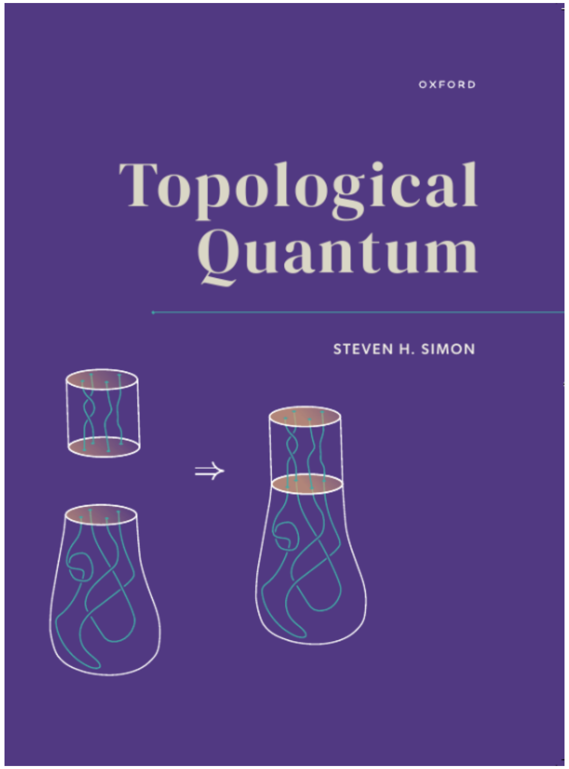
Anyons and Topological Quantum Computation



To learn even more about these topics:

← Read my book!

You can download a draft from my website

https://www-thphys.physics.ox.ac.uk/people/SteveSimon/protobook.html

Draft also has material on Fractional Quantum Hall which is not in the final book.

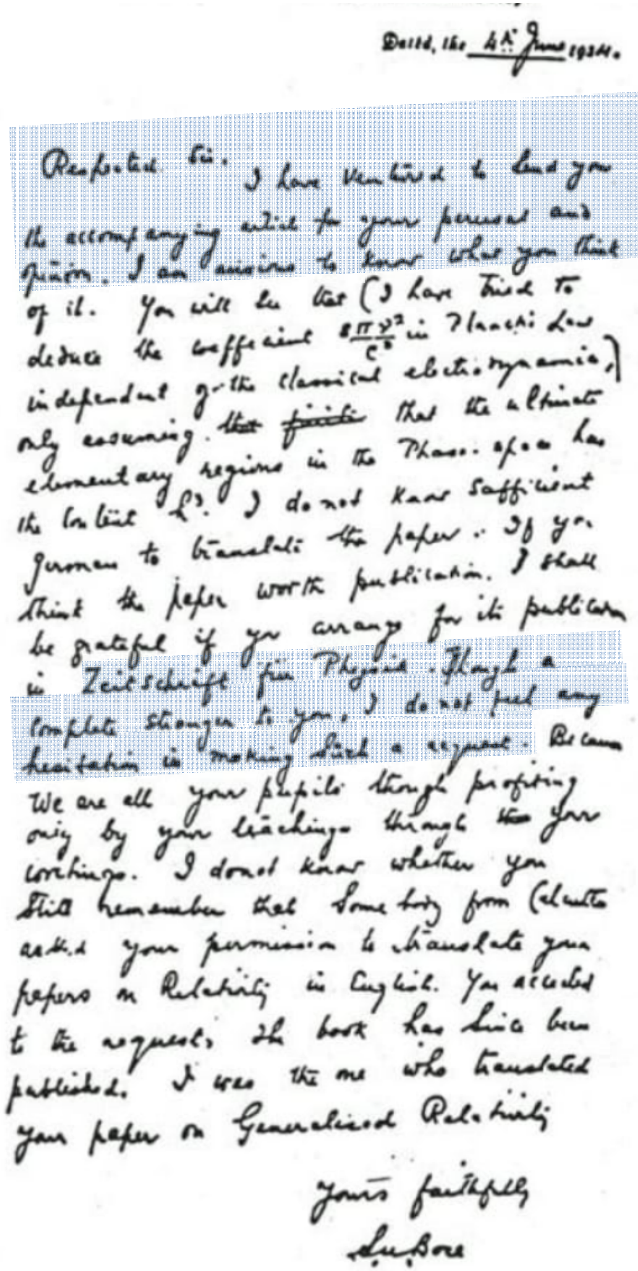
Book does not discuss Majorana physics. But see

https://www-thphys.physics.ox.ac.uk/people/SteveSimon/QCM2025/QuantumMatter2.pdf

Anyons and Topological Quantum Computation

Q: What happens when you exchange two identical particles

4th June 1924

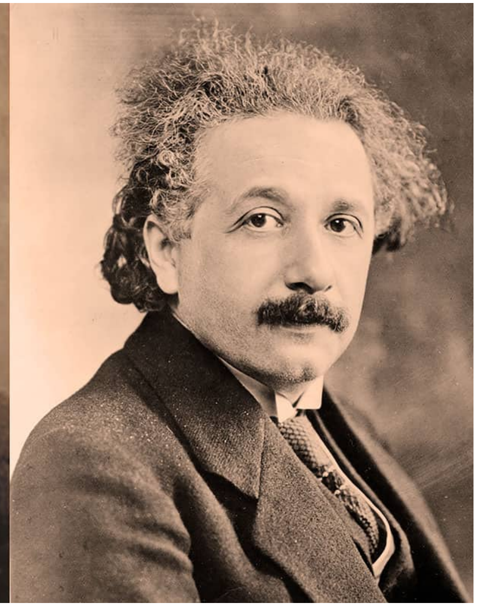


Dated, the 4th June 1924.

Respected Sir, I have ventured to send you the accompanying article for your perusal and opinion. I am anxious to know what you think of it. You will see that (I have tried to deduce the coefficient $\frac{8\pi\nu^2}{c^3}$ in Planck's Law independent of the classical electrodynamics,) only assuming ~~that finite~~ that the ultimate elementary regions in the Phase-space has the content h^3. I do not know sufficient German to translate the paper. If you think the paper worth publication, I shall be grateful if you arrange for its publication in Zeitschrift für Physik. Though a complete stranger to you, I do not feel any hesitation in making such a request. Because we are all your pupils though profiting only by your teachings through your writings. I do not know whether you still remember that somebody from Calcutta asked your permission to translate your papers on Relativity in English. You acceded to the request. The book has since been published. I was the one who translated your paper on Generalised Relativity.

Yours faithfully
S.N. Bose

Satyendra Nath Bose



Albert Einstein

Bose-Einstein Statistics:

photons, pions, gluons, phonons, excitons, Higgs, ...

1925

Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren.

(On the connection between the closure of the electron groups in the atom and the complex structure of the spectra).

January 1925

25

Wolfgang Pauli

Pauli Exclusion Principle

Pauli exclusion principle → **Fermi-Dirac Statistics**

1925

Max Born
Zeitschrift für Physik

Pascual Jordan, 1925

Pauli exclusion principle → **Fermi-Dirac Statistics**

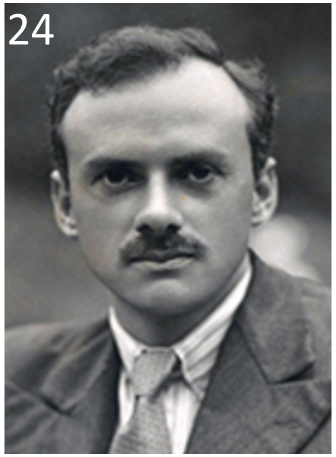
1926

SULLA QUANTIZZAZIONE DEL GAS PERFETTO MONOATOMICO

(On the quantization of the monatomic ideal gas).

January 1926

Enrico Fermi



On the Theory of Quantum Mechanics.

August 1926

Paul A. M. Dirac

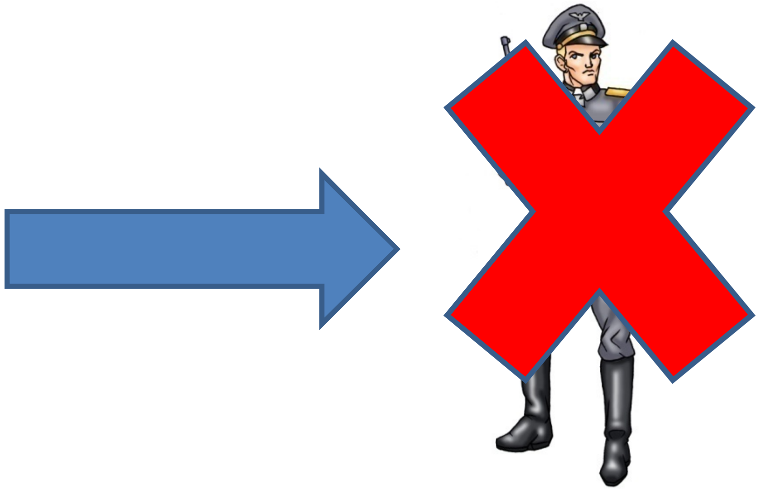
Pauli exclusion principle → **Fermi-Dirac Statistics**

(electrons, muons, quarks, ...)

1926-...

Max Born

Pascual Jordan, 1925

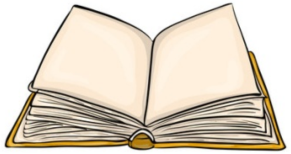


Fermi-Dirac Statistics

1930 - ...

1930: **Basics of Quantum Mechanics Finished** (QFT by ~1948)

Q: Are there other types of particles besides bosons and fermions?



No?

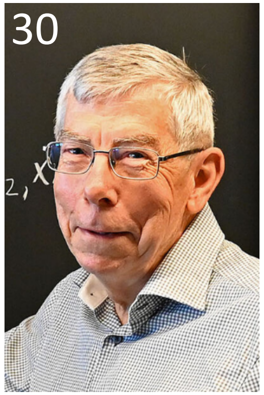
$$\hat{P} = \text{exchange} \quad \Rightarrow \quad \hat{P}\Psi(\mathbf{r}_1, \mathbf{r}_2) = \Psi(\mathbf{r}_2, \mathbf{r}_1)$$

$$\hat{P}^2 = 1 \qquad \text{Exchanging twice is identity}$$

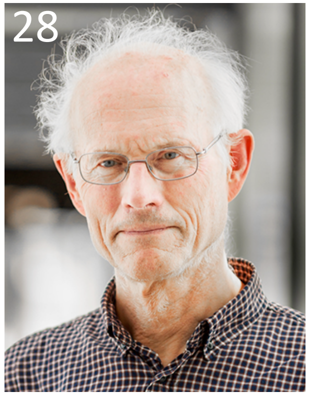
$$\sqrt{1} = \pm 1$$

- Bosons $\Psi(\mathbf{r}_1, \mathbf{r}_2) = +\Psi(\mathbf{r}_2, \mathbf{r}_1)$
- Fermions $\Psi(\mathbf{r}_1, \mathbf{r}_2) = -\Psi(\mathbf{r}_2, \mathbf{r}_1)$

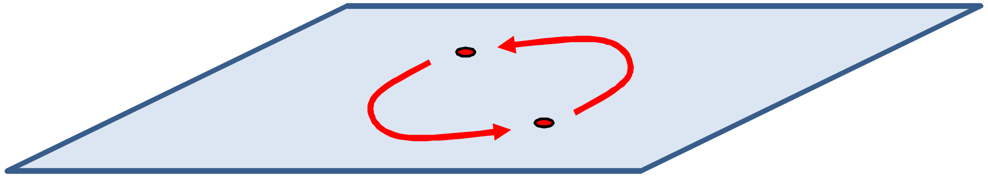
...until 1976



J. M. Leinaas



J. Myrheim



$$\Psi \longrightarrow e^{i\vartheta}\Psi$$

Bosons $\vartheta = 0$

Fermions $\vartheta = \pi$

IL NUOVO CIMENTO VOL. 37 B, N. 1 11 Gennaio 1977

"...in ... two dimensions a continuum of possible intermediate cases connects the boson and fermion cases..."

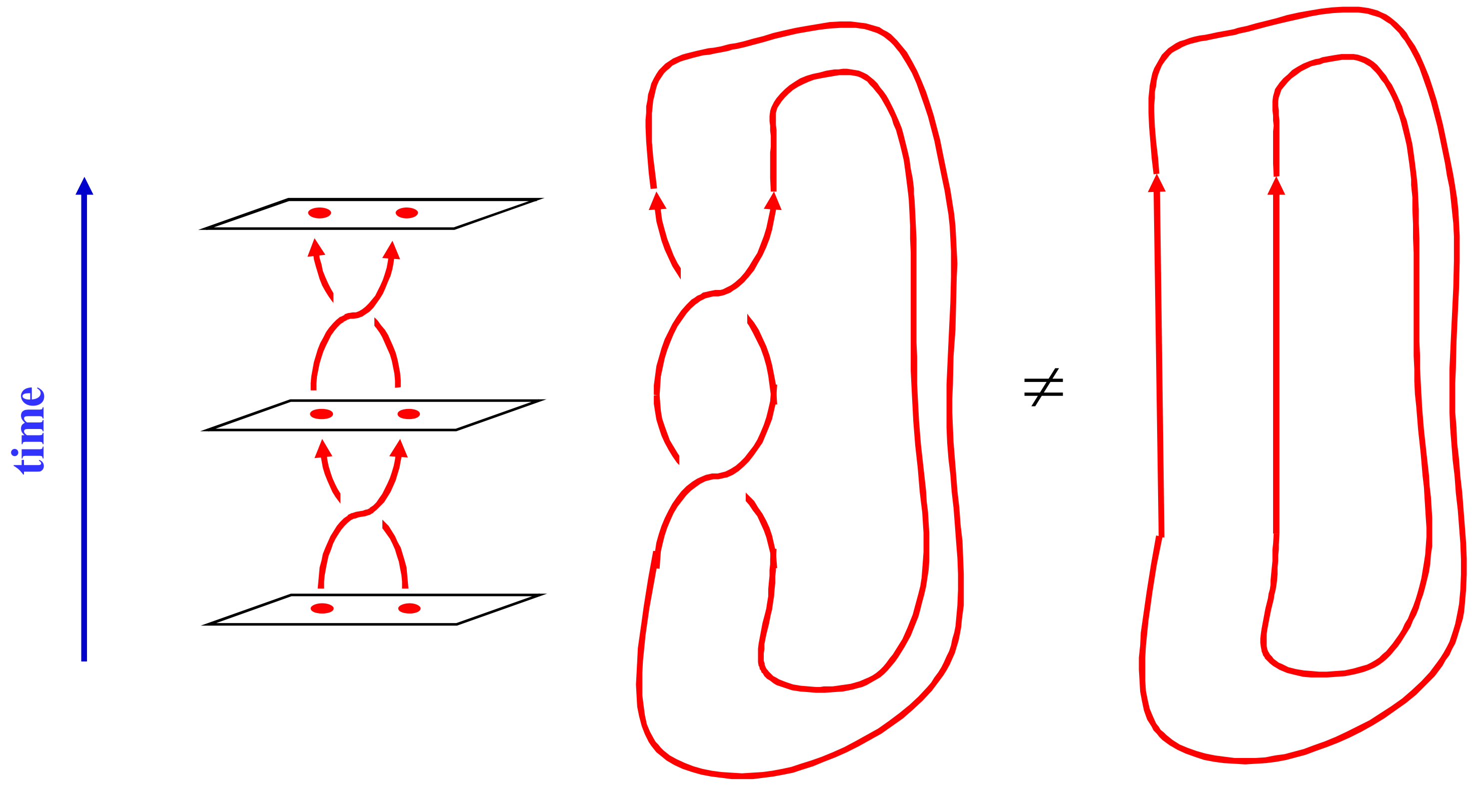
On the Theory of Identical Particles.

J. M. LEINAAS and J. MYRHEIM

Department of Physics, University of Oslo - Oslo

(ricevuto il 16 Agosto 1976)

In 2+1 Dimensions: Two Exchanges $\neq$ Identity

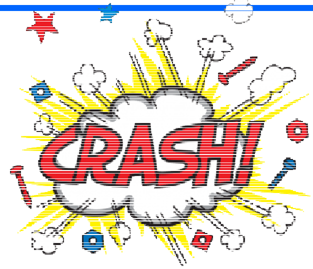


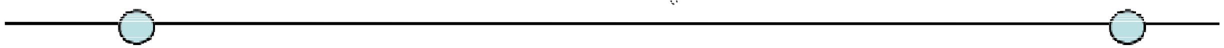
In 3+1 Dimensions: Two Exchanges = Identity

No Knots in (one dimensional) World Lines in 3+1 D !

Why are there no knots in 3+1 dimensions?

1+1 d point particles

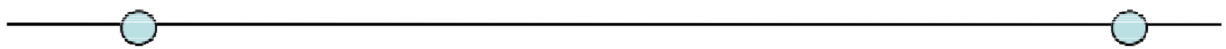




No way to cross without crashing

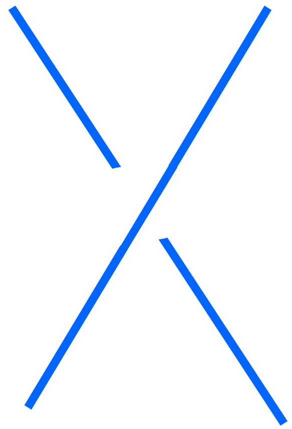
Why are there no knots in 3+1 dimensions?

2+1 d point particles



Can get to the other side without touching

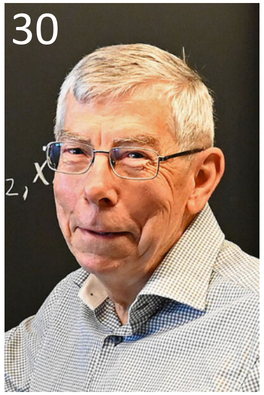
3 +1 d world lines



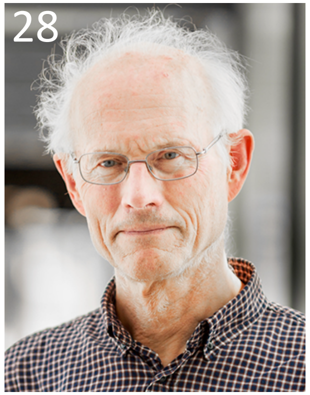
No way to change over-crossing to under-crossing without crashing

But in 3+1d, can get to other side without touching

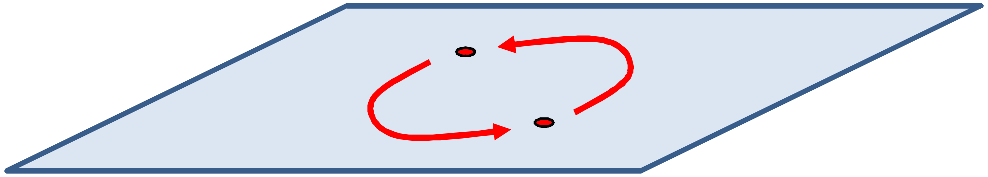
...until 1976



J. M. Leinaas



J. Myrheim



$$\Psi \longrightarrow e^{i\vartheta}\Psi$$

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IL NUOVO CIMENTO VOL. 37 B, N. 1 11 Gennaio 1977

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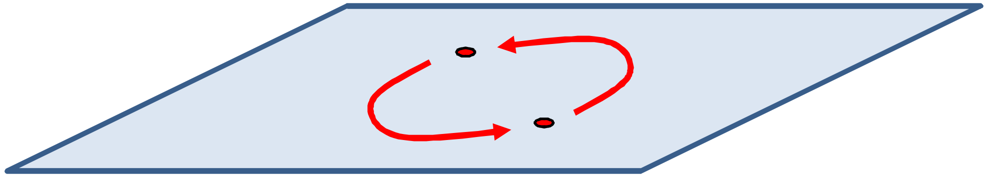
1982

Frank Wilczek



$$\Psi \longrightarrow e^{i\vartheta}\Psi$$

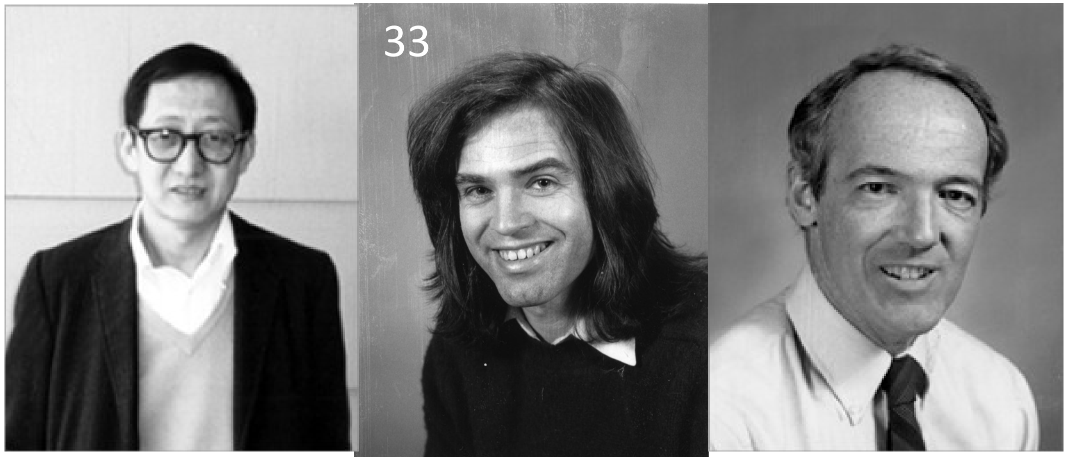
Bosons	$\vartheta = 0$
Fermions	$\vartheta = \pi$
Anyon	$\vartheta =$ **Any**thing else

Quantum Mechanics of Fractional-Spin Particles

Frank Wilczek

Although practical applications of these phenomena seem remote, I think they have considerable methodological interest and do shed light on the fundamental spin-statistics connection.

1982



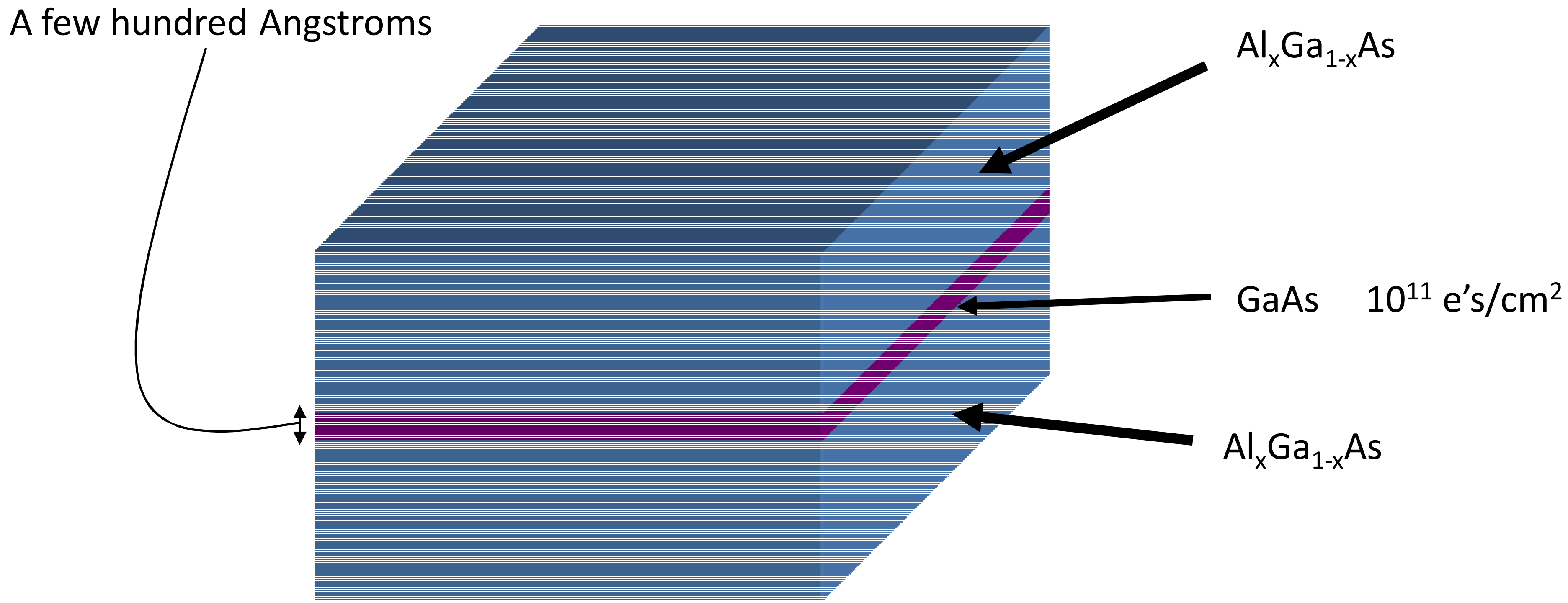
Dan Tsui   Horst Stormer   Art Gossard

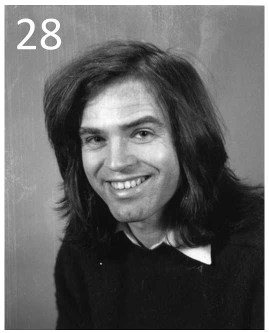
Two-Dimensional Magnetotransport in the Extreme Quantum Limit

Discovery of Fractional Quantum Hall Effect!

2D electrons in high magnetic field at low temperature

Very recently in zero magnetic field too in moiré materials!
(...talk by Jarillo-Herrero, tomorrow!)



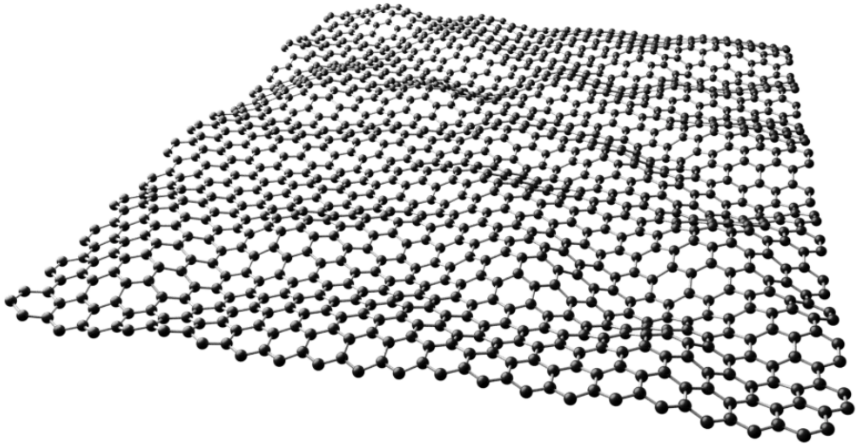


Horst Stormer

2004

Kostya Novoselov

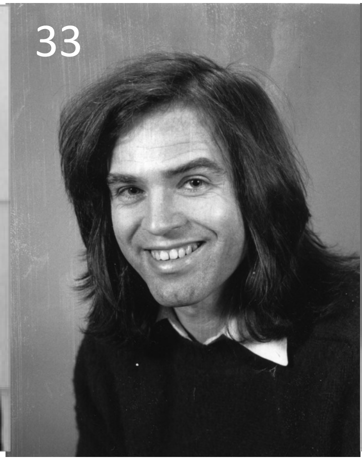
Andrew Geim



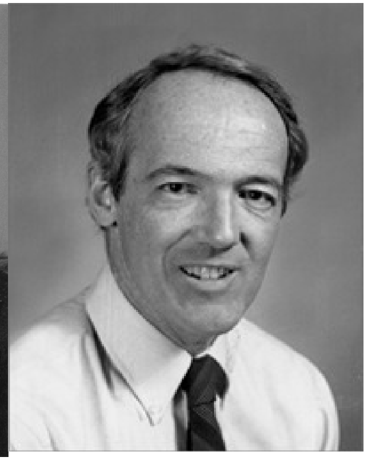
Isolation of Graphene,
Single Layer Carbon
(Nobel 2010)

1982

Dan Tsui



Horst Stormer

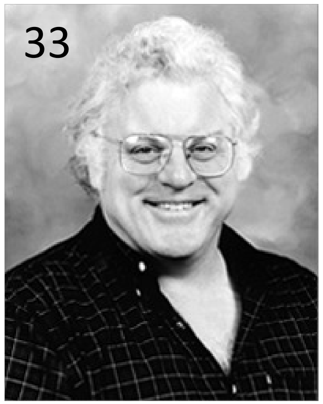


Art Gossard

Two-Dimensional Magnetotransport in the Extreme Quantum Limit

Discovery of Fractional Quantum Hall Effect!

1983



Robert Laughlin

+ Theory of Fractional Quantum Hall Effect

Nobel Prize 1998

1983-84

Low Energy Particles in Fractional Quantum Hall Effect Are Anyons!

Statistics of Quasiparticles and the Hierarchy of Fractional Quantized Hall States

B. I. Halperin

Fractional Statistics and the Quantum Hall Effect

Daniel Arovas
J. R. Schrieffer and Frank Wilczek

A Brief History of Anyons

1920's Bosons and Fermions

1977 First Proposal of Anyons

1982 Discovery of Fractional Quantum Hall Effect
1984 Excitations in Fractional Quantum Hall are Anyons!

2020 Experimental Confirmation

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- Anyon Collider Experiment : Paris, GaAs (2020)
- Anyon Interferometer: Purdue, GaAs (2020) ← Manfra Lecture
  Harvard/UCSB, Kim/Young, Graphene (2024)
- Simulation on Quantum Computers: ← Roushan Lecture
  Zurich/Beijing/Shanghai/Google/Quantinuum/IBM (2020)

Introductions coming!

Questions?

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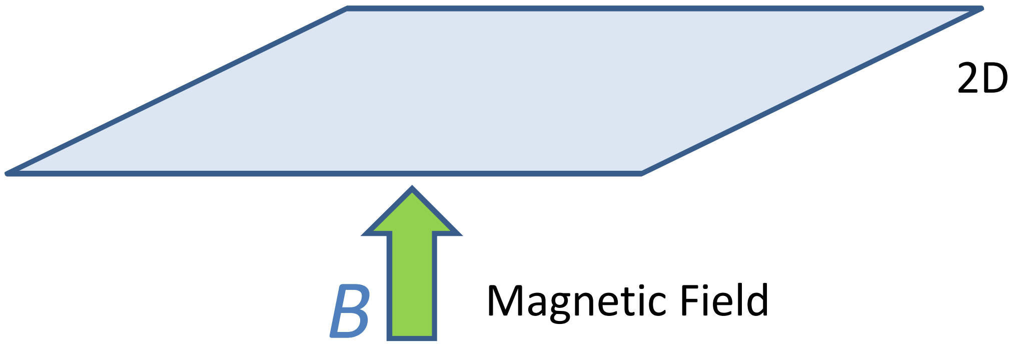
What is Fractional Quantum Hall Effect?



Cool to *very* low temperature (approx 30 mK = Room Temp / 10^4)

Density of electrons

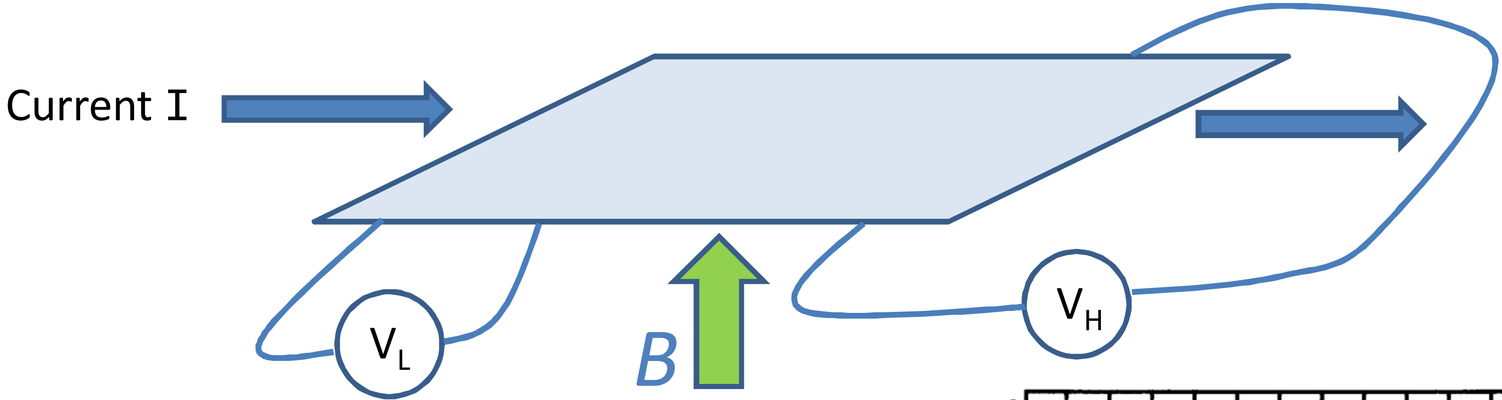
Filling $$\nu = \frac{2\pi\hbar}{e}\frac{n}{B}$$ (dimensionless)

Electron charge

Magnetic Field

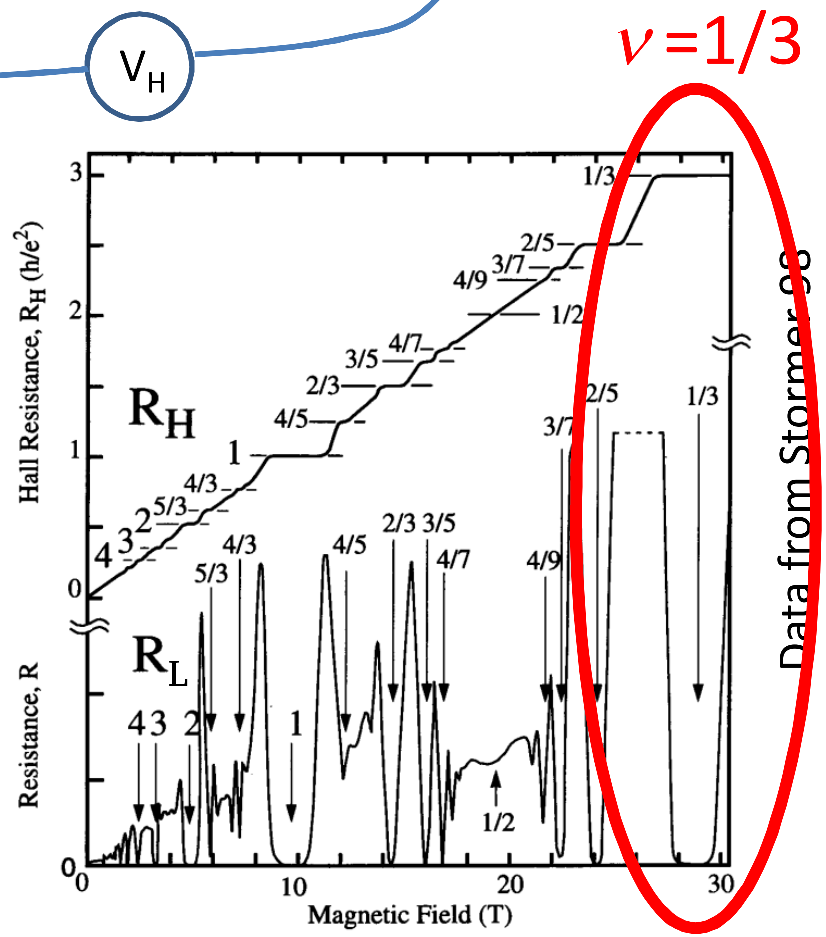
When $\nu \approx p/q \propto n/B$ with p and q coprime small integers, FQHE can occur!

What is Fractional Quantum Hall Effect?



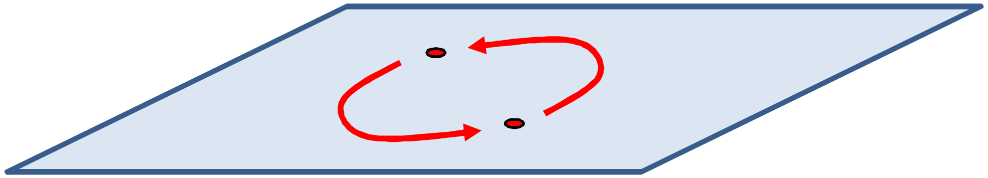
V_L=0

$$V_H / I = R_H = \frac{2\pi\hbar}{e^2}\frac{q}{p}$$ Exactly!!



When $\nu \approx p/q \propto n/B$ with p and q coprime small integers, FQHE can occur!

ν =1/3 Fractional Quantum Hall Effect



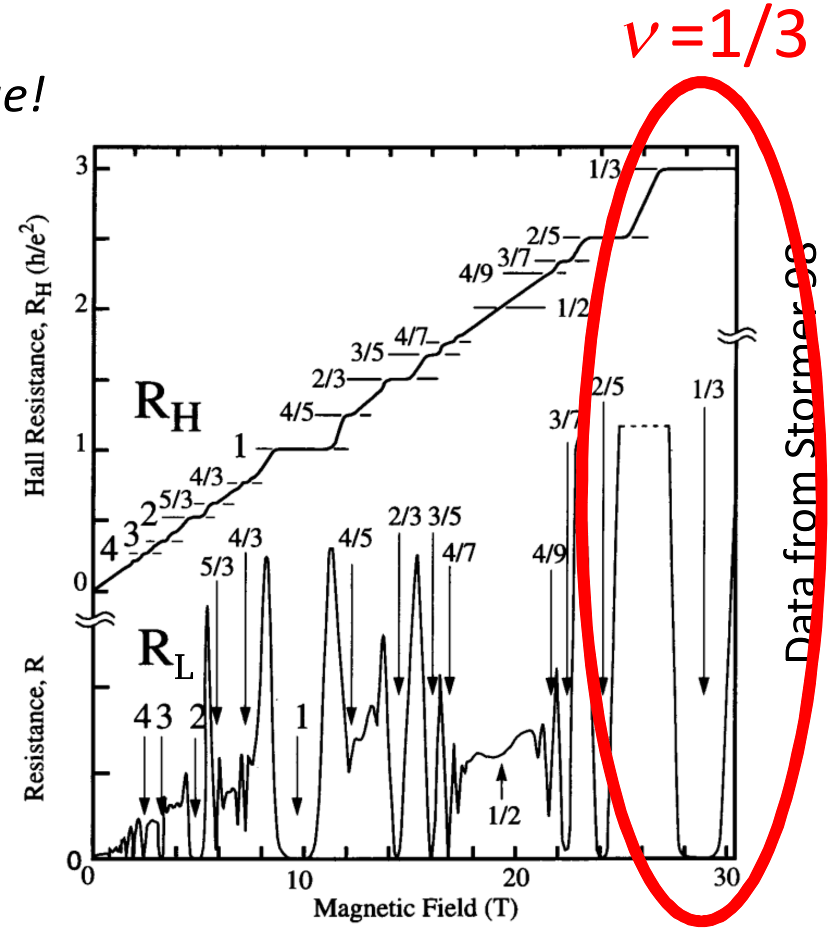
- Low energy particles have *fractional charge!*

$$e^* = \pm e/3$$

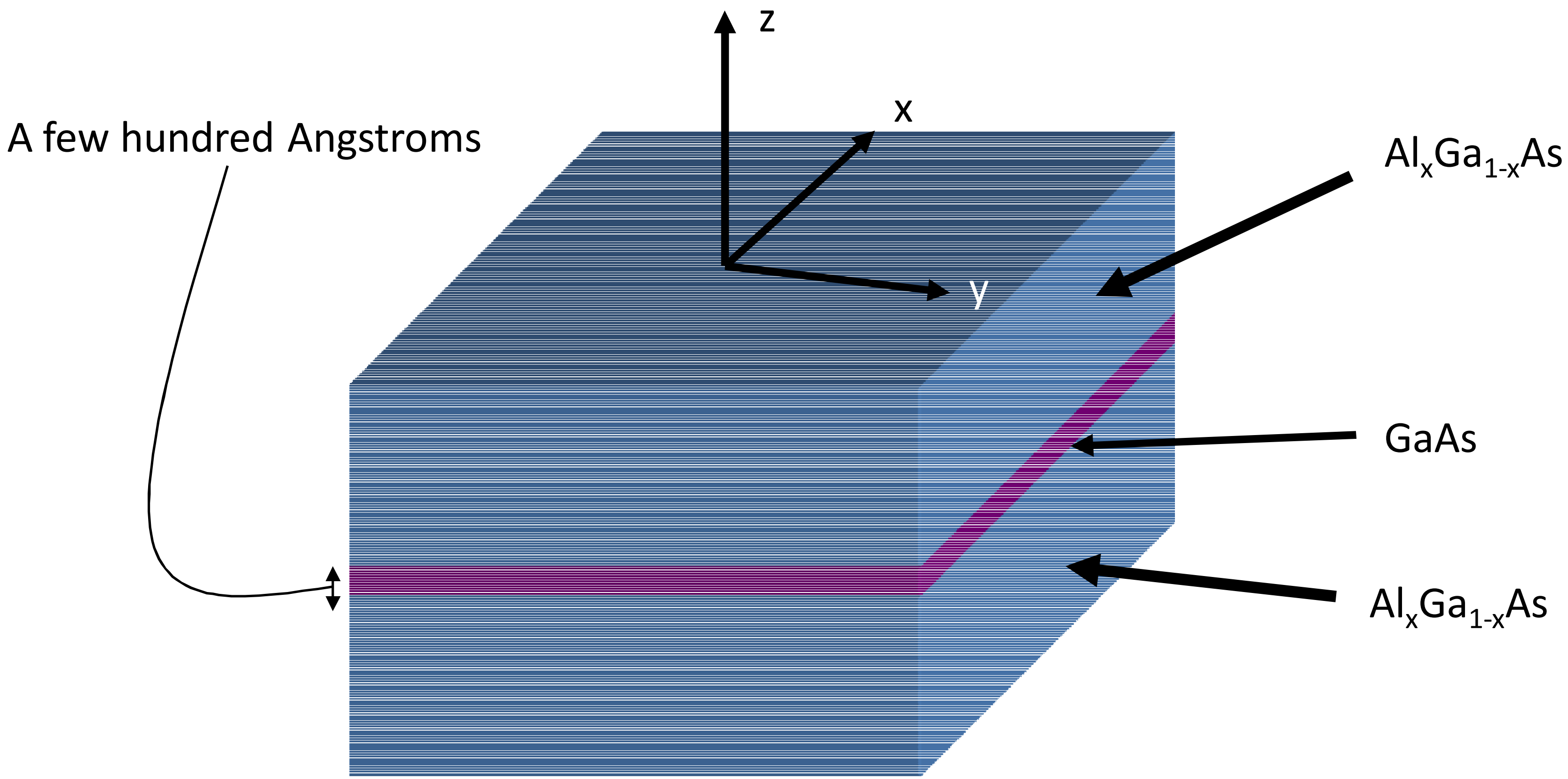
- They are also *anyons*

$$\Psi \longrightarrow e^{i\vartheta}\Psi$$

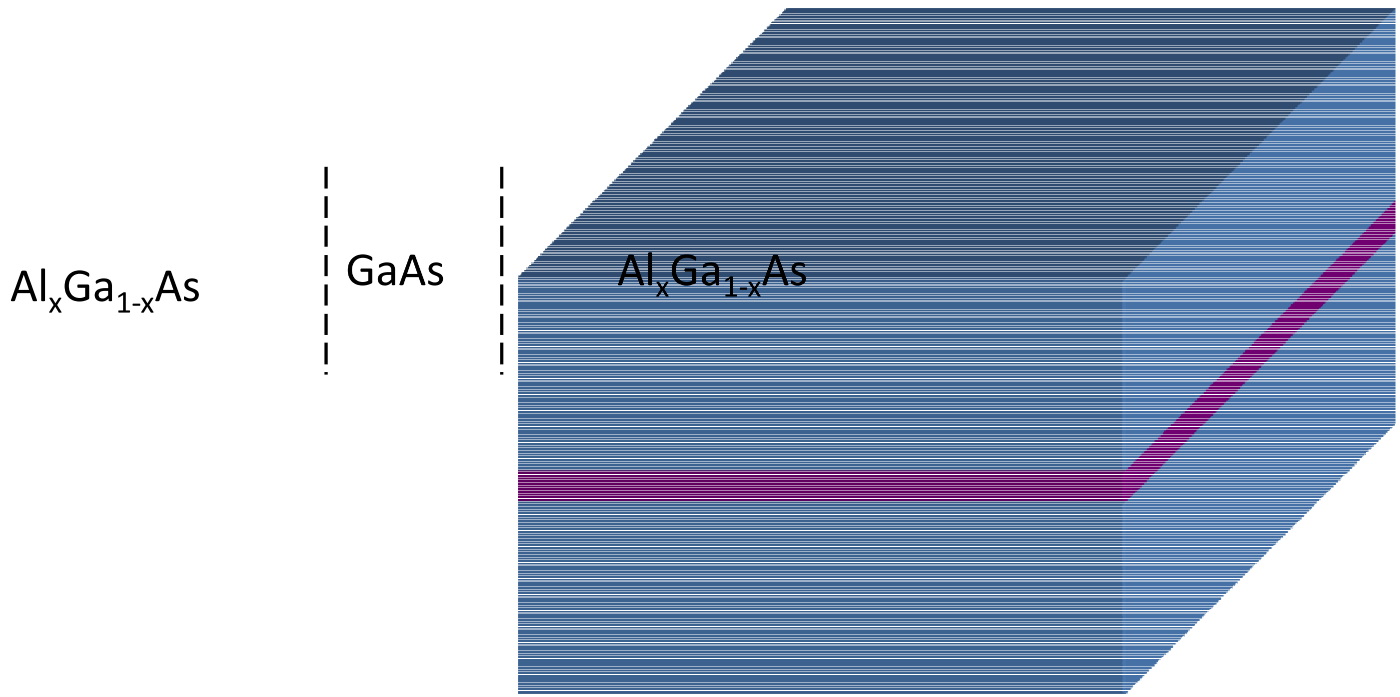
$$\vartheta = 2\pi/3$$

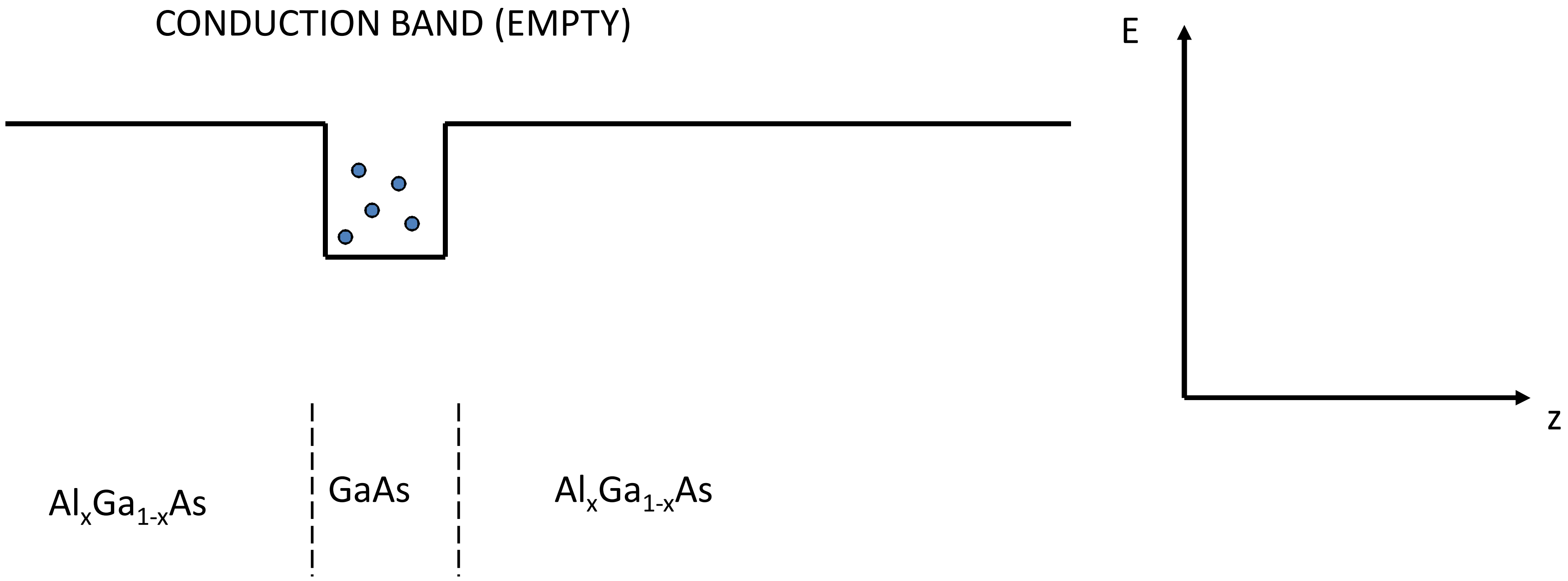


But surely the particles really live in 3D?

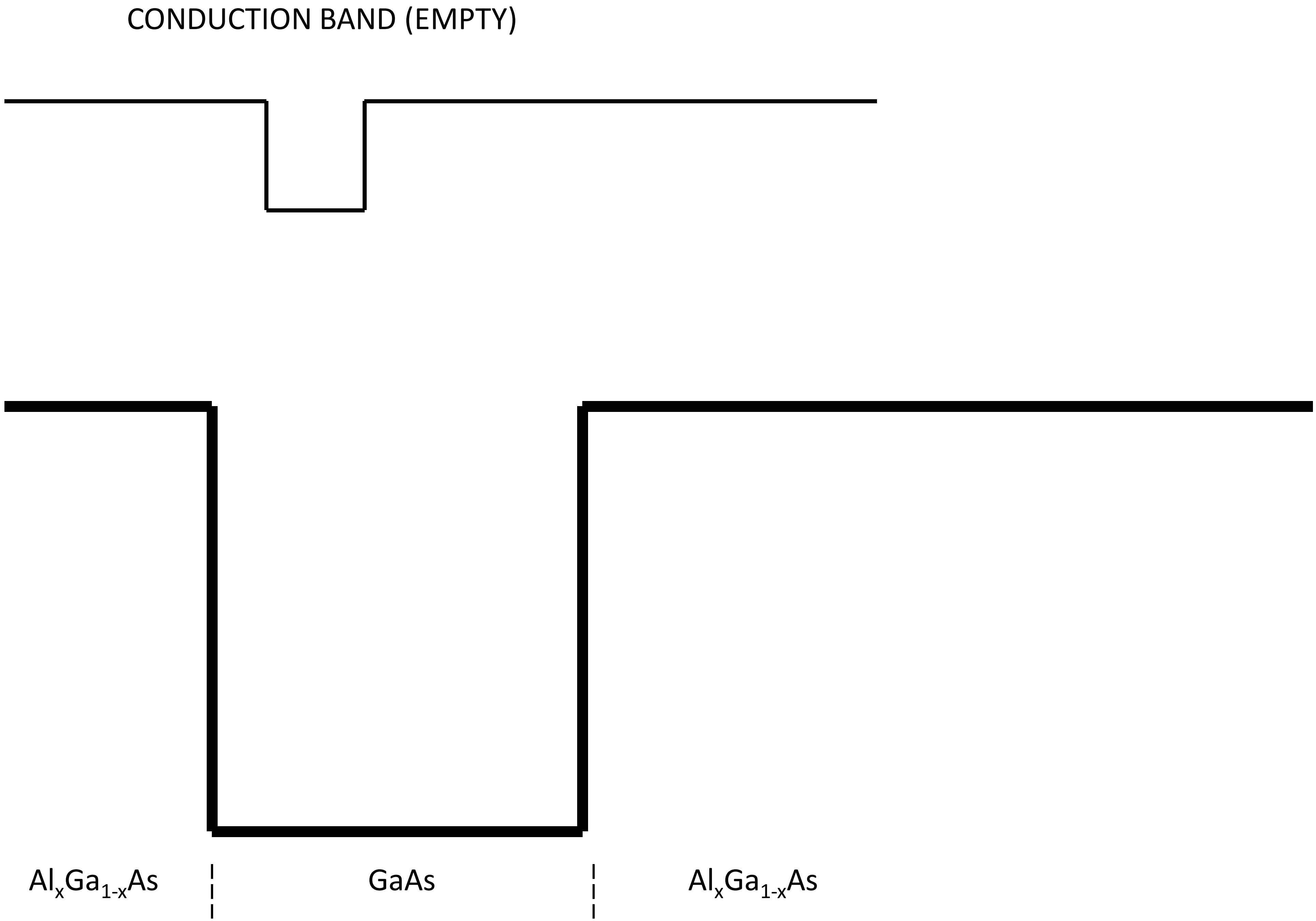


Translational Invariance in x-y plane


$Al_xGa_{1-x}As$
GaAs
$Al_xGa_{1-x}As$



CONDUCTION BAND (EMPTY)
E
z
$Al_xGa_{1-x}As$
GaAs
$Al_xGa_{1-x}As$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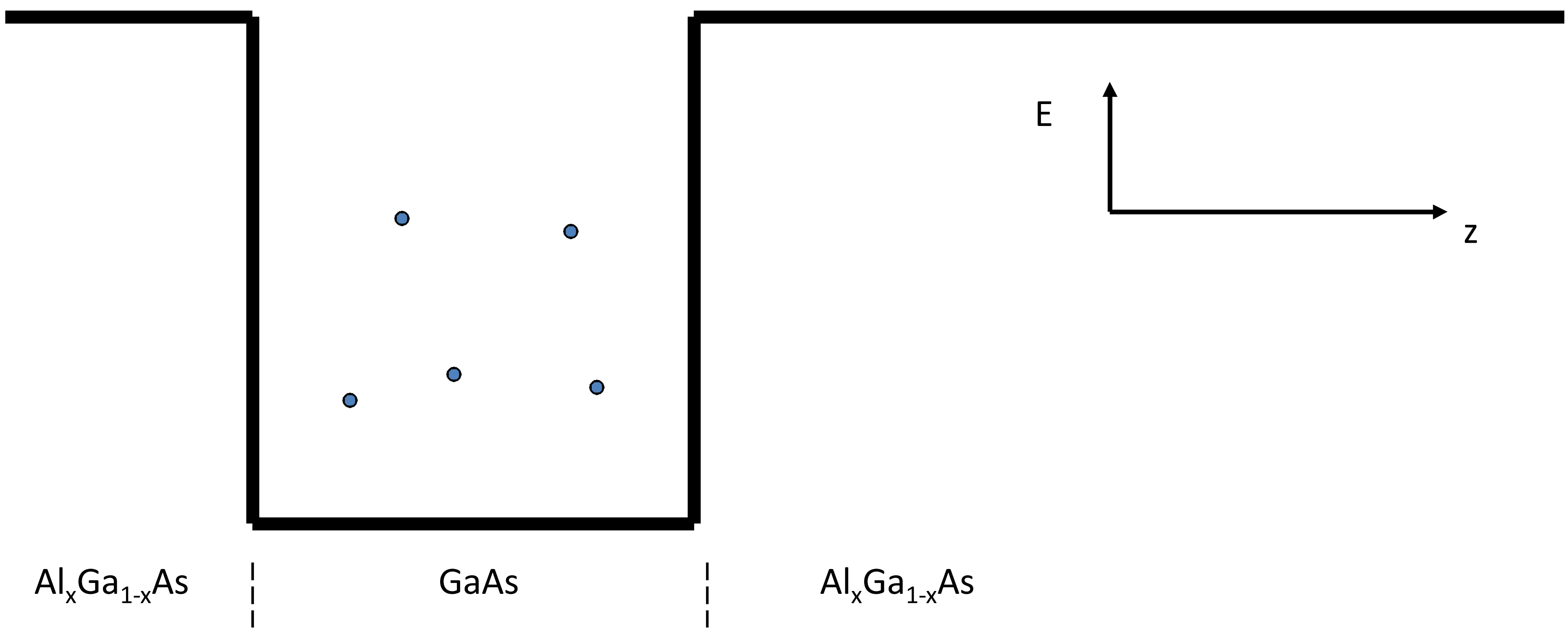
E
z
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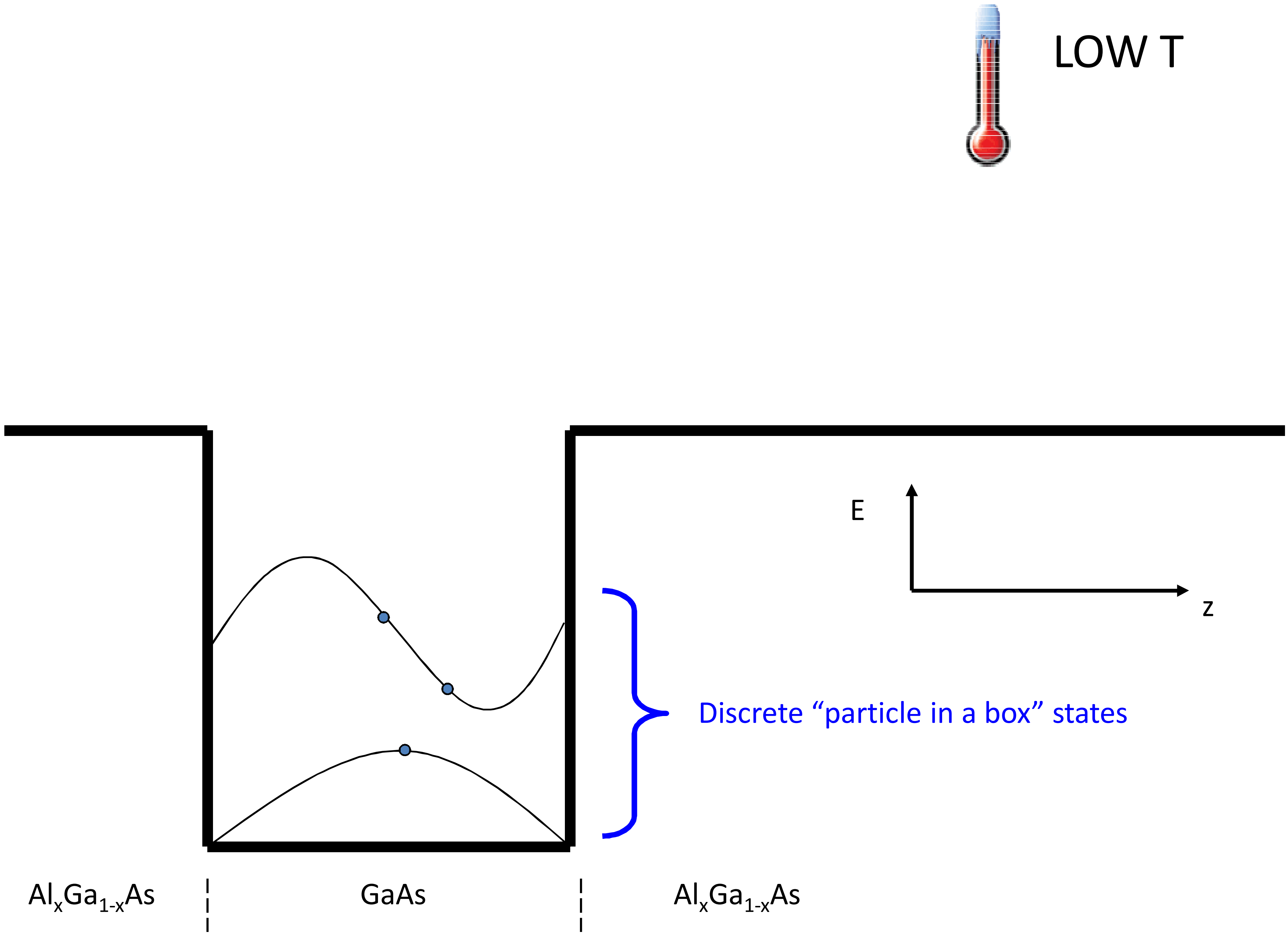
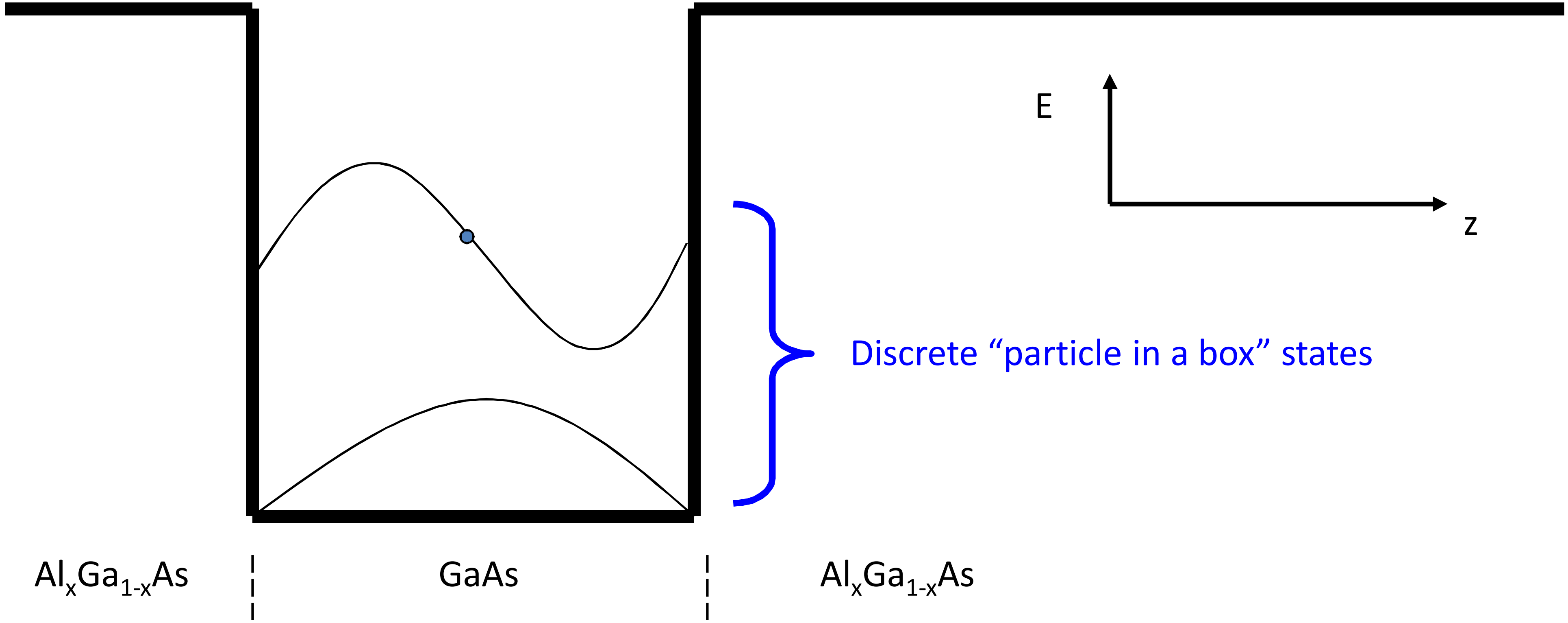

LOW T
E
z
Discrete “particle in a box” states
Al$_x$Ga$_{1-x}$As
GaAs
Al$_x$Ga$_{1-x}$As

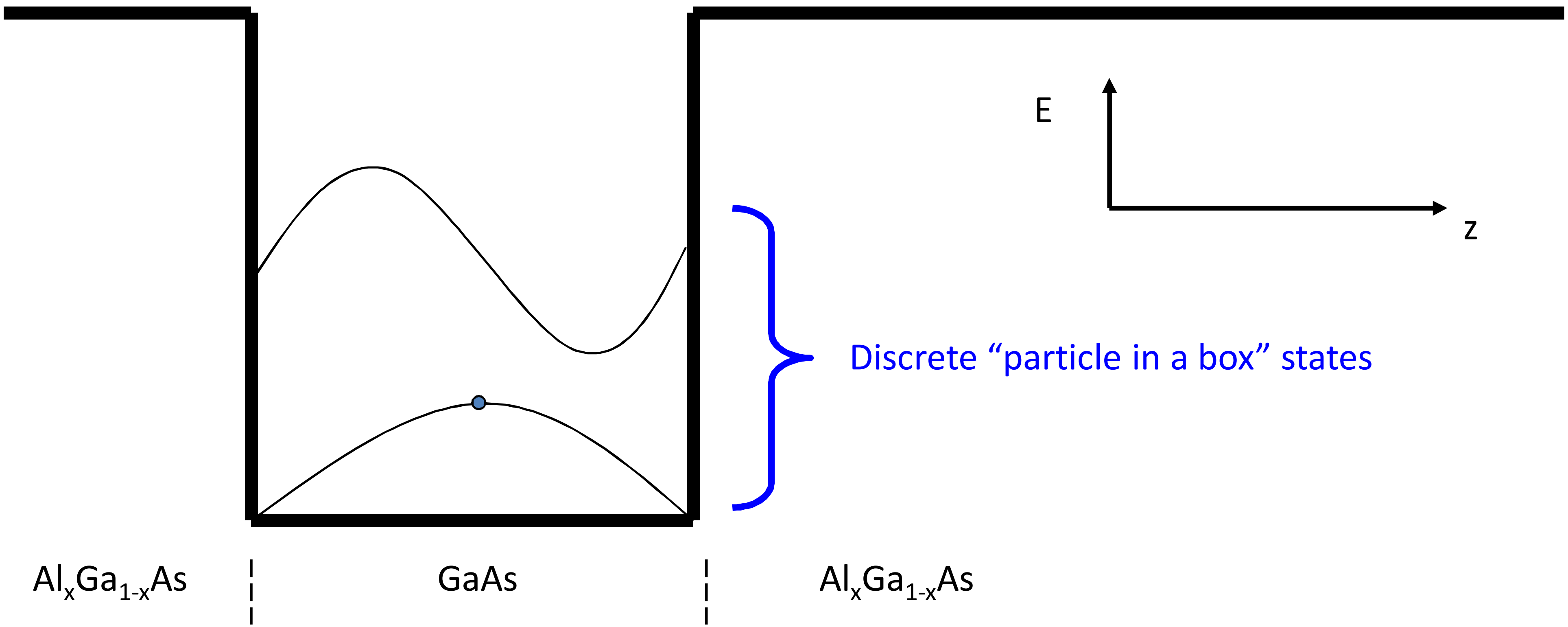


E
z
Discrete "particle in a box" states
$Al_xGa_{1-x}As$
GaAs
$Al_xGa_{1-x}As$

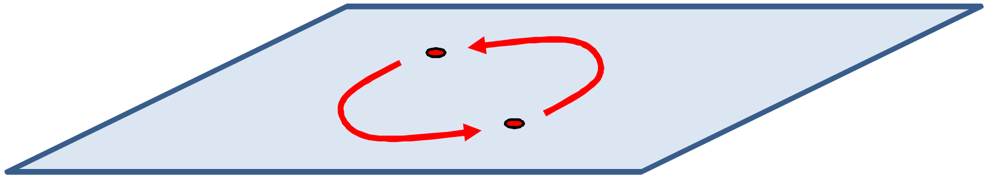
LOW!! T

Z-motion is FROZEN OUT

Strictly 2-D Motion



ν =1/3 Fractional Quantum Hall Effect



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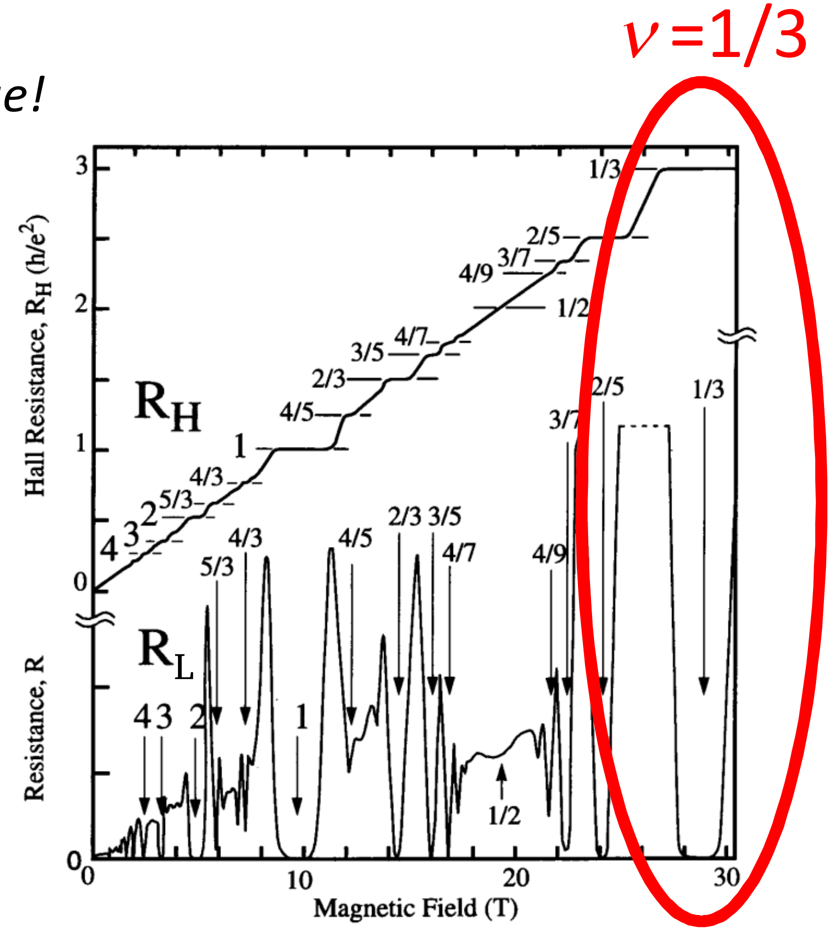

But surely these are not "fundamental" particle?

... well... maybe nothing is.

We are always in the business of describing physical systems on the scales we can access.

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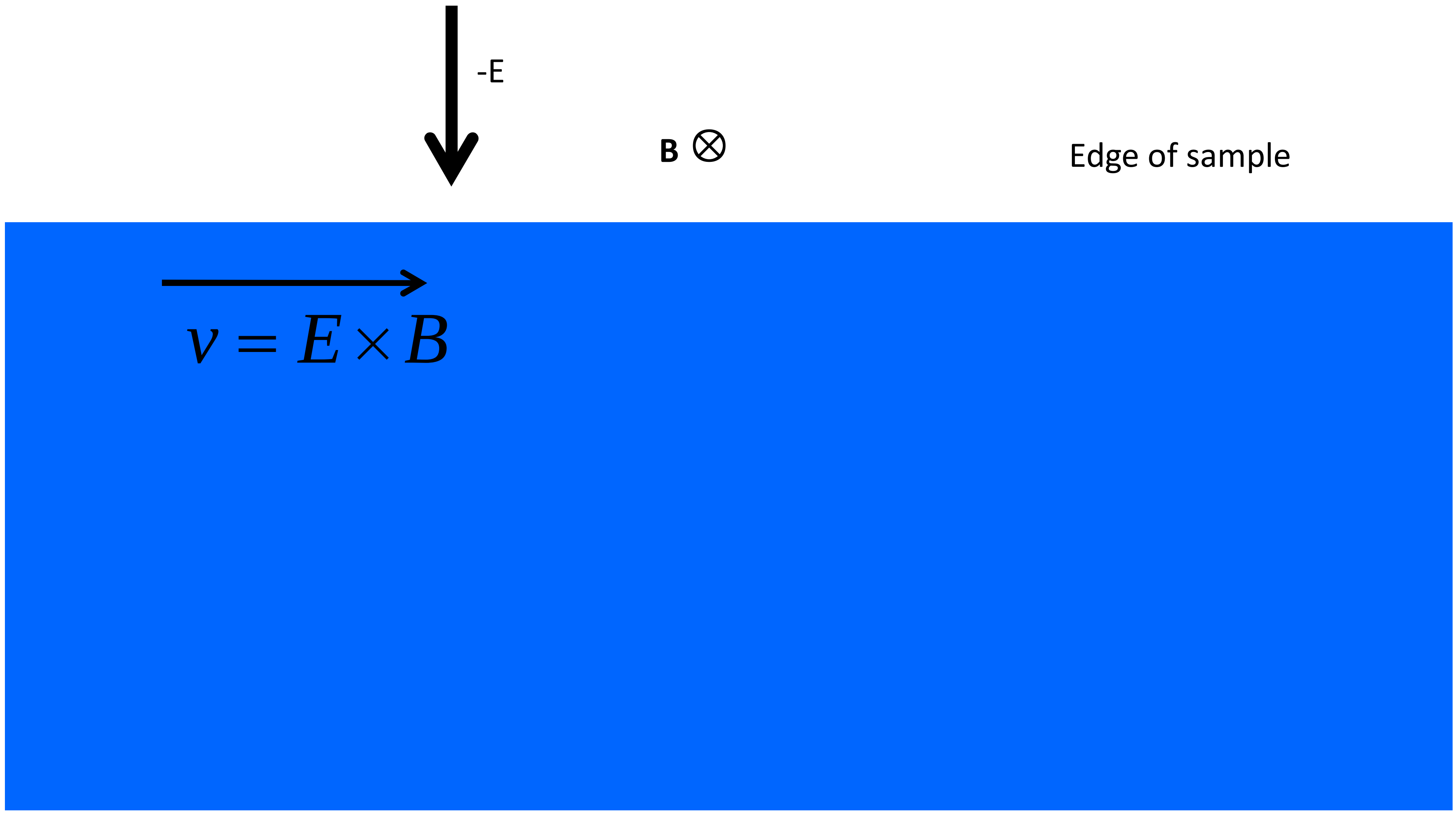
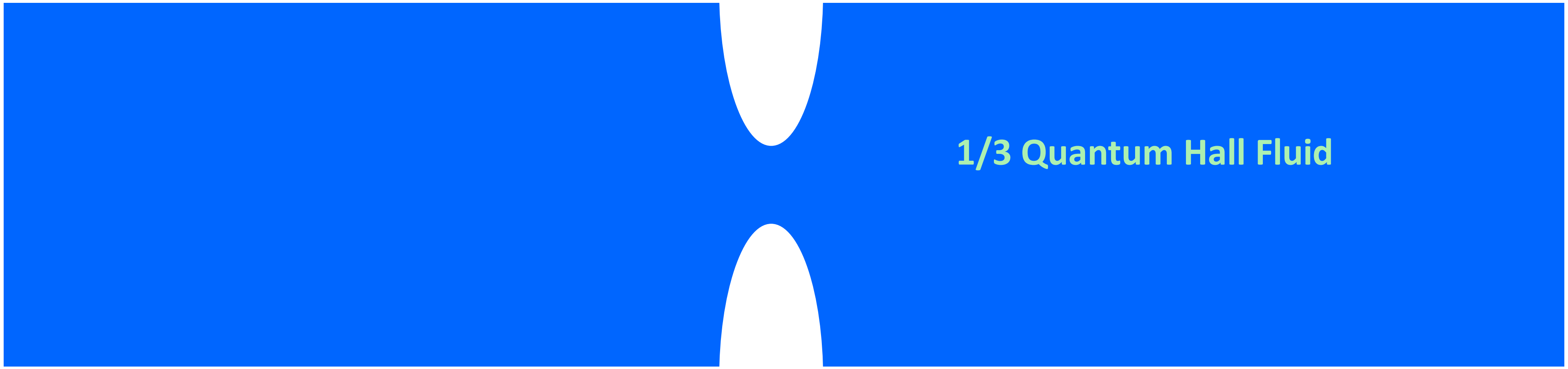
Manfra Lecture

Quantum Hall Edge States



“Quantum” Point Contact



= Half-Silvered Mirror

Noise in backscattered current shows “quantum” of charge

1997

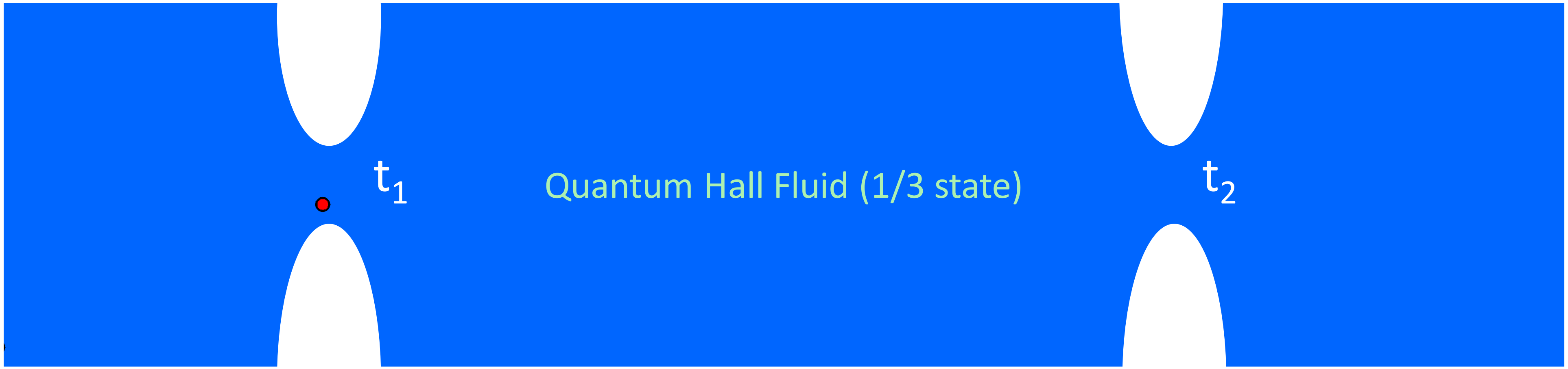
Two point-contact interferometer for quantum Hall systems

Claudio Chamon, Denise Freed, Steve Kivelson, Shivaji Sondhi, Xiao-Gang Wen

(.. based on Kivelson, 1990)

Interference between two paths = Fabry-Pérot Interferometer

Interference of two partial waves



Beam Splitter

Mirror

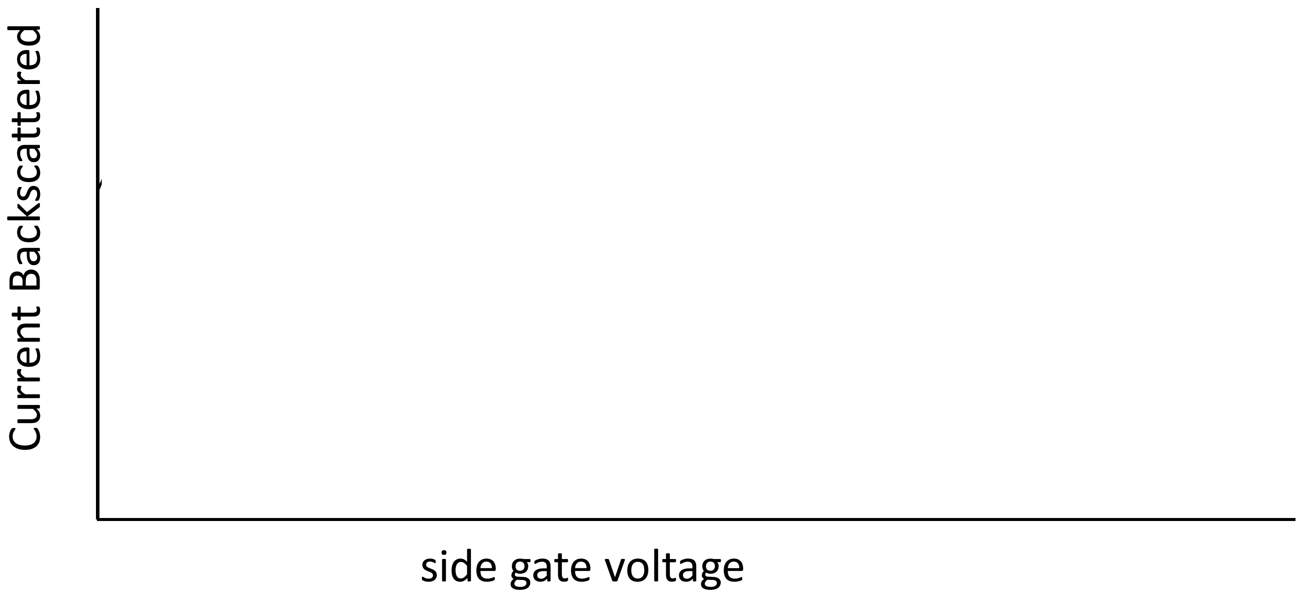
Phase from going around cavity

$$\Psi_{\text{back}} \sim t_1 + e^{i\varphi} t_2$$

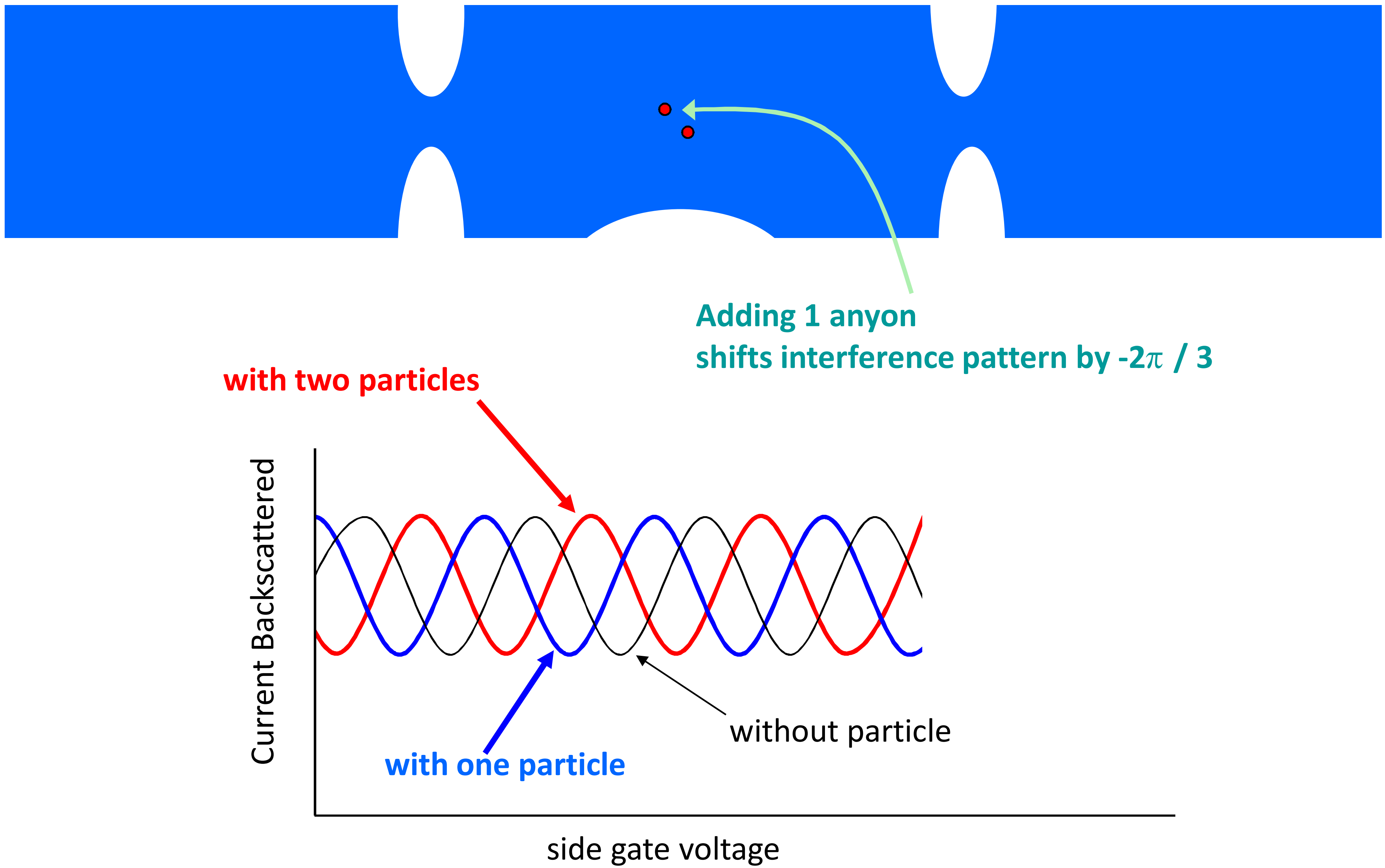
$$I_{\text{back}} \sim |\Psi_{\text{back}}|^2 \sim |t_1|^2 + |t_2|^2 + 2|t_1||t_2|\cos(\varphi)$$

Changing Phase around Cavity with "Side Gate"

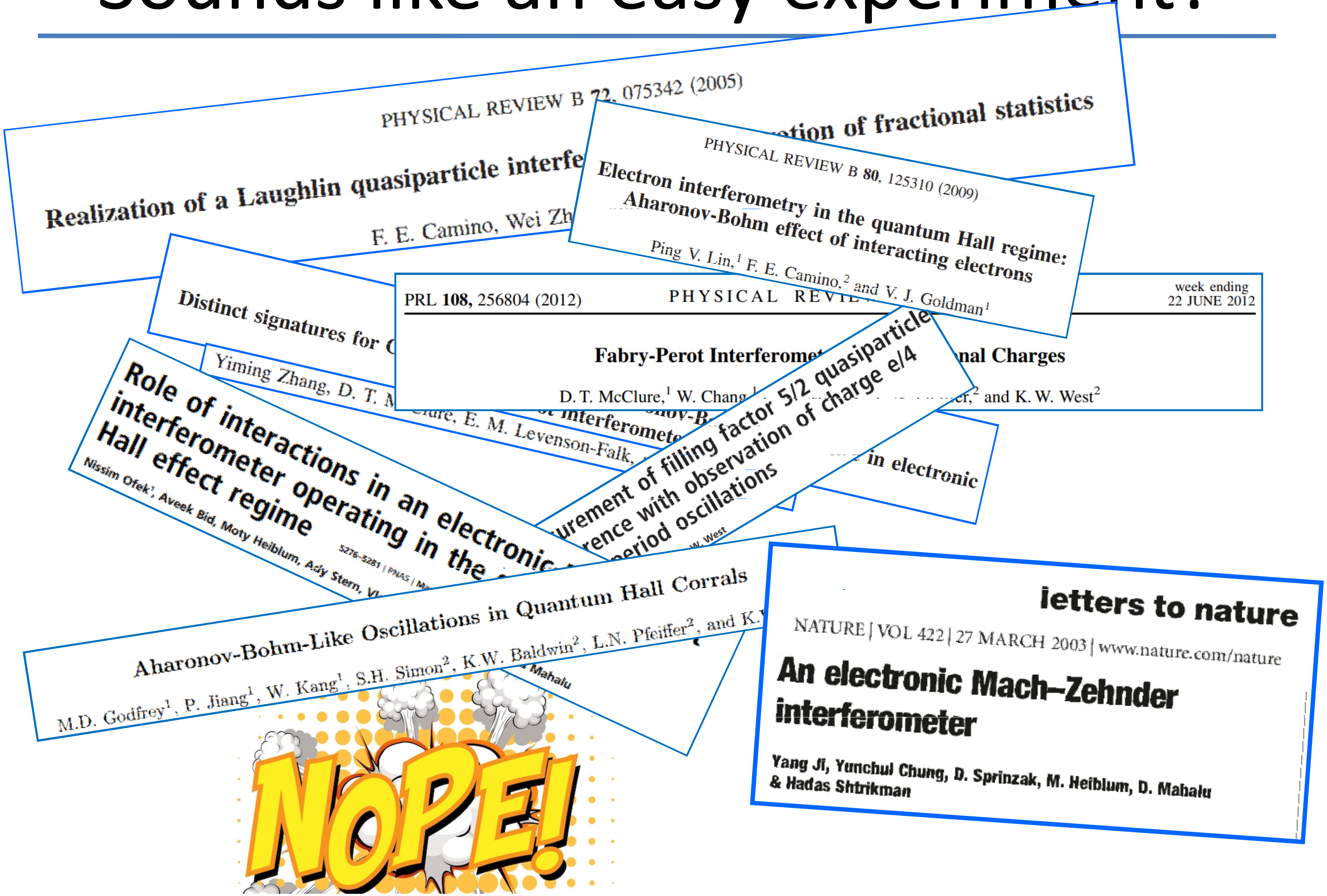
$$I_{\text{back}} \sim |\Psi_{\text{back}}|^2 \sim |t_1|^2 + |t_2|^2 + 2|t_1||t_2|\cos(\varphi)$$



Anyon Braiding Statistics!



Sounds like an easy experiment?



VOLUME 90, NUMBER 22 PHYSICAL REVIEW LETTERS week ending 6 JUNE 2003

Telegraph Noise and Fractional Statistics in the Quantum Hall Effect

C. L. Kane

PRL **96,** 226803 (2006) PHYSICAL REVIEW LETTERS week ending 9 JUNE 2006

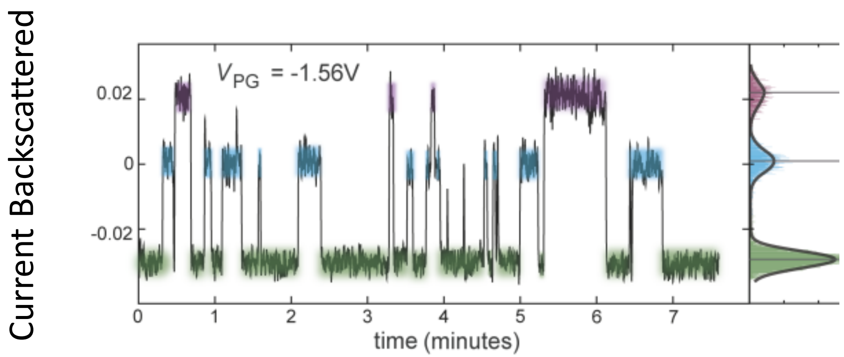
Switching Noise as a Probe of Statistics in the Fractional Quantum Hall Effect

Eytan Grosfeld,[1] Steven H. Simon,[2] and Ady Stern[1]

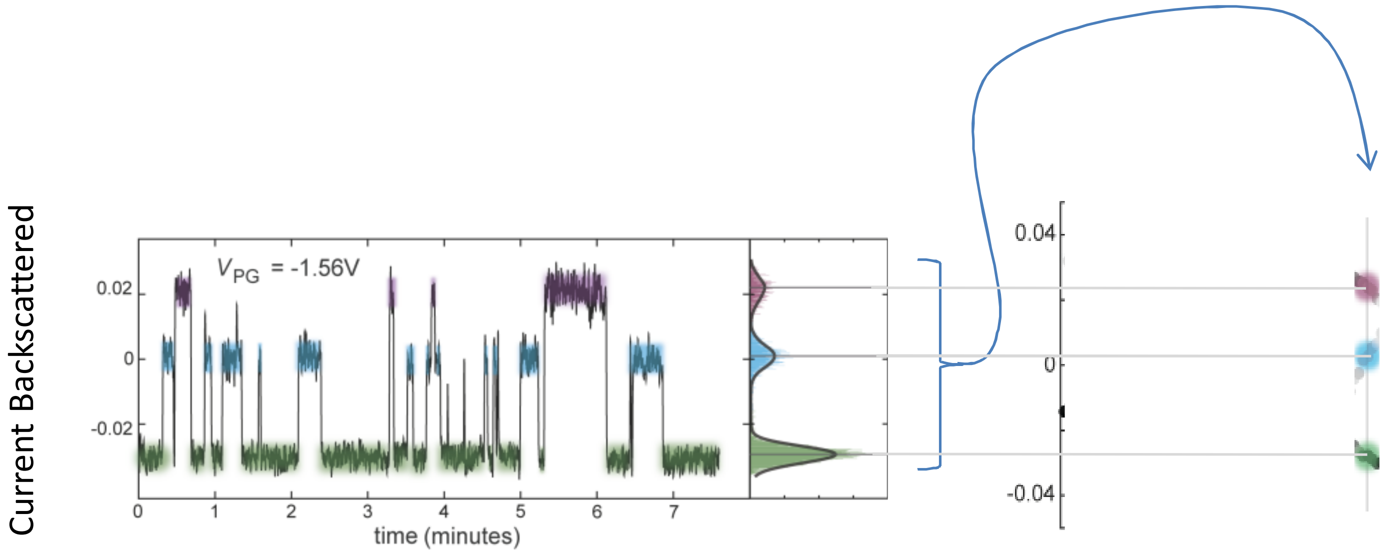
2024

Anyon braiding and telegraph noise in a graphene interferometer

Thomas Werkmeister,[1,†] James R. Ehrets,[2,†] Marie E. Wesson,[1] Danial H. Najafabadi,[3] Kenji Watanabe,[4] Takashi Taniguchi,[5] Bertrand I. Halperin,[2] Amir Yacoby,[1,2] Philip Kim[1,2,*]



Current Backscattered
V_{PG} = -1.56V
0.02
0
-0.02
0
1
2
3
4
5
6
7
time (minutes)



Current Backscattered
V_{PG} = -1.56V
0.02
0
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1
2
3
4
5
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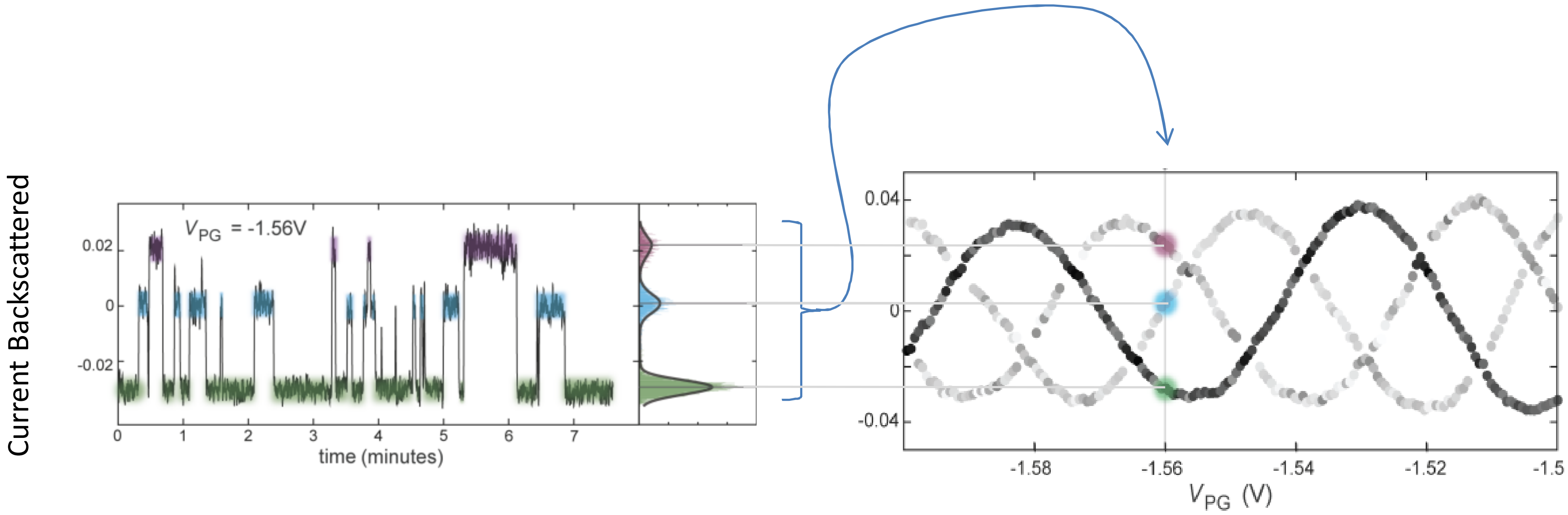
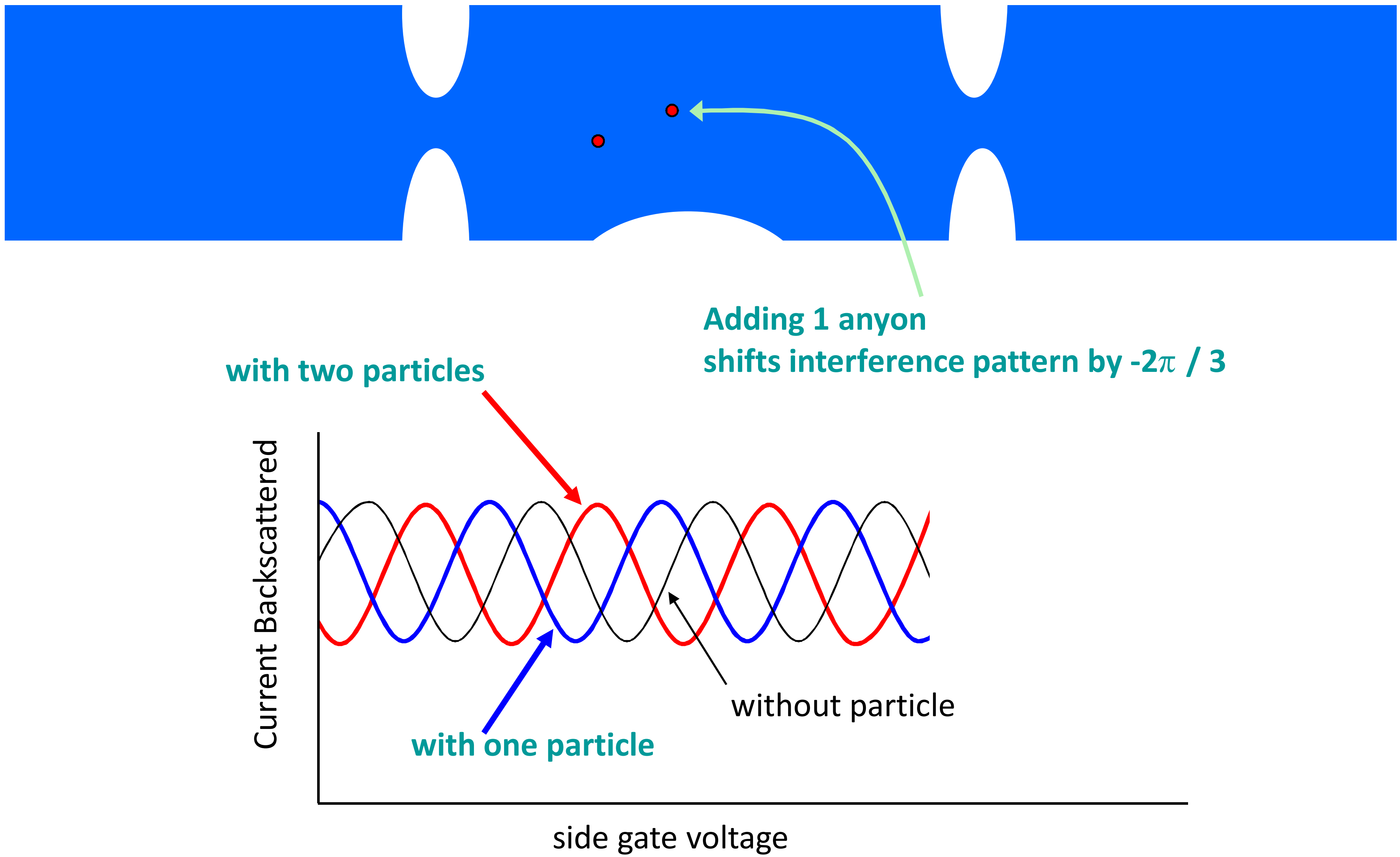
7
time (minutes)
0.04
0
-0.04

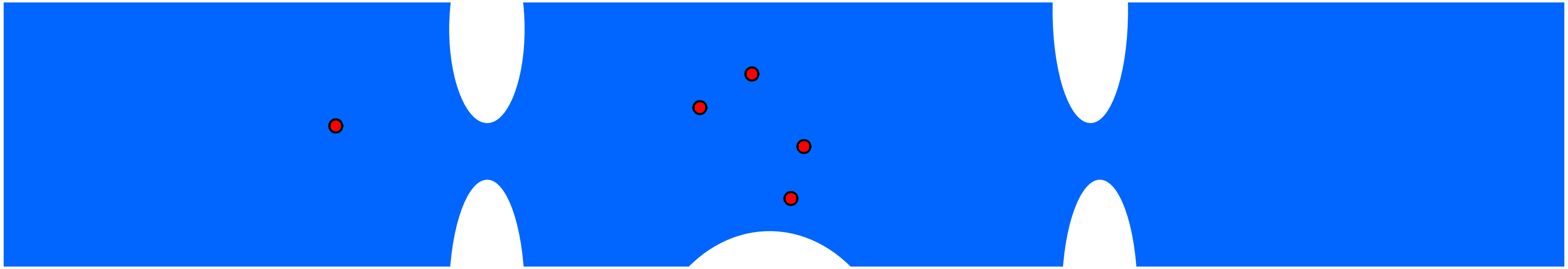


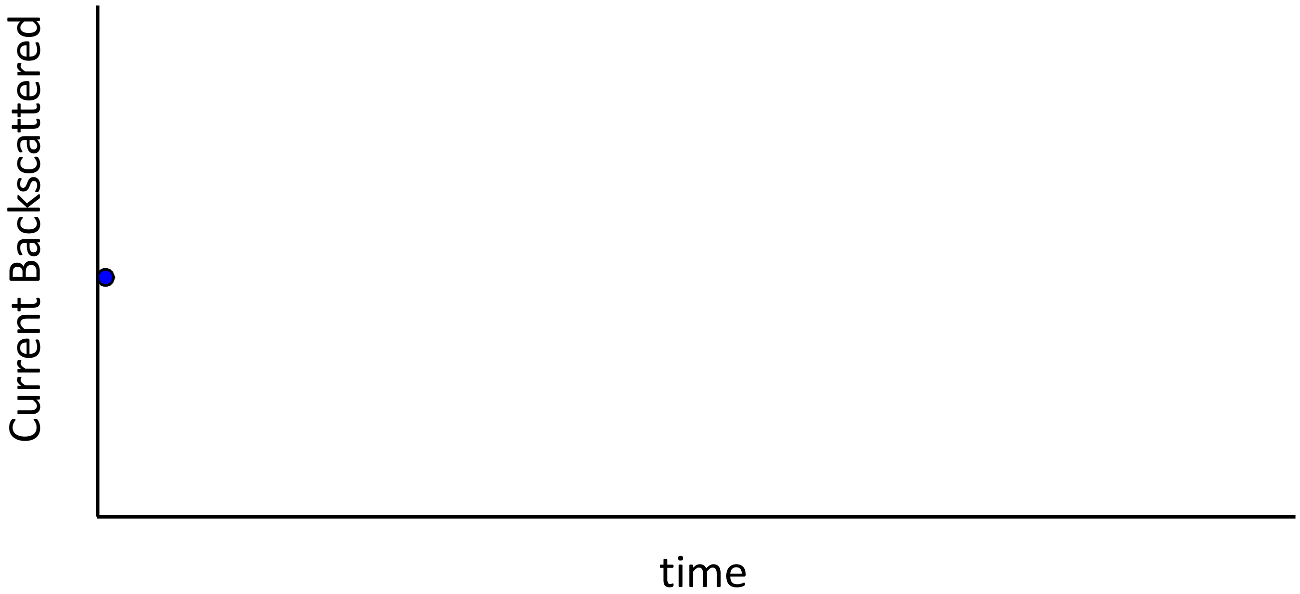
Current Backscattered
V_{PG} = -1.56V
0.02
0
-0.02
0 1 2 3 4 5 6 7
time (minutes)
0.04
0
-0.04
-1.58 -1.56 -1.54 -1.52 -1.5
V_{PG} (V)

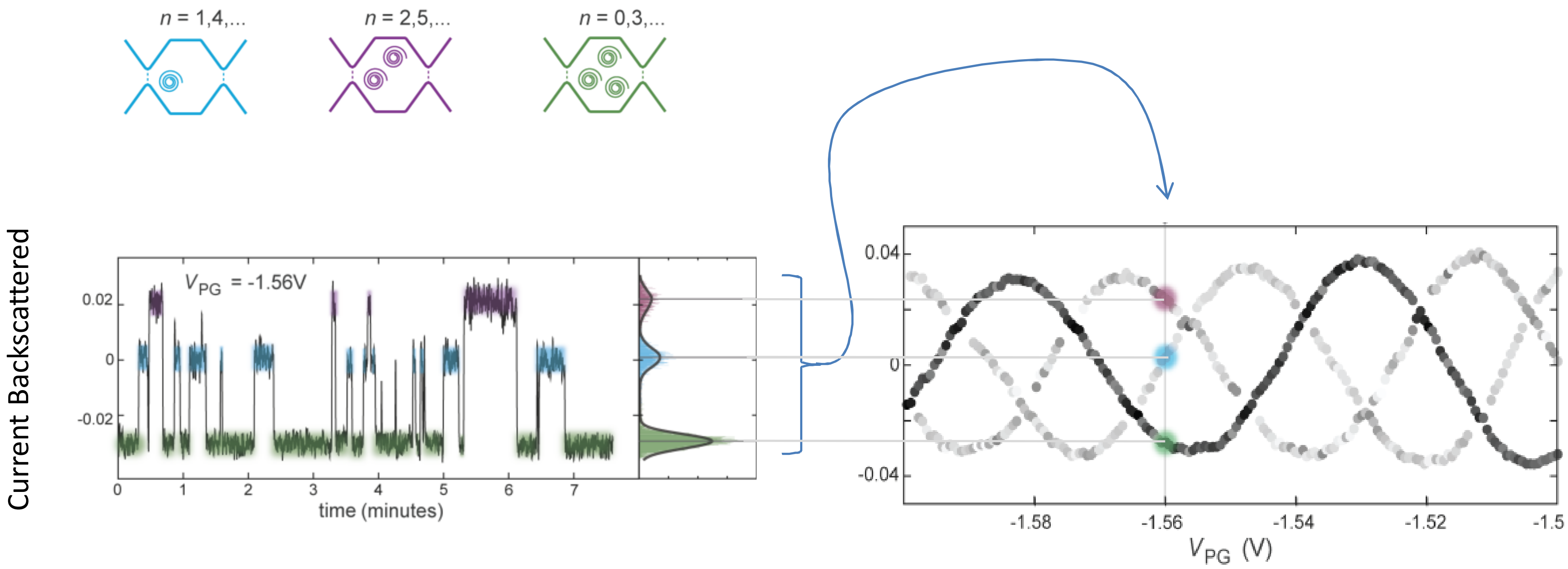
Anyon Braiding Statistics!



Telegraph Noise







n = 1,4,...
n = 2,5,...
n = 0,3,...
Current Backscattered
V_PG = -1.56V
0.02
0
-0.02
0 1 2 3 4 5 6 7
time (minutes)
0.04
0
-0.04
-1.58 -1.56 -1.54 -1.52 -1.5
V_PG (V)

Questions?

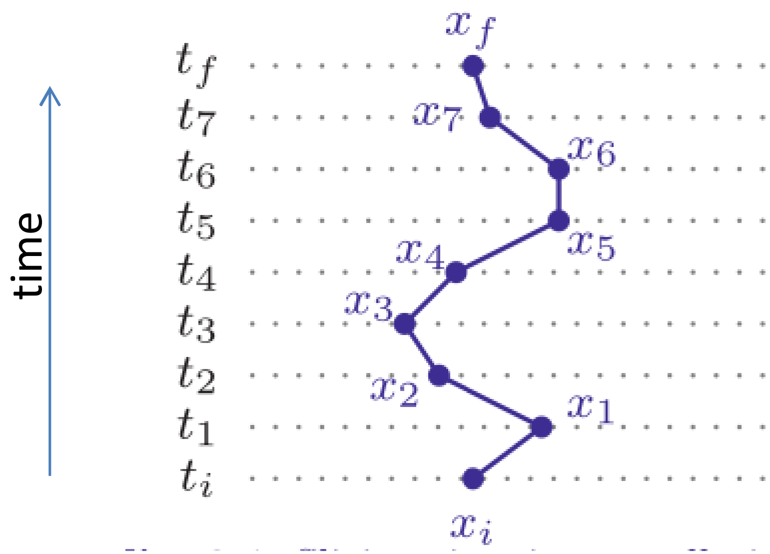
Why can we have anyons?...

.. and what kind of anyons can we have?

Feynman's path integral

$$\langle \mathbf{x}_f | \hat{U}(t_f, t_i) | \mathbf{x}_i \rangle = \mathcal{N} \sum_{\substack{\text{paths } \mathbf{x}(t) \text{ from} \\ (\mathbf{x_i}, t_i) \text{ to } (\mathbf{x_f}, t_f)}} e^{iS[\mathbf{x}(t)]/\hbar} = \mathcal{N} \int_{(\mathbf{x_i}, t_i)}^{(\mathbf{x_f}, t_f)} \mathcal{D}\mathbf{x}(t)\; e^{iS[\mathbf{x}(t)]/\hbar}$$

S =classical action



Consistency under composition:

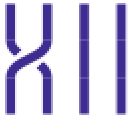
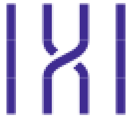
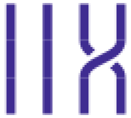
$$\langle \mathbf{x}_f | \hat{U}(t_f, t_i) | \mathbf{x}_i \rangle = \int d\mathbf{x}_m \, \langle \mathbf{x}_f | \hat{U}(t_f, t_m) \underbrace{|\mathbf{x}_m\rangle \langle \mathbf{x}_m|}_{\text{“1”}} \hat{U}(t_m, t_i) | \mathbf{x}_i \rangle$$

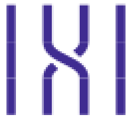
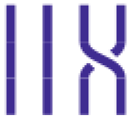
N hard core particles?

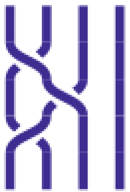
(a) Characterize the space of paths through configuration space
(b) Insist on consistency under composition.

Group of Paths in 2D: The Braid Group

Generators

$\sigma_1 =$  $\sigma_2 =$  $\sigma_3 =$ 

$\sigma_1^{-1} =$  $\sigma_2^{-1} =$  $\sigma_3^{-1} =$ 

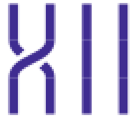
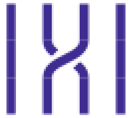
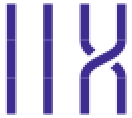
$\sigma_1^{-1}\,\sigma_2^{-1}\,\sigma_1 =$ 

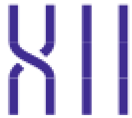
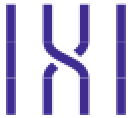
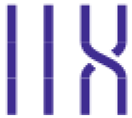
third second first

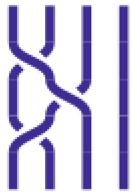
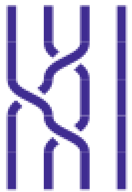
$\sigma_1^{-1}\sigma_1 =$  = identity

Group of Paths in 2D: The Braid Group

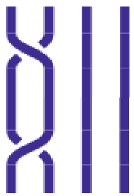
Generators

$\sigma_1 =$  $\sigma_2 =$  $\sigma_3 =$ 

$\sigma_1^{-1} =$  $\sigma_2^{-1} =$  $\sigma_3^{-1} =$ 

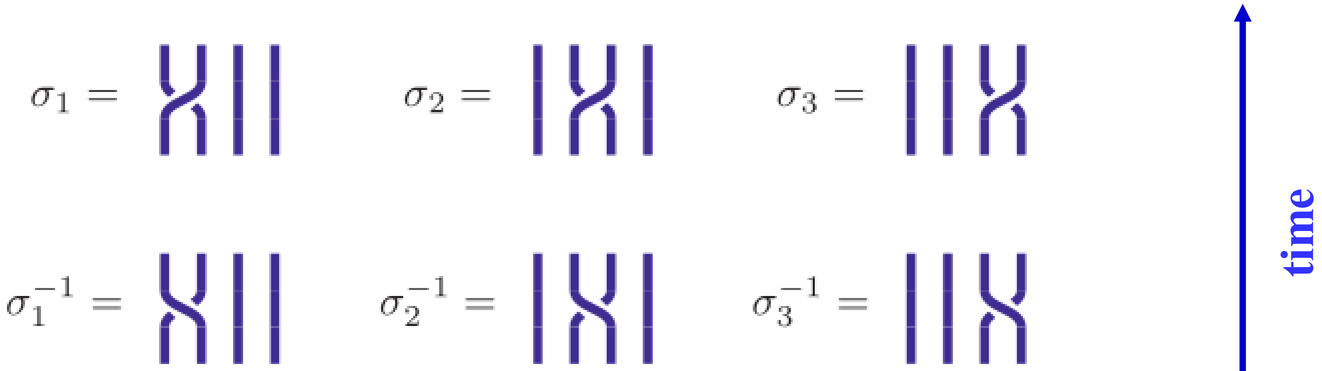
$\sigma_1^{-1}\,\sigma_2^{-1}\,\sigma_1 =$  $=$  $= \sigma_2\,\sigma_1^{-1}\,\sigma_2^{-1}$

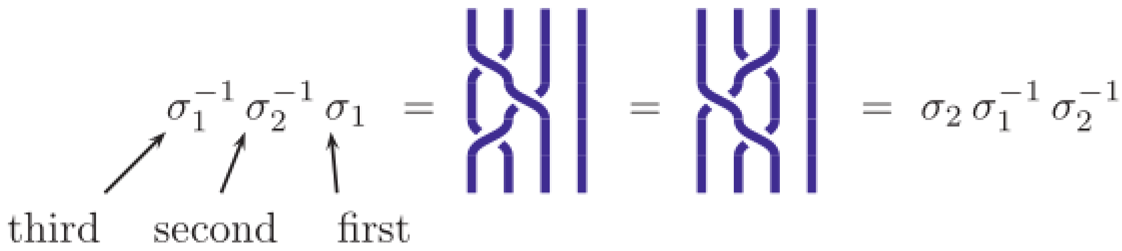
third second first

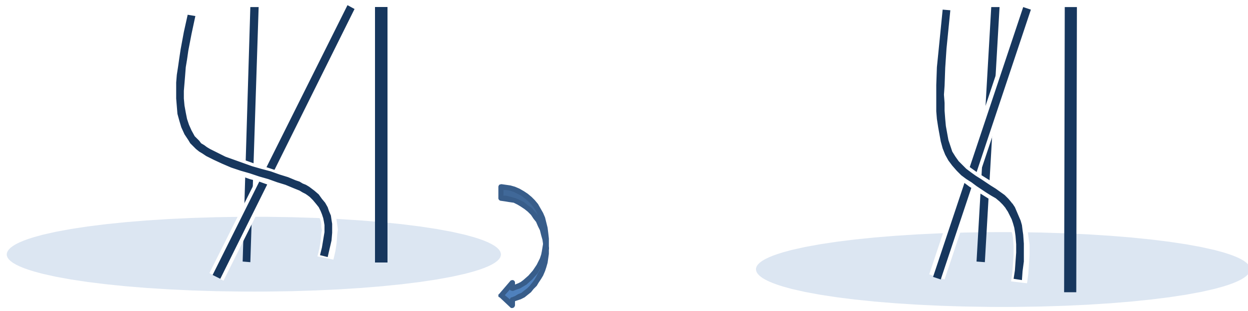
$\sigma_1^{-1}\sigma_1 =$  = identity

Group of Paths in 2D: The Braid Group

Generators

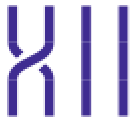
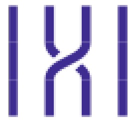


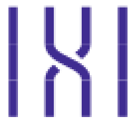


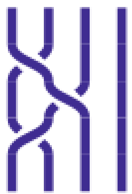
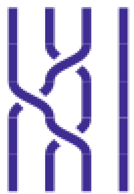


Group of Paths in 2D: The Braid Group

Generators

$\sigma_1 =$  $\sigma_2 =$  $\sigma_3 =$ 

$\sigma_1^{-1} =$  $\sigma_2^{-1} =$  $\sigma_3^{-1} =$ 

$\sigma_1^{-1}\,\sigma_2^{-1}\,\sigma_1 =$  $=$  $= \sigma_2\,\sigma_1^{-1}\,\sigma_2^{-1}$

third second first

All braid word equivalences can be derived from the identity

$$\sigma_n \sigma_{n+1} \sigma_n = \sigma_{n+1} \sigma_n \sigma_{n+1} \quad .$$

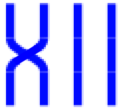
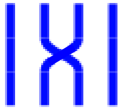
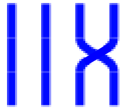
A topological invariant: Winding number W

W = # overcrossings - #undercrossings

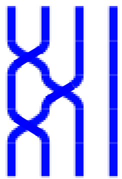
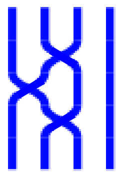
Group of Paths in 3D: The Permutation Group

("symmetric" group)

Generators

$\sigma_1 =$  $\sigma_2 =$  $\sigma_3 =$ 

$$\sigma_i^2 = 1$$

$\sigma_1 \, \sigma_2 \, \sigma_1$ =  =  $= \sigma_2 \, \sigma_1 \, \sigma_2$

third second first

A topological invariant: parity of number of exchanges = P

Constructing the Path Integral

$$\langle \{\mathbf{x}\}_f | \hat{U}(t_f, t_i) | \{\mathbf{x}\}_i \rangle = \mathcal{N} \sum_{g \in G} \qquad \sum_{\substack{\text{paths} \in g \\ i \to f}} e^{iS[\text{path}]/\hbar}$$

G = braid group in 2+1, symmetric group in 3+1

$$= \int d\{\mathbf{x}\}_m \, \langle \{\mathbf{x}\}_f | \hat{U}(t_f, t_m) \overbrace{| \{\mathbf{x}\}_m \rangle \, \langle \{\mathbf{x}\}_m |}^{\text{“1”}} \hat{U}(t_m, t_i) | \{\mathbf{x}\}_i \rangle$$

Consistency requires $\rho(g_1)\rho(g_2) = \rho(g_1 g_2)$

i.e., a representation of the group

N hard core particles?

(a) Characterize the space of paths through configuration space
(b) Insist on consistency under composition.

Must have a unitary rep of the group G of paths

Statistics in 2+1 dimensions: Reps of the braid group

Want a *Representation* of the braid group

$$\rho(\text{braid}_2 \ \text{braid}_1) = \rho(\text{braid}_2)\,\rho(\text{braid}_1)$$

Wavefunction is a scalar, want a unitary scalar rep

Fix some value of θ

All unitary scalar reps are of the form: $\rho(\text{braid}) = e^{i\theta W(\text{braid})}$

Winding number

i.e.,

Counterclockwise exchange gets phase $e^{i\theta}$

Clockwise exchange gets phase $e^{-i\theta}$

(Anyons! Yay!)

... In 3+1 dimensions: Reps of the symmetric group

Want a *Representation* of the symmetric group

$$\rho(\text{perm}_2 \ \text{perm}_1) = \rho(\text{perm}_2)\,\rho(\text{perm}_1)$$

Wavefunction is a scalar, want a unitary scalar rep

Only two scalar unitary reps of the symmetric group exist!

Symmetric representation (bosons): ρ = +1 always

Antisymmetric (sign) rep (fermion) : ρ = +1 for even parity of exchanges

ρ = -1 for odd parity

Is something else possible?

YES!

If there are $M > 1$ degenerate wavefunctions

$$|n; \{\mathbf{x}\}\rangle \text{ for } n = 1 \ldots M$$

Statistics in 2+1 dimensions: Reps of the braid group

Want a *Representation* of the braid group

$$\rho(\text{braid}_2\ \text{braid}_1) = \rho(\text{braid}_2)\,\rho(\text{braid}_1)$$

~~Wavefunction is a scalar, want a unitary scalar rep~~

Fix some value of θ

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(Anyons! Yay!)

Statistics in 2+1 dimensions: Reps of the braid group

Want a *Representation* of the braid group

$$\rho(\text{braid}_2 \ \text{braid}_1) = \rho(\text{braid}_2)\,\rho(\text{braid}_1)$$

~~Wavefunction is a scalar, want a unitary scalar rep~~

Wavefunction is an M-dimensional vector,

$$|\psi_{\{\mathbf{x}\}}\rangle = \sum_{n=1}^{M} A_n |n; \{\mathbf{x}\}\rangle$$

Want an M-dimensional unitary rep

$$\langle n; \{\mathbf{x}\}_f | \hat{U}(t_f, t_i) | n'; \{\mathbf{x}\}_i \rangle = \mathcal{N} \sum_{g \in G} [\rho(g)]_{n,n'} \sum_{\substack{\text{paths} \in g \\ i \to f}} e^{iS[\text{path}]/\hbar}$$

Statistics in 2+1 dimensions: Reps of the braid group

Niels Abel
1802-1829

Nonabelian Anyons

(Nonabelions)

Goldin, Menikov, Sharp '85
....

Noncommutative → Nonabelian

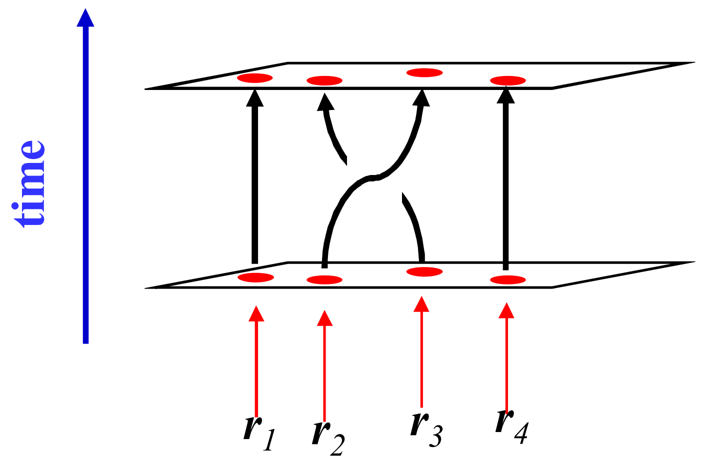
Wavefunction is an M-dimensional vector

$$|\psi_{\{\mathbf{x}\}}\rangle = \sum_{n=1}^{M} A_n |n; \{\mathbf{x}\}\rangle$$

Want an M-dimensional unitary rep → $[\rho(g)]_{n,n'}$

$$\langle n; \{\mathbf{x}\}_f | \hat{U}(t_f, t_i) | n'; \{\mathbf{x}\}_i \rangle = \mathcal{N} \sum_{g \in G} [\rho(g)]_{n,n'} \sum_{\substack{\text{paths} \in g \\ i \to f}} e^{iS[\text{path}]/\hbar}$$

Nonabelian Anyons



Suppose 2 Degenerate Orthogonal States $|\psi_a\rangle \; , \; |\psi_b\rangle$

Vector Represents State

$$\Psi_i = a_i|\psi_a\rangle + b_i|\psi_b\rangle = \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$

$$\Psi_f = a_f|\psi_a\rangle + b_f|\psi_b\rangle = \begin{pmatrix} a_f \\ b_f \end{pmatrix}$$

$$\begin{pmatrix} a_f \\ b_f \end{pmatrix} = U \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$

Unitary Matrix From Braid Operation
Depends only on the Topology of the braid

Exchanging in 3D: The Permutation Group

("symmetric" group)

Are there nonabelian anyons in 3D?

Aren't there nonabelian reps of the symmetric group?

YES.... BUT....

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Exchanging in 3D: The Permutation Group

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Are there nonabelian anyons in 3D?

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YES.... BUT....

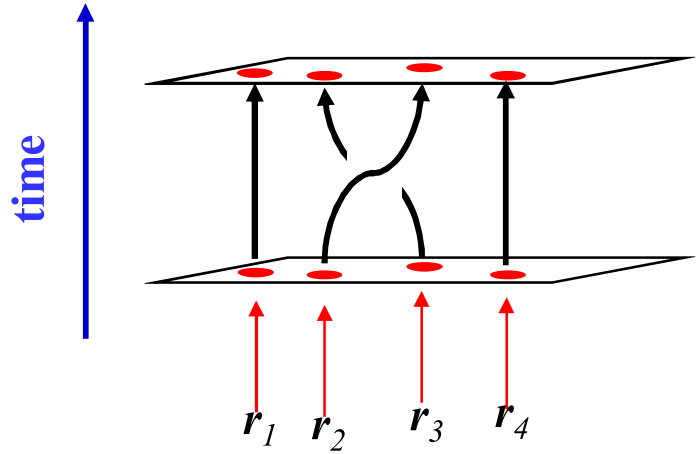
If you add further reasonable constraints:

Locality!!

... then no...

Nonabelian Anyons

Suppose 2 Degenerate Orthogonal States $|\psi_a\rangle \;,\; |\psi_b\rangle$



Vector Represents State

$$\Psi_i = a_i|\psi_a\rangle + b_i|\psi_b\rangle = \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$

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Unitary Matrix From Braid Operation
Depends only on the Topology of the braid

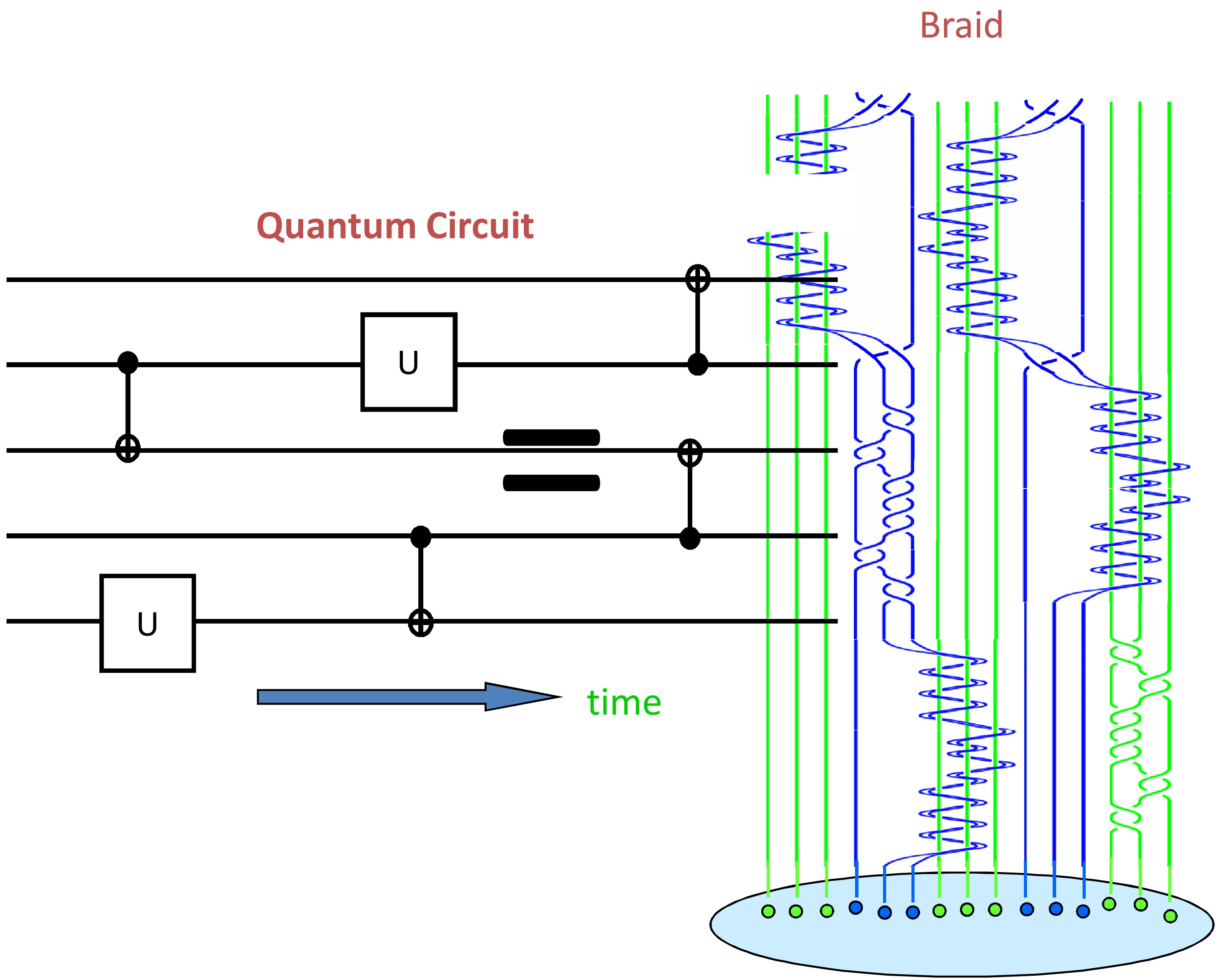
Its a Quantum Computer!

- Building a Topological Quantum Computer

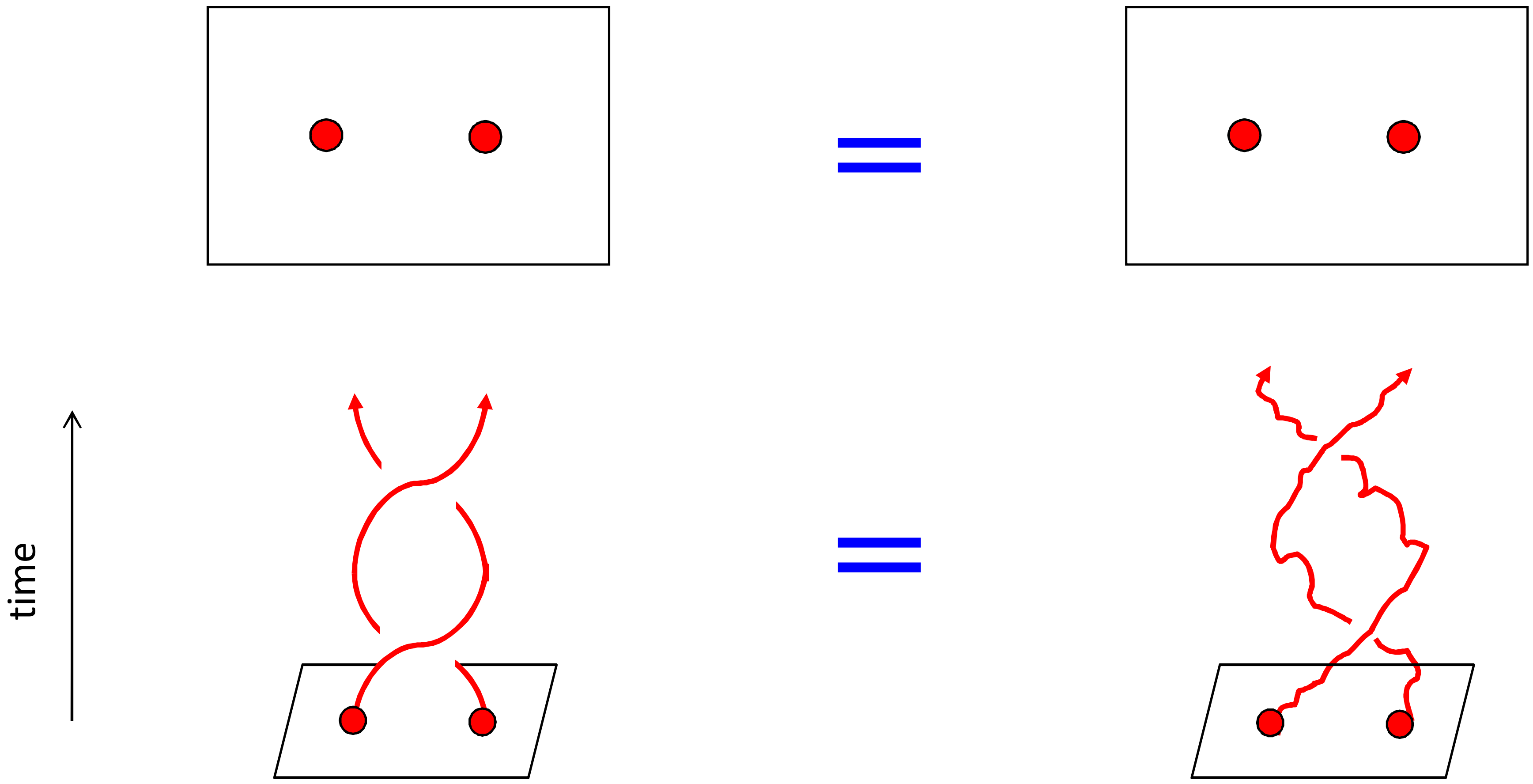
(A. Kitaev and M. Freedman '97) Investment ~1B$ from Microsoft!

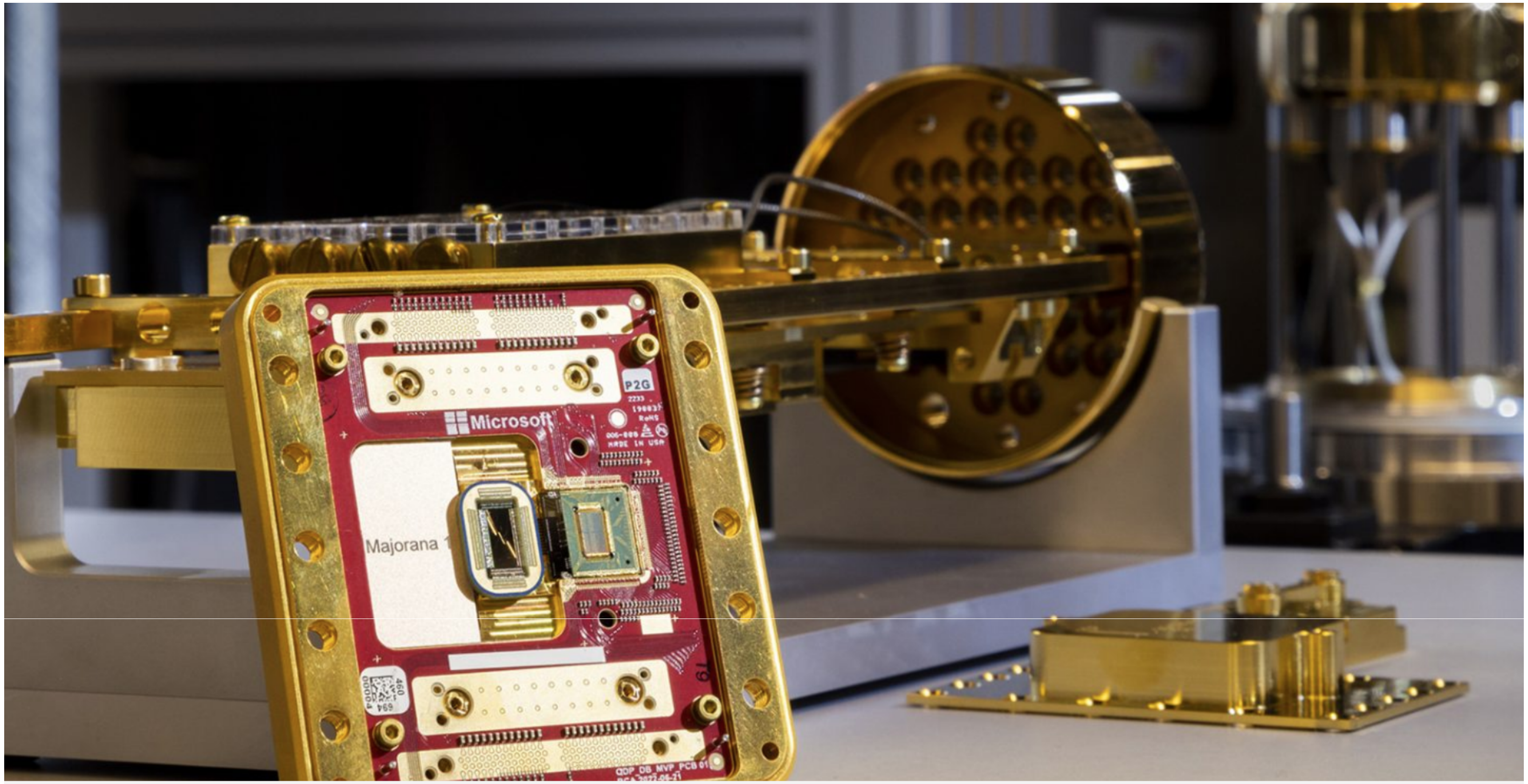
A. Kitaev

Mike Freedman


Braid
Quantum Circuit
U
U
time

Topological Robustness





Introducing Microsoft Majorana 1

+ an insane amount of press

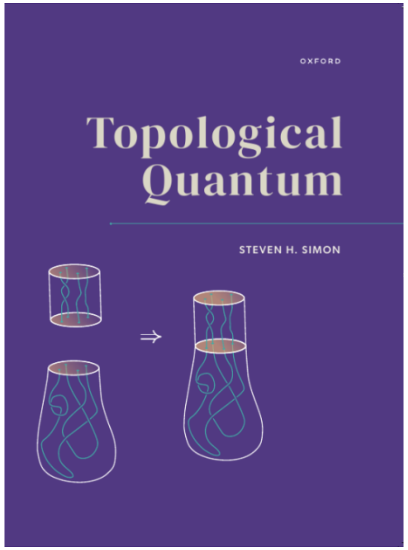
Questions?

Why are we interested in Anyons?

- Fundamental Interest:
  - What can exist, in principle
  - Is it lurking in plain sight?
  - Surprising connections to: High Energy Physics, Quantum Gravity, Pure Maths/Topology, ...

- Connection to Quantum Memories (A. Kitaev '97)
  and the notion of topological order (Wen)

A. Kitaev



Classical Memory

1 bit $\Rightarrow$ 0 or 1

N bit memory $\Rightarrow$ 2^N possible states

Example 5 bit state = 11010

Quantum Memory

1 qubit $\Rightarrow$ $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

N qubit memory $\Rightarrow$ 2^N dimensional Hilbert space

Example 2 qubit State

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

Errors

Error = Any process by which the state of your memory is unintentionally changed.

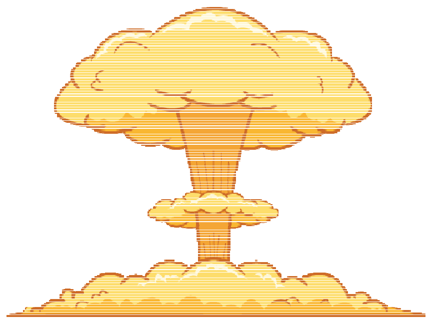
010010001001

Error Correcting Code

One very important bit of information we want to protect

0 →

1 →



Error Correcting Code

One very important bit of information we want to protect

logical bit	physical bits
0	000
1	111

Code Space

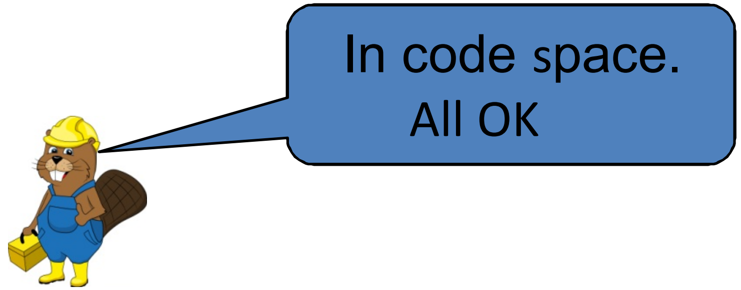
Error Correcting Code

One very important bit of information we want to protect

logical bit	physical bits
0	000
1	111

Code Space

Example:



0 0 0

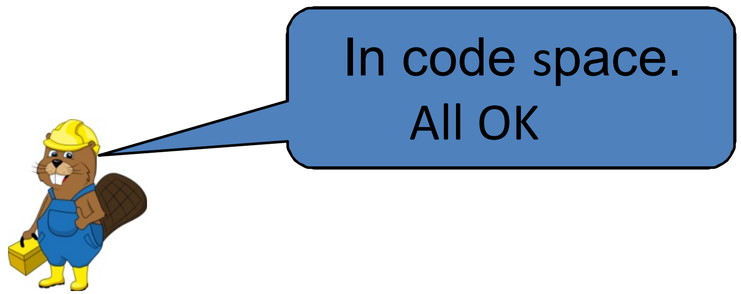
Error Correcting Code

One very important bit of information we want to protect

logical bit	physical bits
0	000
1	111

Code Space

Example:



0 0 0

Error Correcting Code

One very important bit of information we want to protect

logical bit	physical bits
0	000
1	111

Code Space

Example:

0 0 01

Error Correcting Code

One very important bit of information we want to protect

logical bit	physical bits
0	000
1	111

Code Space

Example:

NOT in code space. ERROR!
Must Repair!

0 0 1

Can’t we do the same for qubits?

NOT SO EASY!

1) Quantum No Cloning Theorem!
2) Measuring Disturbs

Can't we do the same for qubits?

Quantum No Cloning Theorem!
(Zurek et al, 1982)

Theorem: Given a qubit in an arbitrary unknown state $|\phi_1\rangle$ and another qubit in an initial state $|\phi_2\rangle$, there does not exist any unitary operator U (i.e., any quantum mecahnical evolution) such that

$$U(|\phi_1\rangle \otimes |\phi_2\rangle) = |\phi_1\rangle \otimes |\phi_1\rangle$$

for all possible input $|\phi_1\rangle$.

Proof of No Cloning Theorem:

Suppose such a copying unitary exists.
Apply unitary to two states |0> and |1>

$$\begin{aligned} U(|0\rangle \otimes |\phi_2\rangle) &= e^{i\chi}|0\rangle \otimes |0\rangle \\ U(|1\rangle \otimes |\phi_2\rangle) &= e^{i\chi}|1\rangle \otimes |1\rangle \end{aligned}$$

Now apply to a superposition $\alpha|0\rangle + \beta|1\rangle$ and use linearity

$$U(\underline{[\alpha|0\rangle + \beta|1\rangle]} \otimes |\phi_2\rangle) = e^{i\chi}(\alpha|0\rangle \otimes |0\rangle + \beta|1\rangle \otimes |1\rangle)$$

But this is *not* a copy of the superposition which would be

$$e^{i\chi}[\alpha|0\rangle + \beta|1\rangle] \otimes [\alpha|0\rangle + \beta|1\rangle]$$

QED

Nonetheless Quantum Error Correction Exists!

Peter Shor:
Quantum Factoring Algorithm 1994
Quantum Error Correction 1995

Toric Code: Kitaev 1997

Anyons and Topological Quantum Computation

THANK YOU FOR LISTENING!