Observing Radiation States

Fluctuations as Signals

Parameters describing acoherence give information about the non-classical properties of radiation fields, and ultimately about their sources.

This issue is especially interesting for gravitational waves, which offer a unique window into some exotic and extreme physical regimes.

In that context, detection of acoherence would provide a meaningful use of quantum gravity, which many regard as an end in itself.

- Counting statistics
    - Ratio Test
- Homodyne and heterodyne
- HBT and active intensity interferometry
- Correlations

Counting Statistics

Simple and Powerful

- Set-up
- Perturbation theory
- P Representation
- General Results

Bar Radiation

$$V_I(t) = \lambda[d(t)a^\dagger + d^\dagger(t)a]$$

$$U_I = e^{-i\lambda \int_t^{t+\Delta t} [d(t)a^\dagger + d^\dagger(t)a]dt}$$

Resonant mode; rotating wave approximation

$$U_I = e^{-i\sqrt{\gamma_0 \Delta t}(a^\dagger b + b^\dagger a)}$$

$\Rightarrow \sim$ Two-mode squeezing of radiation and bar!

Perturbative approach ($\kappa \equiv \sqrt{\gamma_0 \Delta t}$)

$$P_0 = 1 - \kappa^2 \langle s | a^\dagger a | s \rangle + \kappa^4 \langle s | \frac{1}{6} a^{\dagger 2} a^2 + \frac{1}{3} (a^\dagger a)^2 | s \rangle$$

$$- \kappa^6 \langle s | \frac{17}{360} a^\dagger a a^{\dagger 2} a^2 + \frac{17}{360} a^{\dagger 2} a^2 a^\dagger a + \frac{2}{45} (a^\dagger a)^3 + \frac{1}{60} a^{\dagger 3} a^3 + \frac{1}{90} a^{\dagger 2} a a^\dagger a^2 | s \rangle$$

$$P_1 = \kappa^2 \langle s | a^\dagger a | s \rangle - \kappa^4 \langle s | \frac{1}{3} (a^\dagger a)^2 + \frac{2}{3} a^{\dagger 2} a^2 | s \rangle$$

$$+ \kappa^6 \langle s | \frac{8}{45} a^{\dagger 2} a a^\dagger a^2 + \frac{4}{45} a^{\dagger 2} a^2 a^\dagger a + \frac{4}{45} a^\dagger a a^{\dagger 2} a^2 + \frac{2}{45} (a^\dagger a)^3 + \frac{1}{10} a^{\dagger 3} a^3 | s \rangle$$

$$P_2 = \frac{\kappa^4}{2} \langle s | a^{\dagger 2} a^2 | s \rangle$$

$$- \kappa^6 \langle s | \frac{1}{4} a^{\dagger 3} a^3 + \frac{1}{6} a^{\dagger 2} a a^\dagger a^2 + \frac{1}{24} a^{\dagger 2} a^2 a^\dagger a + \frac{1}{24} a^\dagger a a^{\dagger 2} a^2 | s \rangle$$

$$P_3 = \frac{\kappa^6}{6} \langle s | a^{\dagger 3} a^3 | s \rangle$$

Application of P representation

At the end of the day, we are dealing with two coupled harmonic oscillators. The P representation allows us to "integrate out the dynamics":

$$\rho = \int d^2\alpha P(\alpha) | \alpha \rangle \langle \alpha |$$

$$U_I | \alpha \rangle \otimes | 0 \rangle = e^{-i\sqrt{\gamma_0 \Delta t}(a^\dagger b + b^\dagger a)} | \alpha \rangle \otimes | 0 \rangle$$

$$= | \alpha \cos(\sqrt{\gamma_0 \Delta t}) \rangle \otimes | -i\alpha \sin(\sqrt{\gamma_0 \Delta t}) \rangle$$

Global counting statistics

$$P_n = = \frac{[\sin^2(\sqrt{\gamma_0 \Delta t})]^n}{n!} \int d^2\alpha \, P(\alpha) \, |\, \alpha \,|^{2n} e^{-|\alpha|^2 \sin^2(\sqrt{\gamma_0 \Delta t})}$$

$$\bar{n} = \sum_{n=0}^{\infty} n P_n = \sin^2(\sqrt{\gamma_0 \Delta t}) \langle a^\dagger a \rangle_\rho$$

$$\overline{n(n-1)} = \sum_{n=0}^{\infty} n(n-1) P_n = [\sin^2(\sqrt{\gamma_0 \Delta t})]^2 \langle (a^\dagger)^2 a^2 \rangle_\rho$$

$$(\Delta n)^2 \approx \bar{n} + (\gamma_0 \Delta t)^2 Q \langle n \rangle$$

$$Q \equiv \frac{\langle (\Delta \hat{N})^2 \rangle_\rho - \langle \hat{N} \rangle_\rho}{\langle \hat{N} \rangle_\rho} \, , \; \hat{N} \equiv a^\dagger a$$

Mandel Q parameter

For observation to be practical, we need the factor $\langle n \rangle$ (many gravitons) to compensate the small coupling. It can occur for super-Poisson statistics, but not sub-Poisson.

Intuitively, we need to focus on the “balooned” directions, not the “squeezed” ones.

Coherent: $Q = 0$ $\quad (\Delta n)^2 = \bar{n}$ **Poisson**

Thermal: $$P(\alpha) = \frac{1}{\pi \bar{n}_{\rm th}} e^{-|\alpha|^2/\bar{n}_{\rm th}}$$

$$\bar{n} = \gamma_0 \Delta t \bar{n}_{\rm th}, \quad (\Delta n)^2 = \bar{n} + (\gamma_0 \Delta t)^2 \bar{n}_{\rm th}^2, \quad Q = \bar{n}$$ * **super-Poisson**

Squeezed Vacuum: $$|r,0\rangle = \frac{1}{\sqrt{\cosh(r)}} \sum_{m=0}^{\infty} (-\tanh(r))^m \frac{\sqrt{2m!}}{2^m m!} |2m\rangle$$

Note: even only

$$\langle n \rangle = \sinh(r)^2, \quad \bar{n} = \gamma_0 \Delta t \sinh(r)^2$$

$$(\Delta n)^2 = \bar{n} + (\gamma_0 \Delta t)^2 \cosh(2r) \sinh(r)^2 \quad Q = \cosh(2r) \rightarrow 2\bar{n}$$ *

super-Poisson

Fock: $Q = -1$ **sub-Poisson**

Note:

1. Maximally sub-Poisson
2. Independent of n

Simple tests using small counts

Coherent:

$$R \equiv \frac{2P_2P_0}{P_1^2} = 1 \qquad R' \equiv \frac{3P_3P_1}{2P_2^2} = 1$$

Fock:

$$R = 1 - \frac{1}{n} \qquad R' = 1 - \frac{1}{n-1}$$

Here, as in the global statistics, the distinction goes away for large n .

Thermal: $$R = \frac{2P_2P_0}{P_1^2} = 2$$ **(Independent of T!)**

Squeezed Vacuum: $$R \to 3$$

Thermal, displaced (by x_0) and squeezed (amplitude r, angle ϕ) :

$$R \approx \frac{4n_{\mathrm{th}}^2 - 8n_{\mathrm{th}}x_0^2\cos(\phi)\sinh(2r) + 8(2n_{\mathrm{th}}+1)\left(x_0^2-1\right)\cosh(2r)}{2\left((2n_{\mathrm{th}}+1)\cosh(2r) + x_0^2 - 1\right)^2}$$

$$+\frac{3(2n_{\mathrm{th}}+1)^2\cosh(4r) + 4n_{\mathrm{th}} - 8x_0^2\cos(\phi)\sinh(r)\cosh(r) + 2x_0^4 - 8x_0^2 + 5}{2\left((2n_{\mathrm{th}}+1)\cosh(2r) + x_0^2 - 1\right)^2}$$

a.k.a. g_2

$$g^{(2)}(x_a, t_a; x_b, t_b) \equiv \frac{G^{(2)}(x_a, t_a; x_b, t_b)}{|\,G^{(1)}(x_a, t_a)\,|\;|\,G^{(1)}(x_b, t_b)\,|}$$

"Degree of second-order coherence"

$$G^{(2)}(x_a, t_a; x_b, t_b) \equiv \langle\, E^{-}(x_a, t)\, E^{-}(x_b, t+\tau)\, E^{+}(x_b, t+\tau)\, E^{+}(x_a, t)\,\rangle$$

Our R parameter gives $g^{(2)}$ with spatial and time arguments at the place and time it is measured. This is also $Q+1$.

Homodyne and Heterodyne

Complementary Information

We can get additional insight by using interferometry, which gives us access to phase information. In these procedures, we consider that a local oscillator sets up a known baseline state of the bar, to which the signal adds.

When the local oscillator is at the same (resonant) frequency as we use for detection, we say that we have homodyne detection. When the local oscillator is at a different frequency, we say that we have heterodyne detection.

Homodyne operation was Weber's original strategy, and measurement of position fluctuations is still probably simpler to implement than counting.

For homodyne detection, the relevant detector states are $|x\rangle$, satisfying

$$\frac{b + b^\dagger}{\sqrt{2}} |x\rangle = x|x\rangle \ .$$

Once we translate this into the P representation, we can turn the crank to evaluate quantities of interest, notably including the variance, as a function of the radiation state.

$$\langle \hat{x} \rangle = \sqrt{2} x_0 \sin(\sqrt{\gamma_0 \Delta t}) \int d^2\alpha P(\alpha) \mathrm{Im}(\alpha) = \sqrt{2} x_0 \sin(\sqrt{\gamma_0 \Delta t}) \langle \mathrm{Im}(\alpha) \rangle$$

$$\langle x^2 \rangle_D = \frac{1}{2} x_0^2 + 2 x_0^2 \sin^2(\sqrt{\gamma_0 \Delta t}) \int d^2\alpha P(\alpha)\, \mathrm{Im}(\alpha)^2 = \frac{1}{2} x_0^2 + 2 x_0^2 \sin^2(\sqrt{\gamma_0 \Delta t}) \langle\, \mathrm{Im}(\alpha)^2 \rangle$$

$$\langle (\Delta \hat{x})^2 \rangle = \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2 = \frac{1}{2} x_0^2 + 2 x_0^2 \sin^2(\sqrt{\gamma_0 \Delta t}) [\langle \mathrm{Im}(\alpha)^2 \rangle - \langle \mathrm{Im}(\alpha) \rangle^2]$$

$$= x_0^2 \left[\frac{1}{2} + \sin^2(\sqrt{\gamma_0 \Delta t}) \left(\langle (\Delta \hat{P})^2 \rangle - \frac{1}{2} \right) \right]$$

where x_0 = zero point length

This result makes good sense - uncertainty in impulsive momentum transfer gets reflected in displacement uncertainty.

Coherent state: $\langle(\Delta\hat{P})^2\rangle = 1/2 \qquad \langle(\Delta\hat{x})^2\rangle = \frac{x_0^2}{2}$ **Vacuum noise only**

Thermal state: $P_{th}(\alpha) = \frac{1}{\pi n_{th}} e^{-|\alpha|^2/n_{th}}, \quad \langle(\Delta\hat{P})^2\rangle = \frac{2n_{th}+1}{2}$

*** (= not undetectable deviation)**

Fock: $\langle(\Delta\hat{P})^2\rangle = \langle(\Delta\hat{X})^2\rangle = \frac{2n+1}{2}$

*** (= not undetectable deviation) (!)**

Squeezed vacuum, $\phi = 0$:

$$\langle(\Delta\hat{x})^2\rangle = x_0^2\left[\left\{\frac{1}{2}\exp(2r) - \frac{1}{2}\right\}\sin^2(\sqrt{\gamma_0\Delta t}) + \frac{1}{2}\right] \rightarrow \frac{x_0^2}{2}\left[\cos^2(\sqrt{\gamma_0\Delta t}) + \exp(2r)\sin^2(\sqrt{\gamma_0\Delta t})\right]$$

*** (= not undetectable deviation)**

Squeezed vacuum, $\phi = \pi$:

$$\langle(\Delta\hat{x})^2\rangle = x_0^2\left[\left\{\frac{1}{2}\exp(-2r) - \frac{1}{2}\right\}\sin^2(\sqrt{\gamma_0\Delta t}) + \frac{1}{2}\right] \rightarrow \frac{x_0^2}{2}\left[1 - \sin^2(\sqrt{\gamma_0\Delta t})\right]$$

(no detectable deviation)

Heterodyne, briefly

In heterodyne detection, we use an external local oscillator with known amplitude and phase to put the detector in a coherent state..

Thus, the relevant detector state is a coherent state $|\beta\rangle$, i.e. $e^{\beta b^\dagger - \beta^* b}|0\rangle$.

Again, once we express the radiation mode into the P representation, we can turn the crank to evaluate quantities of interest, notably including the variance, as a function of the radiation mode state.

For the variance in power we have

$$\langle J \rangle \ \equiv \langle b^\dagger b \rangle \approx 1 + \gamma_0 \Delta t \langle \hat{N} \rangle_\rho$$

$$\langle (\Delta J)^2 \rangle \approx 2 + 3\gamma_0 \Delta t \langle \hat{N} \rangle_\rho + (\gamma_0 \Delta t)^2 Q \langle \hat{N} \rangle_\rho = (3\langle J \rangle - 1) + (\gamma_0 \Delta t)^2 Q \langle \hat{N} \rangle_\rho$$

There is also significant variance in the phase.

Takeaway messages

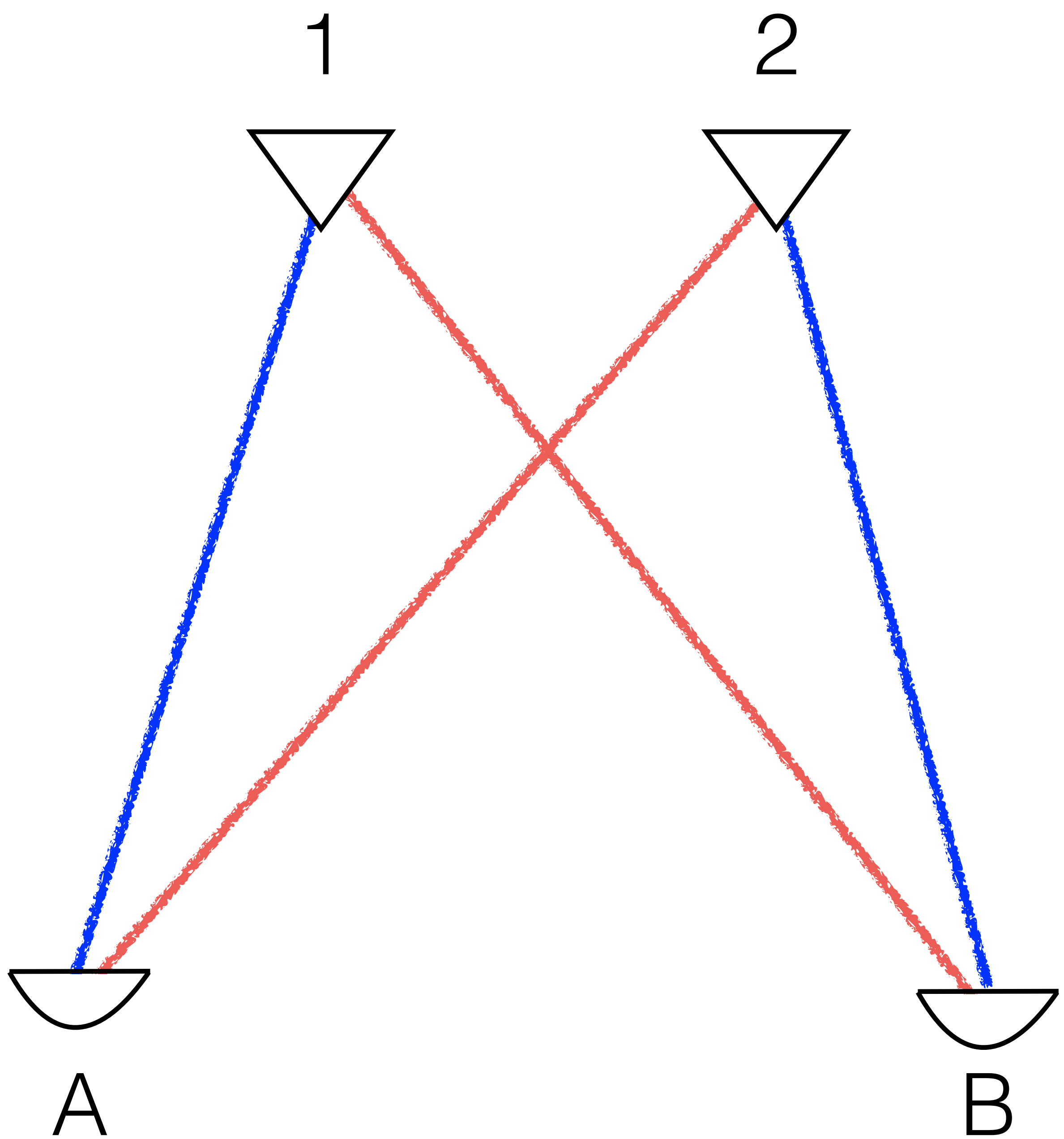
One can realistically aspire to detect acoherence in gravitational radiation through measurements of fluctuations and noise in bar detectors.

Different protocols access different information.

HBT and Active Intensity Interferometry

Concept and Realization

the basic HBT setup



$$| D_{1A} D_{2B} + D_{2A} D_{1B} |^2$$

$$D_{1A} D_{2B} D^*_{2A} D^*_{1B} + D^*_{1A} D^*_{2B} D_{2A} D_{1B}$$

Phase noise at source cancels (1 index and 2 index).

Phase noise in “local” propagation cancels (A index and B index).

Geometry-dependent phase does *not* cancel, and gives - as you vary the distance between A and B - information about the distance between 1 and 2.

In this subject it is easy to draw hasty - and wrong - conclusions. Correct intuition has to build on examples and equations!

A slightly more quantitative discussion

After summing over detector final states,

$$\langle I_A(t)\, I_B(t+\tau)\rangle = \kappa\langle s\,|\,E^-(x_a,t)\,E^-(x_b,t+\tau)\,E^+(x_b,t+\tau)\,E^+(x_a,t)\,|\,s\rangle$$

creation operators (negative frequency) annihilation operators (positive frequency)

We expand the Es into modes, as excited by the source. This evaluation can involve both statistical averaging and propagation effects. It can get complicated (≈ "rich"), but a few general conclusions can be drawn simply:

If we put $x_a = x_b$ and $\tau = 0$, we have

$\langle I^2 \rangle \; = \; \kappa \langle s | \sum_j a_k^{(1)^\dagger} \sum_l a_k^{(2)\dagger} \sum_m a_l^{(2)} \sum_j a_m^{(1)}) | s \rangle$. For the case of $N \gg 1$ independent modes (e.g., many atomic emissions) each mode annihilation operator gets choose, when getting back to $| s \rangle$, from 2 corresponding creation operators. In an equation:

$$\langle I^2 \rangle \; = \; \kappa \langle s | \sum_j a_j^{(1)\dagger} a_j^{(1)} \sum_k a_k^{(2)\dagger} a_k^{(2)} \; + \; \sum_j a_j^{(1)\dagger} a_j^{(2)} \sum_k a_k^{(2)\dagger} a_k^{(1)} | s \rangle$$

$$= \mathbf{2} \langle I \rangle^2$$

This calculation closely embodies the red-blue intuition from "basic HBT set-up".

At the other extreme, if we have only one relevant mode in a coherent state, then the creation and annihilation operators become c-numbers and $\langle I^2 \rangle = \langle I \rangle^2$. In words: no fluctuations.

How did the intuition go so wrong?

A: By thinking in terms of photons, rather than field modes. Some modes don't behave like particles!

Going back to the multimode case: as we let $\tau \neq 0$ or $x_a \neq x_b$ the cross $(1 \leftrightarrow 2)$ term is sensitive to relative phases between the (same) field mode at different space-time points. That is the sort of thing that is discussed in the theory of coherence and diffraction in optics.

Dephasing will eliminate the cross-term, or in other words the non-trivial intensity correlation.

For a finite source, e.g. a disk in the sky (= star) the rate of dephasing, as a function of the detector separation, encodes the size of the disk. (This is how Hanbury Brown and Twiss measured the size of Sirius and several other bright stars.)

We do not get an image, but we do get information we can process into an image …

PHYSICAL REVIEW LETTERS **VOL..XX,** 000000 (XXXX)

Editors' Suggestion

Active Optical Intensity Interferometry

Lu-Chuan Liu,[1,2,3] Cheng Wu,[1,2,3] Wei Li,[1,2,3] Yu-Ao Chen,[1,2,3] Xiao-Peng Shao,[4]
Frank Wilczek,[5,6,7,8,9] Feihu Xu,[1,2,3] Qiang Zhang,[1,2,3] and Jian-Wei Pan[1,2,3]

[1]*Hefei National Research Center for Physical Sciences at the Microscale and School of Physical Sciences, University of Science and Technology of China, Hefei, 230026, China*

[2]*Shanghai Research Center for Quantum Science and CAS Center for Excellence in Quantum Information and Quantum Physics, University of Science and Technology of China, Shanghai, 201315, China*

[3]*Hefei National Laboratory, University of Science and Technology of China, Hefei, 230026, China*

[4]*Xi'an Institute of Optics and Precision Mechanics of CAS, Xi'an 710119, China*

[5]*Center for Theoretical Physics, MIT, Cambridge, Massachusetts 02139, USA*

[6]*T. D. Lee Institute, Shanghai Jiao Tong University, Shanghai, 200240, China*

[7]*Wilczek Quantum Center, School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai, 200240, China*

[8]*Department of Physics, Stockholm University, Stockholm, SE-106 91, Sweden*

[9]*Department of Physics, Arizona State University, Tempe, Arizona 85287, USA*

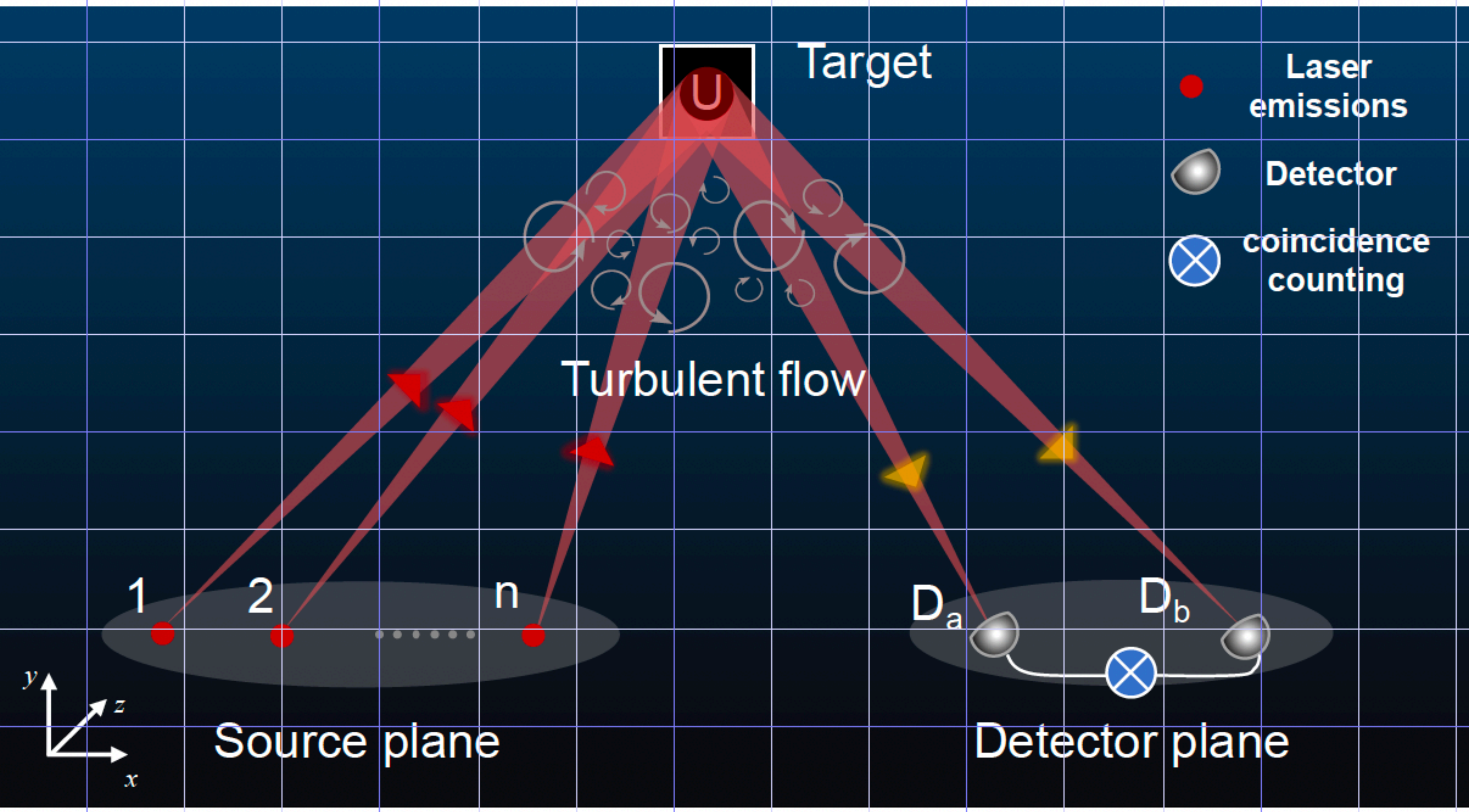
(Received 28 September 2024; revised 6 April 2025; accepted 10 April 2025)

To extend intensity interferometry to non-luminous objects, we need to supply them with light.

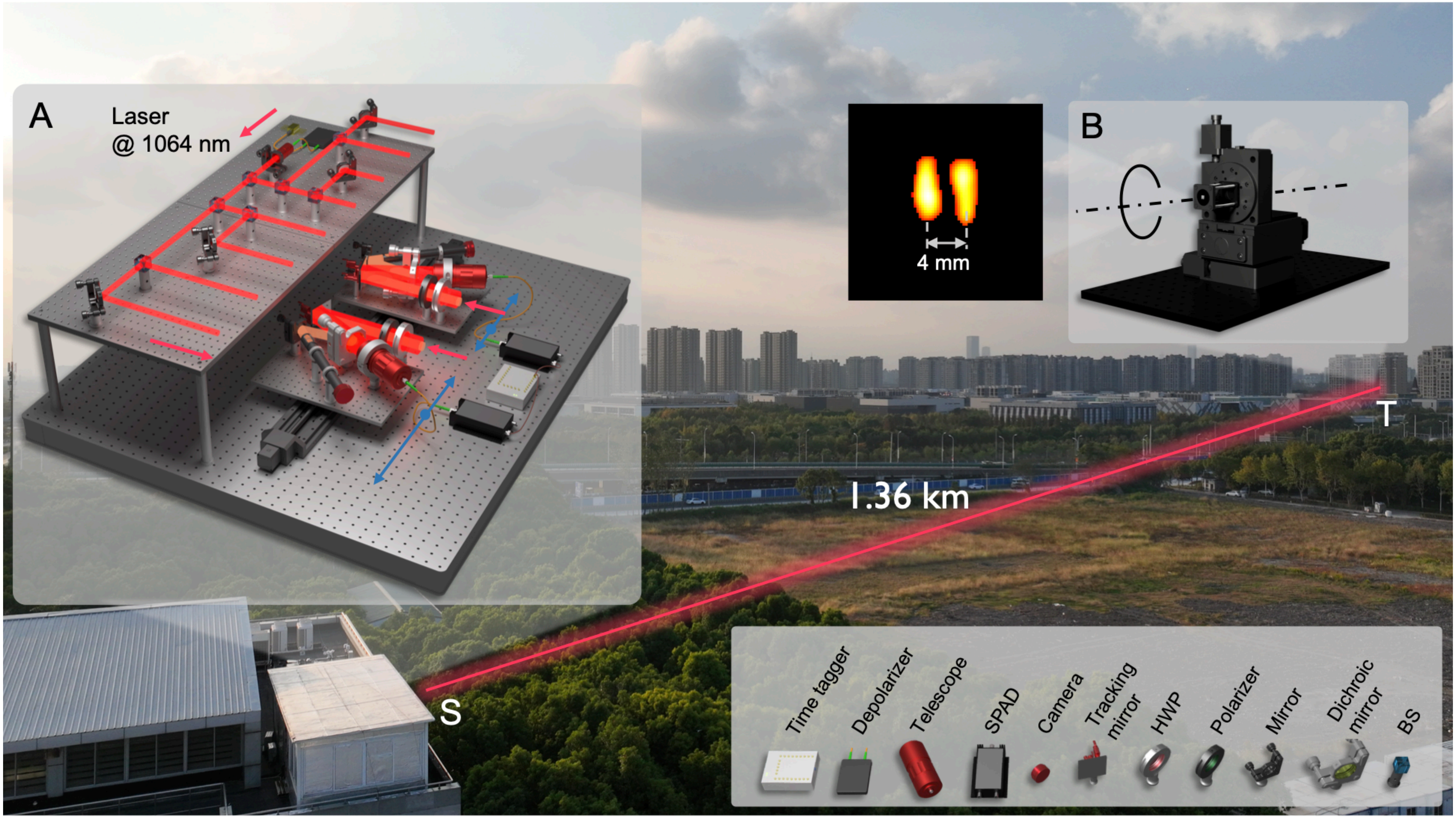
Lasers are good for that, but they supply coherent light, which is “too good”.

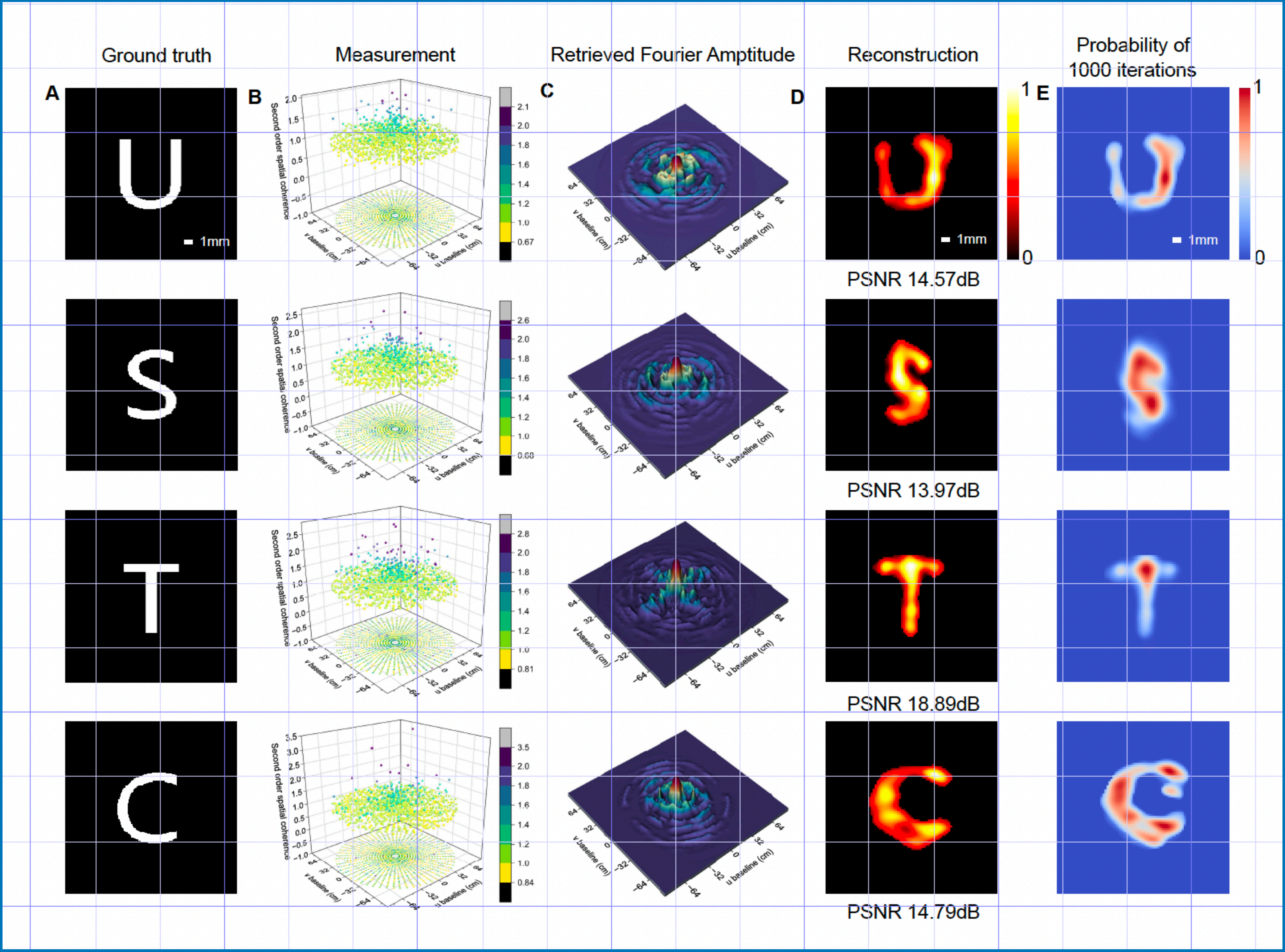
Solution: Use several laser beams and inject phase noise.

Concept: Make Laser Light Decohere Without Spreading



Realization





Result: ~ nm. Resolution at 1.36 km.

Correlations in Noisy Bars

Another Resource

In reality the gravitational wave signal will not be monochromatic, and the bar(s) will have several low-lying modes.

One can not only apply the preceding analysis to each mode, but also study correlations …

Absence of intensity correlations is a direct consequence, and thus provides a test, of the coherent state hypothesis!