

Search for cosmological phase transitions through their gravitational wave signals

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Oscar Klein Centre, University of Stockholm, 7 XI 2024

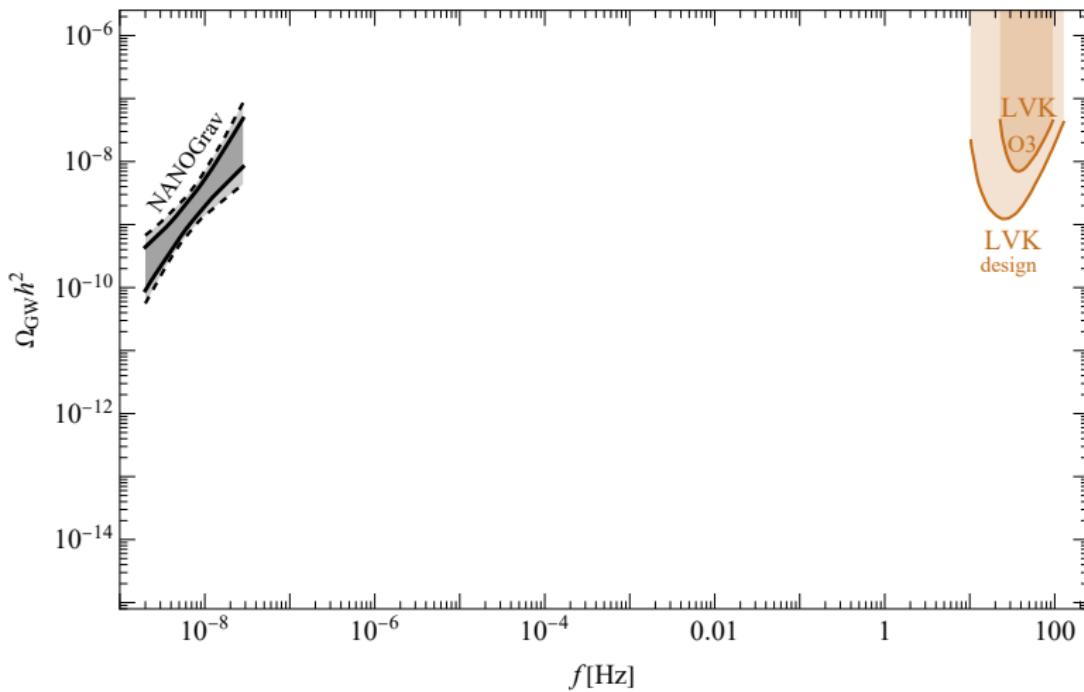
POLSKIE POWROTY
POLISH RETURNS

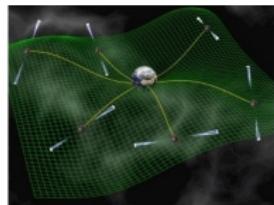
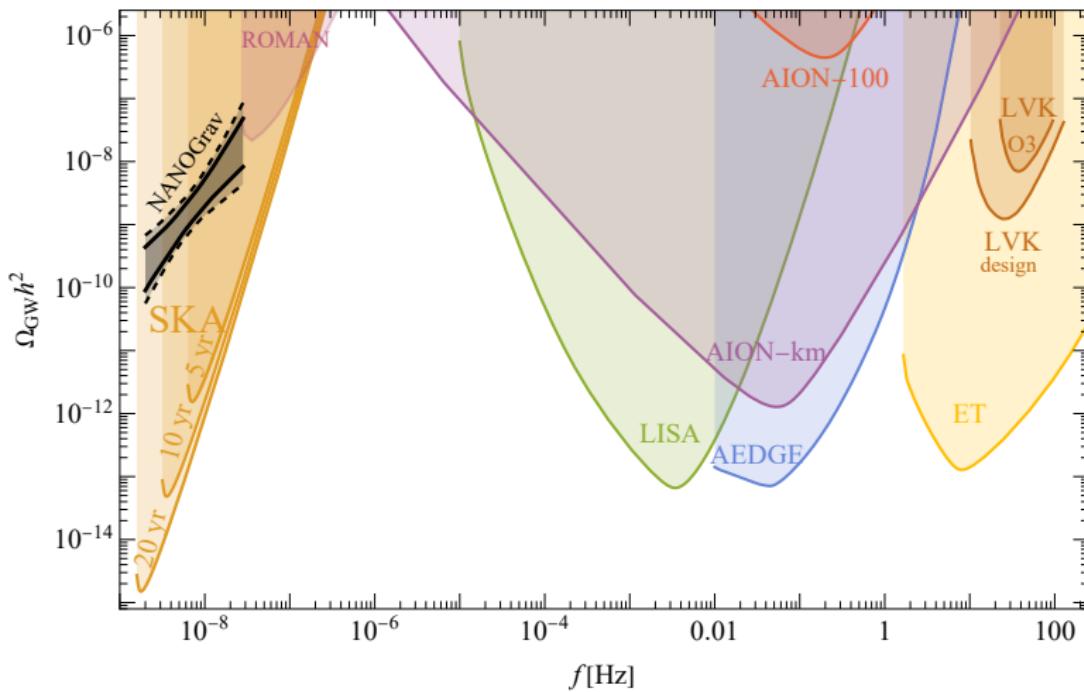


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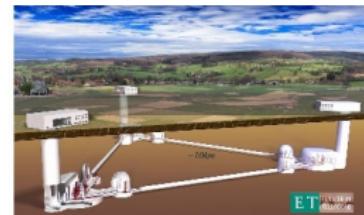


Pulsar Timing

[David Champion/NASA/JPL]

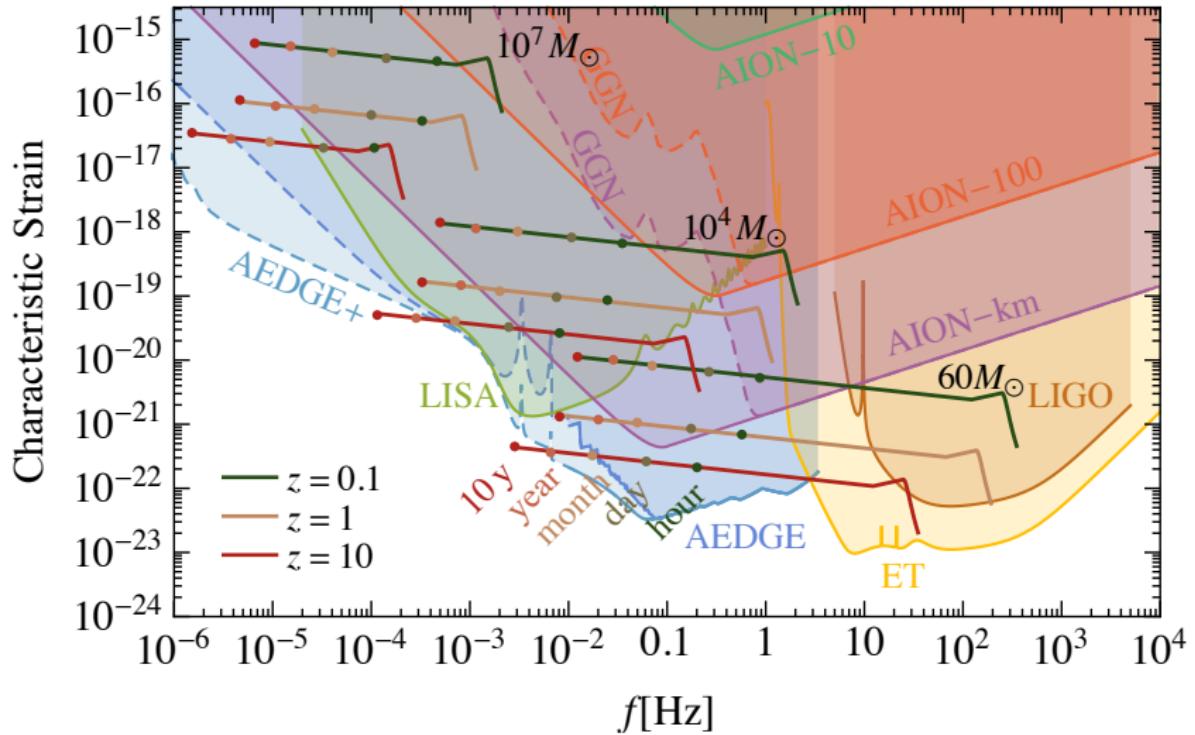


LISA
[wiki/Laser_Interferometer_Space_Antenna](https://en.wikipedia.org/wiki/Laser_Interferometer_Space_Antenna)

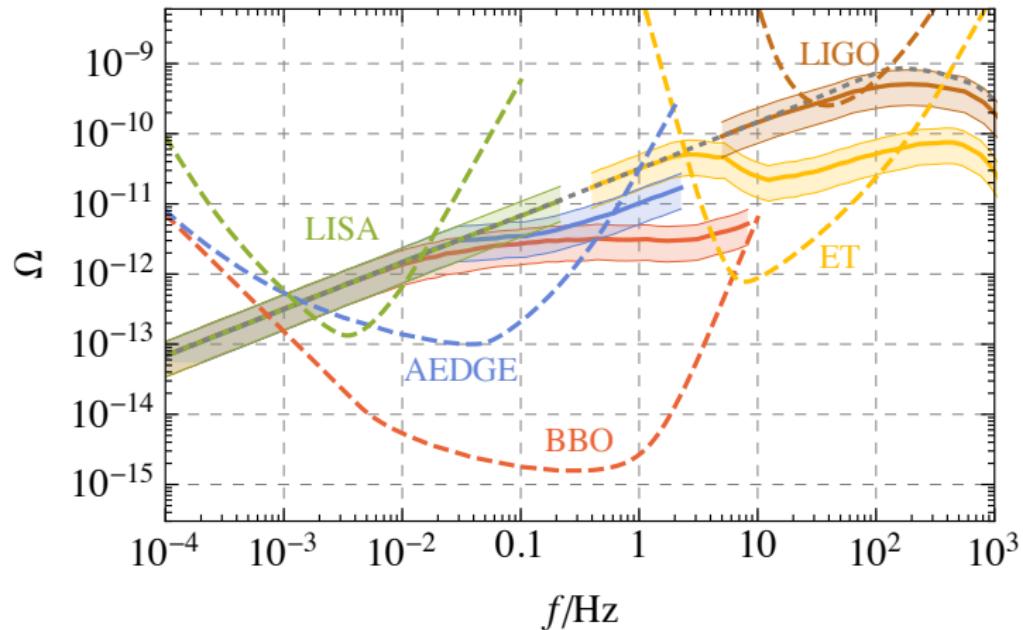


Einstein Telescope
www.et-gw.eu

Sensitivity to binary mergers



Foreground from LIGO-Virgo binaries

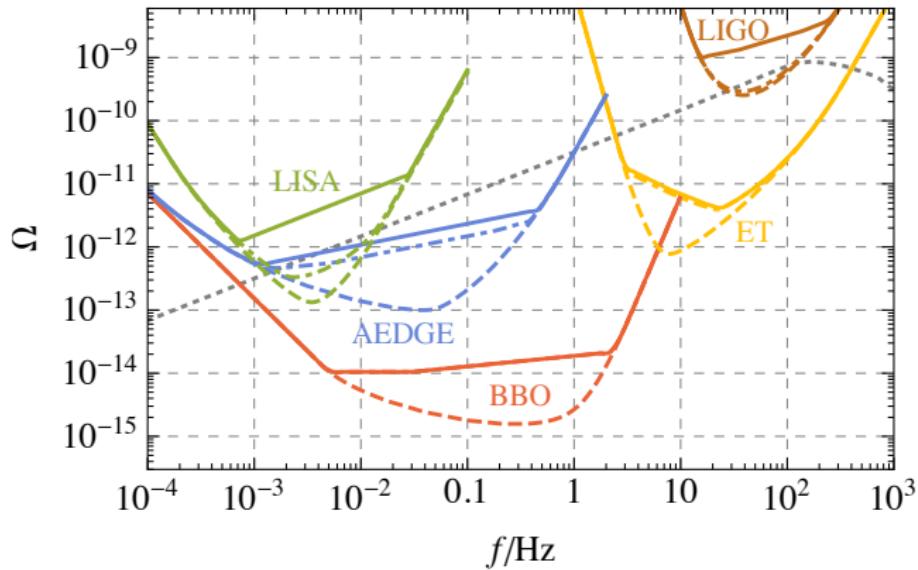


- Dashed gray line: total foreground from LIGO-Virgo binaries
- Thick lines: foreground without individually observable binaries

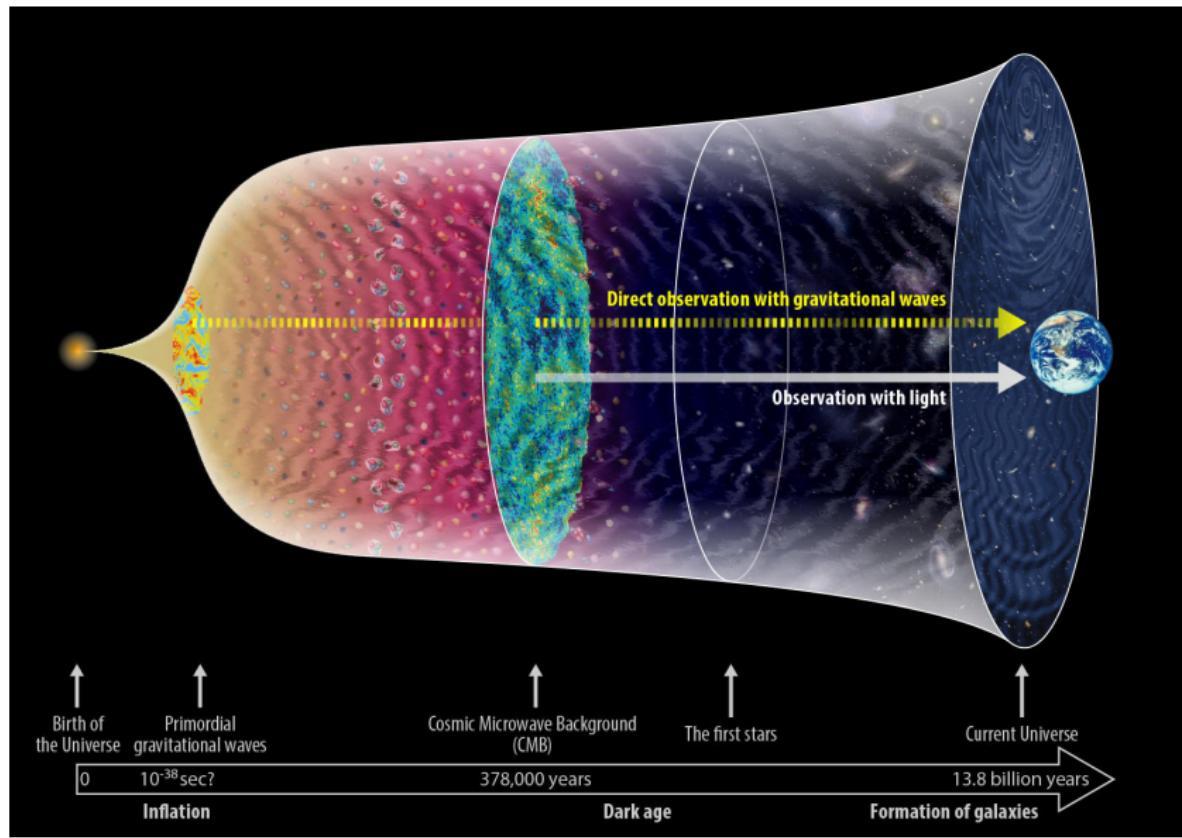
Improved sensitivities from Fisher analysis

- assuming power-law signal as in PI sensitivity

$$\Omega_{\text{GW}}(f) = \Omega \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha} + A \langle \Omega_{\text{BBH}}(f) \rangle + \Omega_{\text{BWD}}(f) + \Omega_{\text{instr}}(f)$$



Early Universe Sources



plot credit:<https://gwpo.nao.ac.jp/en/gallery>

First Order Phase Transition: bubble nucleation

- Temperature corrections to the potential

$$V(\phi, T) = \frac{g_m^2}{24} (T^2 - T_0^2) \phi^2 - \frac{g_m}{12\pi} T \phi^3 + \lambda \phi^4$$

- EOM \rightarrow bubble profile

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} - \frac{\partial V(\phi, T)}{\partial \phi} = 0,$$

$$\phi(r \rightarrow \infty) = 0 \quad \text{and} \quad \dot{\phi}(r=0) = 0.$$

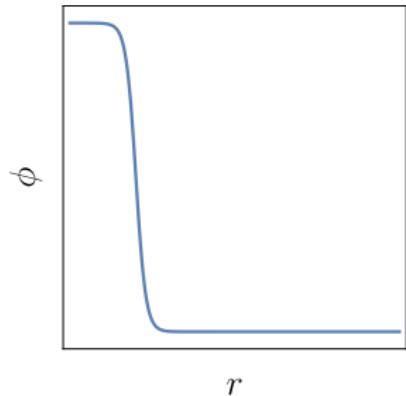
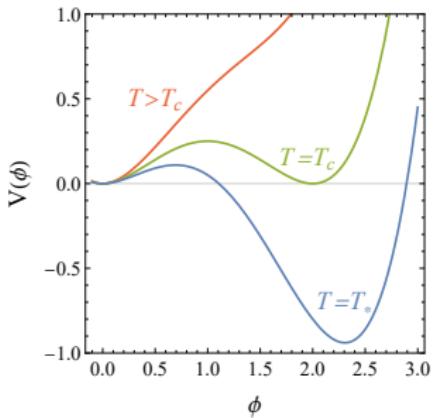
- $\mathcal{O}(3)$ symmetric action

$$S_3(T) = 4\pi \int dr r^2 \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + V(\phi, T) \right].$$

- nucleation temperature

$$\frac{\Gamma}{H^4} \approx \left(\frac{T}{H} \right)^4 \exp \left(- \frac{S_3(T)}{T} \right) \approx 1$$

Linde '81 '83



Gravitational waves from a PT

- Strength of the transition

$$\alpha \approx \left. \frac{\Delta V - T \frac{d\Delta V}{dT}}{\rho_R} \right|_{T=T_*}, \quad \Delta V = V_f - V_t$$

- Characteristic scale

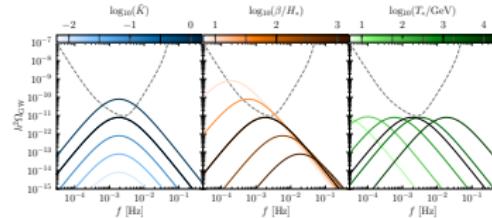
$$\Gamma \propto e^{-\frac{S_3(T)}{T}} = e^{\beta(t-t_0)} \implies \left. \frac{\beta}{H} = T \frac{d}{dT} \left(\frac{S_3(T)}{T} \right) \right|_{T=T_*}$$

- Peak frequency

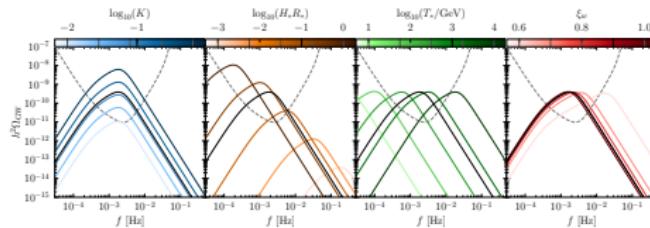
$$f_p \propto T_* \frac{\beta}{H}$$

Three main sources of GWs

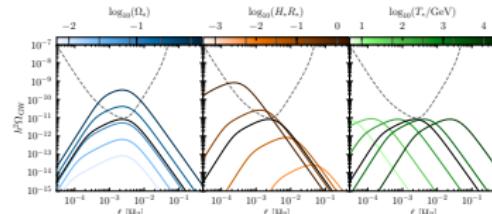
- collisions of bubble walls



- sound waves



- turbulence



Sound Waves (in weak transitions $\alpha \lesssim 0.1$)

- Simulation of a scalar coupled to the plasma

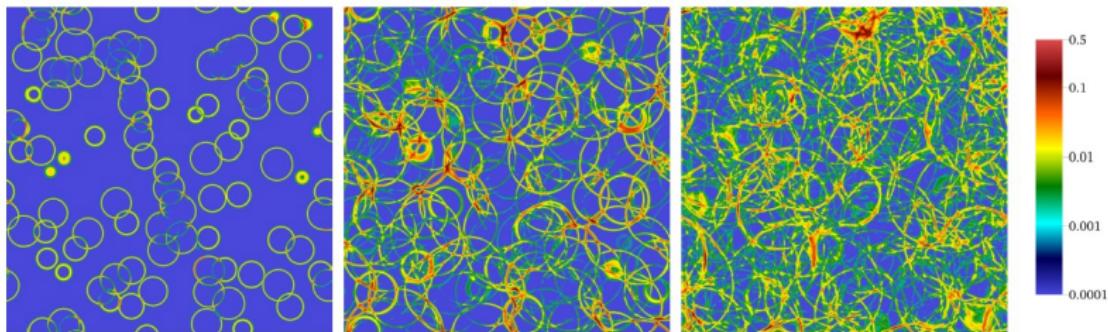


FIG. 4. Slices of fluid kinetic energy density E/T_c^4 at $t = 500 T_c^{-1}$, $t = 1000 T_c^{-1}$ and $t = 1500 T_c^{-1}$ respectively, for the $\eta/T_c = 0.15$, $N_b = 988$ simulation.

- Fit to the GW spectrum

$$\Omega_{\text{gw}} \propto \left(\frac{f}{f_p} \right)^3 \left(\frac{7}{4 + 3(f/f_p)^2} \right)^{\frac{7}{2}}$$

Sound Waves (in weak transitions $\alpha \lesssim 0.1$)

- Higgsless simulation of the plasma

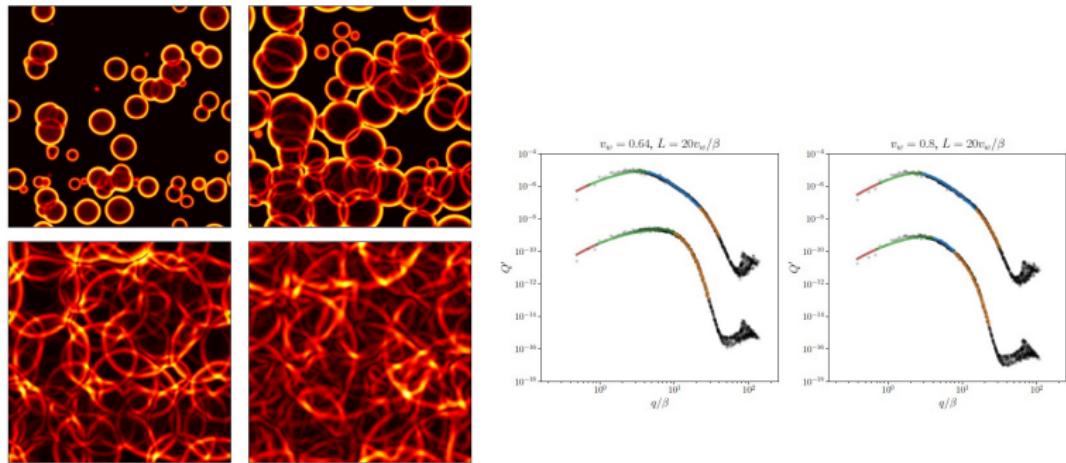


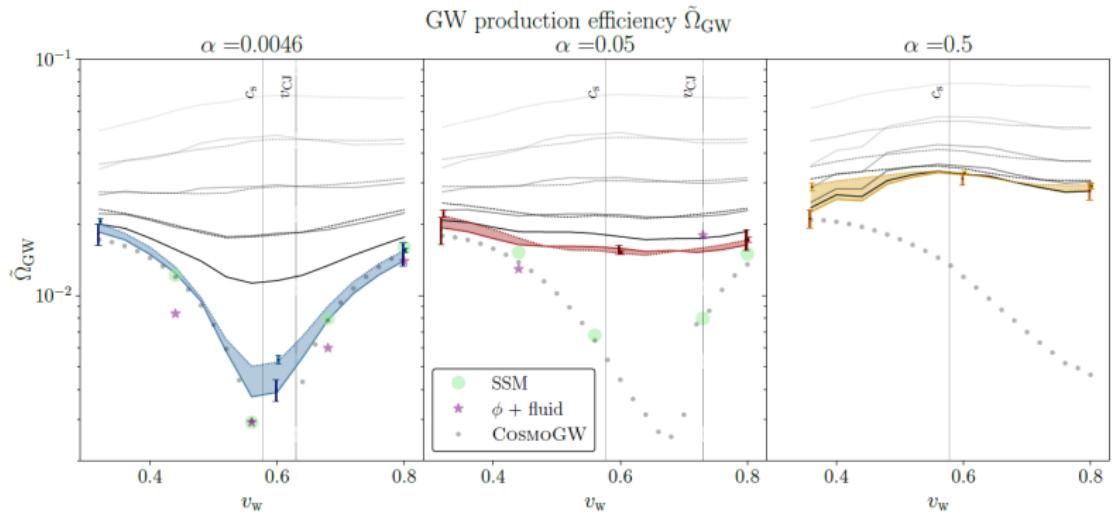
Figure 4: Kinetic energy v^2 in different simulation snapshots: $t = 2.7/\beta$ (top left), $5.4/\beta$ (top right), $10.8/\beta$ (bottom left) and $20.1/\beta$ (bottom right). We use box size $L = 40v_w/\beta$, weak transitions and $v_w = 0.8$.

- Fit to the GW spectrum

$$\Omega_{\text{gw}} \propto \frac{(f/f_1)^3}{1 + (f/f_1)^2[1 + (f/f_2)^4]}, \quad f_2/f_1 \approx 1/\xi_{\text{shell}}$$

Sound Waves (in less weak transitions $\alpha \lesssim 0.5$)

- Higgsless simulation of the plasma



- Fit to the GW spectrum unchanged

$$\Omega_{\text{gw}} \propto \frac{(f/f_1)^3}{1 + (f/f_1)^2[1 + (f/f_2)^4]}, \quad f_2/f_1 \approx 1/\xi_{\text{shell}}$$

Gravitational waves from a very strong PT

- Strength of the transition

$$\textcolor{green}{\alpha} \approx \frac{\Delta V}{\rho_R} \gg 1$$

- Reheating temperature

$$\textcolor{red}{T}_{\text{reh}} \approx \left(\frac{30}{\pi^2} \frac{1}{g_*} \Delta V \right)^{\frac{1}{4}}$$

- Characteristic scale

$$\Gamma \propto e^{-\frac{S_3(T)}{T}} = e^{\beta(t-t_0)} \implies \frac{\beta}{H} = T \frac{d}{dT} \left(\frac{S_3(T)}{T} \right) \Big|_{T=T_p}$$

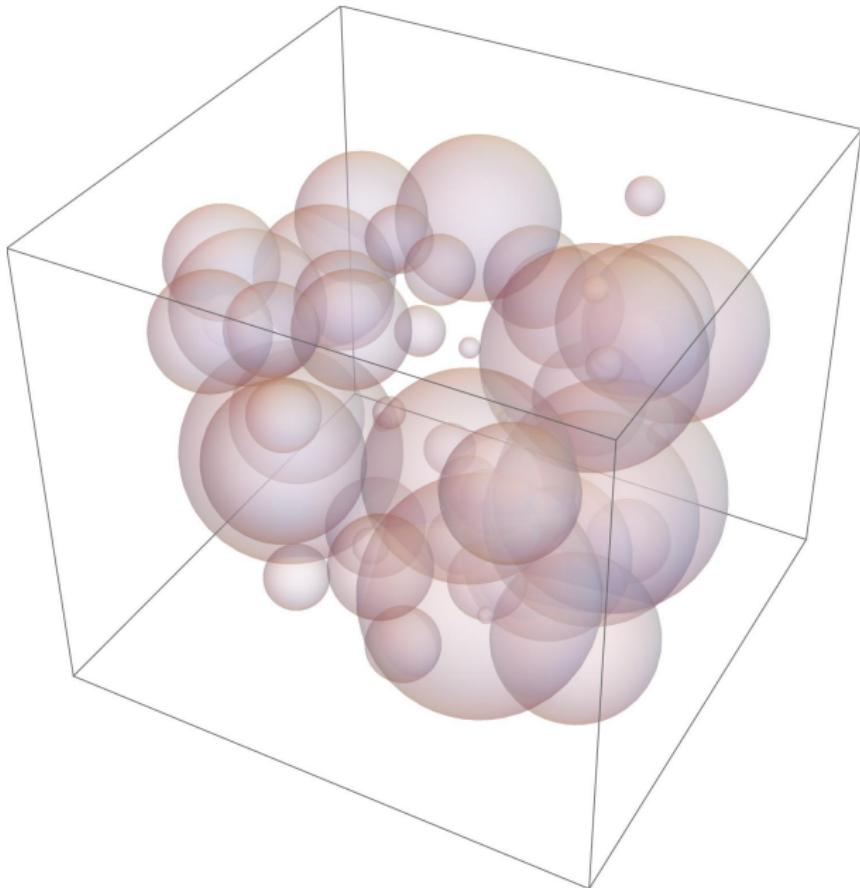
- Amplitude of the spectrum

$$\Omega_p \propto \left(\frac{\textcolor{green}{\alpha}}{\textcolor{green}{\alpha} + 1} \right)^2 \left(\frac{\beta}{H} \right)^{-2}$$

- Peak frequency

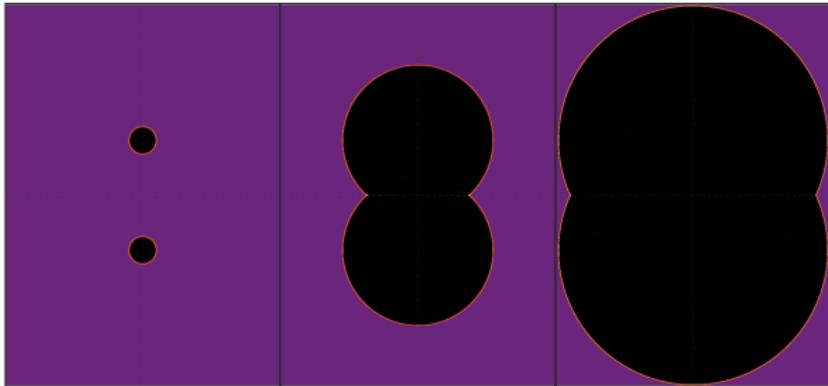
$$f_p \propto \textcolor{red}{T}_{\text{reh}} \frac{\beta}{H}$$

Strong transitions: computation of the GW spectrum



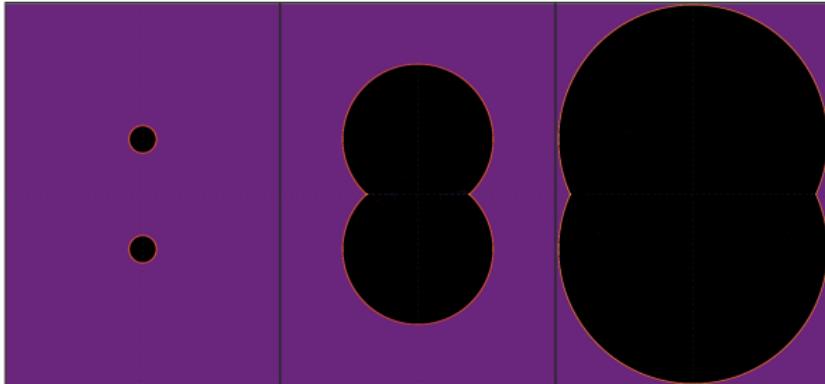
Bubble Collisions

- Envelope

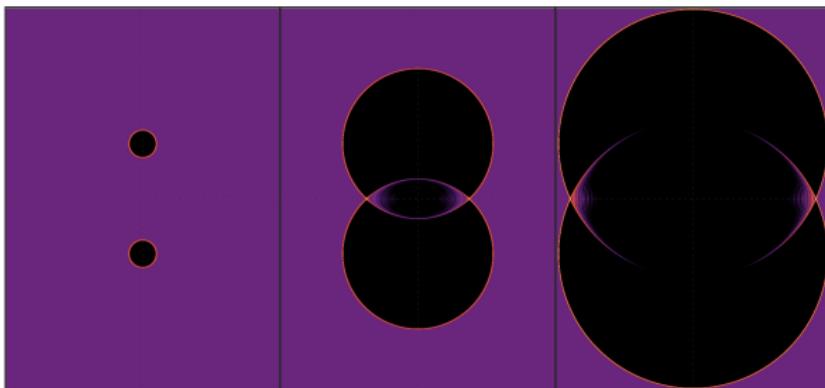


Bubble Collisions

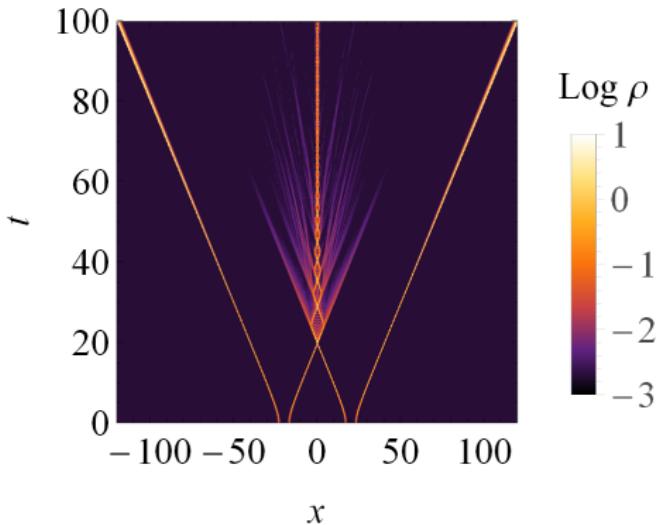
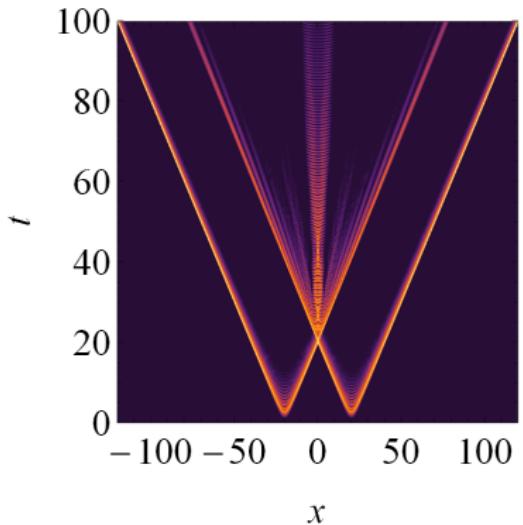
- Envelope



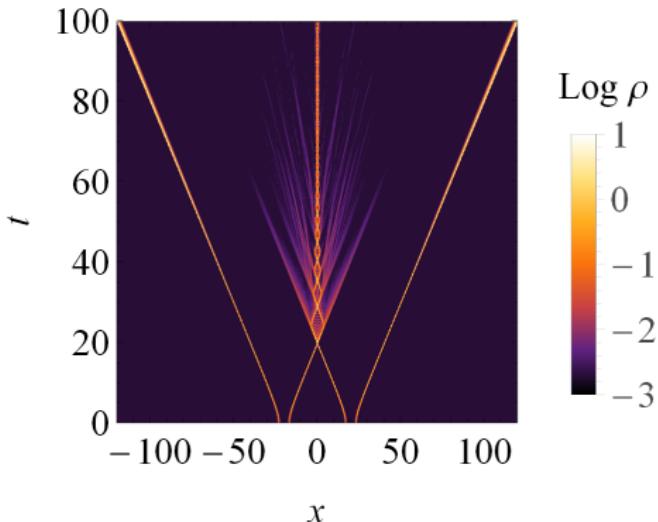
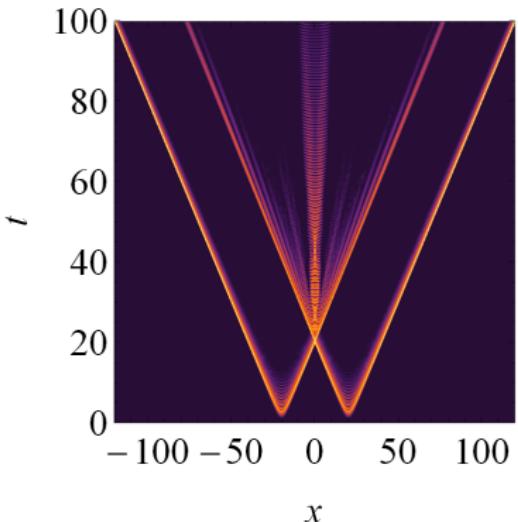
- Simulation



Vacuum Trapping



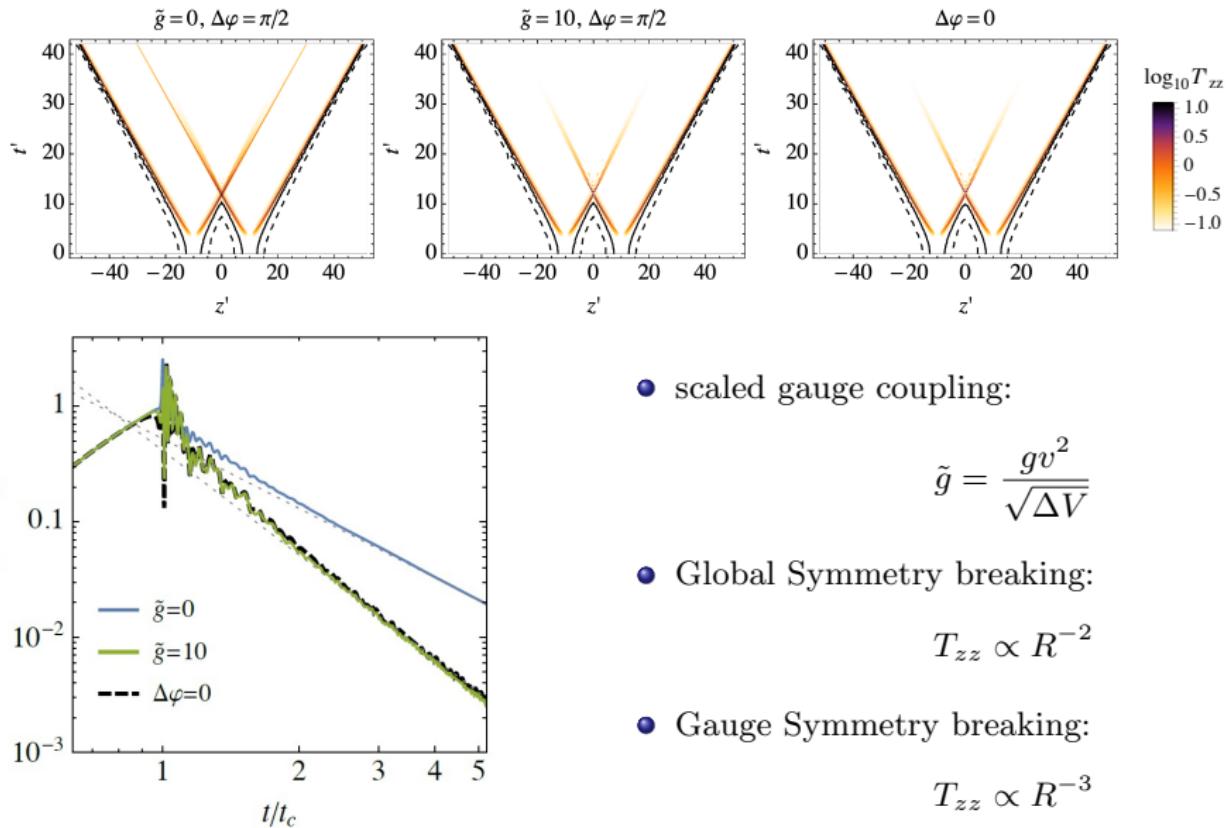
Vacuum Trapping



- scale-invariant vs. polynomial

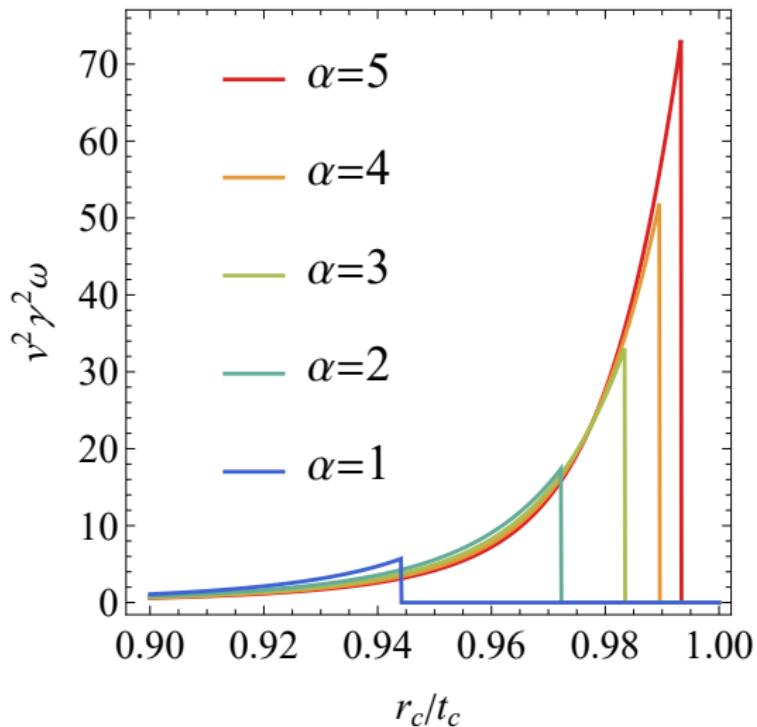
- can also be verified analytically:
R. Jinno, T. Konstandin and M. Takimoto: 1906.02588

Abelian Higgs Model: Energy Scaling



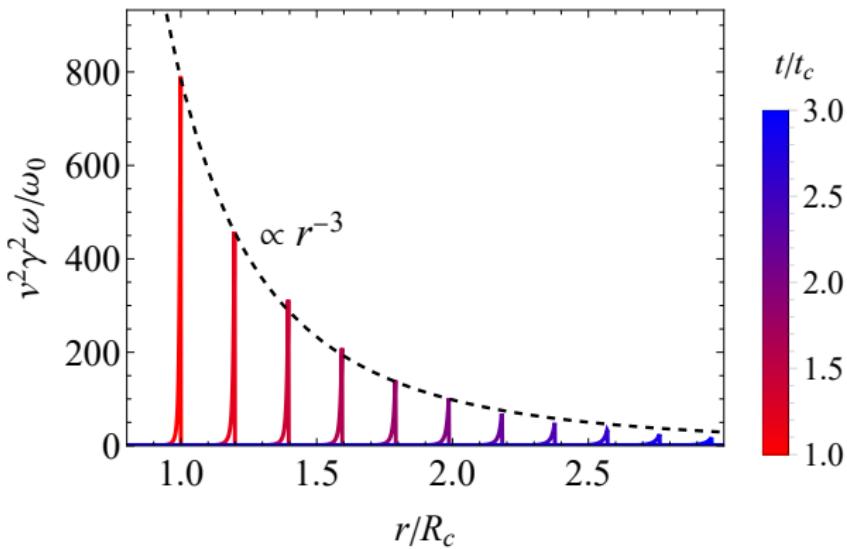
Fluid Shells in strong transitions

- Plasma profiles for $v_w \gtrsim v_J$



Fluid Shell Evolution

- Plasma profile evolution with $\alpha = 20$ and $\gamma_w = 50$

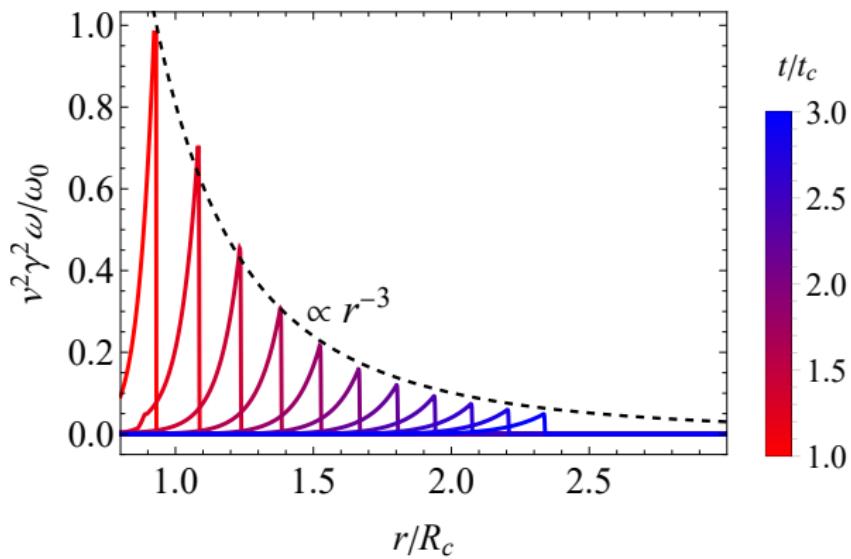


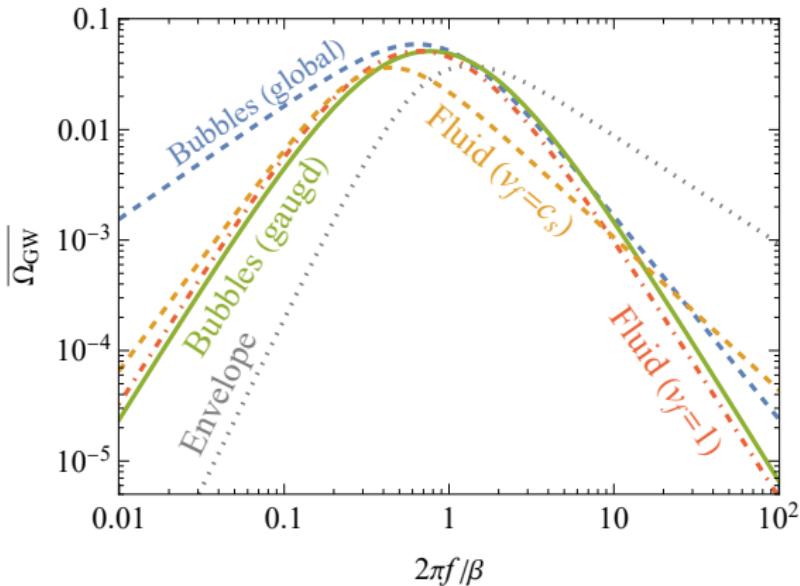
- Fluid shells with $\alpha \gg 1$:

$$T_{zz} \propto R^{-3}$$

Fluid Shell Evolution

- Plasma profile evolution with $\alpha = 0.5$ and $\gamma_w = 3$





- Resulting spectrum:

$$\overline{\Omega_{GW}} = \frac{A (a+b)^c}{\left[b \left(\frac{f}{f_p} \right)^{-\frac{a}{c}} + a \left(\frac{f}{f_p} \right)^{\frac{b}{c}} \right]^c}$$

	Bubbles		Fluid
	Global ($T \propto R^{-2}$)	Gauged ($T \propto R^{-3}$)	$v_{\text{shell}} = 1$
$100 A$	5.93 ± 0.05	5.13 ± 0.05	5.14 ± 0.04
a	1.03 ± 0.04	2.41 ± 0.10	2.36 ± 0.09
b	1.84 ± 0.17	2.42 ± 0.11	2.36 ± 0.09
c	1.91 ± 0.29	1.45 ± 0.34	3.69 ± 0.48
$2\pi f_p / \beta$	1.33 ± 0.19	0.64 ± 0.09	0.66 ± 0.04

ML, Ville Vaskonen arXiv: 2208.11697

ML, Ville Vaskonen, arXiv: 2007.04967

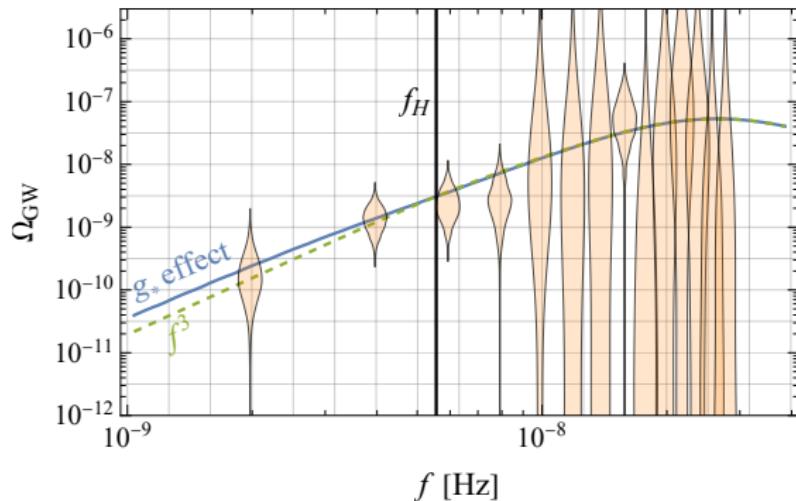
- Spectrum today

$$\Omega_{\text{GW}}(f, T_*) h^2 \approx 1.6 \times 10^{-5} S_H(f, f_H(T_*)) \left[\frac{\beta}{H} \right]^2 \frac{A(a+b)^c}{\left(b \left[\frac{f}{f_p} \right]^{-\frac{a}{c}} + a \left[\frac{f}{f_p} \right]^{\frac{b}{c}} \right)^c}$$

- Superhorizon modes

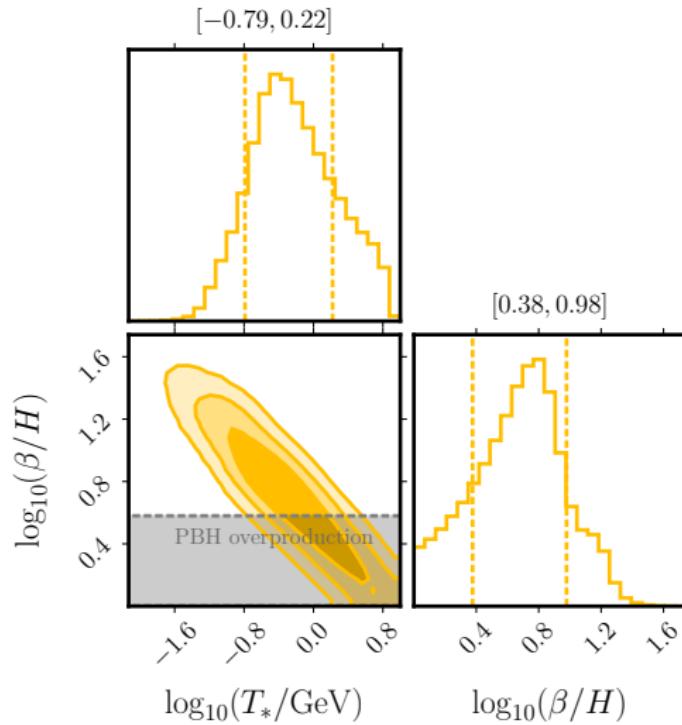
Gabriele Franciolini, Davide Racco, Fabrizio Rompineve arXiv: 2306.17136

$$f_H(T) = \frac{a(T)}{a_0} \frac{H(T)}{2\pi}, \quad S_H(f, f_H) = \left(1 + \left[\frac{\Omega_{\text{CT}}(f)}{\Omega_{\text{CT}}(f_H)} \right]^{-\frac{1}{\delta}} \left[\frac{f}{f_H} \right]^{\frac{a}{\delta}} \right)^{-\delta}$$



Fit to NANOGrav

- Constraint from PBH overproduction $\beta/H \gtrsim 3.9$
ML, Piotr Toczek, Ville Vaskonen arXiv: 2305.04924

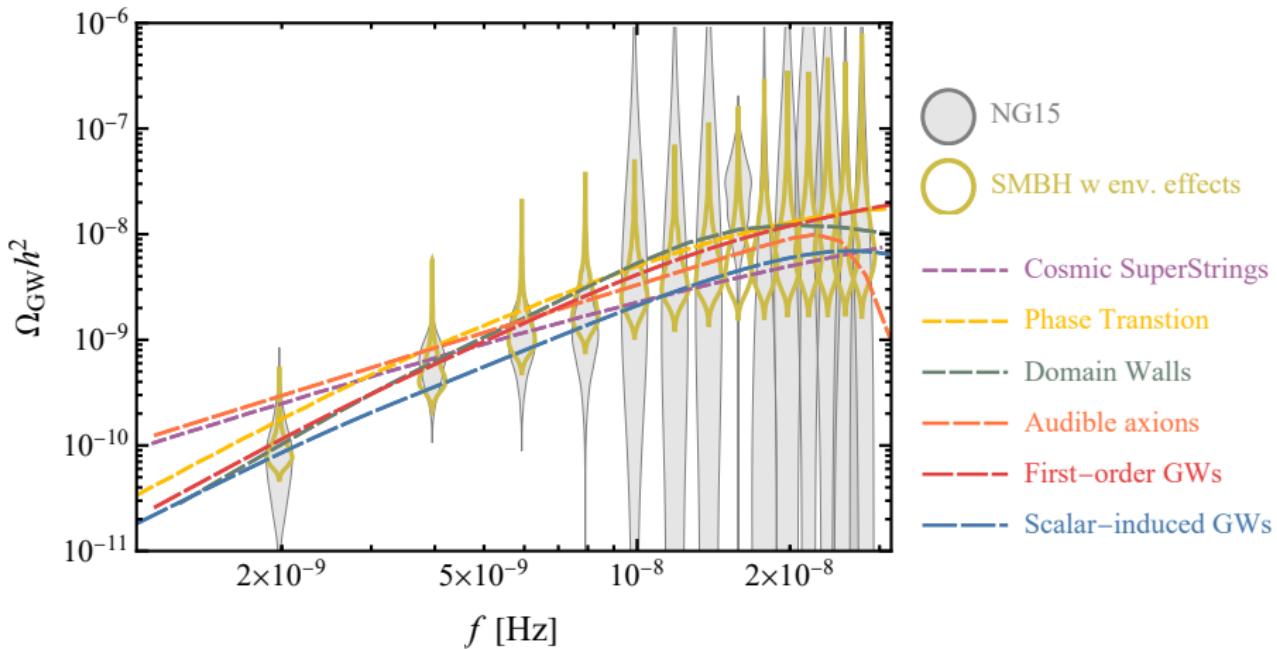


Results from Multi-Model Analysis (MMA)

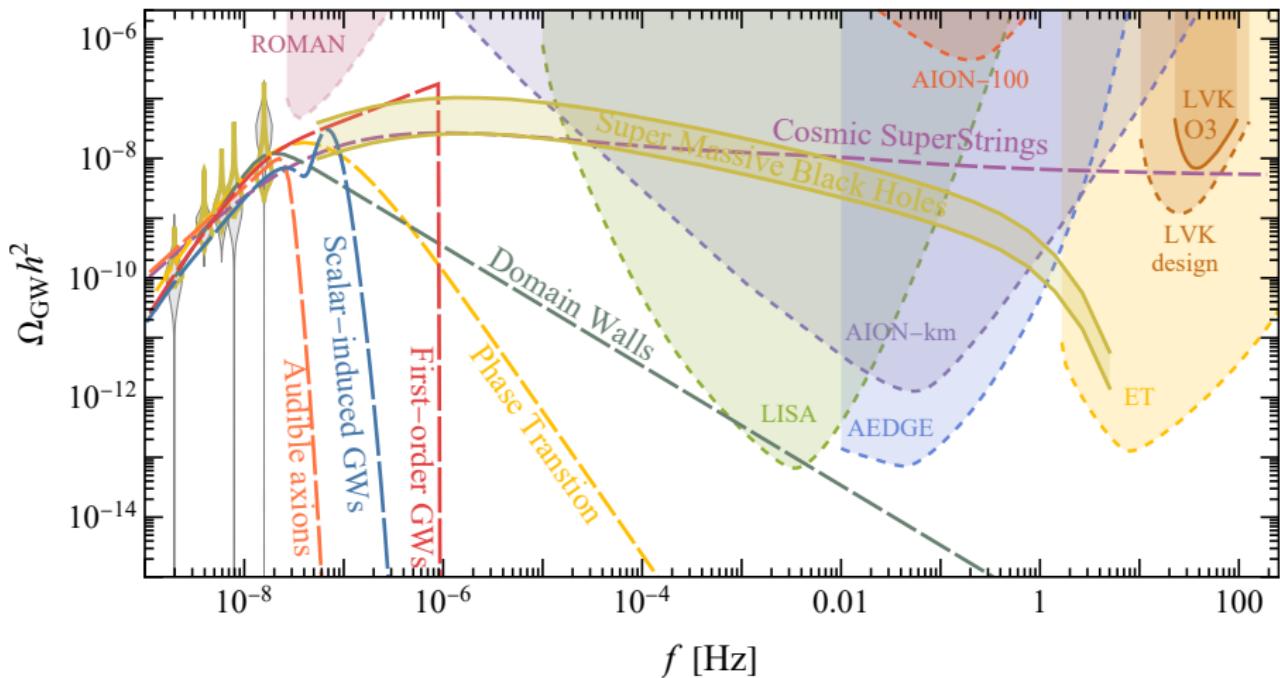
Scenario	Best-fit parameters	ΔBIC
GW-driven SMBH binaries	$p_{\text{BH}} = 0.25$	6.0
GW + environment-driven SMBH binaries	$p_{\text{BH}} = 1, \alpha = 3.8$ $f_{\text{ref}} = 12 \text{ nHz}$	(BIC = 53.9)
Cosmic (super)strings	$G\mu = 2 \times 10^{-12}, p = 6.3 \times 10^{-3}$	-1.2
Phase transition	$T_* = 0.4 \text{ GeV}, \beta/H = 5.5$	-4.9
Domain walls	$T_{\text{ann}} = 0.79 \text{ GeV}, \alpha_* = 0.026$	-5.7
Scalar-induced GWs	$k_* = 10^{7.6}/\text{Mpc}$ $A = 0.08, \Delta = 0.28$	-2.1
First-order GWs	$\log_{10} r = -16, n_t = 2.9$ $T_{\text{rh}} = 0.35 \text{ GeV}$	-2.0
“Audible” axions	$m_a = 3.1 \times 10^{-11} \text{ eV}, f_a = 0.87 M_P$	-4.2

For each model, we tabulate their best-fit values, and the Bayesian information criterion $BIC \equiv -2\Delta\ell + k \ln 14$ relative to that for the purely SMBH model with environmental effects that we take as the baseline. The quantity in the parentheses in the third column shows the ΔBIC for the best-fit combined SMBH+cosmological scenario.

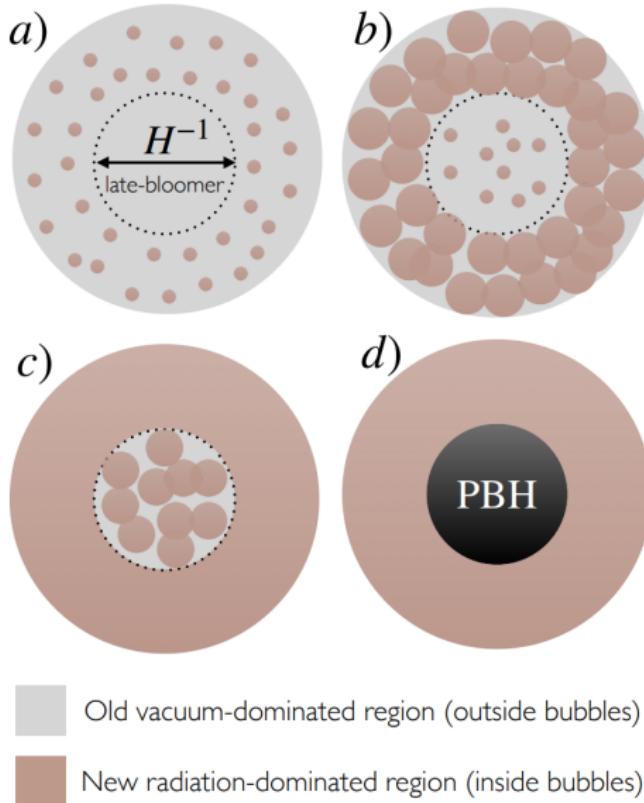
Best Fits to NANOGrav including SMBH



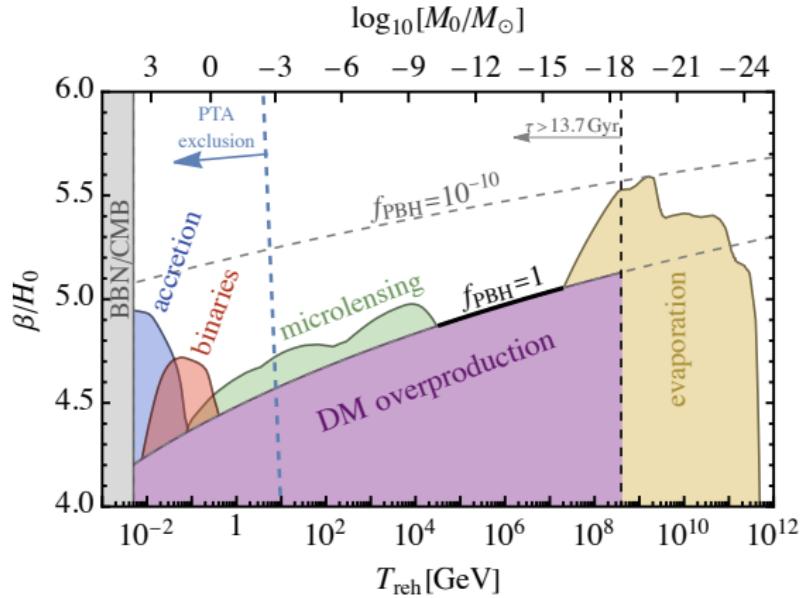
Best Fits to NANOGrav including SMBH



Slow transitions produce PBHs



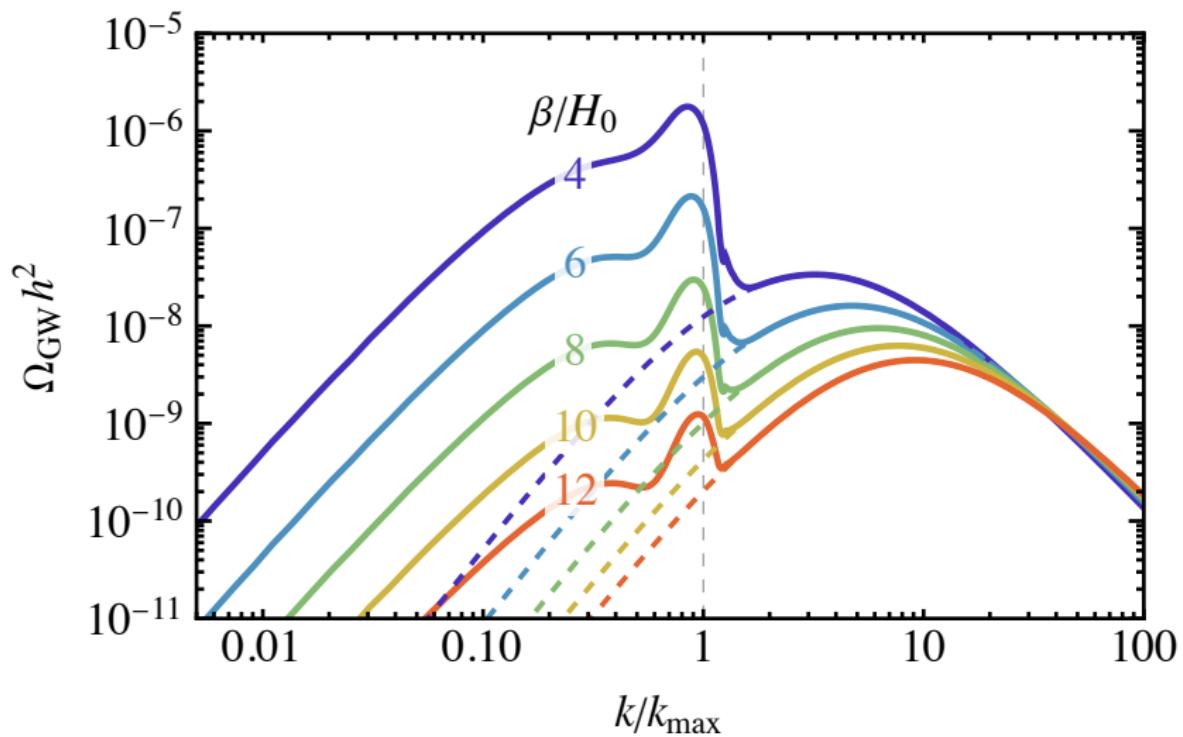
Slow transitions produce PBHs



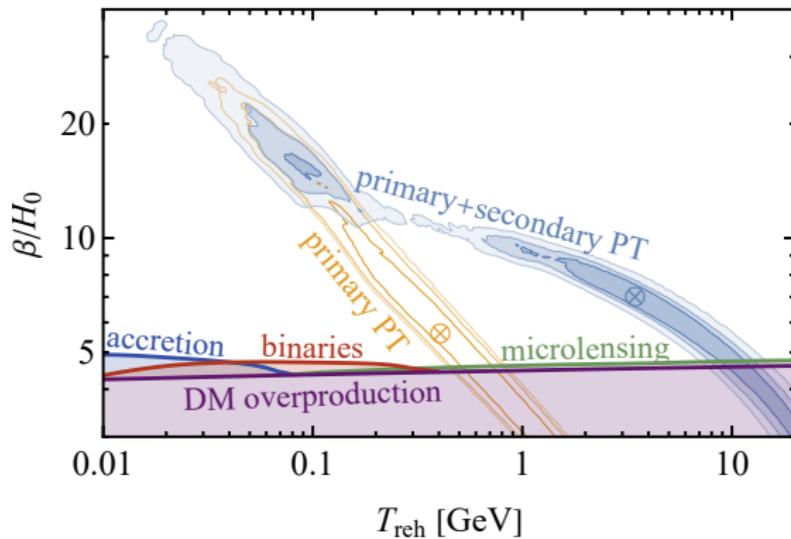
- Mass and abundance of PBHs

$$M_0 \approx 0.05 M_\odot \left[\frac{100}{g_*} \right]^{\frac{1}{2}} \left[\frac{T_{\text{reh}}}{\text{GeV}} \right]^{-2}, \quad f_{\text{PBH}} = 2.8 \times 10^9 \exp \left[-0.246 e^{\beta/H_0} \right] \frac{g_*}{g_{*s}} \frac{T_{\text{reh}}}{\text{GeV}}$$

Secondary GWs dominating for slow transitions

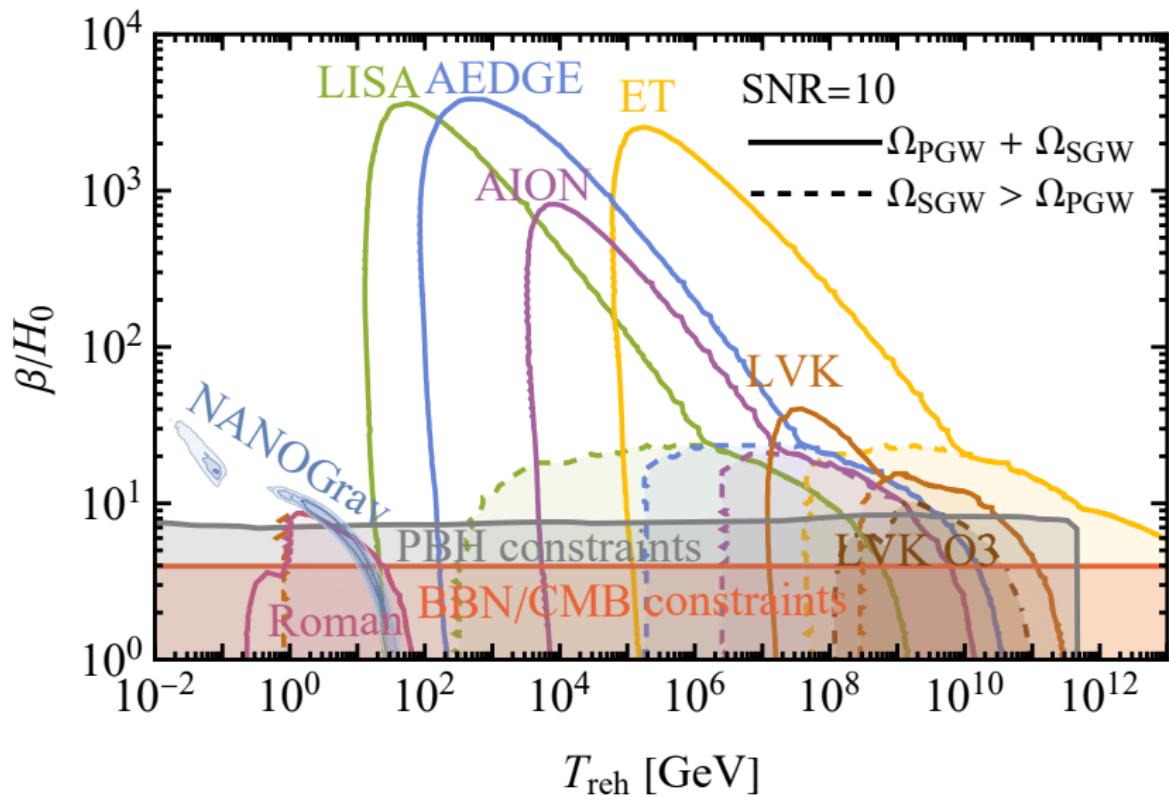


Secondary GWs: Impact on PTA fits



- Best fit with **primary** vs **primary+secondary** GWs
 - $T_{\text{reh}} = 0.4 \text{ GeV}$ $T_{\text{reh}} = 3.4 \text{ GeV}$
 - $\beta/H = 5.5$ $\beta/H = 7.0$

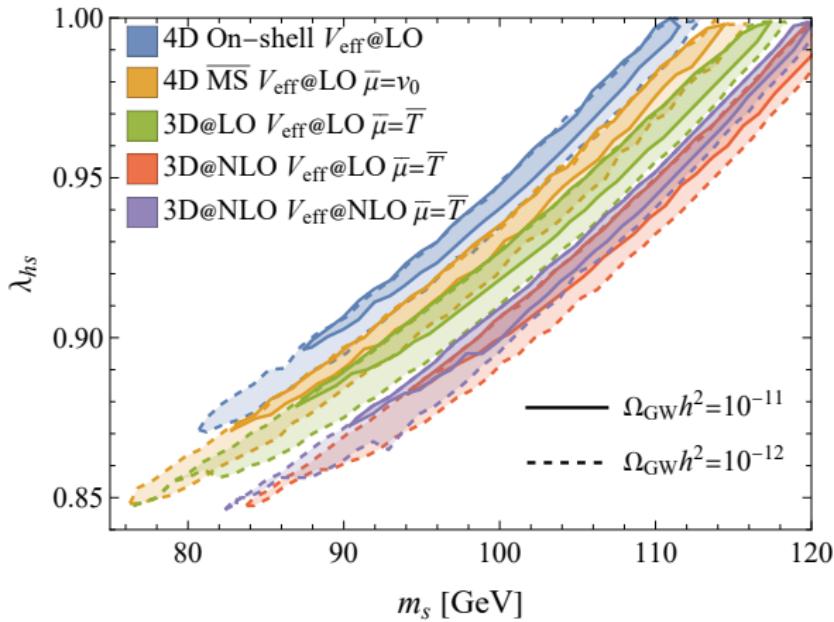
Secondary GWs: Detection prospects



Theoretical uncertainty on the parameter space

- Standard Model with an additional singlet scalar

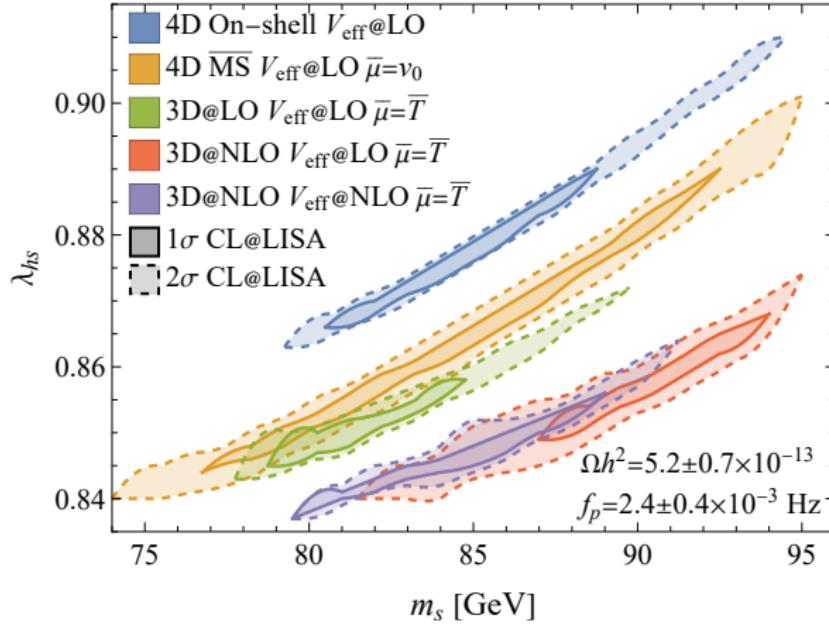
$$V(H, s) = -\mu_h^2 |H|^2 + \lambda |H|^4 + \frac{\lambda_{hs}}{2} S^2 |H|^2 + \left(m_s^2 - \frac{\lambda_{hs} v^2}{2} \right) \frac{s^2}{2} + \frac{\lambda_s}{4} S^4$$



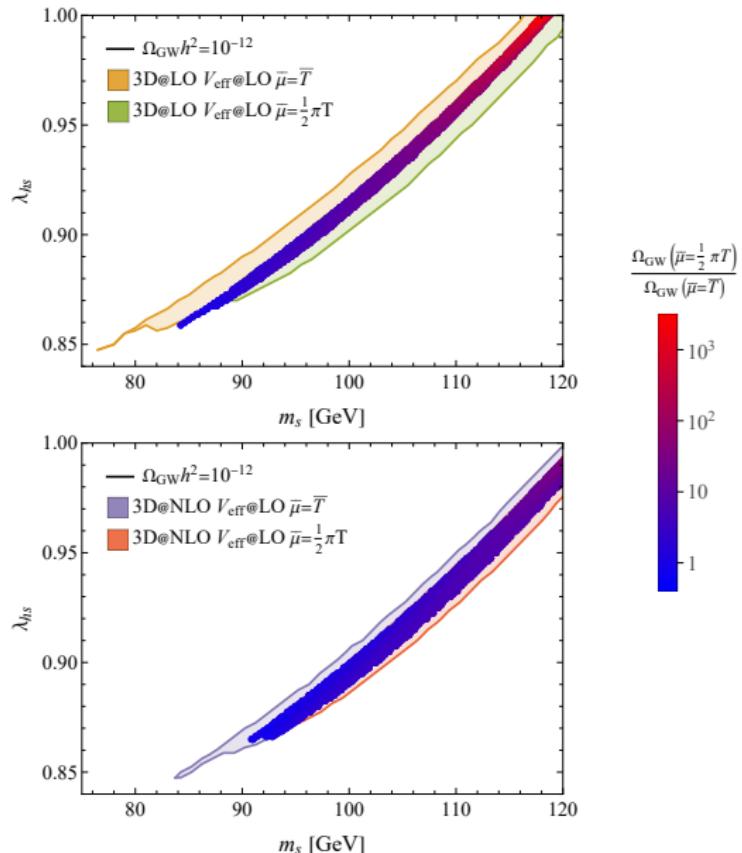
Theoretical uncertainty on the parameter reconstruction

- Standard Model with an additional singlet scalar

$$V(H, s) = -\mu_h^2 |H|^2 + \lambda |H|^4 + \frac{\lambda_{hs}}{2} S^2 |H|^2 + \left(m_s^2 - \frac{\lambda_{hs} v^2}{2} \right) \frac{s^2}{2} + \frac{\lambda_s}{4} S^4$$



Theoretical uncertainty on the parameter space



Conclusions

- Very strong transitions ($\alpha \gg 1$) produce the same GW spectrum regardless of whether the main source is collisions of bubble or motion of the plasma.
- A very strong phase transition is one of the best explanations for the current PTA data.
- If the transition is also slow ($\beta/H \lesssim 10$) a secondary GW contribution will dominate the spectrum and a large population of PBH can be created.
- Large errors on the GW spectra for individual parameter points corresponds to small $O(1\%)$ error on the reconstructed model parameters
- These small reconstruction errors would still dominate the experimental uncertainties for any detectable spectra

Thank you for your attention!