Programmable Quantum Simulators for Condensed Matter and Lattice Gauge Models

Steven Girvin
Yale Quantum Institute

C²QA Co-design Center for Quantum Advantage

Co-design Center for Quantum Advantage (C^2QA)

BROOKHAVEN NATIONAL LABORATORY https://www.bnl.gov/quantumcenter/

Disclosure: SMG is a consultant and equity holder in Quantum Circuits, Inc. and an equity holder in IBM, Inc.

5 federal labs + 19 universities + IBM

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Two flavors of Quantum Simulators:

1. Simple, but non-programmable: use the 'natural' Hamiltonian of a synthetic system with similar degrees of freedom to the system being simulated.

2. Programmable: Requires universal control of all quantum degrees of freedom to synthesize arbitrary Hamiltonian time evolution.
   --Hamiltonian synthesis via 'digital' Trotter-Suzuki + Baker-Campbell-Hausdorff gate sequences and/or analog optimal control theory

ALL simulators require the ability to make accurate and non-trivial measurements (hopefully beyond the capability of traditional experiment).

Error correction/mitigation will ultimately be needed in most cases.

Big Picture Questions:

We know qubits are useful but what about oscillators?
[Superconducting circuits, trapped ions, Rydberg atom arrays]

1) Can we take advantage of their large Hilbert space?

2) Do they offer hardware efficiency in simulating physical models containing bosons [e.g., Bose Hubbard model, lattice gauge theory]?

3) Can we do quantum error correction with bosonic modes?

Big Picture Questions:

We know qubits are useful but what about oscillators?
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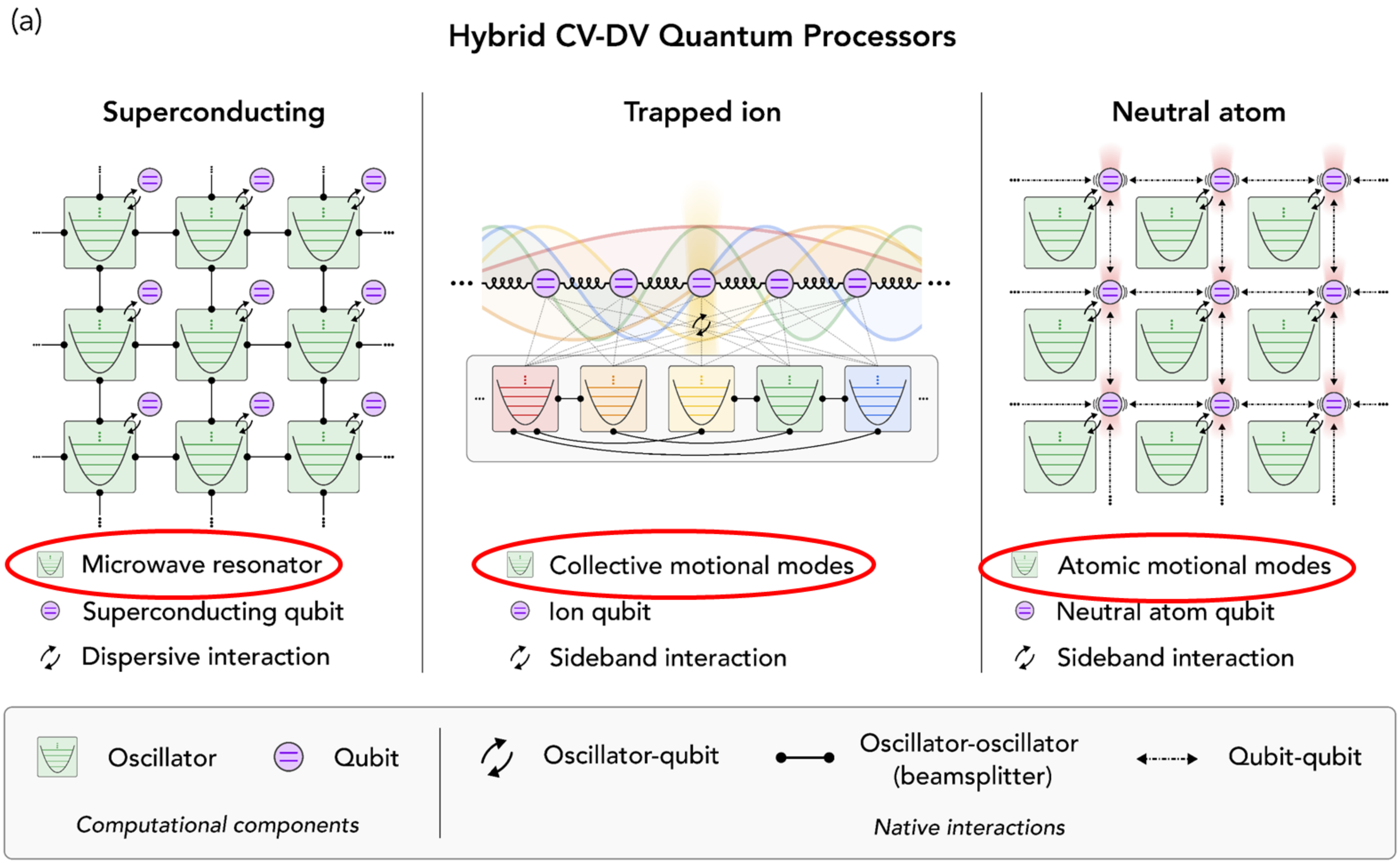
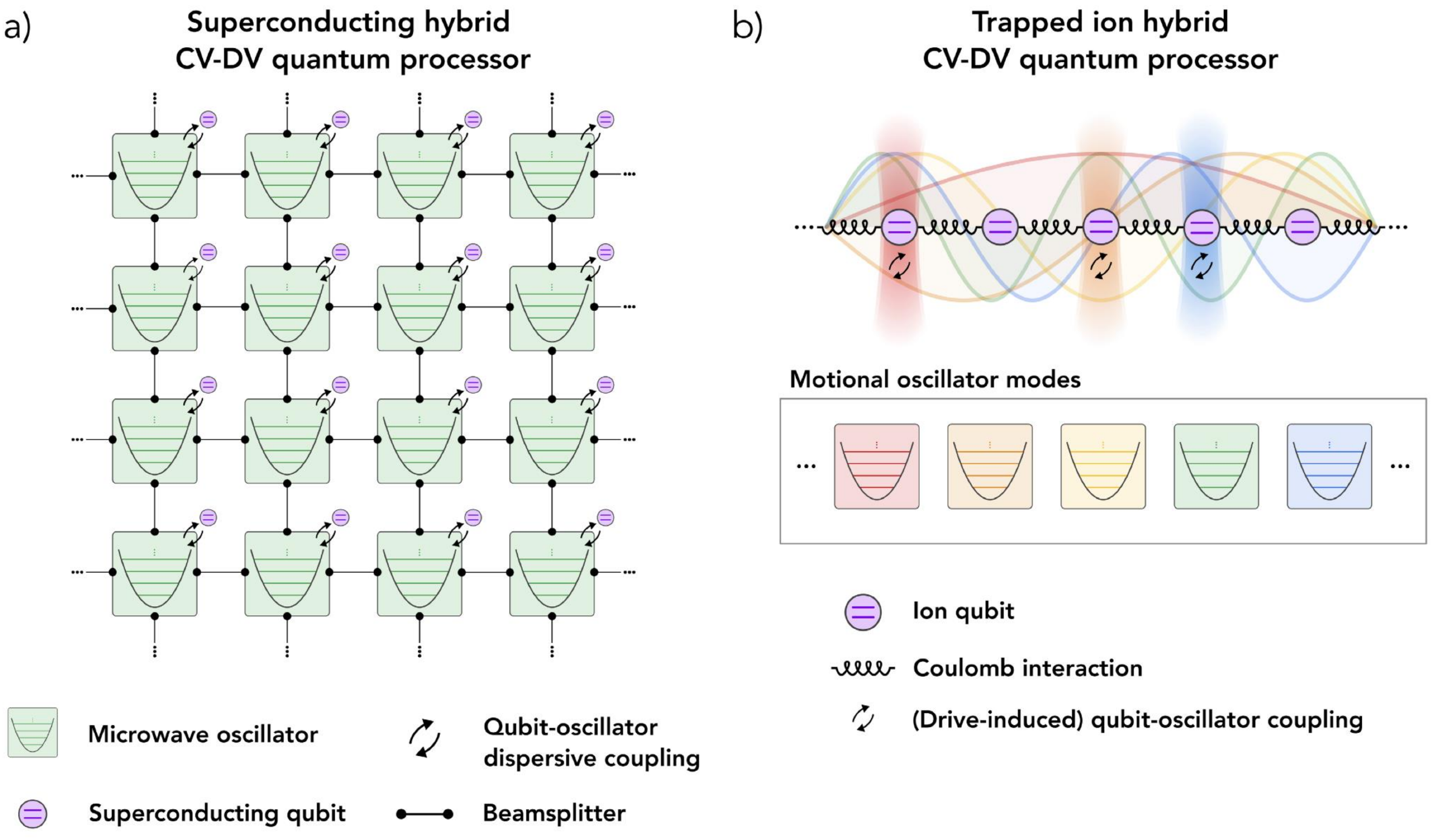
2) Do they offer hardware efficiency in simulating physical models containing bosons [e.g., Bose Hubbard model, lattice gauge theory]? [YES]

3) Can we do quantum error correction with bosonic modes? [YES]

Hybrid Oscillator-Qubit Quantum Processors:
Instruction Set Architectures, Abstract Machine Models and Applications
arXiv:2407.10381





c)

AMM 1: Qubit centric	AMM 2: Bosonic centric	AMM 3: hybrid qubit/oscillator
long-range connectivity through auxiliary bosonic modes	control of bosonic QEC codes through auxiliary qubits	Hybrid algorithms and simulation of physical models w/ spins and bosons

Hybrid qubit/cavity hardware layer

arXiv: 2407.10381

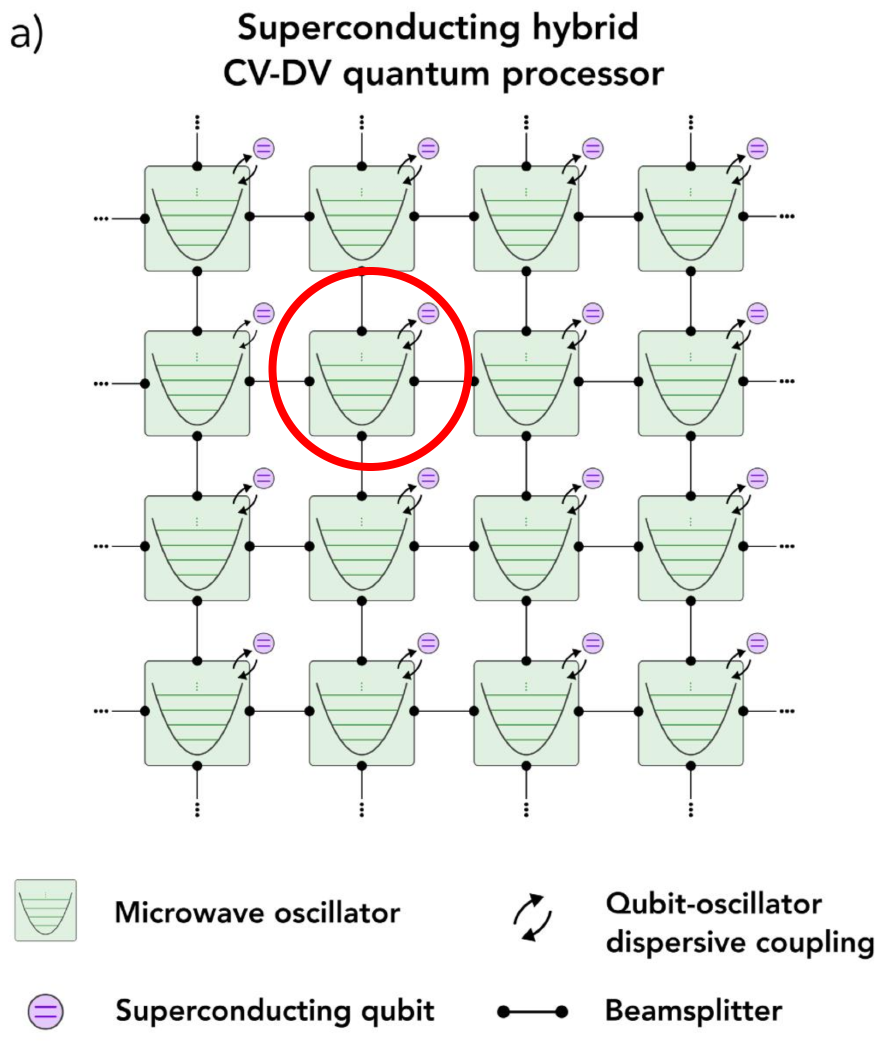
[3D resonator + 1 transmon] local universal control

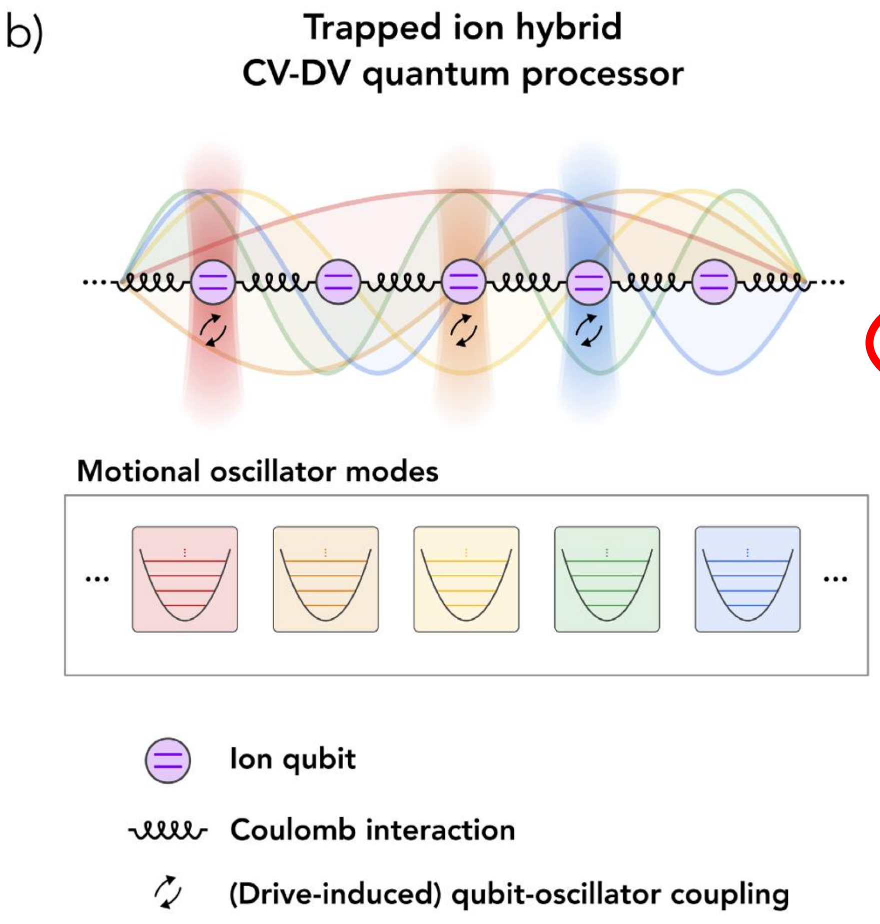
Minimal cross-talk

High-fidelity parallelizable bosonic communication SWAP network

c.f. ion-trap ‘all-to-all’

3 distinct AMMs (abstract machine models)





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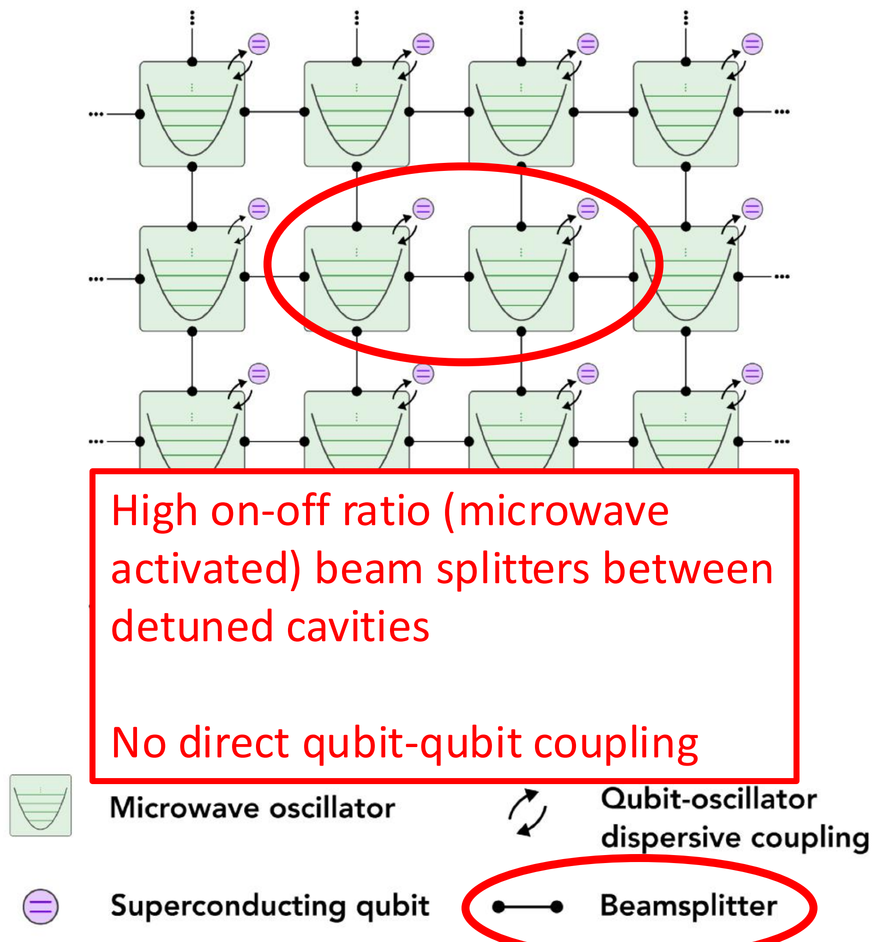
Hybrid qubit/cavity hardware layer

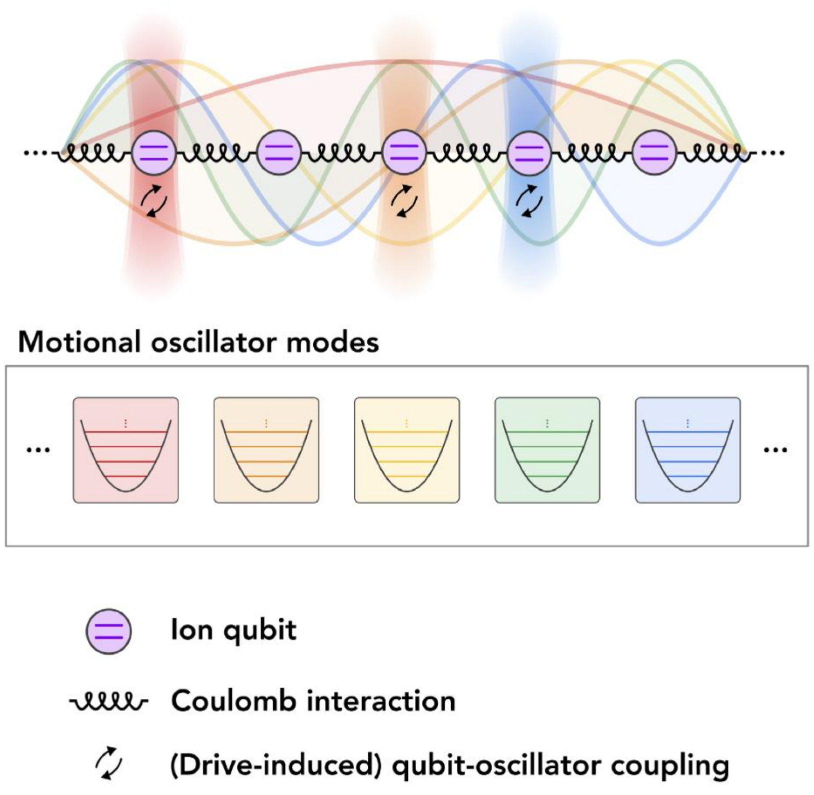
a) **Superconducting hybrid CV-DV quantum processor**



High on-off ratio (microwave activated) beam splitters between detuned cavities

No direct qubit-qubit coupling

b) **Trapped ion hybrid CV-DV quantum processor**



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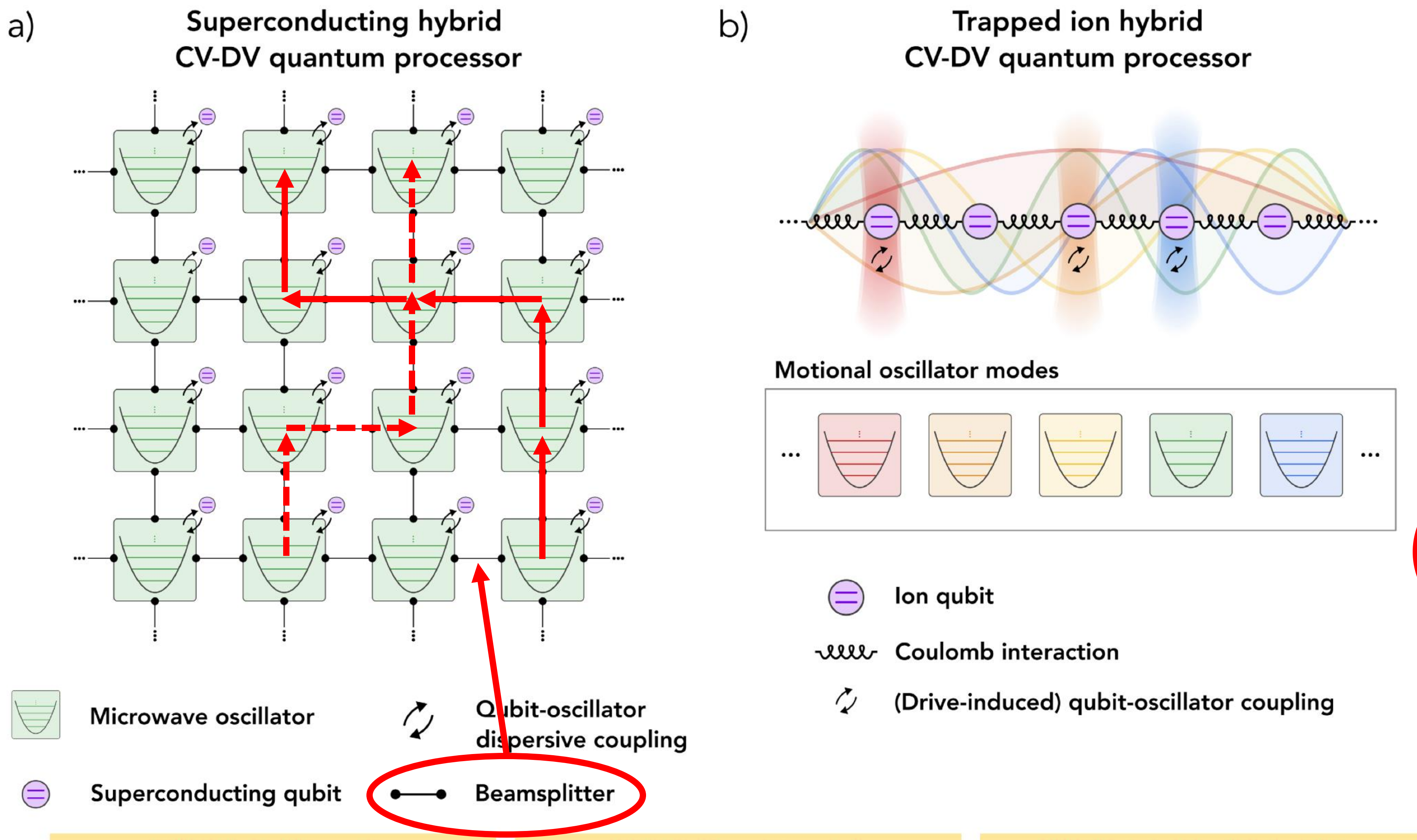

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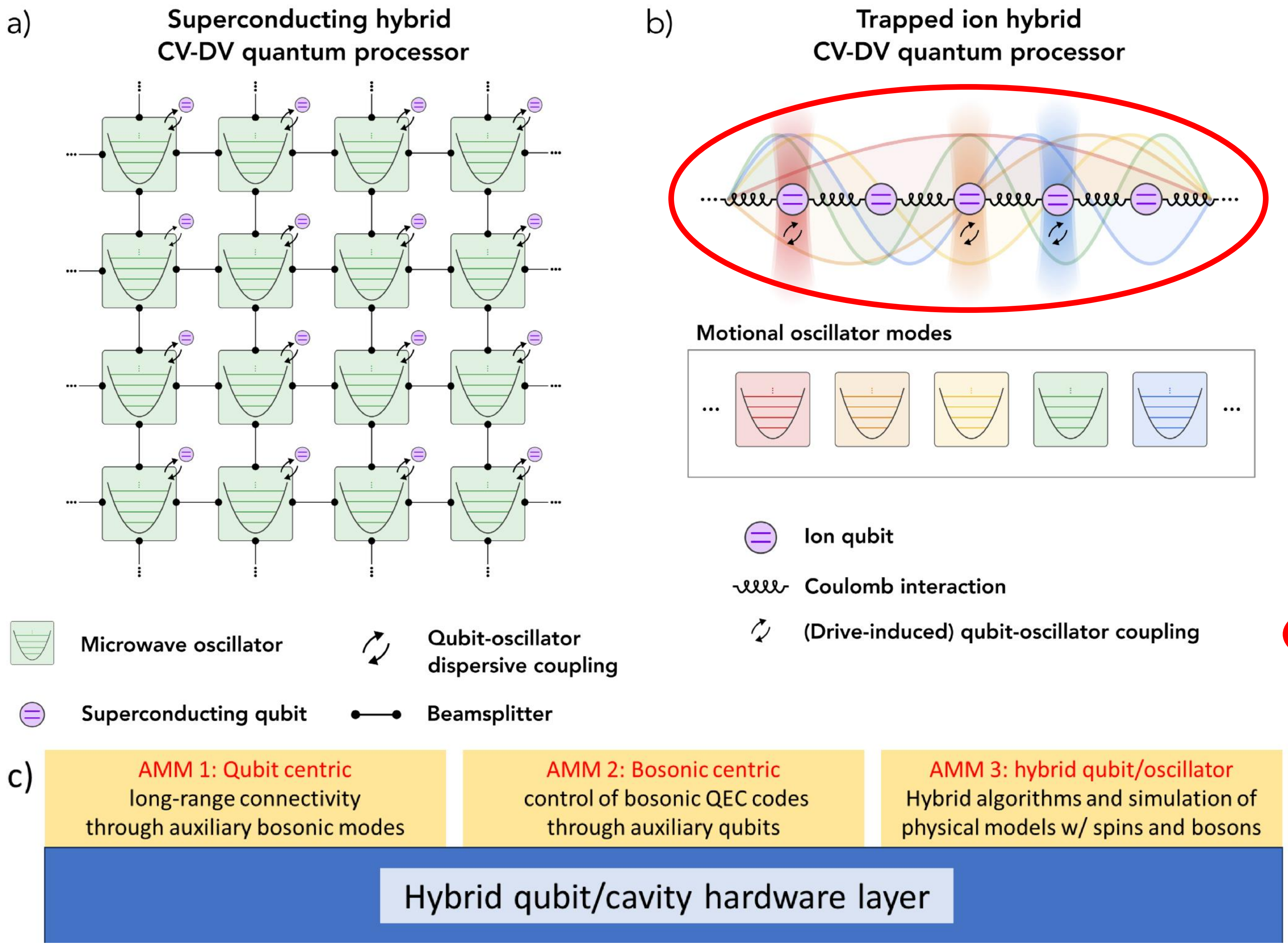

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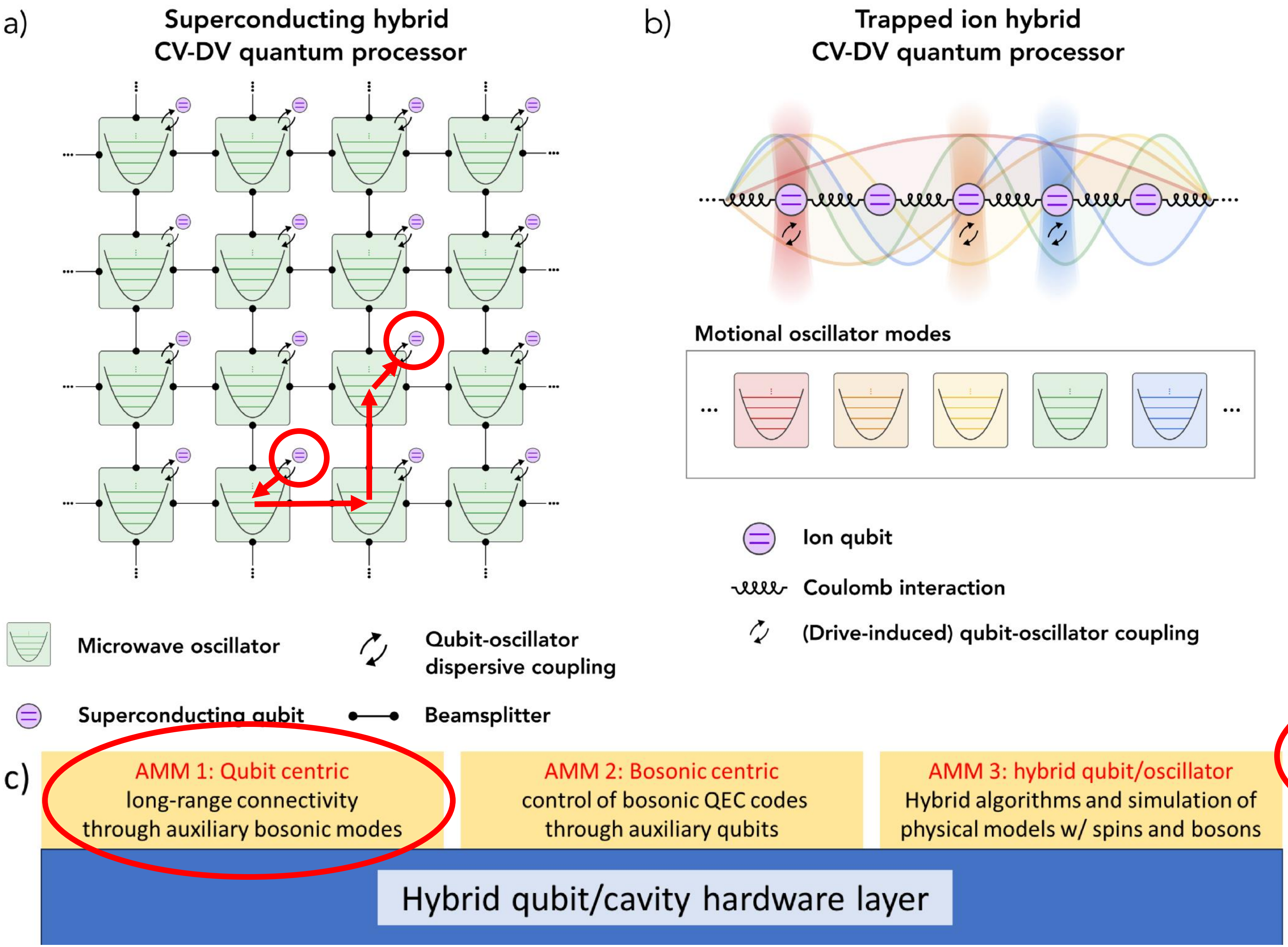

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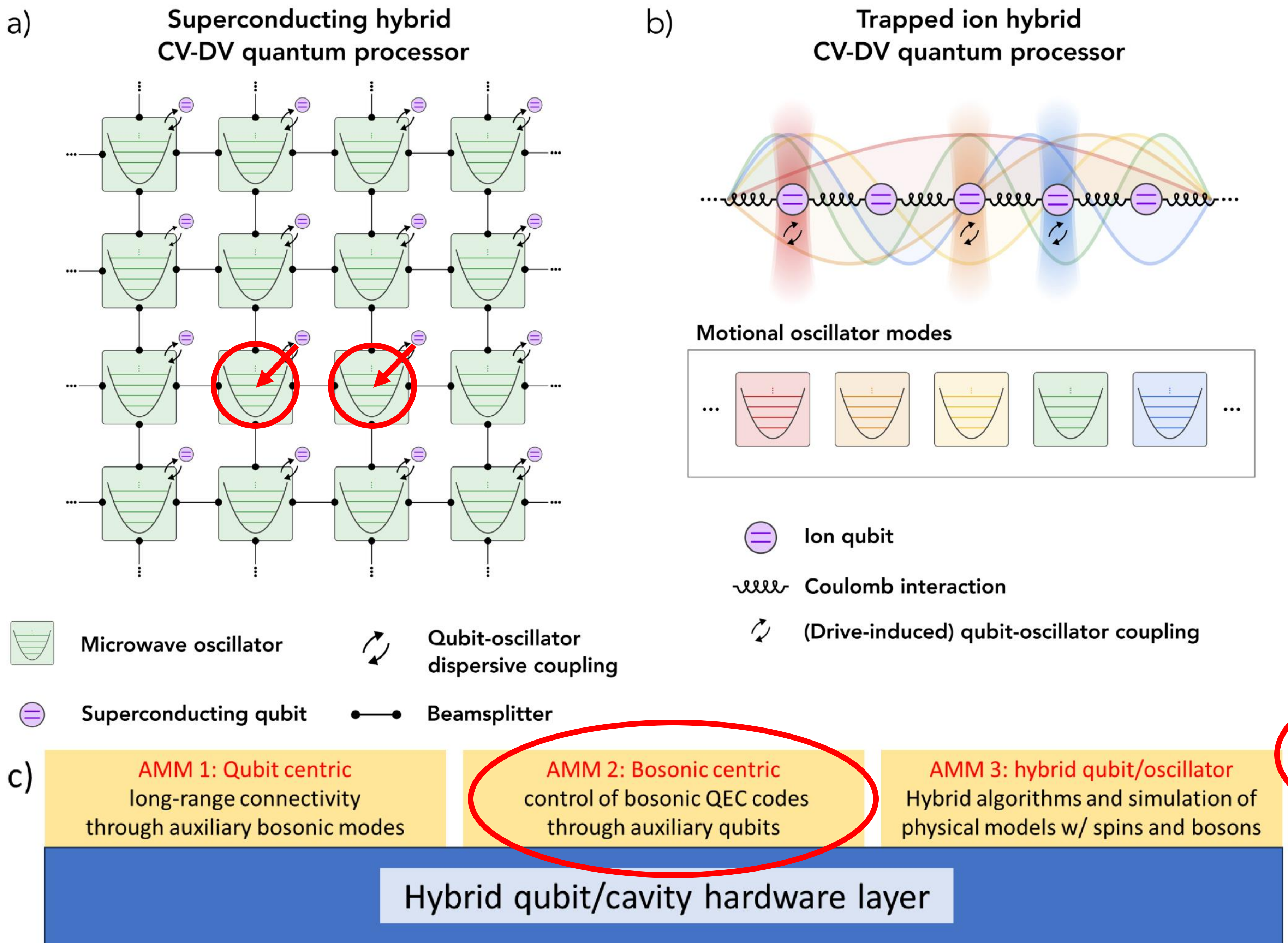

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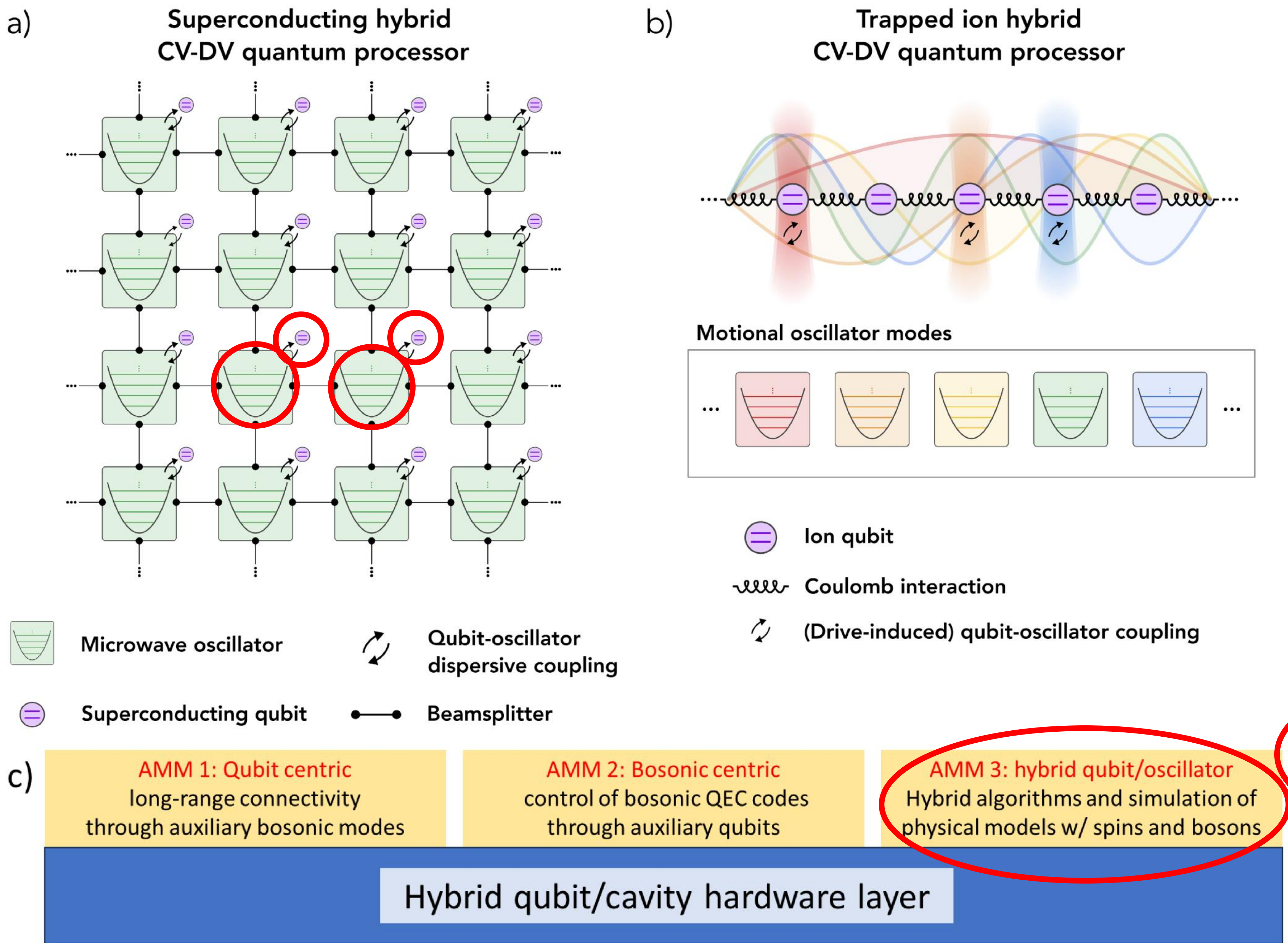

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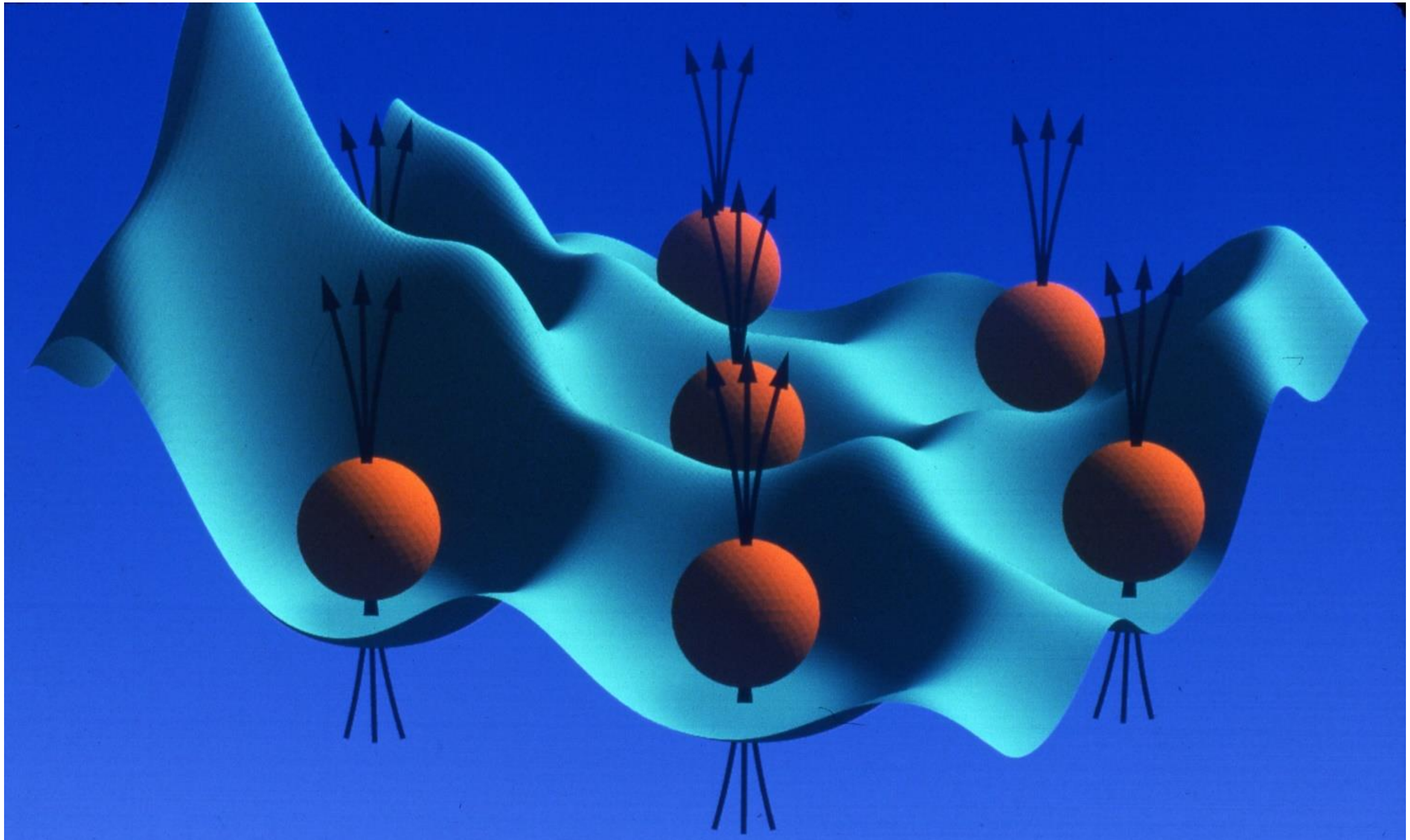
Take-home message:

- **Hardware native bosonic modes offer advantages for:**
  - Efficient quantum error correction
  - Efficient quantum simulation of physical models containing bosons
- **Hybrid qubit/oscillator combinations can achieve universal control**
  - We need a simple instruction set architecture (ISA) in order to be able to develop algorithms and reason about circuit depth/complexity
  - Small ISA can be compiled to the control pulse level via OCT (optimal control theory) but entire algorithms cannot. We need an ISA to compile algorithms, estimate circuit costs and reason about error propagation.
- **Goals:**
  - Develop ISA: instruction set architecture(s);
    apply to quantum simulations, algorithms, and error correction
  - Represent the ISA in an extension of Qiskit that can treat bosonic modes;
    promulgate as a co-design tool for the community arXiv:2209.11153 and https://medium.com/qiskit/introducing-bosonic-qiskit-a-package-for-simulating-bosonic-and-hybrid-qubit-bosonic-circuits-1e1e528287bb

Towards Many-Body Quantum Simulations of Interacting Bosons in Circuit QED



Modern Condensed Matter Physics (Cambridge Press, 2019)

Example target application:

FQHE for bosons (photons)

Can we convince microwave photons that they are charged particles in a magnetic field?

fractional statistics

$\nu = 1/2$ abelian semions

$\nu = 1$ non-abelian

REFERENCES:

K. Fang, Z. Yu and S. Fan, *Realizing effective magnetic field for photons by controlling the phase of dynamic modulation*, Nature Photonics **6**(11), 782 (2012).

E. Kapit, *Quantum simulation architecture for lattice bosons in arbitrary, tunable, external gauge fields*, Phys. Rev. A **87**, 062336 (2013).

M. Hafezi, P. Adhikari and J. M. Taylor, *Engineering three-body interaction and Pfaffian states in circuit QED systems*, Phys. Rev. B **90**, 060503 (2014).

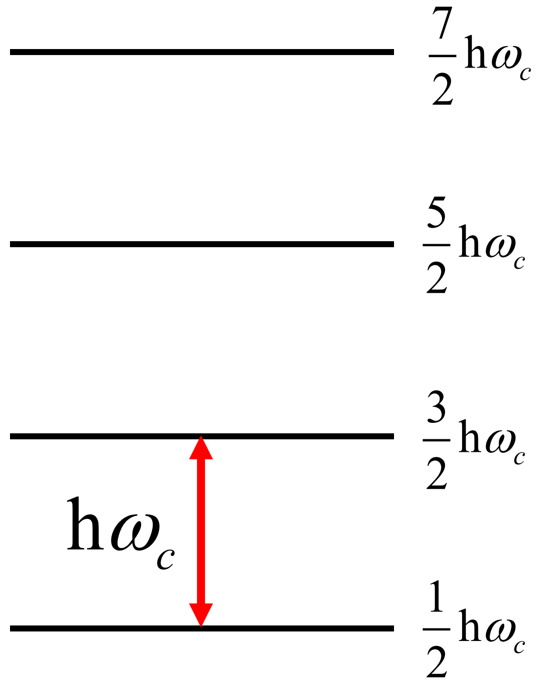
E. Kapit, *Universal two-qubit interactions, measurement, and cooling for quantum simulation and computing*, Phys. Rev. A **92**, 012302 (2015).

P. D. Kurilovich et al., 'Stabilizing the Laughlin state of light: dynamics of hole fractionalization,' arXiv:2111.01157 [*SciPost Phys.* **13**, 107 (2022)].

Outline:

- **Target application I: FQHE for bosons**
  - **Brief review of FQHE physics in 2D continuum and Laughlin wave function/plasma analogy**
  - How do we realize a 2D lattice model version with microwave photons?
  - Bose-Hubbard model in a (pseudo) magnetic field
    - Boson hopping as an optical beam-splitter Hamiltonian
    - Programmable random site disorder via beam-splitter detuning
    - Programmable boson-boson repulsion via SNAP gates
  - Non-equilibrium quantum dynamics: bath engineering to stabilize Laughlin state against boson loss
- **Target application II: Z2 lattice gauge theory**

Single-particle wave functions in the lowest Landau Level (2DEG strong B field)



$$z = [x + iy]/1$$

$$\psi[z] = f[z]e^{-\frac{1}{4}|z|^2} \qquad f[z] = \text{poly}[z]$$

Laughlin correlated many-body ground state for Landau level filling factor $\nu = \frac{1}{m}$

$$\psi_m[z] = \prod_{i<j}(z_i - z_j)^m \, e^{-\frac{1}{4}\sum_k |z_k|^2}$$

$m =$ odd: fermions

$m =$ even: bosons

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$m =$ odd: fermions

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Plasma analogy

2D Coulomb potential ('charge *m* rods')

2D Poisson equation
$\nabla^2 \Phi = -2\pi \rho(\vec{r})$

$$|\Psi_m[z]|^2 = e^{-\beta U_{\text{class}}},$$

where $\beta \equiv \frac{2}{m}$ and

$$U_{\text{class}} \equiv m^2 \sum_{i<j} (-\ln|z_i - z_j|) + \frac{m}{4} \sum_k |z_k|^2.$$

Potential from uniform compensating background charge density $\rho_0 = -\frac{1}{2\pi l^2}$. Charge neutrality sets the electron density ρ: $m\rho + \rho_0 = 0$ corresponding to Landau level filling $\nu = \frac{1}{m}$.

Fractionally charged quasi-hole excitations

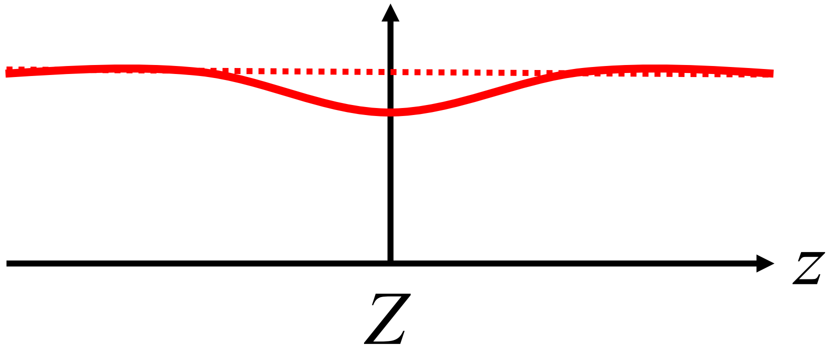
$$\psi_Z^+[z] = \left[\prod_{j=1}^{N}(z_j - Z)\right] \Psi_m[z]$$

Particles avoid the position of quasi-hole

$$|\psi_Z^+|^2 = e^{-\beta U_{\text{class}}}\ e^{-\beta V}$$

$$V \equiv m \sum_{j=1}^{N} (-\ln|z_j - Z|)$$

Charge *m* particles repelled by a charge 1 impurity



Perfect screening in plasma implies local charge neutrality, so the screening cloud has 'charge' $\delta q = m\,\delta n = -1$,

implying that the quasi-hole has net particle number $\delta n = -\dfrac{1}{m}$.

A similar calculation shows the quasi-holes obey fractional statistics.

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Example target application: Bose-Hubbard/FQHE Hamiltonian Simulation

GOALS:

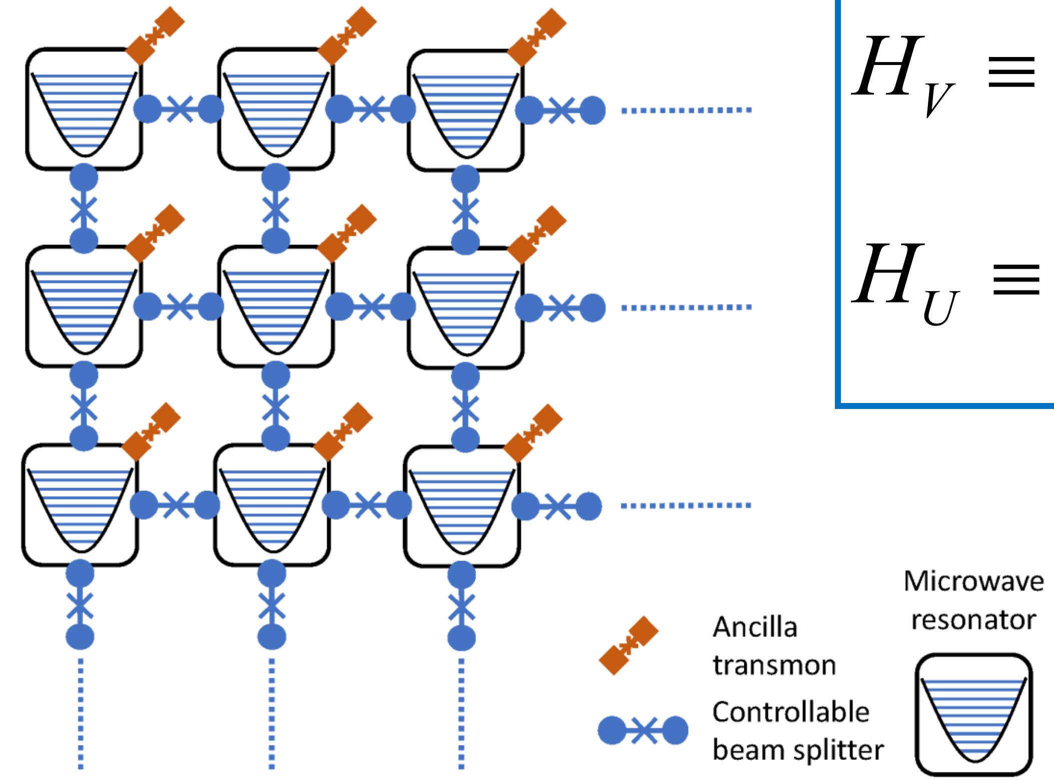
- ❑ Synthesize
  - Ground state (VQE)
  - Dynamics: $U(t) = e^{-iHt}$
- ❑ Measure observables

$$H = H_J + H_V + H_U$$

$$H_J \equiv \sum_{\langle ij \rangle} \left\{ J_{ij} b_i^\dagger b_j + J_{ij}^* b_j^\dagger b_i \right\}$$ boson hopping

$$H_V \equiv \sum_k \epsilon_k b_k^\dagger b_k$$ randomly disordered site energies

$$H_U \equiv U \sum_k b_k^\dagger b_k^\dagger b_k b_k$$ Hubbard *U* boson repulsion



❑ Rich many-body phase diagram:

- Superfluid
- Mott insulator
- Anderson localization/Bose glass
- FQHE with fractional non-abelian excitations

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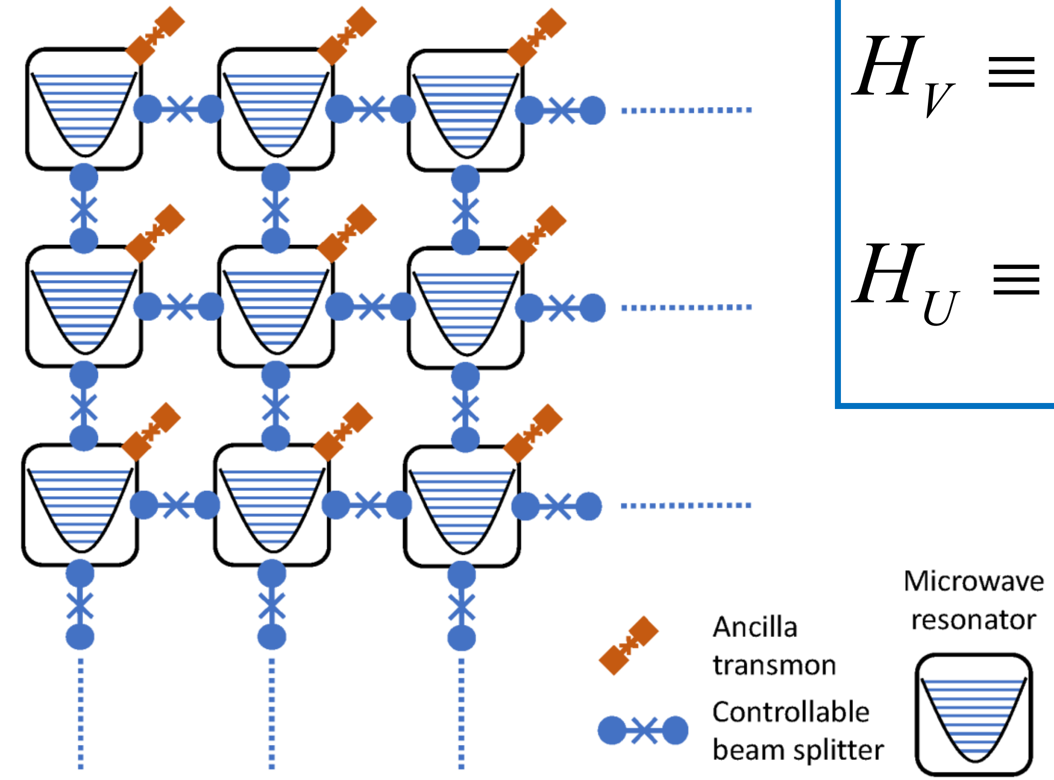

- ❑ Topological states with SC qubits rather than resonators: See M. Gorlach papers:
- ❑ **Phys. Rev. Lett. 128, 213903 (2022)**
- ❑ **Phys. Rev. B 105, L081107 (2022)**

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Controllable beam splitters to realize boson hopping

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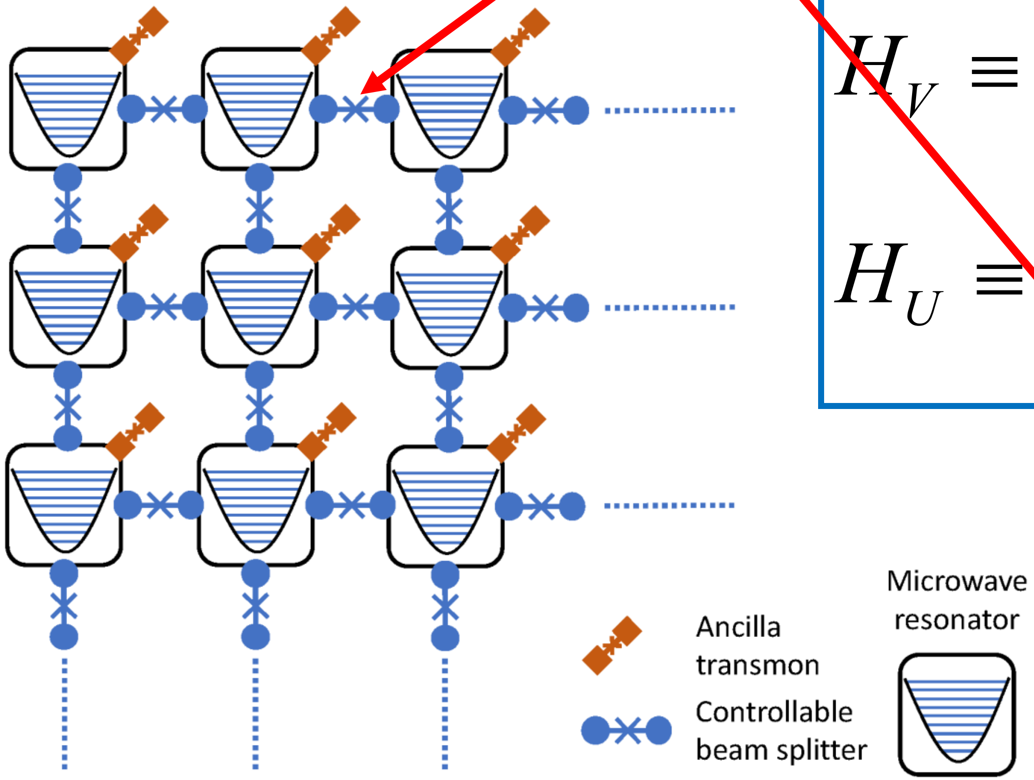
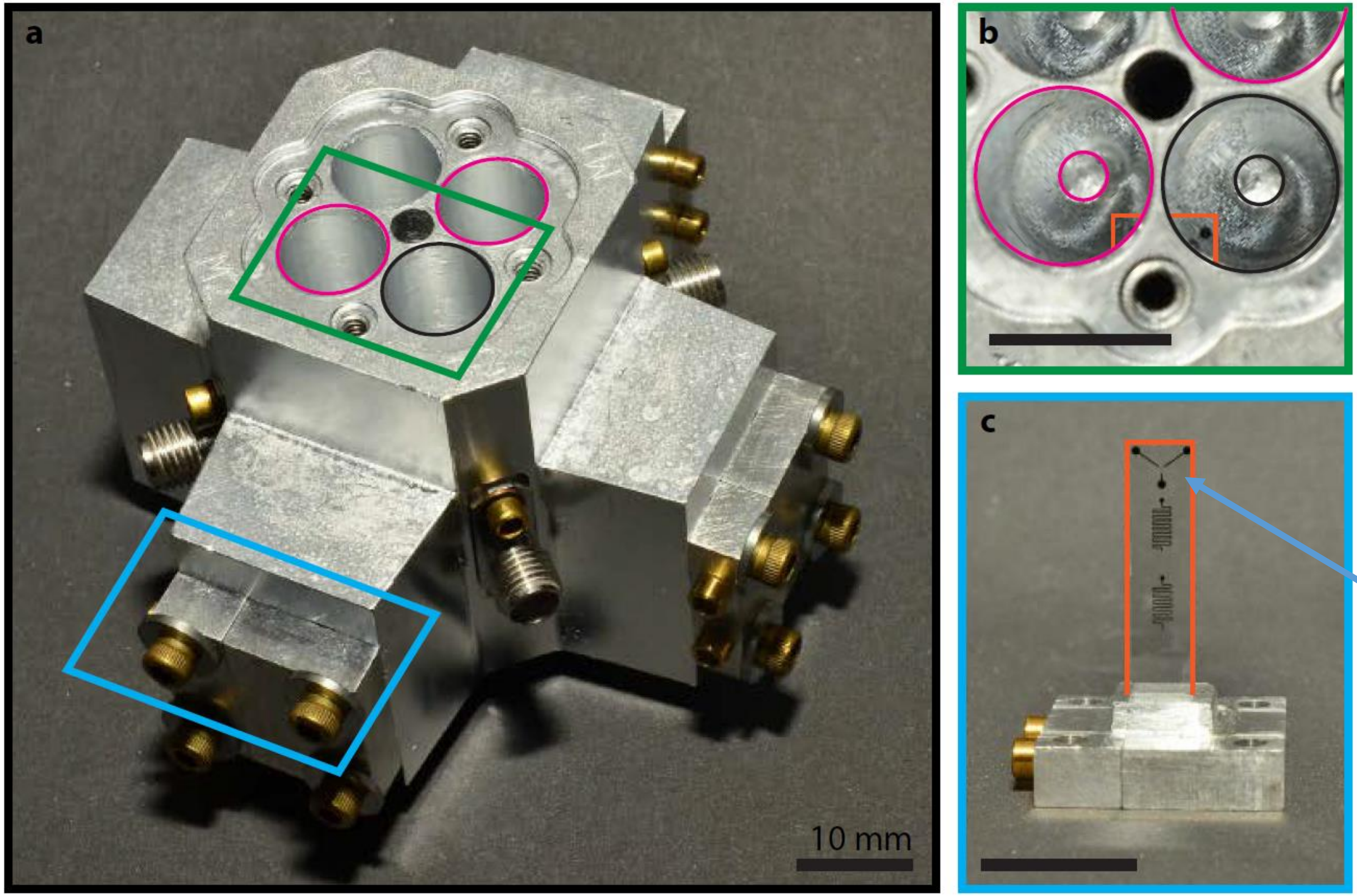
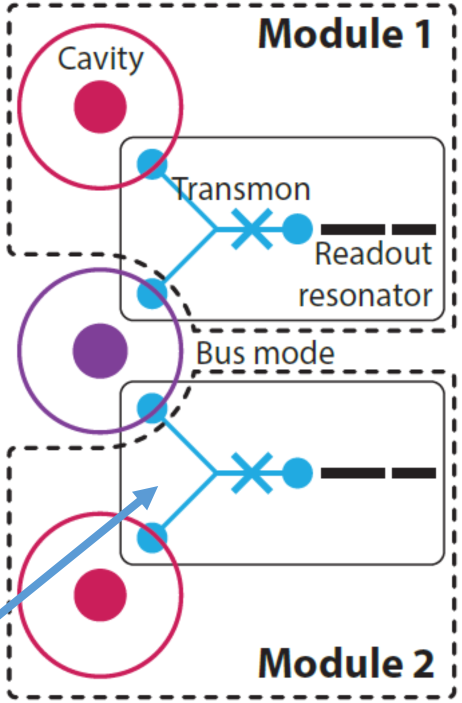


In quantum optics language, this is a beam-splitter Hamiltonian

Realizing a programmable beam splitter Hamiltonian

Deterministic teleportation of a quantum gate between two logical qubits, Kevin S. Chou et al., *Nature* **561**, 368 (2018)

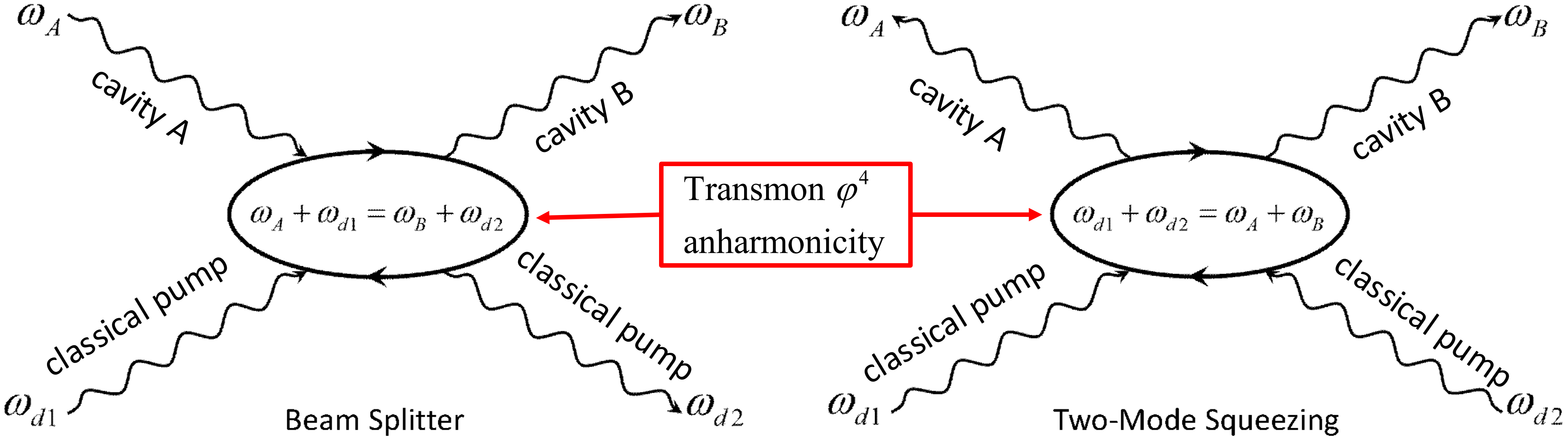




'Y-mon' transmon geometry

JJ is parametrically pumped to turn on the beam splitter between cavities.

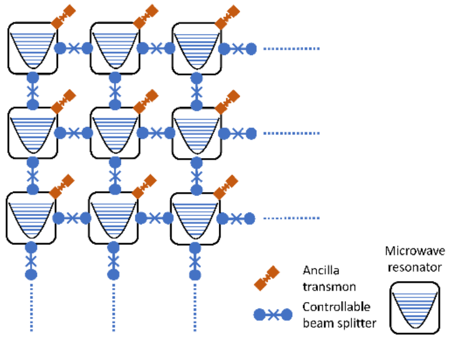
Two-mode Gaussian Operations via 4-wave Mixing with a Transmon Coupler [All bilinear couplers that can be turned on and off by microwave pulses are necessarily pumped non-linear devices.]



$$H_{\mathrm{BS}} = g_{\mathrm{BS}}(t) A B^\dagger + g_{\mathrm{BS}}^*(t) A^\dagger B.$$

$$H_{\mathrm{TMS}} = g_{\mathrm{TMS}}(t) A^\dagger B^\dagger + g_{\mathrm{TMS}}^*(t) A B.$$

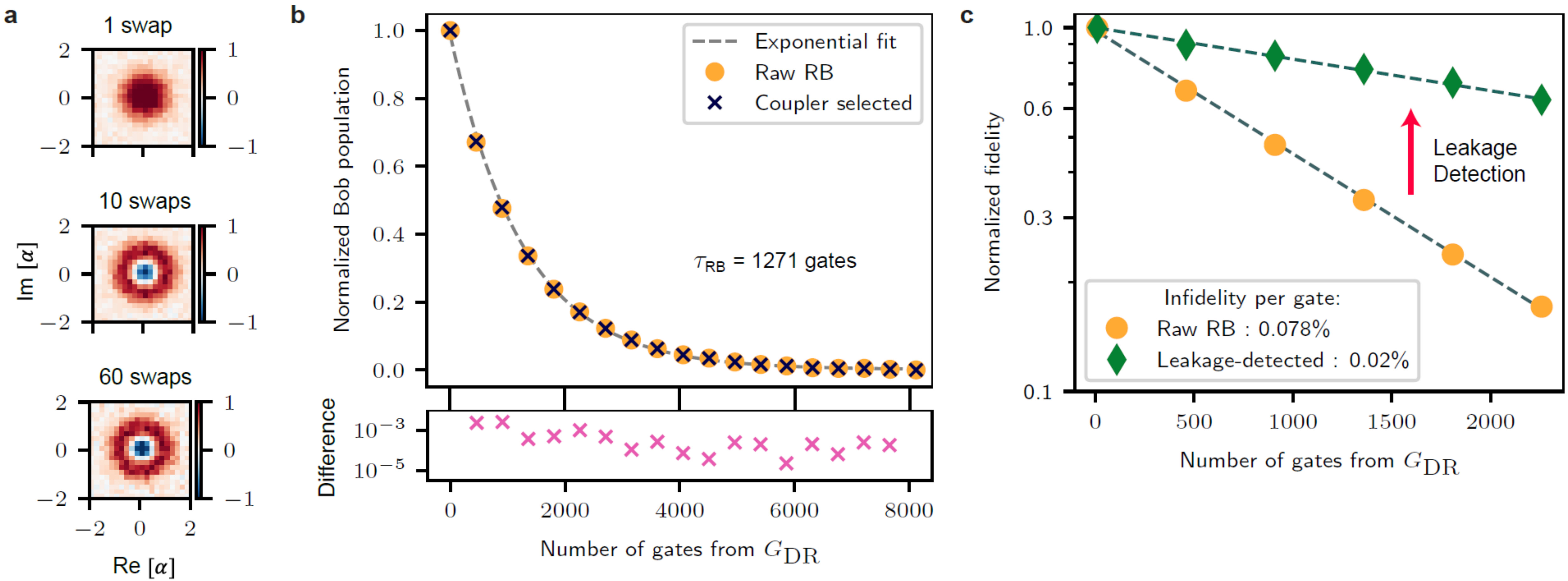
Each resonator has a distinct frequency to reduce cross-talk and enhance on/off ratio.



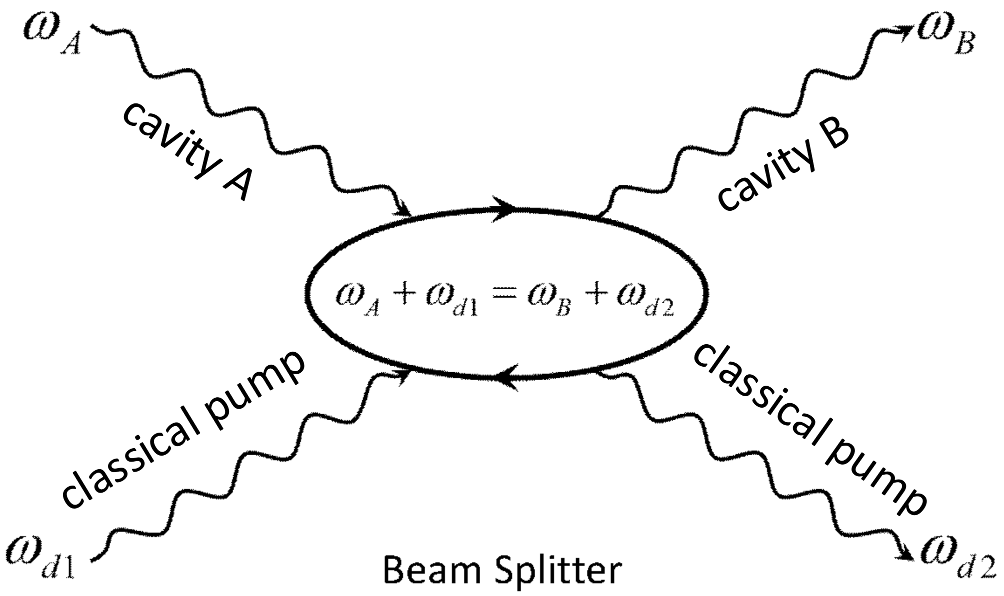
Phase and amplitude of the coupling is controlled by the choice of pump tone phases and amplitudes.
Pump supplies the energy change needed for the process to be resonant (e.g. frequency-converting beam splitter).
No need to fine tune the cavity manufacture.

High-fidelity beam-splitter SWAP gates $|0,1\rangle \rightarrow |1,0\rangle$

Yao Lu et al. (Schoelkopf group), arXiv:2303.00959



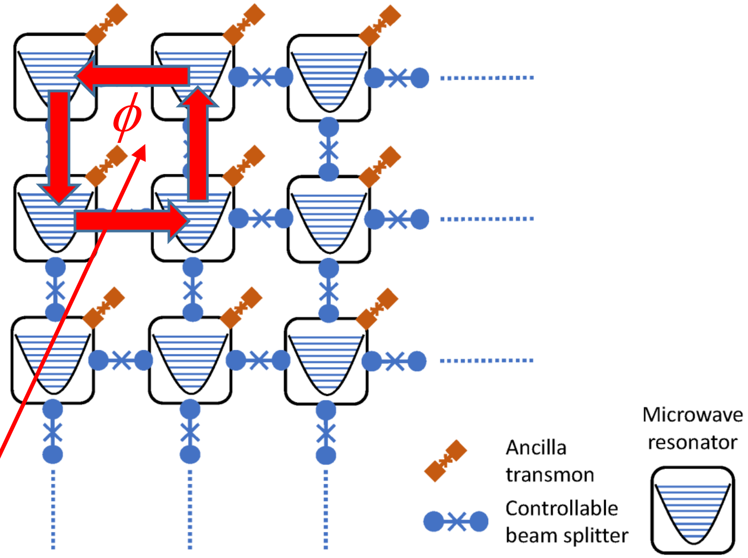
Two-mode Gaussian Operations via 4-wave Mixing



$$H_{\rm BS} = g_{\rm BS}(t) AB^\dagger + g^*_{\rm BS}(t) A^\dagger B.$$

Beam splitter realizes the boson hopping term.

$$H_J \equiv \sum_{\langle ij \rangle} \left\{ J_{ij} b_i^\dagger b_j + J_{ij}^* b_j^\dagger b_i \right\}$$



Phase-locking the pump tones allows complex *J* and Photon can acquire a non-zero phase around each plaquette. Acts like a charged particle in a magnetic field $\phi = \frac{q}{\hbar}\oint d\vec{r} \cdot \vec{A}(\vec{r}) = \frac{q}{\hbar}\Phi$

What does 'phase locking' mean?

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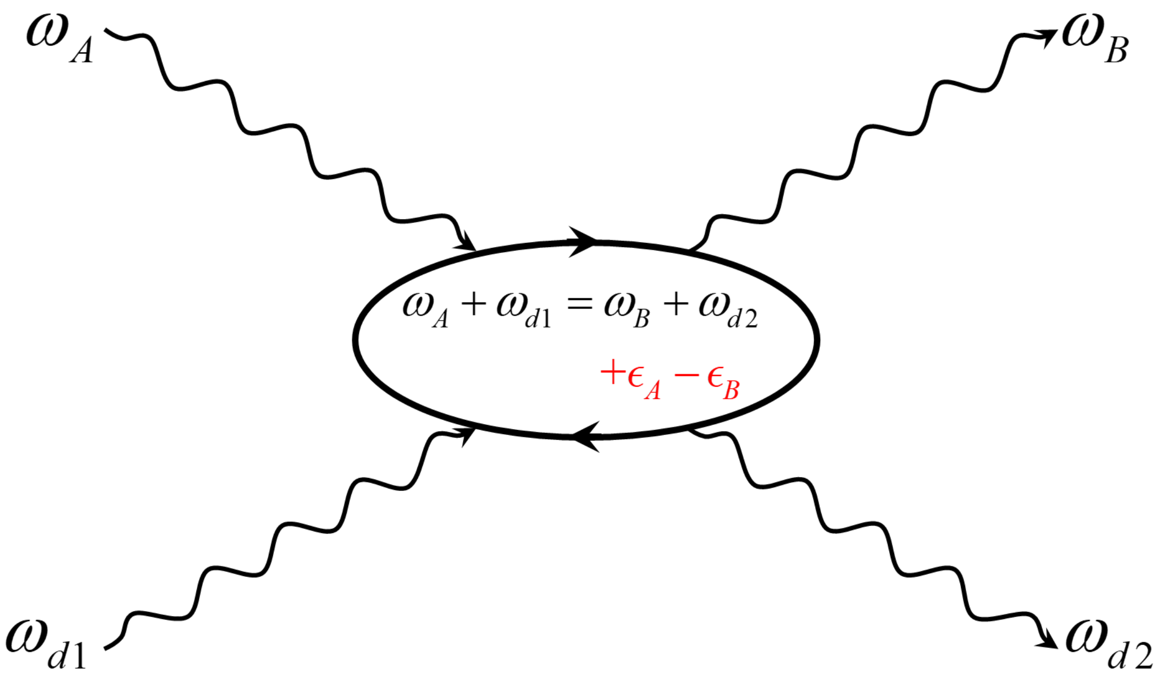
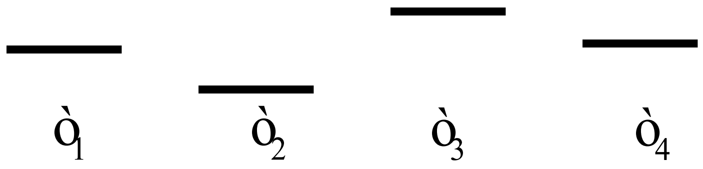
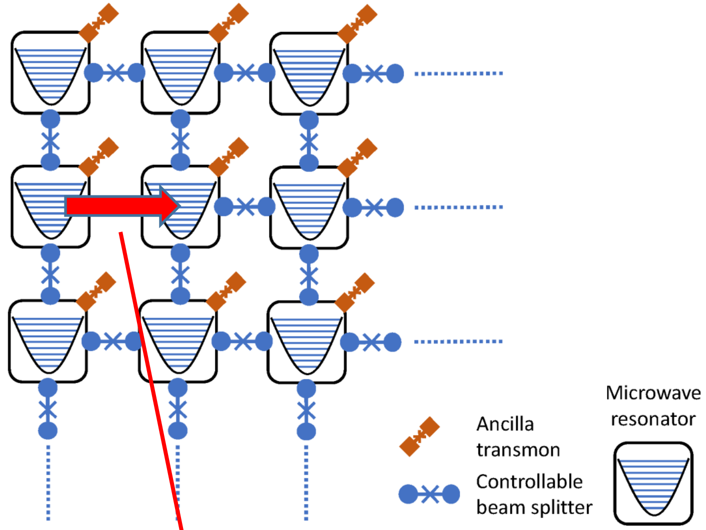
How do we create the randomly disordered site energy terms?

$$H_V \equiv \sum_k \epsilon_k b_k^\dagger b_k$$

Fully in-operando programmable site energies.







Detuning the pump drives means the photon cannot resonantly hop from one cavity to the next: $\omega_{d1} - \omega_{d2} = \omega_B - \omega_A + \epsilon_A - \epsilon_B$.

Analogy to electrodynamics:

$$\vec{E} = -\vec{\nabla} V + \frac{\partial \vec{A}}{\partial t}$$

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? $$H_U \equiv U \sum_k b_k^\dagger b_k^\dagger b_k b_k$$ Hubbard *U* boson repulsion

The quadratic terms in the Hamiltonian are now fully programmable.

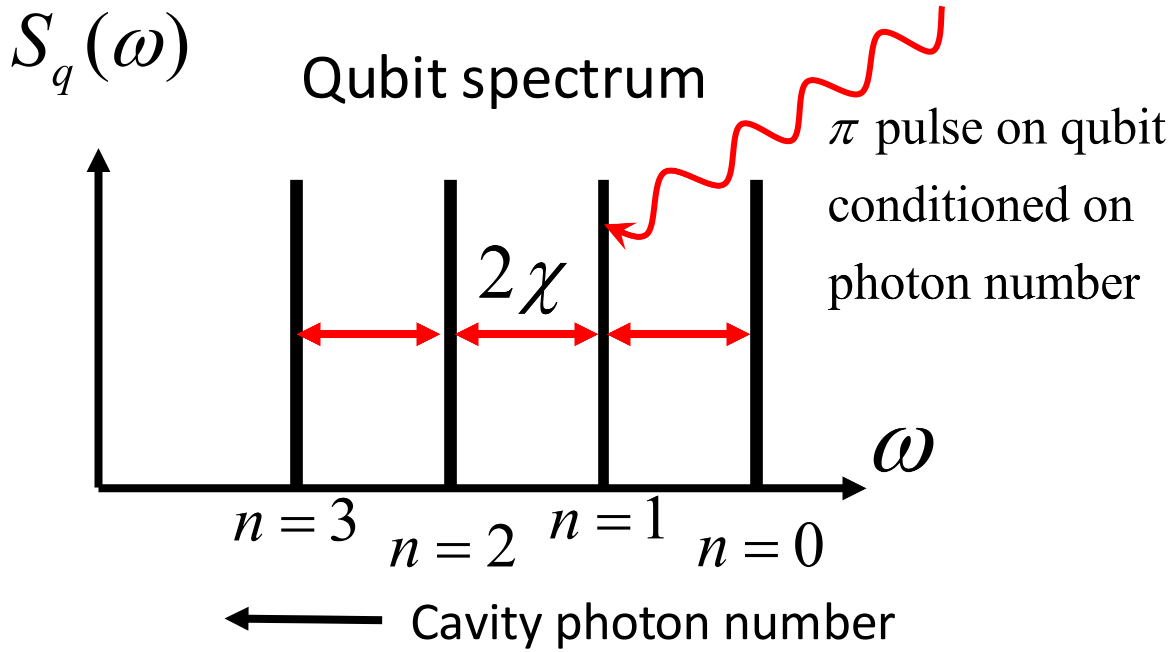
How do we program the boson-boson repulsive interaction term?

Synthesizing the cavity Hubbard *U* interaction using the cavity-qubit dispersive coupling.

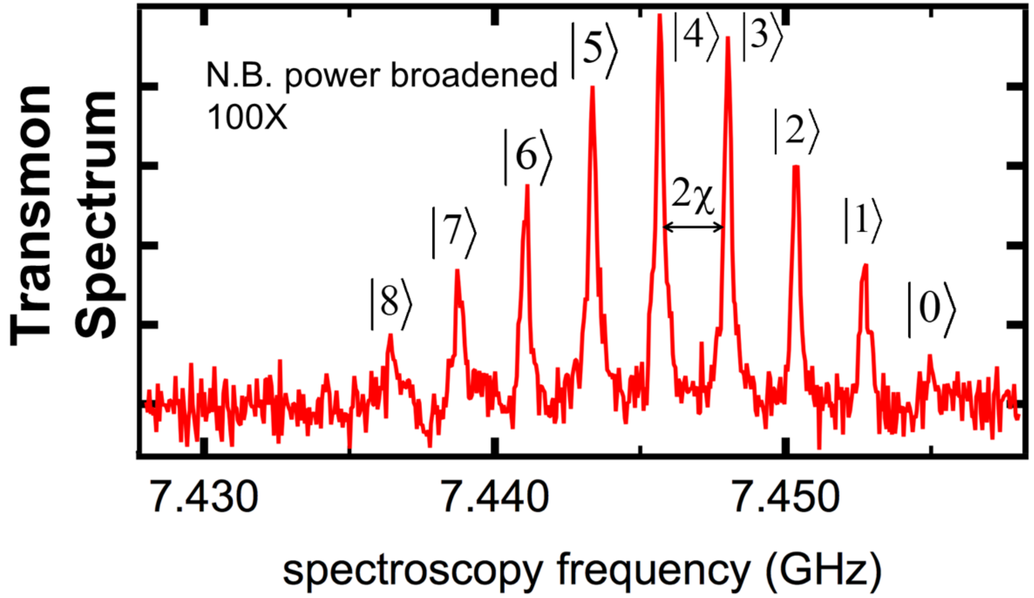
Qubit-Cavity Strong Dispersive Coupling

$$\omega_c \neq \omega_q$$

$$H = \omega_c a^\dagger a + \frac{\omega_q}{2}\sigma^z + \chi\,\sigma^z\, a^\dagger a$$



Microwaves are particles!!

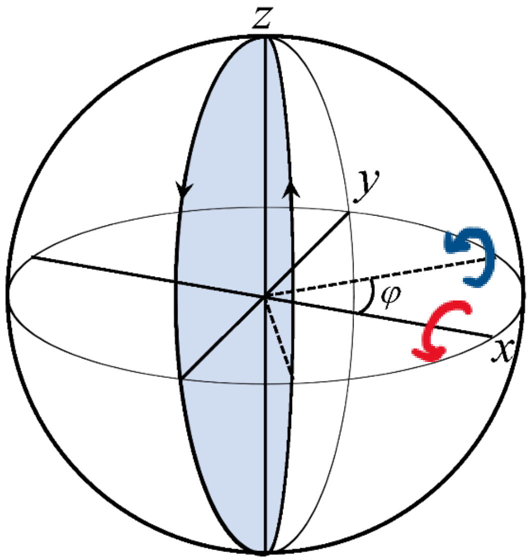


SNAP-gate Instruction Set

Unconditional Displacement Gate

$$D[\alpha] \equiv e^{\alpha a^\dagger - \alpha^* a}$$

$$U_{\mathrm{SNAP}}(\vec{\theta})\left[\sum_m \Psi_m |m\rangle\right] = \left[\sum_m e^{i\theta_m}\Psi_m |m\rangle\right]$$



Use dispersive coupling of qubit to cavity to apply separate independent geometric phases to each photon Fock state.

Provable universal control.

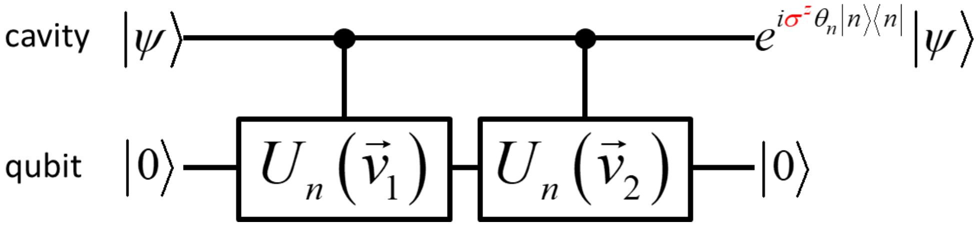
Krastanov et al., *Phys. Rev. A* **92**, 040303(R) (2015)

Heeres et al., *Phys. Rev. Lett.* **115**, 137002 (2015)

$$U_{\mathrm{SNAP}}(\vec{\theta}) \equiv e^{i\sigma^z \sum_{n=0}^{n_{\max}} \theta_n \hat{P}_n}$$

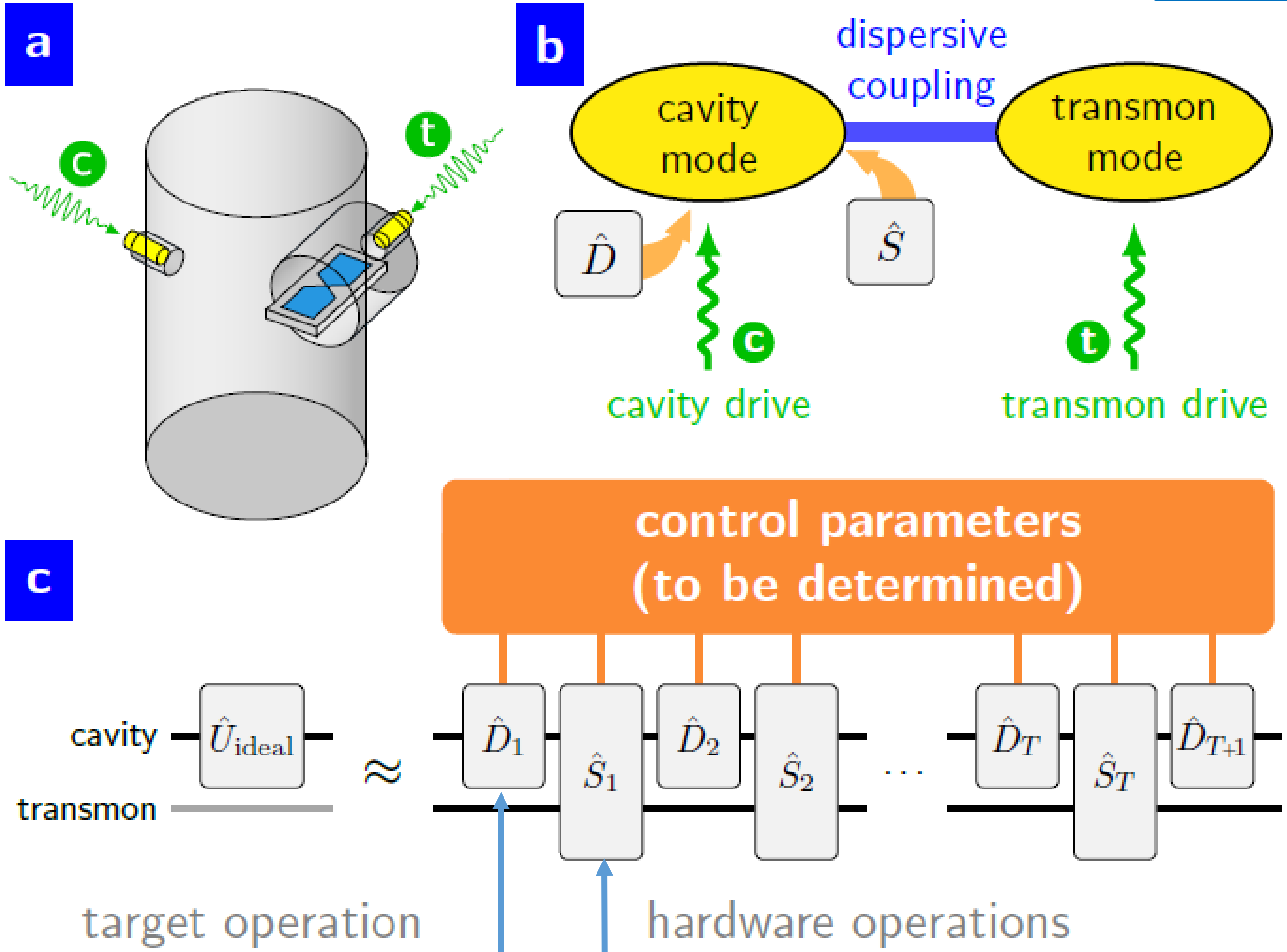
$$\hat{P}_n = |n\rangle\langle n|$$

$$\vec{\theta} = (\theta_0, \theta_1, \ldots, \theta_{n_{\max}})$$



$$U_n(\vec{v}) = e^{-\frac{i}{2}(v_x\sigma^x + v_y\sigma^y)|n\rangle\langle n|}$$

SNAP instruction set is extremely efficient

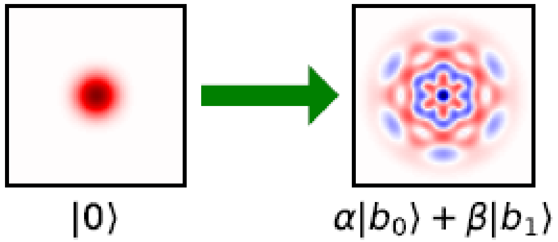


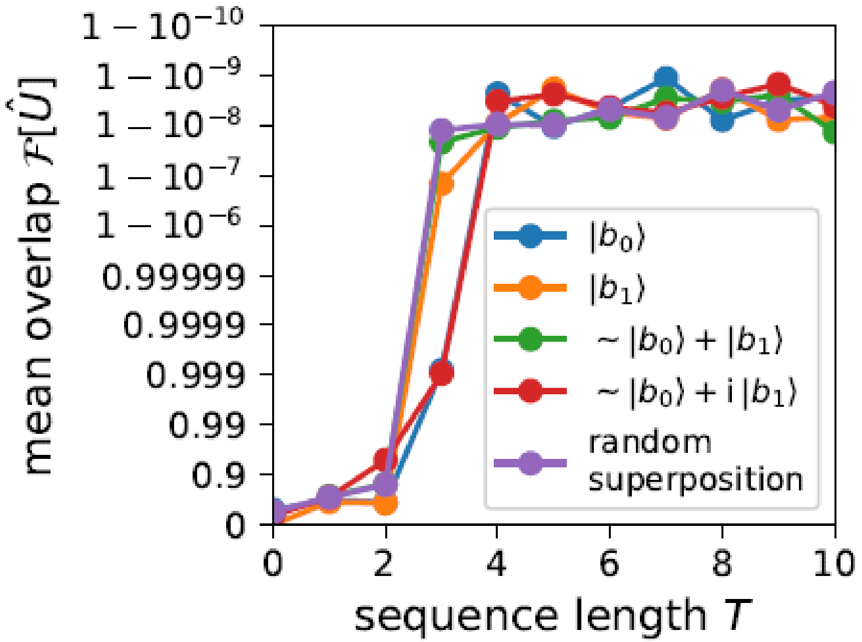
Cavity displacement gate

SNAP gate

Binomial QEC Code State Prep.

$$|b_0\rangle = \frac{|0\rangle + \sqrt{3}|6\rangle}{2} \qquad |b_1\rangle = \frac{\sqrt{3}|3\rangle + |9\rangle}{2}$$





'Efficient cavity control with SNAP gates,' Fösel et al., arXiv:2004.14256

Programming the Hubbard boson repulsion

$$H_U \equiv U \sum_k b_k^\dagger b_k^\dagger b_k b_k$$

On each site *k:*

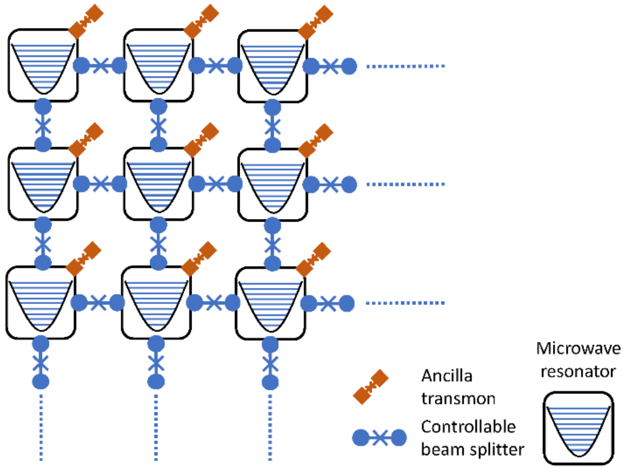
$$e^{-iH_U t} = e^{-iUt\hat{n}_k(\hat{n}_k - 1)} = U_{\text{SNAP}}(\vec{\theta})$$

$$\theta_n = -Ut[n(n-1)]$$

$$U_{\text{SNAP}}(\vec{\theta}) \equiv e^{i\sigma^z \sum_{n=0}^{n_{\max}} \theta_n \hat{P}_n}$$

$$\hat{P}_n = |n\rangle\langle n|$$

$$\vec{\theta} = (\theta_0, \theta_1, \ldots, \theta_{n_{\max}})$$



All required technology has been experimentally demonstrated, but not yet at scale.

$$H = H_J + H_V + H_U$$

☑ $$H_J \equiv \sum_{\langle ij \rangle} \left\{ J_{ij} b_i^\dagger b_j + J_{ij}^* b_j^\dagger b_i \right\}$$ **boson hopping**

☑ $$H_V \equiv \sum_k \epsilon_k b_k^\dagger b_k$$ **randomly disordered site energies**

☑ $$H_U \equiv U \sum_k b_k^\dagger b_k^\dagger b_k b_k$$ **Hubbard *U* boson repulsion**

Experimental status with microwave cavities:

- Two sites ☑
- Two sites ☑
- One site ☑

Outline:

- Target application I: FQHE for bosons
  - Brief review of FQHE physics in 2D continuum and Laughlin wave function/plasma analogy
  - How do we realize a 2D lattice model version with microwave photons?
  - Bose-Hubbard model in a (pseudo) magnetic field
    - Boson hopping as an optical beam-splitter Hamiltonian
    - Programmable random site disorder via beam-splitter detuning
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  - **Non-equilibrium quantum dynamics: bath engineering to stabilize Laughlin state against boson loss**
- Target application II: Z2 lattice gauge theory

These microwave bosons are not strictly conserved.

It is possible to create an engineered quantum bath that will gently (adiabatically) replace missing bosons as long there is an excitation gap.

But for the FQHE:

Novel slow dynamics if the 'hole' (missing photon) fractionalizes into two 'charge'-1/2 quasiholes:

Autonomous stabilization of photonic Laughlin states through angular momentum potentials,
R. O. Umucalılar, J. Simon and I. Carusotto, Phys. Rev. A **104**, 023704 (2021).

Stabilizing the Laughlin state of light: dynamics of hole fractionalization, Kurilovich et al.,
SciPost Phys. **13**, 107 (2022), https://scipost.org/SciPostPhys.13.5.107 .

Laughlin state before a boson has been lost at position *Z*

$$\psi_m[z] = \prod_{i<j} (z_i - z_j)^m \, e^{-\frac{1}{4}\sum_k |z_k|^2} = \prod_{1<k} (z_1 - z_k)^m \prod_{1<i<j} (z_i - z_j)^m \, e^{-\frac{1}{4}\sum_k |z_k|^2}$$

Laughlin state after a boson has been lost at position *Z*

$$\Psi_m[z] = \prod_{1<k} (Z - z_k)^m \prod_{1<i<j} (z_i - z_j)^m \, e^{-\frac{1}{4}\sum_k |z_k|^2}$$

Looks the same but coordinate of particle 1 (say) has been converted from a quantum coordinate to a classical variable *Z* giving location of the *m* quasiholes that have been created. *AN ENERGY EIGENSTATE*.

There is (essentially) no ‘back action’ on the other electrons creating any collective excitations.

We can design an irreversible process to quickly but gently refill the hole (replace the missing photon) thereby stabilizing the state.

Details later. First some pictures.

Kurilovich et al., SciPost Phys. 13, 107 (2022)

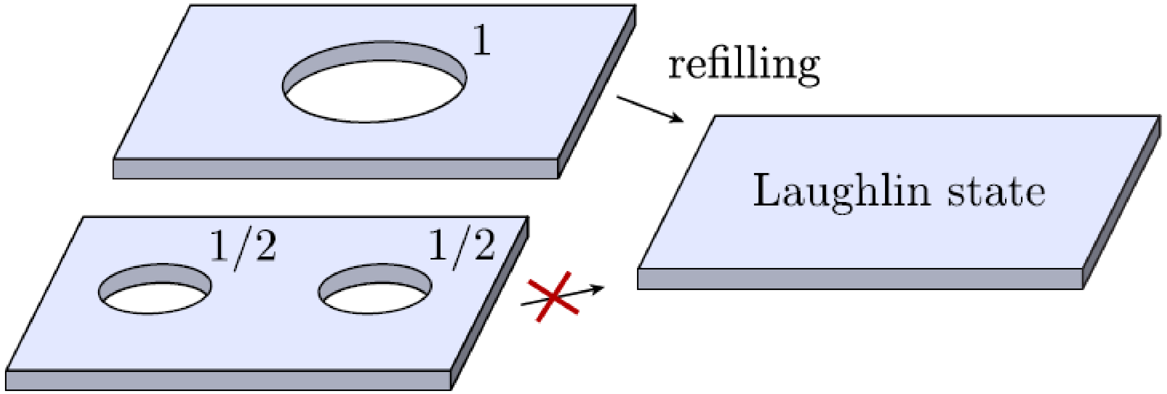
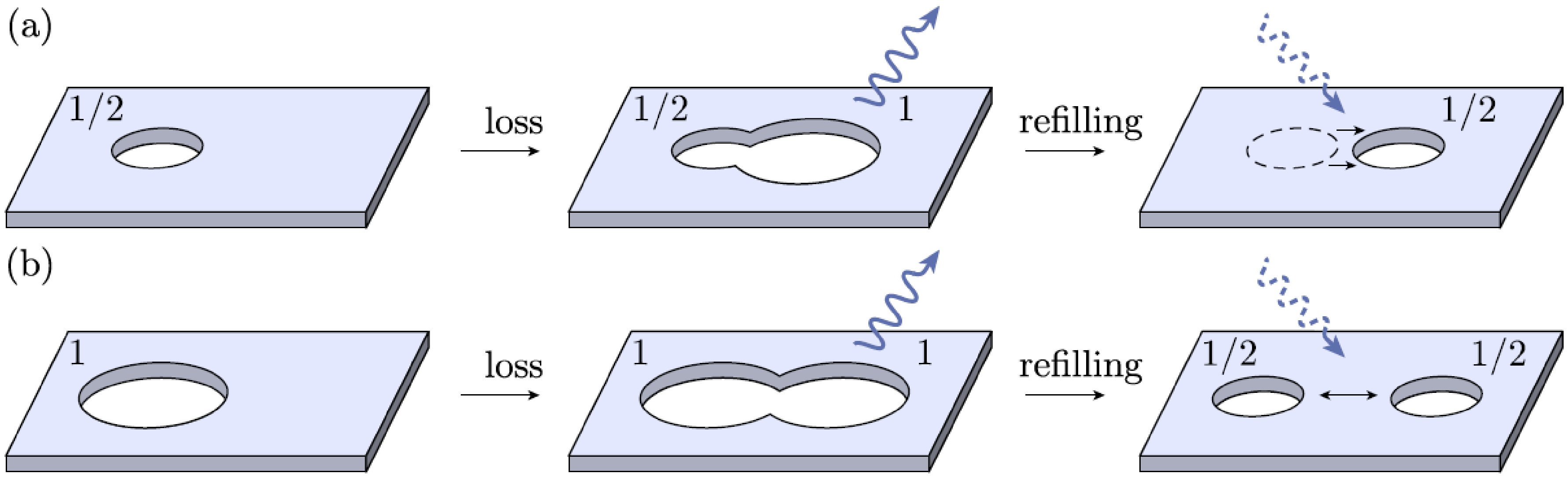


Figure 1: Conceptual picture for the stabilization of the photonic Laughlin state at half-filling. Full hole in the Laughlin state is refilled by adding a photon locally. At the same time two remote quasiholes – which also correspond to the absence of a single photon – cannot be refilled by the stabilization setup. This is because a real (i.e. "bare") photon cannot break into two pieces, in contrast to a hole in the fractional quantum Hall state that can break into two anyons.

Kurilovich et al., SciPost Phys. 13, 107 (2022)

Dynamics dominated by incoherent loss and refilling processes.



Curiously, both the quasi-hole diffusion rate D and the fission/fusion rates are controlled by the loss rate κ (assuming rapid refilling $\Gamma \gg \kappa$).

Figure 2: Dissipative dynamics of quasiholes in the stabilized photonic Laughlin state at half-filling. (a) Diffusion of a single quasihole. A photon is first lost in the vicinity of the quasihole and then quickly refilled by the stabilization setup at a different location. As a result the position of the quasihole is shifted in a random direction. (b) A full hole breaks into two stable quasiholes. Again, this requires loss of an additional photon in the vicinity of the initial hole that is followed by a subsequent refilling at a different location.

Kurilovich et al., SciPost Phys. 13, 107 (2022)

Lindblad master equation for density matrix

$$\frac{d\rho}{dt} = -i[H,\rho] + \mathcal{L}_\kappa \rho + \mathcal{L}_\Gamma \rho \,.$$

$\mathcal{L}_\kappa$ describes the loss of photons due to the dissipation,

$$\mathcal{L}_\kappa \rho = \kappa \int d^2r \left(\psi(\boldsymbol{r})\rho\psi^\dagger(\boldsymbol{r}) - \frac{1}{2}\{\rho, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\} \right),$$

where κ is the photon decay rate.

The superoperator $\mathcal{L}_\Gamma$ describes the action of the stabilization setup that refills the lost photons,

$$\mathcal{L}_\Gamma \rho = \Gamma \int d^2r \left(\tilde{\psi}^\dagger(\boldsymbol{r})\rho\tilde{\psi}(\boldsymbol{r}) - \frac{1}{2}\{\rho, \tilde{\psi}(\boldsymbol{r})\tilde{\psi}^\dagger(\boldsymbol{r})\} \right).$$

Here, Γ is the rate at which the photons are injected into the system

$\tilde{\psi}(\boldsymbol{r}) = \mathcal{P}\psi(\boldsymbol{r})\mathcal{P}$ is the annihilation operator projected on the subspace of the LLL states with zero interaction energy, i.e., the quasihole states [but not quasi-electron states]

This is accomplished by bath engineering, to be described shortly.

Two-component reaction-diffusion equation for the non-equilibrium dynamics

loss refill fission fusion

$$\begin{cases} \partial_t n_{\mathrm{h}} = \frac{1}{2}\kappa - \Gamma n_{\mathrm{h}} - \frac{1}{2}c\kappa n_{\mathrm{h}} + c\kappa n_{\mathrm{qh}}^2 , \\ \partial_t n_{\mathrm{qh}} = D\nabla^2 n_{\mathrm{qh}} + c\kappa n_{\mathrm{h}} - 2c\kappa n_{\mathrm{qh}}^2 . \end{cases}$$

[dimensionless coefficient $c \sim 1$ determined by microscopic details]

Curiously, both the quasi-hole diffusion rate D and the fission/fusion rates are controlled by the loss rate κ (assuming rapid refilling Γ ? κ).

Steady-state solution:

$$n_{\mathrm{h}} = \frac{\kappa}{2\Gamma} \text{ (agrees with naive detailed balance)}$$

$$n_{\mathrm{qh}} = \sqrt{\frac{n_{\mathrm{h}}}{2}} = \sqrt{\frac{\kappa}{4\Gamma}} \text{ ? } n_{\mathrm{h}}$$

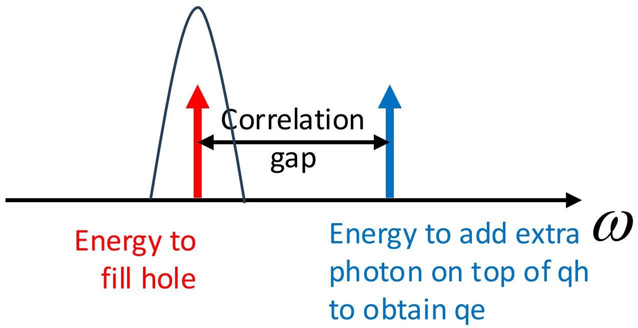
Deviation of photon density from ideal Laughlin state:

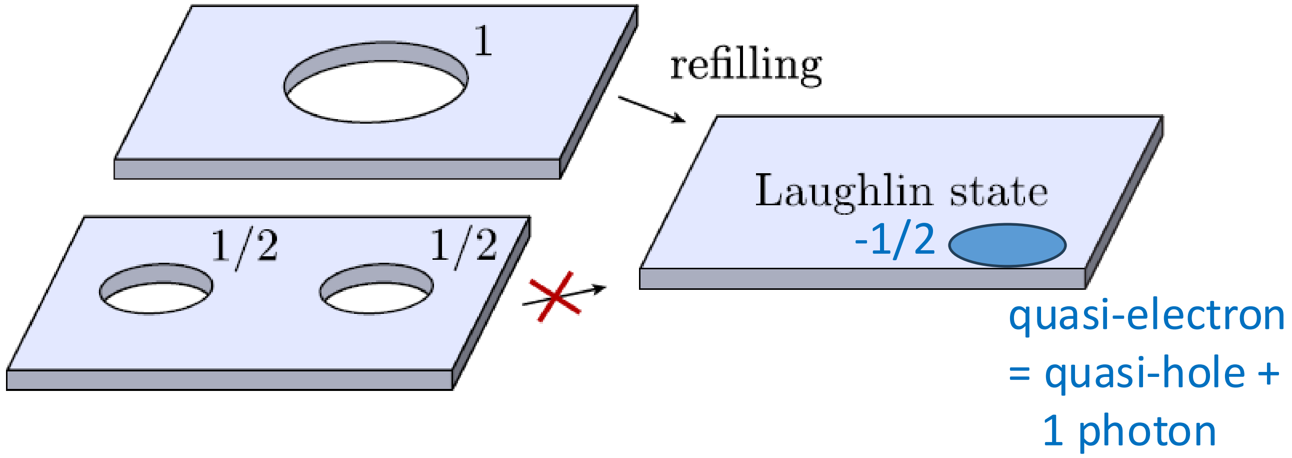
$$\Delta N = n_{\mathrm{h}} + \frac{1}{2}n_{\mathrm{qh}} \sim \frac{1}{4}\sqrt{\frac{\kappa}{\Gamma}} \text{ (dominated by isolated qh particles)}$$

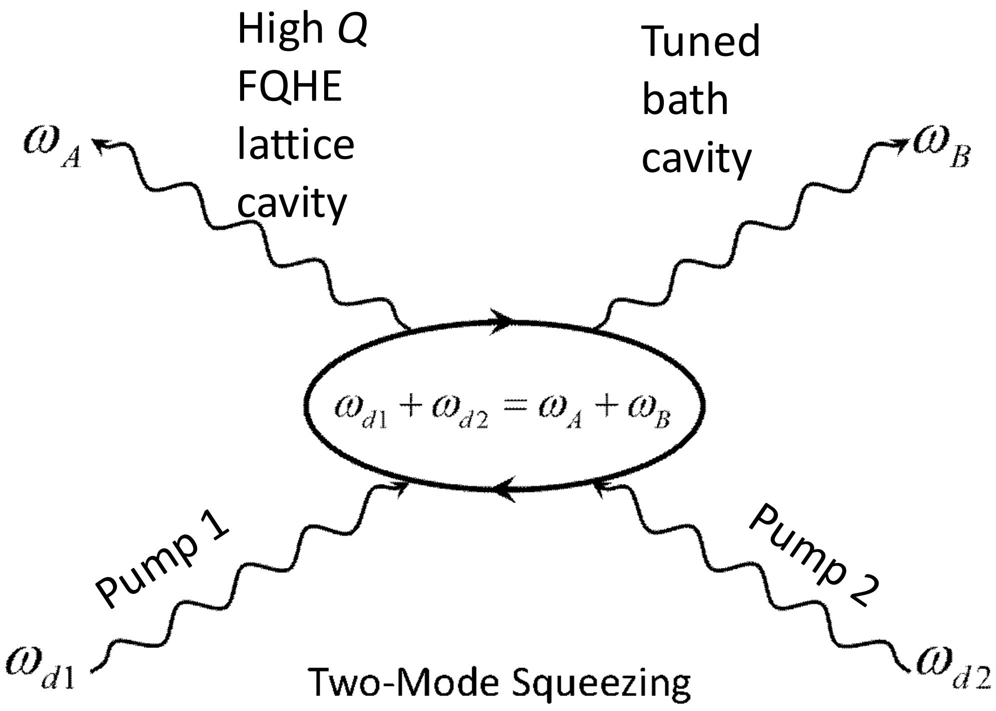
Smallest relaxation rate

$$\tau^{-1} \sim \frac{\kappa^{3/2}}{\Gamma^{1/2}}$$

Bath engineering:
Goal: Gently insert a photon to fill a hole (lost photon) with just the right energy, but not enough energy to cross the FQHE gap and add a photon at a place where there is no hole.







Bath engineering: Linewidth of bath cavity should be large for irreversibility but smaller than the correlation gap

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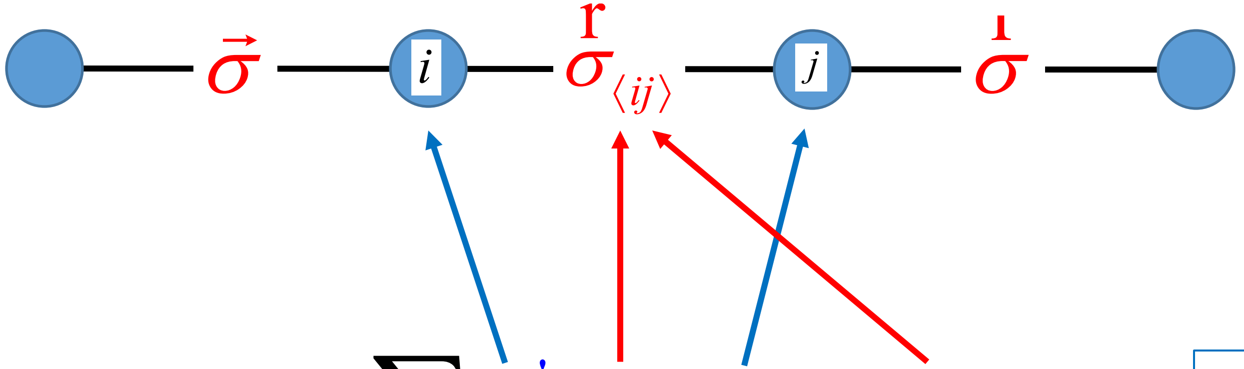
- **Target application II: Z2 lattice gauge theory**

Another target application: ¢$_2$ lattice gauge theory for bosons hopping on a lattice

bosons on lattice sites, ¢$_2$ gauge fields on links



$$H = -J\sum_{\langle ij\rangle} a_i^\dagger \sigma^z_{\langle ij\rangle} a_j - \lambda \sigma^x_{\langle ij\rangle}$$

Physical intuition: each boson hop changes the sign of $\sigma^x_{\langle ij\rangle}$

<u>Dynamical</u> gauge field:

σ^z = vector potential

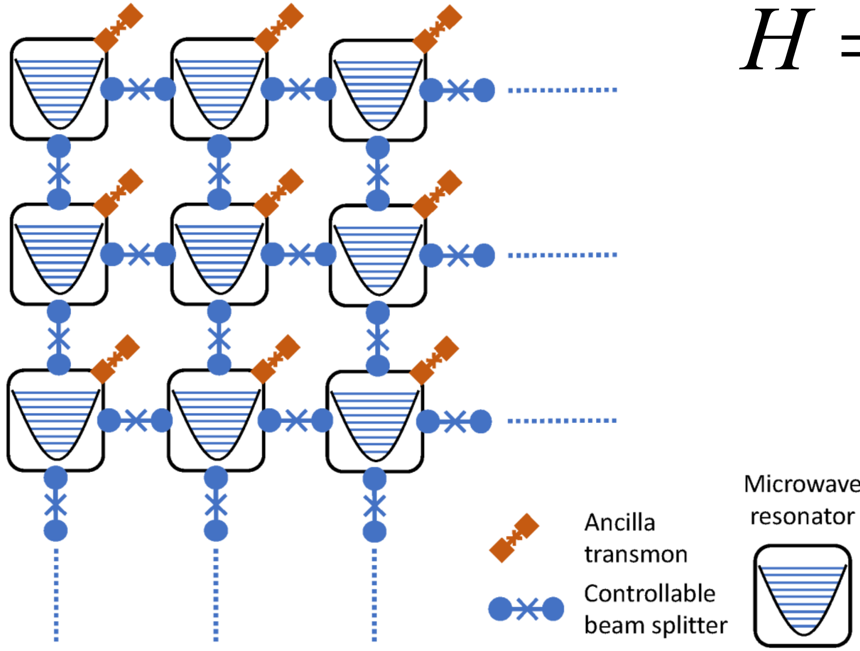
σ^x = conjugate electric field

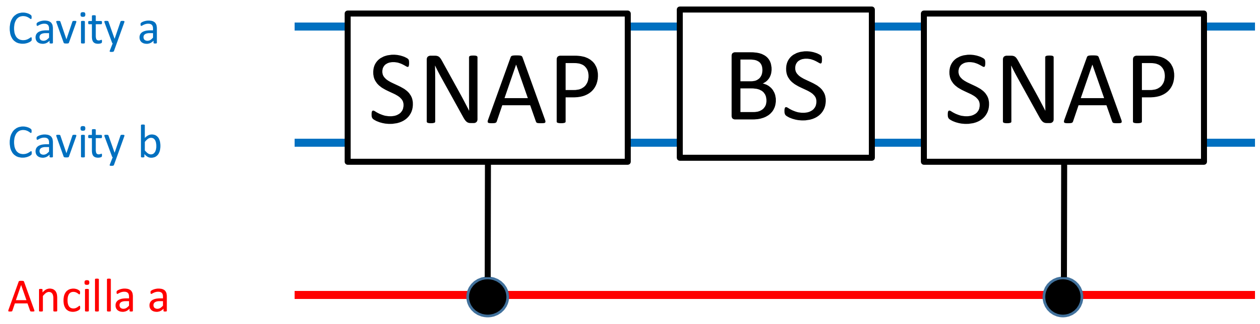
Gauss Law Constraint $G_j \equiv \prod_{i\in\langle ij\rangle} \sigma^x_{\langle i,j\rangle} e^{i\pi a_j^\dagger a_j}$

$$[H, G_j] = 0$$

Realization of ¢$_2$ lattice gauge theory for bosons with SNAP ISA

$$H = -J\sum_{\langle ij\rangle} a_i^\dagger \sigma^z_{\langle ij\rangle} a_j - \lambda \sigma^x_{\langle ij\rangle}$$





Physical intuition: each boson hop changes the sign of $\sigma^x_{\langle ij\rangle}$

$$U(\vec{\theta})e^{Jt[a_i^\dagger a_j - a_j^\dagger a_i]}U^\dagger(\vec{\theta}) = e^{iJt[a_i^\dagger \sigma^z_{\langle ij\rangle} a_j + a_j^\dagger \sigma^z_{\langle ij\rangle} a_i]}$$

(see next slide for algebraic proof)

$$U(\vec{\theta}) = \text{SNAP} = e^{i\frac{\pi}{2} a_i^\dagger a_i \sigma^z_{\langle ij\rangle}}$$ [controlled parity gate]

[In 1D only need ancillae connected to 1 cavity.]

$$U(\vec{\theta})e^{Jt[a_i^\dagger a_j - a_j^\dagger a_i]}U^\dagger(\vec{\theta}) = e^{iJt[a_i^\dagger \sigma^z_{\langle ij\rangle} a_j + a_j^\dagger \sigma^z_{\langle ij\rangle} a_i]}$$

$$U(\vec{\theta}) = \text{SNAP} = e^{i\frac{\pi}{2}a_i^\dagger a_i \sigma^z_{\langle ij\rangle}} \quad \text{[controlled parity gate]}$$

Proof:

$$e^{i\frac{\pi}{2}a_i^\dagger a_i \sigma^z_{\langle ij\rangle}} a_i e^{-i\frac{\pi}{2}a_i^\dagger a_i \sigma^z_{\langle ij\rangle}} = e^{-i\frac{\pi}{2}\sigma^z_{\langle ij\rangle}} a_i = -i\sigma^z_{\langle ij\rangle} a_i$$

and

$$e^{i\frac{\pi}{2}a_i^\dagger a_i \sigma^z_{\langle ij\rangle}} a_i^\dagger e^{-i\frac{\pi}{2}a_i^\dagger a_i \sigma^z_{\langle ij\rangle}} = e^{-i\frac{\pi}{2}\sigma^z_{\langle ij\rangle}} a_i^\dagger = +i\sigma^z_{\langle ij\rangle} a_i^\dagger$$

C²QA ISA & LGT (theory) collaboration


Nathan Wiebe
U. Toronto & PNNL


Tim Stavenger
PNNL


Chris Kang
U. Washington


Eleanor Crane
UCL


Micheline Solely
Yale


Kevin Smith
Yale

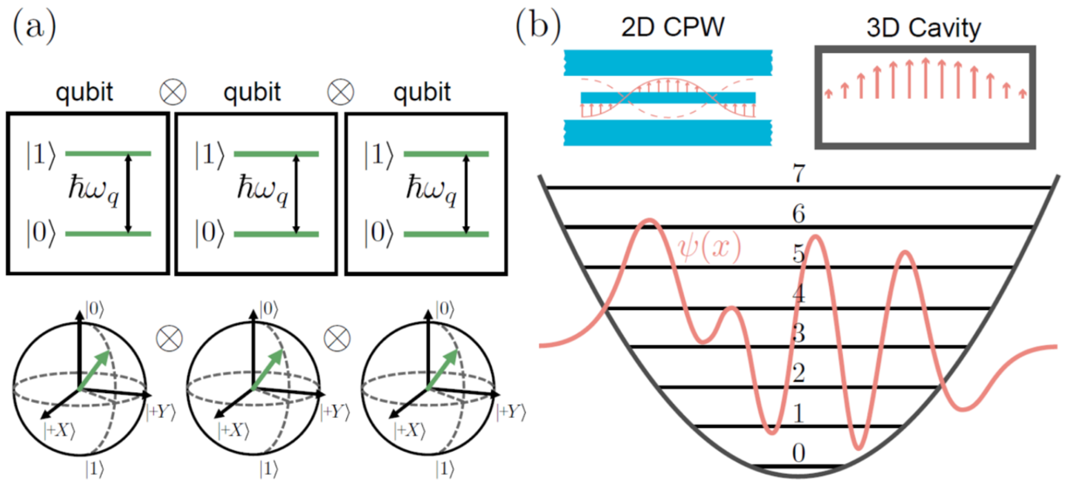
+ Ike Chuang (MIT)
+ Ali Javadi (IBM)

+ Alec Eickbusch and Devoret Lab

+Michael DeMarco
Teague Tomesh
Lena Funke
Stefan Kuehn

Discrete variable (transmon qubits)

Continuous variable (microwave or mechanical oscillators)



$$|\psi\rangle = a_0|000\rangle + a_1|001\rangle + a_2|010\rangle + a_3|011\rangle + a_4|100\rangle + a_5|101\rangle + a_6|110\rangle + a_7|111\rangle$$

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle + a_2|2\rangle + a_3|3\rangle + a_4|4\rangle + a_5|5\rangle + a_6|6\rangle + a_7|7\rangle$$

Boson Fock (photon number) states

- Instruction Set Architecture for hybrid qubit/oscillator systems
- Qiskit extension to oscillators
  - Represent $\Lambda = 2^n$ levels of oscillator with a register of $n = \log_2 \Lambda$ qubits
  - Access ISA and Wigner tomography toolkit within Qiskit

Physics and Computer Science Foundations of

Hybrid Oscillator-Qubit Quantum Processors:

Instruction Set Architectures, Abstract Machine Models and Applications

Steven M. Girvin
Yale University

Yuan Liu, Shraddha Singh, Kevin C. Smith, Eleanor Crane, John M. Martyn, Alec Eickbusch, Alexander Schuckert, Richard D. Li, Jasmine Sinanan-Singh, Micheline B. Soley, Takahiro Tsunoda, Isaac L. Chuang, Nathan Wiebe, Steven M. Girvin

arXiv: 2407.10381
155 pages, 51 figures, 652 references

Disclosure: SMG is a consultant and equity holder in Quantum Circuits, Inc.