# Thermodynamics of Large AdS Black Holes

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AdS/CFT

large black hole dual to high temperature thermal state

IR regulated string thermodynamics

large black holes dominate at high energy density

# AdS-Schwarzschild metric $(d+1$ dim.)

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2$$

$$f(r) = \frac{r^2}{\ell^2} + 1 - \frac{\mu}{r^{d-2}}$$

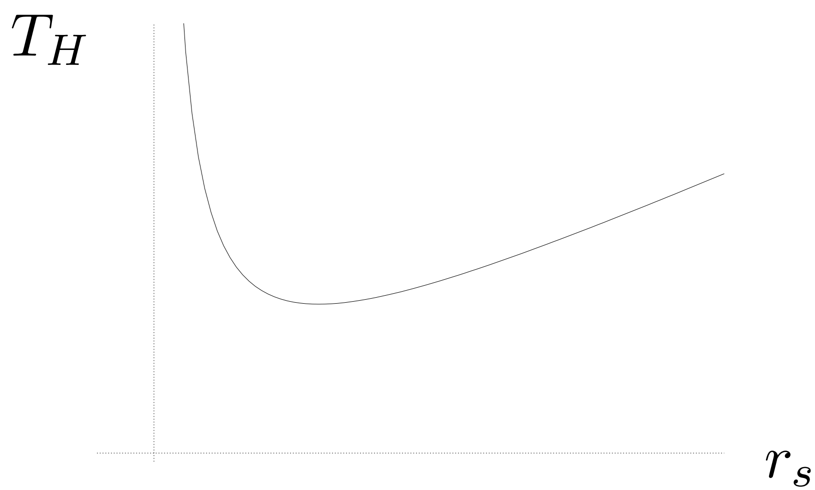
$$\Lambda = -\frac{d(d-1)}{2}\ell^2$$ cosm. const.

$$\mu = \frac{16\pi G_N M}{(d-1)a_{d-1}}$$ black hole mass

$$r = r_s \qquad \text{event horizon}$$

$$f(r_s) = \frac{r_s^2}{\ell^2} + 1 - \frac{\mu}{r_s^{d-2}} = 0 \qquad \Longleftrightarrow \qquad \mu = r_s^{d-2} + \frac{r_s^d}{\ell^2}$$

$$T_H = \frac{1}{4\pi} f'(r_s) = \frac{1}{4\pi}\left(\frac{d-2}{r_s} + \frac{d}{\ell^2}\, r_s\right) \qquad \text{Hawking temperature}$$



$$r_s > \sqrt{\frac{d}{d-2}}\,\ell \quad \text{large black hole}$$

$$r_s < \sqrt{\frac{d}{d-2}}\,\ell \quad \text{small black hole}$$

Minimum Hawking temperature

$$T_H \geq \frac{d(d-2)}{2\pi\ell}$$

Positive specific heat?

$$\frac{d\mu}{dT_H} > 0 \quad \text{large black hole}$$

## Length scales in the problem

$$\ell_s \ll \ell_o \ll \ell\,, r_s$$

| | |
|---|---|
| $\ell_s$ | microscopic scale (string length) |
| $\ell_o$ | microscopic observer |
| $\ell$ | adS length |
| $r_s$ | horizon size |

Very large black holes $\quad r_s \approx (\mu\ell^2)^{1/d} \gg \ell$

$$R_{abcd}R^{abcd}\,|_{r=r_s} = 12\left(\frac{2}{\ell^4} + \frac{\mu^2}{r_s^6}\right) \rightarrow \frac{36}{\ell^4}\left(1 + O(\ell/\mu)^{2/3}\right) \qquad (d=3)$$

Black holes emit Hawking radiation

Confining gravitational potential in asymptotic adS background

$\longrightarrow$ Hawking radiation is reflected back

a small black hole evaporates

$\longrightarrow$ final state is an adS 'star'

a large black hole reabsorbs reflected radiation

$\longrightarrow$ black hole + 'atmosphere'

Does high $T_H$ imply a 'hot' atmosphere?

# AdS stars

Page & Phillips ’84
Hubeny, Liu, & Rangamani ’06

spherically symmetric, self-gravitating radiation $(d = 3)$

perfect fluid $T_{ab} = \rho(r) u_a u_b + p(r)(g_{ab} + u_a u_b)$

equation of state $\rho(r) = 3\, p(r)$

$\longrightarrow$ IR regulated string gas at low energy density

## Einstein's equations reduce to two ODE's

mass equation $\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$

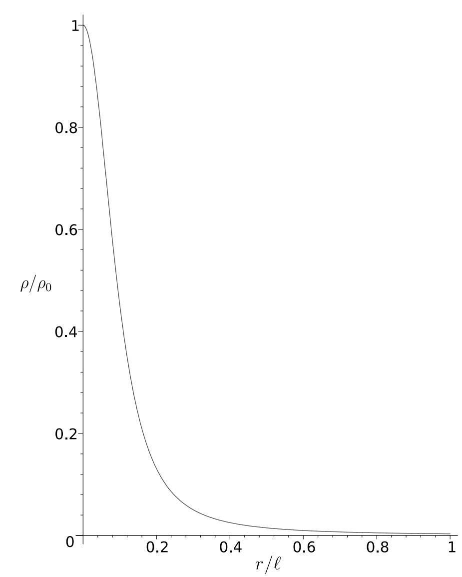
O–V equation $\frac{d\rho(r)}{dr} = -\frac{4\rho(r)}{r}\frac{(\frac{4}{3}\pi r^3\rho(r)+m(r)+\ell^{-2}r^3)}{(\ell^{-2}r^3+r-2m(r))}$

asymptotic behavior $r \gg \ell$

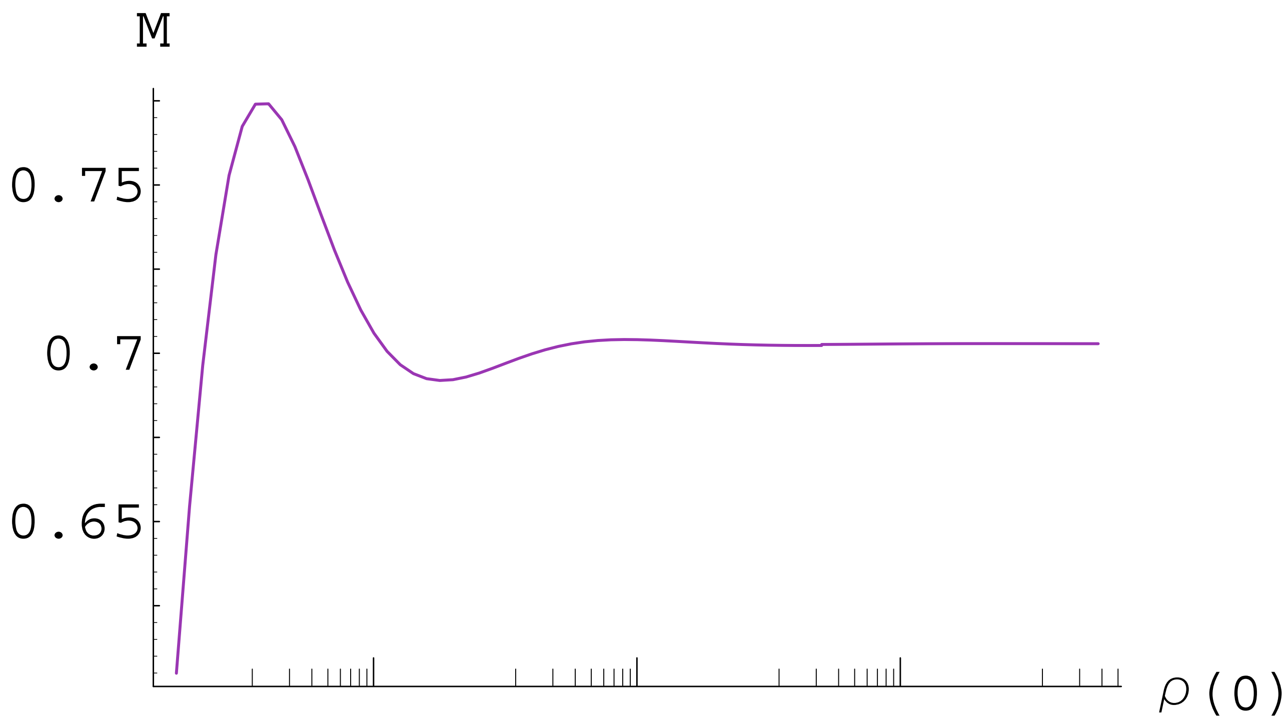
$$\rho(r) = O(r^{-4})$$

$$m(\infty) - m(r) = O(r^{-1})$$

one-parameter family of solutions labeled by $\rho(0)$



total mass of adS star is bounded $M = m(\infty) < O(\ell)$




from Hubeny et al. '06

## Large adS black hole + Hawking radiation

semi-classical geometry including back-reaction?

local temperature is observer dependent

– fiducial observer at fixed $r, \theta, \phi$ $\quad T_{\text{fid}}(r) = \frac{T_H}{\sqrt{f(r)}}$

$T_{\text{fid}}(r) \to 0$ as $r \to \infty$

$T_{\text{fid}}(r) \to \infty$ as $r \to r_s$

- observer in free fall $T_{\text{ff}}(r) = ?$

- relation between $T_{\text{ff}}(r)$ and $T_H$ ?

- Unruh detector in free fall
  $\longrightarrow$ compute acceleration in GEMS

- estimate order of magnitude of $T_{\text{ff}}$ from energy density of Hawking radiation
  $\longrightarrow$ static black hole solutions of semiclassical eom's

## Spherical reduction of 3+1 dim Einstein gravity

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g^{(4)}} \left( R^{(4)} + \frac{6}{\ell^2} \right)$$

metric ansatz $\quad ds^2 = g_{ij}(x^0, x^1) dx^i dx^j + r(x^0, x^1)^2 d\Omega_2^2$

$$S_0 = \frac{1}{4G} \int d^2x \sqrt{-g} \left( r^2 R + 2(\nabla r)^2 + 2 + \frac{6r^2}{\ell^2} \right)$$

Add $N$ scalar fields (minimally coupled in 3+1 dim)

$$S_m = -2 \int d^2x \sqrt{-g}\, r^2 \sum_{i=1}^{N} (\nabla f_i)^2$$

# One-loop effective action

Mukhanov et al. '94

Bousso & Hawking '97

$\vdots$

$$S_q = -\frac{N}{48\pi}\int d^2x\sqrt{-g}\left(\frac{1}{2}R\Box^{-1}R - \frac{6(\nabla r)^2}{r^2}\Box^{-1}R - 6R\ln r\right)$$

rewrite in local form using auxiliary fields $\psi$, $\chi$

Buric et al '98

$$\begin{aligned} S_q &= -\frac{N}{48\pi}\int d^2x\sqrt{-g}\Big(R(\psi - 6\chi - 6\ln r) + \frac{1}{2}(\nabla\psi)^2 \\ &\qquad -6\nabla\psi\nabla\chi - 6\psi\frac{(\nabla r)^2}{r^2}\Big) \end{aligned}$$

small semi-classical corrections if $\frac{NG}{\ell^2} \ll 1$

look for static solutions in conformal gauge $g_{ij} = f(x^1)\,\eta_{ij}$

$$
\begin{aligned}
\psi'' + \partial_x^2 \ln f &= 0 \\
\chi'' - \frac{(r')^2}{r^2} &= 0 \\
r^2 \left( \frac{1}{2}\partial_x^2 \ln f - 3\ell^{-2} f + \frac{r''}{r} \right) &= -NG \left( \partial_x (\psi \frac{r'}{r}) + \frac{1}{2}\partial_x^2 \ln f \right) \\
(r')^2 + r r'' - f(1 + 3\ell^{-2} r^2) &= -NG \left( \frac{1}{6}\partial_x^2 \ln f + \frac{r''}{r} \right)
\end{aligned}
$$

work order by order in $\alpha = \frac{NG}{\ell^2}$

$\alpha^0$ $\quad \frac{dr}{dx} = f_0(r) = \frac{r^2}{\ell^2} + 1 - \frac{\mu}{r}$

$\alpha^1$ $\quad \frac{dr}{dx} = f_0(r) + \alpha\, C_1(r)\,, \qquad f(r) = f_0(r) + \alpha\, f_1(r)$

$\longrightarrow$ linear ODE's for $C_1$, $f_1$, $\chi_0$, $\psi_0$

smoothness at $r = r_s$ and asymptotic adS behavior fix most of the integration constants

asymptotic expansions at large $r$

$$C_1(r) = -3\ln r - \frac{5}{6} + \frac{3}{2}\psi_0 + c_0\, r^{-1} + O(r^{-2}\ln r)$$

$$f_1(r) = -4\ln r - \frac{4}{3} + 2\psi_0 + c_0\, r^{-1} + O(r^{-2}\ln r)$$

$$\psi_0(r) = -2\ln r + \psi_0 - \left(\frac{\ell^2}{r_s} + 3r_s\right) r^{-1} + O(r^{-2})$$

$$\partial_x \chi_0 = \frac{r}{\ell^2} + \frac{1}{2}\left(\frac{1}{r_s} - \frac{3r_s}{\ell^2}\right) - \frac{1}{r} + \frac{\mu}{2r^2}$$

Boundary stress tensor construction for adS gravity

Balasubramanian & Kraus '99

add counterterms at $r \to \infty$ boundary

semi-classical terms in action require additional counterterms

$O(\alpha)$ correction to the total energy

$$\mu = \mu_{\mathsf{cl}} + c\, NGr_s/\ell^2 + \ldots \qquad c = O(1) \text{ constant}$$

energy density of radiation within proper distance $\ell$ of horizon

$$\rho \sim \frac{N\, r_s/\ell^2}{\ell\, r_s^2} \sim \frac{N}{\ell^3\, r_s} \to 0 \quad \text{as} \quad r_s \to \infty$$

## Global Embedding Minkowski Space (GEMS) for $d = 3$ Schwarzschild black hole

$$(t, r, \theta, \phi) \longrightarrow (Z^0, \ldots, Z^5)$$

$$Z^0 = 4m\sqrt{1 - \tfrac{2m}{r}}\,\sinh(t/4m) \qquad Z^1 = 4m\sqrt{1 - \tfrac{2m}{r}}\,\cosh(t/4m)$$

$$Z^2 = \int^r dr' \sqrt{\tfrac{2m}{r'} + \tfrac{4m^2}{r'^2} + \tfrac{8m^3}{r'^3}}$$

$$Z^3 = r \sin\theta \cos\phi \qquad Z^4 = r \sin\theta \sin\phi \qquad Z^5 = r\cos\theta$$

$$g_{ij} = \eta_{MN} \frac{\partial Z^M}{\partial x^i} \frac{\partial Z^N}{\partial x^j}$$

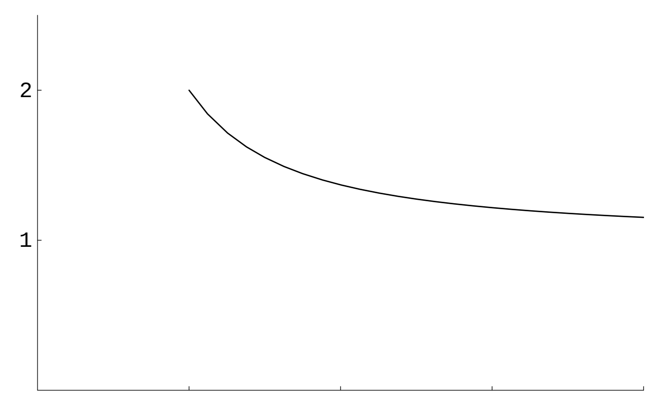
A fiducial observer in b.h. geometry is a Rindler observer in GEMS

$$T_{\text{fid}}(r) = \frac{a_6}{2\pi} = T_{\text{Unruh}}$$ Deser & Levin '98

$a_6 =$ magnitude of 6-acceleration in GEMS

Free fall observer in b.h. geometry is non-uniformly accelerated in GEMS

instantaneous $a_6 \longleftrightarrow 2\pi\, T_{\text{ff}}(r)$



# AdS-Schwarzschild geometry requires 7-dim GEMS