Black Holes as the next crucial step towards understanding the Planck world of the small

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part 2

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The correct answer will be revolutionary, and it will change our views as rigorously as what the *Higgs particle* brought about in our standard understanding of the subatomic particles.

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string theory might reemerge anyway !

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Is there anything we can learn about this question, from studying

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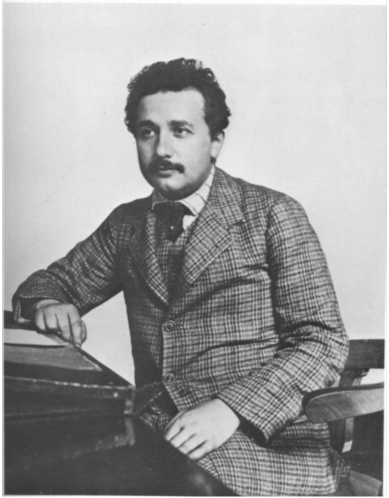

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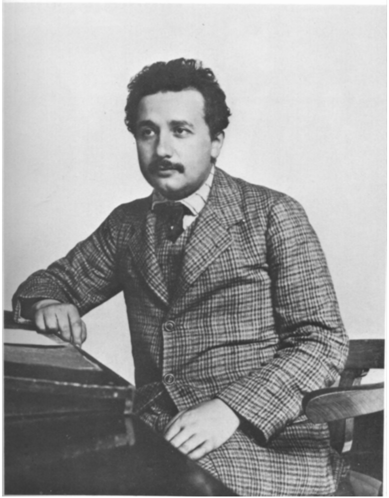

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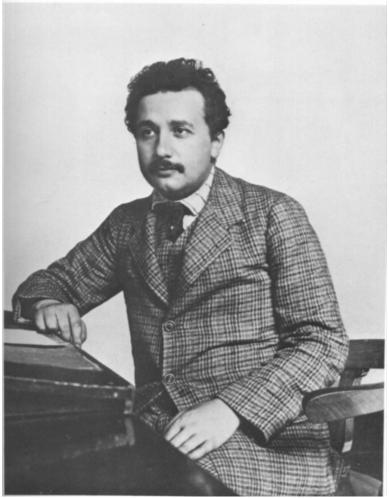

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rephrase the boundary conditions at the black hole horizon.

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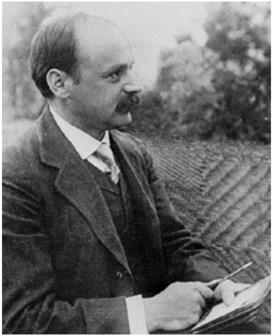
Keep in mind when

reformulating Quantum Gravity such that it allows application in black holes,

neither String Theory nor AdS/CFT are fool-proof !

Schwarzschild Metric in spacetime



Karl Schwarzschild

(r, t, θ, φ):

$$\mathrm{d}s^2 \;=\; \frac{1}{1-\frac{2GM}{r}}\mathrm{d}r^2 - \left(1-\tfrac{2GM}{r}\right)\mathrm{d}t^2 + r^2\mathrm{d}\Omega^2 \;;$$

$$\Omega \;\equiv\; (\theta,\, \varphi)\;,$$

$$\mathrm{d}\Omega \;\equiv\; (\mathrm{d}\theta,\, \sin\theta\,\mathrm{d}\varphi)\;.$$

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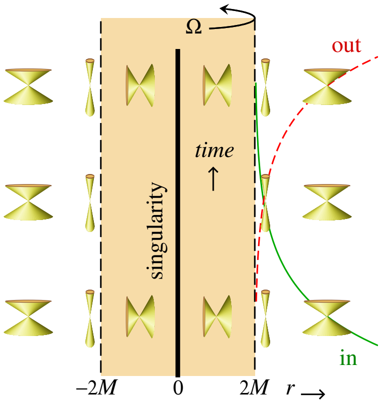

In perturbative QFT, interaction between particles always vanish at lowest order. So those you can't use those to retrieve the information of the in-particles by inspecting the out-particles !

But gravitational interactions are non-perturbative.

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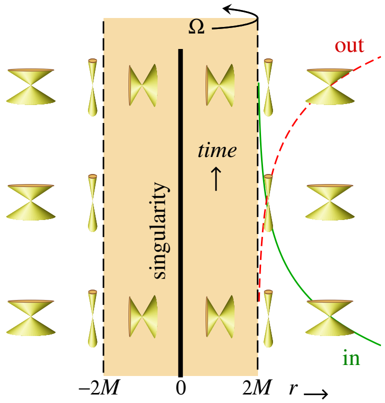

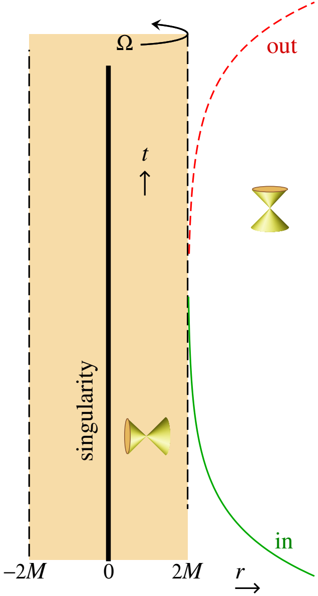

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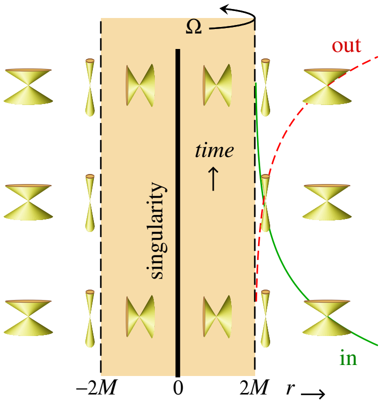

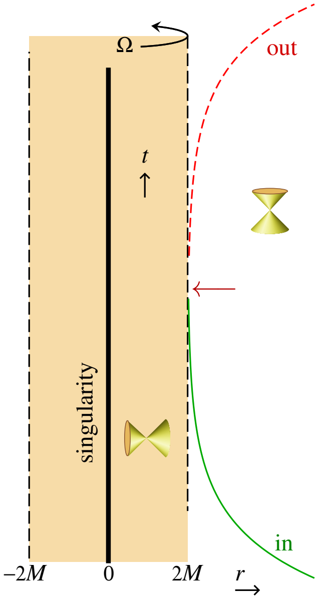

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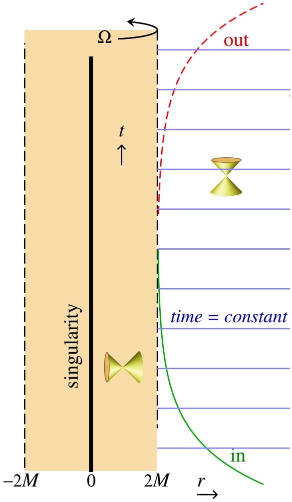
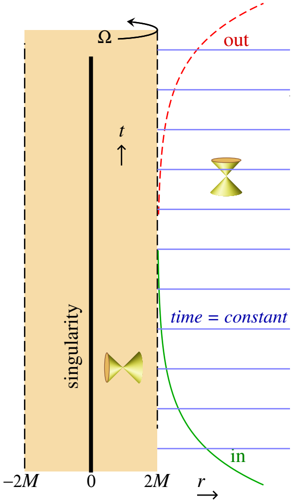
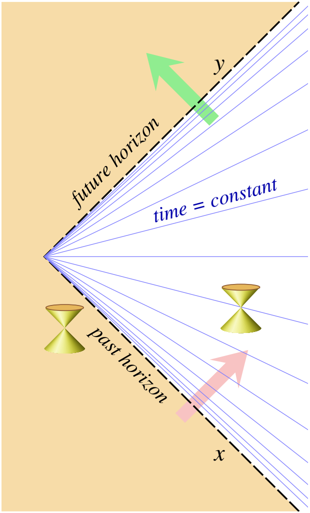

Ω
out
t
time = constant
singularity
in
−2M
0
2M
r





The Black Hole Information problem: *How can matter coming in (green arrow) affect matter coming out (red arrow)?*

Kruskal-Szekeres (or 'tortoise') coordinates x, y, are defined by

$$\begin{aligned} x\,y &= \left(\frac{r}{2GM} - 1\right) e^{r/2GM}; \\ y/x &= e^{t/2GM}. \\ \mathrm{d}s^2 &= \frac{32(GM)^3}{r}\, e^{-r/2GM}\, \mathrm{d}x\, \mathrm{d}y + r^2 \mathrm{d}\Omega^2 \ . \end{aligned}$$

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$-\infty < x, y < \infty.$

$x = \tan(u^-)\,, \quad y = \tan u^+\,.$

$-\frac{1}{2}\pi < u^{\pm} < \frac{1}{2}\pi$

At $r \to 2GM$, we have

$x = 0$: future event horizon,
$y = 0$: past event horizon.

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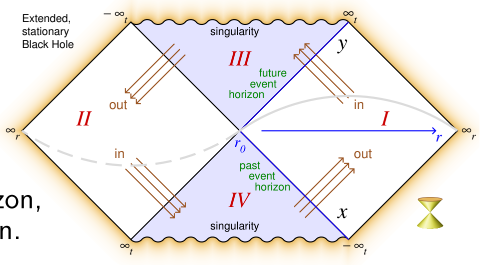

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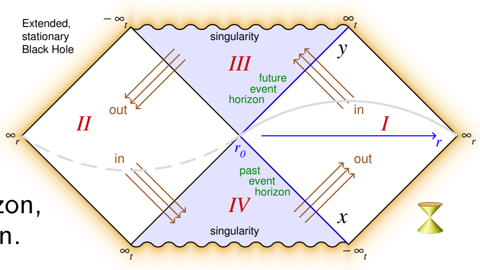
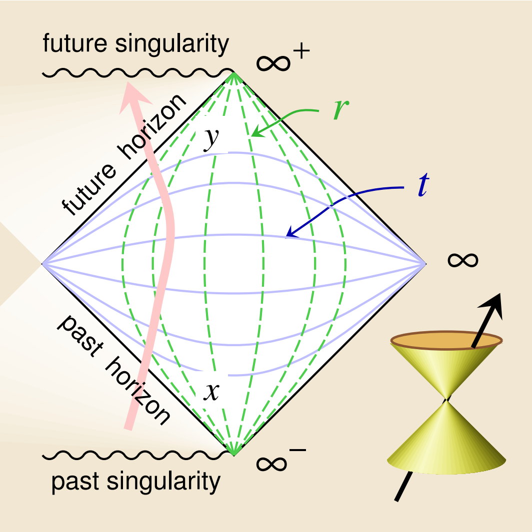
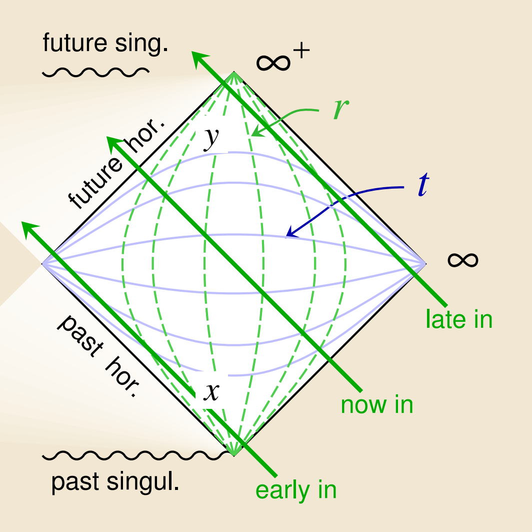
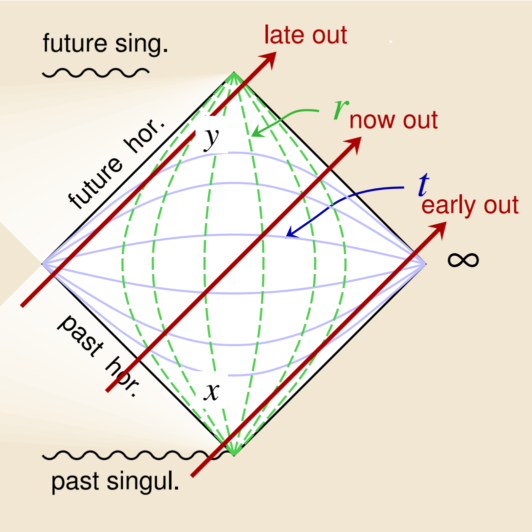


For every point (r, t, θ, φ), there are *two* points in these new coordinates: with every (x, y, θ, φ) there is also $(-x, -y, \theta, \varphi)$.

$(x,\ y)$: $x = \tan u^-$, $y = \tan u^+$.




future sing.
∞^+
r
future hor.
y
t
∞
late in
past hor.
x
now in
past singul.
early in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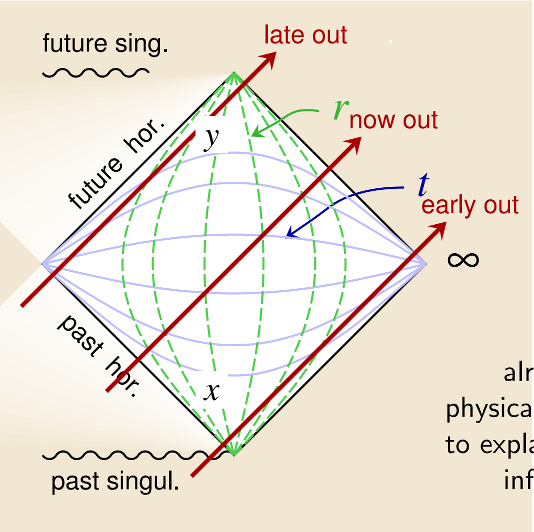
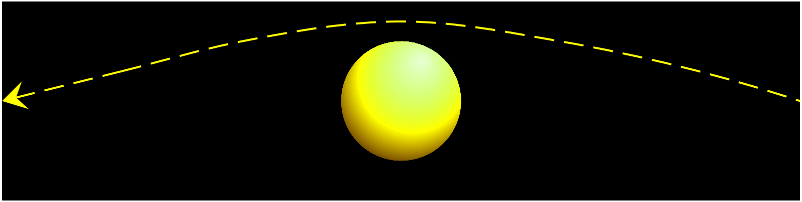
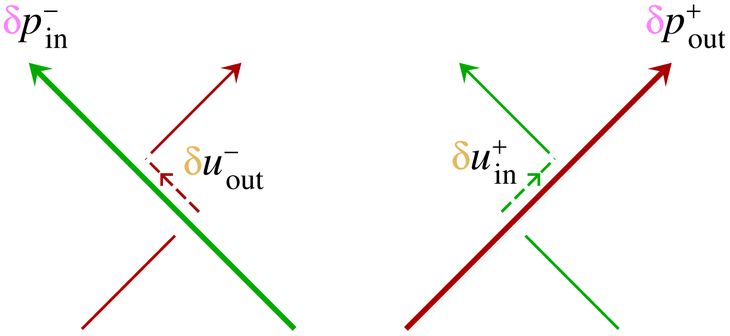


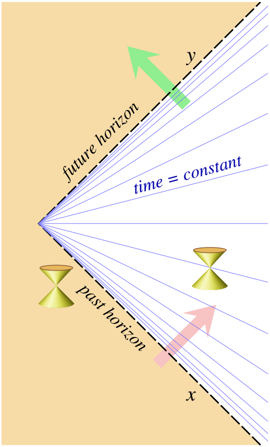
Which
already existing
physical laws can be used
to explain the flow of
information?

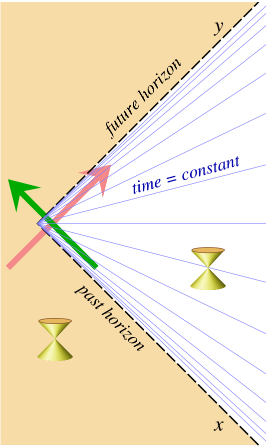


The most important gravitational interaction is now
the Shapiro effect.



The *positions* u of the out going particles will be associated to the *momenta* p of the in going particles, and *vice versa* .


y
future horizon
time = constant
past horizon
x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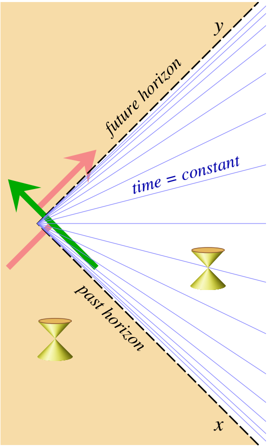
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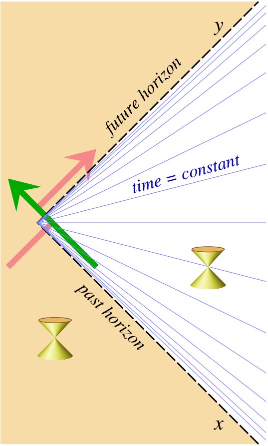
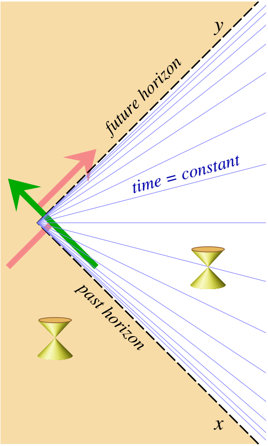


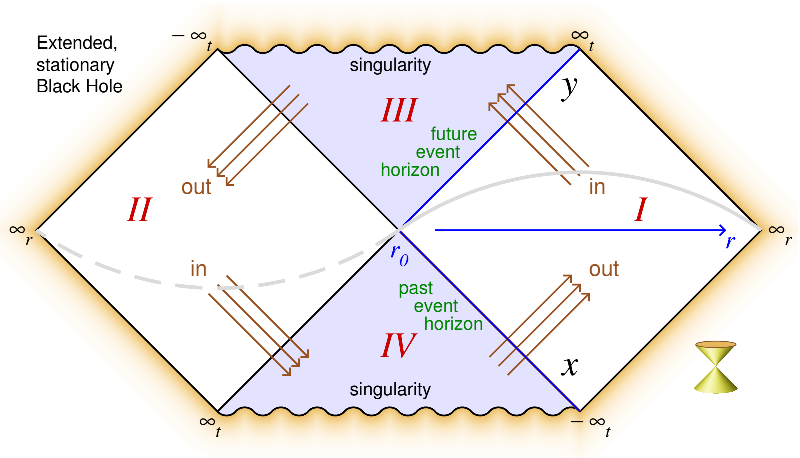
We see that information does enter in the forbidden domain!

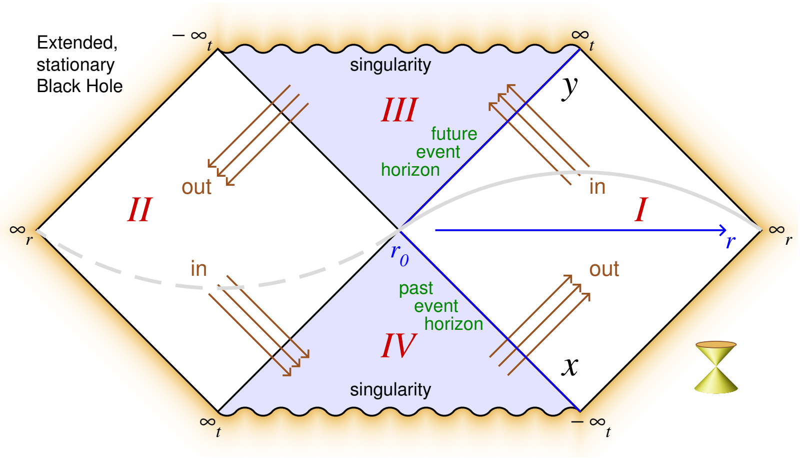


We see that information does enter in the forbidden domain!

And it returns if you annihilate a visible particle!

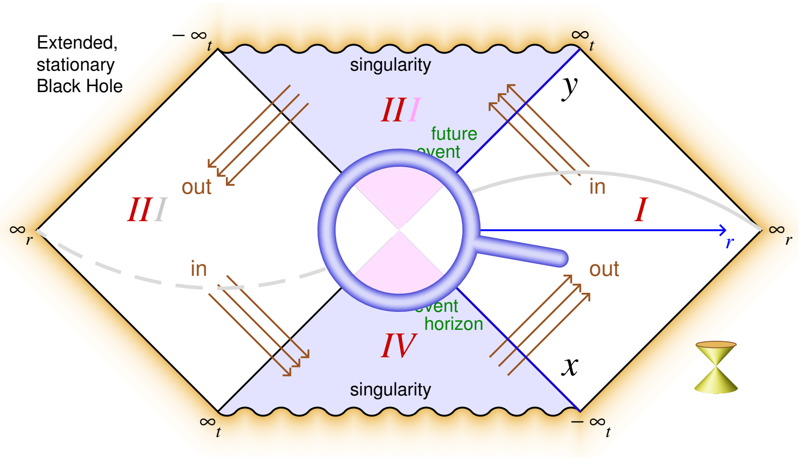
Extended,
stationary
Black Hole





This purely mathematical coordinate transformation suggests that a second, phantom universe, fits with our universe!

As time runs forward in our universe (right), time goes *backwards* in the phantom universe !



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As time runs forward in our universe (right), time goes *backwards* in the phantom universe !

At first sight, the Shapiro shift generates a *non-locality* of the kind we don't want !

Instead of a non-locality, this is a simple manifestation of *curvature*. This keeps the e.o.m. local.

What we must do is allow space-time to be cut apart at the horizon, and glue things together with a shift.

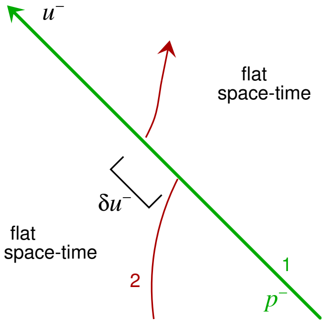
This recovers continuity of the motion of particles.

The Shapiro effect.

Partial waves:
$$\delta u^-_{\rm out} \approx \delta p^-_{\rm in}\ .$$
$$\delta u^-_{\ell m} \approx \delta p^-_{\ell m}\ .$$

This gives algebra:

$$[u^+,\, u^-] = i\lambda\ ,$$
$$\lambda \equiv \frac{8\pi G}{\ell^2+\ell+1}\ .$$



We expand everything in spherical harmonics $Y_{\ell,m}$.

Therefore, u^- is the Fourier transform of u^+. In only one dimension.

This is how quantum mechanics comes in!

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However, he assumed that all particle configurations in region II can be chosen independently from region I.

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However, he assumed that all particle configurations in region II can be chosen independently from region I.

This means that he assumed the hidden region II to be an entire, new universe, but this would not agree with our assumption that a black hole should behave like rocks or planets.

It should not interact directly with *one* other distant yet identical rock or planet.

We'll return to that!

How to calculate this:

Consider quantised fields, $\Phi(\vec{x}, t)$ near the horizon. Since all points in region I are space-like separated from all points in region II, we can derive, from Φ, creation operators $a^\dagger$ and annihilation operators a in region I that commute with those of region II.

Write in locally flat apace-time, near the origin:
$\Phi = A_{\rm Mink}(T) + A^\dagger_{\rm Mink}(T),$ where T is the local time parameter.

The relation between the time coordinate t and T is exponential:
$e^t = f(T, \vec{X})$.

We calculate the creation and annihilation operators $a_I(\omega)$, $a_{II}(\omega)$ and their Hermitean conjugates, in terms of $A_{\rm Mink}(\omega)$ and its hermitean conjugate;

here, ω is a variable conjugate to the energy parameter in regions I and II.

When we express $A_{\mathrm{Mink}}(k)$ and $A^{\dagger}_{\mathrm{Mink}}(k)$ in terms of the creation and annihilation operators seen by a distant observer, we find them to be different, in terms of a *Bogolyubov transformation* (transformation mixing $a_{I,II}$ and $a^{\dagger}_{I,II}$).
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In short hand:

$$\begin{pmatrix} A_{\mathrm{Mink}}(\omega) \\ A_{\mathrm{Mink}}(-\omega) \\ A^{\dagger}_{\mathrm{Mink}}(\omega) \\ A^{\dagger}_{\mathrm{Mink}}(-\omega) \end{pmatrix} = C(\omega) \begin{pmatrix} 1 & 0 & 0 & -e^{-\pi\omega} \\ 0 & 1 & -e^{-\pi\omega} & 0 \\ 0 & -e^{-\pi\omega} & 1 & 0 \\ -e^{-\pi\omega} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_I(\omega) \\ a_{II}(\omega) \\ a^{\dagger}_I(-\omega) \\ a^{\dagger}_{II}(-\omega) \end{pmatrix}$$

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Thus we have: $A_{\rm Mink}(\omega) = C(\omega)\left(a_I(\omega) - e^{-\pi\omega} a^\dagger_{II}(-\omega)\right)$, etc.

See arXiv:2410.16891

If region I would be independent of region II, you would be describing the universe surrounding a black hole as if it consists of two universes. Consider the state $|\Omega\rangle$ obeying $A|\Omega\rangle = 0$.
Solution:

in $|\Omega\rangle$, $\quad n_1 = n_2 \quad$ and

$$\langle n_1,\, n_2|\Omega\rangle = C(n)\delta_{n_1,\,n}\delta_{n_2,\,n}\ ; \qquad C(n) = \frac{e^{-\pi\omega n}}{\sqrt{1-e^{-2\pi\omega}}}\ .$$

One may now calculate the probability of having n particles in region I, assuming that that region represents our universe:

$$P_n = \sum_{n_2=0}^{\infty} |\langle n|\Omega\rangle|^2 = C(n)^2 = \frac{e^{-2\pi\omega n}}{1-e^{-2\pi\omega}}.$$

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Here, ω represents the energy of the particles that can be created or annihilated. $2\pi\omega$ stands for the inverse temperature of the particles just outside the black hole.

It is the value derived by Hawking: $T = \frac{1}{8\pi M}$

But are we really dealing with a thermal state?

$$\langle n_1,\, n_2|\Omega\rangle = C(n)\delta_{n_1,\,n}\delta_{n_2,\,n}\ ; \qquad C(n) = \frac{e^{-\pi\omega n}}{\sqrt{1-e^{-2\pi\omega}}}\ .$$

We see that $n_1 = n_2$. For a thermal state this is somewhat odd. What are those "negative energy particles" doing? The states with $n_1 \neq n_2$ disappear when nothing falls in.

The state we have when nothing falls in resembles a thermal *density matrix :* $|n\rangle\, e^{-\beta n}\langle n|$, it seems to indicate a probability mapping *without squaring* :
$P_n = \langle\ |n\rangle\, e^{-\beta n}\langle n|\ \rangle = C(n) = e^{-\pi\omega n}$.

Its inverse is also a temperature, but *twice* the temperature derived by Hawking.

Now, it is $T = \frac{1}{4\pi M}$.

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To be continued in discussion session.

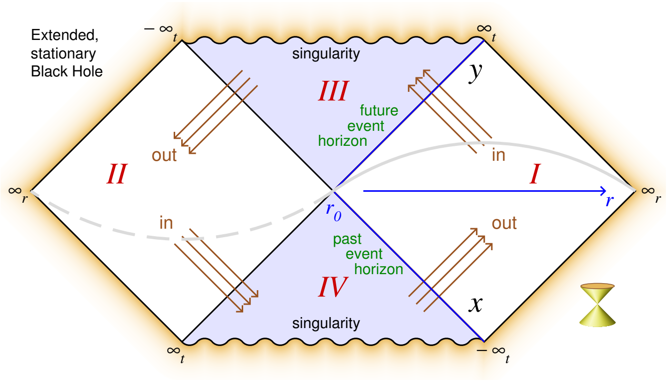
Region II is now redundant. It is fixed by what happens in region I.

Essential and, I claim, necessary departure from Hawking:

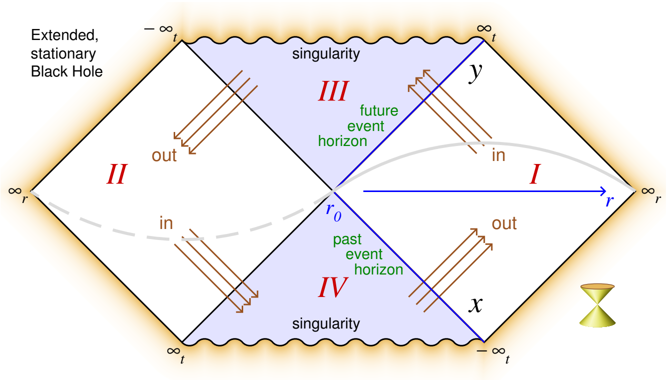
u^+ and u^- are not the 'good' coordinates, but r and t are. They do not change as you go from (u^+, u^-) to $(-u^+, -u^-)$.

Also, the Schwarzschild metric formed by collapse cannot generate different coordinates for one space-time point.

We can use this only if: *Region II is identical to region I.*
Everything in these two regions matches exactly
(including the observers themselves !)



Extended, stationary Black Hole
$-\infty_t$
∞_t
singularity
y
III
future event horizon
out
in
II
I
∞_r
r
∞_r
r_0
in
out
past event horizon
IV
x
singularity
∞_t
$-\infty_t$



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How is information from the in=particles recovered by the out-particles?

The Shapiro effect makes in- and out-particles interact strongly.

The momenta p by which the in-particles enter the future horizon, determines how far the coordinates u of the out-particles move, at the past horizon:

$$\delta u_{\,\mathrm{out}} = \lambda p_{\,\mathrm{in}} \ , \quad \lambda = \frac{8\pi G}{\ell^2+\ell+1} \ .$$

Because region II = region I, the functions $\psi(x)$ will be even functions:

$$\psi(x) = \psi(-x) \; ; \quad \text{also} \quad \hat{\psi}(p) = \hat{\psi}(-p).$$

Therefore, we need to know only the functions for $x > 0$ and $p > 0$.

In our calculations, we may therefore limit ourselves to purely even or purely odd functions !

We must simply ignore region II and the particles there, since they are exact copies of region I.

The *Fourier transform* is usually assumed to map functions on the infinite real line onto functions on another infinite real line. So:

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ipx}\, \hat{\psi}(p) \;;$$

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But, all our observables are on the half-lines. This allows us to use *"cosine Fourier transformations"* on our half-lines:

$$\psi(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty \cos(px)\, \hat{\psi}(p)\ ;$$

inverse: $\hat{\psi}(p) = \sqrt{\frac{2}{\pi}} \int_0^\infty \cos(px)\, \psi(p)$

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This solves two problems in one blow!

First, half-space Fourier transformations from future to past horizon keep the particles on the same half-line. Now the transformation is unitary on the half-line. *No information is lost !*

We found the transformation that does that !

If applied to region *II*, we see that wave functions are indeed sent backwards in time ...

(merely because we replaced space-time curvature effects by pseudo-non-local field transformations)

In the formalism just developed,

note: the in- and out-going particles are *not second-quantised* in the r, t direction; there is always exactly one particle on every ℓ, m point in angular momentum space!

That particle can *either* be regarded as in-going *or* as out-going, we cannot have both an in- and an out-particle on one given ℓ, m point !

This makes the math very easy! This is the second problem solved!

(But the overall picture can be much improved! - see first attempts on last slide.)

Consequently, the computation of the transmission effect from future to past horizon only works if we have single, scalar particles of only one kind whose momenta and positions are handled.

And, since gravity does not discriminate between fields of different colors, the information regarding colors' of fields, cannot be transferred this way.

Suggestion: perhaps this can be used to prove that *no two observable* particles in the SM can be entirely identical.

Time to discuss the following?

The coordinate substitution $x, y \;\rightarrow\; r, t$ returns to us the single universe, but at the cost of a singularity at $(x, y) \rightarrow (0, 0)$, now seen to be a *conical singularity* (angle $= 180^\circ$). But this is a mild singularity. We could remove it by *smearing* the metric a bit, at $(x, y) \rightarrow (0, 0)$, accidentally revealing a dense blob of matter there?

The singularity is locally observable, and it resembles a *Euclidean* **string** *world sheet.* What is its dynamics?

And to discuss:

Since now all information needed to describe the black hole is in region I, *and it stays there;* we suspect it to be conserved. This implies we must be able now to use an *action principle* with a *boundary Lagrangian* added, to map future on past horizon and back. We can project the entire Standard Model in region I. Thus, black holes may get completely integrated in the Standard Model, simply by a more careful analysis of the boundary conditions connecting past to future horizon.

However, the computation of the information-transmission effect from future to past horizon seems to work only if we have single, scalar particles of only one kind whose momenta and positions are handled.

(First quantisation)

On the boundary conditions at future and past horizons

For 1^{st} quantised scalar particles, this is straightforward (we omit the ℓ dependence): Just use the 'half-line Fourier transformation':

$$\psi(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty \mathrm{d}y\ \cos(xy)\hat{\psi}(y)\ ; \tag{1}$$
$$\hat{\psi}(y) = \sqrt{\frac{2}{\pi}} \int_0^\infty \mathrm{d}x\ \cos(xy)\psi(x)\ .$$

But what to do if we have several species of scalar particles?

How should we handle particles with spin ($\frac{1}{2}$ or 1, like in the SM)?

Or, how should we replace the single particle states by 2^{nd}-quantised SM-particles?

—∞—

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THE END