delta_\parallel + \Delta_a + \Delta_{\rm pl}}{2} \pm \frac{\Delta\lambda}{2} \Rightarrow \Delta\lambda = \sqrt{\left(\Delta_a - \Delta_\parallel - \Delta_{\rm pl}\right)^2 + 4\Delta_M^2} \overset{\rm res}{\approx} 2\Delta_M$$

- In case of a **resonance** $\Delta\lambda = 2\Delta_M$. Thus, the eigenvalues never cross $\Rightarrow$ **avoided level-crossing** and the system cannot go from one eigenvalue to another continuously by varying parameters.
- Instead, the system has to jump from one eigenstate to another, with probability given by the **adiabaticity** of the conversion at the resonance.

Adiabaticity

- The resonant conversions are akin to neutrino oscillations, governed by non-adiabatic conversions at a single resonant radius
- In case of a **resonance** $\Delta\lambda = 2\Delta_M$. The adiabaticity is given by

$$\gamma_{res} \equiv \frac{\Delta_M^2}{|\frac{d}{dr}(\Delta_a - \Delta_{||} - \Delta_{\mathrm{pl}})|_{\mathrm{res}}|}$$

- Heuristically adiabaticity=gap size/resonance speed
- Large (small) $\gamma_{res} \Rightarrow$ small (large) conversion probability.

Adiabaticity

The probability to **NOT CONVERT** is called the **jump probability**, given by the **Landau-Zener** (LZ) formula

$$P_{\rm jump} = e^{-\pi\gamma_{\rm res}/2}$$

One finds in vacuum resonance

$$\gamma_{res} \propto g_{a\gamma}^2 \frac{r_{\rm res}}{3\omega}$$

and in plasma resonance

$$\gamma_{res} \propto g_{a\gamma}^2 B_{\rm res}\omega \frac{n_e}{|n_e'|}$$

Thus the plasma resonance is stronger for higher frequencies while for smaller frequencies vacuum resonance is dominant.

Constraining axions by polarization measurements

The photon polarization is described by the Stokes parameters

$$I(z) \equiv \langle A_\perp(z)A_\perp^*(z)\rangle + \left\langle A_{||}(z)A_{||}^*(z)\right\rangle$$

$$Q(z) \equiv \langle A_\perp(z)A_\perp^*(z)\rangle - \left\langle A_{||}(z)A_{||}^*(z)\right\rangle$$

$$U(z) \equiv 2\mathrm{Re}\left(\left\langle A_{||}(z)A_\perp^*(z)\right\rangle\right), \quad V(z) \equiv -2\mathrm{Im}\left(\langle A_{||}(z)A_\perp^*(z)\rangle\right)$$

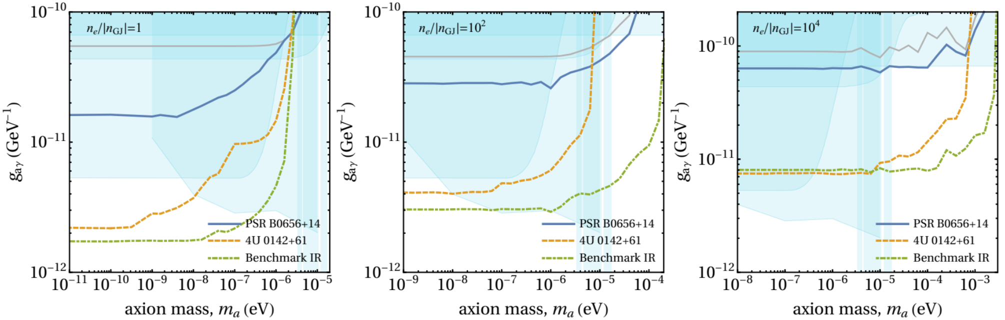
The observable is then the degree of linear polarization defined as

$$\Pi_L(r) = \frac{\sqrt{Q^2(r) + U^2(r)}}{I(r)}$$

Constraining axions by polarization measurements

- Evolve initially unpolarized photons at the star surface through the magnetosphere
- Photons interact only with $A_{||}$ so originally unpolarized light will obtain an induced linear polarization
- If the linear polarization induced by the axion is too large, $\Pi_L(\infty) > P\mathrm{obs}$, these axion parameters are ruled out.

Current resonant polarization constraints (2402.15144)



Clouds around compact objects

- What about the effect of large axion densities around stars and BHs?
- Many ways to create, including
  - polar cap cascades via dynamical $\vec{E} \cdot \vec{B}$ sourcing ($m_a \sim 10^{-9} - 10^{-4}$ eV)
  - superradiance, magnetospheric or in-medium ($m_a \sim 10^{-12}$ eV). Interesting to consider also in SMBHs (stellars have no B fields).
  - Classical axion configurations from static $\vec{E} \cdot \vec{B}$ ($m_a \sim 10^{-12}$ eV)

Clouds around compact objects

These axion "clouds" can grow to large energy densities/field values ($a = \sqrt{2\rho/m_a}$)

- $a_{max} \sim 10^{16}$ eV (polar cap, 2307.11811)
- $a_{max} \sim 10^{23}$ eV (BH SR, $5M_\odot$)
- $a_{max} \sim 10^{26}$ eV (BH SR, $10^6 M_\odot$)
- $a_{max} \sim 10^{21}$ eV (axion configurations, 1804.04224)

Quadratic interaction

- If the axion field is large, there comes a point where the quadratic interaction becomes relevant and the theory becomes

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a a^2 - \frac{g_{a\gamma}}{4}aF\tilde{F} - \\ - c_{F^2}\frac{\alpha}{16\pi^2}\left(\frac{a}{f_a}\right)^2 F^2 + \frac{\alpha^2}{90m_e^4}\left[\left(F^2\right)^2 + \frac{7}{4}(F\tilde{F})^2\right]$$

- The quadratic coupling and linear coupling are a priori independent (2307.10362)
- Note: the quadratic term couples like a scalar to EM!

Cloud-induced "plasma mass"

- The quadratic coupling behaves as an axion mass term $m_c \sim \frac{B^2}{2\Lambda^2}$ with $\Lambda = \frac{4\pi}{\alpha c_{F^2}} f_a \sim 10^3 f_a \Rightarrow$ shifted resonances
- the prefactor is of order $c_{F^2} \sim \mathcal{O}(0.1)$ for QCD axion (2307.10362)
- One finds the cloud induced "plasma mass" to be

$$m_c \sim \left(c_{F^2} \frac{\alpha}{4\pi^2} \left(\frac{|\boldsymbol{B}|}{f_a} \right)^2 \right)^{1/2}$$

$$\sim 10^{-9}\mathrm{eV} \times \left(\frac{|\boldsymbol{B}|}{10^{12}\mathrm{G}} \right) \left(\frac{g_{a\gamma}}{5 \times 10^{-12}\mathrm{GeV}^{-1}} \right)$$

Cloud effects

The cloud system leads to a three state system described by

$$\left[i\partial_z + \begin{pmatrix} \Delta_a + \Delta_\Lambda & \Delta_\mathrm{M} & \Delta_c \\ \Delta_\mathrm{M} & \Delta_\parallel + \Delta_\mathrm{pl} & 0 \\ \Delta_c & 0 & \Delta_\perp \end{pmatrix} \right] \begin{pmatrix} a \\ A_\parallel \\ A_\perp \end{pmatrix} = 0$$

with

$$\Delta_\mathrm{M} = \frac{1}{2} g_{a\gamma} B \sin\Theta, \quad \Delta_a = -\frac{m_a^2}{2\omega}, \quad \Delta_\mathrm{pl} = -\frac{\omega_\mathrm{pl}^2}{2\omega}, \quad \Delta_c = \frac{a_B}{\Lambda^2} \frac{B_B}{2} \sin\Theta$$

$$\Delta_\parallel = \frac{7}{2}\omega\xi \sin^2\Theta, \quad \Delta_\perp = \frac{4}{2}\omega\xi\sin^2\Theta, \quad \Delta_\Lambda = -\frac{m_\Lambda^2}{2\omega} \equiv -\frac{B_B^2}{2\omega\Lambda^2}$$

Note: the energy scale Λ generally differs from f_a.

3-state system

$$\left[i\partial_z + \begin{pmatrix} \Delta_a + \Delta_\Lambda & \Delta_{\mathrm{M}} & \Delta_c \\ \Delta_{\mathrm{M}} & \Delta_{\|} + \Delta_{\mathrm{pl}} & 0 \\ \Delta_c & 0 & \Delta_\perp \end{pmatrix} \right] \begin{pmatrix} a \\ A_{\|} \\ A_\perp \end{pmatrix} = 0$$

- The effect of the cloud is to couple the axion to both polarizations. The effect could be to wash out polarization signals, produce elliptic polarization, or make pseudoscalars look like scalars.
- The effect of the quadratic term is two-fold:
  - New effective mass term on the diagonal akin to plasma mass $\Rightarrow$ moves resonance
  - New offdiagonal term in the $1, 3$-sector, coupling the axion also to $A_\perp$

A work in progress...

- The 3-state system can not be diagonalized with just one mixing angle. Instead, we solve the Schrödinger equation numerically
- We classify the resonances of the system
- We consider magnetars for now but SMBHs can be an interesting target.
- Easy to generalize everything to scalars.

Thank you