



# Hard Condensed Matter



**NORDITA**

The Nordic Institute for Theoretical Physics



# Theoretical Quantum Matter



Alexander  
Balatsky



D. Kuzmanovski

H. Yerzhakov

Tien-Tien Yeh



Dynamical Order  
Odd-frequency  
superconductivity

A. Tyner

A. Pathapati



Material  
Informatics  
Organic Materials  
Database (OMDB)

J. Schaltegger

Dirac Materials  
Dark Matter  
Detection

Light –  
Superconductors  
interaction

[Homepage](#): for more info



Ivan Khaymovich

NORDITA Ass. Prof.  
in TQM group

# Ergodicity breaking phases States



ergodic metal

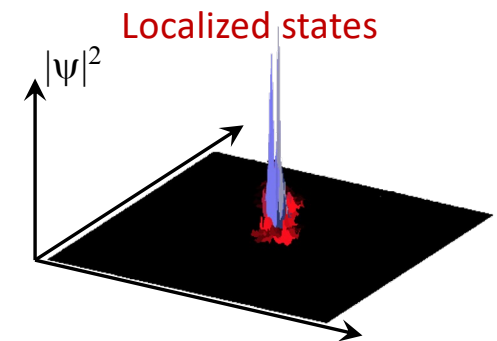
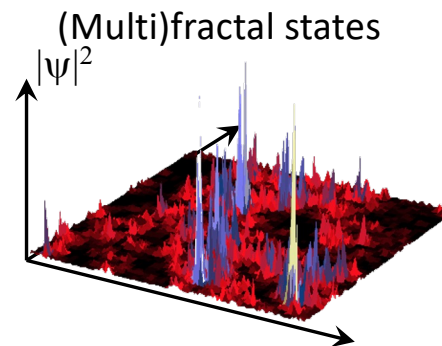
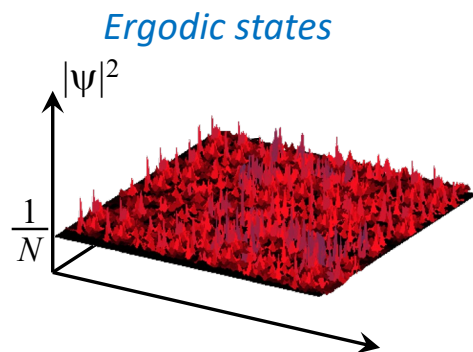


non-ergodic phase



insulator

disorder



[Homepage](#): for more info



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# Ergodicity breaking phases

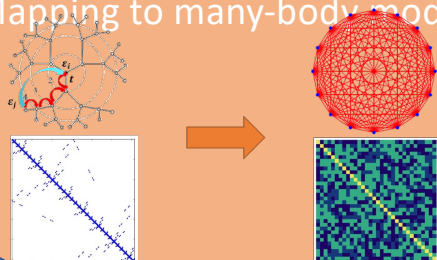


SCAN ME

Ivan Khaymovich

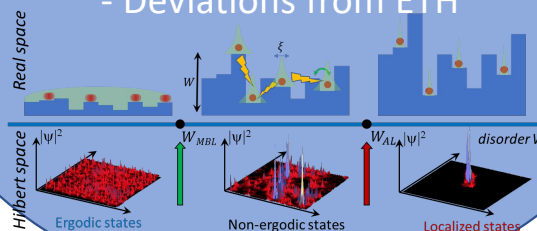
## Random-matrix theory:

- Multifractal phases
- Mapping to many-body models



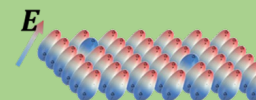
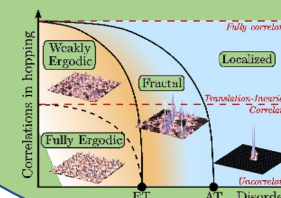
## Many-body systems:

- Many-body localization vs Hilbert-space multifractality,
- Multifractality/entanglement
- Deviations from ETH

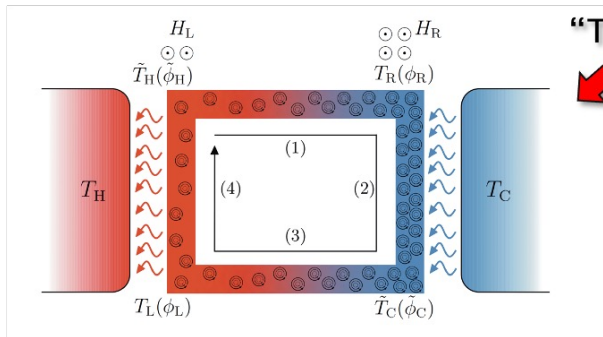


## Long-range systems:

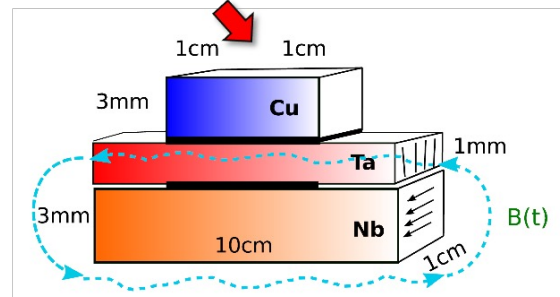
- Integrability, correlations,
- non-Hermitian/open models
- Localization and multifractality



# Principles of solid-state quantum refrigeration and thermodynamic control in the nanoscale

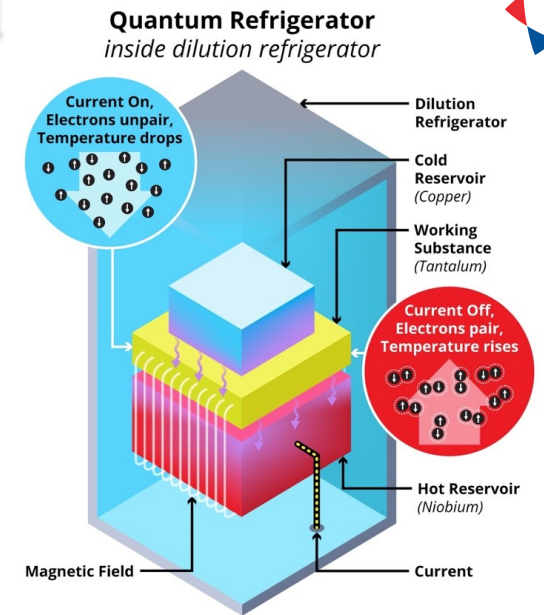


“The qubit goes here”



Fluxons in type II superconductors can act as puddles carrying heat in circulation.

Adiabatic magnetization of type I superconductor for cooling below 10mK



(University of Rochester illustrations / Michael Osadciw)



Sreenath Manikandan

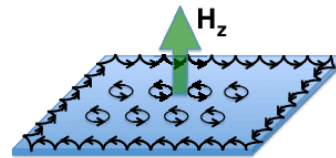
Email: [sreenath.k.manikandan@su.se](mailto:sreenath.k.manikandan@su.se)

# Topological Insulators Revolution

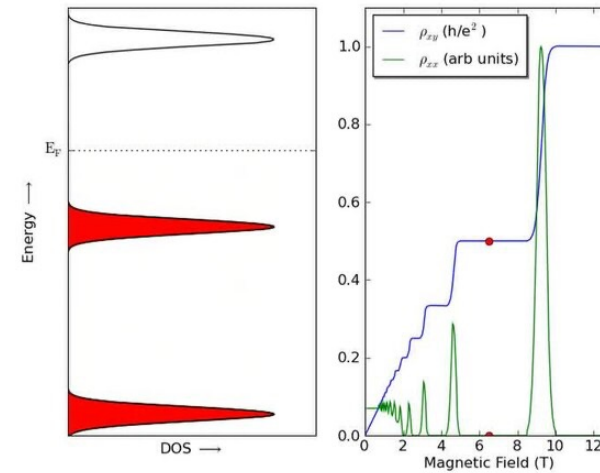
Adrien Bouhon  
Assistant Professor  
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Quantum Hall Effect  
(Klaus von Klitzing., 1980)

External Magnetic Field



(b) Quantum Hall effect



# Topological Insulators Revolution

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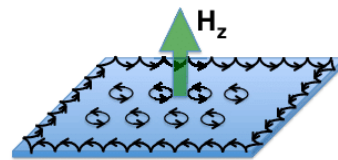
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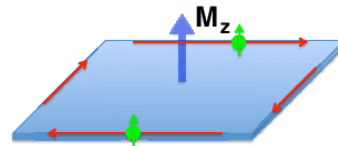


Anomalous QHE  
Haldane (1988)

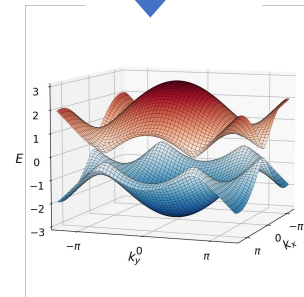
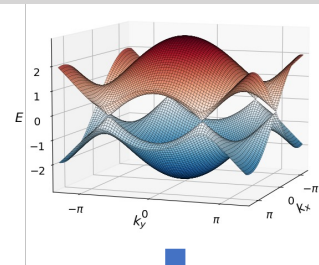
Intrinsic Magnetization



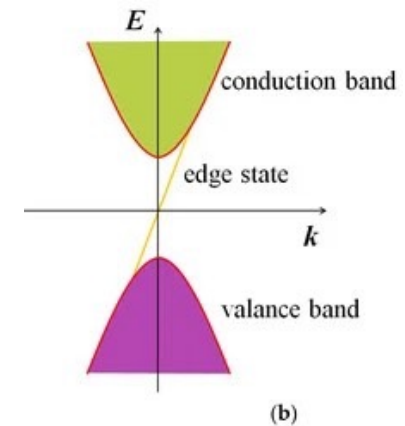
(b) Quantum Hall effect



(d) Quantum Anomalous Hall effect



Graphene



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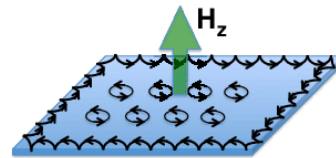
Intrinsic Magnetization



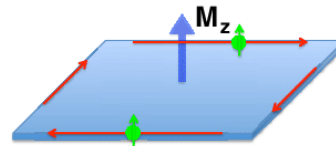
Quantum Spin Hall Effect  
Kane & Mele (2005)

protected by

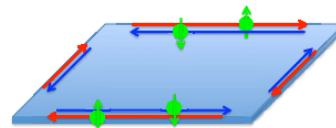
Fermionic  
Time Reversal Symmetry



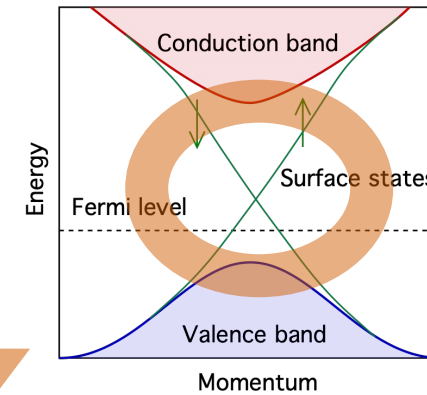
(b) Quantum Hall effect



(d) Quantum Anomalous Hall effect



(f) Quantum Spin Hall effect





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Quantum Hall Effect  
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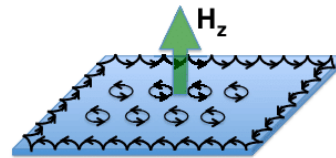
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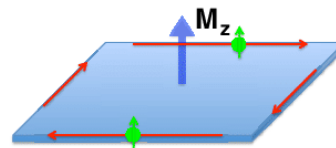


Quantum Spin Hall Effect  
Kane & Mele (2005)

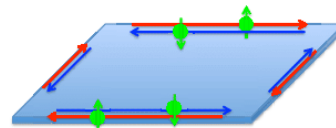
protected by  
Fermionic  
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(b) Quantum Hall effect



(d) Quantum Anomalous Hall effect



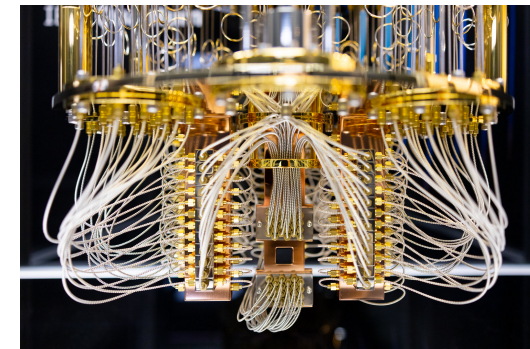
(f) Quantum Spin Hall effect

**Robust quantum surface states**

Substrate for QBits of Quantum Computers

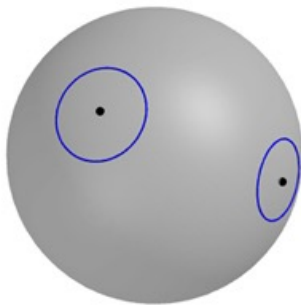
**Important Investments**

Race for the Quantum Supremacy States (USA, Chine, Emirates, etc.) and companies (Google, etc.)



# Topological?

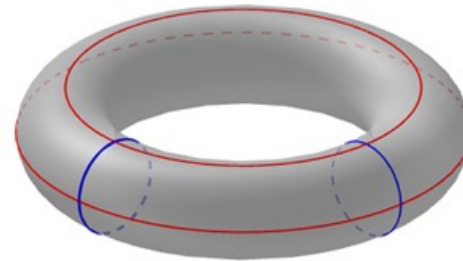
Topology: classification of manifolds with respect to their global features



$g = 0$

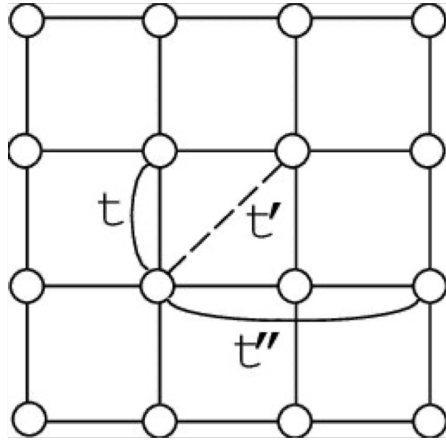


Exemple of topological invariant:  
the genus  $g$

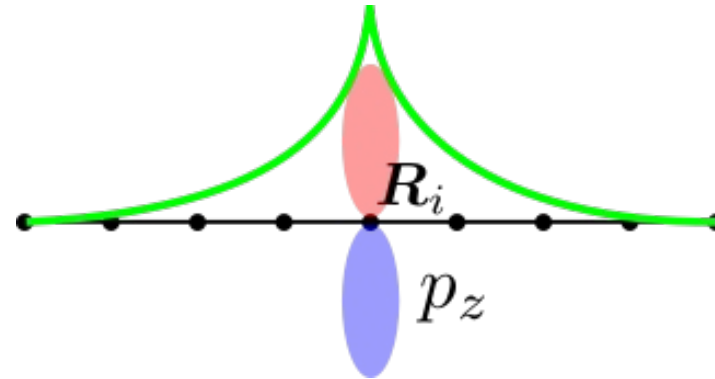


$g = 1$

# Quantum Materials Modeling

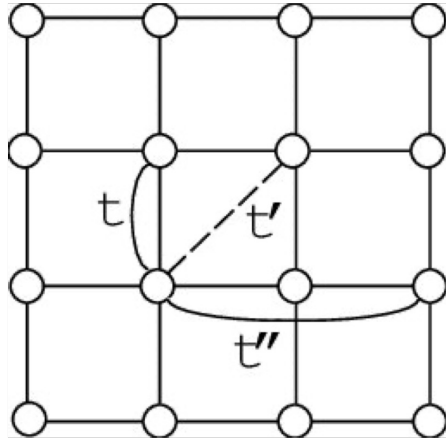


Localized Wannier functions



$$t_{\alpha\sigma,\beta\sigma'}(\mathbf{R}_i - \mathbf{R}_j) = \int d\mathbf{x} w_{\alpha\sigma}(\mathbf{x} - \mathbf{R}_i - \mathbf{r}_\alpha)^* \overset{\text{Kohn-Sham Hamiltonian}}{\mathfrak{h}(\mathbf{x})} w_{\beta\sigma'}(\mathbf{x} - \mathbf{R}_j - \mathbf{r}_\beta)$$

# Quantum Materials Modeling



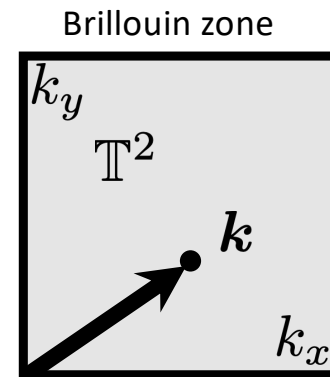
$$H(\mathbf{k}) \in \mathbb{C}^N \times \mathbb{C}^N$$

$N$  degrees of freedom per unit cell:  
Wyckoff positions, sub-lattice sites,  
electronic orbitals, spins

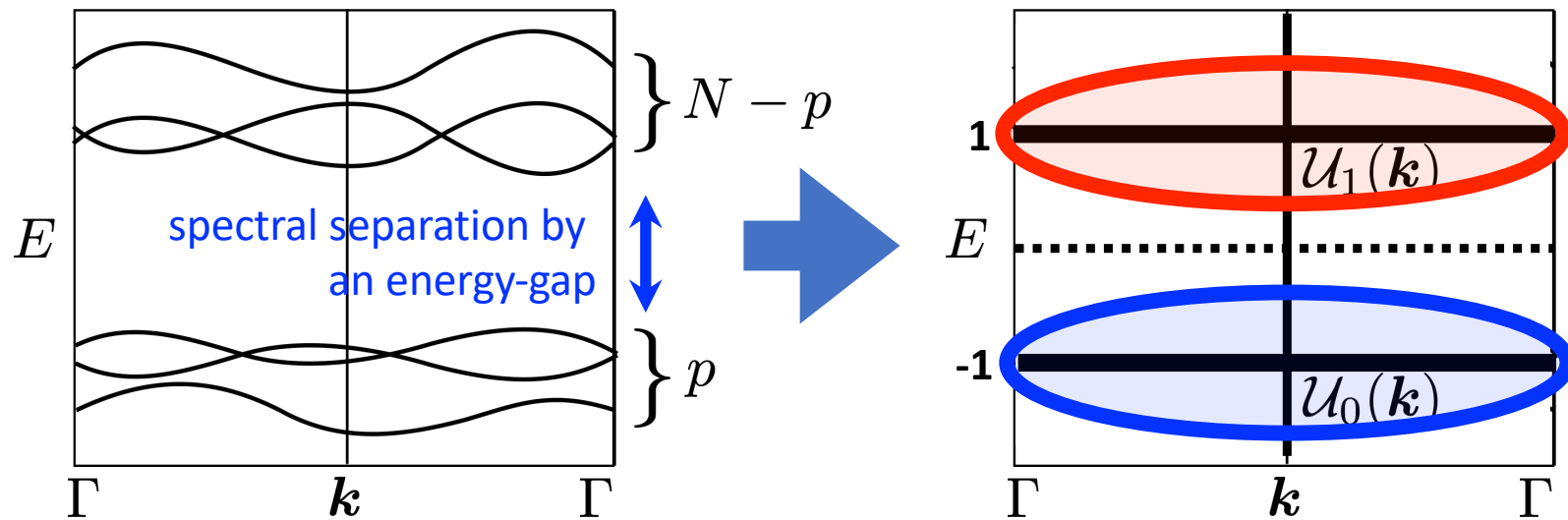
Bloch picture:

$$H_{\alpha\sigma,\beta\sigma'}(\mathbf{k}) = \sum_j e^{i\mathbf{k}\cdot(\mathbf{R}_j+\mathbf{r}_\beta-\mathbf{0}-\mathbf{r}_\alpha)} t_{\alpha\sigma,\beta\sigma'}(\mathbf{0}-\mathbf{R}_j)$$

Localized Wannier  $\rightarrow$  Finite Fourier series (analytical)



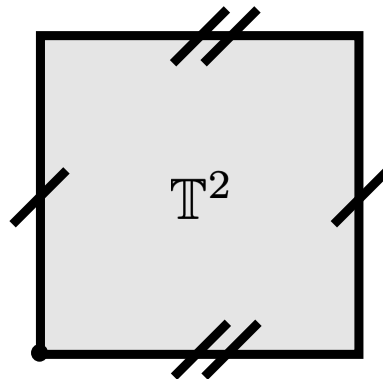
# Space of gapped Hamiltonian



Grassmannian  
(gauge structure)

$$\text{Gr}_{p,N}^{\mathbb{C}} \cong \text{U}(N) / [\text{U}(p) \times \text{U}(N-p)]$$

## Homotopy classification of gapped Hamiltonian


$$\xrightarrow{H(\mathbf{k})} \text{Gr}_{p,N}^{\mathbb{C}} \cong \text{U}(N) / [\text{U}(p) \times \text{U}(N-p)]$$

(Bott periodicity)

$$\begin{aligned} [\mathbb{T}^2, \text{Gr}_{p,N}^{\mathbb{C}}] &\cong \pi_2(\text{Gr}_{p,N}^{\mathbb{C}}) \cong \pi_1(\text{U}(p)) \\ &= \pi_1(\text{SU}(p) \times \text{U}(1)) \\ &= \pi_1(\text{SU}(p)) \times \pi_1(\text{U}(1)) \\ &= \pi_1(\text{U}(1)) = \mathbb{Z} \end{aligned}$$

(Chern number)

# Chern number as the chirality of a Weyl node

Flow of Berry phase = Chern number

$$\Delta\gamma_B = \gamma_B[\text{NP}] - \gamma_B[\text{SP}] = 2\pi c_1[\mathcal{S}]$$

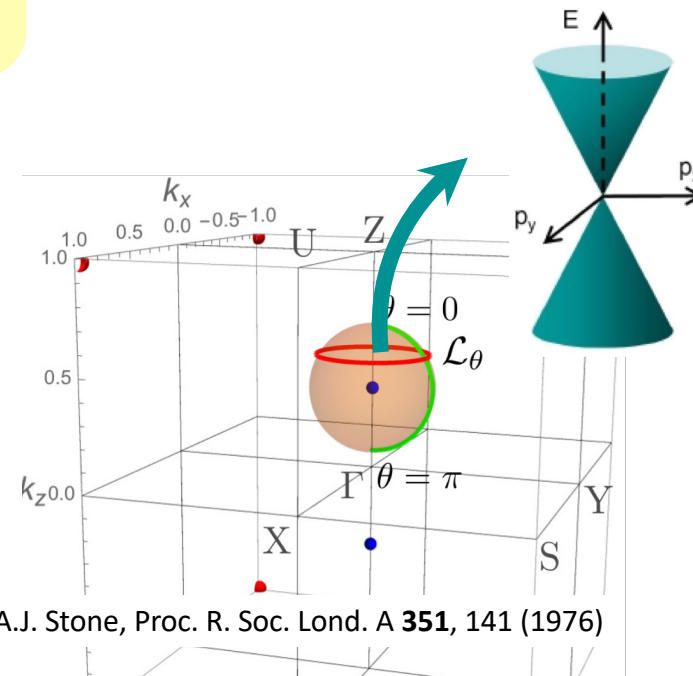
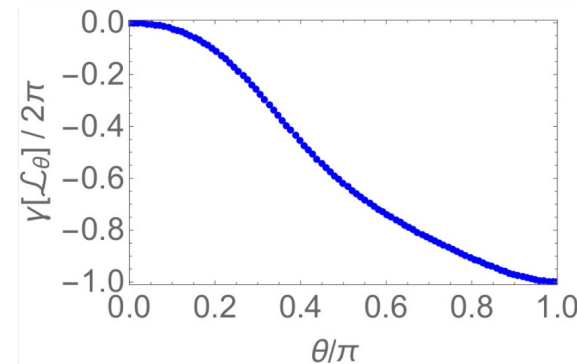
Loop integral over the occupied band subspace:

→ Wilson loop

$$\mathcal{W}[l_\theta] = \exp \left\{ \oint_{l_\theta} \mathcal{A}(\mathbf{k}) \cdot d\mathbf{l} \right\} \in U(p)$$

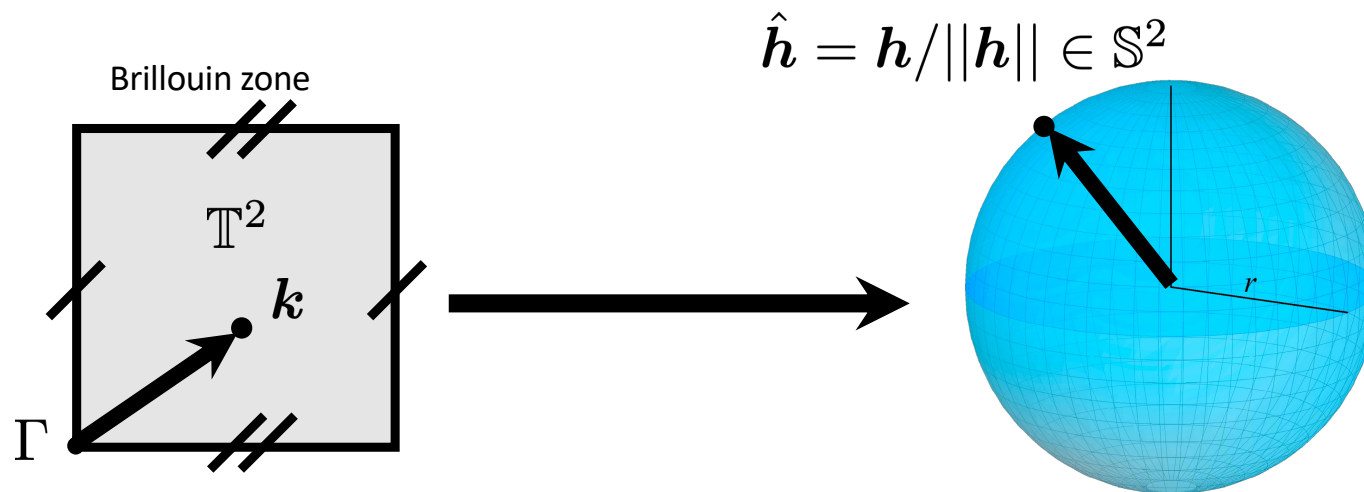
→ Berry phase

$$e^{i\gamma_B[l_\theta]} = \det \mathcal{W}[l_\theta] \in U(1)$$



A.J. Stone, Proc. R. Soc. Lond. A **351**, 141 (1976)

## Gapped graphene: Chern insulator



Chern number: (Skyrmion number)

$$c_1 = \frac{1}{4\pi} \int_{\mathbb{T}^2} \hat{\mathbf{h}} \cdot \partial_{k_x} \hat{\mathbf{h}} \times \partial_{k_y} \hat{\mathbf{h}} dk_x dk_y \in \mathbb{Z}$$

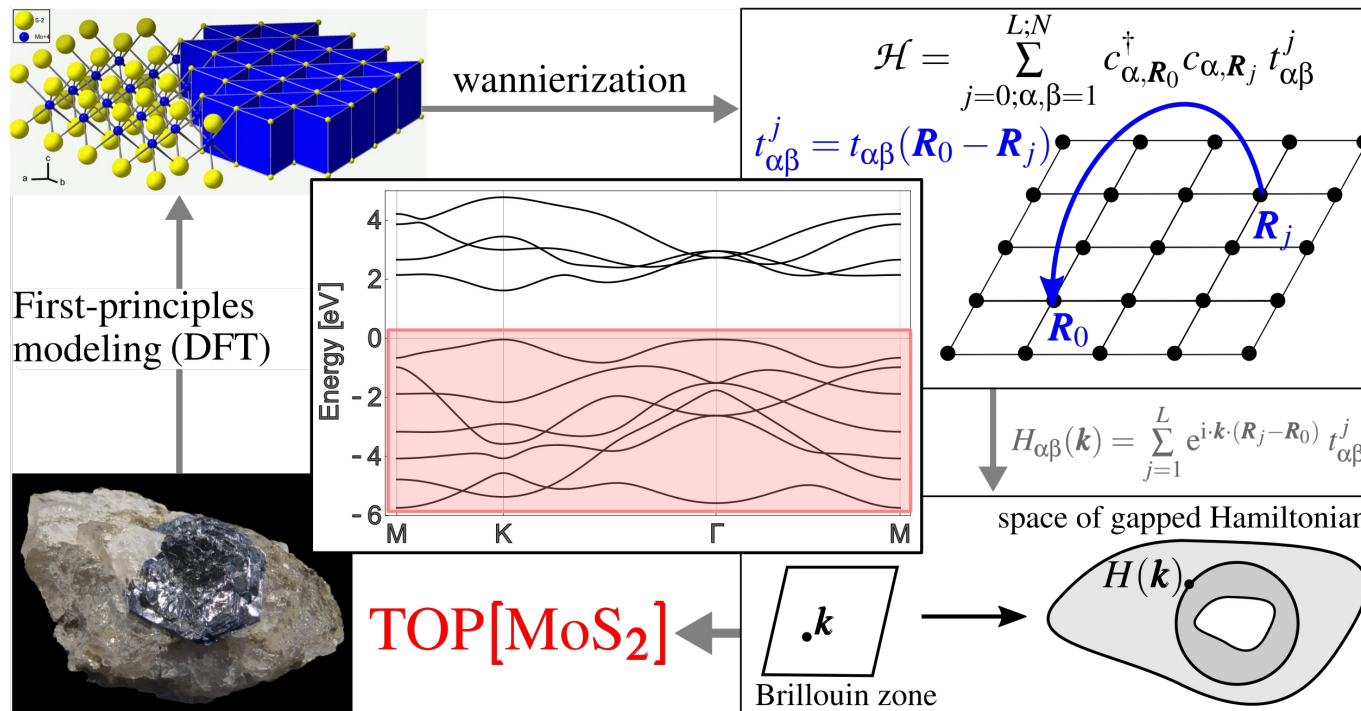


## Zoo of Topological Insulators

AZ	s\d	0	1	2	3	4	5	6	7
A	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
<b>AIII</b>	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
<b>BDI</b>	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
D	2	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
<b>DIII</b>	3	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
AII	4	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
<b>CII</b>	5	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
C	6	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
<b>CI</b>	7	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

**2D Chern insulator**

# Topological Inverse Problem





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# Optimized Topological Quantum Systems

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December 2024



Marie Curie  
Fellowship



Swedish  
Research Council