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Introduction to Circuit QED

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Disclosure: SMG is a consultant and equity holder in Quantum Circuits, Inc. and an equity holder in IBM, Inc.

Lecture notes on circuit QED (150 pages)
2011 Les Houches Summer School

https://girvin.sites.yale.edu/lectures

Lecture series on quantum error correction and fault tolerance

arXiv:2111.08894: Introduction to Quantum Error Correction and Fault Tolerance

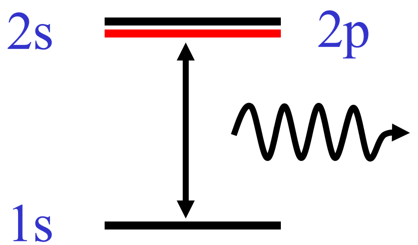
OUTLINE:

Introduction to Circuit QED

- **What is Cavity QED?**
- Quantum LC Oscillators
- Josephson Junctions & Transmon Qubits
- Qubits coupled to microwave cavities

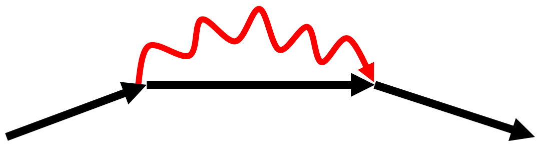
QED: Atoms Coupled to Photons

Zero-Point Fluctuations of the Vacuum Affect Atomic Spectra

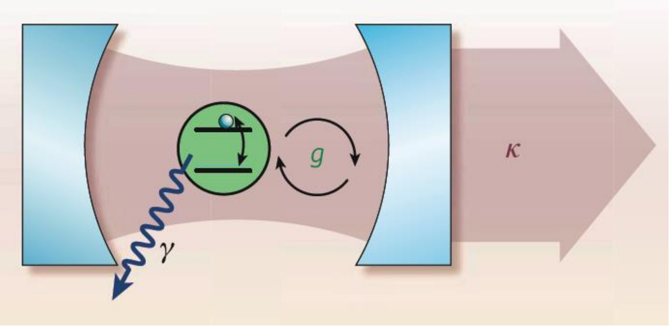


Irreversible spontaneous decay into the photon continuum:

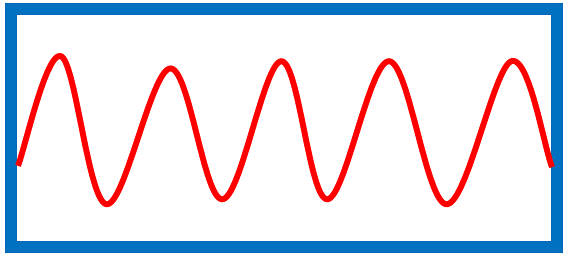
$$2p \rightarrow 1s + \gamma \qquad T_1 \sim 1 \text{ ns}$$



Vacuum Fluctuations: electron mass renormalization;
Virtual photon emission and reabsorption,
Lamb shift lifts 2s-2p degeneracy



Optical cQED



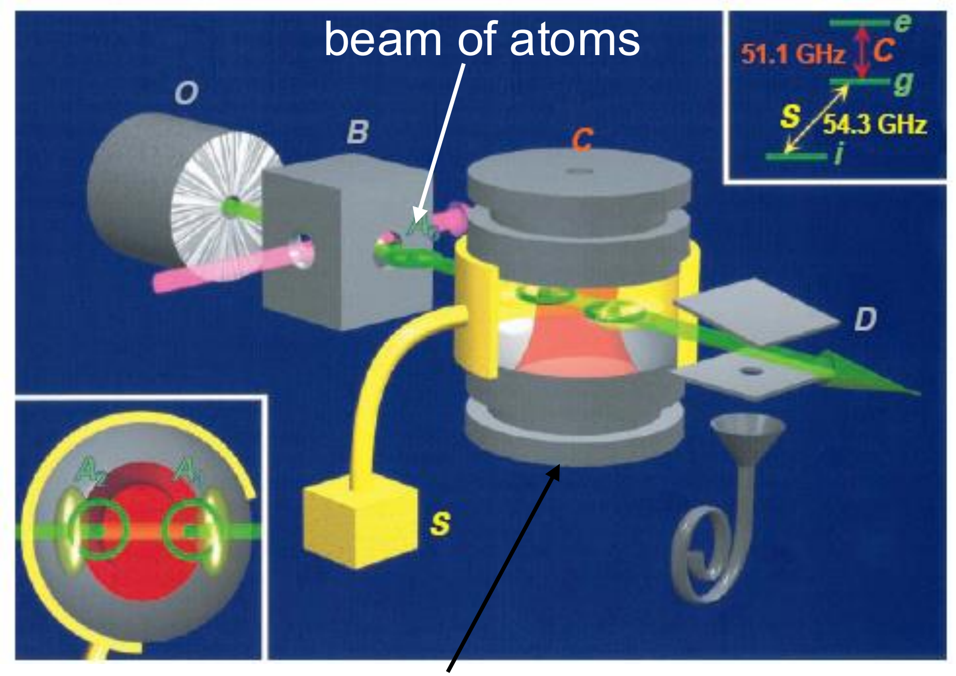
μwave cQED

Cavity QED: What happens if we trap the photons in engineered discrete modes inside a cavity?

$$T_1^{cQED} \rightarrow 10^3 T_1$$

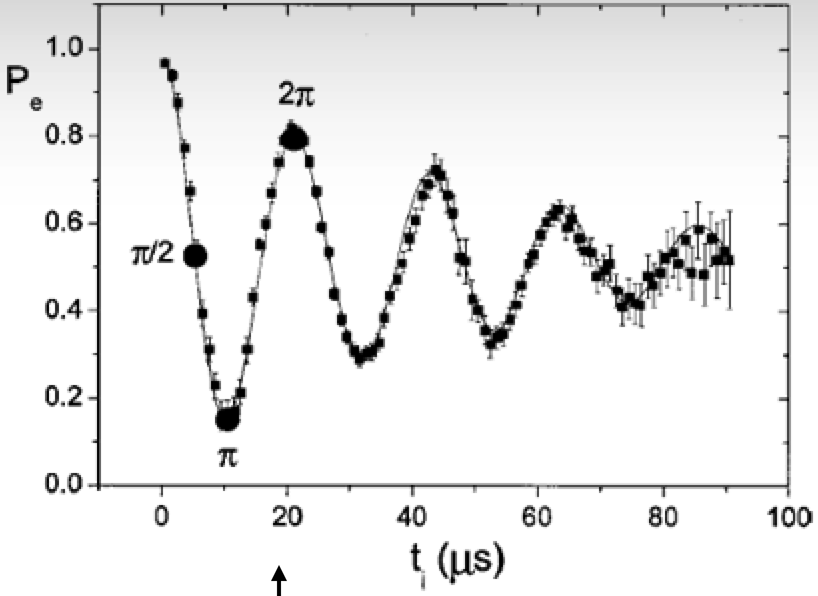
If cavity has no mode at atom's frequency.

μwave cQED with Rydberg Atoms



3-d superconducting cavity
(50 GHz)

vacuum Rabi oscillations

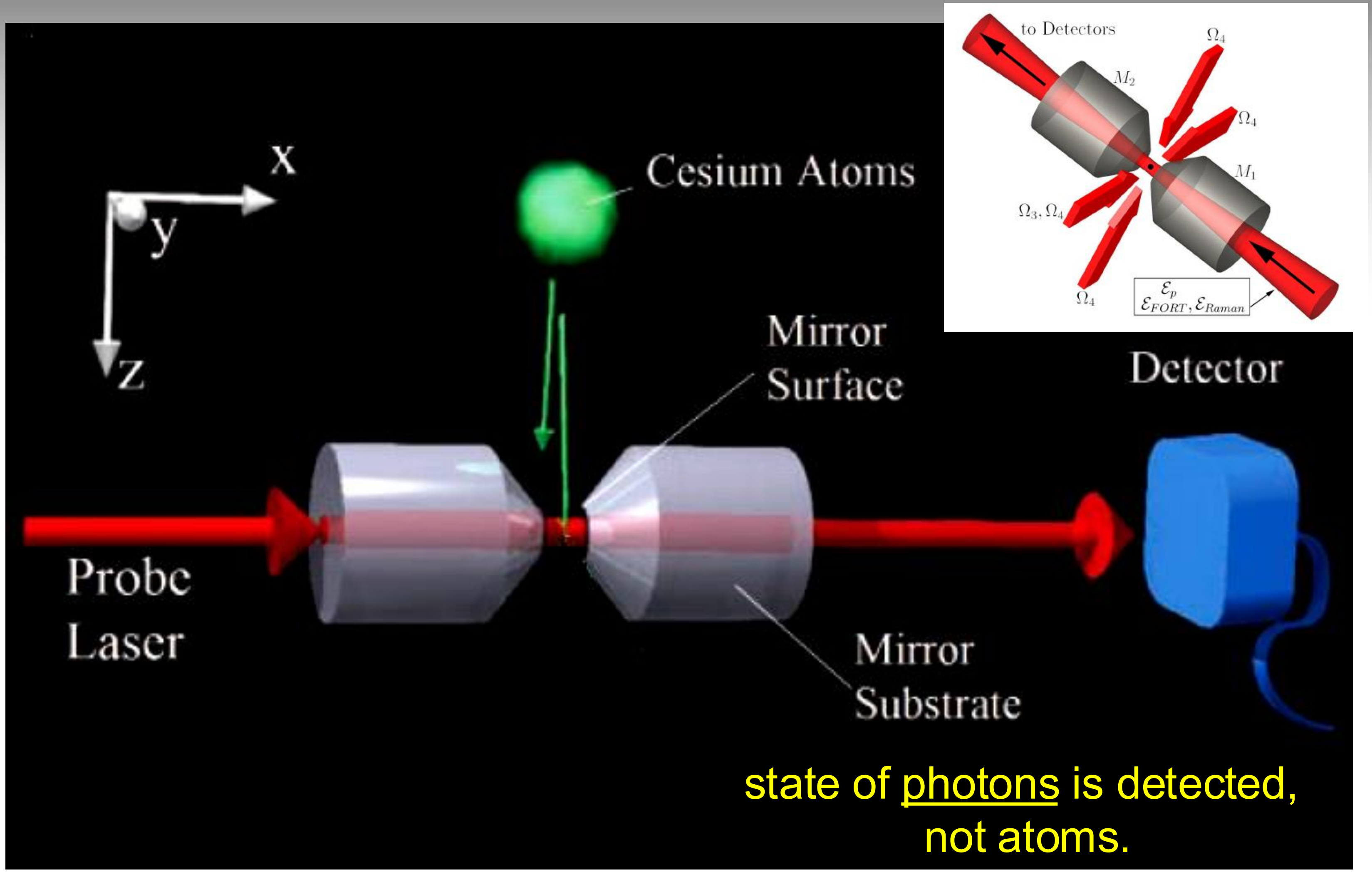


observe dependence of atom final state on time spent in cavity

measure atomic state, or …

Review: S. Haroche Nobel Lecture, Rev. Mod. Phys. 85, 1083 (2013)

cQED at optical frequencies



state of photons is detected,
not atoms.

… measure changes in transmission of optical cavity

(H. J. Kimble, H. Mabuchi)

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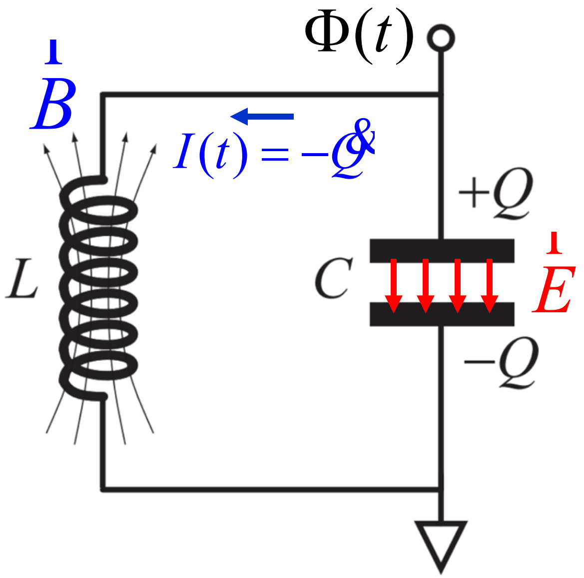
How to be a quantum electrical engineer: Quantization of the LC Oscillator

Lumped element LC



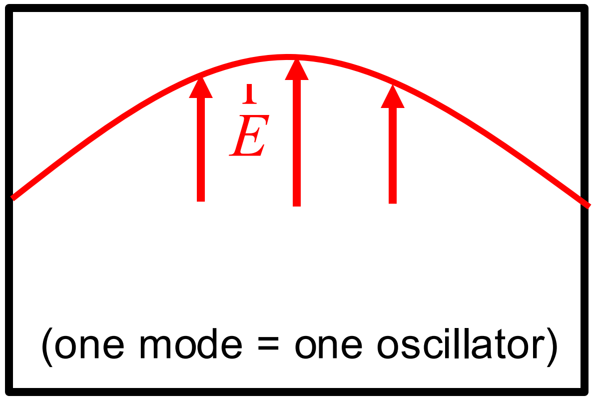
Define generalized flux

$$\Phi(t) = \int^{t} d\tau V(\tau)$$

$$\dot{\Phi}(t) = V(t)$$

(Faraday induction up to a minus sign)

3D cavity



Kinetic energy (aka electrostatic potential energy)

$$K = \frac{1}{2}CV^2 = \frac{1}{2}C\dot{\Phi}^2$$

Faraday: $V = L\dot{I} = \dot{\Phi} \Rightarrow \Phi = LI$

Potential energy (aka magnetic energy)

$$U = \frac{1}{2}LI^2 = \frac{1}{2L}(LI)^2 = \frac{1}{2L}\Phi^2$$

$$\mathcal{L} = \frac{1}{2}C\dot{\Phi}^2 - \frac{1}{2L}\Phi^2$$

Conjugate momentum:

$$Q = \frac{\delta \mathcal{L}}{\delta \dot{\Phi}} = C\dot{\Phi} = CV$$

$$H = Q\dot{\Phi} - \mathcal{L} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

Hamilton Eqns of Motion

$$\dot{\Phi} = \frac{\partial H}{\partial Q} = \frac{Q}{C} \quad \text{(Faraday)}$$

$$\dot{Q} = -\frac{\partial H}{\partial \Phi} = -\frac{\Phi}{L} = -I \quad \text{(charge conservation)}$$

$$\ddot{\Phi} = -\frac{1}{LC}\Phi \quad \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

How to be a quantum electrical engineer: Quantization of the LC Oscillator

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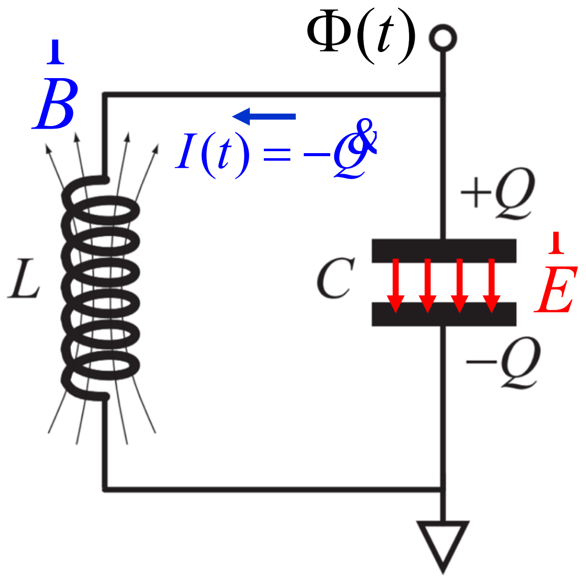

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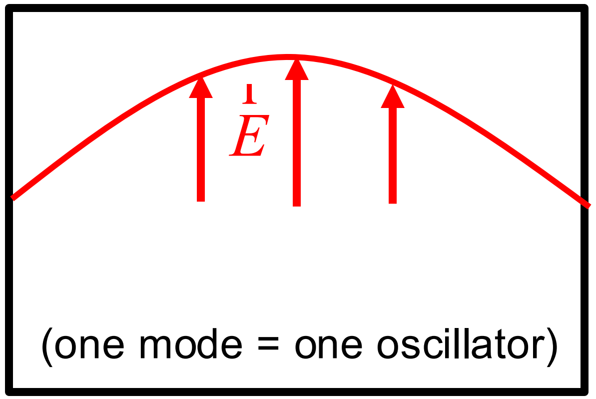

$$I = I_0 \sin\left(\omega_R t + \theta\right)$$

$$V = I_0 Z_R \cos\left(\omega_R t + \theta\right)$$

$Z_R \equiv$ characteristic impedance

(nothing to do with dissipation: I, V 90 degrees out of phase)

$$I = -\dot{Q} = -C\dot{V} = +I_0 \underbrace{\omega_R C Z_R}_{=1} \sin\left(\omega_R t + \theta\right)$$

$$\Rightarrow Z_R = \sqrt{\frac{L}{C}}$$

Hamilton Eqns of Motion

$$\dot{\Phi} = \frac{\partial H}{\partial Q} = \frac{Q}{C} \quad \text{(Faraday)}$$

$$\dot{Q} = -\frac{\partial H}{\partial \Phi} = -\frac{\Phi}{L} = -I \quad \text{(charge conservation)}$$

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How to be a quantum electrical engineer: Quantization of the LC Oscillator

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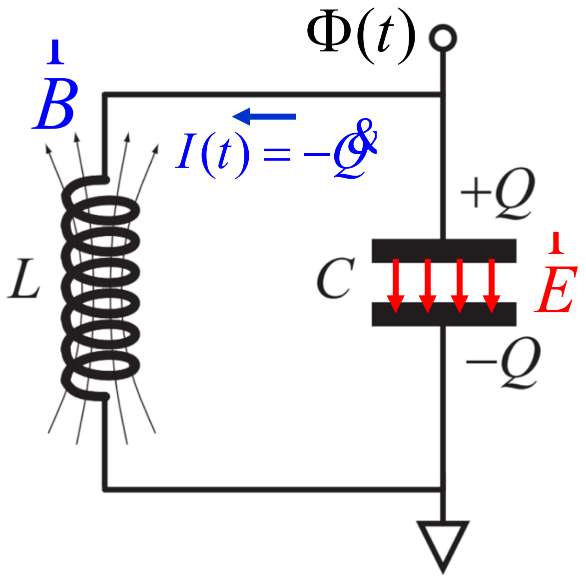
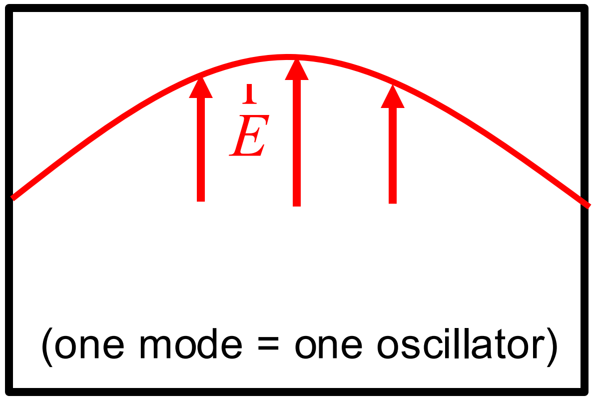


3D cavity



$$Z_R = \sqrt{\frac{L}{C}} \sim 50 - 500\,\Omega$$

because impedance of free space is

$$Z_0 \equiv \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 376.730\,313\,412\,(59)\,\Omega$$

Quantum of impedance (Klitzing constant)

$$Z_K \equiv \frac{h}{e^2} \equiv 25{,}812.807\,\Omega \text{ (exact)}$$

Fine structure constant:

$$\alpha \equiv \frac{e^2}{\hbar c}\underbrace{\left[\frac{1}{4\pi\epsilon_0}\right]}_{\text{SI units}} = \frac{e^2}{2h}\frac{\sqrt{\epsilon_0\mu_0}}{\epsilon_0} = \frac{Z_0}{2Z_K} \approx \frac{1}{137.035\,999\,177\,(21)}$$

Quantizing the electromagnetic oscillator

$$H = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

$$\left[\hat{Q}, \hat{\Phi}\right] = -i\hbar$$

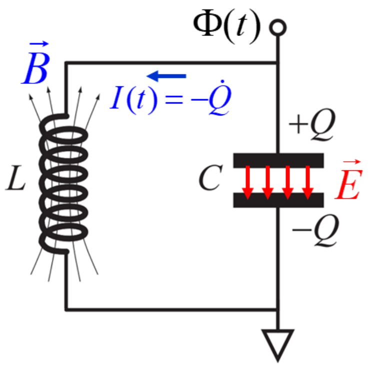
$$\hat{\Phi} = \Phi_{\text{ZPF}}\left(a + a^\dagger\right)$$

$$\hat{Q} = -iQ_{\text{ZPF}}\left(a - a^\dagger\right)$$

photon creation and destruction operators

$$\left[a, a^\dagger\right] = 1 \Rightarrow Q_{\text{ZPF}}\Phi_{\text{ZPF}} = \frac{\hbar}{2}$$

Lumped element LC



vacuum fluctuations

$$a|0\rangle = 0 \quad \Rightarrow \quad \langle 0|\hat{Q}^2|0\rangle = Q_{\text{ZPF}}^2 \langle 0|(a + a^\dagger)(a + a^\dagger)|0\rangle = Q_{\text{ZPF}}^2$$

zero-point energy:

$$\langle 0|H|0\rangle = \frac{\langle 0|\hat{Q}^2|0\rangle}{2C} + \frac{\langle 0|\hat{\Phi}^2|0\rangle}{2C} = \frac{1}{2}\hbar\omega = \frac{h}{4\pi}\frac{1}{\sqrt{LC}}$$

$$\frac{Q_{\text{ZPF}}^2}{2C} = \frac{1}{2}\left(\frac{1}{2}\hbar\omega\right) \Rightarrow Q_{\text{ZPF}} = \sqrt{\frac{\hbar}{2Z_{\text{R}}}} = e\sqrt{\frac{Z_{\text{K}}}{4\pi Z_{\text{R}}}}$$

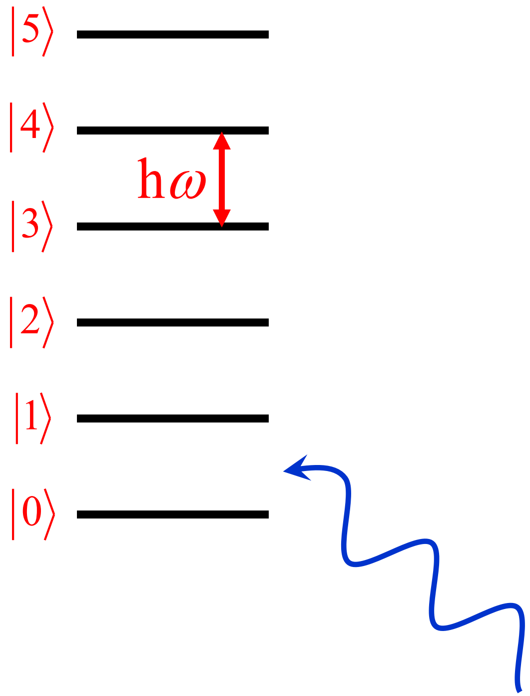
$$Q_{\text{ZPF}} = e\sqrt{\frac{Z_{\text{K}}}{4\pi Z_0}\left(\frac{Z_0}{Z_{\text{R}}}\right)} = e\sqrt{\frac{1}{8\pi\alpha}\left(\frac{Z_0}{Z_{\text{R}}}\right)} \approx 2.3e\sqrt{\left(\frac{Z_0}{Z_{\text{R}}}\right)} \approx 2-6 \text{ electrons}$$

$$Z_{\text{K}} = \frac{h}{e^2} \approx 25\ \text{k}\Omega \qquad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\Omega \qquad Z_{\text{R}} = \sqrt{\frac{L}{C}} \approx 50-500\Omega$$

Quantum Harmonic Oscillators have many important uses but:

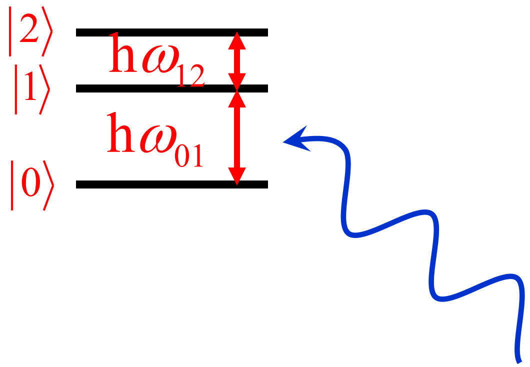
Their level spacing is uniform making them impossible to achieve full *quantum* control with *classical* signals.

$$H = \hbar\omega\, a^\dagger a$$



We need anharmonicity to make *qubits* and *auxiliary controllers* for oscillators:

$$H = \hbar\left[\omega\, a^\dagger a - \frac{K}{2} a^\dagger a^\dagger a a\right]$$



$$\omega_{12} - \omega_{01} = K$$

Quantum control paradox:

Microwave resonators
- can have very long lifetimes (1ms – 1 s) compared to qubits
- contain a large Hilbert space in a simple empty box
- can replace multiple qubits

But:
- require ancilla non-linear element (e.g. a qubit) to provide universal control

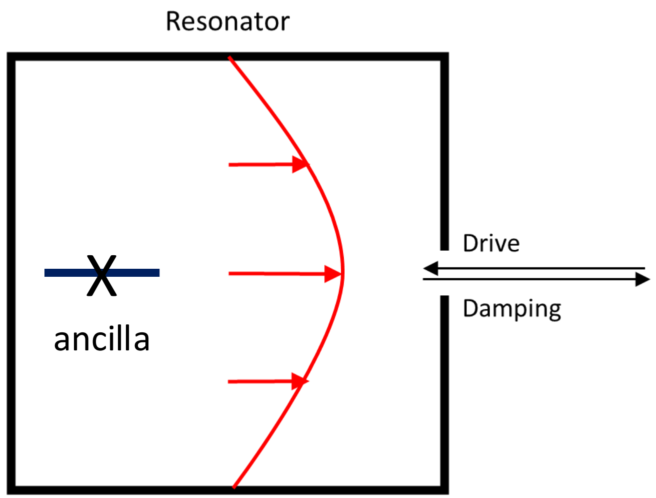
Recent theory papers:

'Hybrid Oscillator-Qubit Quantum Processors: Instruction Set Architectures, Abstract Machine Models, and Applications,' Y. Liu et al., arXiv:2407.10381

'Quantum control of bosonic modes with superconducting circuits,' Wen-Long Ma et al., *Science Bulletin* **66**, 1789 (2021)

'Photon-Number-Dependent Hamiltonian Engineering for Cavities,' Chiao-Hsuan Wang et al. *Phys. Rev. Applied* **15**, 044026 (2021)

'Constructing Qudits from Infinite Dimensional Oscillators by Coupling to Qubits,' Yuan Liu et al., *Phys. Rev. A* **104**, 032605 (2021)



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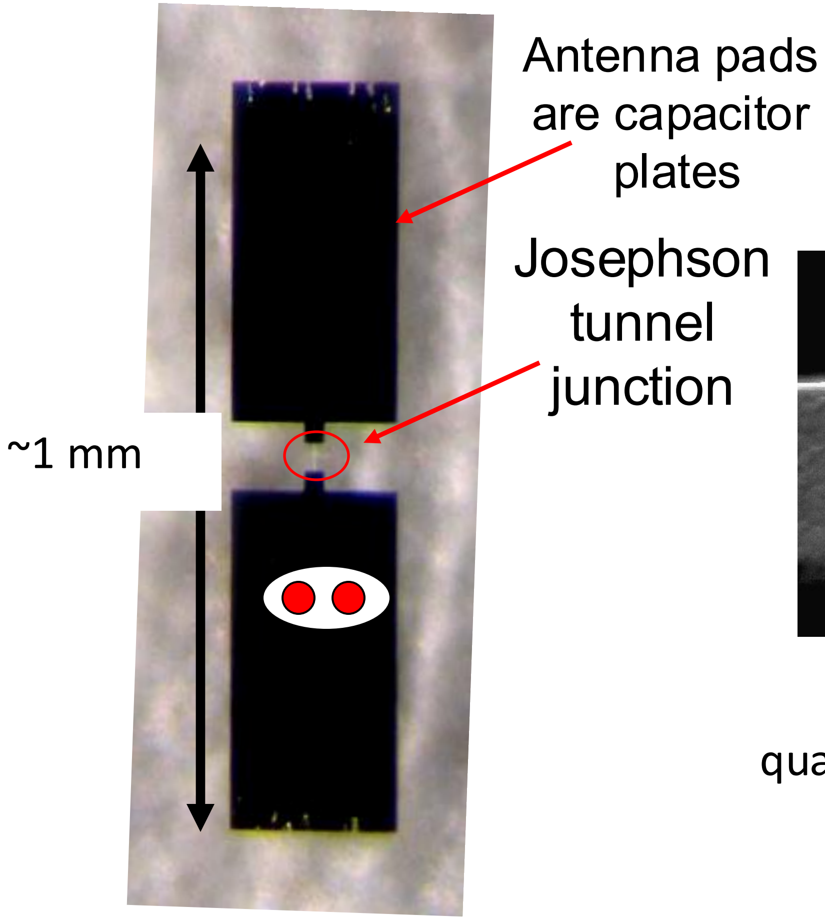
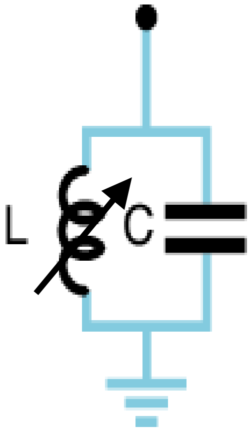
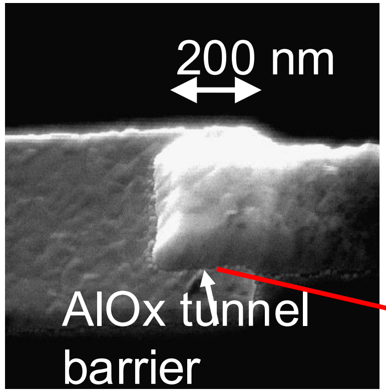
Joseph tunnel junctions act as non-linear inductors to produce anharmonic oscillators and qubits

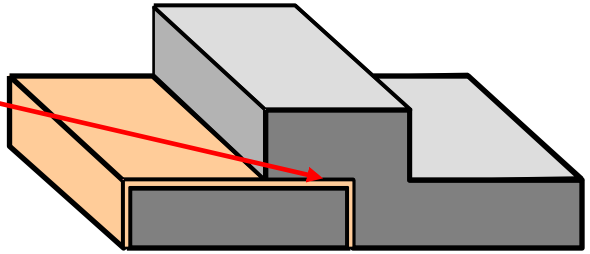
'Transmon' Qubit

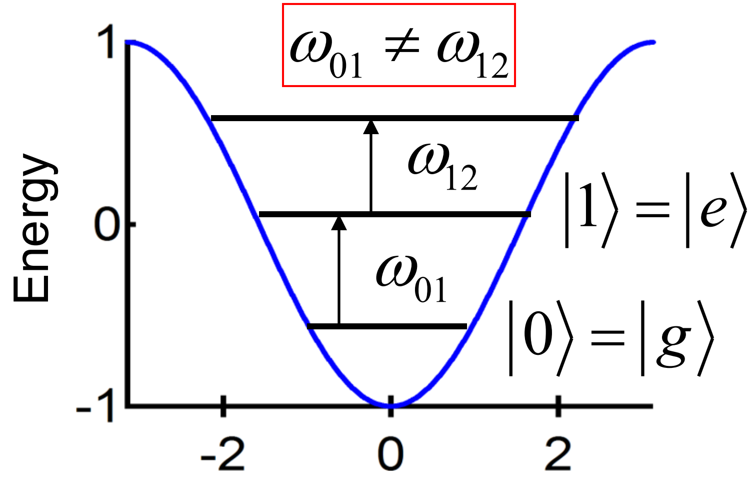






'transistor of quantum computing'





$$\omega_{01} \sim 5-10\ \mathbf{GHz}$$

‘Circuit QED:’

-microwave photons inside superconducting circuits

-artificial atoms (Josephson junction qubits)

Ultra-strong photon-‘atom’ coupling:

-non-linear quantum optics at the single photon level

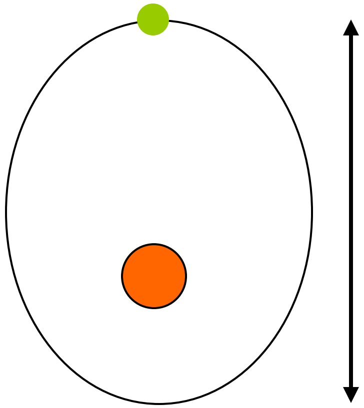
Hydrogen atom

$f_{1S-2P} \approx 2.46\times10^{15}\,\mathrm{Hz}$

$\tau_{2P} \approx 1.6\mathrm{ns}$

$\mathrm{Q}/2\pi \approx 4\times10^{6}$

$\mathrm{dipole} \sim 1\ \mathrm{Debye}$



$10^{-10}\,m$

(Not to scale!)

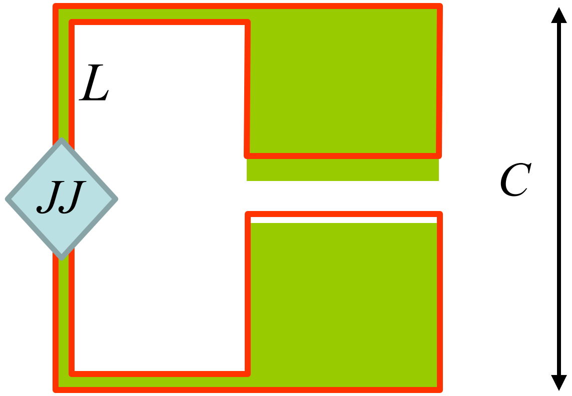
Superconducting oscillator/qubit

$f_{01} \approx 7\times10^{9}\,\mathrm{Hz}$

$\tau_{2P} \approx 300\,\mu\mathrm{s}$

$\mathrm{Q}/2\pi \approx 2\times10^{6}$

$\mathrm{dipole} \sim 3\times10^{7}\ \mathrm{Debye}$

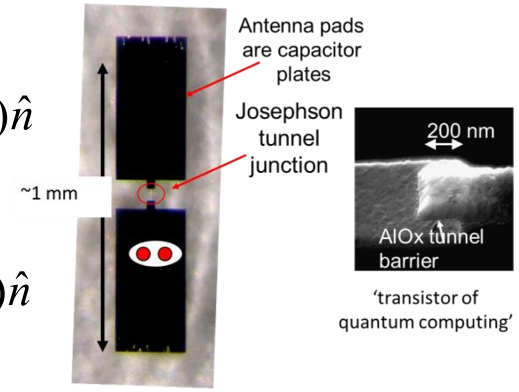


$10^{-3}\,m$

'Transmon' Qubit

$$\hat{Q} = +(2e)\hat{n}$$

$$\hat{Q} = -(2e)\hat{n}$$

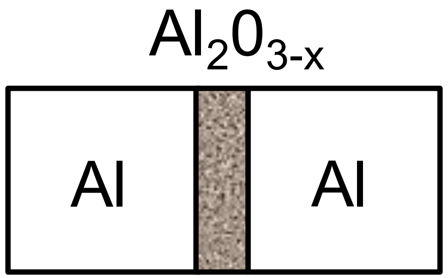


Transmon qubit is a synthetic atom with 'atomic number' $\sim 10^{13}$

But the spectrum is simple.
Superconductivity gaps out single-particle excitations.
Coulomb interaction gaps out plasma waves in the antenna pads.

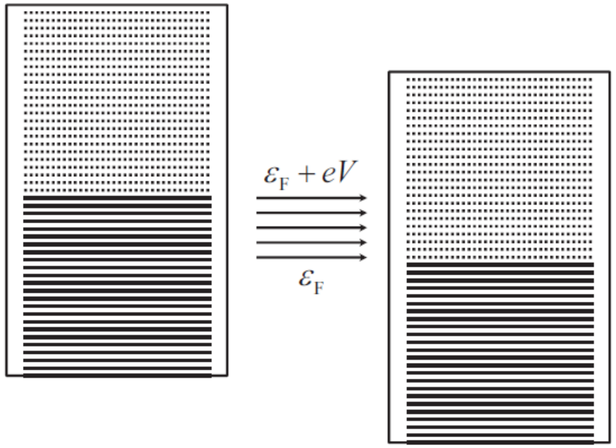
States in the low-energy Hilbert space are specified simply by the integer number *n* of Cooper pairs that have tunneled through the junction.

Josephson Tunnel Junctions

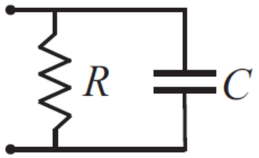


Normal tunnel junction

a)

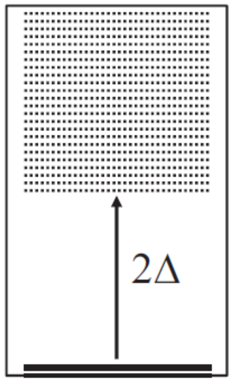


b)



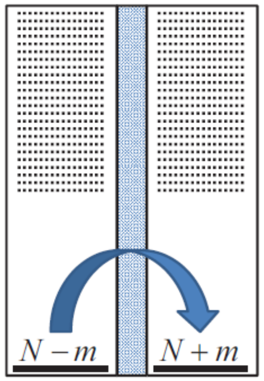
Superconducting tunnel junction

a)

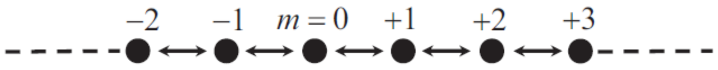


Unique ground state for N pairs on an island

b)



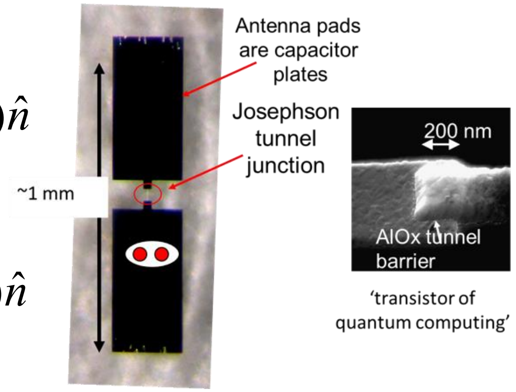
c)



Total number of Cooper pairs that have tunneled uniquely determines the low-energy quantum state of a pair of islands.

‘Transmon’ Qubit

$\hat{Q} = +(2e)\hat{n}$



$\hat{Q} = -(2e)\hat{n}$

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But the spectrum is simple.
Superconductivity gaps out single-particle excitations.
Coulomb interaction gaps out plasma waves in the antenna pads.

States in the low-energy Hilbert space are specified simply by the integer number *n* of Cooper pairs that have tunneled through the junction.

Coulomb charging energy

$$U = \frac{\hat{Q}^2}{2C} = 4E_C\hat{n}^2$$

$$E_C \equiv \frac{e^2}{C}$$

square of a +/- integer

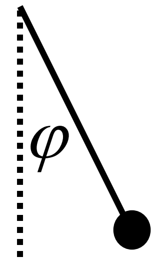
Notice that the (single axis) quantum rotor has integer angular momentum

$\hat{L} = \hbar\hat{n}$ and kinetic energy

$$T = \frac{\hat{L}^2}{2I_0} = 4E_C\hat{n}^2$$

(if we relate the moment of inertia to the capacitance).

Quantum Rotor



Charge (aka angular momentum) eigenstates:

$$\psi_m(\varphi) = e^{im\varphi} = \langle\varphi|m\rangle$$

$$\hat{n} = -i\frac{d}{d\varphi}$$

$$\hat{n}|m\rangle = m|m\rangle$$

φ represents the condensate phase difference across the JJ

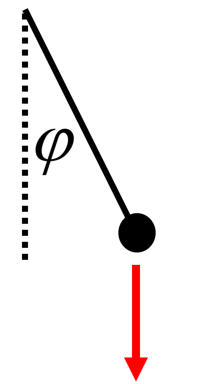
Representing Josephson Tunneling in the Rotor Model

Notice that the operator $T_\pm = e^{\pm i\varphi}$ changes the rotor angular momentum by one unit

$T_\pm |m\rangle = |m \pm 1\rangle$ which changes the tunneled charge $Q = (2e)\ m$ by one Cooper pair

$$T_\pm = \sum_{n=-\infty}^{+\infty} |n \pm 1\rangle\langle n|$$

Quantum Rotor



We can represent the Josephson tunneling (in both directions) by

$$T = -\frac{E_J}{2}\left(T_+ + T_-\right) = -E_J \cos\varphi$$

where E_J is the Josephson energy.

Represents a torque due to gravity

that alters the angular momentum!

Strong gravity is:

E_J ? E_C

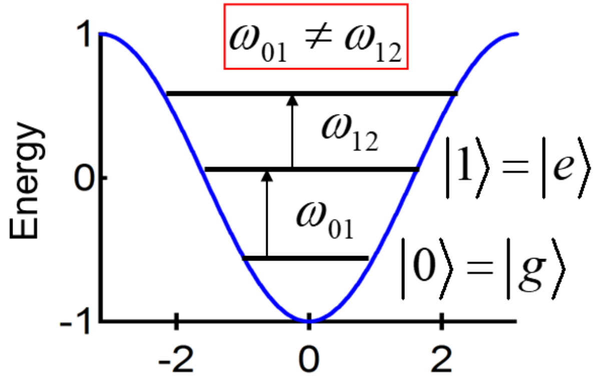
Cooper Pair Box: $E_J \leq E_C$

Transmon: $E_J \sim 10^2 E_C$

Transmon Hamiltonian

$$H = 4E_C\hat{n}^2 - E_J \cos\varphi$$

$$\hat{n} = -i\frac{d}{d\varphi}$$



$\omega_{01} \sim 5-10$ **GHz**

$$\psi_m(\varphi) = e^{im\varphi} = \langle\varphi|m\rangle$$

$$\hat{n} = -i\frac{d}{d\varphi}$$

$$\hat{n}|m\rangle = m|m\rangle$$

Transmon Hamiltonian

$$H = 4E_C\hat{n}^2 - E_J\cos\varphi$$

$$\dot{\varphi} = \frac{\partial H}{\hbar\partial\hat{n}}$$

$$\hbar\dot{\varphi} = 8E_C\hat{n} = 8\frac{e^2}{2C}\hat{n} = (2e)\frac{(2e)\hat{n}}{C}$$

Voltage Josephson Relation:

$$\hbar\dot{\varphi} = (2e)V \quad \Rightarrow \quad \dot{\varphi} = \frac{2e}{\hbar}\dot{\Phi}$$

$$\varphi = 2\pi\frac{\Phi}{\Phi_0}$$

← Flux

← Flux quantum

$$\Phi_0 \equiv \frac{h}{2e}$$

Transmon as a non-linear LC oscillator

$$H = \frac{\hat{Q}^2}{2C} - E_J\cos\left(2\pi\frac{\Phi}{\Phi_0}\right)$$

$$\approx \frac{\hat{Q}^2}{2C} - E_J + \frac{\Phi^2}{2L_J} + \lambda\Phi^4 + \ldots$$

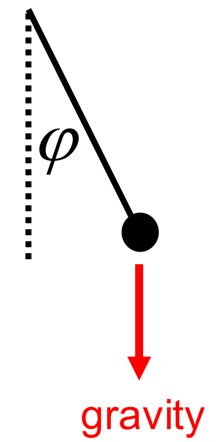
$$L_J \equiv \frac{1}{E_J}\left(\frac{\Phi_0}{2\pi}\right)^2$$

Current Josephson Relation:

$$I = \frac{\partial H}{\partial\Phi} = I_c\sin\left(2\pi\frac{\Phi}{\Phi_0}\right)$$

$$I_c = \left(\frac{2\pi}{\Phi_0}\right)E_J \quad \text{critical current}$$

Quantum Rotor



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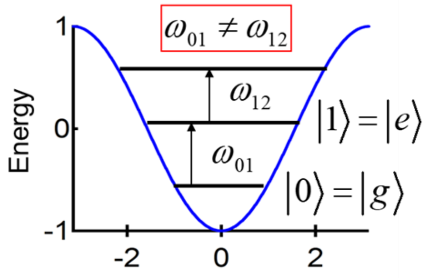
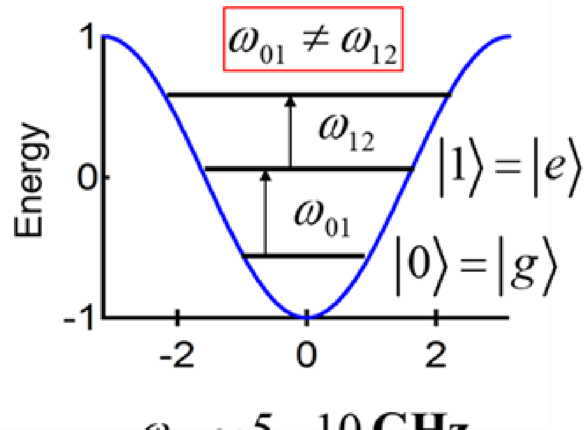
Transmon: $E_J \sim 10^2 E_C$



$\omega_{01} \sim 5-10$ **GHz**

Anharmonic transmon as a two-level qubit

$$H_0 = \frac{\omega_{01}}{2}\sigma^z$$



$\omega_{01} \sim 5-10$ **GHz**

Classical control tones at ω_{01} can rotate the qubit between $|0\rangle$ and $|1\rangle$ without exciting the higher lying levels:

$$V_{\text{drive}}(t) = 2\left[\Omega_x \cos(\omega_{01}t) + \Omega_y \sin(\omega_{01}t)\right]\sigma^x$$

In the interaction picture (frame rotating with the qubit)

where $H_0 \to 0, \quad V_{\text{drive}} \to \Omega_x\sigma^x + \Omega_y\sigma^y$,

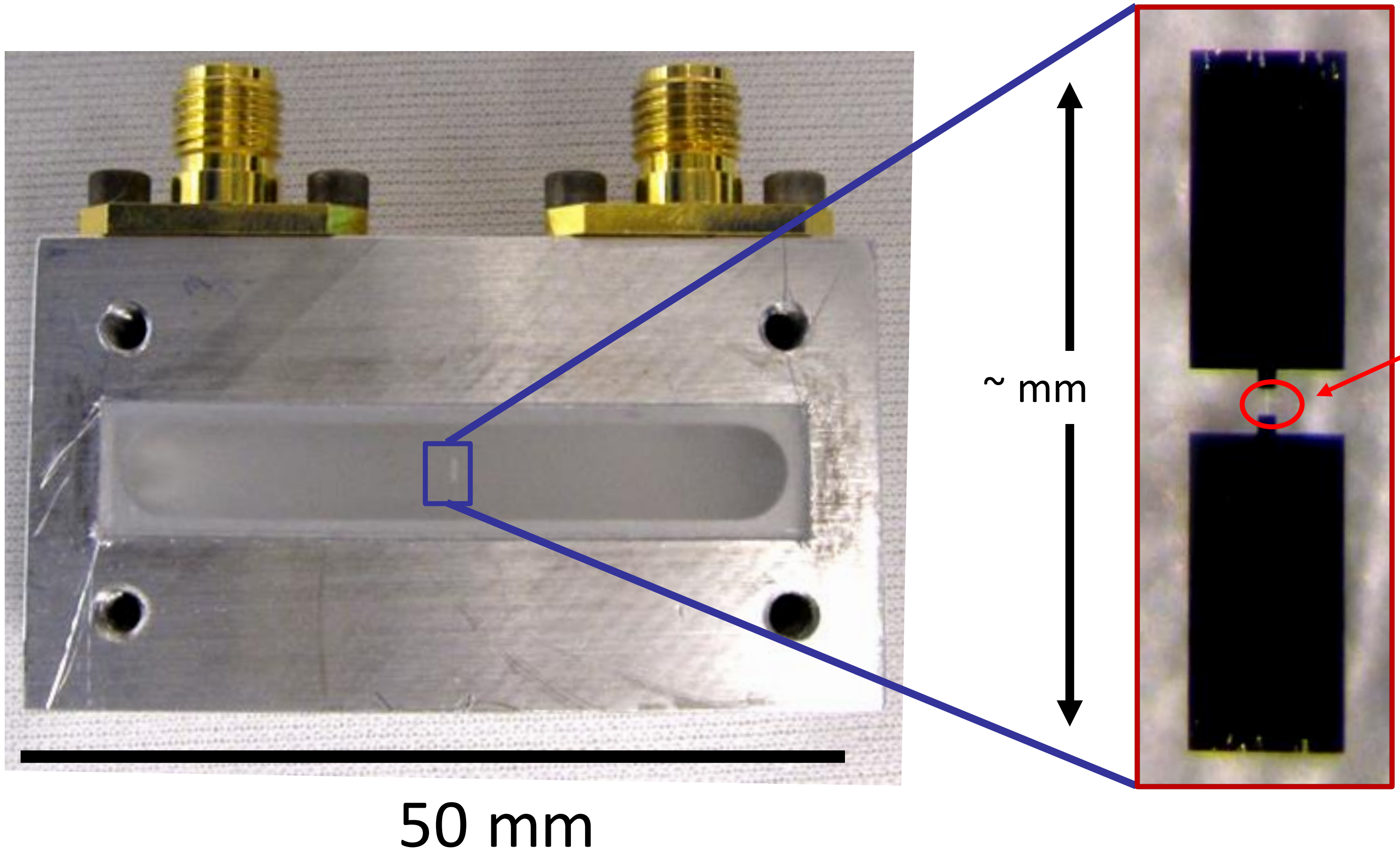
giving universal single-qubit control since $\{\sigma^x, \sigma^y\}$ generate SU(2).

OUTLINE:

Introduction to Circuit QED

- What is Cavity QED?
- Quantum LC Oscillators
- Josephson Junctions & Transmon Qubits
- **Qubits coupled to microwave resonators**

Transmon Qubit in 3D Cavity



$$\frac{Q_{\text{ZPF}}}{2e} \sim \frac{1}{\sqrt{16\pi\alpha}} \sim 1-3$$

bit flip

$$g = \frac{\vec{d} \bullet \vec{E}_{rms}}{h}$$

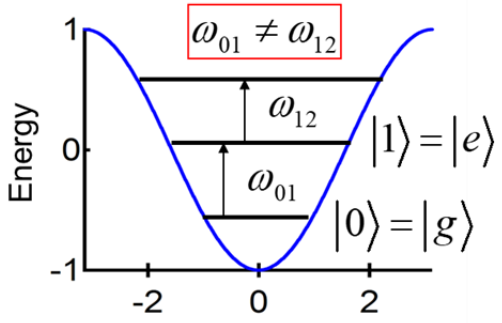
$$\left|\vec{d}\right| = 2e \times 1 \text{ mm} \approx 10^7 \text{Debye!!}$$

$$V_{\text{dipole}} = g\sigma^x (a + a^\dagger)$$

Huge dipole moment: strong coupling

$g \sim 100$ MHz

Use anharmonicity to neglect higher excited states and treat transmon as a two-level system



$\omega_{01} \sim 5-10$ **GHz**

Microwave resonator

Transmon qubit

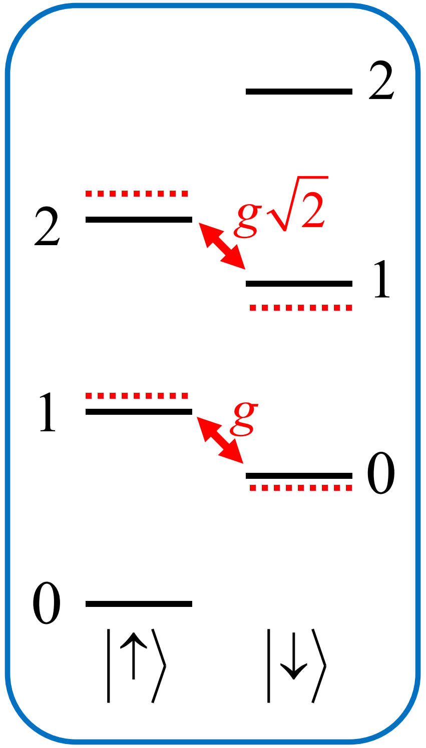
Dipole Coupling

$$H = \omega_r a^\dagger a + \frac{\omega_q}{2}\sigma^z + g\sigma^x[a + a^\dagger] + H_{\text{damping}} \qquad \text{[Rabi]}$$

$$H = \omega_r a^\dagger a + \frac{\omega_q}{2}\sigma^z + g[a\sigma^+ + a^\dagger\sigma^-] + H_{\text{damping}} \qquad \text{[Jaynes-Cummings]}$$

$$H = \omega_r a^\dagger a + \frac{\omega_q}{2}\sigma^z + \chi\sigma^z a^\dagger a + H_{\text{damping}} \qquad \text{[Dispersive]}$$

$$\chi \sim \frac{g^2}{\omega_r - \omega_q} \quad \text{(strong anharmonicity)} \qquad \chi \propto E_C \quad \text{(weak anharmonicity)}$$



'Dressed atom'

Strong Dispersive Hamiltonian

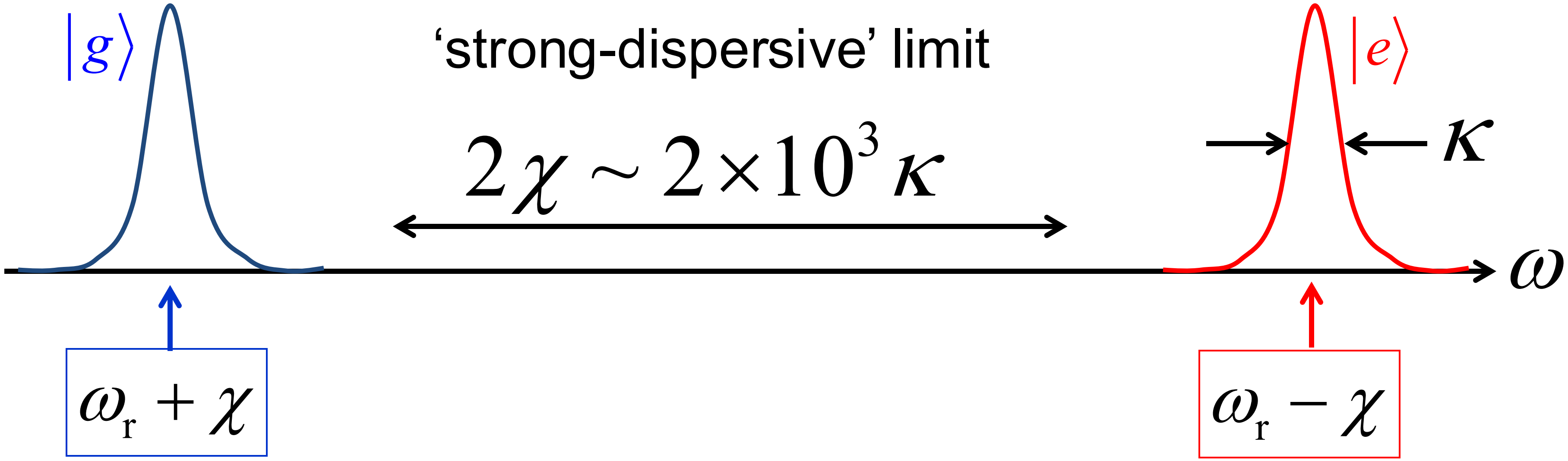
$$H = \omega_{\mathrm{r}} a^{\dagger} a + \frac{\omega_{\mathrm{q}}}{2} \sigma^{z} + \chi \sigma^{z} a^{\dagger} a + H_{\mathrm{damping}}$$

$$\chi >> \kappa, \Gamma$$

resonator qubit dispersive coupling

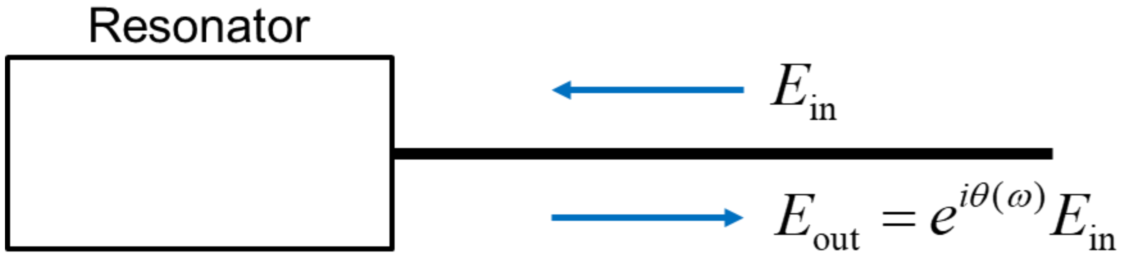
$$\text{cavity frequency} = \omega_{\mathrm{r}} + \chi \sigma^{z}$$

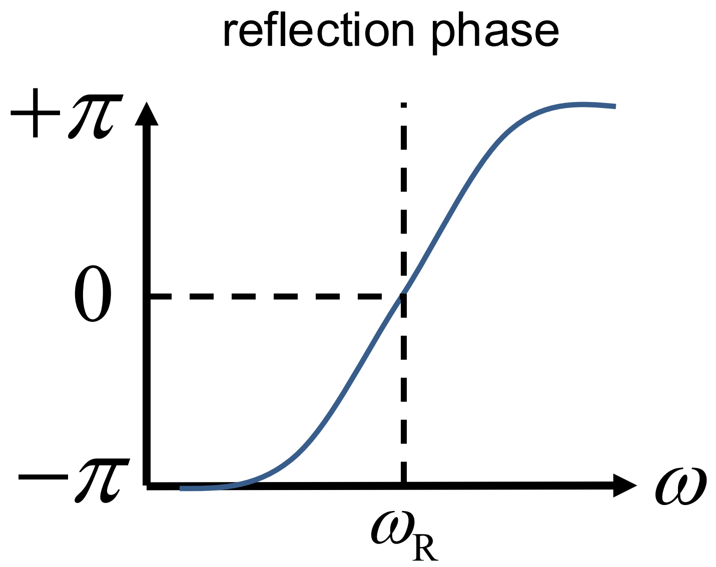
[Cavity frequency can be used to readout state of qubit]



Using (not so) strong dispersive coupling to measure the state of the qubit

The S matrix for reflection of microwaves from a resonator:

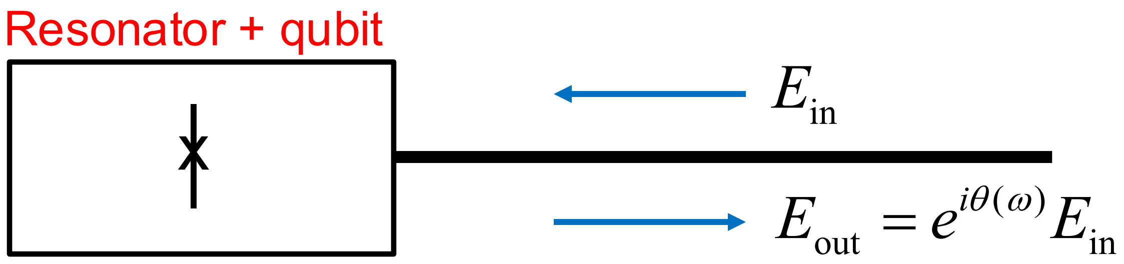




$$S(\omega) = e^{i\theta(\omega)} = -\frac{\omega - \omega_R - i\kappa/2}{\omega - \omega_R + i\kappa/2}$$

Probe frequency (ω), Resonator frequency (ω_R), Resonator damping (κ)

Recall that an oscillator driven on resonance responds internally with a phase shift of $\frac{\pi}{2}$. However, the externally reflected drive wave has a different phase shift.



$$S(\omega) = e^{i\theta(\omega)} = -\frac{\omega - \omega_{\text{R}} - i\kappa/2}{\omega - \omega_{\text{R}} + i\kappa/2}$$

Probe frequency — Resonator frequency — Resonator damping

When we include the dispersive coupling of the resonator to the qubit

$$\omega_{\text{R}} \to \omega_{\text{R}} + \chi\sigma^z$$

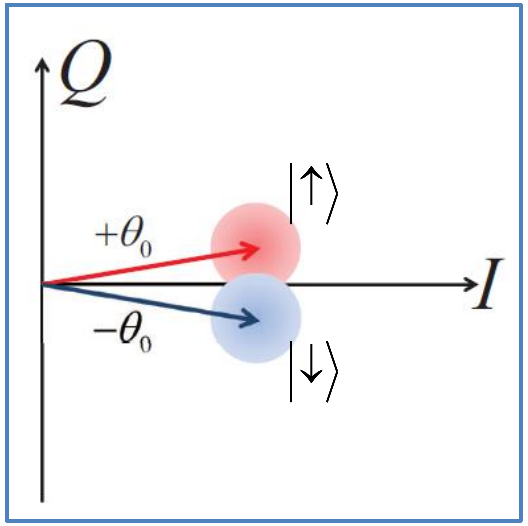
Choosing the drive frequency $\omega = \omega_{\text{R}}$ yields

$$S(\omega_{\text{R}}) = e^{i\theta(\omega_{\text{R}})} = -\frac{\chi\sigma^z - i\kappa/2}{\chi\sigma^z + i\kappa/2} = e^{i\theta_0\sigma^z}$$

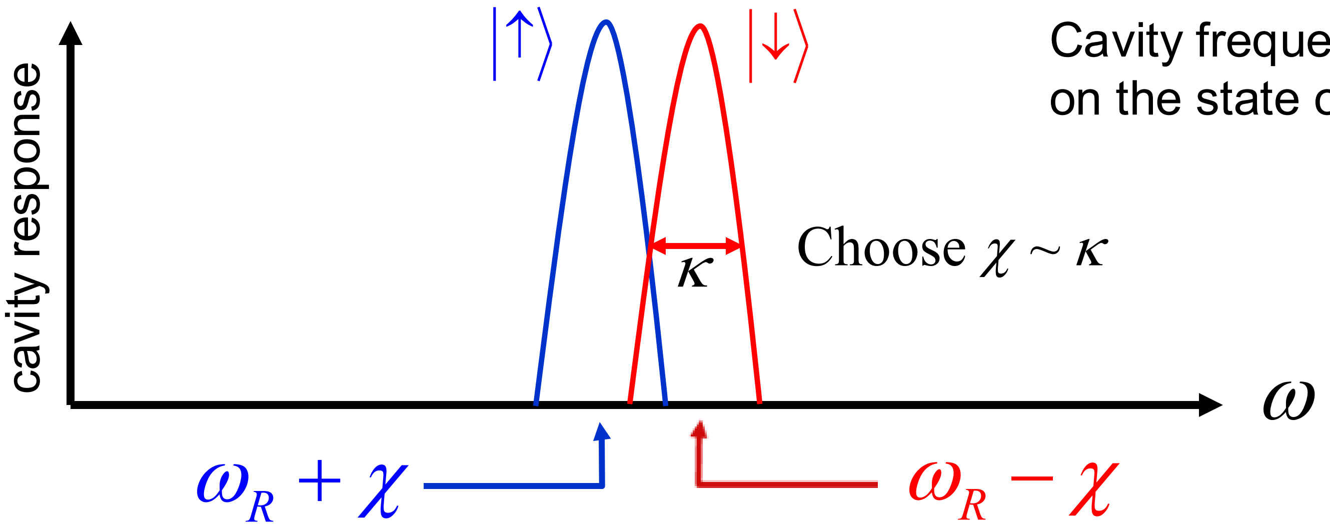
with $\tan\dfrac{\theta_0}{2} = \dfrac{\kappa}{2\chi}$.

Quadrature amplitudes:
I is oscillator position.
Q is oscillator momentum.

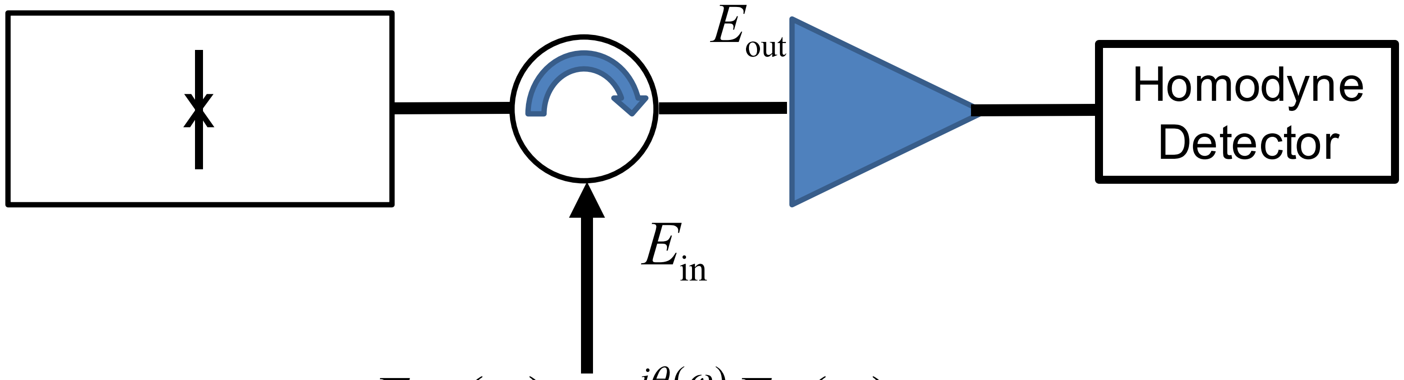
Fuzzy due to uncertainty principle.



Can read out qubit state by measuring cavity resonance frequency (which is shifted by the qubit)

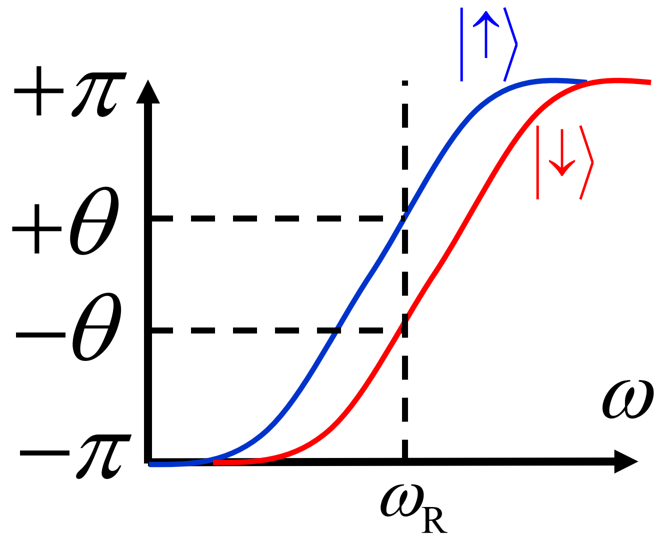


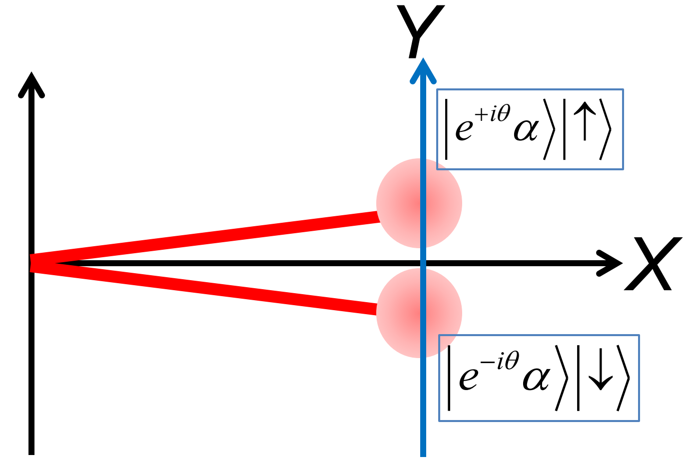
Cavity frequency depends on the state of the qubit



$$E_{\text{out}}(\omega) = e^{i\theta(\omega)} E_{\text{in}}(\omega)$$

Cavity reflection phase depends on the state of the qubit

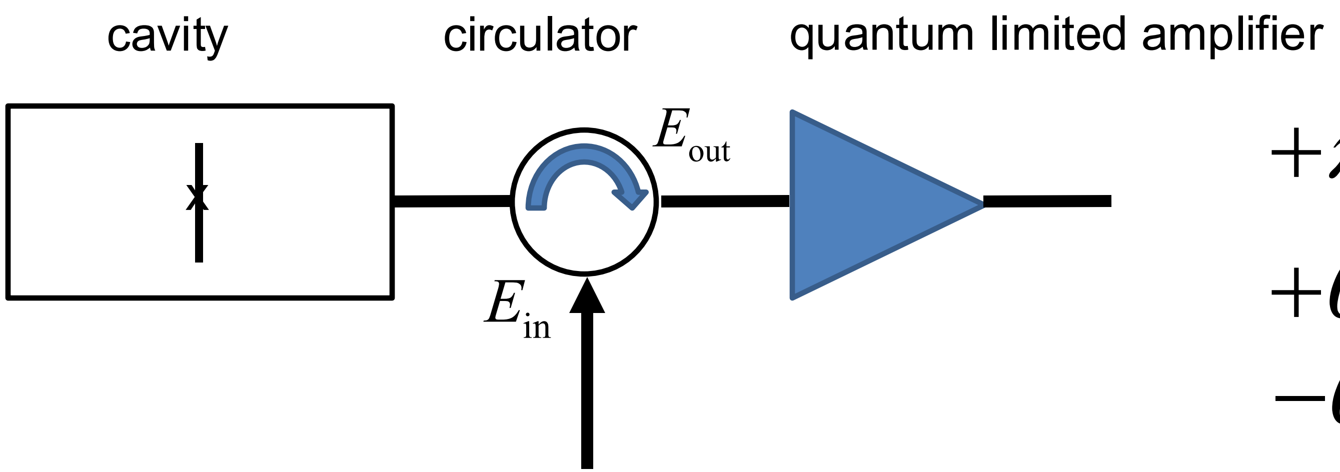




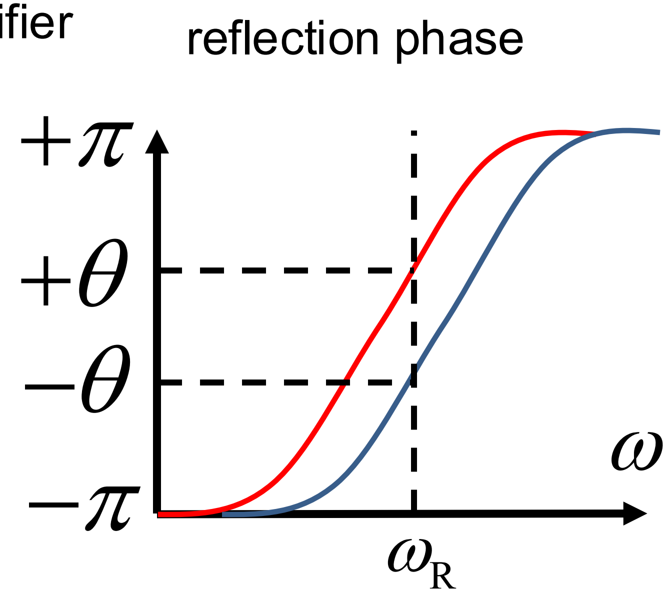
$$|\psi_{\text{in}}\rangle = \{a|\uparrow\rangle + b|\downarrow\rangle\}|\alpha\rangle$$

$$|\psi_{\text{out}}\rangle = a|e^{+i\theta}\alpha\rangle|\uparrow\rangle + b|e^{-i\theta}\alpha\rangle|\downarrow\rangle$$

State of qubit is entangled with the 'meter' (microwave phase)
Then 'meter' is read with amplifier.

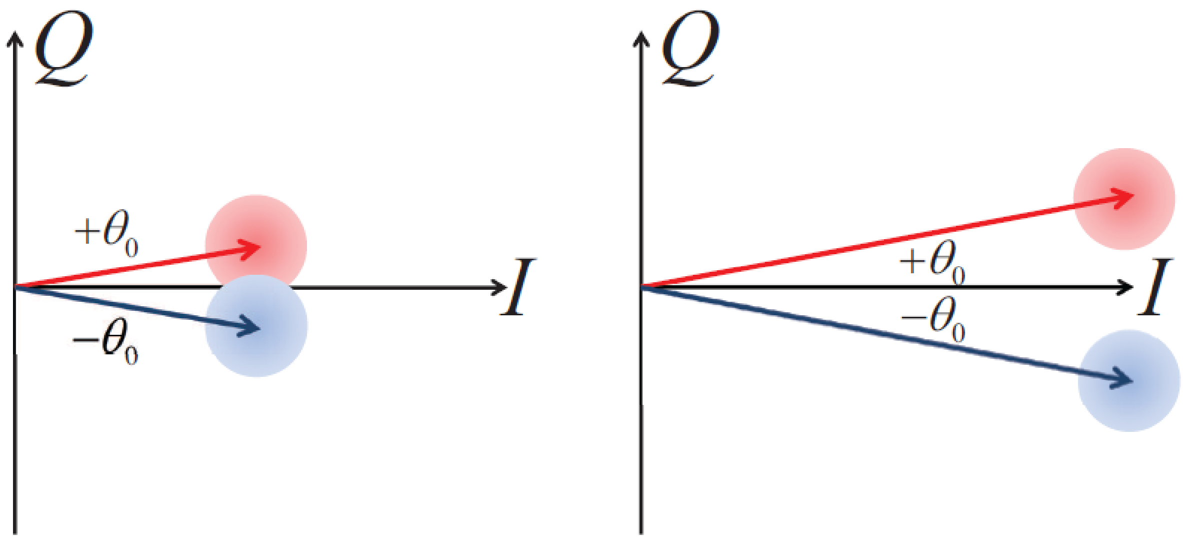


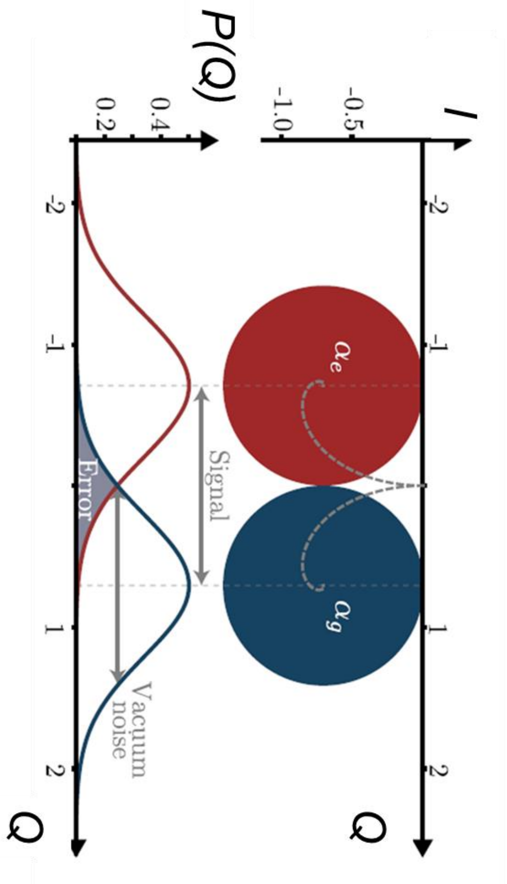
$$E_{\text{out}}(\omega) = e^{i\theta(\omega)} E_{\text{in}}(\omega)$$



Readout fidelity
vacuum noise → shot noise

Measure Q to readout qubit state

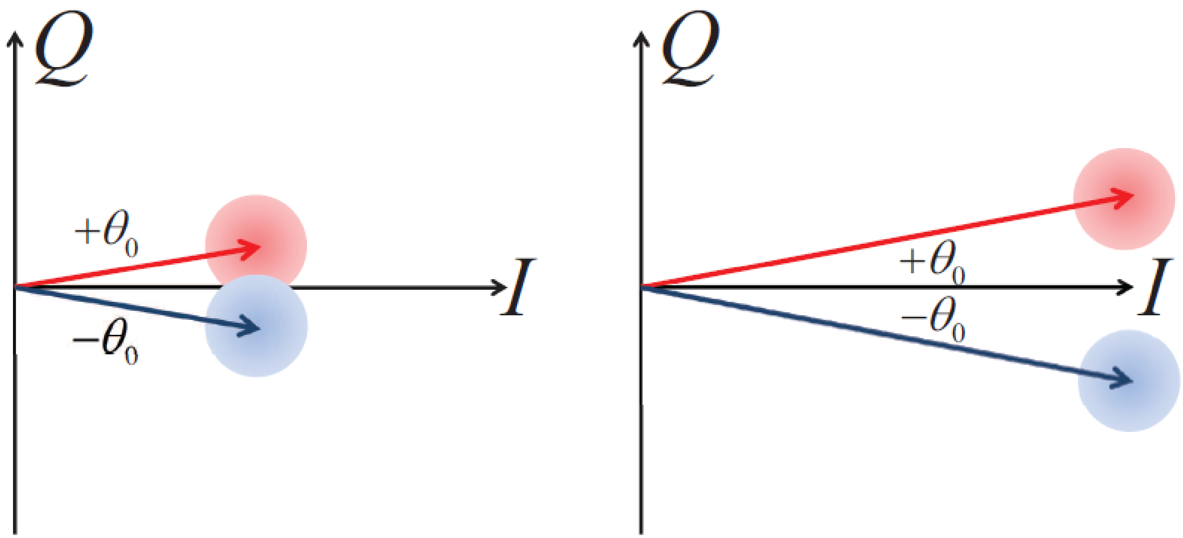




Quadrature amplitudes
I,*Q* are canonically
conjugate, leading to
quantum vacuum noise

Readout fidelity
vacuum noise → shot noise

Measure Q to readout qubit state



Quadrature amplitudes I, Q are canonically conjugate, leading to quantum vacuum noise

We need a readout drive with definite phase so we can see the reflection phase shift. But this means the incoming microwave pulse has indefinite photon number:

Number - Phase Uncertainty Principle

$$\Delta N\,\Delta\theta = \frac{1}{2}$$

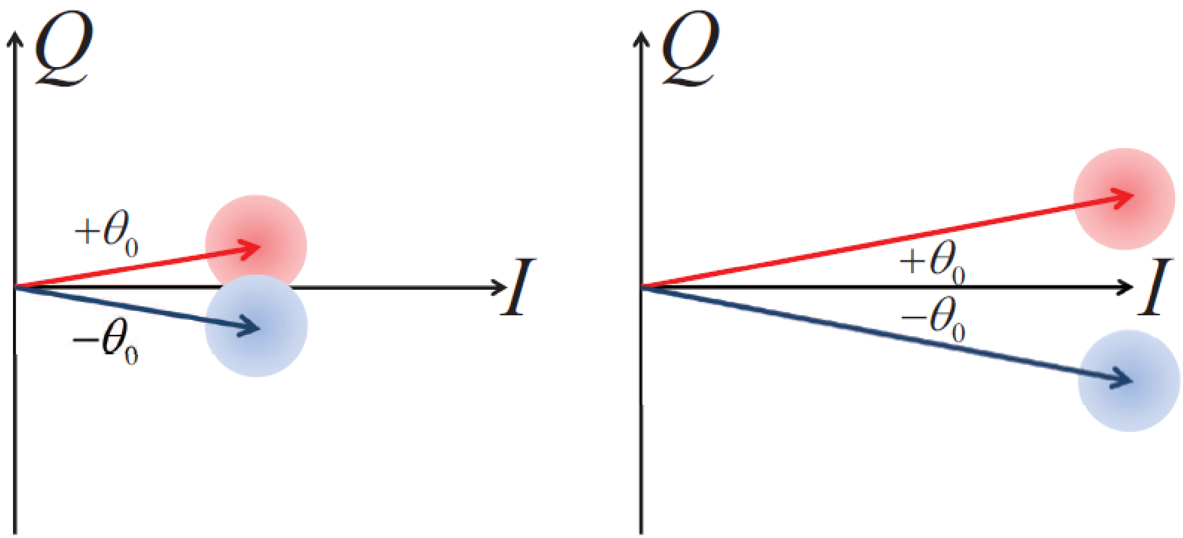
$$\Delta\theta < \theta_0 \Rightarrow \Delta N > \frac{1}{2\theta_0}$$

For coherent states $\Delta N = \sqrt{\bar{N}} \Rightarrow$

$$\bar{N} > \frac{1}{4\theta_0^2}$$

Readout fidelity
vacuum noise → shot noise

Measure Q to readout qubit state



Quadrature amplitudes I, Q are canonically conjugate, leading to quantum vacuum noise

Formal derviation (for small θ_0)

Coherent state:

$$\left|\alpha e^{i\theta_0}\right\rangle = e^{-\frac{\alpha^2}{2}} e^{\alpha e^{i\theta_0} a^\dagger} \left|0\right\rangle = e^{-\frac{\alpha^2}{2}} \sum_{n=0}^{\infty} \frac{\left(\alpha e^{i\theta_0}\right)^n}{\sqrt{n!}} \left|n\right\rangle$$

$$\bar{N} = \alpha^2$$

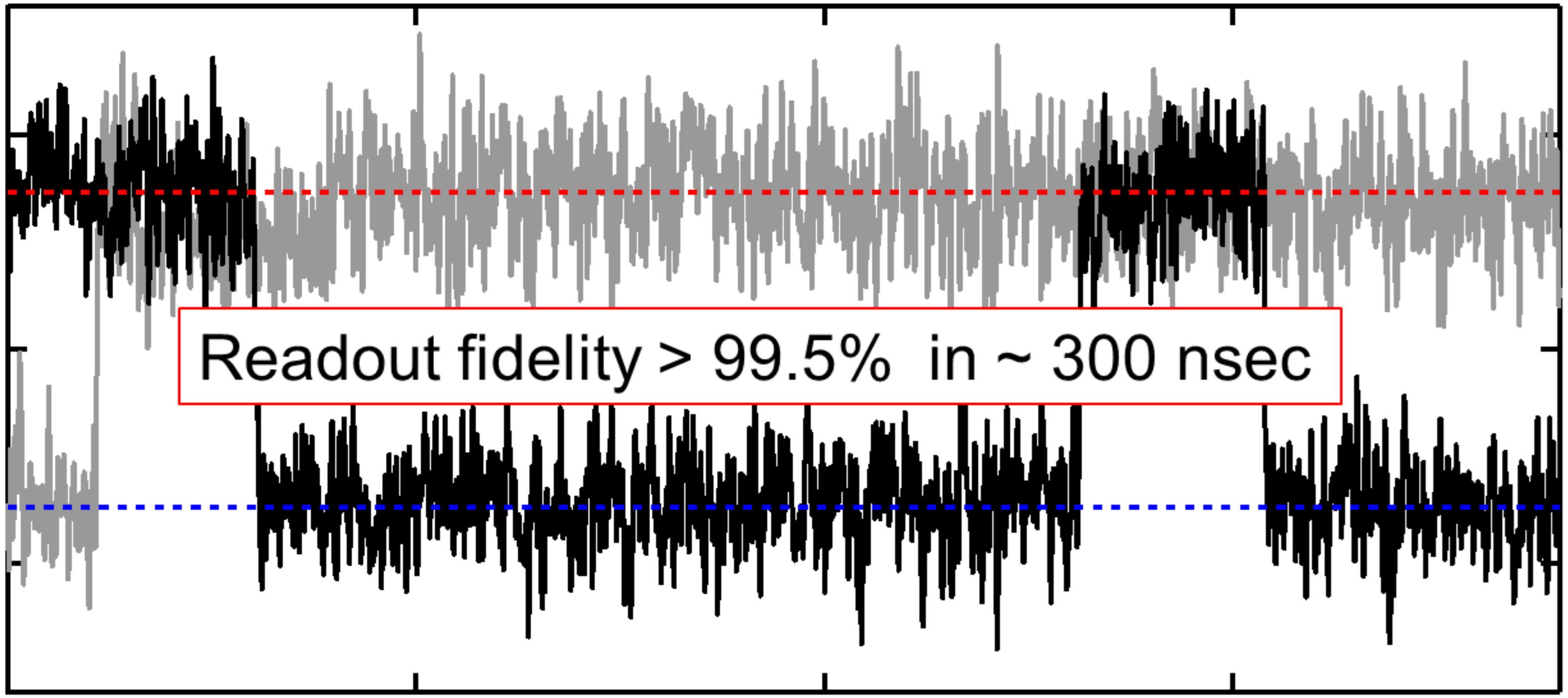
$$\left|\left\langle \alpha e^{-i\theta_0} \middle| \alpha e^{i\theta_0} \right\rangle\right|^2 = e^{2\alpha^2[\cos(2\theta_0)-1]} \approx e^{2\alpha^2[-2\theta_0^2]} = e^{-4\bar{N}\theta_0^2}$$

The two reflected waves become orthogonal

(and therefore distinguishable) for $\bar{N}$? $\frac{1}{4\theta_0^2}$.

Using (not so) strong dispersive coupling to measure the state of the qubit

Dispersive readout proposed in: Blais et al., Phys. Rev. A 69, 062320 (2004)
First experiment: Wallraff et al., Nature 431, 162 (2004)
Quantum limited amplifiers developed…
First single-shot quantum jumps observed: R. Vijay et al., Phys. Rev. Lett. 106, 110502 (2011)



Data from: M. Hatridge et al., Science 339, 178 (2013)

Photon shot noise

Using strong-dispersive coupling to measure the photon number distribution in a cavity

Strong Dispersive Hamiltonian

$$H = \omega_{\mathrm{r}} a^{\dagger} a + \frac{\omega_{\mathrm{q}}}{2}\sigma^{z} + \chi\sigma^{z} a^{\dagger} a + H_{\mathrm{damping}}$$

$$\chi >> \kappa, \Gamma$$

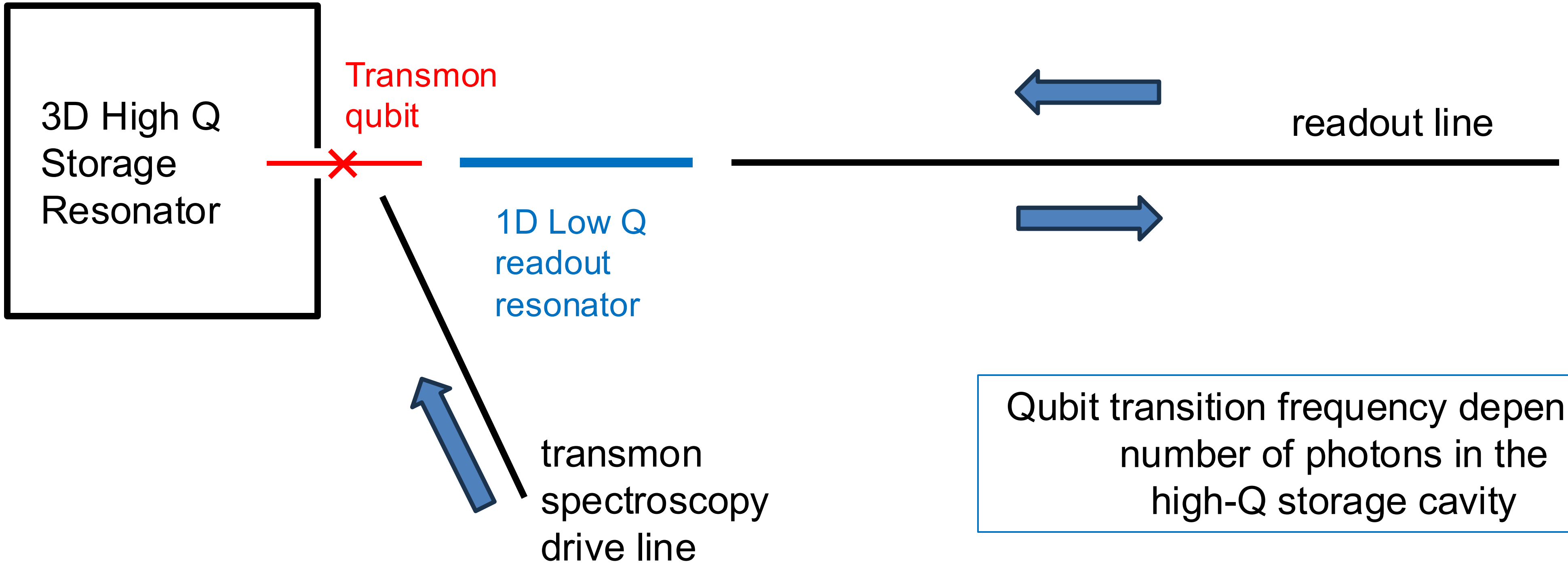
resonator qubit dispersive coupling

Reinterpretation of same Hamiltonian: Quantized Light Shift of Qubit Transition Frequency

$$H = \omega_{\mathrm{r}} a^{\dagger} a + \frac{1}{2}\sigma^{z}\left[\omega_{\mathrm{q}} + 2\chi a^{\dagger} a\right] + H_{\mathrm{damping}}$$

Spectrum of qubit depends on cavity photon number

Using strong-dispersive coupling to measure the photon number distribution in a cavity



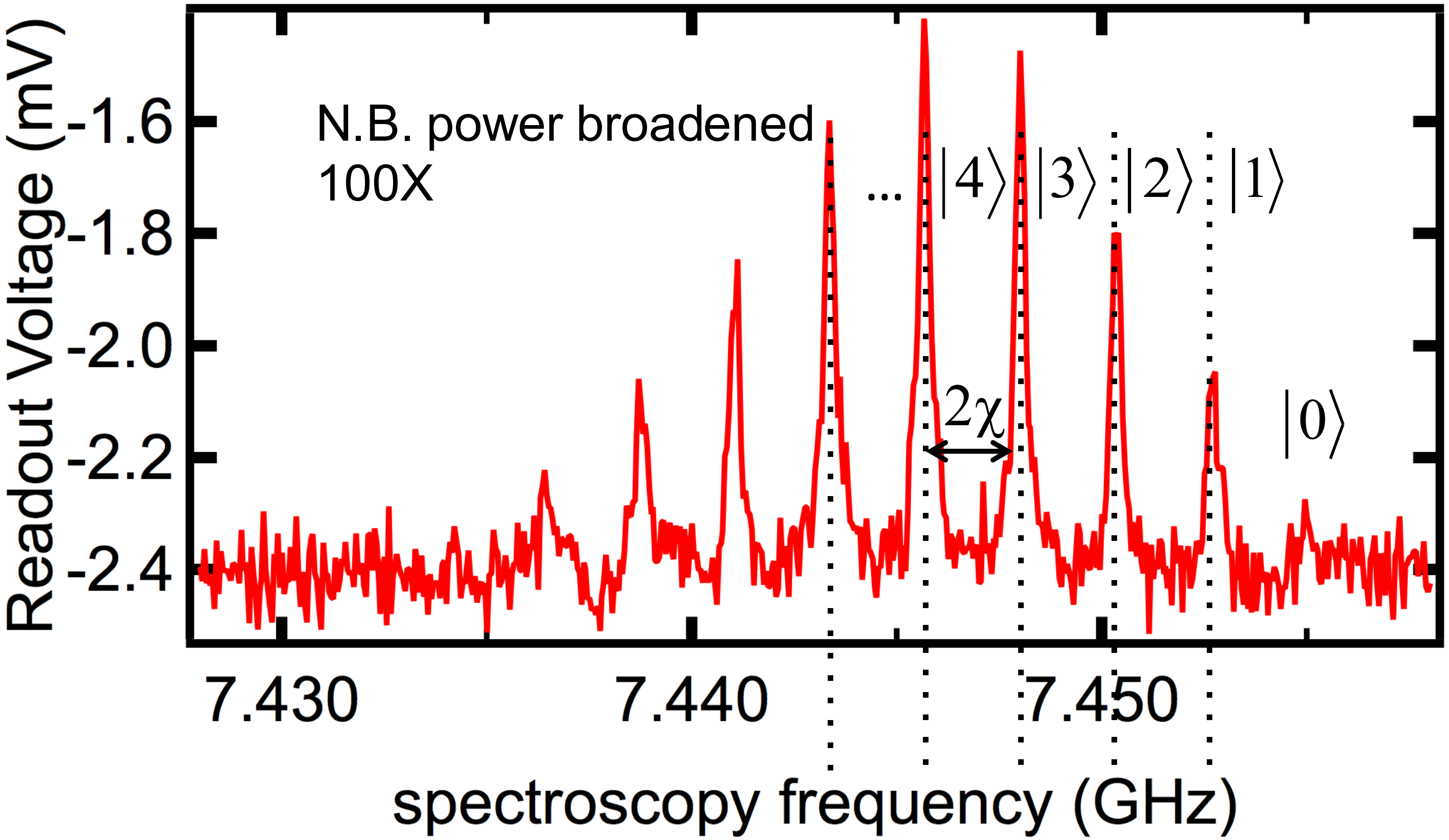
Qubit transition frequency depends on number of photons in the high-Q storage cavity

Measure photon number in high Q storage cavity via dispersive coupling to transmon.

Readout transmon state via dispersive coupling to low Q readout resonator.

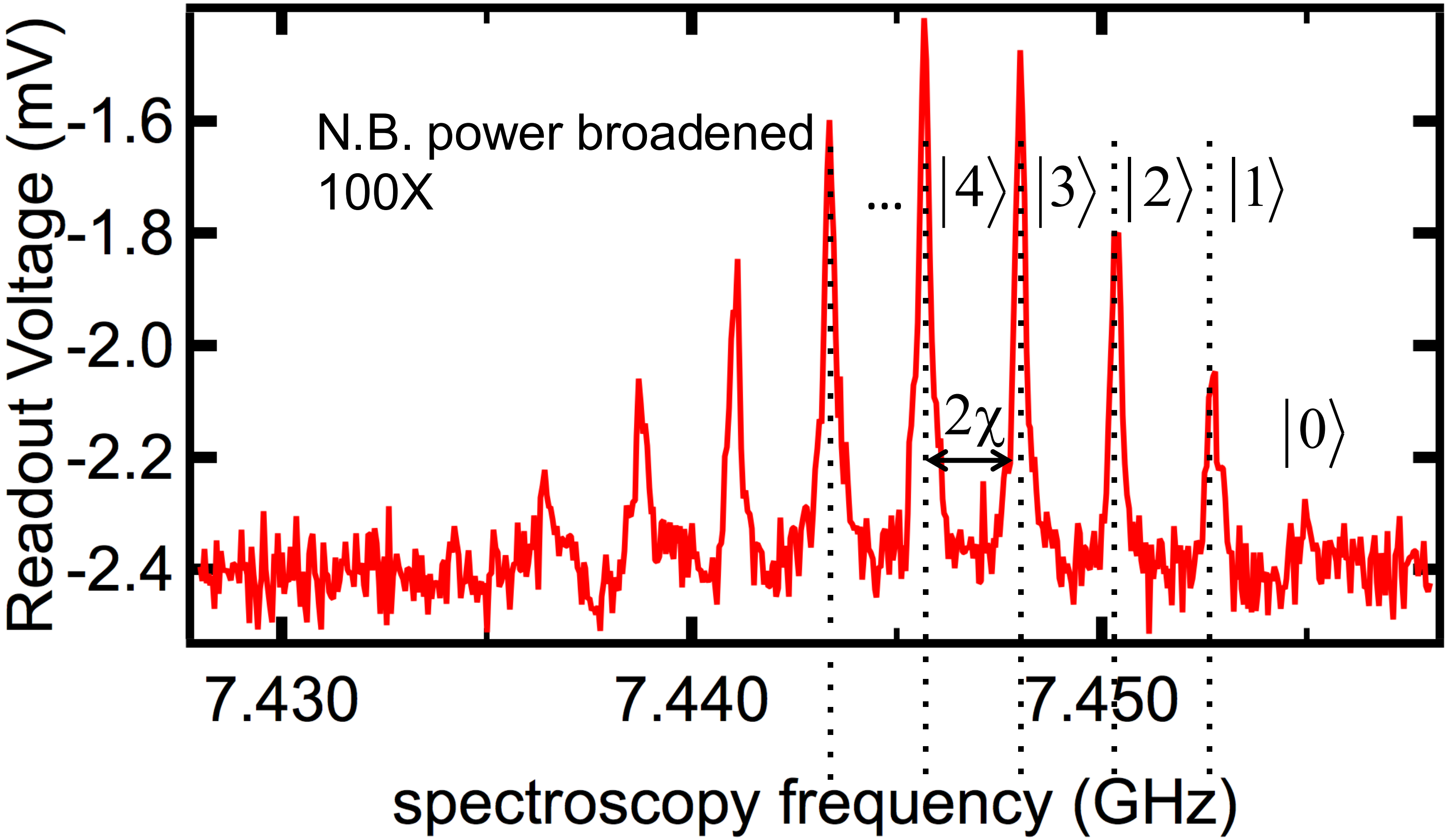
- quantized light shift of qubit frequency
  (coherent microwave state)

$$\frac{\omega_q + 2\chi a^\dagger a}{2}\sigma^z$$



- quantized light shift of qubit frequency (coherent microwave state)

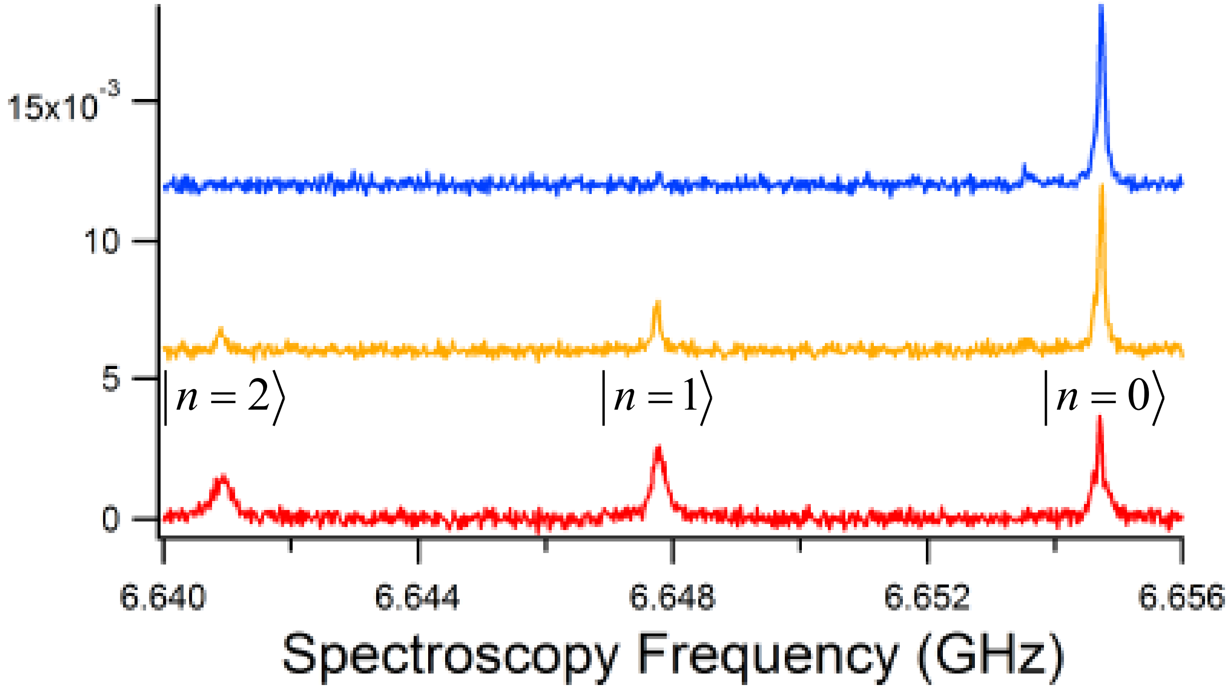
$$\frac{\omega_q + 2\chi a^\dagger a}{2}\sigma^z$$



Microwaves are particles!

- quantized light shift of qubit frequency (coherent microwave state)

$$\frac{\omega_q + 2\chi a^\dagger a}{2}\sigma^z$$



New low-noise way to do axion dark matter detection by QND photon counting

Zheng et al. arXiv:1607.02529 → A. Chou: PRL **126**, 141302 (2021)

Photon number parity

$$\hat{P} = (-1)^{a^\dagger a} = \sum_{n=0}^{\infty} |n\rangle (-1)^n \langle n|$$

Remarkably easy to measure using our quantum engineering toolbox

and

Measurement is 99.8% QND

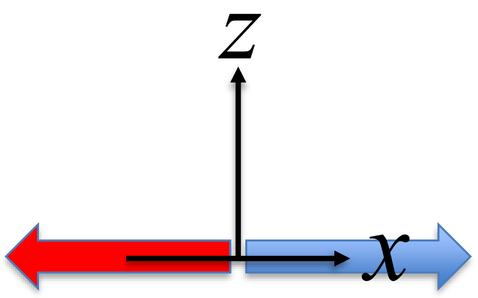
Measuring Photon Number Parity

- use quantized light shift of qubit frequency

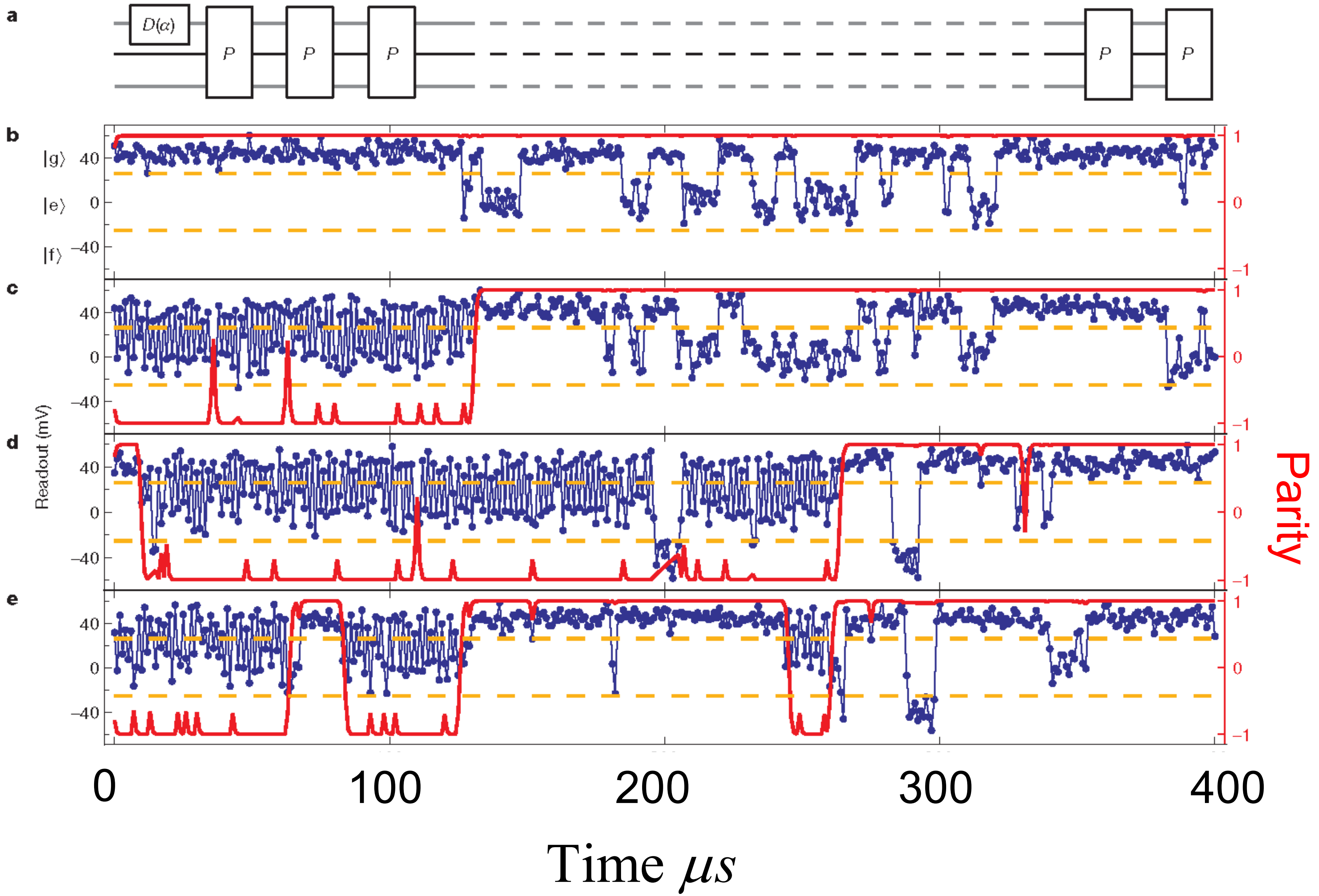
$$\frac{\omega_q + 2\chi a^\dagger a}{2}\sigma^z$$

$$e^{-i2\chi\hat{n}t\frac{\sigma^z}{2}} = \mathrm{e}^{-i\pi\hat{n}\frac{\sigma^z}{2}}$$

$\hat{n} = 1, 3, 5, \ldots$



$\hat{n} = 0, 2, 4, \ldots$



Nature **511**, 444 (2014)

400 consecutive parity measurements (99.8% QND)

Summary of Lecture I:

Introduction to Circuit QED

- What is Cavity QED?
- Quantum LC Oscillators
- Josephson Junctions & Transmon Qubits
- Qubits coupled to microwave cavities
    - Control and QND measurement of both qubit and cavity