Revising Schwinger pair production in a constant Electric field in de-Sitter

Theoretical Cosmology Working Group Meeting- Stockholm

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Ongoing work with

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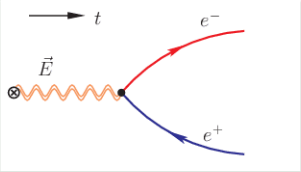
Soon on 2412.XXXXX

Spontaneous **particle creation**

- The Schwinger effect has for long time been known
Fritz, Sauter (1931); W. Heisenberg, H. Euler (1936); J. Schwinger (1951)
  - Pairs of charged particles and anti-particle created by background $\vec{E}$
  - Strong Electric fields are required
  - Effects are exponentially suppressed by the mass



$$log\,\Gamma \propto \frac{m^2 c^3}{ehE} \quad \rightarrow \quad E_{CR} \simeq 1.32 \times 10^{18}\,\mathrm{Vm}^{-1}$$

- It can happen for constant $\vec{E}$, but requires time dependent vector potential $\vec{A}$

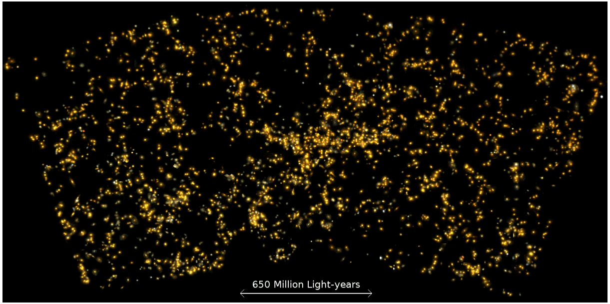
Spontaneous **particle creation**

- A similar effect can happen in Curved backgrounds

  L. Parker (1966); S. W. Hawking (1975)

  - Particle production from vacuum under **time dependent gravitational field**
  - Effects are already important in cosmology
    - LSS might be seeded by accelerated expansion during inflation

  Cosmological Schwinger effect, J. Martin 0704.3540



Sloan Digital Sky Survey, in Saraswati supercluster. Credit: IUCAA

Spontaneous **particle creation** by **time-varying backgrounds**

- Besides cosmic structure, **magnetic fields** observed in the Universe might also have a cosmological origin
  - Proper conditions for **pair production** might have existed during inflation

Exact setting to combine the two examples for particle production

Spontaneous **particle creation** by **time-varying backgrounds**

- Besides cosmic structure, **magnetic fields** observed in the Universe might also have a cosmological origin
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Exact setting to combine the two examples for particle production

Concrete applications for:

- Inflationary Magnetogenesis
  - Generate the **observed magnetic fields** present in voids our Universe
- Generation of Dark Sectors
  - Candidates for non-thermal **dark matter**

Spontaneous **particle creation** by **time-varying backgrounds**

During inflation (Φ), in practice, this could be realized with

$$S = -\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + V(\Phi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{4f}\Phi F_{\mu\nu}\tilde{F}^{\mu\nu} + \mathcal{L}_{ch}(\chi, A_\nu)\right],$$

$$\ddot{A}_\pm + (H+\sigma)\dot{A}\pm + \left(\frac{k^2}{a^2} \mp \frac{\alpha}{f}\dot{\Phi}\frac{k}{a}\right)A_\pm = 0$$

$$\dot{\rho}_\chi + 4H\rho_\chi = \sigma\langle E^2\rangle \qquad\qquad \sigma = \frac{J_\chi}{E}$$

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- No analytical solutions, difficult to test if (renormalization) results make sense
- **Forget about inflation**
  - Fix a de-Sitter background
  - Constant electric field $\vec{E}$ (along z direction)

Scalar QED in de-Sitter

$$S = \int d^4x\sqrt{-g}\left\{-g^{\mu\nu}\left(\partial_\mu - ieA_\mu\right)\phi^*\left(\partial_\nu + ieA_\nu\right)\phi - (m_\phi^2 + \xi R)\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right\},$$

- Set a constant a electric field

$$A_\mu = \frac{E}{H^2\tau}\delta_\mu^z, \qquad F_{\mu\nu}F^{\mu\nu} = -2E^2$$

- After canonically normalizing the scalar field ϕ e.o.m. for $q \equiv a\phi$

$$q_k'' + \omega_{\boldsymbol{k}}^2 q_{\boldsymbol{k}} = 0,$$

- Analytical solution with Whittaker functions

$$q_k = \frac{\mathrm{e}^{-\pi\lambda r/2}}{\sqrt{2k}} W_{i\lambda r,\mu}(2ik\tau)$$

Scalar QED in de-Sitter

- A_ν e.o.m.

$$\nabla^\nu F_{\mu\nu} = J^\phi_\mu \quad \text{with} \quad J^\phi_\mu = \frac{ie}{2}\left\{\phi^\dagger\left(\partial_\mu + ieA_\mu\right)\phi - \phi\left(\partial_\mu - ieA_\mu\right)\phi^\dagger\right\} + \text{ h.c. } .$$

- **Divergent expectation value**. With a cut off momentum ζ

$$\left\langle J^\phi_z \right\rangle = aH\frac{e^2 E}{4\pi^2}\lim_{\zeta\to\infty}\left[\frac{2}{3}\left(\frac{\zeta}{aH}\right)^2 + \frac{1}{3}\ln\frac{2\zeta}{aH} - \frac{25}{36} + \frac{\mu^2}{3} + \frac{\lambda^2}{15} + F_\phi(\lambda,\mu)\right].$$

$$\lambda = \frac{eE}{H^2}, \qquad \mu^2 = \frac{9}{4} - \frac{m_\xi^2}{H^2} - \lambda^2 \quad \text{and} \quad m_\xi^2 = m_\phi^2 + 12\,\xi H^2.$$

Scalar QED in de-Sitter

- The **renormalized** current has been found, with different prescriptions, to be

$$\langle J_z \rangle_{\rm ren} = aH \frac{e^2 E}{4\pi^2} \left[\frac{1}{6} \ln \frac{m_\xi^2}{H^2} - \frac{2\lambda^2}{15} + F_\phi(\lambda, \mu) \right].$$

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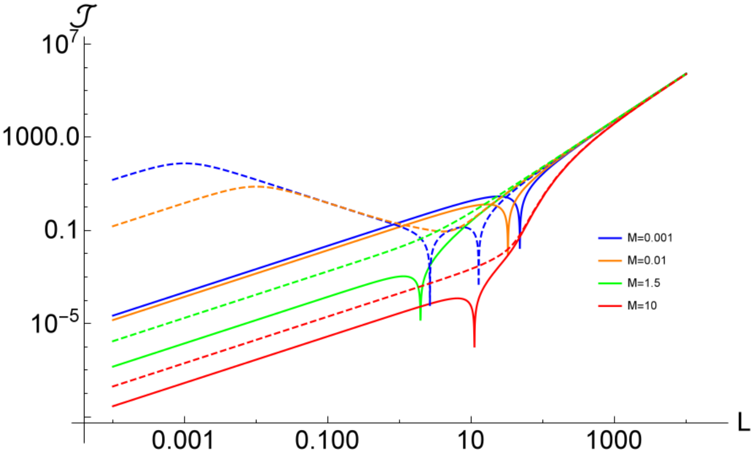
- Adiabatic Subtraction (AS) — *T. Kobayashi, N. Afshordi 2014*
- Point Splitting (PS) — *T. Hayashinaka, J. Yokoyama 2016*
- Pauli Villars (PV) — *M. Banyeres, G. Domenèch, J. Garriga 2018*

- Similar expressions for **fermions**
  - Adiabatic Subtraction — *T. Hayashinaka, T. Fujita, J. Yokoyama 2016*

State of the art



$L = \lambda = \frac{eE}{H^2}$

Dashed: Scalars $\xi = 0$

Solid: Fermions

T. Hayashinaka, T. Fujita, J. Yokoyama 2016

Revising the Renormalization

- Schwinger effect with **classical** $\vec{E}$
  - A_μ not quantized
  - Only charged particles (ϕ/ψ) are quantized
    - No photon loops $\rightarrow$ ϕ/ψ propagator not corrected at loop level
    - No running for ϕ/ψ and $m_{\phi/\psi}$
    - Running of e charge $\Longleftrightarrow$ A_μ (from Ward Identity)

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    - No photon loops $\rightarrow$ ϕ/ψ propagator not corrected at loop level
    - No running for ϕ/ψ and $m_{\phi/\psi}$
    - Running of e charge $\Longleftrightarrow$ A_μ (from Ward Identity)
  - Need to introduce **only one counter-term** in Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}\delta_3(F_{\mu\nu})^2 - eA_\mu J^\mu + \dots \ ,$$

And the corrected equations of motion will be

$$(\delta_3 + 1)\,\nabla^\nu F_{\mu\nu} = \langle J_\mu \rangle \,.$$

Revising the Renormalization

- For a constant electric field in de-Sitter the left hand side gives,

$$(\delta_3 + 1)\,\nabla^{\nu} F_{\mu\nu} = (\delta_3 + 1)\,(-2aHE\delta^{z}_{\nu})\,.$$

- From this we define the **renormalized current**

$$\nabla^{\nu} F_{\mu\nu} = \langle J_{\mu} \rangle_{\mathrm{ren}}$$
$$\langle J_{\mu} \rangle_{\mathrm{ren}} = \langle J_{\mu} \rangle - (-2aHE\delta^{z}_{\nu})\delta_3\,.$$

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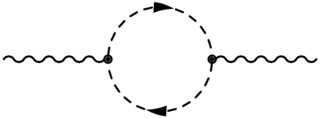
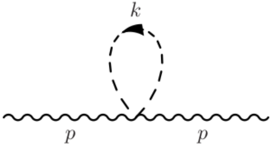
$$\langle J_\mu \rangle_{\rm ren} = \langle J_\mu \rangle - (-2aHE\delta^z_\nu)\delta_3\,.$$

- To get ***physical* renormalized current** we want **on-shell counter-term**! not MS

$$\Pi(p^2 = m_A^2) = 0 \;\rightarrow\; \delta_3 = -e^2\Pi_2(m_A^2)$$

- With classical A_μ , Π_2 fully defined by





Revising the Renormalization: Constant $\vec{E}$ in dS?

- Standard literature result is obtained with by treating Π_2 as in Minkowski

$$\delta_3 = -e^2\Pi_2(p^2 = 0) \;\rightarrow\; \delta_3^{PV} = -\frac{e^2}{48\pi^2}\ln\frac{\Lambda^2}{m^2}$$

- This results in $\ln m/H$ term that creates **negative conductivies** when $m \ll H$

- But does this condition actually hold for our setting?

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$$S = -\int d^4x \sqrt{-g}\, \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \;\rightarrow\; g^{\alpha\nu} g^{\beta\sigma} \nabla_\alpha F_{\nu\sigma} = 0\,.$$

Taking $A_\mu = \frac{E}{H^2 \tau} \delta^z_\mu$, in e.o.m. we find

$$g^{\alpha\nu} g^{\beta\sigma} \nabla_\alpha F_{\nu\sigma} = -2a^{-4} \frac{E}{\tau^3 H^2} \delta^z_i \neq 0\,.$$

- Just a kinetic term is not compatible with constant $\vec{E}$ in de-Sitter

Revising the Renormalization: Constant $\vec{E}$ in dS?

- Introduce an effective mass in Lagrangian

$$S = -\int d^4x\sqrt{-g}\left(\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_A^2 A_\mu A^\mu\right) .$$

e.o.m. gives

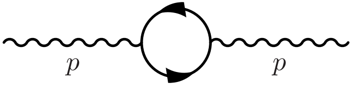
$$-a^{-4}2\frac{E}{\tau^3 H^2}\delta_i^z - m_A^2 a^{-2}\frac{E}{\tau H^2}\delta_i^z = 0 \;\rightarrow\; m_A^2 = -2H^2 .$$

- Get effective tachyonic mass
- Interpreted as effective source that ensures that $\vec{E}$ is not diluted with expansion
- **Consistency with constant electric field** background implies

$$\Pi(p^2 = m_A^2) = 0 \;\rightarrow\; \delta_3 = -e^2\Pi_2(p^2 = -2H^2)$$

Revising the Renormalization: Constant $\vec{E}$ in dS?

Computing δ_3 as in Minkowski with the external momentum fixed by $p^2 = m_A^2 = -2H^2$



- Corrected $\ln m/H$ factor $\rightarrow$ Currents in the massless limit become **finite**
- But for fermions and conformal scalars ($\xi = 1/6$), **when** $eE \ll H^2$ **they are negative**

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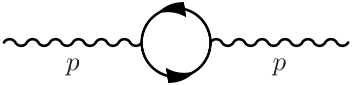


- Corrected $\ln m/H$ factor $\rightarrow$ Currents in the massless limit become **finite**
- But for fermions and conformal scalars ($\xi = 1/6$), **when $eE \ll H^2$ they are negative**

- Most likely Minkowski treatment of propagators in the loop is not accurate
- It does not capture correctly **IR effects**

- We try an approximation as exact de-Sitter does not seem doable (to us)

Correction to propagator: scalar fields

- In QFT, for any spin s, **dispersion relations** come from the Klein-Gordon equation

$$(\Box + m^2)\phi_s = 0 \implies \omega_p^2 = |p|^2 + m^2,$$

- In de-Sitter ($R = 12H^2$), and for a scalar field

$$(\Box + m^2 + \xi R)\phi = (\Box + m_\xi^2)\phi = 0,$$

- In a FLRW background

$$\ddot{\phi} + \frac{\nabla^2}{a^2}\phi + 3H\dot{\phi} + m_\xi^2\phi = 0,$$

- We neglect the friction term while keeping the constant ξR contribution to the mass

$\Box$ is taken to flat space limit and

$$\langle 0|T\{\phi(0)\bar{\phi}(x)\}|0\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2 - m_\xi^2 + i\varepsilon}e^{ipx}.$$

Correction to propagator: What about fermions?

From the Dirac equation

$$(i\underline{\gamma}^\mu \nabla_\mu + m_f)(i\underline{\gamma}^\mu \nabla_\mu - m_f)\psi = 0$$

$$\left(\Box + m_f^2 + \frac{1}{4}R\right)\psi = \left(\Box + \bar{m}_f^2\right)\psi = 0$$

- Take same approximation by treating the kinematics as in Minkowski $\Box \simeq \partial^2$,
- Keep correction to the mass $\bar{m}_f^2 = m_f^2 + 3H^2$

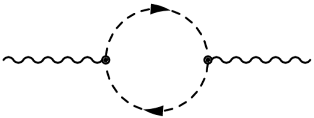
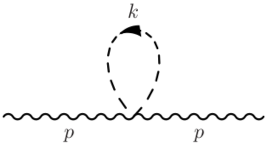
$$\langle 0|T\{\psi(0)\bar{\psi}(x)\}|0\rangle = \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p} + \bar{m}_f)}{p^2 - \bar{m}_f^2 + i\varepsilon} e^{ipx}.$$

Finally δ_3

Taking the previously stated modifications :

- Vector field external momentum fixed to ensure constant electric field in de-Sitter
- R corrected masses to improve IR behavior

  We calculate δ_3 and regularize divergent diagrams

- We have used Pauli-Villars regularization to regularize both δ_3 and $\langle J_\mu \rangle$

Renormalized currents with PV

- Introduce non-dynamical **auxiliary fields to cancel divergences**

  For the **regularized** current we need 3 extra fields

$$\langle J_z \rangle_{\text{reg}} = \lim_{\Lambda\to\infty} \sum_{i=0}^{3} (-1)^i \langle J_z \rangle_i = aH \frac{e^2 E}{4\pi^2} \lim_{\Lambda\to\infty} \left[\frac{1}{6} \ln \frac{\Lambda^2}{H^2} - \frac{2\lambda^2}{15} + F_\phi(\lambda, \mu, r) \right] .$$

- In Λ/H **divergence** to be reabsorbed with **renormalization** of the charge

$$\langle J_\mu \rangle_{reg} = (\delta_3 + 1)\, \nabla^\nu F_{\mu\nu}$$

$$\nabla^\nu F_{\mu\nu} = \langle J_\mu \rangle_{ren} = \langle J_\mu \rangle_{reg}^{PV} - (-2aHE\delta_\nu{}^z)\delta_3^{PV}$$

Renormalized currents with PV

With

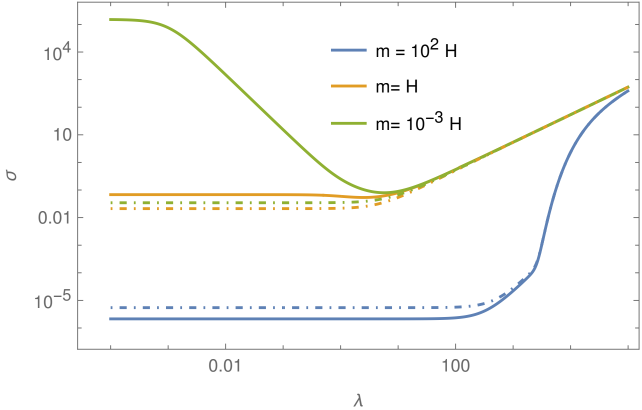
$$\delta_3 = \left(\frac{e}{12\pi}\right)^2 \left(3\ln\left(\frac{m^2}{\Lambda^2}\right) - 12\left(\frac{m}{H}\right)^2 + 6\left(2\left(\frac{m}{H}\right)^2+1\right)^{3/2}\coth^{-1}\left(\sqrt{2\left(\frac{m}{H}\right)^2+1}\right) - 8\right)$$

- We find the **renormalized** current to be

$$\left\langle J_z^\phi \right\rangle_{\text{ren}}^{PV} = aH\frac{e^2 E}{4\pi^2}\left[\frac{1}{3}\ln\frac{m}{H} - \frac{4}{9} - \frac{2}{3}\left(\frac{m}{H}\right)^2 - \frac{2\lambda^2}{15} + \frac{\left(1+2\left(\frac{m}{H}\right)^2\right)^{3/2}}{3}\coth^{-1}\left(\sqrt{2\left(\frac{m}{H}\right)^2+1}\right) + F_\phi\right]$$

- As we will see, $\log m/H$ will "cancel out" when $m \to 0$ and $\left\langle J_z^\phi \right\rangle_{\text{ren}}$ is always positive

Renormalized Conductivities (PV) Fermions vs Scalars



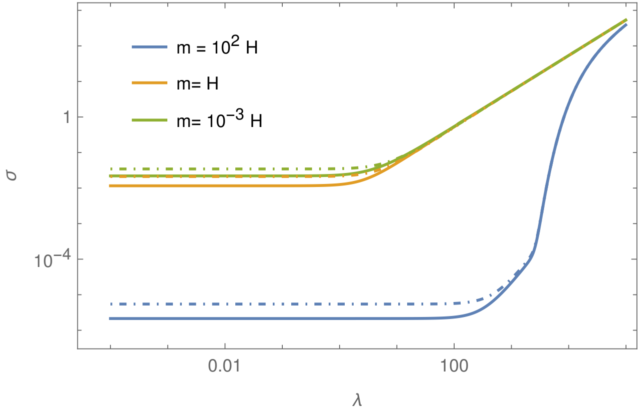
$$\sigma_z \equiv \frac{1}{aH}\frac{\langle J_z \rangle}{e^2 E}$$

Solid: Scalars with $\xi = 0$

DotDashed: Fermions

- **Removed the infrared divergences** ($\ln m/H$) that lead to negative conductivities
- **Corrected negative conductivities**

Renormalized Conductivities (PV) Fermions vs Scalars



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- **Corrected conformal conductivities**

Conclusion & Outlook

- We have revised Schwinger pair production for constant **E** in de-Sitter
- We were able to address and clarify literature's negative conductivities in $H > m$ case
  - **Unphysical** result comes from **wrong physical** conditions
  - Minkowski propagators are inadequate for IR behavior

Next steps

- Submit paper
- Application for the generation of Dark Sectors during inflation
- Study Gravitational wave spectrum in Dark matter compatible scenarios

Renormalizing currents with AS

- The subtraction is done **mode by mode** removing the expectation evaluated in the adiabatic approx

WKB expansion $$q_{\boldsymbol{k}}(\tau) = \frac{1}{\sqrt{2W_{\boldsymbol{k}}(\tau)}} \exp\left\{-i\int^{\tau} d\tilde{\tau}\, W_{\boldsymbol{k}}(\tilde{\tau})\right\}$$

- **Running / Physical Scale AS** with an **arbitrary adiabatic expansion scale** $\bar{m}$

A. Ferreiro, S. Monin, J. Navarro Salas, F. Torrenti 2018, 2022, 2023

- Value of $\bar{m}$ has to be set to obtain the **appropriate adiabatic vacuum** evolution

$$\left\langle J_z^{\phi}\right\rangle_{\rm reg}^{AS} = \left\langle J_z^{\phi}\right\rangle - \left\langle J_z^{\phi}\right\rangle^{(2)} = aH\frac{e^2 E}{4\pi^2}\left[\frac{1}{3}\ln\frac{\bar{m}}{H} - \frac{2\lambda^2}{15} + F_{\phi}(\lambda,\mu,r)\right]$$

- If $m > H \quad \bar{m} = m$;
- If $m < H \quad \bar{m} = H$

Renormalizing currents with DR

- Applying DR, in the Whitaker function we have a **scaleless argument** and integral gives zero
- **Expanding** the argument for a **large energy-like quantity**,

$$e_k = \sqrt{k^2 + a^2 x^2}$$

  Isolates the divergent pieces and **introduce an artificial IR regulator**.

  A. V. Lysenko, O. O. Sobol, E. V. Gorbar, A. I. Momot, and S. I. Vilchinskii 2020, 2023

- We just **regularize the asymptotic piece** $\left\langle J_z^{\phi} \right\rangle^{e_k}$ with DR

Renormalizing currents with DR

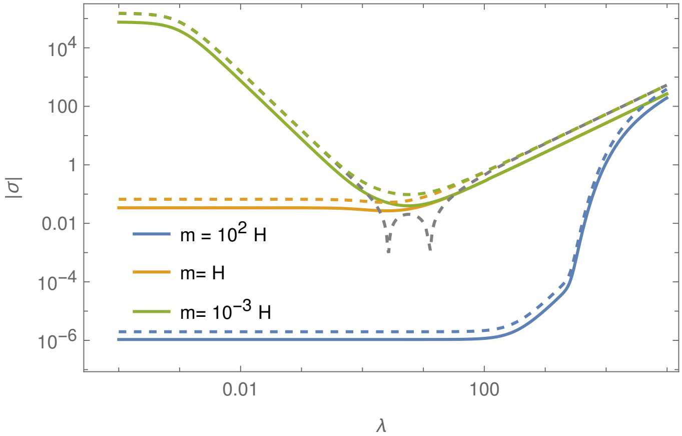
- We obtain a different regularization (minimal ?)

$$\left\langle J_z^{\phi} \right\rangle_{\text{reg}}^{DR} = \left\langle J_z^{\phi} \right\rangle - \left\langle J_z^{\phi} \right\rangle^{e_k} + \left\langle J_z^{\phi} \right\rangle_{reg}^{e_k}$$

$$\left\langle J_z^{\phi} \right\rangle_{\text{ren}}^{DR} = \left\langle J_z^{\phi} \right\rangle_{\text{reg}}^{DR} - (-2aHE\delta_{\nu}^{\ z})\delta_3^{DR}$$
$$= aH\frac{e^2 E}{4\pi^2}\left[\frac{1}{3}\ln\frac{2m}{H} - \frac{7}{18} - \left(\frac{m}{H}\right)^2 - \frac{4\lambda^2}{15} + \frac{\left(1 + 2\left(\frac{m}{H}\right)^2\right)^{3/2}}{3}\coth^{-1}\left(\sqrt{2\left(\frac{m}{H}\right)^2 + 1}\right) + F_{\phi}\right]$$

Renormalized Conductivities PV vs AS

- Successfully **removed the infrared divergences** ($\ln m/H$) that lead to negative conductivities



$$\sigma_z \equiv \frac{1}{aH}\frac{\langle J_z \rangle}{e^2 H}$$

Solid: PV
Dashed: AS
Grey: Old results

Renormalized Conductivities PV vs DR

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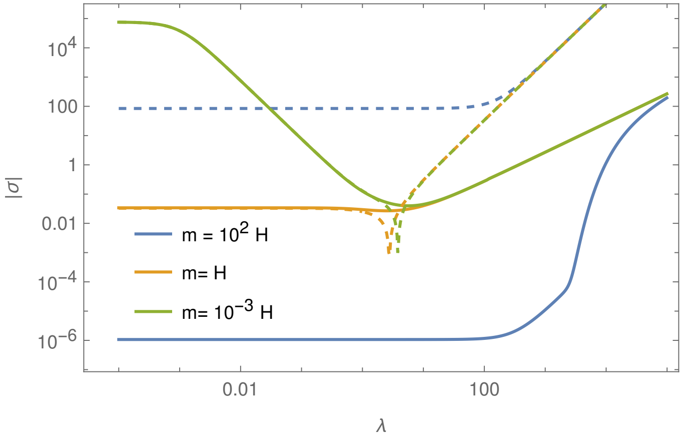

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DR **disagrees** when:

- $\lambda \gg 1$
- $m \gg H$

Non physical results

Solid: PV
Dashed: DR

Conclusion & Outlook

- We have revised PV, AS and DR renormalization in the literature
- We were able to address and clarify literature's negative conductivities in $H > m$ case
  - **Unphysical** result comes from **wrong physical** conditions
  - Minkowski propagators are inadequate for IR behavior
- With both PV and AS we have always recovered physically sensible results
  - Currents show **small deviations**
  - In PV we seem to have a **better knowledge on the physical system**.
  - With the the physical scale AS criteria to determine the scale $\bar{m}$ seems more unsatisfactory.

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Backup

Revising PV

M. Banyeres, G. Domenèch, J. Garriga 2018

- An arbitrary number of additional **auxiliary fields are introduced to cancel divergences**
- The mass of these extra fields will then be sent to infinity, making them **non-dynamical**

Introduce **3** fields $$\sum_{i=0}^{3}(-1)^i = 0 \quad \text{and} \quad \sum_{i=0}^{3}(-1)^i m_i^2 = 0,$$

$$m_0 = m\,, \quad m_2^2 = 4\Lambda^2 - m^2 \quad \text{and} \quad m_1^2 = m_3^2 = 2\Lambda^2, \quad \Lambda \to \infty$$

The **regularized** current $$\langle J_z \rangle_{\rm reg} = \lim_{\Lambda\to\infty} \sum_{i=0}^{3}(-1)^i \langle J_z \rangle_i \,.$$

$$\left\langle J_z^{\phi} \right\rangle_{\rm reg} = aH\frac{e^2 E}{4\pi^2} \lim_{\Lambda\to\infty} \left[\frac{1}{6}\ln\frac{\Lambda^2}{H^2} - \frac{2\lambda^2}{15} + F_\phi(\lambda,\mu,r)\right]$$

- $\ln \Lambda/H$ **divergence** to be reabsorbed with **renormalization** of the charge

$$(\delta_3 + 1)\,\nabla^\nu F_{\mu\nu} = \langle J_\mu \rangle_{reg}$$
$$\langle J_\mu \rangle_{ren} = \nabla^\nu F_{\mu\nu} = \langle J_\mu \rangle_{reg} - (-2aHE\delta_\nu^{\ z})\delta_3$$

Revising AS

- In a **time-dependent background** the **vacuum** of the theory is generally **evolving** making the concept of "vacuum contribution" ambiguous

- The subtraction is done **mode by mode** removing the expectation evaluated in the adiabatic approx

WKB expansion $$q_{\boldsymbol{k}}(\tau) = \frac{1}{\sqrt{2W_{\boldsymbol{k}}(\tau)}} \exp\left\{-i\int^{\tau} d\tilde{\tau}\, W_{\boldsymbol{k}}(\tilde{\tau})\right\}$$

$$\left\langle J_z^{\phi} \right\rangle = -\frac{2e}{(2\pi)^3 a^2}\int d^3k\,(k_z + eA_z)\,\frac{1}{2W_{\boldsymbol{k}}}$$

Inserting the mode function q in the e.o.m.

$$W_{\boldsymbol{k}}^2 = \omega_{\boldsymbol{k}}^2 + \frac{3}{4}\left(\frac{W_{\boldsymbol{k}}'}{W_{\boldsymbol{k}}}\right)^2 - \frac{1}{2}\frac{W_{\boldsymbol{k}}''}{W_{\boldsymbol{k}}}$$

Expanded at the n^{th} order

$$W_{\boldsymbol{k}} = W_{\boldsymbol{k}}^{(0)} + W_{\boldsymbol{k}}^{(1)} + W_{\boldsymbol{k}}^{(2)} + \dots$$

Running / Physical Scale AS

- Take $\Omega_{\boldsymbol{k}}^{\bar{m}}$ with **arbitrary adiabatic expansion scale** $\bar{m}$ (opposed to automatically set $\bar{m} = m$)
  A. Ferreiro, S. Monin, J. Navarro Salas, F. Torrenti 2018, 2022, 2023

$$\Omega_{\boldsymbol{k}}^{\bar{m}2} = (k_z + eA_z)^2 + k_x^2 + k_y^2 + a^2\bar{m}^2 = \omega_{\boldsymbol{k}}^2 + a^2(\bar{m}^2 - m^2) + \frac{a''}{a}$$

And set $W_{\boldsymbol{k}}^{2\,(0)} = \Omega_{\boldsymbol{k}}^{\bar{m}2}$

Find second order $W_{\boldsymbol{k}}^2$ with e.o.m. $W_{\boldsymbol{k}}^{2\,(2)} = \Omega_{\boldsymbol{k}}^{\bar{m}2} - a^2(\bar{m}^2 - m^2) - \frac{a''}{a} + \frac{3}{4}\left(\frac{\Omega_{\boldsymbol{k}}^{\bar{m}\prime}}{\Omega_{\boldsymbol{k}}^{\bar{m}}}\right)^2 - \frac{1}{2}\frac{\Omega_{\boldsymbol{k}}^{\bar{m}\prime\prime}}{\Omega_{\boldsymbol{k}}^{\bar{m}}}$

$$\left\langle J_z^{\phi}\right\rangle^{(2)} = \lim_{\zeta\to\infty}\frac{eaH^3}{(2\pi)^2}\left[\frac{2\lambda}{3}\left(\frac{\zeta}{aH}\right)^2 - \frac{2\lambda^3}{15} - \frac{\lambda}{3}\left(\frac{m}{H}\right)^2 + \frac{\lambda}{3}\ln\left(\frac{2\zeta}{a\bar{m}}\right) + \frac{\lambda}{18}\right]$$

- And the **renormalized** current is given by

$$\left\langle J_z^{\phi}\right\rangle_{\text{ren}}^{AS} = \left\langle J_z^{\phi}\right\rangle - \left\langle J_z^{\phi}\right\rangle^{(2)} = aH\frac{e^2E}{4\pi^2}\left[\frac{1}{3}\ln\frac{\bar{m}}{H} - \frac{2\lambda^2}{15} + F_{\phi}(\lambda,\mu,r)\right]$$

(Similar to Banyeres et al)

- Value of $\bar{m}$ has to be set to obtain the **appropriate adiabatic vacuum** evolution