Entanglement, global symmetries and topological contributions

Marina Huerta

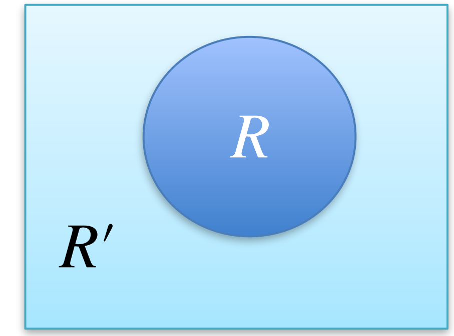
Instituto Balseiro - Centro Atómico Bariloche - CONICET
In collaboration with H. Casini, J.M.Magan and D. Pontello

WINQ Program on Complex and Quantum Systems
Nordita, Stockholm
May 2024

Based on

Entanglement entropy and superselection sectors I: Global symmetries
Entropic order parameters for the phases of QFT

Preliminaries



Entanglement Entropy in QFT

Region R and state ρ $\longrightarrow$ $S(R) = -\,\mathrm{tr}\rho_R \log \rho_R$

$\mathscr{H}_R \otimes \mathscr{H}_{R'}$

The entropy is divergent in the continuum but…. admits an expansion in powers of ϵ

In d dimensions

$$S(R) = \frac{g_{d-2}}{\epsilon^{d-2}} + \frac{g_{d-4}}{\epsilon^{d-4}} + \cdots + g_0 \log(\epsilon) + S_0$$

Area law $S \sim \frac{R^{d-2}}{\epsilon^{d-2}}$

Non universal

Finite term

Depends on the region and the state

CFT+ Spheres $\begin{cases} (-)^{\frac{d}{2}-1} 4A \log(R/\epsilon) & d \text{ even} \\ (-)^{\frac{d-1}{2}} F & d \text{ odd} \end{cases}$

The coefficients g_i regularization dependent
extensive and local on the boundary

Entropic theorems
RG flow irreversibility

Preliminaries

Perspective:

Algebraic approach to QFT based on algebras of operators corresponding to causal spacetime regions

↓ "described by a net of von Neumann algebras"

strong indication that properties of the assignation $\mathscr{A}(\mathscr{R})$ is in the core of the EE

QFT : Entanglement Entropy of a region

↓ AA

Region ⟷ Local algebra

Global symmetry → Subset of invariant operators → DHR formalism

[Doplicher, Haag, Roberts, 1969]

DHR formalism ↓ Superselection sectors

Subset of invariant operators ↓ non unique assignation!

Region $\overset{?}{\longleftrightarrow}$ Local algebra

Global symmetry ↓ $S(R)$ is not unique ← non unique assignation!

Motivations

- Anomaly mismatch for gauge theories [Dowker, 2010]

- Boundary dof $\longleftrightarrow$ Centers/require fine-tuning

[Buividovich, Polikarpov 2008]
[Donnelly 2011]
[Donnelly, Wall 2015]
[Ghosh, Soni, Trivedi 2015]
[Huang 2015]

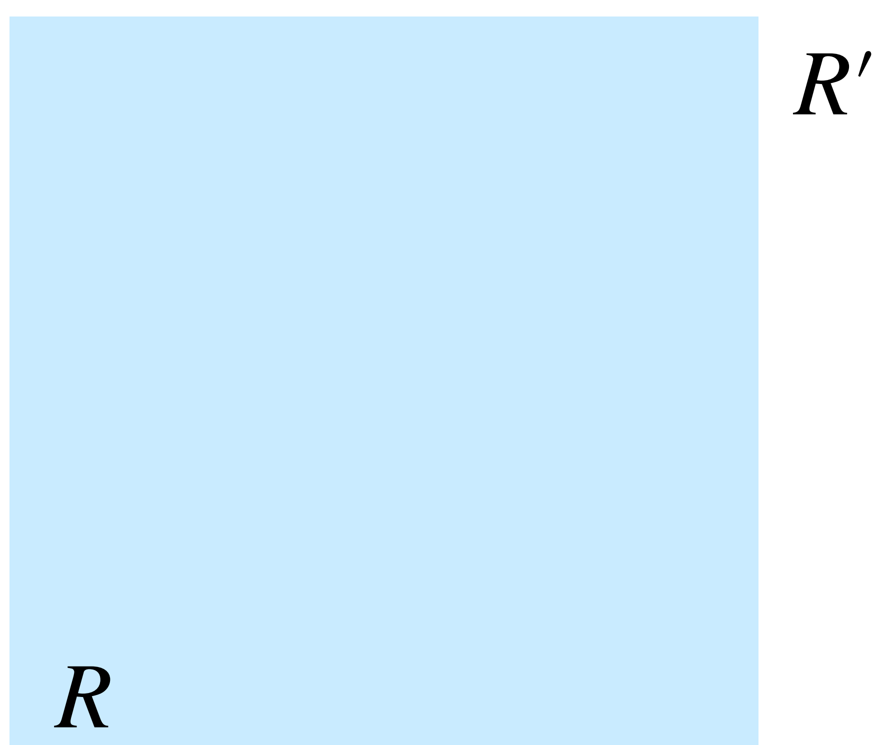
- Mutual Information seems to fail $a_{MI} \neq a_{\langle T^\mu_\mu \rangle}$ [Casini, MH., 2015]

Motivations

- A different perspective: Algebraic approach

Region ⟷ Local algebra

- Algebra/Region ambiguities on the lattice [Casini, MH, Rosabal, 2014]

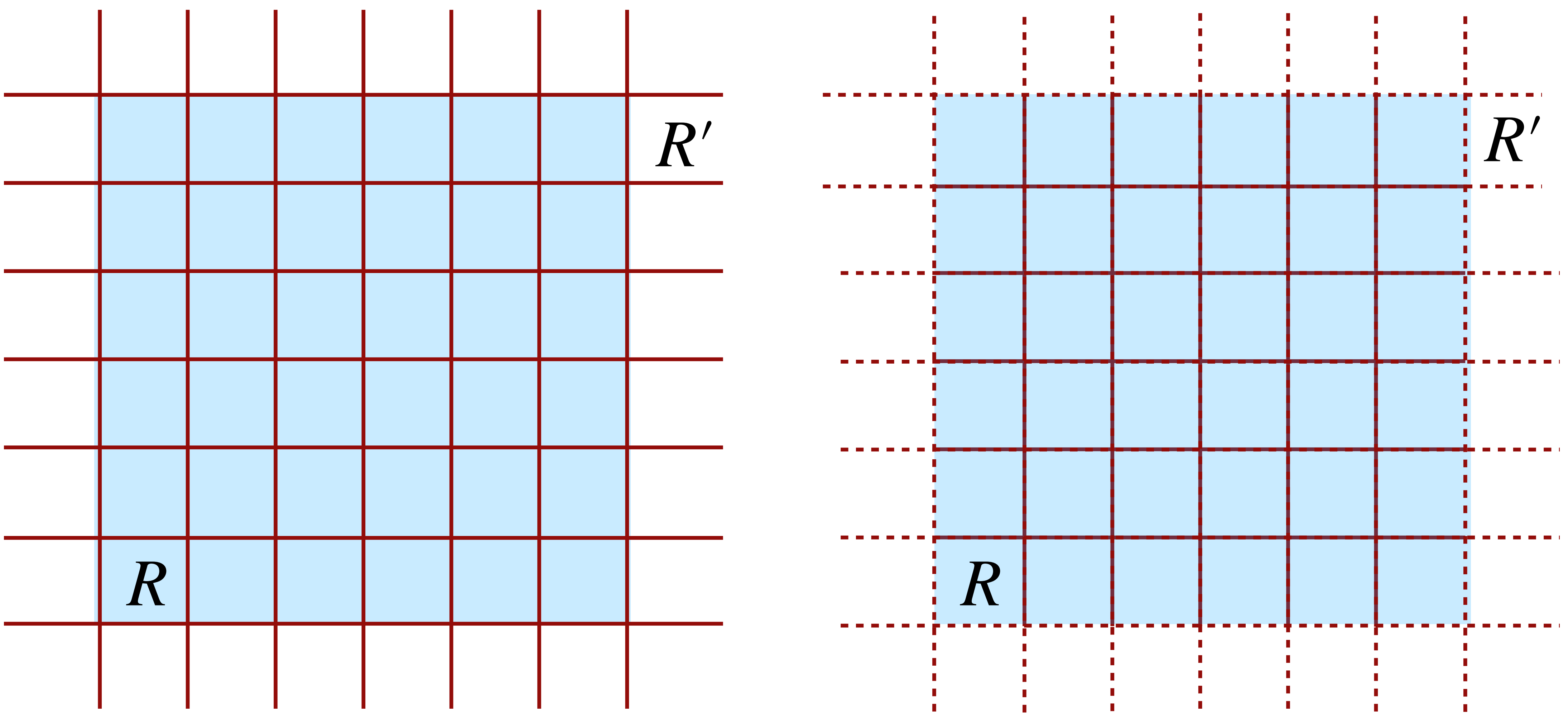


Motivations

- A different perspective: Algebraic approach

Region ⟷ Local algebra

- Algebra/Region ambiguities on the lattice [Casini, MH, Rosabal, 2014]



Infinite number of choices…the same mutual information

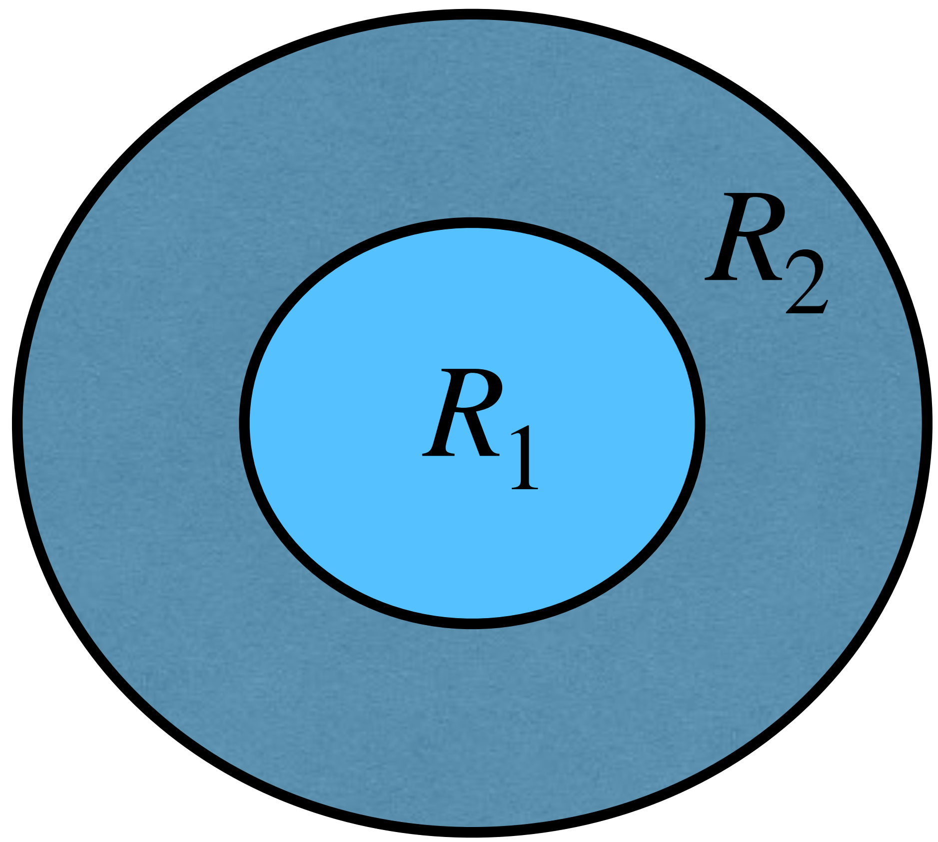
Plan of the talk

- Algebras and regions in QFT
- QFT with global symmetries
- Relative entropy and conditional expectations
- Novel universal terms in the entanglement entropy
- Chiral Scalar in two dim

Algebras and regions in QFT

Algebras and regions in QFT

- Isotony



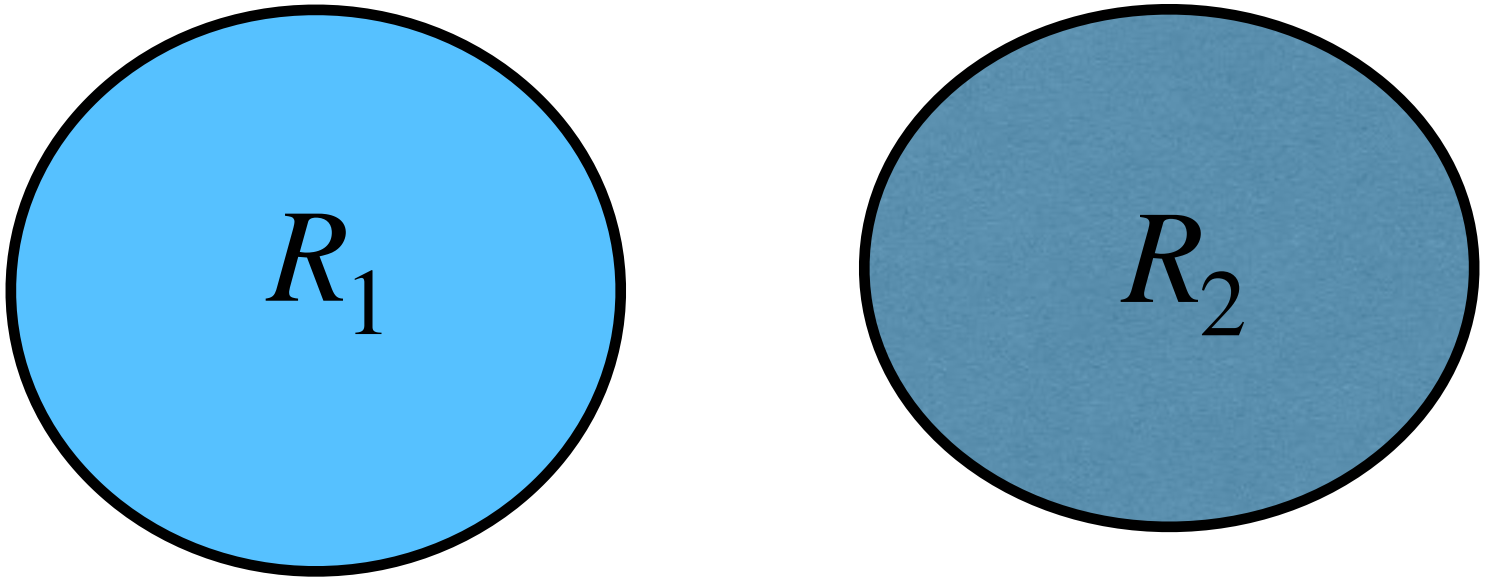
$$R_1 \subseteq R_2$$

$$\Downarrow$$

$$\mathscr{A}_{R_1} \subseteq \mathscr{A}_{R_2}$$

Algebras and regions in QFT

- Isotony $R_1 \subseteq R_2 \longrightarrow \mathscr{A}_{R_1} \subseteq \mathscr{A}_{R_2}$
- Additivity



$$\mathscr{A}_{R_1 \vee R_2} = \mathscr{A}_{R_1} \vee \mathscr{A}_{R_2}$$

Algebras and regions in QFT

- Isotony $R_1 \subseteq R_2 \longrightarrow \mathscr{A}_{R_1} \subseteq \mathscr{A}_{R_2}$
- Additivity $\mathscr{A}(R_1 \vee R_2) = \mathscr{A}(R_1) \vee \mathscr{A}(R_2)$
- Causality

$$[\mathscr{A}(R), \mathscr{A}(R')] = 0$$

$$\Big\downarrow$$

$$\mathscr{A}(R) \subseteq \mathscr{A}(R')'$$

Algebras and regions in QFT

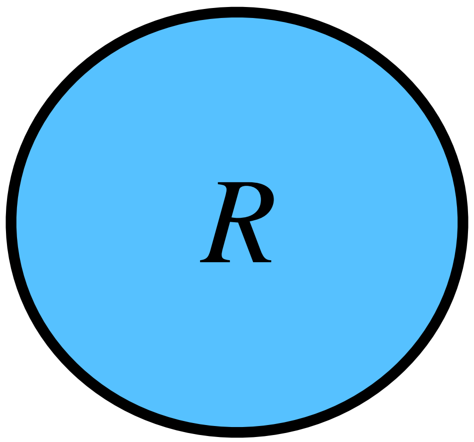
- Isotony $A \subseteq B \longrightarrow \mathcal{O}_A \subseteq \mathcal{O}_B$
- Additivity $\mathscr{A}(R_1 \vee R_2) = \mathscr{A}(R_1) \vee \mathscr{A}(R_2)$
- Causality $\mathscr{A}(R) \subseteq \mathscr{A}(R')'$
- Duality

$$\mathscr{A}(R) \stackrel{?}{=} \mathscr{A}(R')'$$

Algebras and regions in QFT

- Isotony $A \subseteq B \longrightarrow \mathcal{O}_A \subseteq \mathcal{O}_B$
- Additivity $\mathscr{A}(R_1 \vee R_2) = \mathscr{A}(R_1) \vee \mathscr{A}(R_2)$
- Causality $\mathscr{A}(R) \subseteq \mathscr{A}(R')'$
- Duality

For simply connected regions (most QFT's)

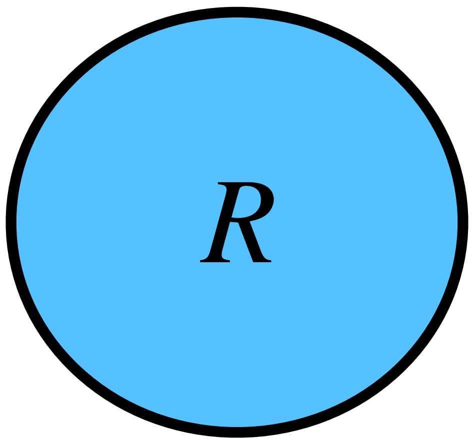


$$\mathscr{A}(R) = \mathscr{A}(R')'$$

Algebras and regions in QFT

- Isotony $A \subseteq B \longrightarrow \mathcal{O}_A \subseteq \mathcal{O}_B$
- Additivity $\mathscr{A}(R_1 \vee R_2) = \mathscr{A}(R_1) \vee \mathscr{A}(R_2)$
- Causality $\mathscr{A}(R) \subseteq \mathscr{A}(R')'$
- Duality

For simply connected regions (most QFT's)



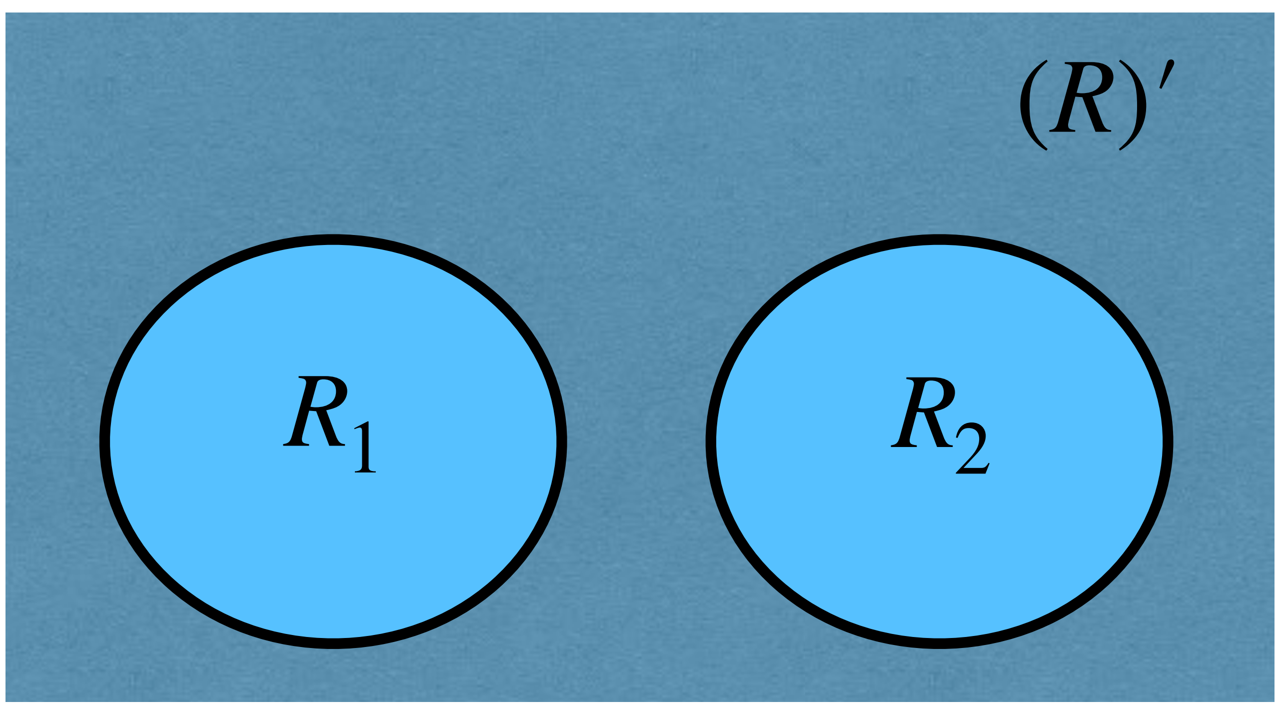
$$\mathscr{A}(R) = \mathscr{A}(R')'$$

But what about regions with non-trivial topology?

Algebras and regions in QFT

- Isotony $A \subseteq B \longrightarrow \mathcal{O}_A \subseteq \mathcal{O}_B$
- Additivity $\mathscr{A}(R_1 \vee R_2) = \mathscr{A}(R_1) \vee \mathscr{A}(R_2)$
- Causality $\mathscr{A}(R) \subseteq \mathscr{A}(R')'$
- Duality $\mathscr{A}(R) = \mathscr{A}(R')'$ simply connected regions (most QFT's)

Consider the regions $R \equiv R_1 \vee R_2$ and R'



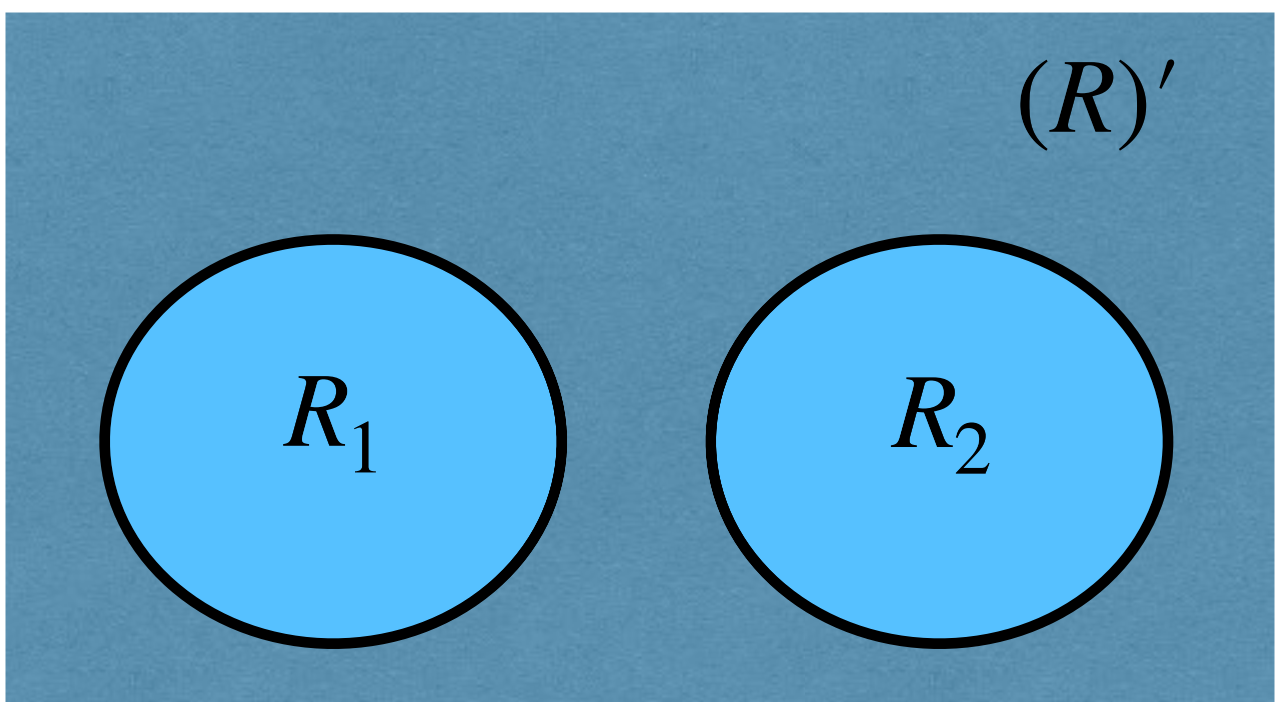
From causality

$$\mathscr{A}_R \subseteq \mathscr{A}'_{(R)'}$$

The region R has non trivial $\pi_0(R)$. The region R' has non trivial $\pi_{d-2}(R)$

Algebras and regions in QFT

- Isotony $A \subseteq B \longrightarrow \mathcal{O}_A \subseteq \mathcal{O}_B$
- Additivity $\mathscr{A}(R_1 \vee R_2) = \mathscr{A}(R_1) \vee \mathscr{A}(R_2)$
- Causality $\mathscr{A}(R) \subset \mathscr{A}(R')'$
- Duality $\mathscr{A}(R) = \mathscr{A}(R')'$ simply connected regions (most QFT's)



From causality

$$\mathscr{A}_R \subset \mathscr{A}'_{(R)'}$$

$$\mathscr{A}(R) \stackrel{?}{=} \mathscr{A}(R')'$$

Algebras and regions in QFT

$$\mathscr{A}(R) \stackrel{?}{=} \mathscr{A}(R')'$$

If duality is not satisfied for certain region

$$\mathscr{A}_{max}(R) \equiv (\mathscr{A}(R'))' = \mathscr{A}(R) \vee \{a\}$$

Interestingly, the breaking of duality in region R forces a dual breaking in region R'

$$\mathscr{A}_{max}(R') \equiv (\mathscr{A}(R))' = \mathscr{A}(R') \vee \{b\}$$

It also implies that the dual sets of non-local operators are complementary

$$[a, b] \neq 0$$

★ To construct QFT nets satisfying duality requires introducing some non local operators that close an algebra that can be associated to a symmetry

↓

generalized symmetry

QFT with global symmetries

Simple example: Free Dirac field restricted to the algebra of bosonic operators

$$\mathcal{F} \equiv 1, \psi(x) , \cdots$$

$$\mathcal{O} \equiv 1, \psi(x)\psi(y) , \psi^\dagger(x)\psi^\dagger(y) , \psi(x)\psi^\dagger(y) , \cdots$$

This is a Z_2 symmetry for which the fermion has charge one.

In the model $\mathcal{F}$ we can consider the following localized operator

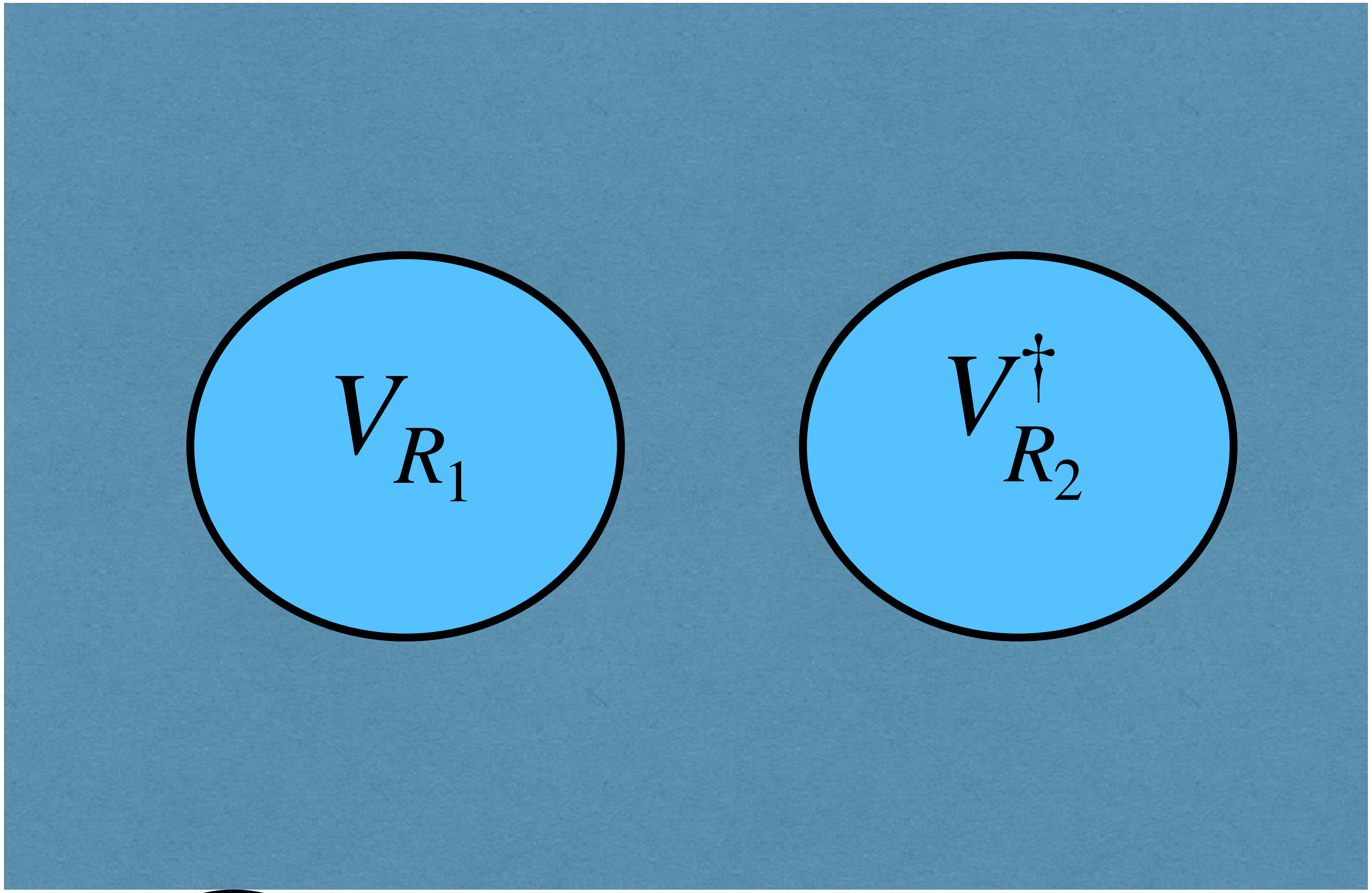
$$V_A = \int_A d^{d-1}x\, \alpha(x)\, (\psi(x) + \psi^\dagger(x))$$

If we have two regions we can construct the "intertwiner"

$$\mathcal{I}_{R_1 R_2} = V_{R_1} V_{R_2}^\dagger \quad \in \mathcal{O}$$

QFT with global symmetries

With respect to region $R \equiv R_1 \vee R_2$



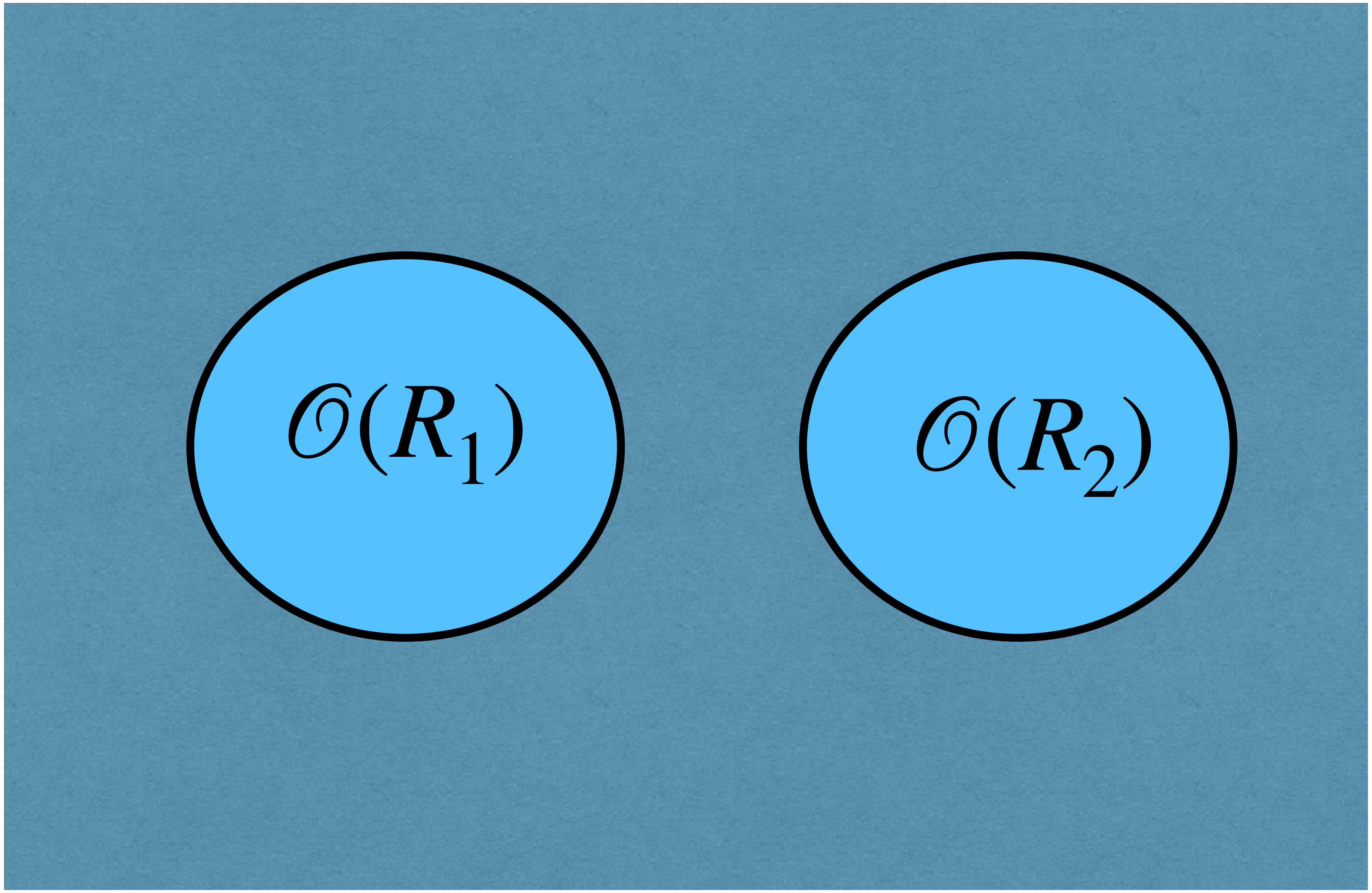
$(\mathscr{A}(R'))' = \mathscr{A}(R) \vee \{a\}$

The intertwiner belongs to the commutant $\mathcal{O}(R')'$

But does the intertwiner belong to the algebra of the union in $\mathcal{O}$?

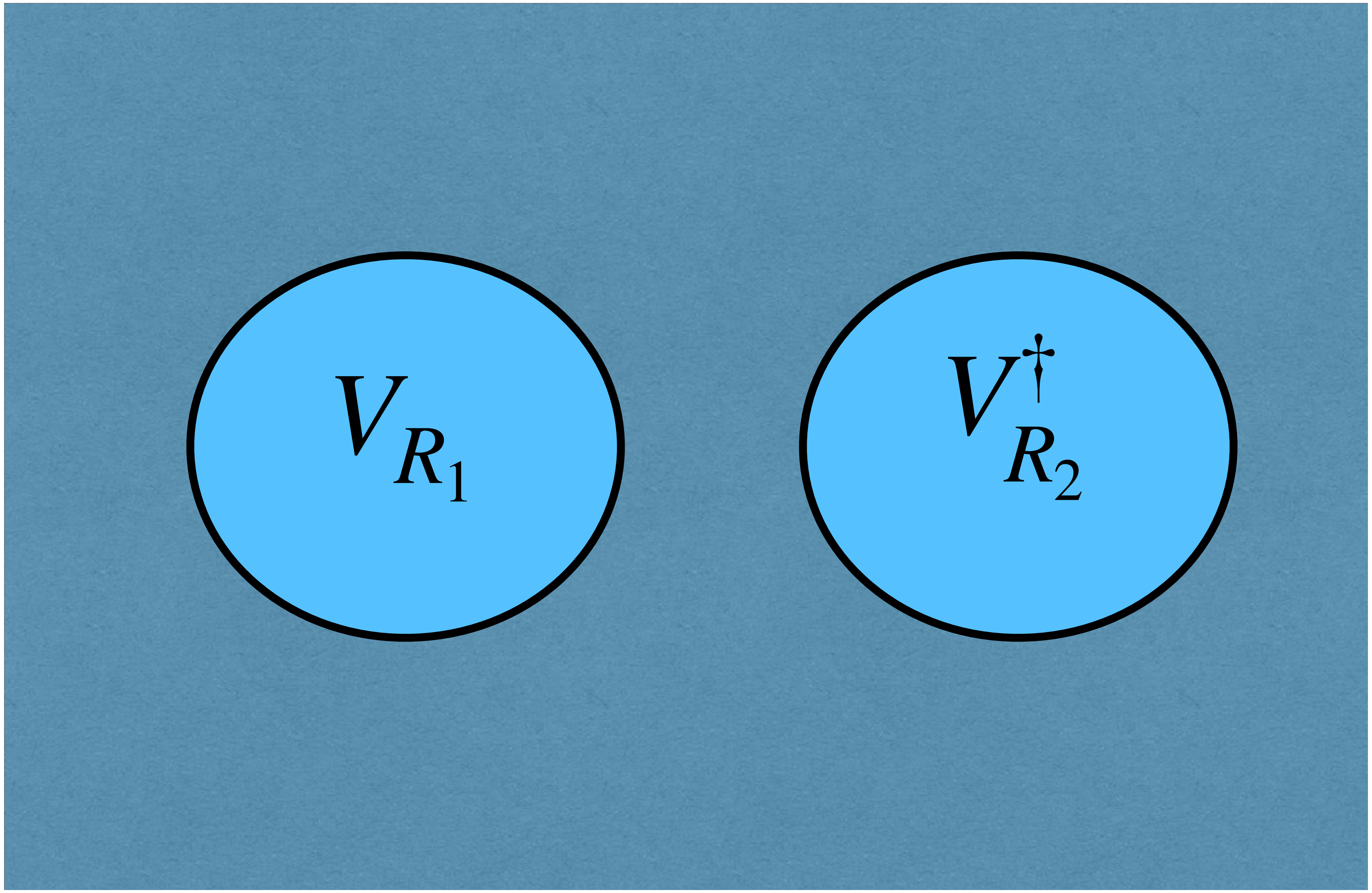
$$\mathcal{O}(R_1 \vee R_2) = \mathcal{O}(R_1) \vee \mathcal{O}(R_2)$$

QFT with global symmetries



The additive algebra is the product of even operators in the right and in the left

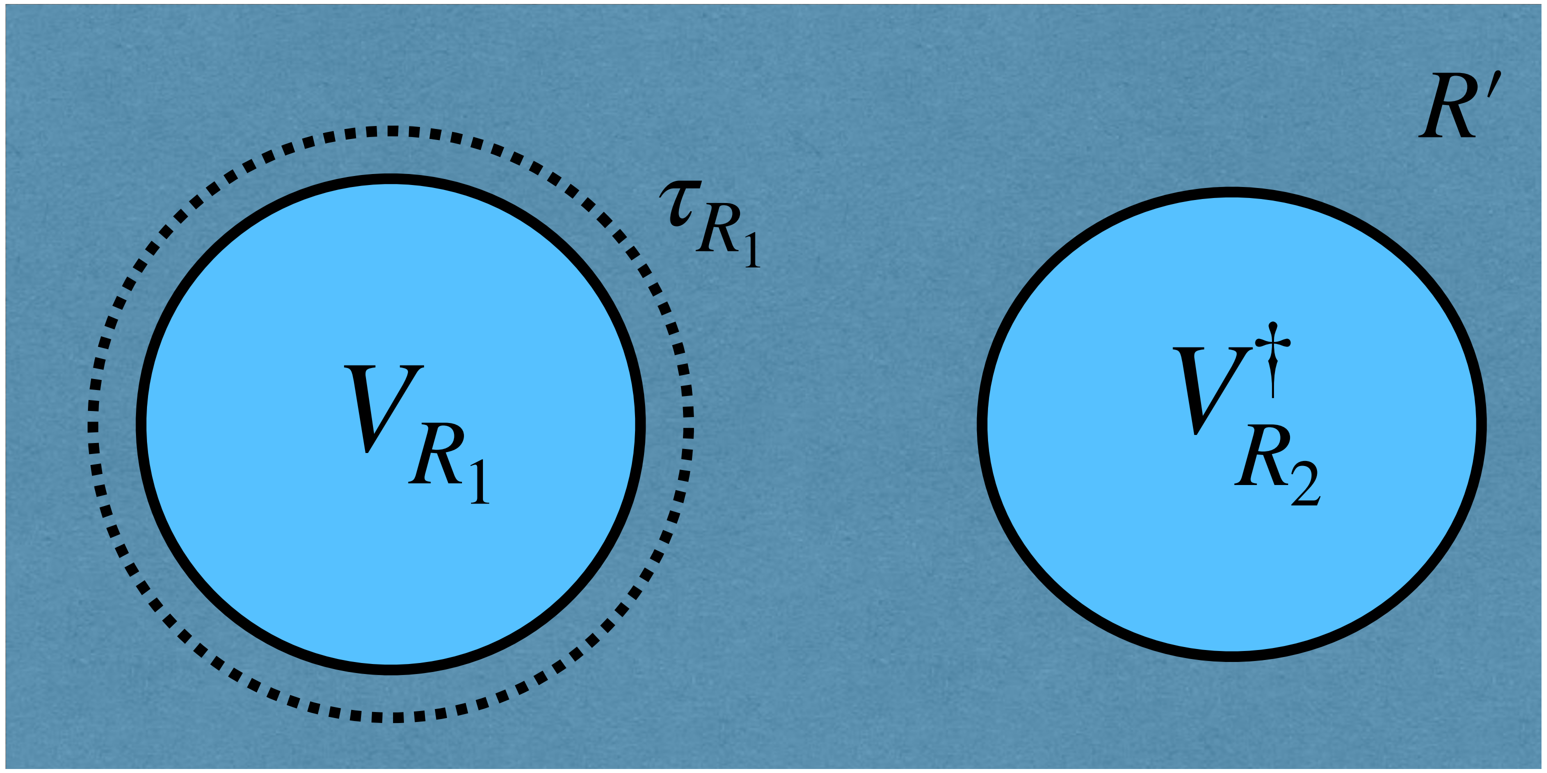
QFT with global symmetries



It does not belong to the local algebra…

QFT with global symmetries

With respect to region R'



$\mathscr{A}(R)' = \mathscr{A}(R') \vee \{b\}$

The commutant $\mathcal{O}(R)'$ contains "twist" operators that implement the symmetry transformations locally

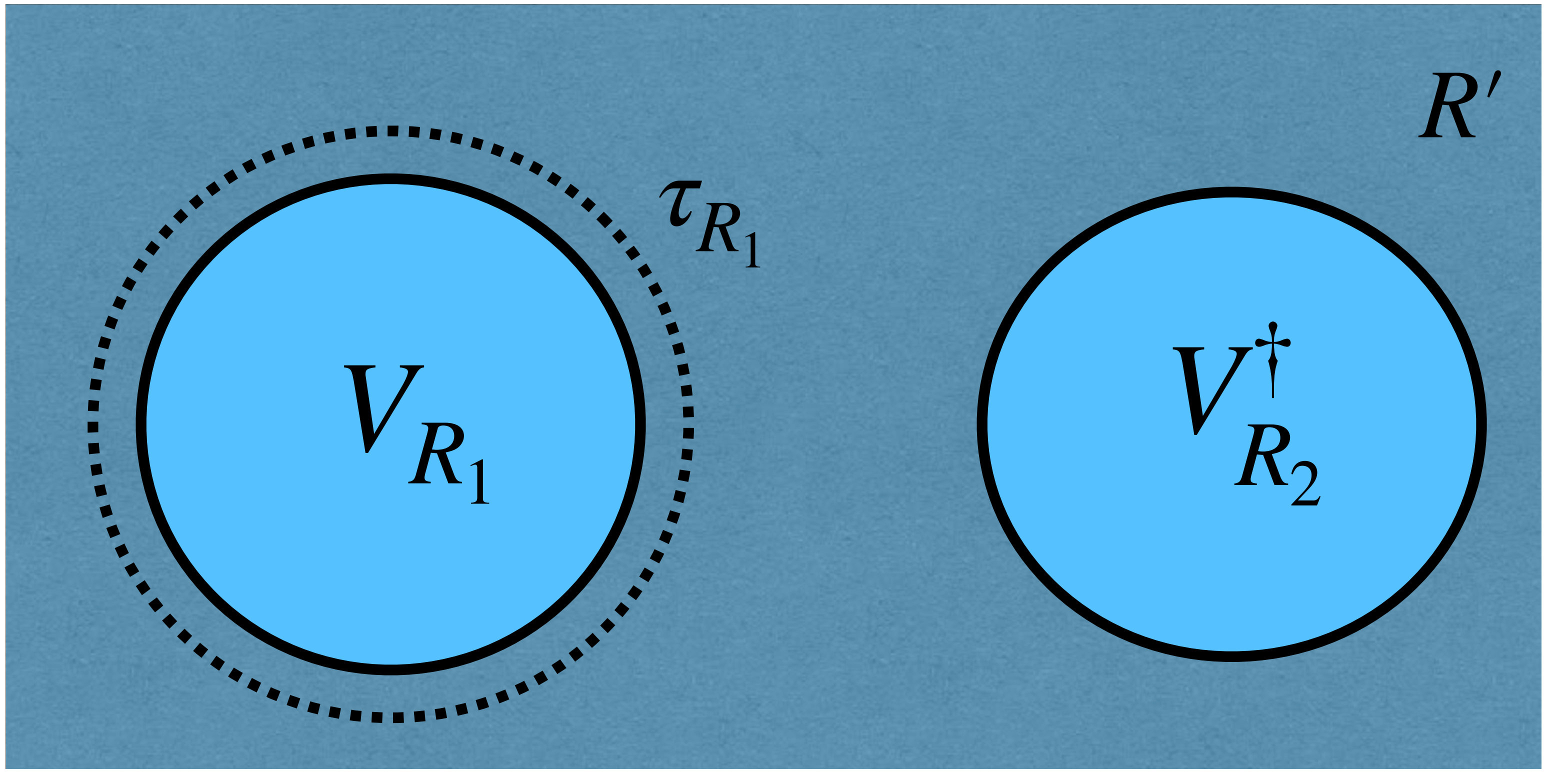
$$\tau_{R_1} = e^{i\pi \int dt\, d^{d-1}x\, \gamma(t)\, \beta_{R_1}(\vec{x}) J^0(x)}$$

The spatial test function is zero in region R_2, and one in R_1 so that

$$\tau V_{R_1} \tau^{-1} = -V_{R_1} \qquad \tau V_{R_2} \tau^{-1} = V_{R_2}$$

QFT with global symmetries

With respect to region R'

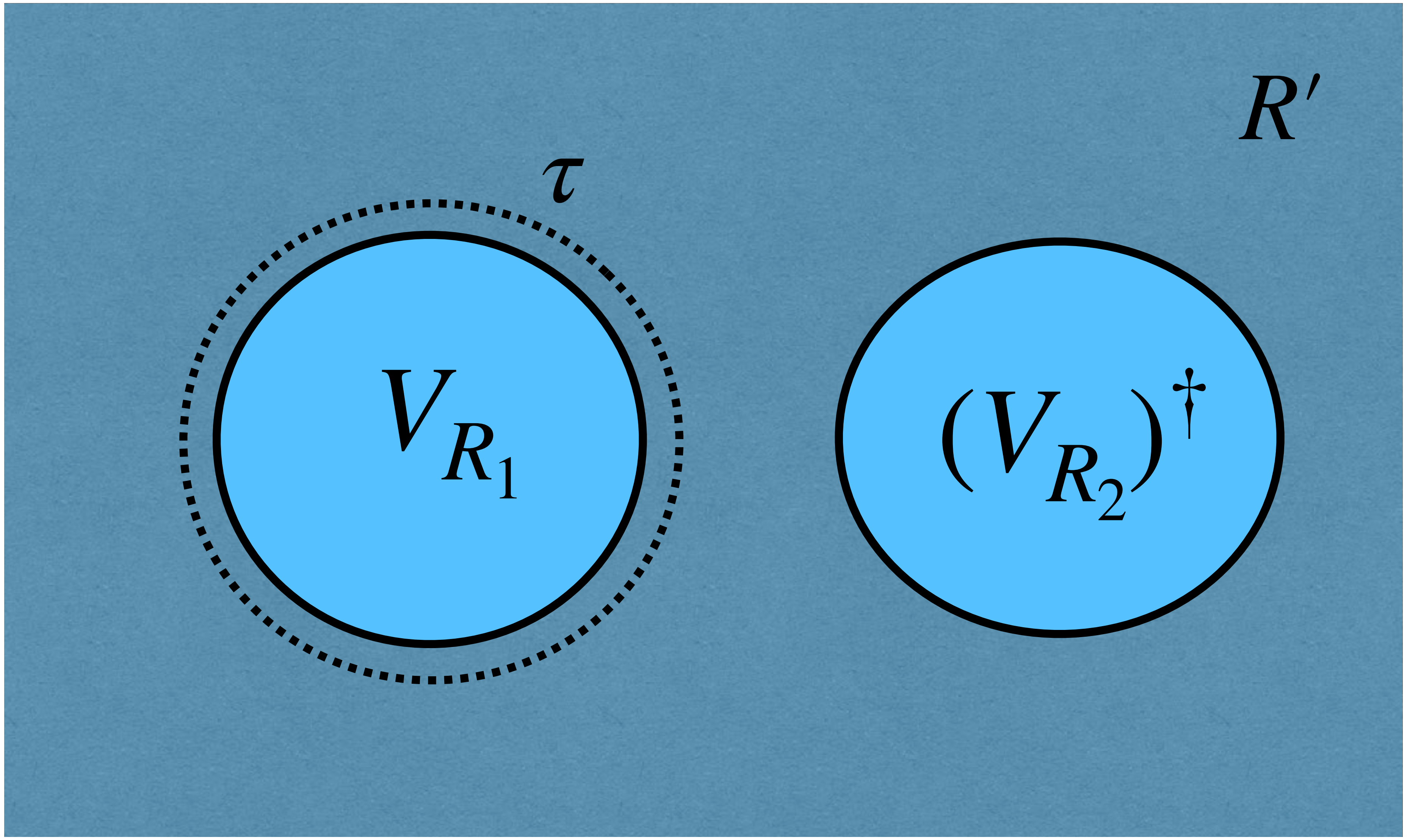


The twists belong to the commutant $\mathcal{O}(R)'$

Crucially, this implies that

$$[\tau, \mathcal{I}_{AB}] \neq 0$$

QFT with global symmetries



$$\mathcal{O}(R) \subset \mathcal{O}_{max}(R) \equiv \mathcal{O}(R) \vee \mathcal{I}_{R_1R_2}$$

$$\mathcal{O}(R') \subset \mathcal{O}_{max}(R') \equiv \mathcal{O}(R') \vee \tau$$

The global symmetry manifests itself in the difference between the maximal algebras and the local algebras of regions with specific topologies

Relative entropy and conditional expectations

Given an inclusion of algebras

$$\mathcal{O} \subset \mathcal{F}$$

A conditional expectation E is a linear map from $\mathcal{F}$ to $\mathcal{O}$ satisfying

$$E(b_1\, a\, b_2) = b_1\, E(a)\, b_2 \qquad b_1, b_2 \in \mathcal{O}\,, a \in \mathcal{F}$$

Relative entropy and conditional expectations

Given an inclusion of algebras

$$\mathscr{O} \subset \mathscr{F}$$

A conditional expectation E is a linear map from $\mathscr{F}$ to $\mathscr{O}$ satisfying

$$E(b_1\, a\, b_2) = b_1\, E(a)\, b_2 \qquad b_1, b_2 \in \mathscr{O}\,, a \in \mathscr{F}$$

Example: Tracing out a factor is a conditional expectation

$$\mathscr{F} = \mathscr{O} \otimes \mathscr{A}$$

Relative entropy and conditional expectations

Given an inclusion of algebras

$$\mathcal{O} \subset \mathcal{F}$$

A conditional expectation E is a linear map from $\mathcal{F}$ to $\mathcal{O}$ satisfying

$$E(b_1\, a\, b_2) = b_1\, E(a)\, b_2 \qquad b_1, b_2 \in \mathcal{O}\,, a \in \mathcal{F}$$

Example: Tracing out a factor is a conditional expectation

$$\mathcal{F} = \mathcal{O} \otimes \mathcal{A} \qquad\qquad E(O \otimes A) = Tr\,(A)\; O \otimes 1_{\mathcal{A}}$$

Another example (our case): Quotient by a symmetry group

$$\mathcal{O} = \frac{1}{G} \sum_g \tau_g\, \mathcal{F}\, \tau_g^{-1} = E(\mathcal{F})$$

Relative entropy and conditional expectations

Conditional expectations can be composed with states

$$\omega_{\mathscr{O}} \to (\omega_{\mathscr{O}} \circ E)_{\mathscr{F}}$$

Relative entropy and conditional expectations

Conditional expectations can be composed with states

$$\omega_{\mathscr{O}} \to (\omega_{\mathscr{O}} \circ E)_{\mathscr{F}}$$

Relative entropy: Let us remind the relative entropy definition

$$S_{\mathscr{F}}(\omega \,|\, \phi) = Tr\, \omega \log \omega - Tr\, \omega \log \phi$$

It can be used to define Mutual Information

$$I_{AB} = S(\omega_{AB} \,|\, \omega_A \otimes \omega_B)$$

Relative entropy and conditional expectations

Conditional expectations can be composed with states

$$\omega_{\mathcal{O}} \to (\omega_{\mathcal{O}} \circ E)_{\mathcal{F}}$$

Relative entropy: Let us remind the relative entropy definition

$$S_{\mathcal{F}}(\omega \,|\, \phi) = Tr\,\omega \log \omega - Tr\,\omega \log \phi$$

It can be used to define Mutual Information

$$I_{AB} = S(\omega_{AB} \,|\, \omega_A \otimes \omega_B)$$

RE+CE The following key equation can be proven [Petz, 1993]

$$S_{\mathcal{F}}(\omega \,|\, \phi \circ E) = S_{\mathcal{O}}(\omega \,|\, \phi) + S_{\mathcal{F}}(\omega \,|\, \omega \circ E)$$

Relative entropy and conditional expectations

Conditional expectations can be composed with states

$$\omega_{\mathcal{O}} \to (\omega_{\mathcal{O}} \circ E)_{\mathcal{F}}$$

Relative entropy: Let us remind how relative entropy is defined

$$S_{\mathcal{F}}(\omega \,|\, \phi) = Tr\, \omega \log \omega - Tr\, \omega \log \phi$$

It can be used to define Mutual Information

$$I_{AB} = S(\omega_{AB} \,|\, \omega_A \otimes \omega_B)$$

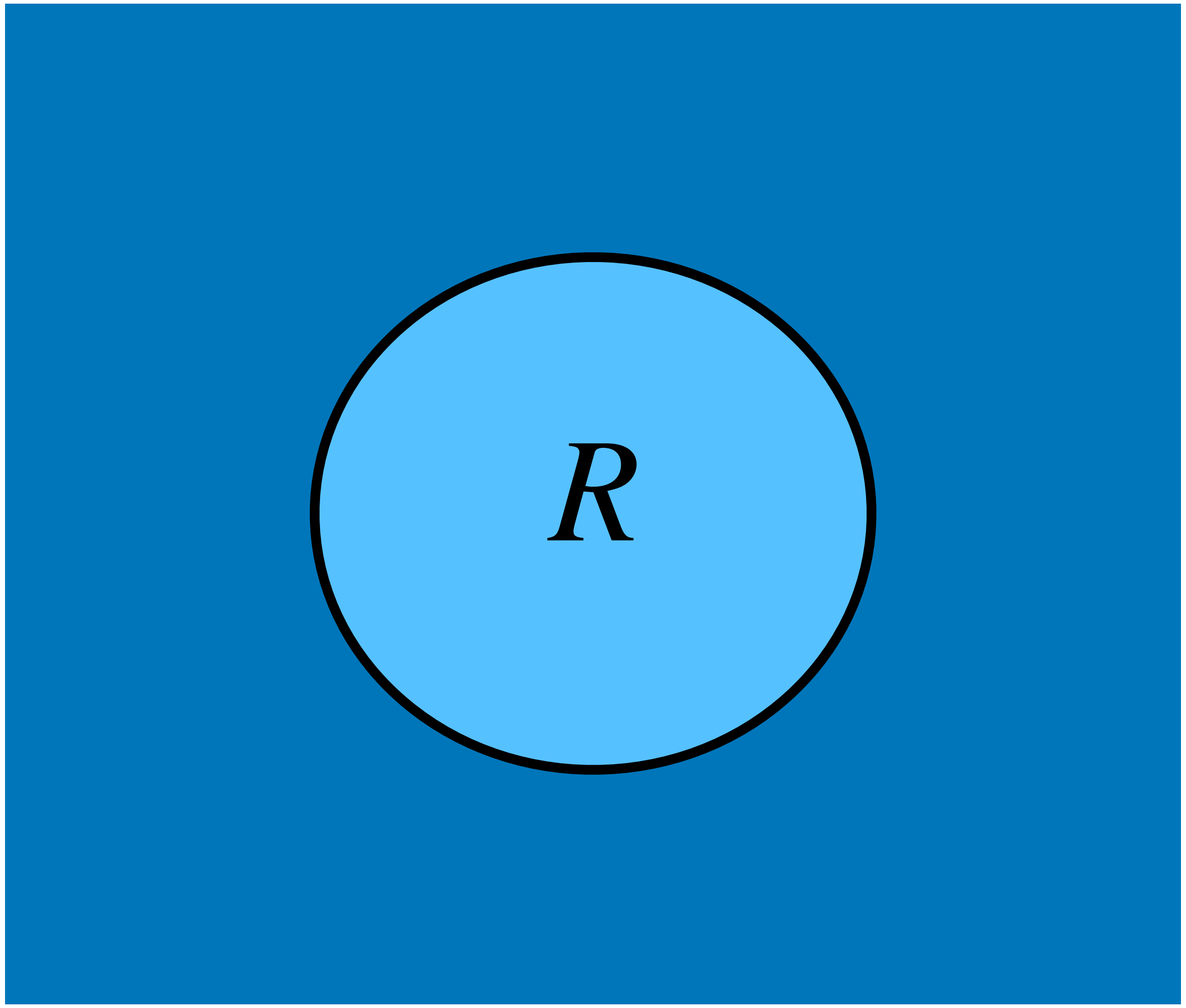
RE+CE The following key equation can be proven [Petz, 1993]

$$S_{\mathcal{F}}(\omega \circ E \,|\, \phi \circ E) \overset{*}{=} S_{\mathcal{O}}(\omega \,|\, \phi)$$

$$* \quad S_{\mathcal{F}}(\omega \,|\, \phi \circ E) = S_{\mathcal{O}}(\omega \,|\, \phi) + S_{\mathcal{F}}(\omega \,|\, \omega \circ E)$$

Relative entropy and conditional expectations

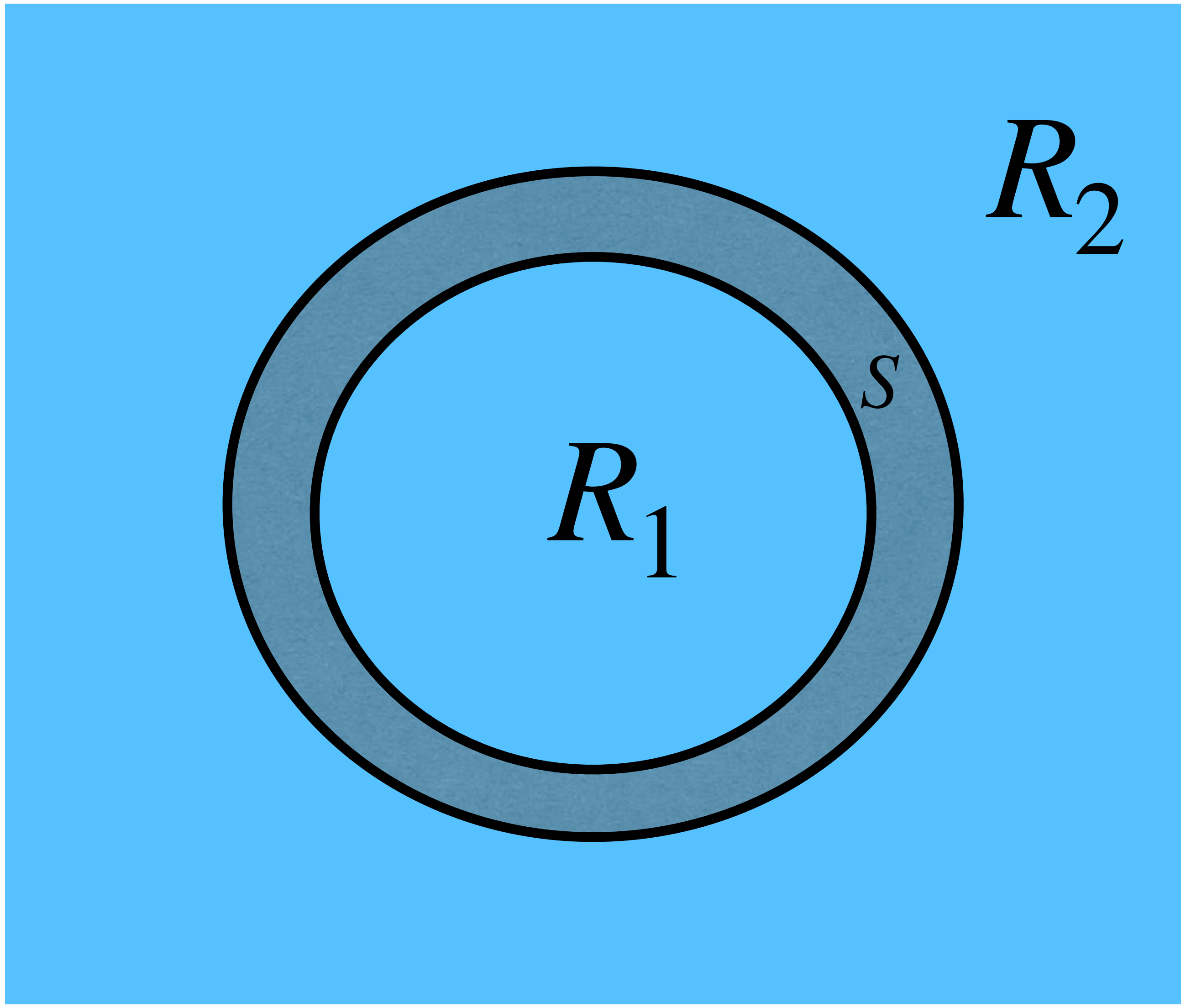
Entanglement entropy does not properly exists in QFT



Relative entropy and conditional expectations

Entanglement entropy does not properly exists in QFT

Using Mutual Information to define EE in QFT introduces a non-trivial topological configuration.



In the presence of symmetries we have two choices

$$\mathcal{O}(R) \qquad \mathcal{O}(R) \vee \mathcal{I}_{R_1 R_2}$$

leading to

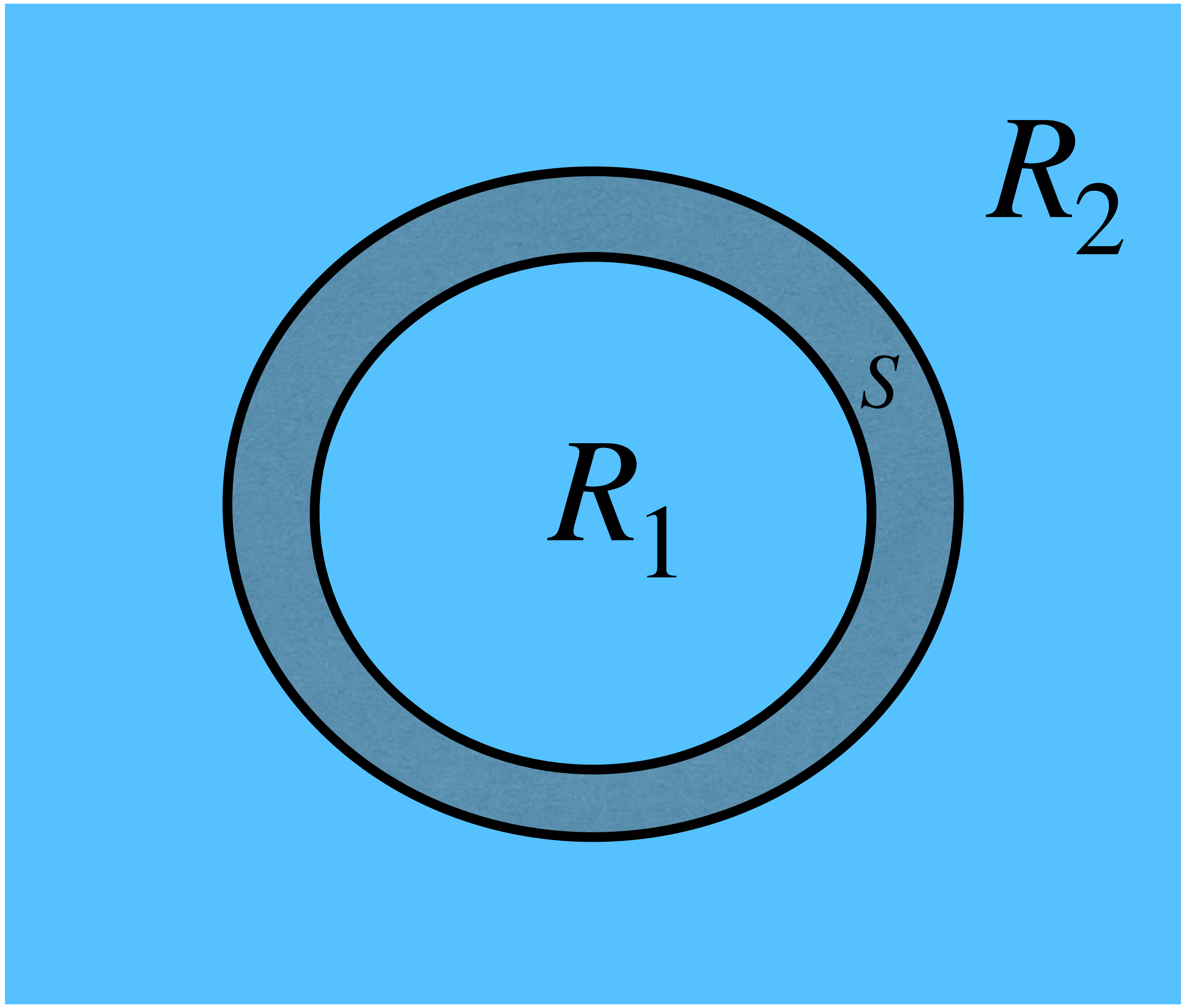
$$S_{\mathcal{O}(R)}(\omega, \omega_{R_1} \otimes \omega_{R_2}) = I_{\mathcal{O}}(R_1, R_2)$$

$$S_{\mathcal{O}(R')'}(\omega, (\omega_{R_1} \otimes \omega_{R_2}) \circ E) = I_{\mathcal{F}}(R_1, R_2)$$

Relative entropy and conditional expectations

Entanglement entropy does not properly exists in QFT. It is just infinite.

Using Mutual Information to define EE in QFT introduces non-trivial topological configurations.



In the presence of superselection sectors we have two choices

$$\mathcal{O}(R) \qquad \mathcal{O}(R) \vee \mathcal{I}_{R_1R_2}$$

Leading to two relative entropies

$$S_{\mathcal{O}(R)}(\omega, \omega_{R_1} \otimes \omega_{R_2}) = I_{\mathcal{O}}(R_1, R_2)$$

$$S_{\mathcal{O}(R')'}(\omega, (\omega_{R_1} \otimes \omega_{R_2}) \circ E) = I_{\mathcal{F}}(R_1, R_2)$$

The algebras are related by

$$E : \mathcal{O}(R) \vee \mathcal{I}_{R_1R_2} \to \mathcal{O}(R)$$

The previous formula involving RE and CE implies

$$I_{\mathcal{F}}(R_1, R_2) - I_{\mathcal{O}}(R_1, R_2) = S_{\mathcal{F}}(\omega, \omega \circ E)$$

Novel universal terms in the entanglement entropy

We are led to compute

$$I_{\mathcal{F}}(R_1, R_2) - I_{\mathcal{O}}(R_1, R_2) = S_{\mathcal{F}}(\omega, \omega \circ E)$$

Difference between both states only come from the intertwiners

$$\mathcal{I}_{R_1 R_2} \equiv \sum_i V^i_{R_1} (V^i_{R_2})^\dagger$$

$$\omega \circ E(\mathcal{I}_{R_1 R_2}) = 0$$

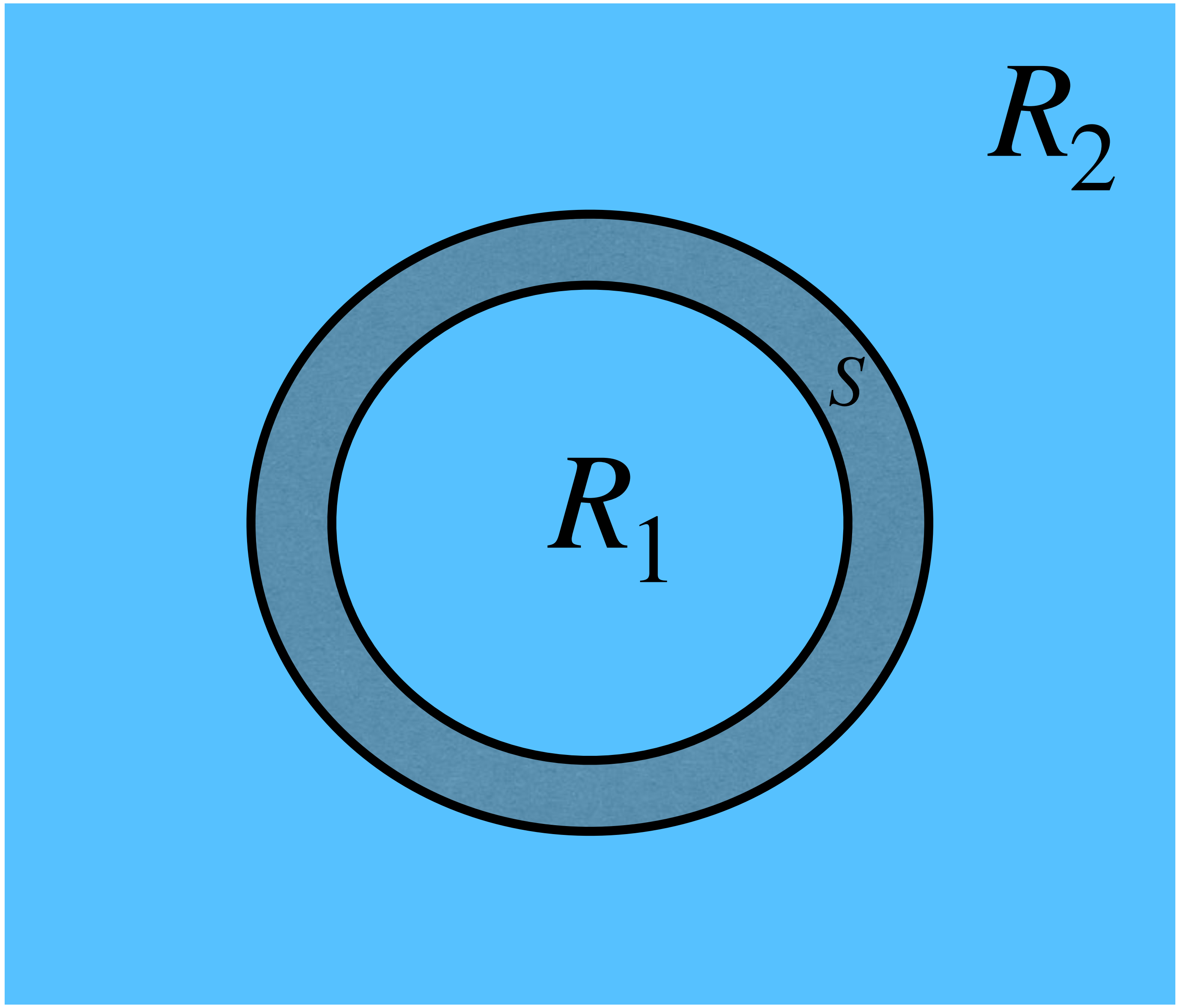
We approach the computation by means of monotonicity of relative entropy.
A lower bound arises by restricting to the “intertwiner algebra”

$$I_{\mathcal{F}}(R_1, R_2) - I_{\mathcal{O}}(R_1, R_2) = S_{\mathcal{F}}(\omega, \omega \circ E) \geq S_{\mathcal{I}_{R_1 R_2}}(\omega, \omega \circ E)$$

Novel universal terms in the entanglement entropy

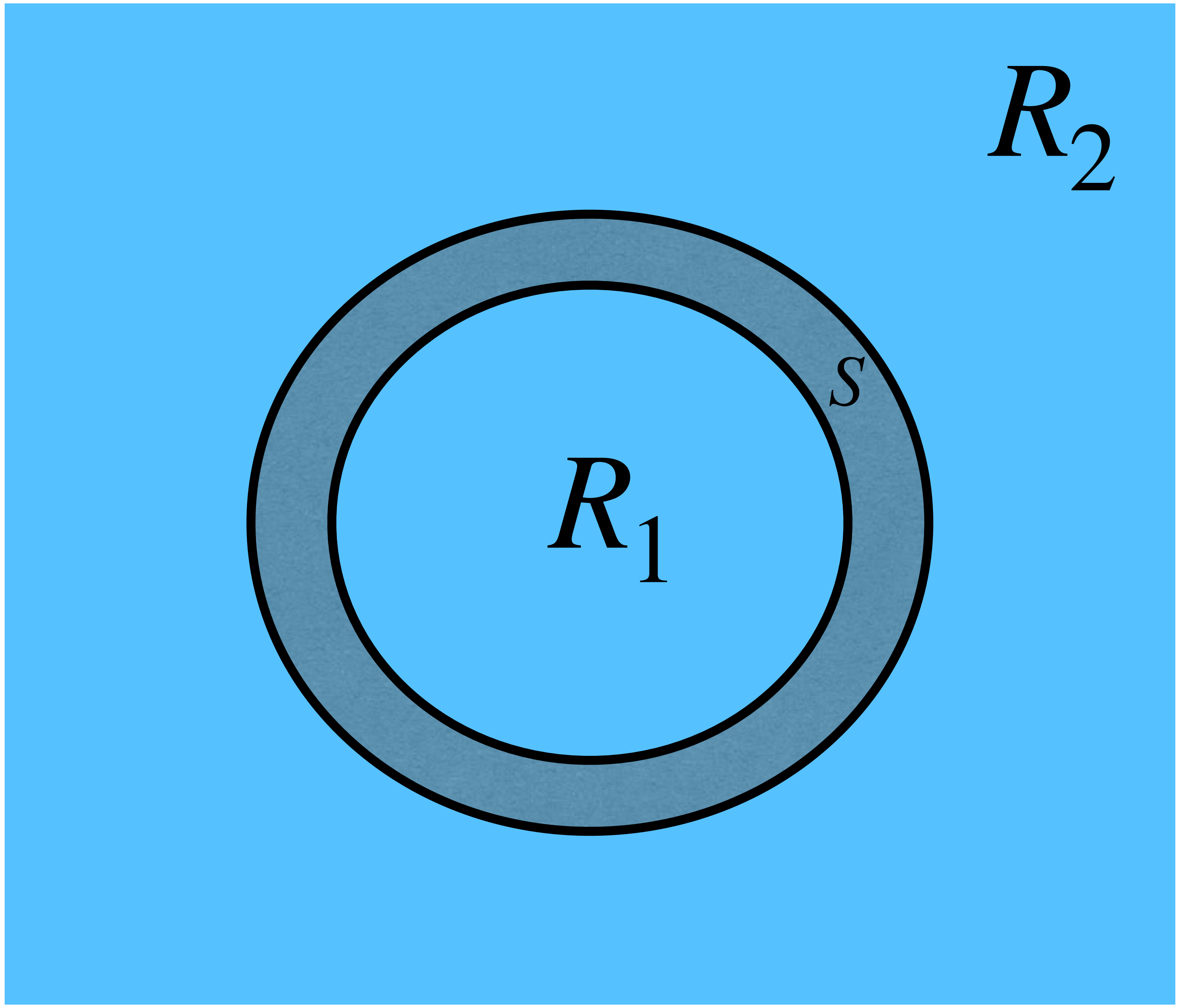
And the higher bound?

The story repeats itself for the spherical shell region.



Novel universal terms in the entanglement entropy

The story repeats itself for the spherical shell region.



We have two algebras, with or without the twist algebra

$$\mathcal{O}_S \qquad \mathcal{O}_S \vee \tau_{[g]}$$

There is a conditional expectation killing the twists

$$\tilde{E} : \mathcal{O}_S \vee \tau_{[g]} \to \mathcal{O}_S$$

And an associated relative entropy

$$S_{\mathcal{O}_S \vee \tau_{[g]}}(\omega, \omega \circ \tilde{E})$$

Novel universal terms in the entanglement entropy

For finite groups the following entropic certainty relation can be derived

$$S_{\mathcal{O}_R \vee \mathcal{I}_{R_1 R_2}}(\omega, \omega \circ E) + S_{\mathcal{O}_S \vee \tau}(\omega, \omega \circ \tilde{E}) = \log |G|$$

We finally find the higher bound

$$S_{\mathcal{I}_{R_1 R_2}}(\omega \,|\, \omega \circ E) \leq I_{\mathcal{F}}(R_1, R_2) - I_{\mathcal{O}}(R_1, R_2) \leq \log |G| - S_\tau(\omega \,|\, \omega \circ \tilde{E})$$

Novel universal terms in the entanglement entropy

- Finite groups $\Delta I = \log G$

- Lie groups $$\Delta I \simeq \frac{1}{2}(d-2)\,\mathcal{G}\,\log\frac{R}{\epsilon}$$

$$\Delta I \simeq \frac{1}{2}\,\mathcal{G}\,\log\left(\log\frac{R}{\epsilon}\right)\;;\;d=2$$

Novel universal terms in the entanglement entropy

Chiral free scalar in two dim.

Conformal, with $c = 1/2$

$j(x^+) = \partial_+ \phi$ $\quad$ x^+null coordinate, is an operator in a line.

The algebra of the current (or the chiral scalar) is exactly formed by the operators of the fermion algebra that are invariant under charge transformations $\psi(x) \to e^{i\alpha}\psi(x)$. So there is a $U(1)$ symmetry in the fermion such that the *orbifold*, the part of the algebra invariant under the symmetry, is the scalar.

$$H = \frac{1}{2}\int dx\, j(x)^2 \ , \ [j(x), j(y)] = i\delta(x-y)$$

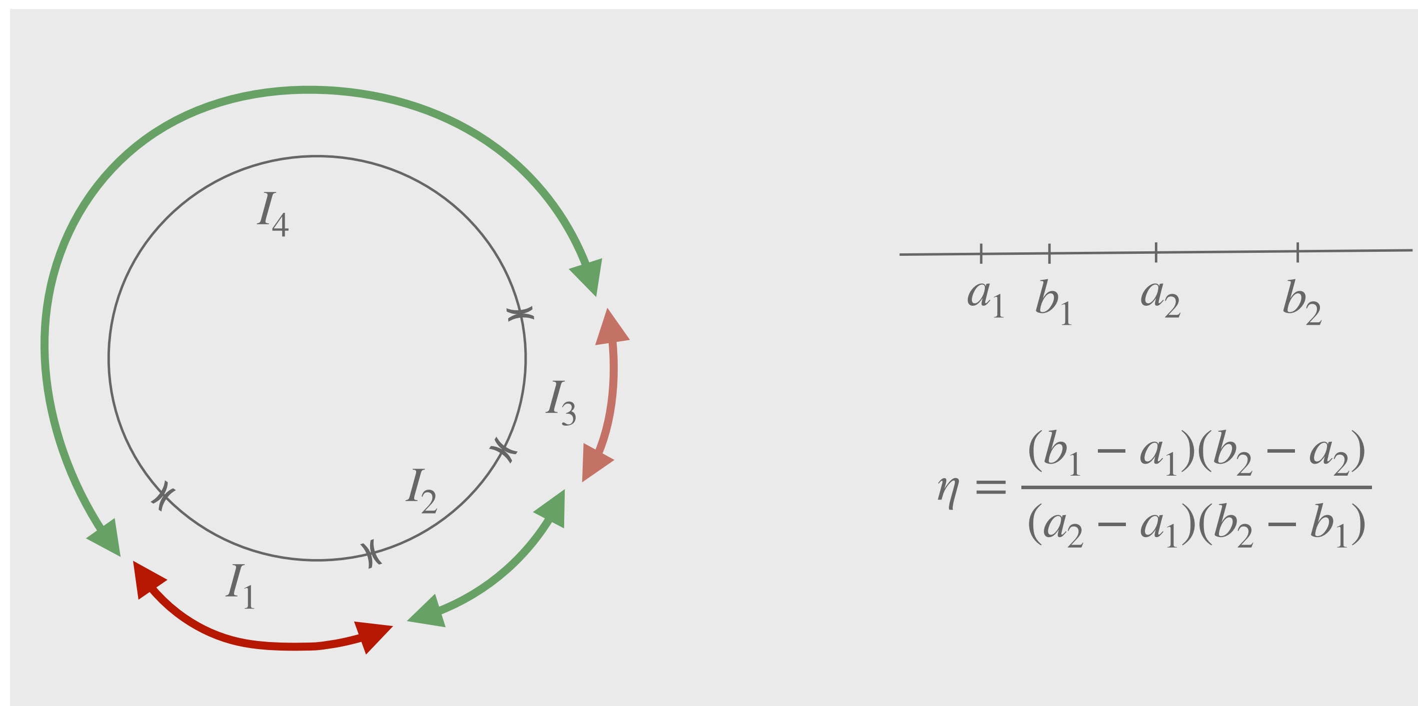
one interval

Checking duality in EE

$$S(I_1) = S(I_2 \cup I_3 \cup I_4)$$

$$S(I_1 \cup I_3) = S(I_2 \cup I_4)$$

two intervals



In the line $S(R) = \frac{c}{3}\log(R)$ for any CT

Novel universal terms in the entanglement entropy

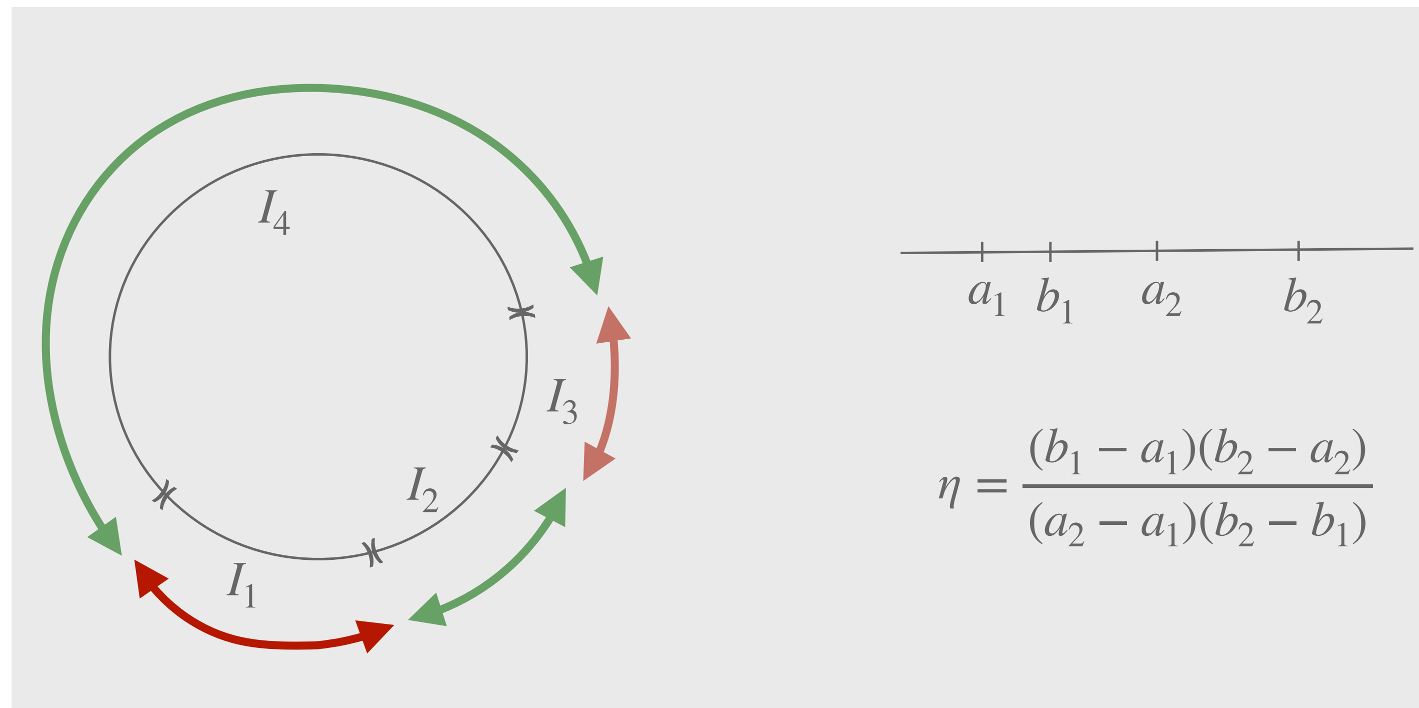
Chiral free scalar in two dimensions

$$j(x^+) = \partial_+ \phi \qquad H = \frac{1}{2}\int dx\, j(x)^2 \;,\; [j(x), j(y)] = i\delta(x-y)$$

Checking duality in mutual information

$$I(I_1, I_3) = S(I_1) + S(I_3) - S(I_1 \cup I_3)$$

$$I(I_2, I_4) = S(I_2) + S(I_4) - S(I_2 \cup I_4)$$



Assuming duality $S(I_1 \cup I_3) = S(I_2 \cup I_4)$

$$I(I_1, I_3) = I(I_2, I_4) + S(I_1) + S(I_3) - S(I_4) - S(I_2)$$

$$\downarrow \star$$

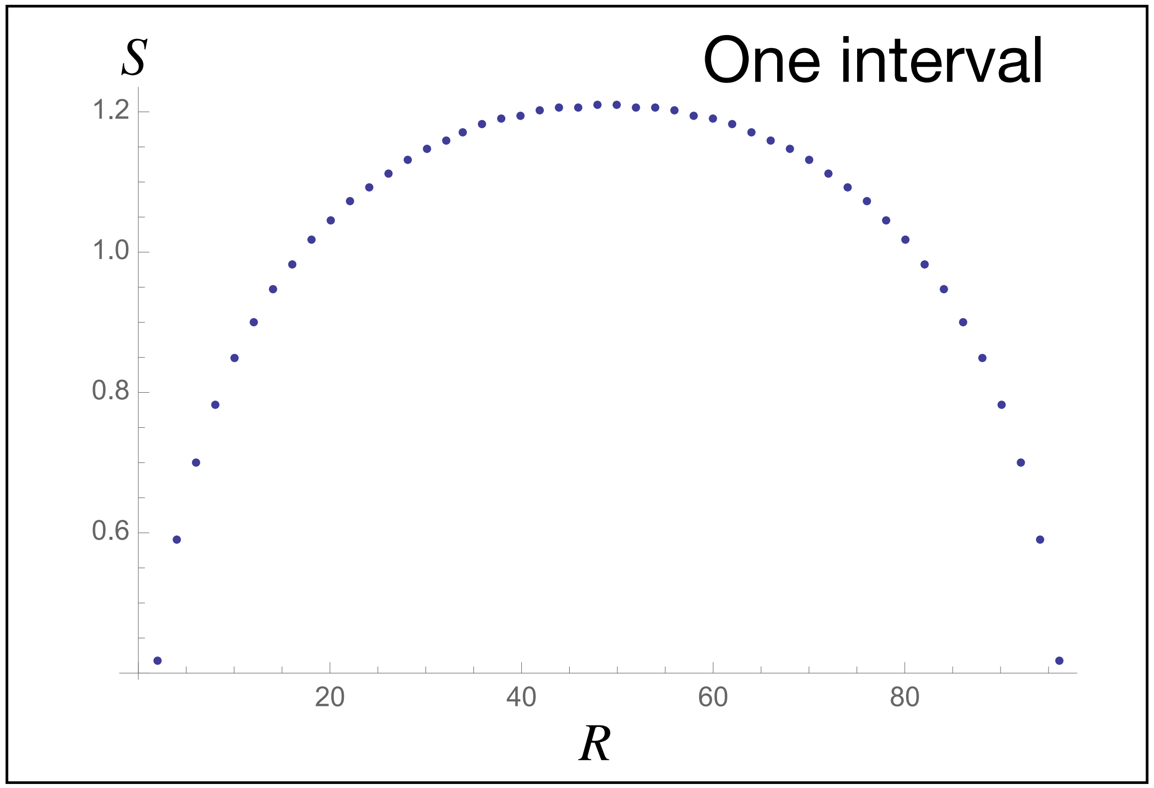
$$I(\eta) = I(1-\eta) - \frac{c}{3}\log\left(\frac{1-\eta}{\eta}\right) \longleftrightarrow$$

$U(\eta) = U(1-\eta)$ Haag duality

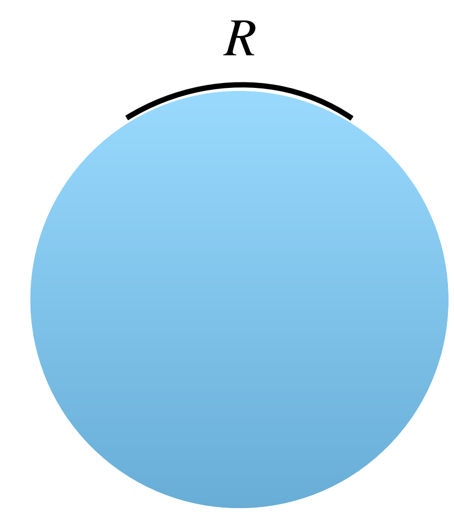
$$\star \begin{cases} S(R) = \frac{c}{3}\log(R) \\ I(\eta) = -\frac{c}{3}\log(1-\eta) + U(\eta) \end{cases}$$

for any CT

Novel universal terms in the entanglement entropy



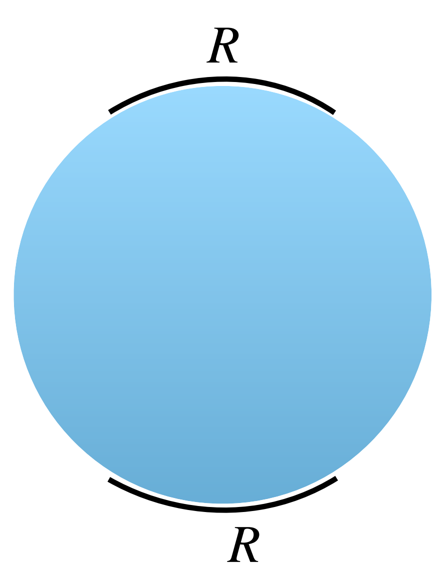
Circle Length $100\ \epsilon$

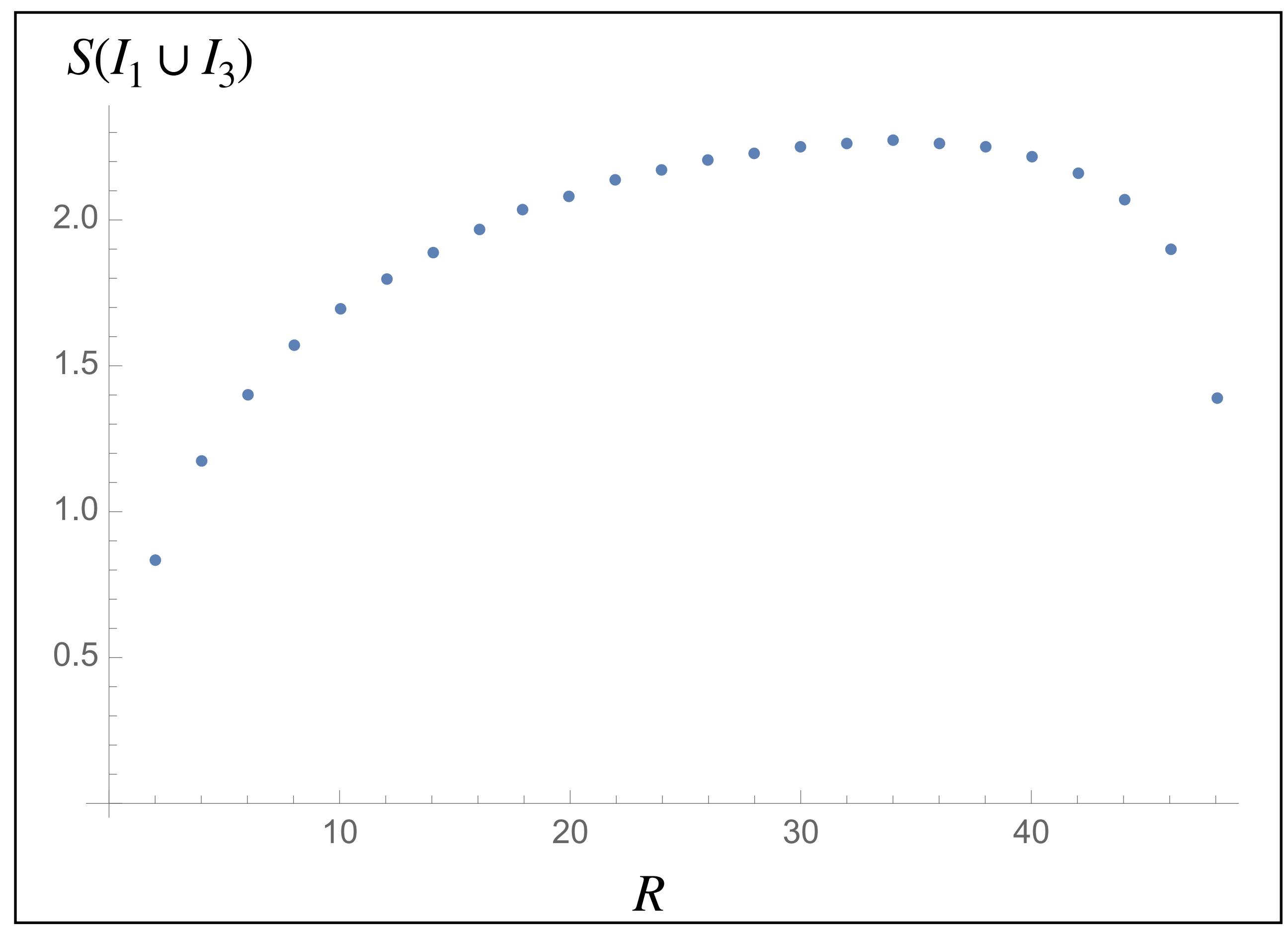


$$S(I_1) = S(I_2 \cup I_3 \cup I_4)$$

$$S(I_1 \cup I_3) \neq S(I_2 \cup I_4)$$

$$b_1 - a_1 = b_2 - a_2 = R\ ,\ a_2 - a_1 = n/2$$



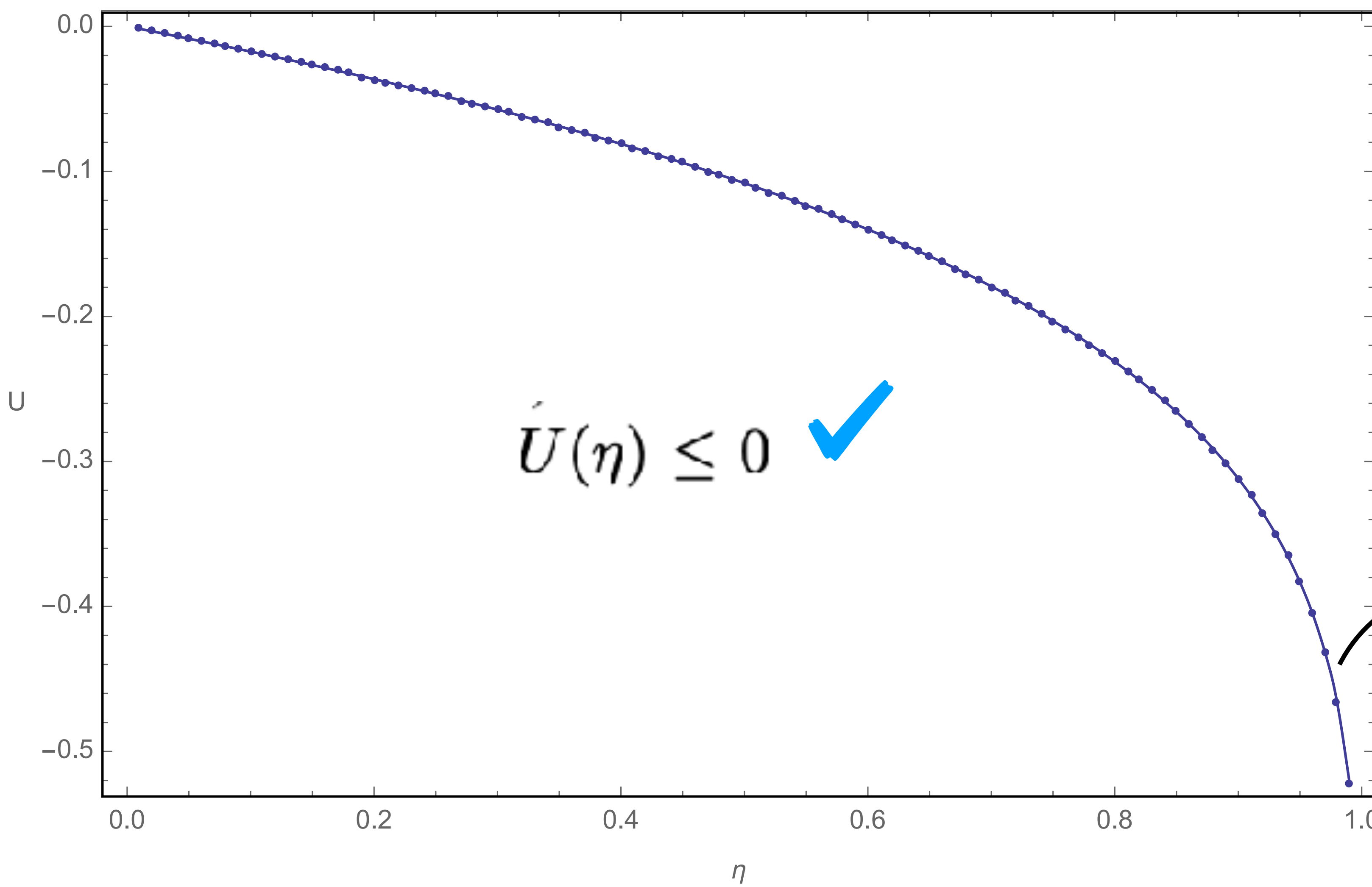


Novel universal terms in the entanglement entropy

Mutual Two intervals

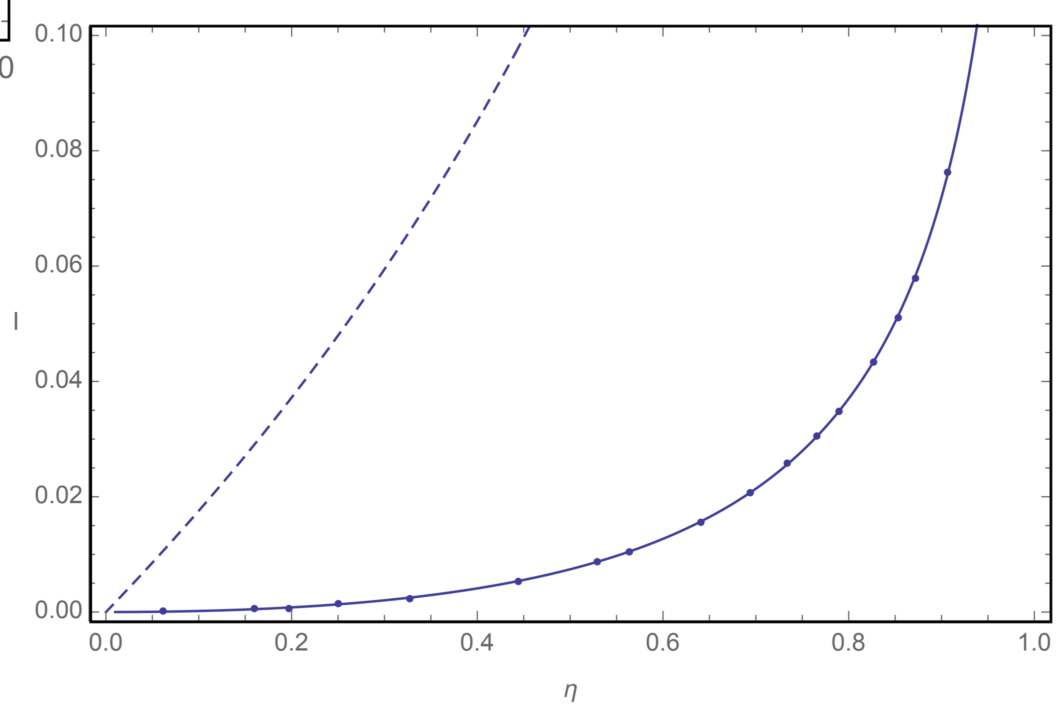
$$U(\eta) = U(1-\eta)$$ ✗

$$\Delta I \simeq \frac{1}{2}\, \mathscr{G} \log\left(\log \frac{R}{\epsilon}\right)$$



$$U(\eta) = -\frac{i\pi}{2}\int_0^\infty ds \frac{s}{\sinh^2(\pi s)} \log\left(\frac{{}_2F_1(1+is,-is,1,\eta)}{{}_2F_1(1-is,is,1,\eta)}\right)$$

hypergeometric functions



Novel universal terms in the entanglement entropy

Twist and intertwines?

$$O_{13} = \phi(x_1) - \phi(x_3) = \int_{x_1}^{x_3} dx\, \partial_x \phi(x)\,, \quad x_1 \in I_1 \text{ and } x_3 \in I_3$$

$O_{13} \in \mathscr{O}$	$O_{13} \in (\mathscr{O}_2 \cup \mathscr{O}_3)'$	$O_{13} \not\in \mathscr{O}_1 \cup \mathscr{O}_3$

$$[O_{13}, O_{24}] = i\,,$$

$$(\mathcal{A}_{\rm add}(I_1 I_3))' = (\mathcal{A}(I_1) \vee \mathcal{A}(I_3))' = \mathcal{A}(I_2) \vee \mathcal{A}(I_4) \vee O_{24} = \mathcal{A}_{\rm add}(I_2 I_4) \vee O_{24}\,,$$

$$(\mathcal{A}_{\rm add}(I_2 I_4))' = (\mathcal{A}(I_2) \vee \mathcal{A}(I_4))' = \mathcal{A}(I_1) \vee \mathcal{A}(I_3) \vee O_{13} = \mathcal{A}_{\rm add}(I_1 I_3) \vee O_{13}\,.$$

$\mathscr{F}$: Chiral fermion with c = 1/2

$\mathscr{O}$: Chiral scalar is a subalgebra of the chiral fermion generated by the current

$$j(x) = \psi^\dagger \psi \xrightarrow{\text{bosonization}} j(x^+) = \partial_+ \phi$$

$$I(\eta) = -\frac{c}{3}\log(1-\eta) + U(\eta)$$

$\mathscr{O}$ → $U(\eta) \leq 0$

$\mathscr{F}$ → $U(\eta) = 0$

Conclusion

- Theories based on subsets of local operators invariant under some global symmetry lead to a Haag duality/additivity violation

- Why? Existence of twists and intertwiners / generalized symmetry

- Assignation of algebra to a region is Non unique

- Novel topological contributions to MI

- Comment:
  Local symmetries give rise to the same structure: violation of additivity/duality, existence of non locally generated operators, wilson and ´t Hooft loops. Solution to the mismatch of the Maxwell anomaly

Thanks!