Gravitational Wave Detection

Foundations and Survey

- Fundamentals
- Interferometers
- Resonant bars
- Observations
- Universal spontaneous decoherence

Fundamentals

What is a gravitational wave?

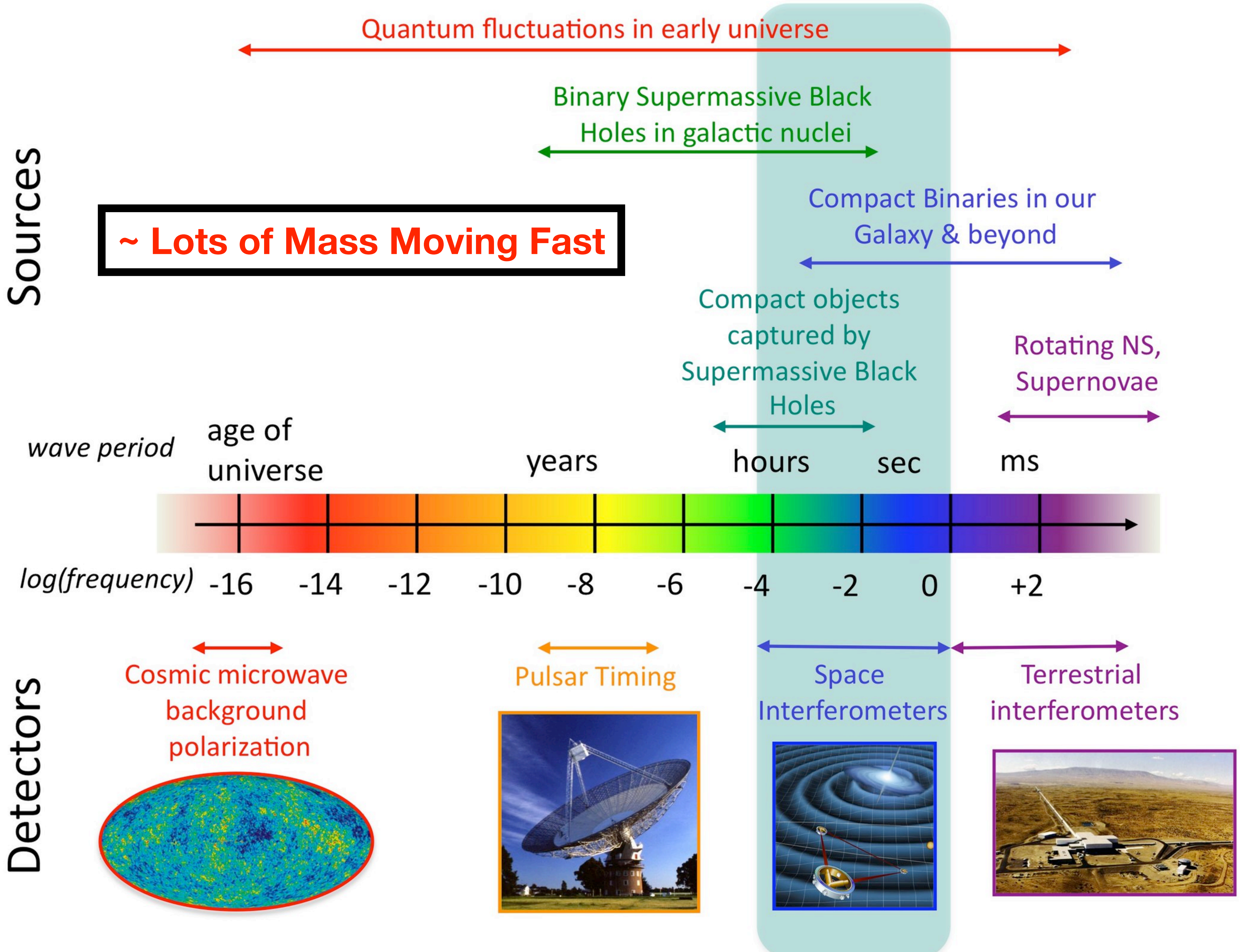
Gravitational waves are distortions of space-time that propagate through empty space at the speed of light.

They are a prediction of the general theory of relativity, where they arise as solutions of the matter-free Einstein equations.

Gravitational waves have been observed both indirectly, through their effect on pulsar spin-down rates, and - starting in 2012 - directly, at LIGO and several other facilities.

Big questions in astrophysics, cosmology, and high-energy physics can addressed through observation (or non-observation) of gravitational radiation.

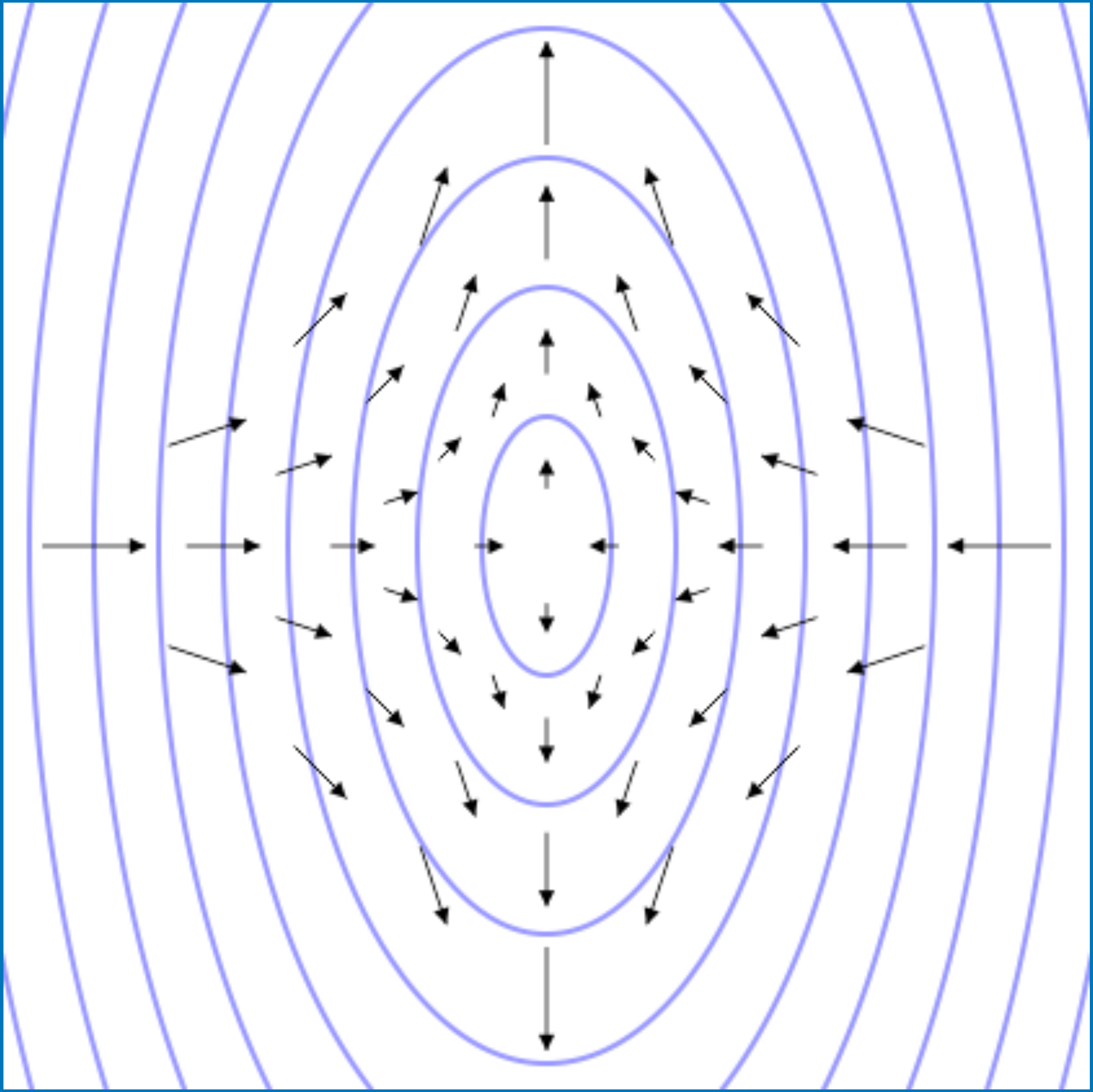
The Gravitational Wave Spectrum



After a broad over-view, my special focus in these lectures will be on characteristically quantum effects in gravitational radiation.

Gravitational radiation is in many ways similar to electromagnetic radiation. In the electromagnetic case, *quantum* optics is a thriving field. A lot of what we'll be doing is transfer of theoretical technology.

Sreenath Manikandan



Distortion in Space-Time

$R_{\mu\nu} \;=\; 0$ (or $G_{\mu\nu} \;=\; 0$)

With $g_{\mu\nu} \;\approx\; \eta_{\mu\nu} + \kappa h_{\mu\nu}$, $\kappa \equiv \sqrt{32\pi G} = 1.6 \times 10^{-32}$ cm., $h_{\mu\nu}$ becomes a field with conventionally normalized kinetic energy $\sim \frac{1}{2}\partial_\alpha h_{\mu\nu}\partial^\alpha h^{\mu\nu}$. Expand in plane waves

$$h_{\mu\nu}(x,t) \;=\; \int \frac{d^3k}{(2\pi)^3} \sum_{\alpha=1,2} \pi^{\alpha}_{\mu\nu}(k)\; h_\alpha(k)\; e^{i(kx-\omega t)}$$

There are only two dynamical degrees of freedom; the rest parameterize gauge transformations, and are uncoupled.

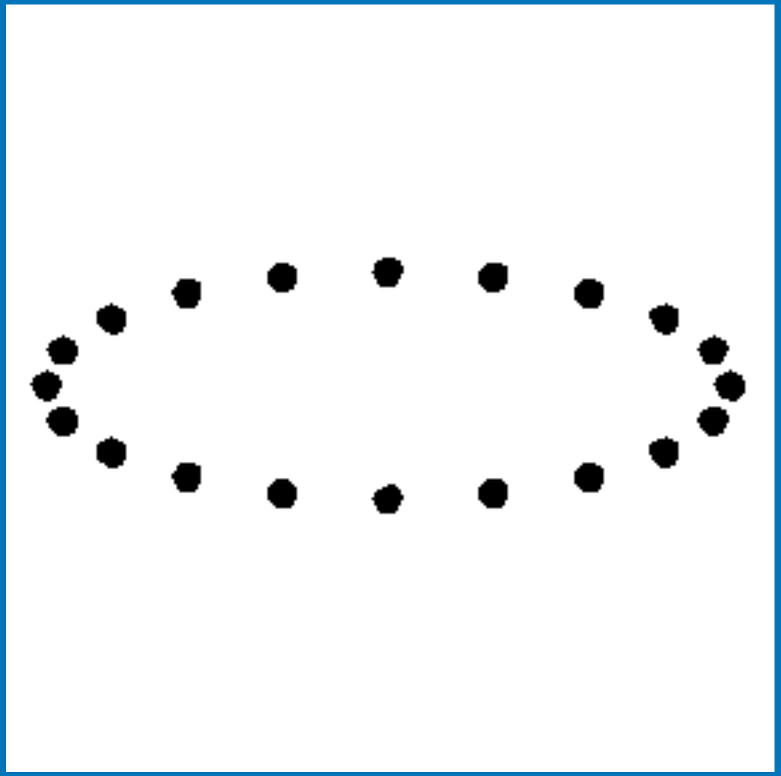
Transverse traceless gauge:

$\pi^{\alpha}_{0\mu}(k) \; = \; 0$, $k^j \pi^{\alpha}_{ij}(k) \; = \; 0$, $\pi^{\alpha}_{ii} = 0$, $k^j \pi^{\alpha}_{ij}(k) \; = \; 0$, and of course $\pi^{\alpha}_{\mu\nu}(k) \; = \; \pi^{\alpha}_{\nu\mu}(k)$.

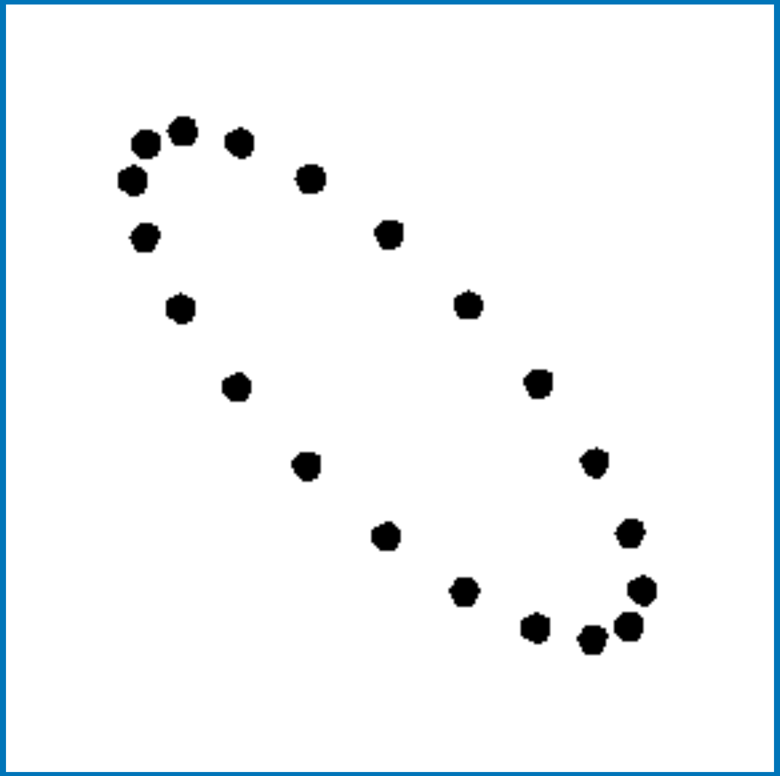
For $\vec{k} \;\; \propto \;\; \hat{z}$, a basis is

Plus polarization: $\pi^{+}_{11} = -\pi^{+}_{22} = \dfrac{1}{\sqrt{2}}$; $\pi^{+}_{12} \; = \; \pi^{+}_{21} \; = \; 0$

Cross polarization: $\pi^{\times}_{11} = -\pi^{\times}_{22} = 0$; $\pi^{\times}_{12} \; = \; \pi^{\times}_{21} \; = \; \dfrac{1}{\sqrt{2}}$



Effect of Plus-Polarized Wave



Effect of Cross-Polarized Wave

In terms of quanta, these waves represent massless spin-2, helicity ± 2 particles. The gauge conditions project out the longitudinal and lower spin components!

For weak sources - i.e., when we can linearize throughout - we have the quadrupole radiation formula

$$P = \frac{G}{5c^5}\frac{d^3\,Q_{ij}}{dt^3}\frac{d^3\,Q^{ij}}{dt^3}$$

$$Q_{ij} = \int d^3x\,\rho(x)\,(3x_i x_j - x^2\delta_{ij})$$

An outstanding fact about gravitational waves is that their amplitude, regarded as a fractional distortion of space-time, is very small. This arises from the basic energetics.

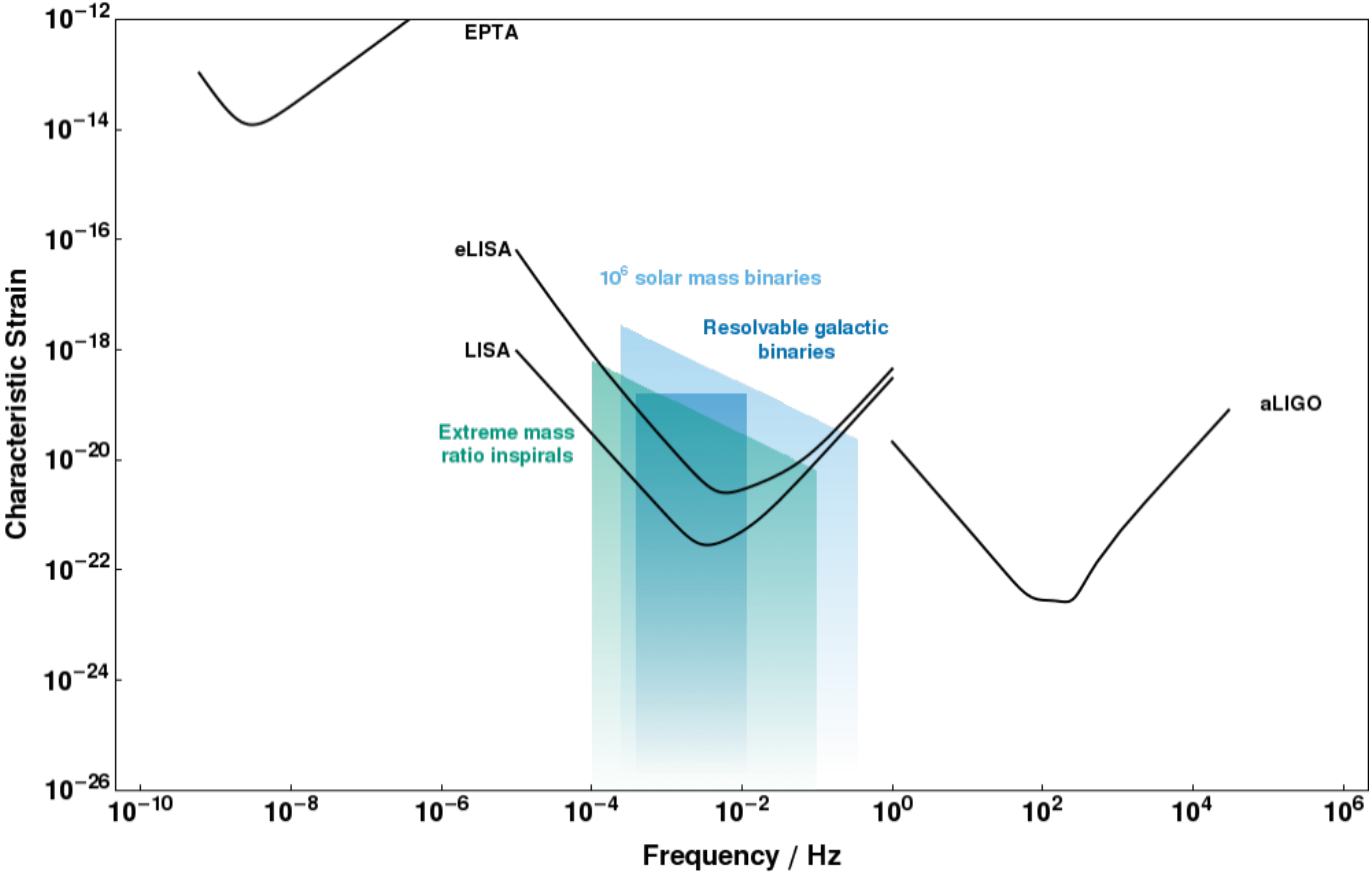
Let's put in rough numbers for the space-time distortion caused by gravitational radiation from a black hole merger observed at distance R . The only parameters are $M_{\mathrm{Pl.}} \sim 10^{19}$ GeV , $R = 100\,\mathrm{Mpc}. \sim 10^{24}\mathrm{cm}. \sim 10^{-38}\,\mathrm{GeV}^{-1}$, and $M_{\mathrm{BH}} \sim 10^{57}\,\mathrm{GeV}$. Assuming that the burst duration is roughly the Schwarzschild time and that the energy released is a finite fraction of the total mass, from

$$\frac{\text{Energy}}{\text{Volume}} = \left(\frac{\Delta l}{l}\right)^2 M_{\mathrm{Pl.}}^2 \omega^2$$ and

$$\text{Energy} \sim M_{\mathrm{BH}} \;;\; \text{Volume} = R^2 \frac{M_{\mathrm{BH}}}{M_{\mathrm{Pl.}}^{\;2}} \;;\; \omega = \frac{M_{\mathrm{Pl.}}^{\;2}}{M_{BH}}$$

we find $\left(\frac{\Delta l}{l}\right)^2 \sim \frac{M_{\mathrm{BH}}^2}{M_{\mathrm{Pl.}}^4 R^2} \sim 10^{114-76-76} = 10^{-38}$

This is not a ridiculous estimate …



EPTA
eLISA
LISA
10^6 solar mass binaries
Resolvable galactic binaries
Extreme mass ratio inspirals
aLIGO
Characteristic Strain
Frequency / Hz
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
10^{-26}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}

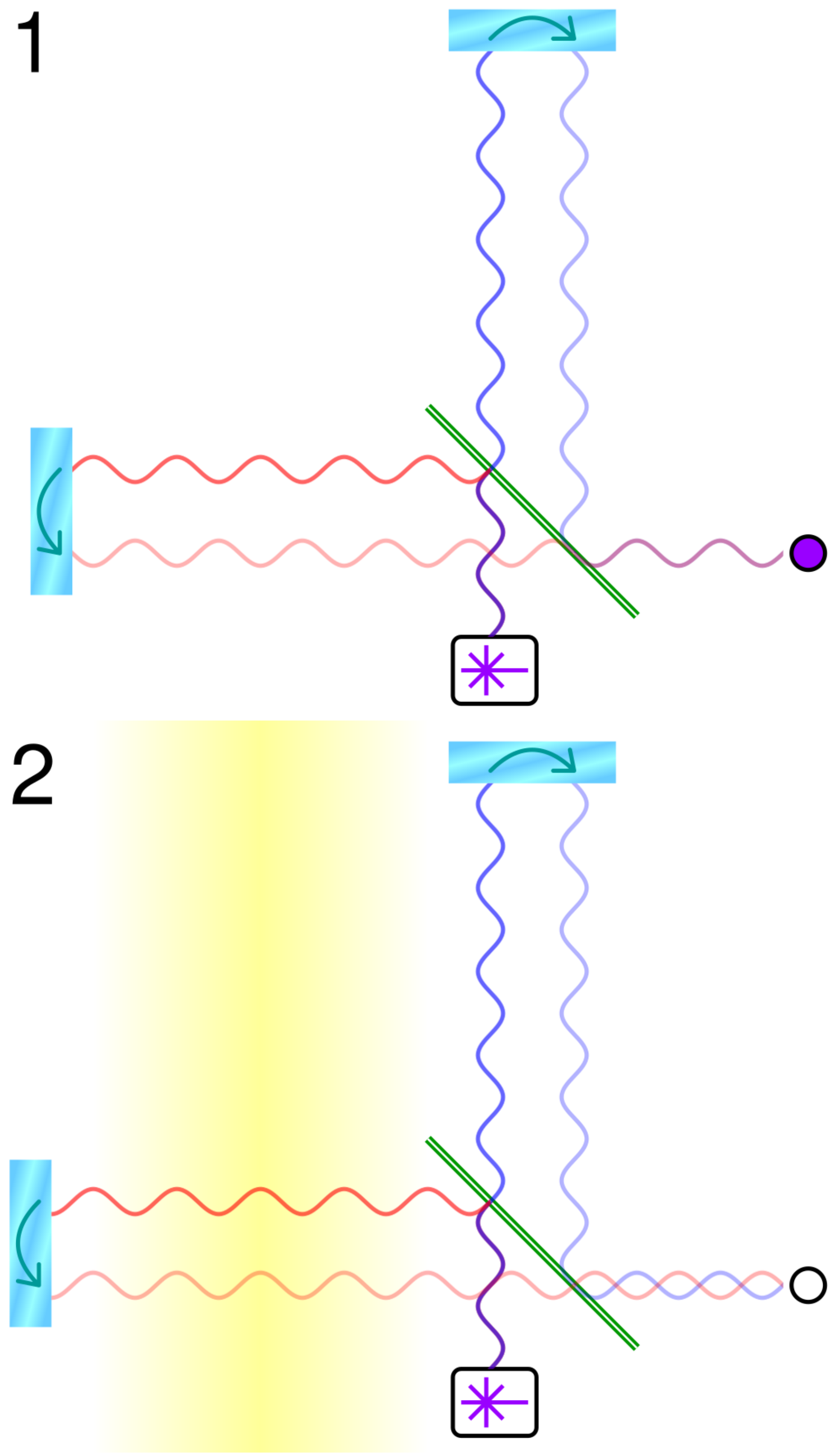
Space-time is *very* stiff.

Sorry wormhole fans and aspiring space-time engineers …

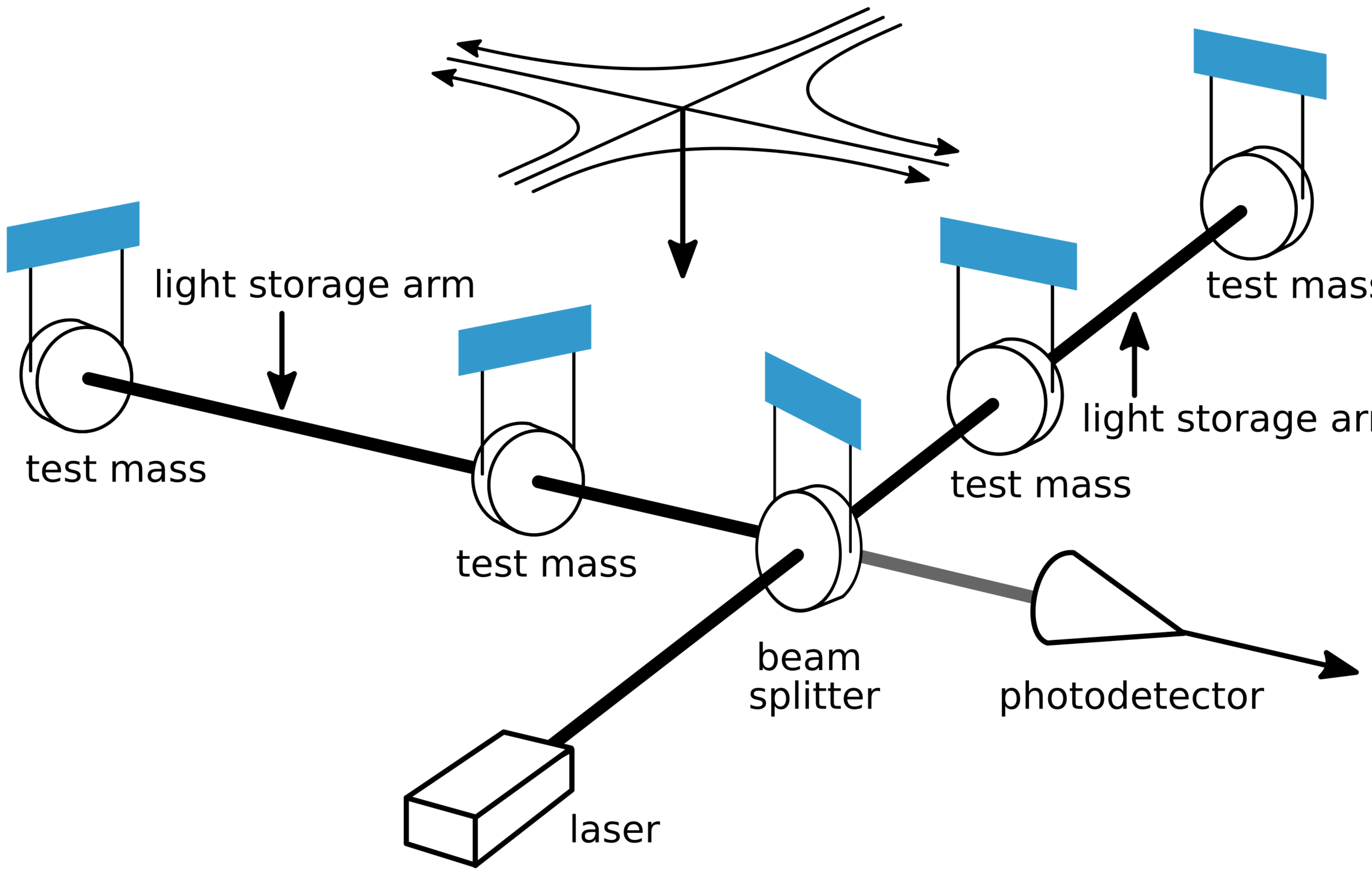
OTOH, gravitational wave might provide a convincing way for a super-advanced technological civilization to show its chops and advertise its existence!

Interferometers

LIGO and others



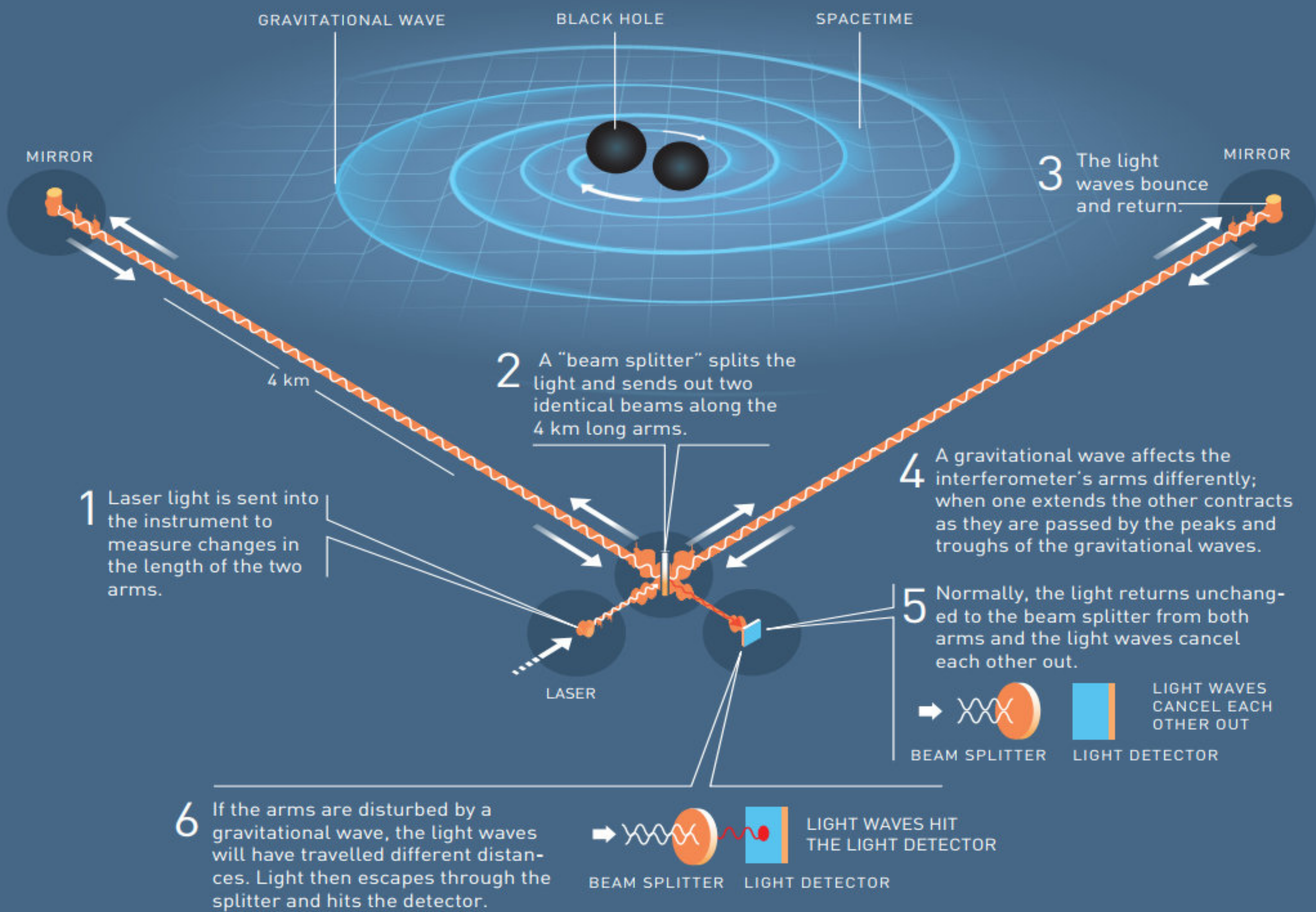
Interferometer Principle



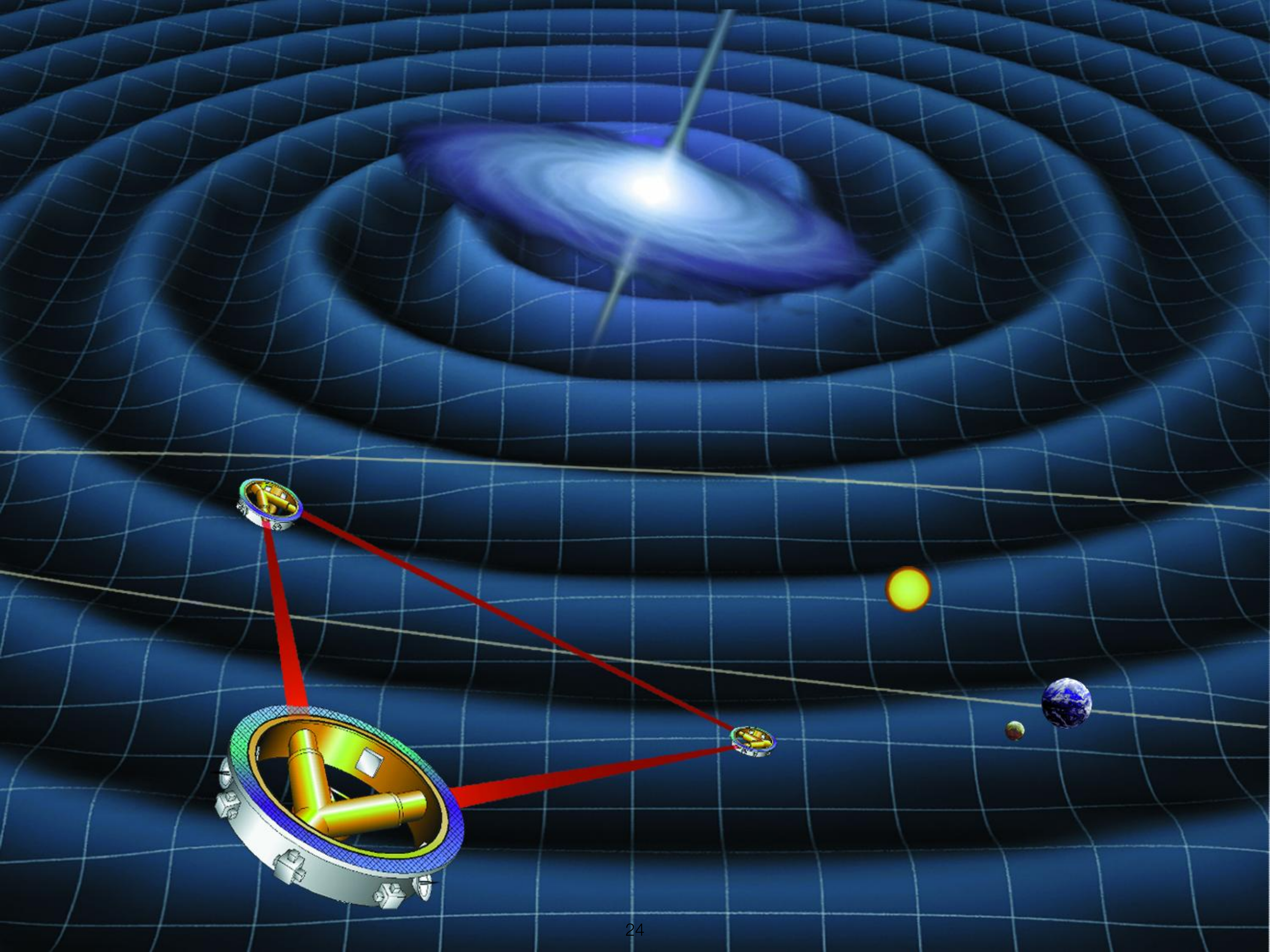
Interferometer Schematic

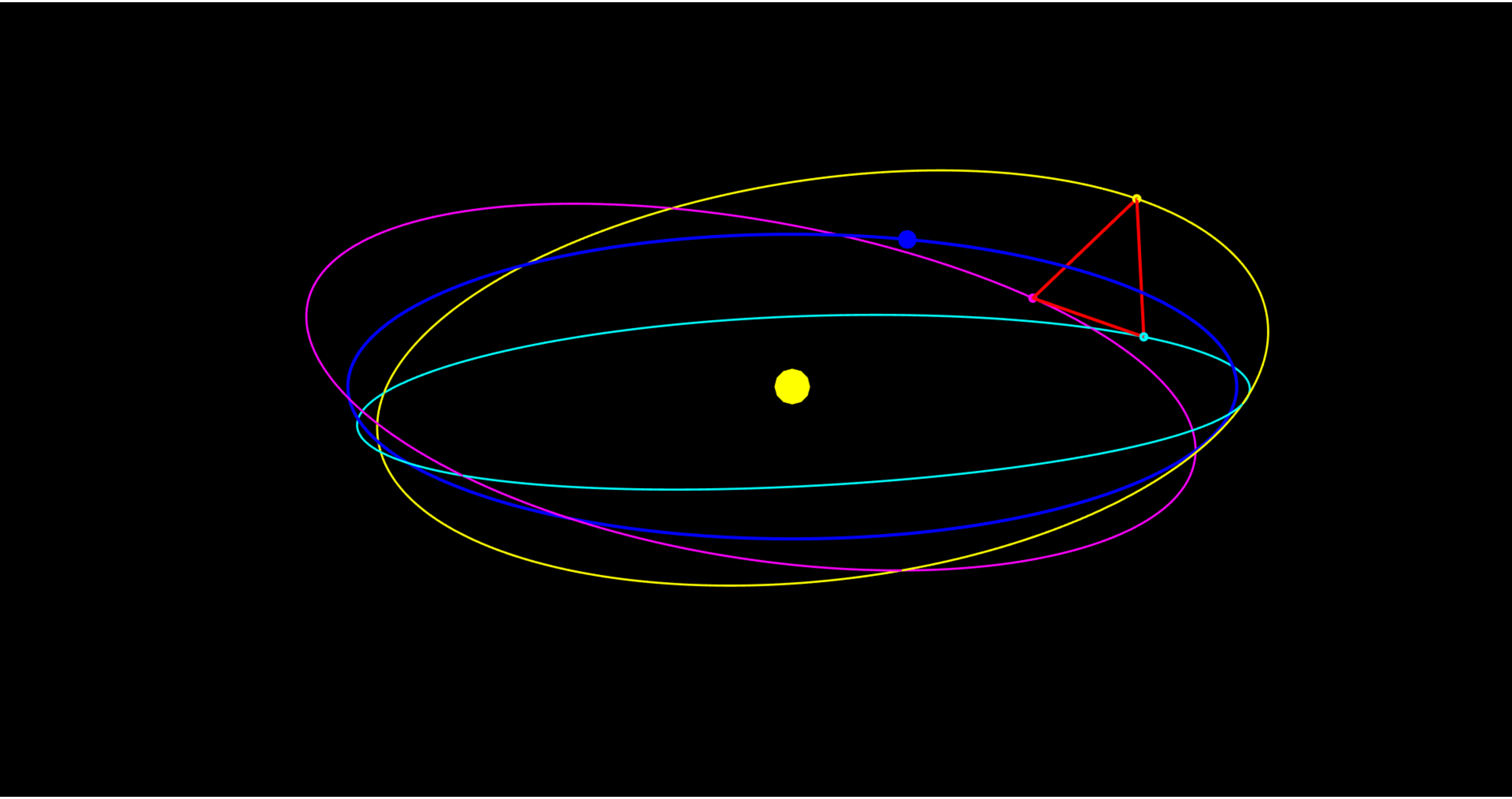
LIGO – A GIGANTIC INTERFEROMETER

Nature "Explainer"




Livingston
Hanford





Resonant Bars

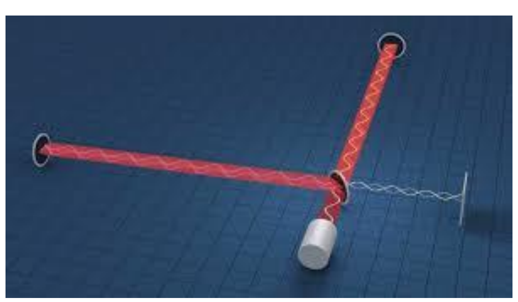
History, Coupling, Utility

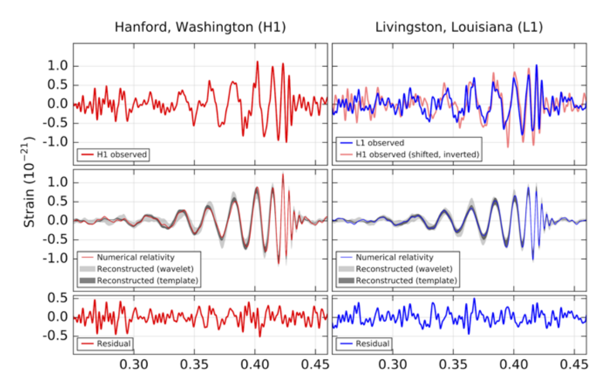
Massive acoustic resonators (Weber bars) for detecting classical gravitational waves

❖ Joseph Weber was a pioneer in gravitational wave detection.

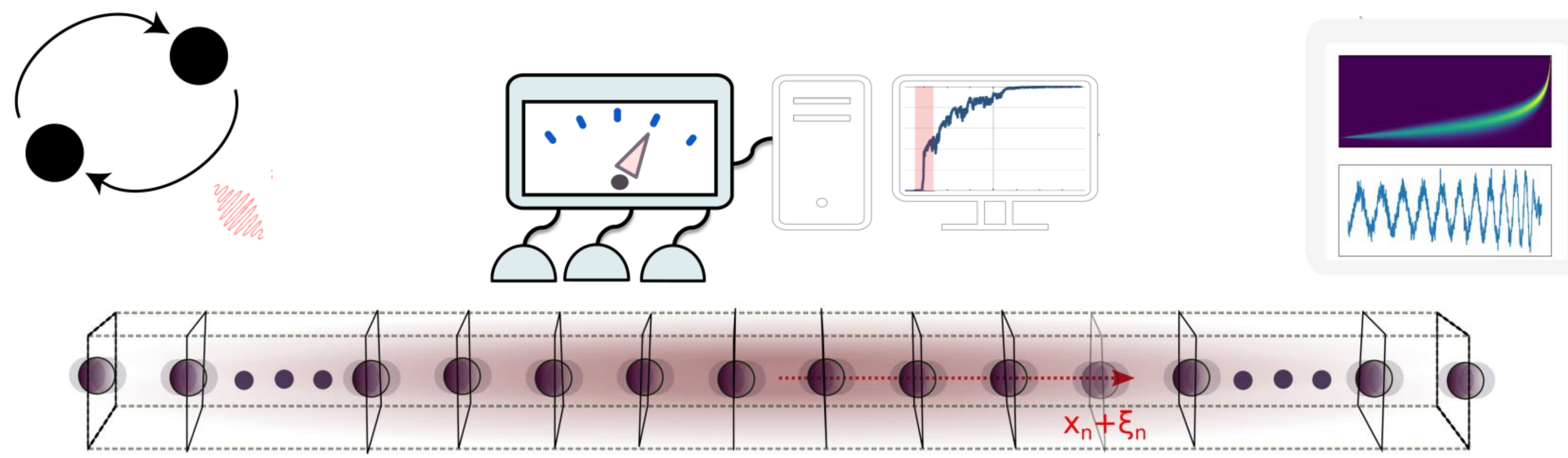
❖ The interferometry idea by LIGO was conceptually different, and is now the main-stream approach to detecting classical gravitational waves (GW150914 shown):

Cho, Adrian. "Remembering joseph weber, the controversial pioneer of gravitational waves." Science 12 (2016).





These can be the “photodetectors” of gravitational radiation.



Detecting Single Gravitons with Quantum Sensing

Speaker: Sreenath K. Manikandan, Researcher in theoretical physics

Nordita, Stockholm University and KTH Royal Institute of Technology, Stockholm, Sweden

Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, & Igor Pikovski. Nature Communications 15, 7229 (2024)

Germain Tobar

SK Manikandan

Thomas Beitel

Igor Pikovski

Swedish Research Council

General Sir John Monash Foundation

The Branco Weiss Fellowship Society in Science

NSF

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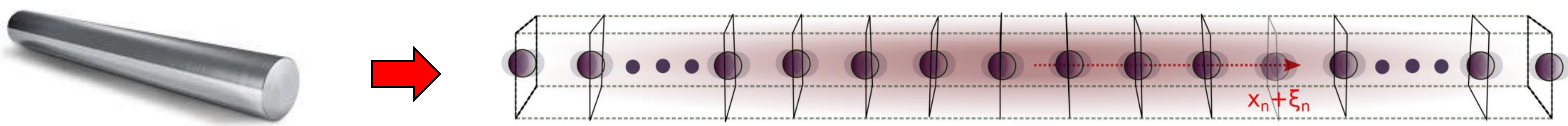
Acoustic modes of a Weber bar



- ❖ $N+1$ atoms with mass m, distance a apart, $M = m(N+1)$
- ❖ Vibrate with Debye frequency ω_D around their mean positions $x_j = aj/2$, j odd
- ❖ Local displacements $x = x_j + \xi_j$
- ❖ $\xi_j = \sum_{l=0,2\ldots}^{N-1} \chi_l(t) \cos\frac{jl\pi}{2N+2} + \sum_{l=1,3\ldots}^{N} \chi_l(t) \sin\frac{jl\pi}{2N+2}$, new collective modes $\ddot{\chi}_l = -\omega_l^2 \chi_l$
- ❖ Total $E = \frac{1}{2} m \sum_{j=-N}^{N} \dot{\xi}_j^2 + \frac{1}{2} m\omega_D^2 \sum_{j=-N}^{N-2} \left(\xi_{j+2} - \xi_j\right)^2 = \frac{M}{4} \sum_{l=0}^{N} \left(\dot{\chi}_j^2 + \omega_l^2 \chi_l^2\right)$
- ❖ Collective oscillators with mass $M/2$

- ❑ Grishchuk, L. P. (1992). Quantum mechanics of a solid-state bar gravitational antenna. Physical Review D, 45(8), 2601.
- ❑ Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing."
- ❑ Nature Communications 15, 7229 (2024)`

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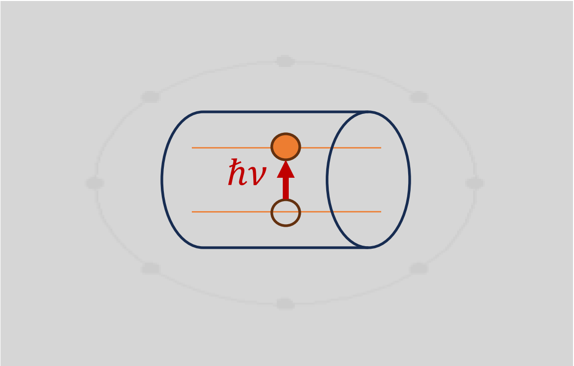
Rates for spontaneous and stimulated processes



Field in a coherent state $|\alpha\rangle \to |\alpha\rangle$.

$$\Gamma_{spon}\,(1 \to 0) = \frac{2\pi}{\hbar}\left|\langle 1|\,\langle 0|\hat{H}_{int}|1\rangle\,|0\rangle\right|^2 \rho = \frac{8GML^2\omega_l^4}{l^4\pi^4c^5} = \frac{8\pi G\rho_m R^2 v_s^4}{Lc^5}$$

$v_s = \frac{L\omega_l}{l\pi}$: sound speed, ρ_m: mass density

$$\Gamma_{stim}\,(1 \to 0) = \frac{2\pi}{\hbar}\left|\langle 1|\,\langle \alpha|\hat{H}_{int}|\alpha\rangle\,|0\rangle\right|^2 \rho = \frac{|\alpha|^2 8GML^2\omega_l^4}{l^4\pi^4c^5} \qquad |\alpha|^2 \approx N = \frac{h_0^2c^5}{32\pi G\hbar\omega_l^2}$$

$$\boxed{\Gamma_{stim} = \frac{ML^2\omega_l^2}{4l^4\pi^5\hbar}h_0^2 = \frac{Mv_s^2}{4l^4\pi^3\hbar}h_0^2}$$

Spontaneous emission rate for a Niobium cylinder:

$\rho_m = 8570\frac{kg}{m^3}$ $\quad 2R = L = 1m$ $\quad \Gamma_{spon} = 10^{-33}s^{-1}$

Much better than Weinberg (atom), but still small!

Stimulated emission rate for an aluminium cylinder:

$h_0 = 5 \times 10^{-22}$ (GW150914) $\quad v_s = 5.4\frac{km}{s}$

$M = 1800\,kg \quad \Gamma_{stim} = 1\,Hz$

One graviton emitted/absorbed per second.

- Tobar, Germain*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)

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Requirements:

GW Source	GW170817 (NS-NS merger)	GW170817 (NS-NS merger)	GW170608 (BH-BH merger)	GW150914 (BH-BH merger)	J1301+0833 (black-widow pulsar)	J1748−2446ad (fast-spinning pulsar)	A0620-00 (BH Super-radiance)	Primordial (rare BH-BH merger)
$f = \frac{\omega}{2\pi}$	100 Hz	150 Hz	175 Hz	200 Hz	1085 Hz	1433 Hz	33 kHz	5.5 MHz
$h_0(f)$	2×10^{-22}	2×10^{-22}	2×10^{-22}	10^{-21}	$< 10^{-25}$	$< 10^{-25}$	3×10^{-21}	10^{-16}
M_c	$1.19\ M_\odot$	$1.19\ M_\odot$	$7.9\ M_\odot$	$28.6\ M_\odot$	Continuous	Continuous	Continuous	$5 \times 10^{-4}\ M_\odot$
Material	Beryllium	Aluminum	Niobium	CuAl6%	Niobium	Superfluid He-4	Sapphire	Quartz
v_0	13 km/s	5.4 km/s	5 km/s	4.1 km/s	5 km/s	238 m/s	10 km/s	6.3 km/s
T	1 mK	1 mK	1 mK	1 mK	0.1 μK	0.1 μK	0.6 K	0.6 mK
Q-factor	10^{10}	10^{10}	10^{10}	10^{10}	10^{10}	10^{13}	10^{10}	10^{10}
M	~ 15 kg	~ 250 kg	~ 9 t	~ 6 t	> 52 t	> 20 t	~ 100 kg	~ 10 g

- Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)

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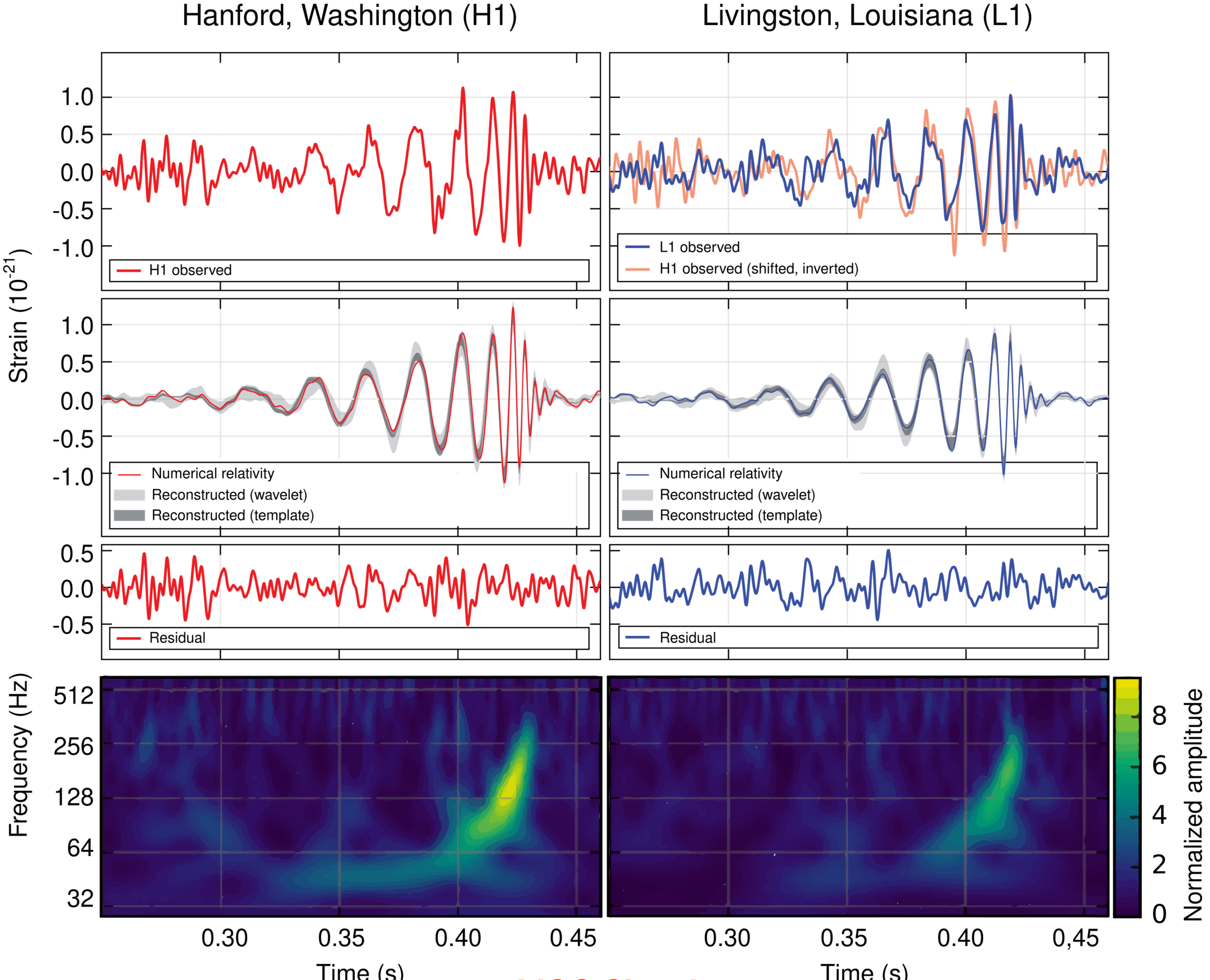
Bar detectors are not as sensitive as interferometers, for detection purposes.

But they may be cheaper and easier to play with, and of course they can be triggered in coincidence.

We will be considering things that might be done using several such detectors and diverse protocols.

Observations

Present - and Future?

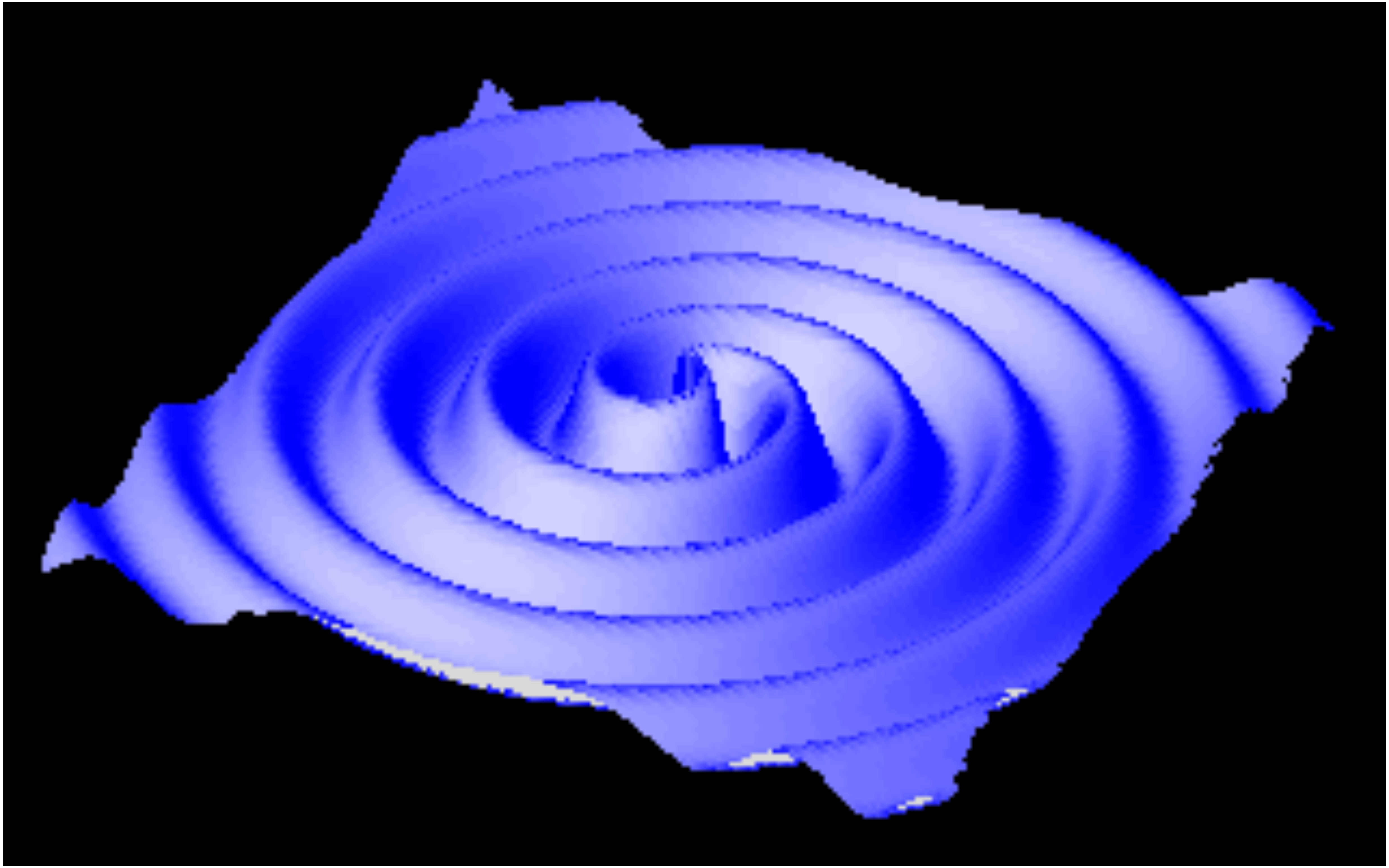


Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
512
256
128
64
32
Normalized amplitude
0.30
0.35
0.40
0.45
Time (s)

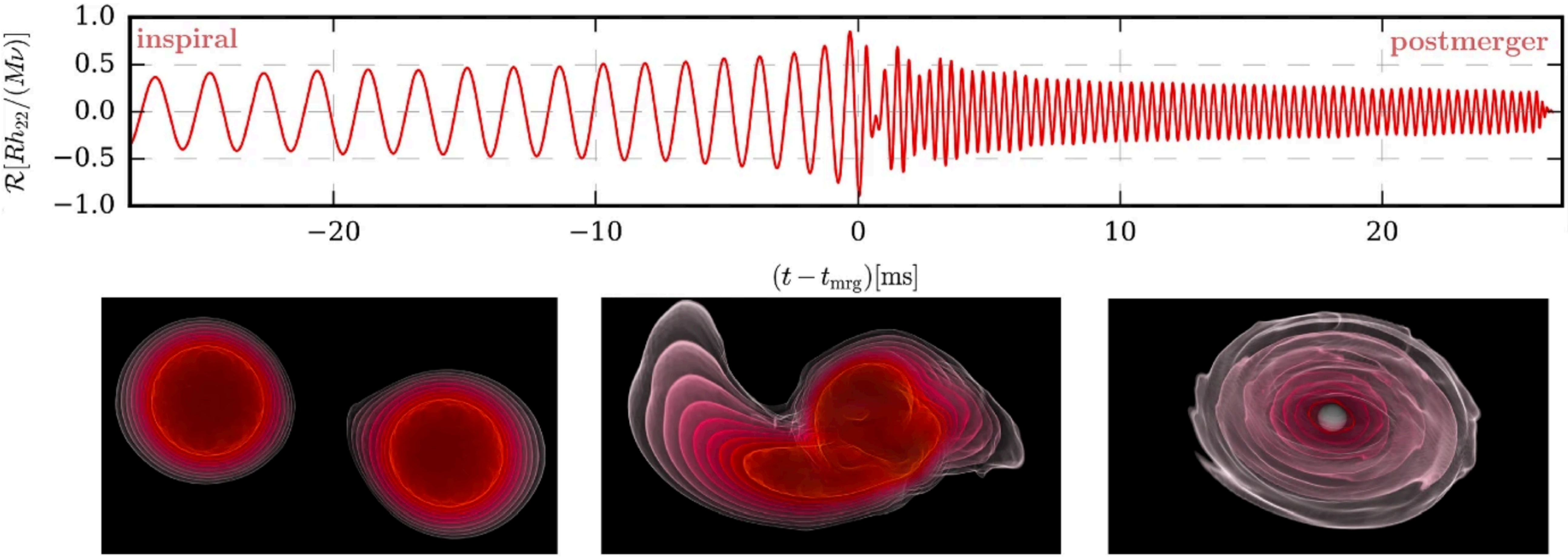
LIGO Signal

Neutron Star Merger (Artistic Impression)

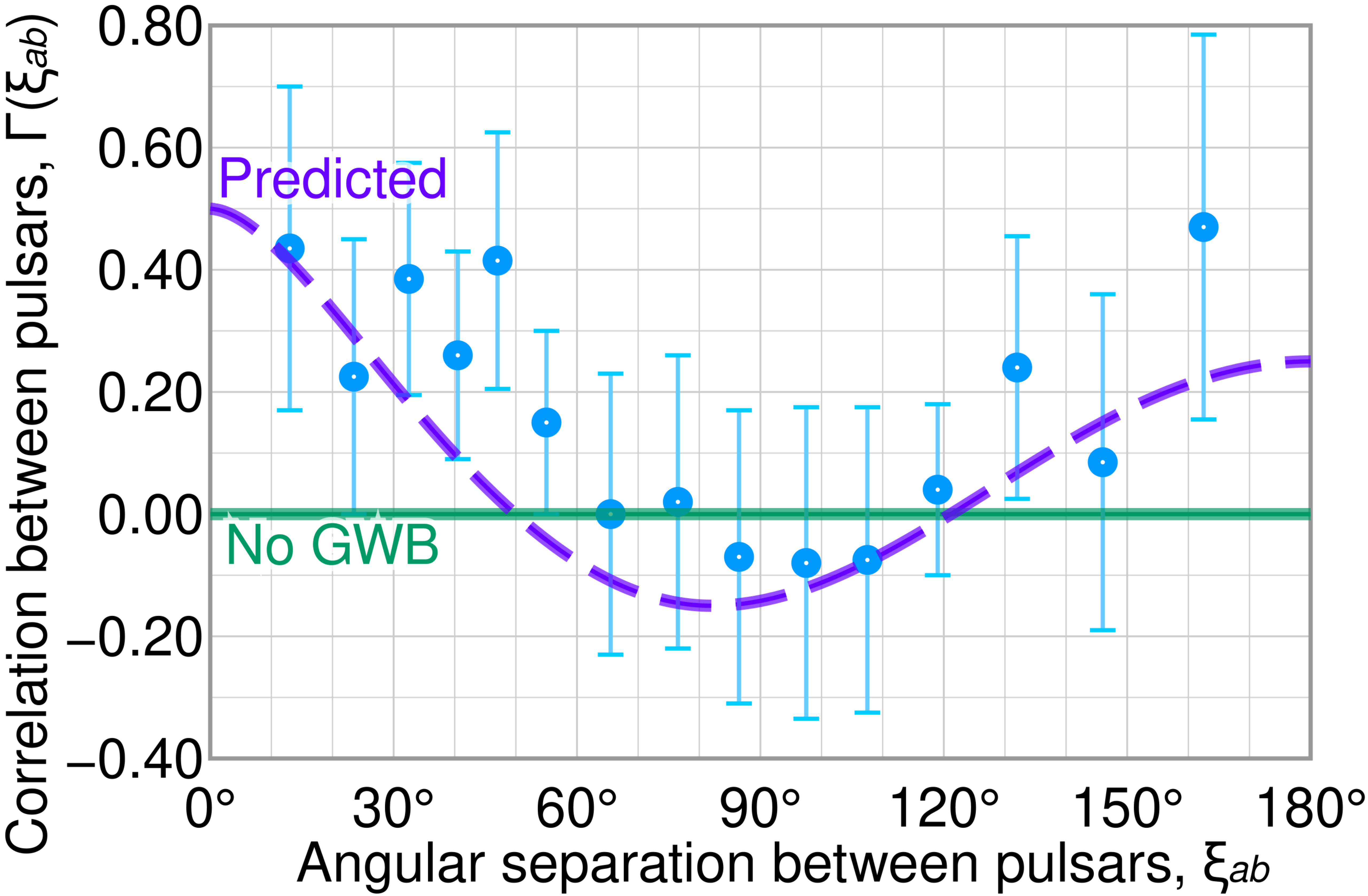
Gravitational Waves from Neutron Star Binary



From: Interpreting binary neutron star mergers: describing the binary neutron star dynamics, modelling gravitational waveforms, and analyzing detections



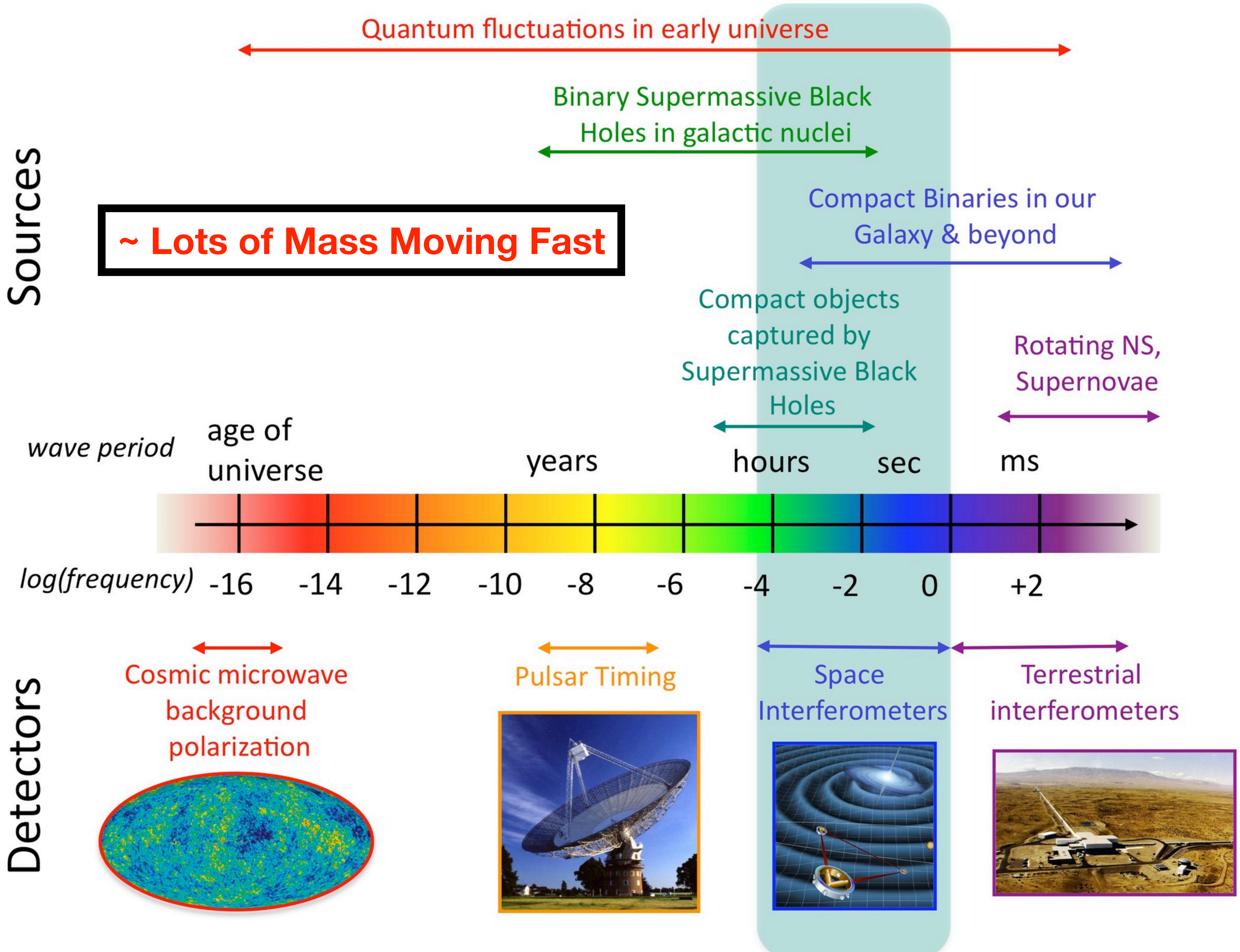
NR simulation of a BNS merger showing the GW signal and the matter evolution. Top panel: GW signal emitted during the last orbits before the merger (late-inspiral phase) and during the postmerger phase of the BNS coalescence. Bottom panel: Rest-mass density evolution for the inspiral (first panel), the merger (second panel) and the postmerger phase after the formation of the black hole (third panel)



Hints from Pulsar Timing Project


Gravitational Waves
NSF

The Gravitational Wave Spectrum



Universal Spontaneous Decoherence

Fragility of Macroscopic Quantum Coherence

Decoherence can be induced by measurement, i.e. interaction with observers, or by interaction with stray particles in the environment.

It is natural to ask: Do the quantum fields of “empty” space make measurements?

Answer: Of course they do!

Berkeley's Idealism

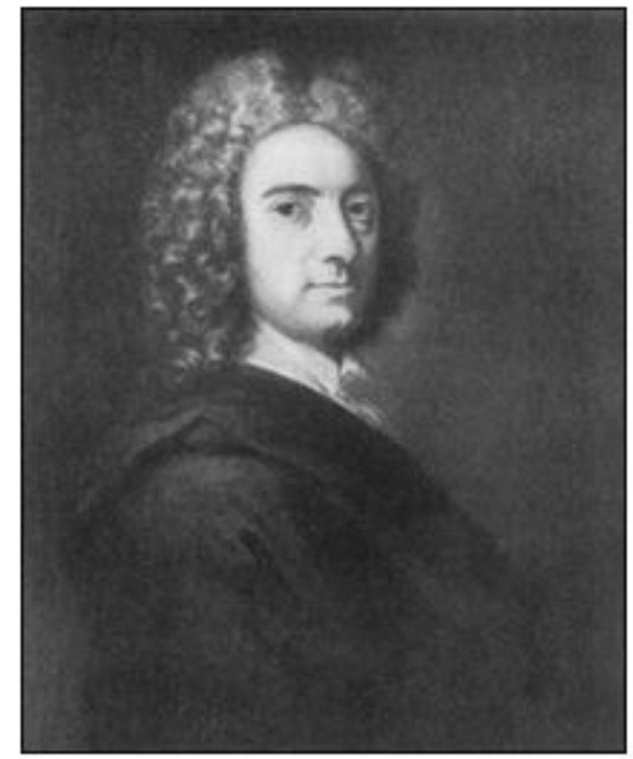
George Berkeley

God in the Quad

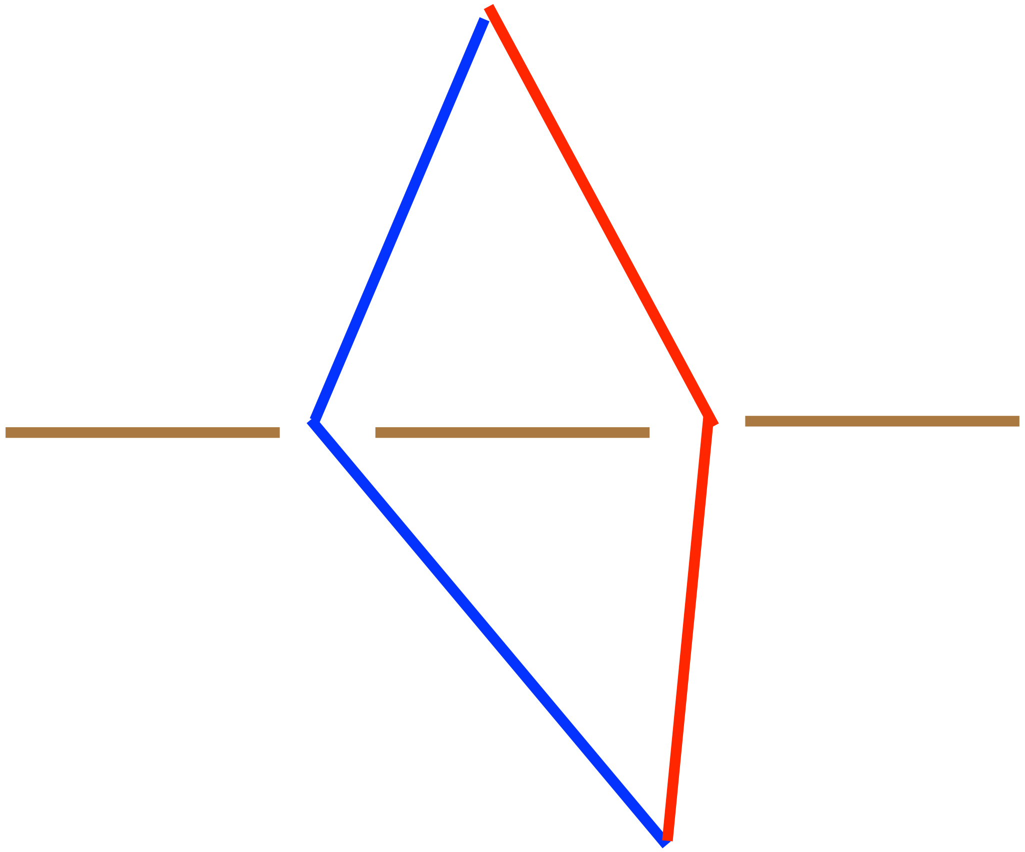
There was a young man who said "God
Must find it exceedingly odd
To think that the tree
Should continue to be
When there's no one about in the quad."

Reply:
"Dear Sir: Your astonishment's odd;
I am always about in the quad.
And that's why the tree
Will continue to be
Since observed by, Yours faithfully, God."

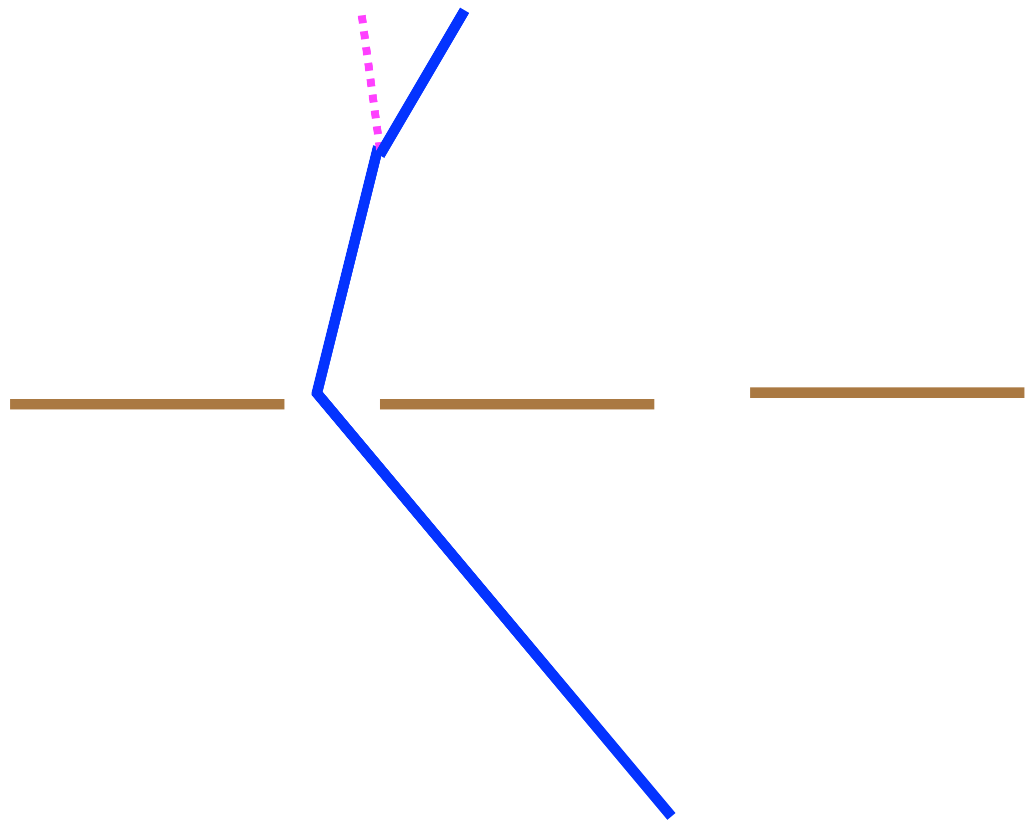
(poem actually written by: Monsignor Ronald Knox)



Consider two-slit diffraction of a charged quarticle. It can be considered as a rather exotic scattering process.



(Spontaneous) emission of photons is the photon field "observing" the difference between the two paths!



It produces an incoherent background to the interference pattern.

Most important here, quantitatively and conceptually, is soft radiation.

It’s pleasant, too, that there’s a universal formula for it!

(Recommended reference: Weinberg, Quantum Theory of Fields, Chapter 13 - Infrared effects)

$$M^{\mu}_{\alpha\beta}(q) \;\rightarrow\; M_{\alpha\beta} \sum_{n} \frac{\eta_n Q_n p_n^{\mu}}{p_n \cdot q + i\eta_n \epsilon}$$

$\eta_n = \pm 1$ for incoming, outgoing

Emission with “undetectable” radiation:

$$\Gamma(\leq E) \approx \left(\frac{E}{\Lambda}\right)^{A} \Gamma_{\Lambda}$$

$$A = \frac{\alpha}{2\pi}\left(\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\right)$$

β = relative velocity; Λ = cutoff (conventional)

Because of the $\dfrac{\alpha}{2\pi}$, A tends to be small, and for small β the energy factor becomes $1 - \dfrac{\alpha\beta^2}{3\pi}\ln\dfrac{\Lambda}{E}$.

So $\dfrac{\alpha\beta^2}{3\pi}\ln\dfrac{\Lambda}{E}$ (x2) is the incoherent background fraction.

There is a similar formula for soft radiation of gravitons.

$$M^{\mu\nu}_{\alpha\beta}(q) \to M_{\alpha\beta} \sum_n \frac{\eta_n p_n^\mu p_n^\nu}{p_n \cdot q + i\eta_n \epsilon} \sqrt{8\pi G_N}$$

Thus, part of the process goes incoherent.

Where previously we had α, now we have Gm^2.

This decoherence mechanism is universal.

It becomes quantitatively significant for energy-momentum transfers that approach (or exceed) the Planck scale.

That will happen routinely for collective variables associated with (barely) macroscopic objects, e.g., the center-of-mass coordinates for moving bodies with mass $\geq \sim 10^{-6}\,\mathrm{gm}$.

I think that this perfectly orthodox quantum-mechanical process corresponds to Penrose's heuristic discussion of decoherence due to space-time fluctuations.

This universal mechanism is adequate to dispose of literal Schrödinger ~~qu~~cats …

… as are many other, non-universal but in practice much larger decoherence effects. Semi-macroscopic objects typically have *many* readily excited low-lying states, so they're difficult to create or maintain in pure states*.

Here the challenge for testing quantum gravity is not signal, but background!

*But keep Weber bars in mind!

In the electromagnetic case, if we take the logarithm to be 10, and $\beta = .1$, the incoherent fraction is $\sim 10^{-4}$.

Beams of particles in well-defined (discrete) excited states, that decay spontaneously on convenient time-scales, could enable “easy” practical experiments.

Relevant here:

Decoherence of matter waves by thermal emission of radiation

Lucia Hackermueller, Klaus Hornberger, Bjoern Brezger, Anton Zeilinger, Markus Arndt

Emergent quantum technologies have led to increasing interest in decoherence - the processes that limit the appearance of quantum effects and turn them into classical phenomena. One important cause of decoherence is the interaction of a quantum system with its environment, which 'entangles' the two and distributes the quantum coherence over so many degrees of freedom as to render it unobservable. Decoherence theory has been complemented by experiments using matter waves coupled to external photons or molecules, and by investigations using coherent photon states, trapped ions and electron interferometers. Large molecules are particularly suitable for the investigation of the quantum-classical transition because they can store much energy in numerous internal degrees of freedom; the internal energy can be converted into thermal radiation and thus induce decoherence. Here we report matter wave interferometer experiments in which C70 molecules lose their

An interesting possibility is to enhance the relevant density of states, using cavities (“Purcell factor”).

One should also consider how spontaneous decoherence constrains quantum information processing.

In that context, error rates of $\sim 10^{-4}$ could be not only detectable, but annoying.