Quantum geometry in Superconductivity
Slides 2

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Aalto University

Quantum Connections in Sweden-15 Summer School 2025, Stockholm, Sweden

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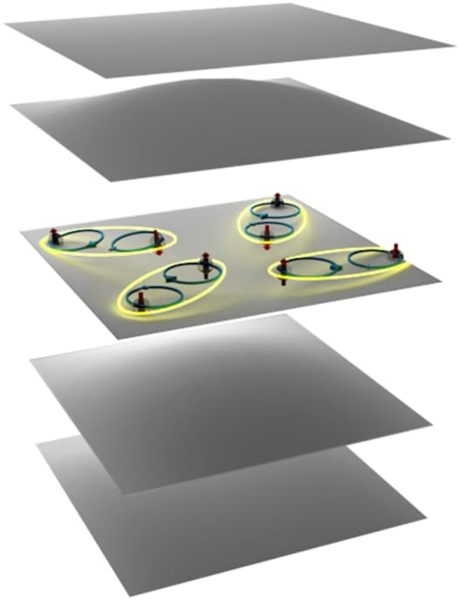

Superconducting transport in flat bands

Nearest-neighbor pairing and quantum geometry in Lieb and kagome lattice flat bands and van Hove singularities

Rhombohedral graphite and LLL quantum geometry

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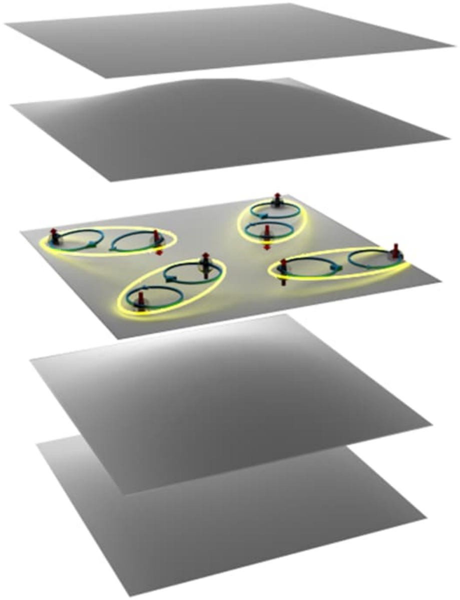


Superconducting transport in flat bands

Nearest-neighbor pairing and quantum geometry in Lieb and kagome lattice flat bands and van Hove singularities

Rhombohedral graphite and LLL quantum geometry

Normal-superconductor interfaces and Josepson junctions (JJs)

arXiv > cond-mat > arXiv:2410.23121

Condensed Matter > Superconductivity

[Submitted on 30 Oct 2024]

Superconducting junctions with flat bands

P. Virtanen, R. P. S. Penttilä, P. Törmä, A. Díez-Carlón, D. K. Efetov, T. T. Heikkilä

Pauli Virtanen

arXiv > cond-mat > arXiv:2502.04785

Search... Help |

Condensed Matter > Superconductivity

[Submitted on 7 Feb 2025]

Probing the flat-band limit of the superconducting proximity effect in Twisted Bilayer Graphene Josephson junctions

A. Diez-Carlon, J. Diez-Merida, P. Rout, D. Sedov, P. Virtanen, S. Banerjee, R. P. S. Penttila, P. Altpeter, K. Watanabe, T. Taniguchi, S.-Y. Yang, K. T. Law, T. T. Heikkila, P. Torma, M. S. Scheurer, D. K. Efetov

Andrés Diéz-Carlón

Reko Penttilä

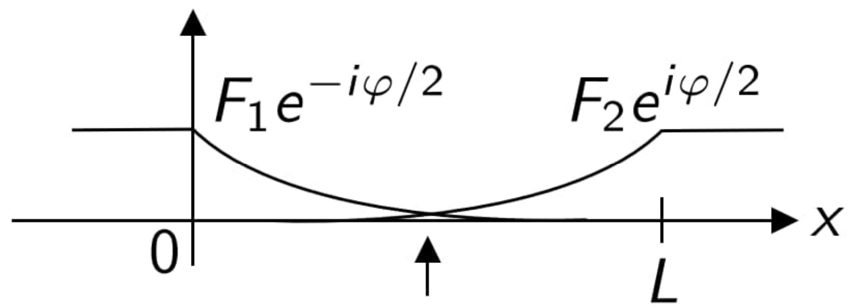
Tero Heikkilä

Dmitri Efetov

Existing theory assumes Fermi energy as a large energy scale - what about (almost) flat bands?

S/N/S supercurrent

flatband-N $\quad F(x) = \langle c_\uparrow c_\downarrow \rangle = \Pi(x, x')\Delta(x')$



$$I_c \propto F_1 F_2 \sin\varphi$$

Conventional: $\quad I_c \propto \Pi(0, L) \propto e^{-L/\xi} \qquad \xi = \hbar v_F/(2\pi T)$

$$v_F \to 0 \Rightarrow I_c \to 0$$

Flat bands: $\quad \xi = ?? \qquad$ role of interactions in FB?

Interactions in FB-N (MF)

$$F = \int dx \frac{|\Delta|^2}{g} - T\sum_{\omega_n} \mathrm{tr}\log[i\omega_n - H_{\mathrm{BdG}}(\Delta, \Delta_S)] \simeq \ldots + \int dx dx' \Delta(x)^* V(x-x')\Delta(x')$$

$$V^{-1}(q) = \frac{1}{1/g - \Pi(q)}$$

Π(x) short-range
$V^{-1}(x)$ doesn't have to be
g can be small (not intrinsic S)

SNS supercurrent:

$$\Delta(0) = \Delta_0 e^{-i\varphi/2}, \qquad \Delta(L) = \Delta_0 e^{i\varphi/2}$$

$$I_{S,\mathrm{int}} = \frac{2e}{\hbar}\partial_\varphi F[\Delta_{\min}] \simeq \frac{4\Delta_0^2}{|V^{-1}(x=0)|^2} V^{-1}(x=L)\sin(\varphi)$$

$$I_S \simeq I_{S,\mathrm{int}} + I_{S,\mathrm{nonint}}$$

$$F = \ldots + \int_{x<0}\int_{x'>L} \Delta_S(x)\Pi(x,x')\Delta_S(x')$$

Length scales in S/FB-N/S

$$\xi_g^2 = a \int_{-\pi/a}^{\pi/a} \frac{dk}{2\pi} g^{(0)}(k)$$

$$\Pi(q) = \frac{1}{4T}[1 - \xi_g^2 q^2 + \ldots]$$

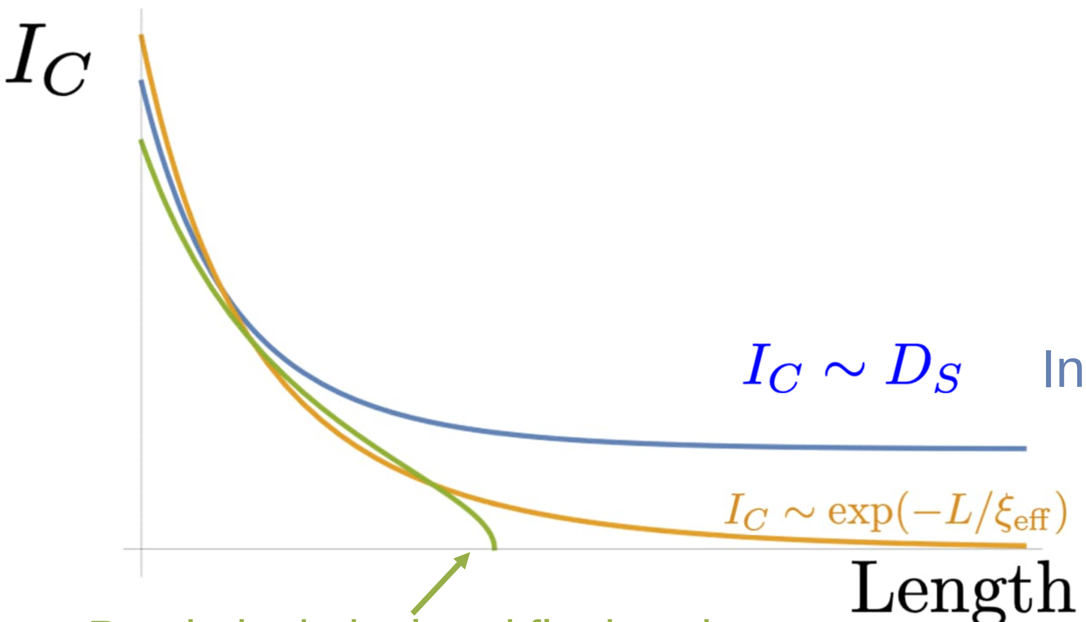
$$\langle r^2 \rangle \equiv \frac{\int dx\, x^2\, \Pi(0,x)}{\int dx\, \Pi(0,x)} = \xi_g^2 = \langle g^{(\mathrm{FB})}(k) \rangle_{\mathrm{BZ}}$$

$$I_{c,\mathrm{int}} \propto V^{-1}(0,L) \propto e^{-L/L_g}$$

$$L_g = \xi_g \sqrt{\frac{T_c}{T - T_c}}$$

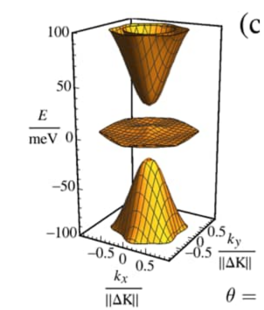
Proximity effect in long SNS junctions: $L \to \infty$

(close to Tc)



Intrinsic SC

Proximity SC



Proximity in isolated flat bands,
no interaction $\xi_{\mathrm{eff}} \sim \xi_g$

$$\xi_{\mathrm{eff}} \sim \max(v_F/T, 1/\sqrt{2mE_g}, \xi_g\sqrt{T_c/|T_c - T|})$$

c.f. Chen and Law PRL 2024; Li et al. arXiv:2404.09211, Iskin arXiv:2409.14921; Thumin+Bouzerar arXiv:2405.06215

Condensed Matter > Superconductivity

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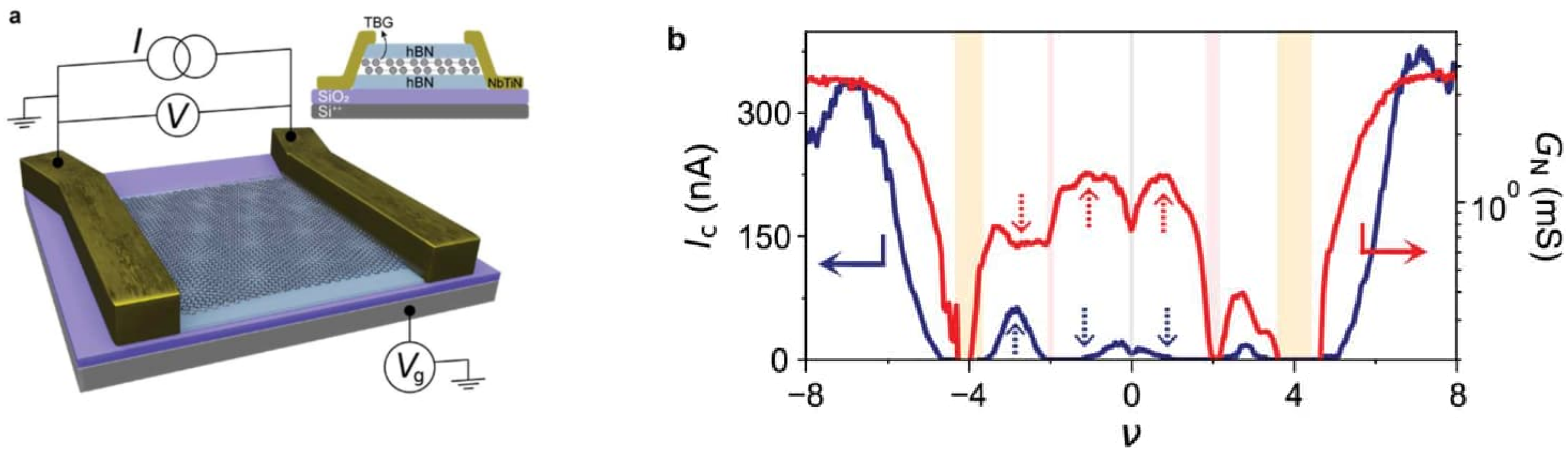

Interaction-induced critical current possible by an order of magnitude estimate

$$T > T_c$$

$$I_c^{\text{int}} = \frac{8e}{\hbar}\frac{W L_g}{A_m}\frac{\Delta_S^2}{U}\left(1 - \frac{U}{4k_B T}\right) e^{-L/L_g}$$

$$L_g = \xi_g \sqrt{U/(4k_B T - U)}$$

Typically, $I_c \sim G_N$

But not here!

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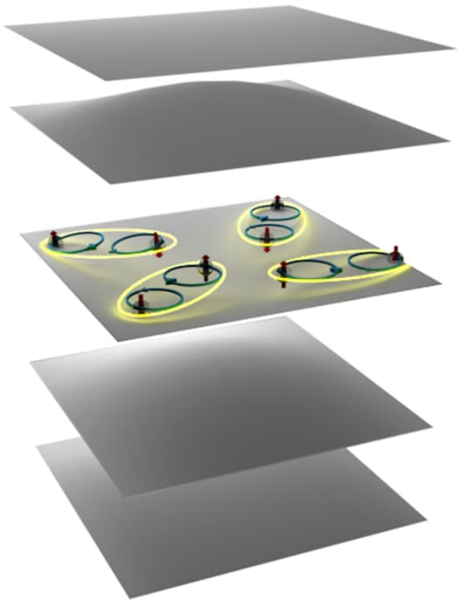


Superconductivity and pair density waves from nearest-neighbor interactions in frustrated lattice geometries

Eeli Lamponen

Sofia Pöntys

Lamponen, Pöntys, PT, arXiv:2502.20911 (2025)

Generic nearest neighbor interactions J

$$\hat{H} = \sum_{i\alpha,j\beta} \sum_{\sigma} t^{\sigma}_{i\alpha j\beta} \hat{c}^{\dagger}_{i\alpha\sigma} \hat{c}_{j\beta\sigma} - \mu \sum_{i\alpha\sigma} \hat{c}^{\dagger}_{i\alpha\sigma} \hat{c}_{i\alpha\sigma} + \hat{H}_{\text{int}}$$

$$\hat{H}_{\text{int}} = -\sum_{i\alpha,j\beta} J_{i\alpha j\beta} \hat{b}^{\dagger}_{i\alpha j\beta} \hat{b}_{i\alpha j\beta}$$

$$\hat{b}_{i\alpha j\beta} = \sum_{\sigma} A_{\sigma} \hat{c}_{i\alpha\sigma} \hat{c}_{j\beta-\sigma}$$

i, j unit cells

α, β orbitals

Pairing susceptibility in mean field

μ, ν = orbital (etc) indices

$$\chi = -\frac{J}{2sN_c} \begin{pmatrix} \mathrm{Re}X & \mathrm{Im}X \\ -\mathrm{Im}X & \mathrm{Re}X \end{pmatrix}$$

$$X_{\mu\nu} = \sum_{\boldsymbol{k}mn} \frac{n_F(\xi_{\boldsymbol{k}+\boldsymbol{q}m}) + n_F(\xi_{\boldsymbol{k}-\boldsymbol{q}n}) - 1}{\xi_{\boldsymbol{k}+\boldsymbol{q}m} + \xi_{\boldsymbol{k}-\boldsymbol{q}n}} \times 2 \langle m_{\boldsymbol{k}+\boldsymbol{q}} | \delta\Delta_{\mu}(\boldsymbol{k}) | n_{\boldsymbol{k}-\boldsymbol{q}} \rangle \langle n_{\boldsymbol{k}-\boldsymbol{q}} | \delta\Delta^{\dagger}_{\nu}(\boldsymbol{k}) | m_{\boldsymbol{k}+\boldsymbol{q}} \rangle$$

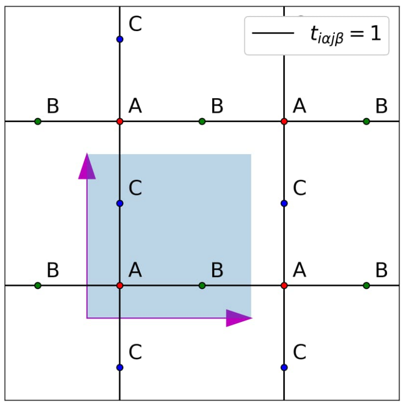
$$\xi_{\boldsymbol{k}m} \equiv \epsilon_{\boldsymbol{k}m} - \mu$$

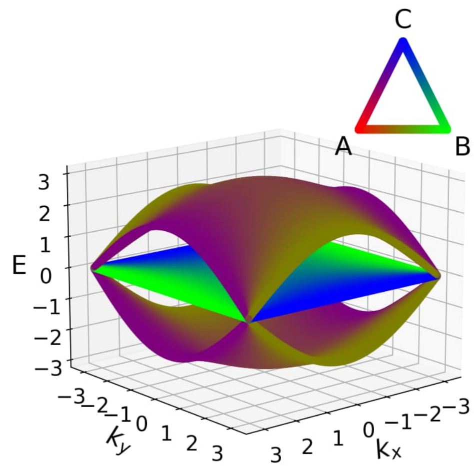
$|m_{\boldsymbol{k}}\rangle$ Bloch states

$$\delta\Delta_{\mu}(\boldsymbol{k}) \equiv \partial_{\Delta^R_{\mu}} \Delta_{\boldsymbol{k}}$$

$$[\Delta_{\boldsymbol{k}}]_{\alpha\beta} = \sum_{j} \left(A_{\downarrow} \overline{\Delta}_{0\beta j\alpha} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}^{\Delta}_{0\beta j\alpha}} - A_{\uparrow} \overline{\Delta}_{0\alpha j\beta} e^{i\boldsymbol{k}\cdot\boldsymbol{r}^{\Delta}_{0\alpha j\beta}} \right)$$

In bipartite lattice flat bands, the form factor vanishes for nearest-neighbor interaction





Lieb lattice: flat band completely localized on orbitals B and C

Bipartite form of the Hamiltonian

$$H_{\boldsymbol{k}} = \begin{array}{cc} & \begin{array}{cc} n_L & n_S \end{array} \\ & \begin{pmatrix} 0 & S_{\boldsymbol{k}} \\ S_{\boldsymbol{k}}^{\dagger} & 0 \end{pmatrix} \begin{array}{c} n_L \\ n_S \end{array} \end{array}$$

=>

$$\langle m_{\boldsymbol{k}+\boldsymbol{q}} | \delta\Delta_{\mu}(\boldsymbol{k}) | n_{\boldsymbol{k}-\boldsymbol{q}} \rangle$$
$$= \begin{pmatrix} \phi^{\dagger}_{\boldsymbol{k}+\boldsymbol{q}m} & 0 \end{pmatrix} \begin{pmatrix} 0 & \partial_{\Delta_{\mu}^{R}} \Delta_{\boldsymbol{k}}^{\mathrm{LS}} \\ \partial_{\Delta_{\mu}^{R}} \Delta_{\boldsymbol{k}}^{\mathrm{SL}} & 0 \end{pmatrix} \begin{pmatrix} \phi_{\boldsymbol{k}-\boldsymbol{q}n} \\ 0 \end{pmatrix} = 0$$

Calugaru… Bernevig
Nat Phys 2022

Consider a specific form of J (spin-exchange interaction)

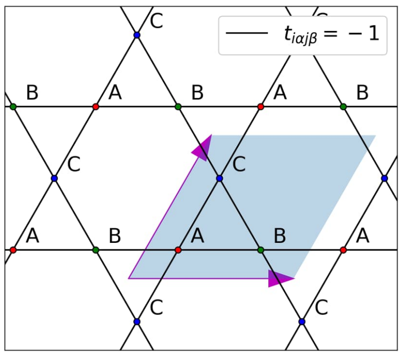
$$\hat{H} = \sum_{i\alpha,j\beta} \sum_{\sigma} t^{\sigma}_{i\alpha j\beta} \hat{c}^{\dagger}_{i\alpha\sigma} \hat{c}_{j\beta\sigma} - \mu \sum_{i\alpha\sigma} \hat{c}^{\dagger}_{i\alpha\sigma} \hat{c}_{i\alpha\sigma} + \hat{H}_{\text{int}}$$

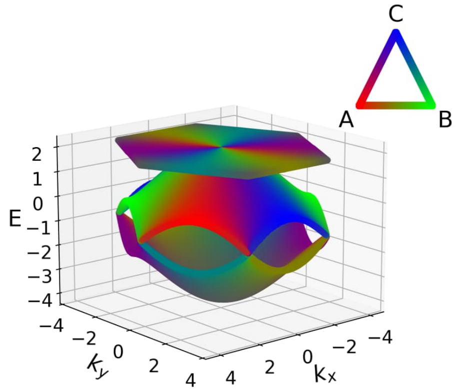
$$\hat{H}_{\text{int}} = -\frac{J}{2} \sum_{\langle i\alpha,j\beta \rangle} \hat{h}^{\dagger}_{\boldsymbol{i}\alpha\boldsymbol{j}\beta} \hat{h}_{\boldsymbol{i}\alpha\boldsymbol{j}\beta}$$

$$\hat{h}_{\boldsymbol{i}\alpha\boldsymbol{j}\beta} = \hat{c}_{i\alpha\uparrow} \hat{c}_{j\beta\downarrow} - \hat{c}_{i\alpha\downarrow} \hat{c}_{j\beta\uparrow}$$

Pairing susceptibility predicts a 2***q*** = ***M*** pair-density wave (PDW) at kagome vHs, due to sub-lattice interference

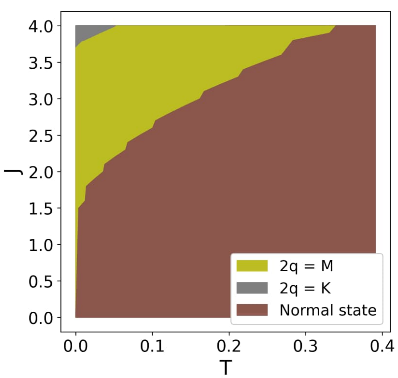
$$\Delta_{i\alpha(i+j)\beta} = \overline{\Delta}_{0\alpha j\beta} e^{i2\boldsymbol{q}\cdot(\boldsymbol{r}_{i\alpha}+\boldsymbol{r}_{(i+j)\beta})/2}$$

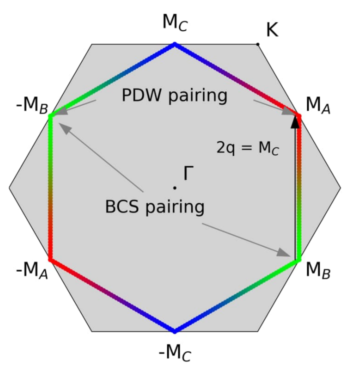




M-PDW or M-CDW predicted due to this effect in different interaction models

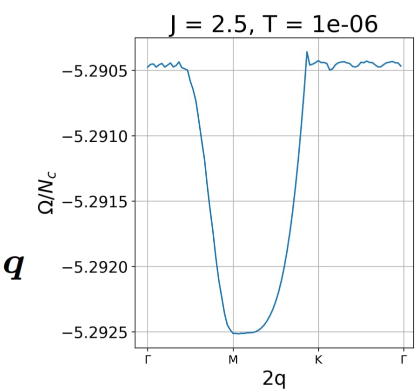
Kiesel, Thomale, PRB 86, 121105 (2012)
Wang et al., PRB 87, 115135 (2013)
Dong, Wang, Zhou, PRB 107, 045127 (2023)
Wu, Thomale, Raghu, PRB 108, L081117 (2023)
Fu et al., arXiv:2405.09451 (2024)





However, the superfluid weight is zero in our interaction model

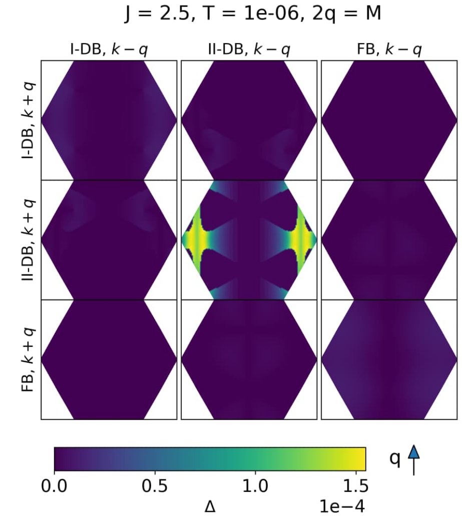
$$[D_s]_{ij} = \frac{1}{V}\frac{\mathrm{d}^2\Omega(\boldsymbol{q}')}{\mathrm{d}q'_i \mathrm{d}q'_j}\bigg|_{\boldsymbol{q}'=\boldsymbol{q}}$$

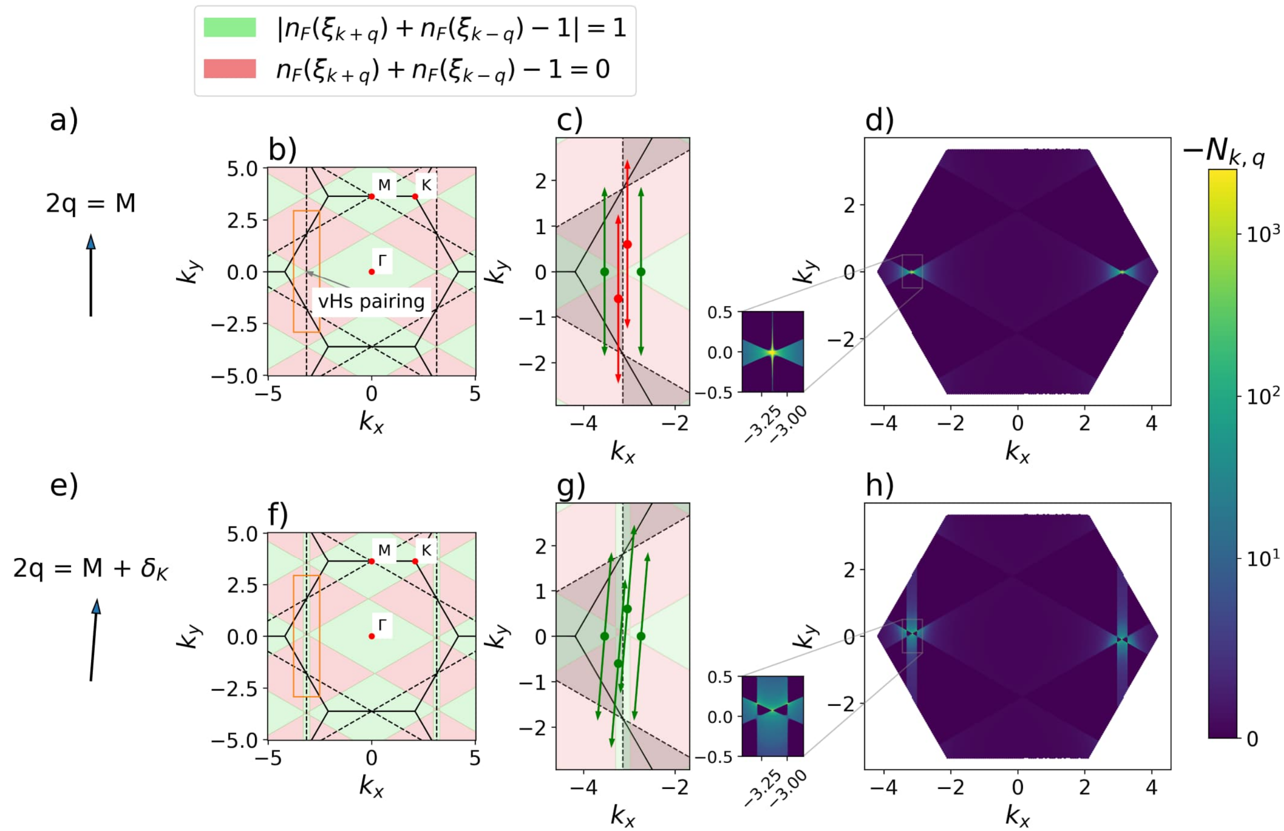


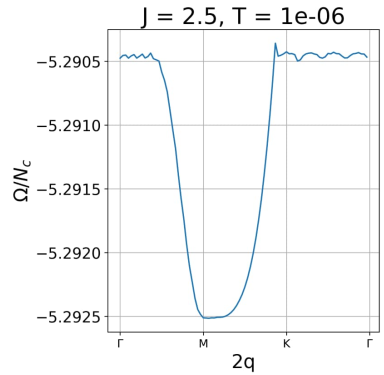
Kagome vHs M-PDW: insensitivity of the energy to the order parameter phase ($\boldsymbol{q}$)

Band-resolved gap equation integrand

$$\Delta_{i\alpha(i+j)\beta} = \overline{\Delta}_{0\alpha j\beta} e^{i2\boldsymbol{q}\cdot(\boldsymbol{r}_{i\alpha}+\boldsymbol{r}_{(i+j)\beta})/2}$$





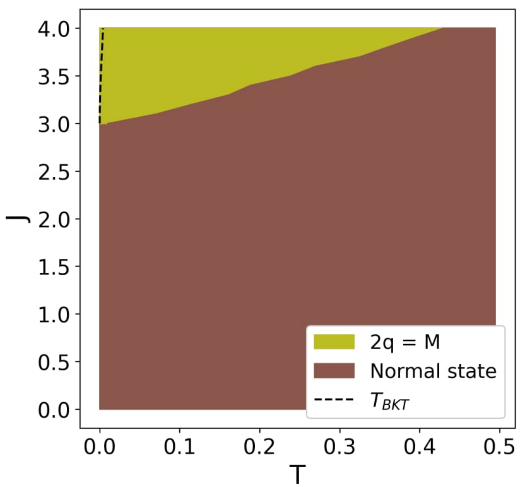


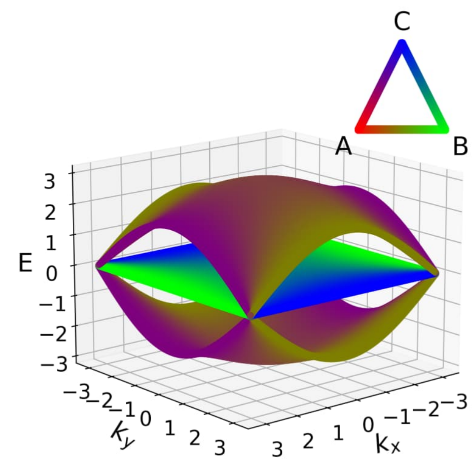
$$N_{\boldsymbol{k},\boldsymbol{q}} \equiv (n_F(\xi_{\boldsymbol{k}+\boldsymbol{q}}) + n_F(\xi_{\boldsymbol{k}-\boldsymbol{q}}) - 1)/(\xi_{\boldsymbol{k}+\boldsymbol{q}} + \xi_{\boldsymbol{k}-\boldsymbol{q}})$$

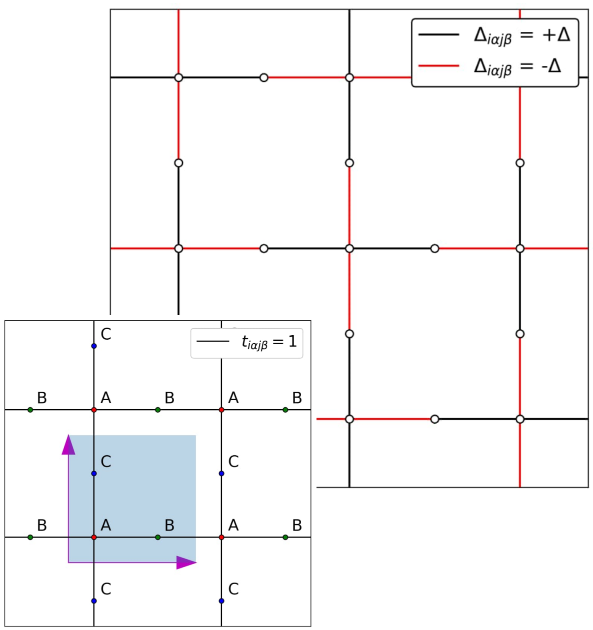
Susceptibility without the form factor

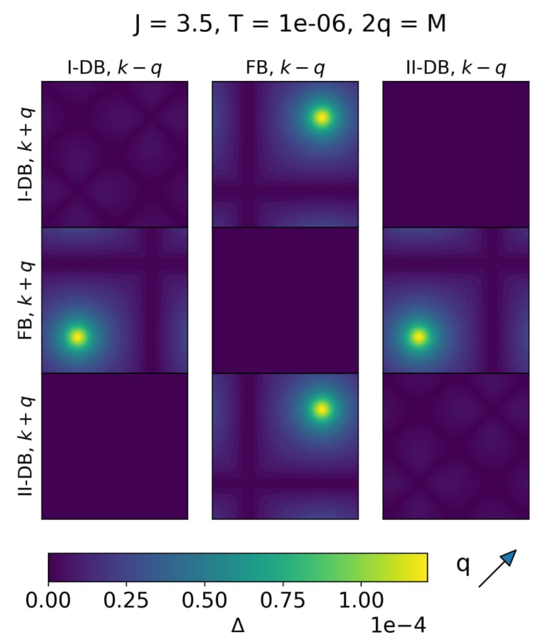
At the bipartite Lieb lattice flat band, critical pairing interaction needed

The ground state is a M-PDW; why?

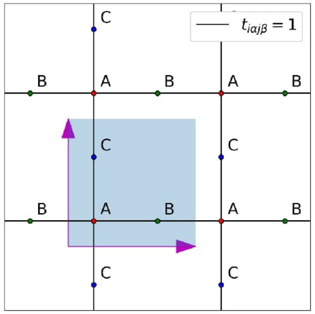


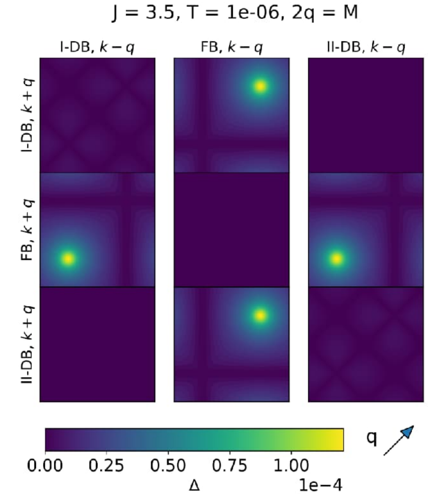


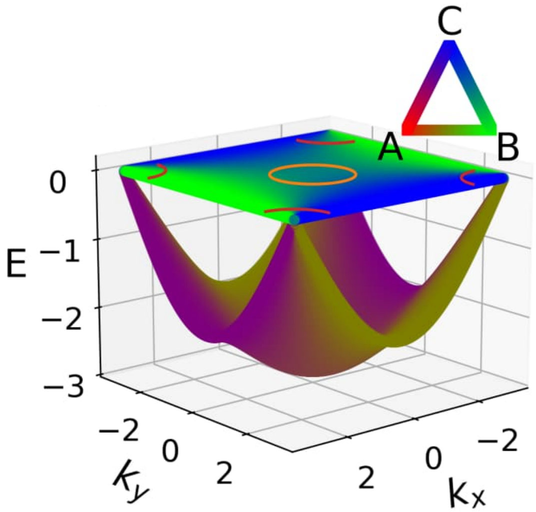




The ground state at Lieb flat band is M-PDW; why?







Susceptibility around the Dirac point of the dispersive bands

$$S_{\mu\nu}^{\theta} = -\frac{J}{4}\frac{n_F(ck_0) - \frac{1}{2}}{ck_0} \cdot 2\langle m_\theta|\delta\Delta_\mu(\boldsymbol{k})|n_\theta^{\boldsymbol{q}}\rangle\langle n_\theta^{\boldsymbol{q}}|\delta\Delta_\nu^\dagger(\boldsymbol{k})|m_\theta\rangle$$

$$|m_\theta\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ f_B^m(\theta) \\ f_C^m(\theta) \end{pmatrix} \qquad |n_\theta^{\boldsymbol{q}}\rangle = \begin{pmatrix} 0 \\ f_B^n(\theta, \boldsymbol{q}) \\ f_C^n(\theta, \boldsymbol{q}) \end{pmatrix}$$

$$\sqrt{[F_{BB}^n(\boldsymbol{q}) - F_{CC}^n(\boldsymbol{q})]^2 + 4|F_{BC}^n(\boldsymbol{q})|^2} \qquad F_{\alpha\beta}^n(\boldsymbol{q}) = \frac{1}{2\pi}\int_0^{2\pi} f_\alpha^n(\theta, \boldsymbol{q})^* f_\beta^n(\theta, \boldsymbol{q})\, \mathrm{d}\theta$$

$$|F_{BC}^n(\boldsymbol{q})|^2 \leq F_{BB}^n(\boldsymbol{q}) F_{CC}^n(\boldsymbol{q})$$

In this case, quantum geometry is detrimental for pairing!

BKT temperatures with NN interactions often low compared to on-site Hubbard-U

Nearest-neighbour (NN) $\hat{H}_{\text{int}} = -\frac{J}{2}\sum_{\langle i\alpha, j\beta\rangle} \hat{h}^{\dagger}_{\boldsymbol{i}\alpha\boldsymbol{j}\beta}\hat{h}_{\boldsymbol{i}\alpha\boldsymbol{j}\beta}$ $\hat{h}_{\boldsymbol{i}\alpha\boldsymbol{j}\beta} = \hat{c}_{i\alpha\uparrow}\hat{c}_{j\beta\downarrow} - \hat{c}_{i\alpha\downarrow}\hat{c}_{j\beta\uparrow}$

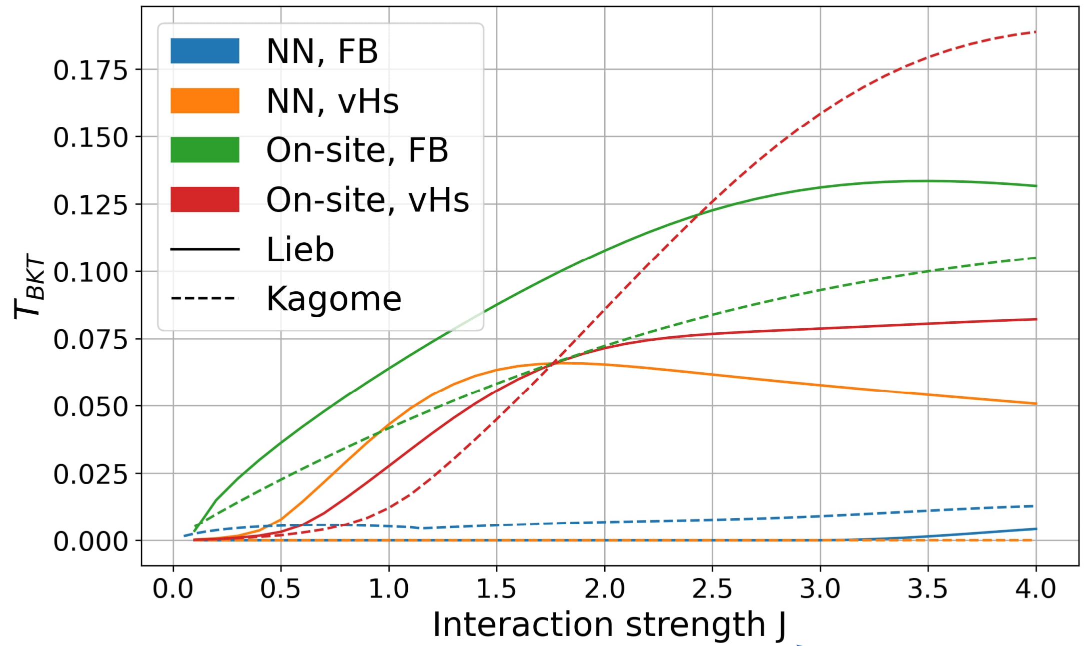
On-site $\hat{H}_{\text{int}} = -U\sum_{\boldsymbol{i}\alpha}\hat{c}^{\dagger}_{i\alpha\uparrow}\hat{c}^{\dagger}_{i\alpha\downarrow}\hat{c}_{i\alpha\downarrow}\hat{c}_{i\alpha\uparrow}$

$$T_{\text{BKT}} = \frac{\pi}{8}\sqrt{\det(D_s(T_{\text{BKT}}))}$$

$$[D_s]_{ij} = \frac{1}{V}\frac{\mathrm{d}^2\Omega(\boldsymbol{q}')}{\mathrm{d}q'_i\mathrm{d}q'_j}\bigg|_{\boldsymbol{q}'=\boldsymbol{q}}$$

For a single-band system

$$D_s(T) = \frac{e^2 n_s(T)}{m}$$



Effective mass – infinite on a flat band, finite at the vHs

Hubbard-U for on-site

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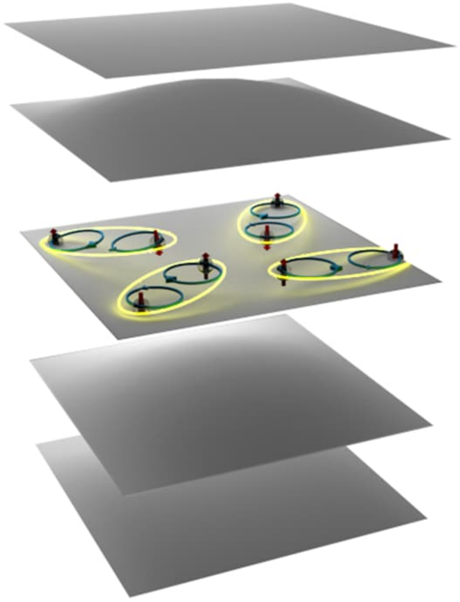


Superconducting transport in flat bands

Nearest-neighbor pairing and quantum geometry in Lieb and kagome lattice flat bands and van Hove singularities

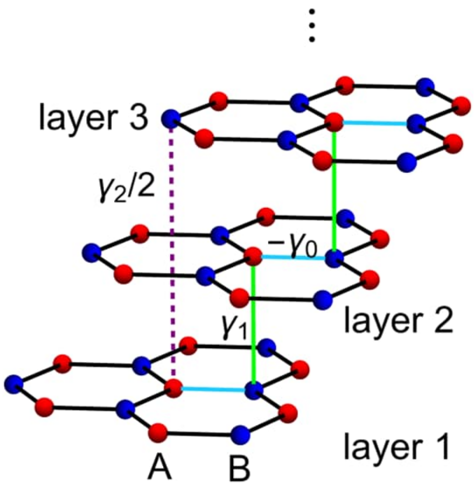
Rhombohedral graphite and LLL quantum geometry

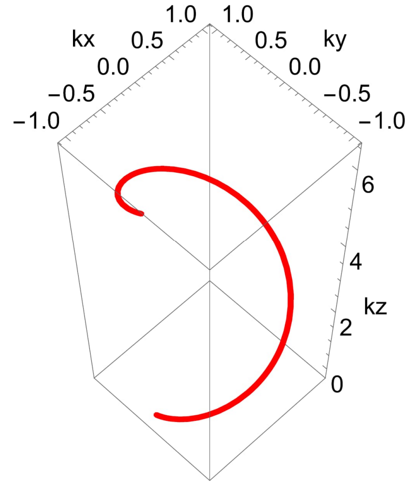
Quantum geometry of the surface states of rhombohedral graphite and its effects on the surface superconductivity

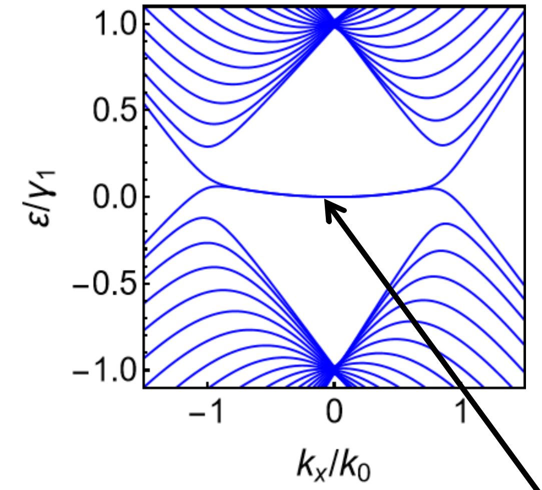
Guodong Jiang

Tero Heikkilä

Jiang, Heikkilä, PT, arXiv:2504.03617 (2025)







Drumhead surface states

$$\hat{H} = \{-\gamma_0 \sum_{n,\langle iA,jB\rangle} c^{+}_{niA} c_{njB} + \gamma_1 \sum_{n} c^{+}_{n+1,iA} c_{niB}\} + h.c.$$

F. T. $$h(k_x,k_y,k_z) = \begin{pmatrix} 0 & c.c. \\ k_x + ik_y + e^{ik_z} & 0 \end{pmatrix}$$

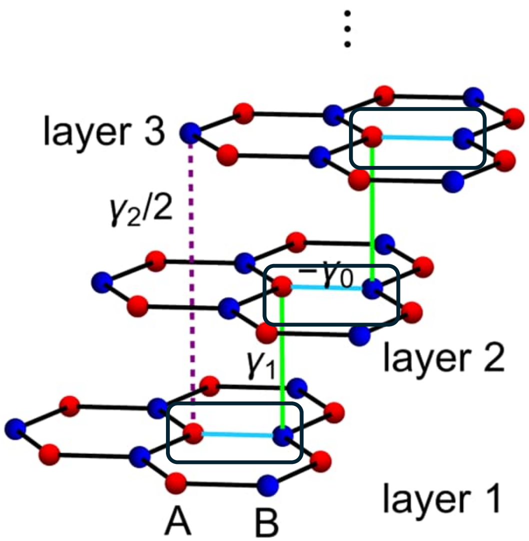
In scaled units: γ_1 (energy), γ_0/v_f (momentum).

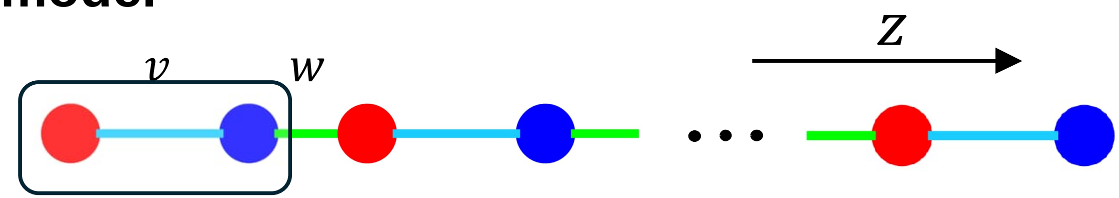
Degeneracies:

$$k_x + ik_y + e^{ik_z} = 0 \Longrightarrow \begin{cases} k_x = -\cos k_z \\ k_y = -\sin k_z \end{cases}$$ nodal spiral

Two incomplete pictures of RG surface states:

(1) Su-Schrieffer-Heeger model



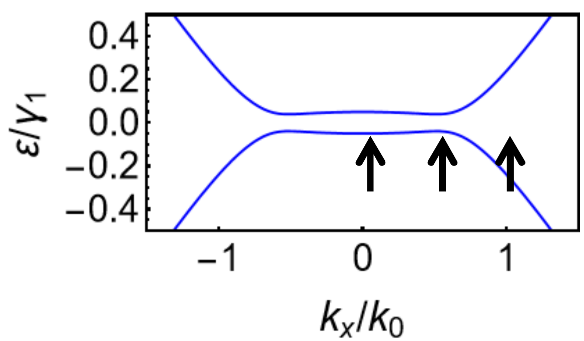


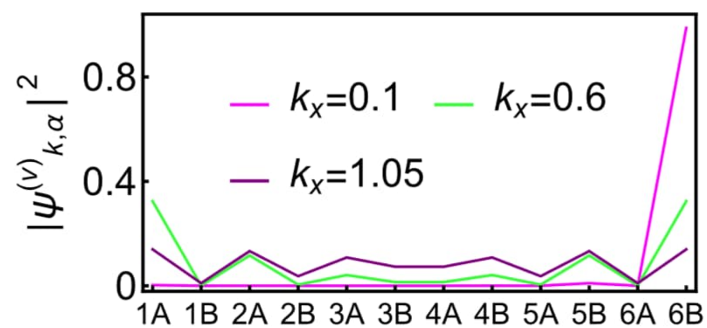
$|w| > |v|$: topological phase, two E=0 edge states

$$h(k_z) = \begin{pmatrix} 0 & c.c. \\ v + we^{ik_z} & 0 \end{pmatrix}$$

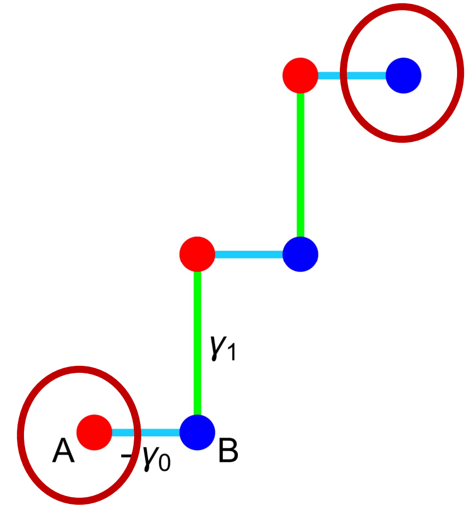
SSH model corresponds to only one point $(k_x, k_y) = (\frac{v}{w}, 0)$ in the drumhead region.

For RG, surface state decay length varies with $\mathbf{k}$!





(2) Two-orbital effective model



From perturbation theory ($H = H_0 + H'$)

$$H_{\text{eff}} = E_0 + P_0 H' \sum_{j=1}^{\infty} (\tilde{G}_0 H')^j P_0 \,,$$

$$P_0 = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix}$$ is the projection to 1A and NB,

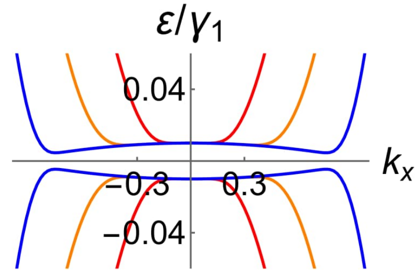
$$\tilde{G}_0 = P_1 \frac{1}{E_0 - H_0} P_1,\ P_1 = 1 - P_0 \quad \Longrightarrow \quad H_{\text{eff}} = (-1)^{N-1} \begin{pmatrix} m & \pi^{*N} \\ \pi^N & -m \end{pmatrix}$$

$$\pi = k_x + i k_y$$ McCann 2006, Guinea 2006, Min 2008, etc.

limitations:

- **good dispersion near the center**
- **good quantum geometry at the rim**
- **topologically incorrect**

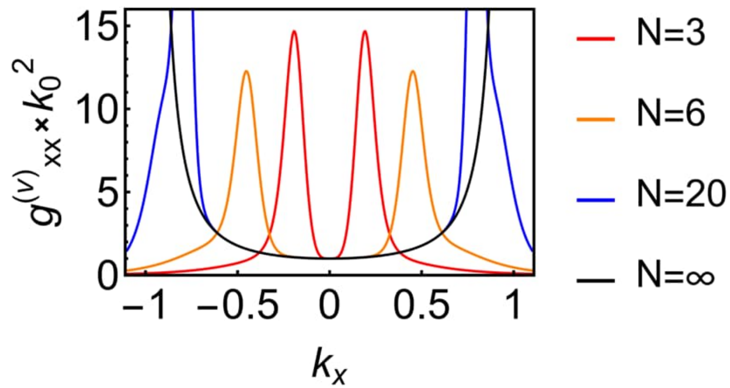
Quantum metric of RG surface states

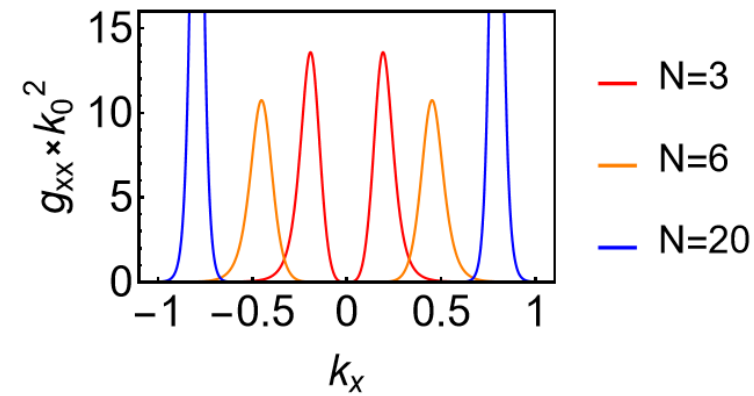


Dispersion with small surface potential

Tight binding (✓) VS Two-orbital effective (×)

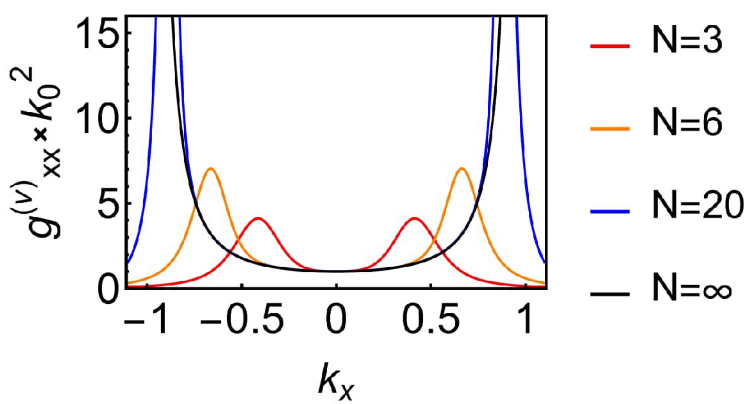
$m = 0.01\gamma_1$

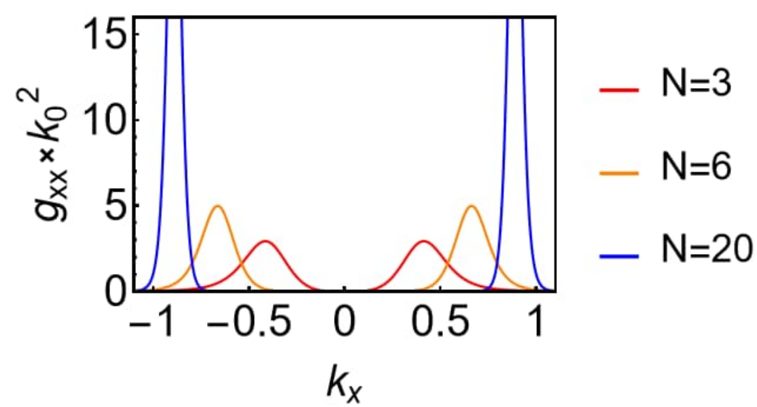




Increase surface potential m

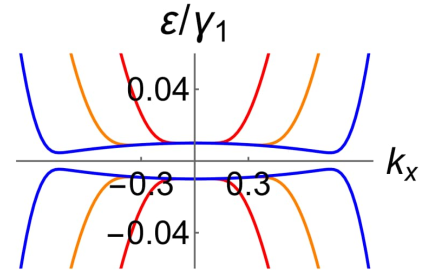
$m = 0.1\gamma_1$





v: valence band

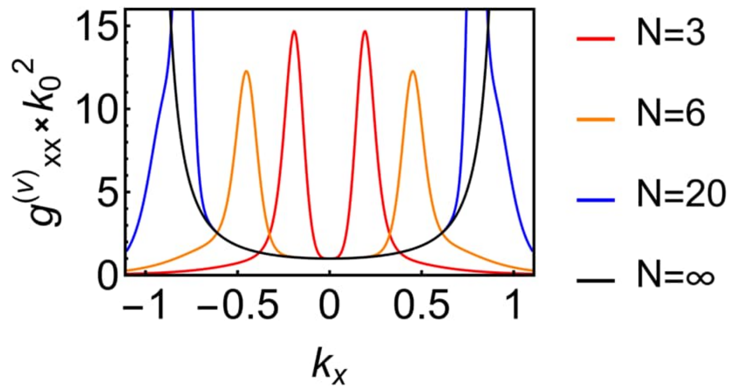
Quantum metric of RG surface states

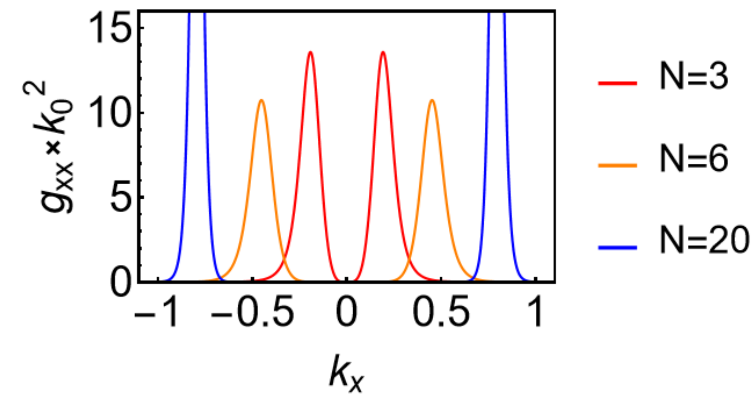


Dispersion with small surface potential

Tight binding (✓) VS Two-orbital effective (×)

$m = 0.01\gamma_1$





Increase surface potential m

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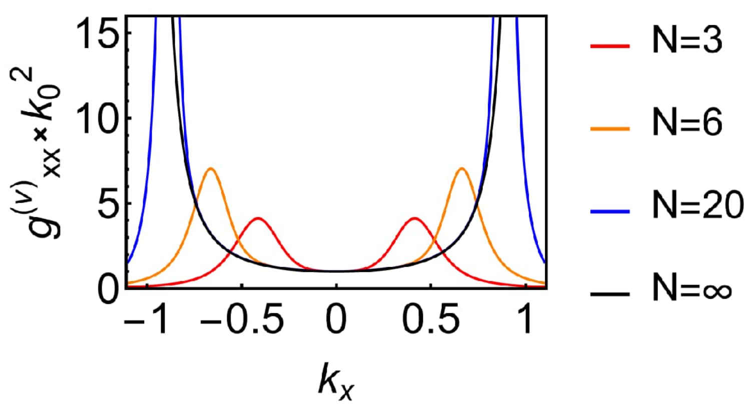
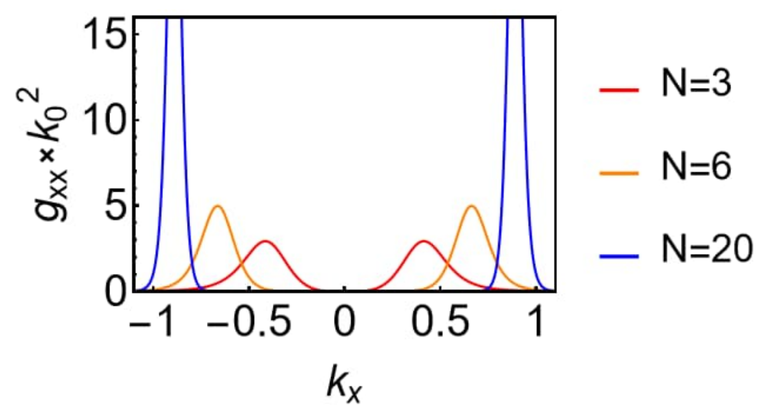
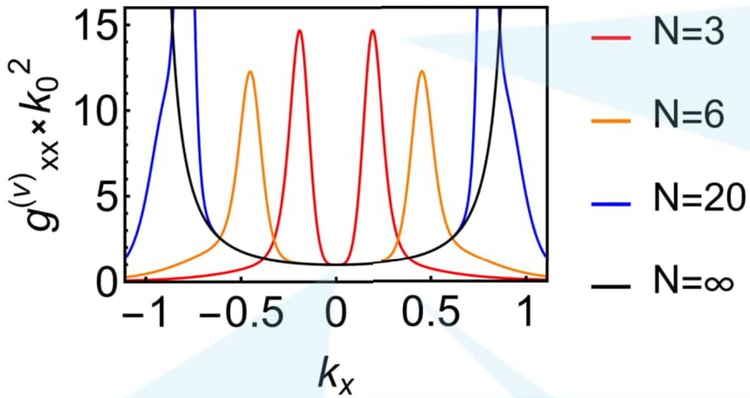




(1) Nonzero at the center;
(2) Peak at the rim;
(3) Converge to N=∞ as N increases

v: valence band

QGT of RG surface states



(1) Nonzero at the center

N=∞ limit:

$$\psi_{\mathbf{k}}^{(v)}(z) = \frac{\sqrt{1-k^2}}{k} e^{\kappa(\mathbf{k})z} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\kappa(\mathbf{k}) = \ln[-(k_x + ik_y)]$$

QGT $B_{\mu\nu}^{(v)}(\mathbf{k}) = \frac{1}{(1-k^2)^2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$

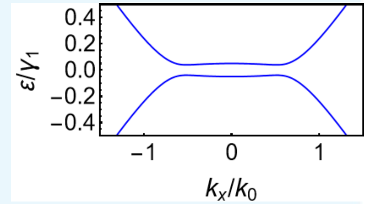
The nonzero value is from the **momentum-dependent decay, i.e. $\nabla_{\mathbf{k}}\kappa$**

(2) Peak at the rim

A surface hybridization effect. At some momentum, $\lambda(\mathbf{k}) = 1/\kappa(\mathbf{k}) \sim$ N.

$$B_{\mu\nu}^{(v)}(\mathbf{k}) \equiv \sum_l \langle \partial_\mu \psi_{\mathbf{k}}^{(v)} | \psi_{\mathbf{k}}^{(l)} \rangle \langle \psi_{\mathbf{k}}^{(l)} | \partial_\nu \psi_{\mathbf{k}}^{(v)} \rangle$$

Focus on the $l = c$ term



$$\Longrightarrow \frac{\langle \psi_{\mathbf{k}}^{(v)} | \partial_\mu H_{\mathbf{k}} | \psi_{\mathbf{k}}^{(c)} \rangle \langle \psi_{\mathbf{k}}^{(c)} | \partial_\nu H_{\mathbf{k}} | \psi_{\mathbf{k}}^{(v)} \rangle}{\left(\varepsilon_{v,\mathbf{k}} - \varepsilon_{c,\mathbf{k}}\right)^2}$$

Jiang, Heikkilä, PT, arXiv: 2504.03617
Bernevig, Kwan, arXiv: 2503.09692
(Different approaches)

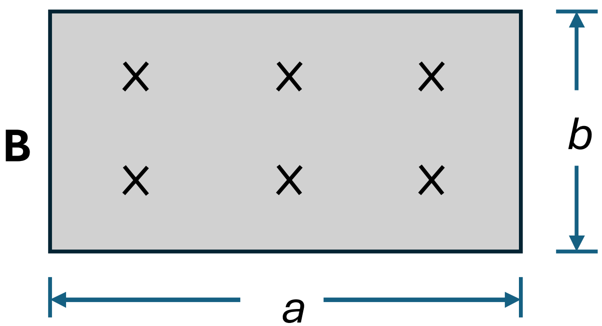
(3) N-independent at the center

No surface hybridization
$\Longrightarrow$ wavefunctions for different N are the same.

LLL quantum geometry

$$B = g - \frac{i}{2}\Omega, \qquad \mathrm{Tr}[g] \geq |\Omega|$$

$$B_{\mu\nu}^{(v/c)}(\mathbf{k}) = \frac{1}{(1-k^2)^2}\begin{pmatrix} 1 & \pm i \\ \mp i & 1 \end{pmatrix}$$ Quantum geometric tensor of RG



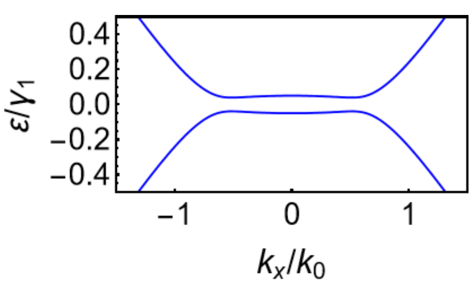
A fictitious unit cell

Hamiltonian of the LL problem:

$$H = \frac{(\mathbf{p} + \mathbf{A})^2}{2m}, \qquad (e = \hbar = 1)$$

Quantum geometric tensor of LL

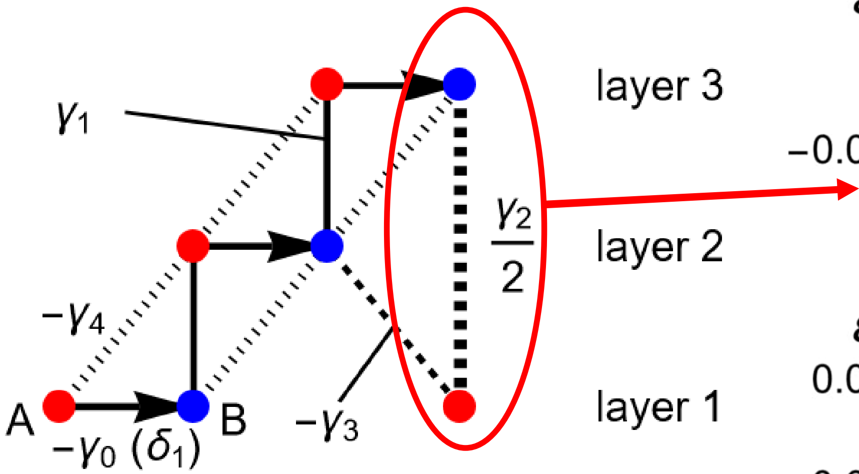
$$B_{\mu\nu}^{n}(\mathbf{k}) = \frac{l_B^2}{2}\begin{pmatrix} 2n+1 & -i \\ i & 2n+1 \end{pmatrix}, \quad n = 0, 1, \ldots$$



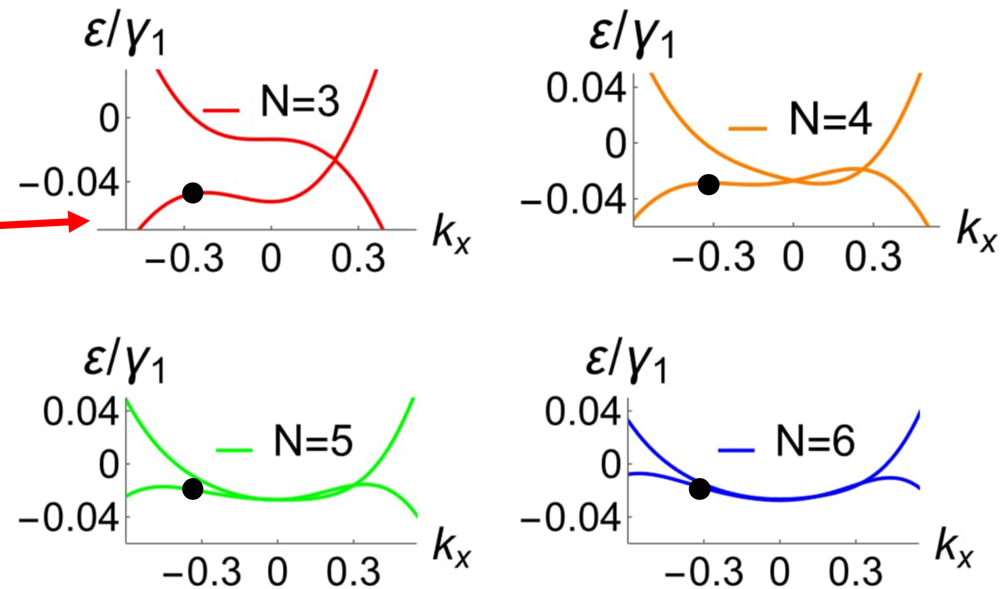
The two surface bands of RG resemble a pair of decoupled LLLs with opposite B fields

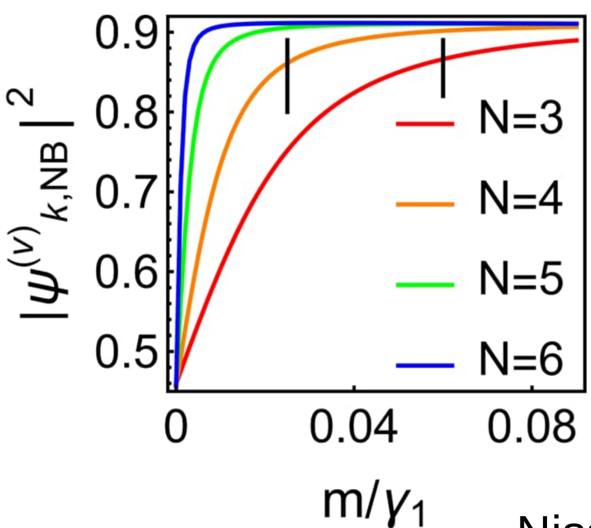
Superconductivity and surface polarization

Effects of long-range hoppings:



Dispersion at m=0:





Displacement field has two effects:

(1) Flatten the band and enhance DOS;

(2) Polarize electron density to one surface

Once m is above the m_s, QGT is the same as previously discussed !

Nissinen, Heikkilä, Volovik (2021) PRB 103, 245115

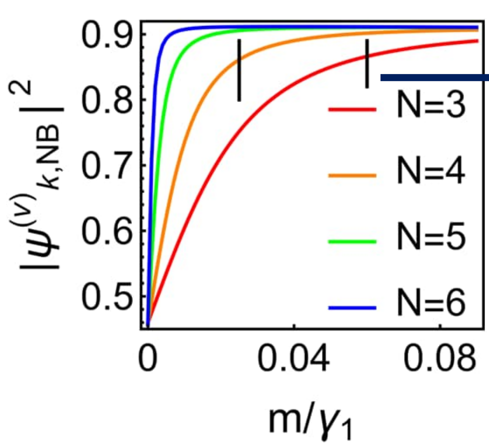
Superconductivity and surface polarization

Gap equation in orbital basis (onsite pairing, s-wave and doped to the valence band)

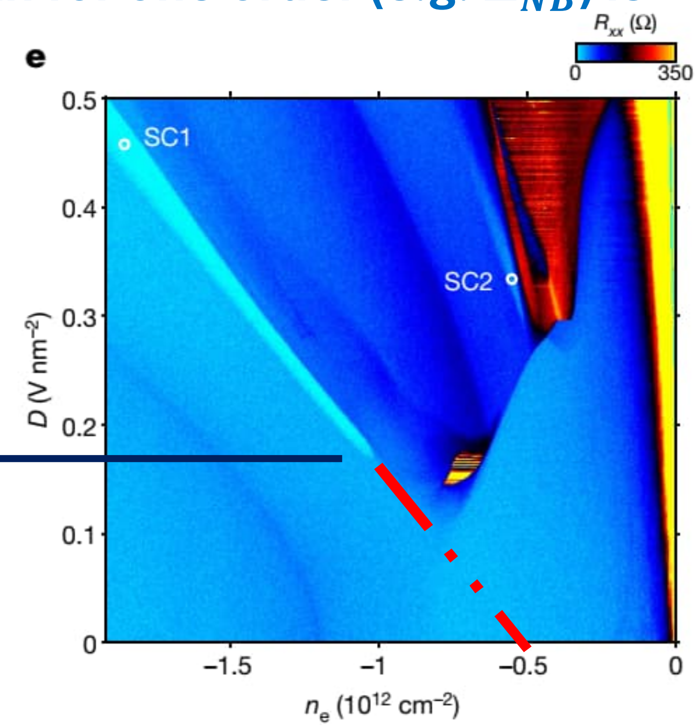
$$\Delta_\alpha = -U\langle c_{i\alpha\downarrow} c_{i\alpha\uparrow}\rangle \qquad \alpha\text{=1A or NB}$$

$$\Delta_v(k) = \left|\psi^{(v)}_{k,1A}\right|^2 \Delta_{1A} + \left|\psi^{(v)}_{k,NB}\right|^2 \Delta_{NB}$$

At full polarization, **effective coupling strength for one order (e.g. Δ_{NB}) is doubled**.



m_s of trilayer ~23 meV



Zhou… Young, Nature 2021

Superconductivity and topological heavy-fermion model

Fictitious superconducting state in large N-layer RG:

Distribution of superfluid stiffness in **k**-space:

$$D_{s,\mu\nu} = 2\sum_{\mathbf{k}}[f_{\mu\nu}^{\mathrm{conv}}(\mathbf{k}) + f_{\mu\nu}^{\mathrm{geo}}(\mathbf{k})]$$

$$f_{\mu\nu}^{\mathrm{conv}}(\mathbf{k}) = -\left(\frac{\xi_{v,\mathbf{k}}}{E_{v,\mathbf{k}}} + 1\right)\partial_\mu\partial_\nu\xi_{v,\mathbf{k}}$$

dispersion

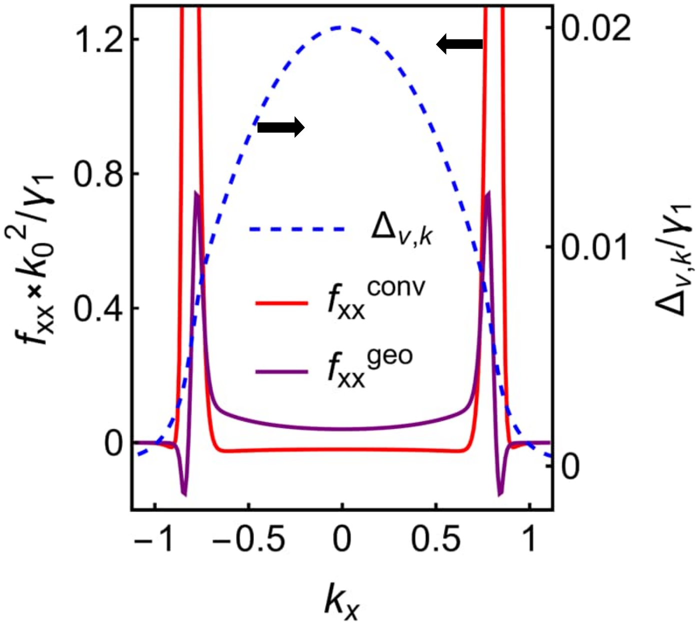
$$f_{\mu\nu}^{\mathrm{geo}}(\mathbf{k}) = \frac{1}{E_{v,\mathbf{k}}}\mathrm{Tr}\{\partial_\mu P_{\mathbf{k}}^{(v)}\hat{\Delta}\partial_\nu P_{\mathbf{k}}^{(v)}\hat{\Delta} - \partial_\mu\partial_\nu P_{\mathbf{k}}^{(v)}\hat{\Delta}P_{\mathbf{k}}^{(v)}\hat{\Delta}\}$$

Pairing matrix in orbital basis

Fully polarized order parameter:

$$\hat{\Delta} = \begin{pmatrix} 0 & & & \\ & \ddots & & \\ & & 0 & \\ & & & \Delta_{NB} \end{pmatrix}$$

$$f_{\mu\nu}^{\mathrm{geo}}(\mathbf{k}) = 2\Delta_{NB}(\delta_{\mu\nu} + \frac{2}{1-k^2}k_\mu k_\nu)$$

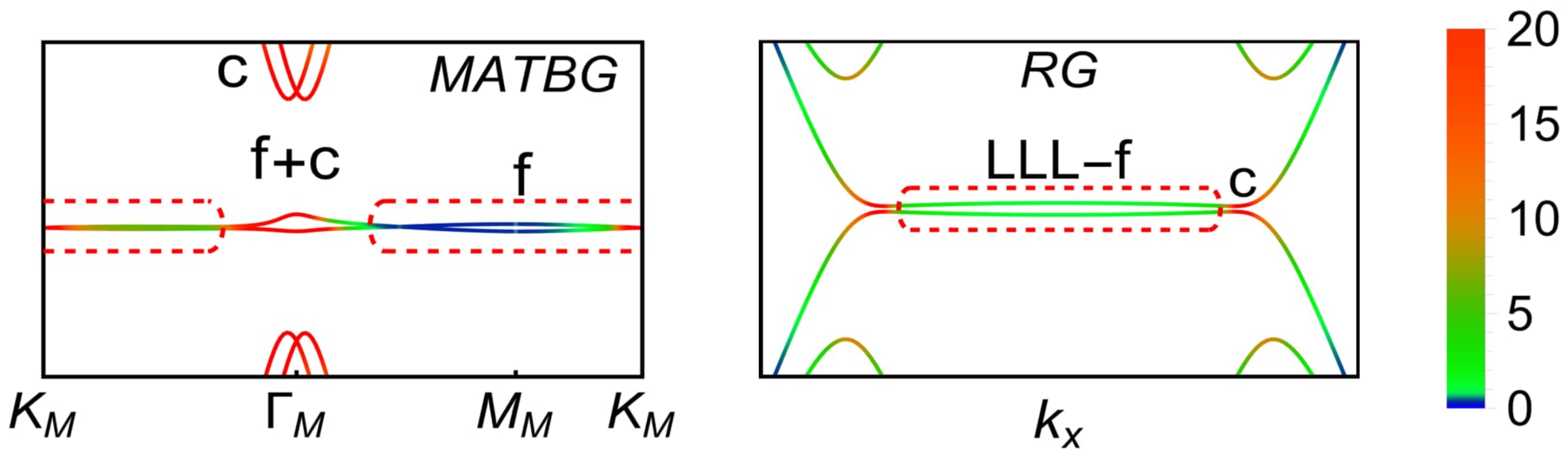


Superconductivity and topological heavy-fermion model

Unusual heavy-fermion picture of RG

Usual topological HF (Song, Bernevig, PRL 2022)
f-electron localized in all dimensions;
Wannierizable;

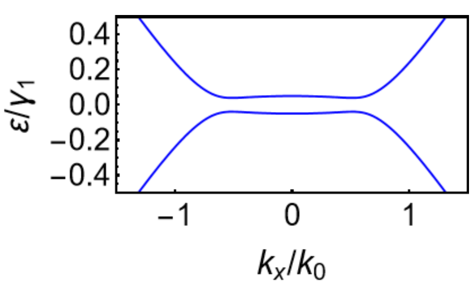
We find: Unusual RG HF
only localized in z direction, and delocalized in x-y direction (LLL QG);
non-Wannierizable (topological)



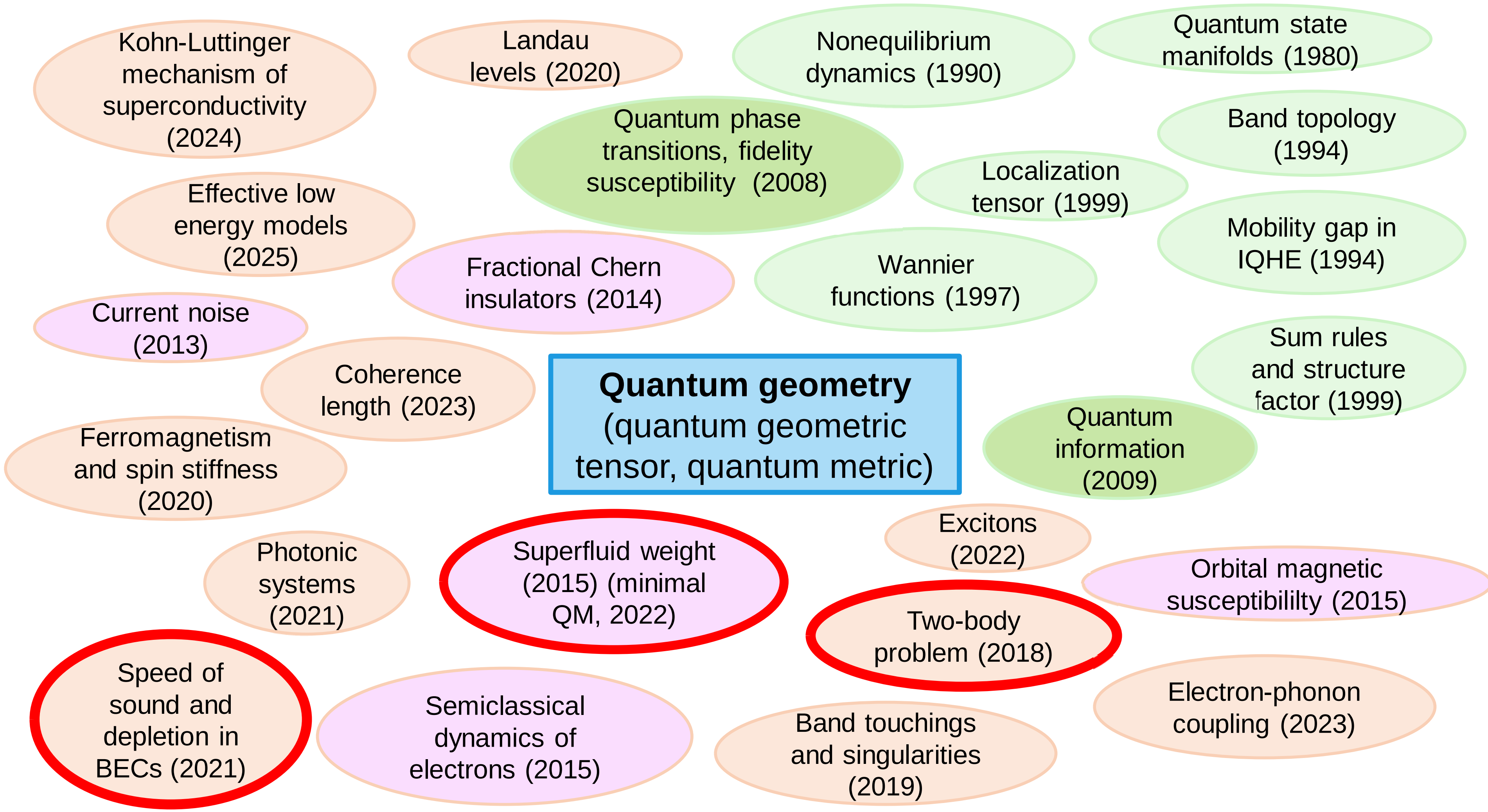
Are there other examples of unusual HFs in condensed matter?

Summary

- RG surface bands have similar QGT to LLL
- Unusual HFs localized in reduced dimensions
- Implications for other correlated phases (fractional topological phases, etc.?)



$$\frac{1}{(1-k^2)^2}\begin{pmatrix} 1 & \pm i \\ \mp i & 1 \end{pmatrix}$$



Perspective on quantum geometry
PT, PRL 2023
Review on quantum geometry
Yu, Bernevig, Queiroz, Rossi, PT, Yang, arXiv 2025

Review on quantum geometric superconductivity
PT, Peotta, Bernevig, Nat. Rev. Phys. 2022

Perspective

PHYSICAL REVIEW LETTERS **131,** 240001 (2023)

Essay: Where Can Quantum Geometry Lead Us?

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Quantum geometry defines the phase and amplitude distances between quantum states. The phase distance is characterized by the Berry curvature and thus relates to topological phenomena. The significance of the full quantum geometry, including the amplitude distance characterized by the quantum metric, has started to receive attention in the last few years. Various quantum transport and interaction phenomena have been found to be critically influenced by quantum geometry. For example, quantum geometry allows counterintuitive flow of supercurrent in a flat band where single electrons are immobile. In this Essay, I will discuss my view of the important open problems and future applications of this research topic and will try to inspire the reader to come up with further ideas. At its best, quantum geometry can open a new chapter in band theory and lead to breakthroughs as transformative as room-temperature superconductivity. However, first, more experiments directly showing the effect of quantum geometry are needed. We also have to integrate quantum geometry analysis in our most advanced numerical methods. Further, the ramifications of quantum geometry should be studied in a wider range, including electric and electromagnetic responses and interaction phenomena in free- and correlated-electron materials, bosonic systems, optics, and other fields.

Part of a series of Essays which concisely present author visions for the future of their field.

DOI: 10.1103/PhysRevLett.131.240001

properties of the eigenfunctions. Now, it seems that topological physics was perhaps only one aspect of a wider and possibly even more influential concept, namely, quantum geometric physics.