Quantum simulations with superconducting qubits

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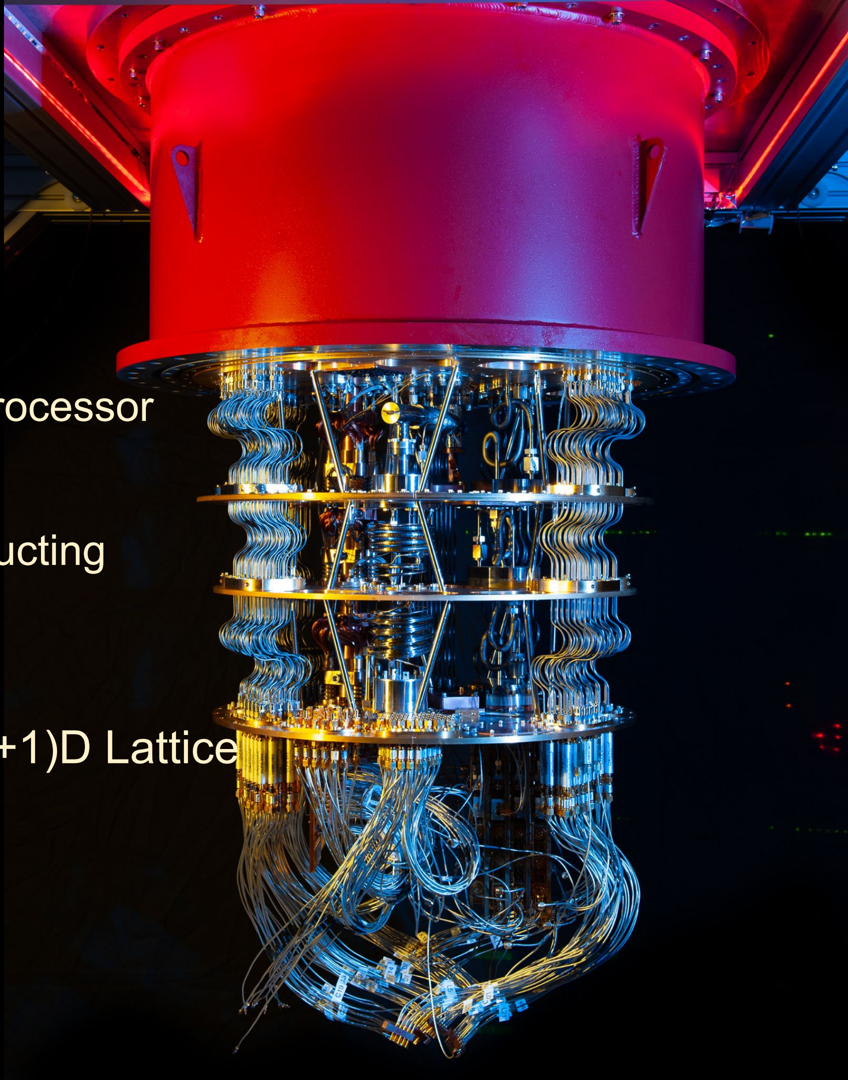
Google Quantum AI,

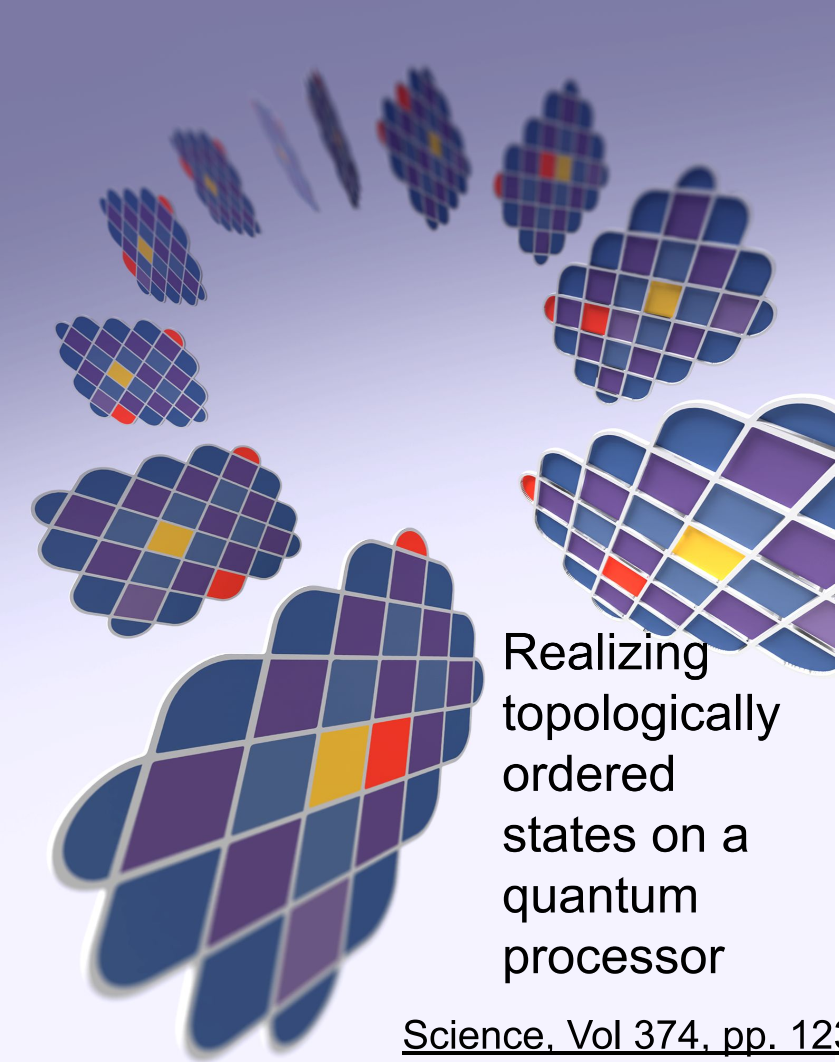
Realizing topologically ordered states on a quantum processor
Science 374, 1237-1241 (2021)

Non-Abelian braiding of graph vertices in a superconducting processor
Nature 618, 264–269 (2023)

Visualizing Dynamics of Charges and Strings in (2+1)D Lattice Gauge Theories
Nature 642, June 2025



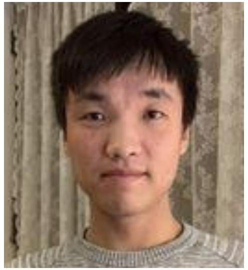
Quantum connection , Stockholm June 2025



Realizing topologically ordered states on a quantum processor

Science, Vol 374, pp. 1237-1241 (2021)

Kevin Satzinger



Yujie Liu

Adam Smith

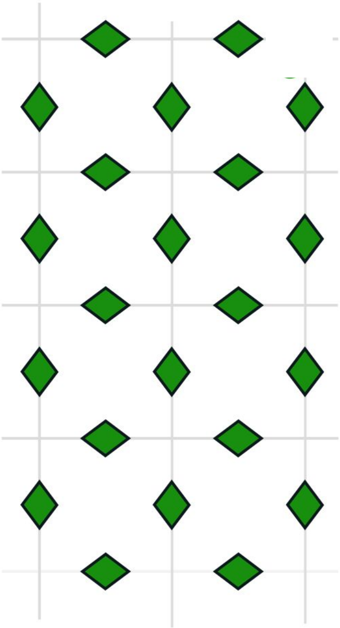
Chrstina Knapp

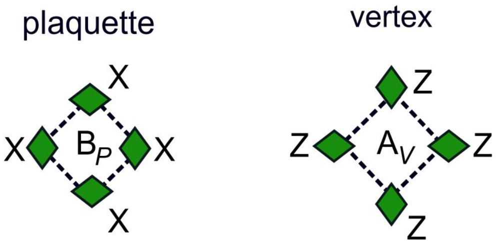
Michael Knap

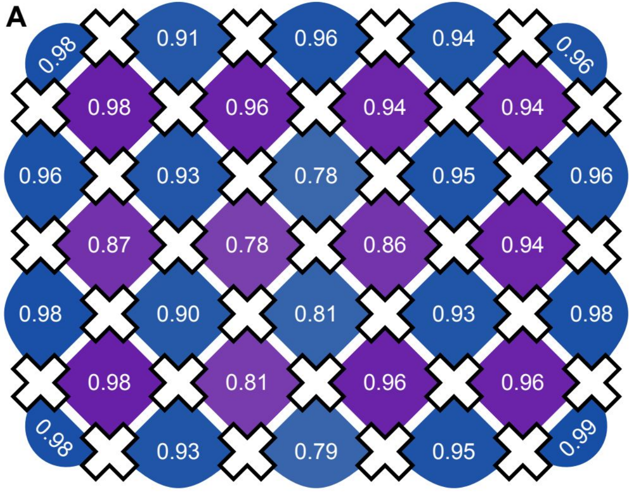
Frank Pollmann

Realizing topologically ordered states on a quantum processor

$$\mathcal{H} = -\sum_p \underbrace{\prod_{i\in p} X_i}_{B_p} - \sum_v \underbrace{\prod_{i\in v} Z_i}_{A_v}$$







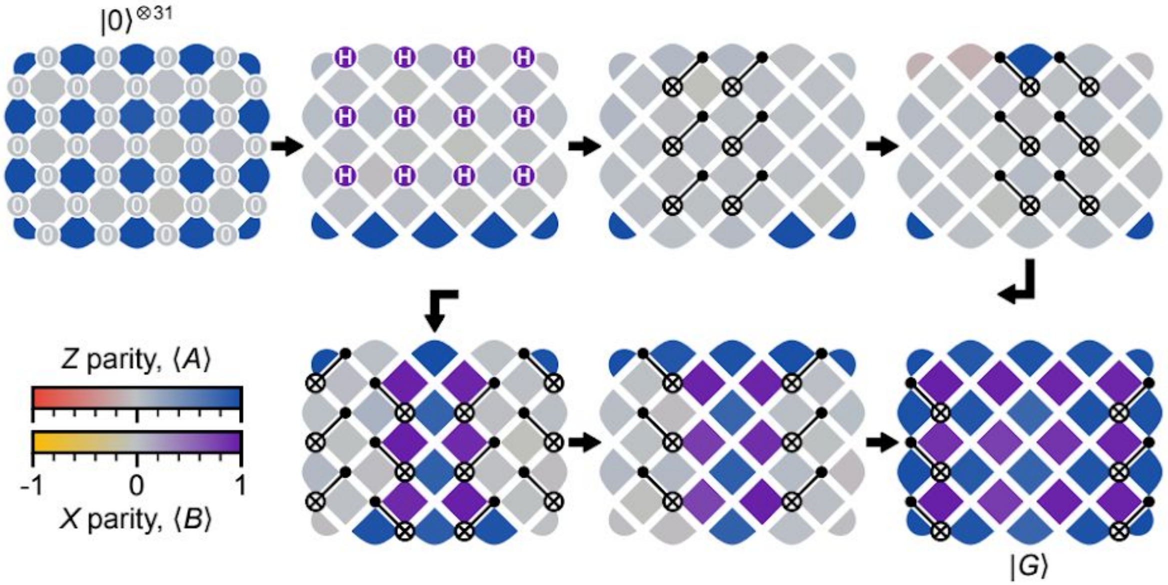
A. Y. Kitaev, Ann. Phys. (N. Y). 303, 2 (2003).

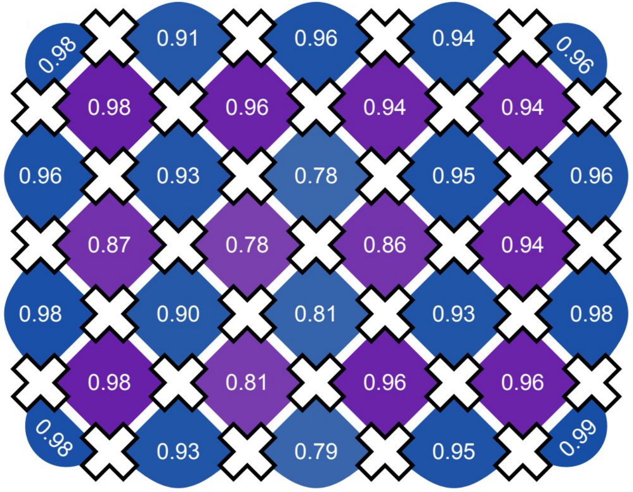
Satzinger *et al.* Science 374, 1237-1241 (2021)

Gate sequence to create ground-states

$$[B_p, A_v] = 0 \longrightarrow |G\rangle \propto \prod_p \underbrace{(\mathbb{I} + B_p)}_{\text{projector}} |0\rangle^{\otimes 31}$$

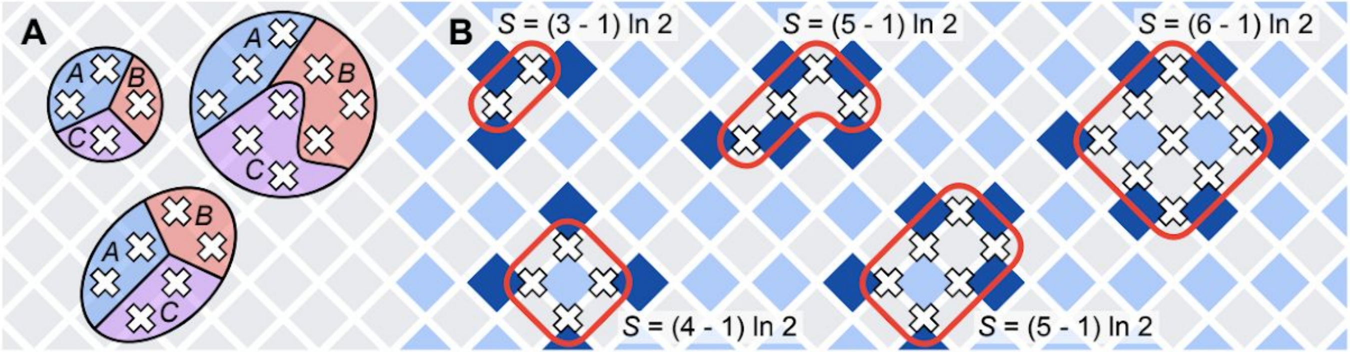
superposition of all plaquette configurations





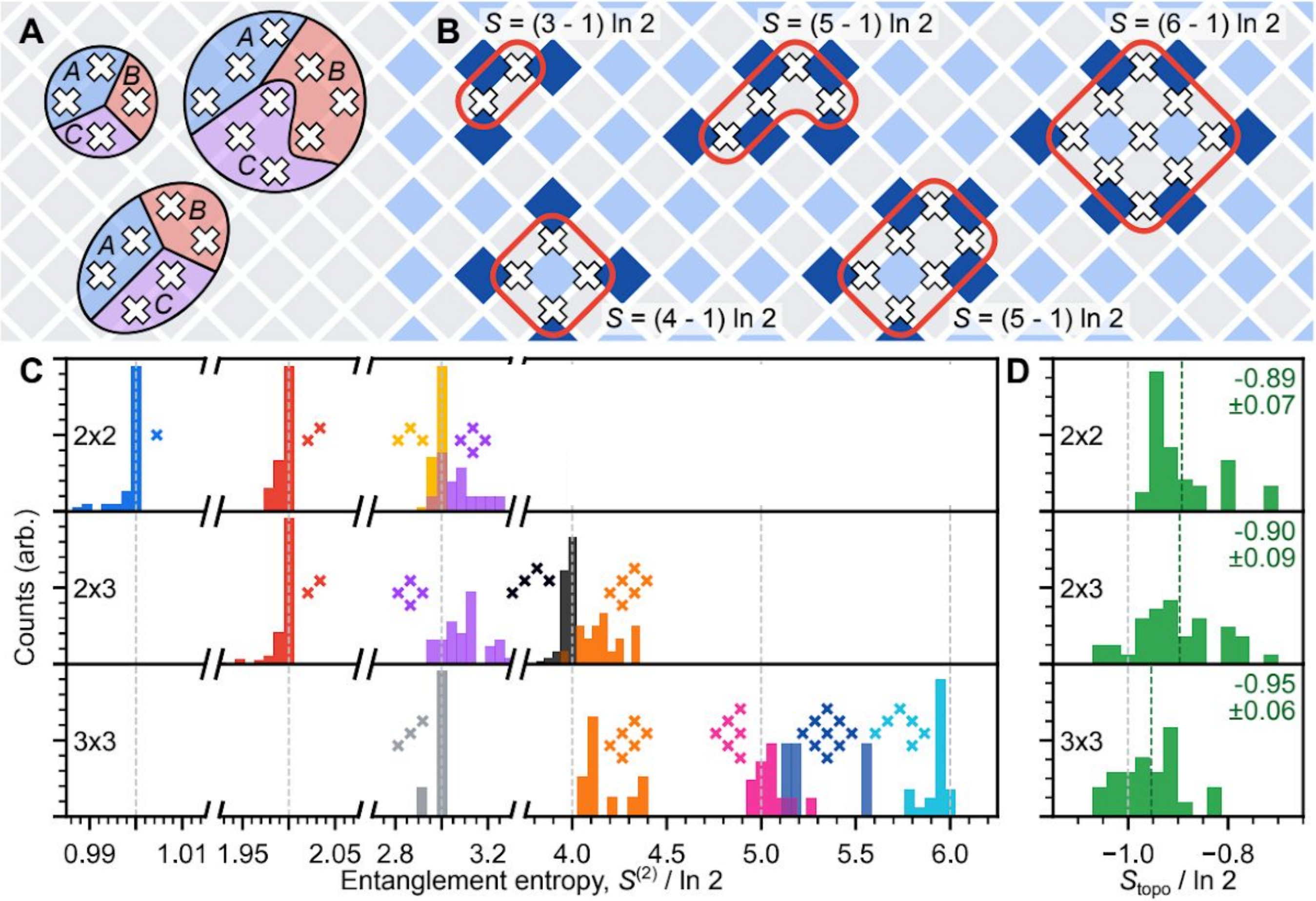
Topological entanglement entropy

$$S_{\text{topo}} = S_A + S_B + S_C - S_{AB} - S_{BC} - S_{AC} + S_{ABC}$$



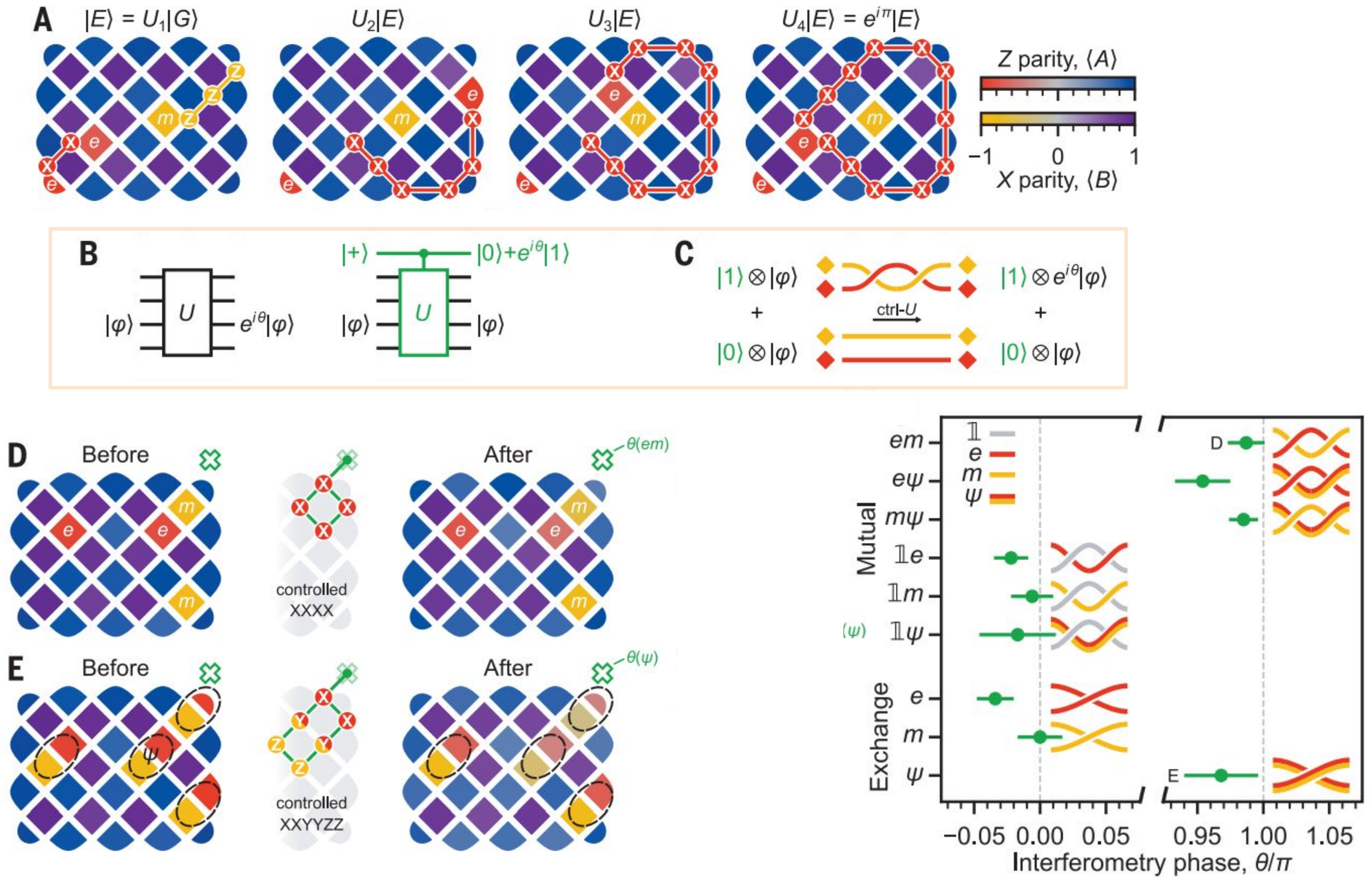
Topological entanglement entropy

$$S_{\rm topo} = S_A + S_B + S_C - S_{AB} - S_{BC} - S_{AC} + S_{ABC}$$

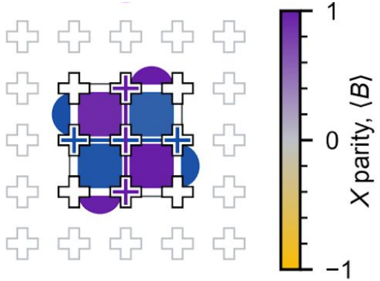


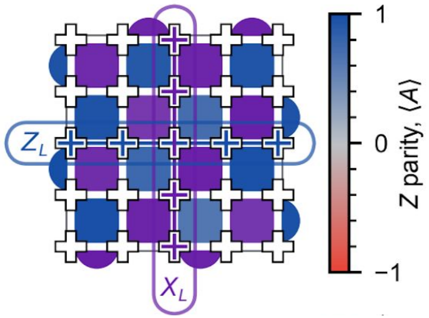
Randomized measurement:
Tiff *et al.*, Science (2019),
Vermersch *et al.*, PRA (2018)

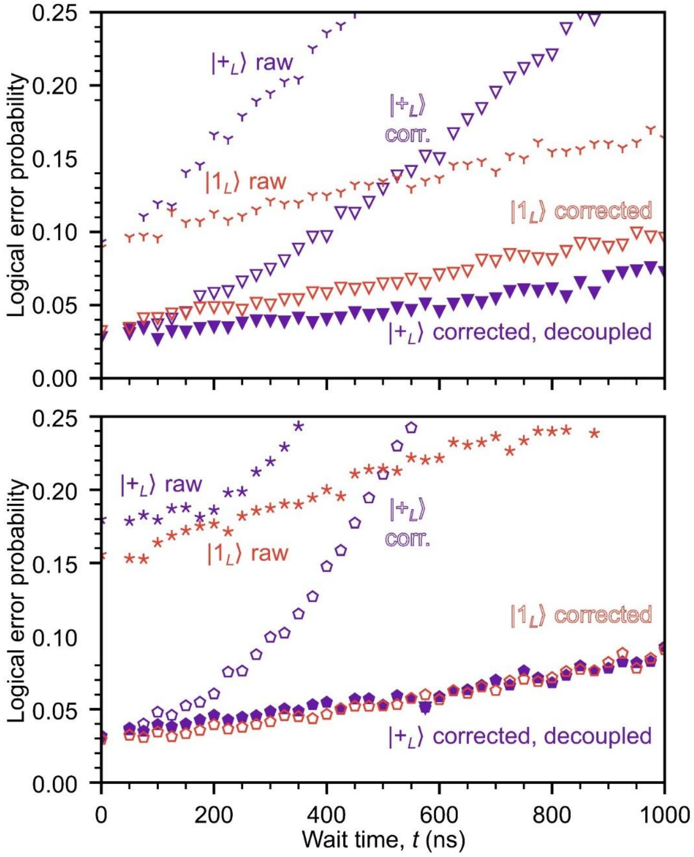
Abelian braiding statistics by Ramsey interferometry

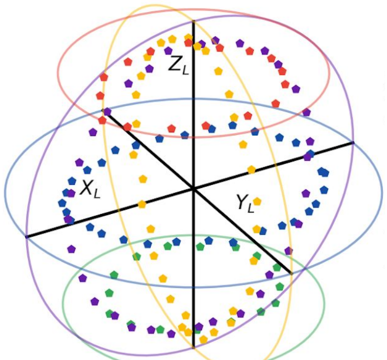


Fidelity of logical states

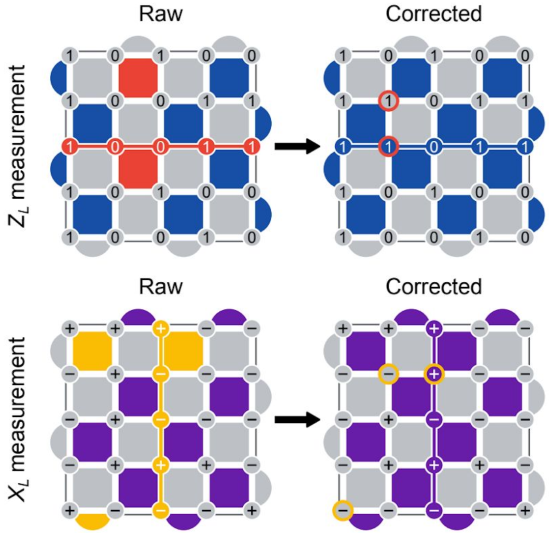




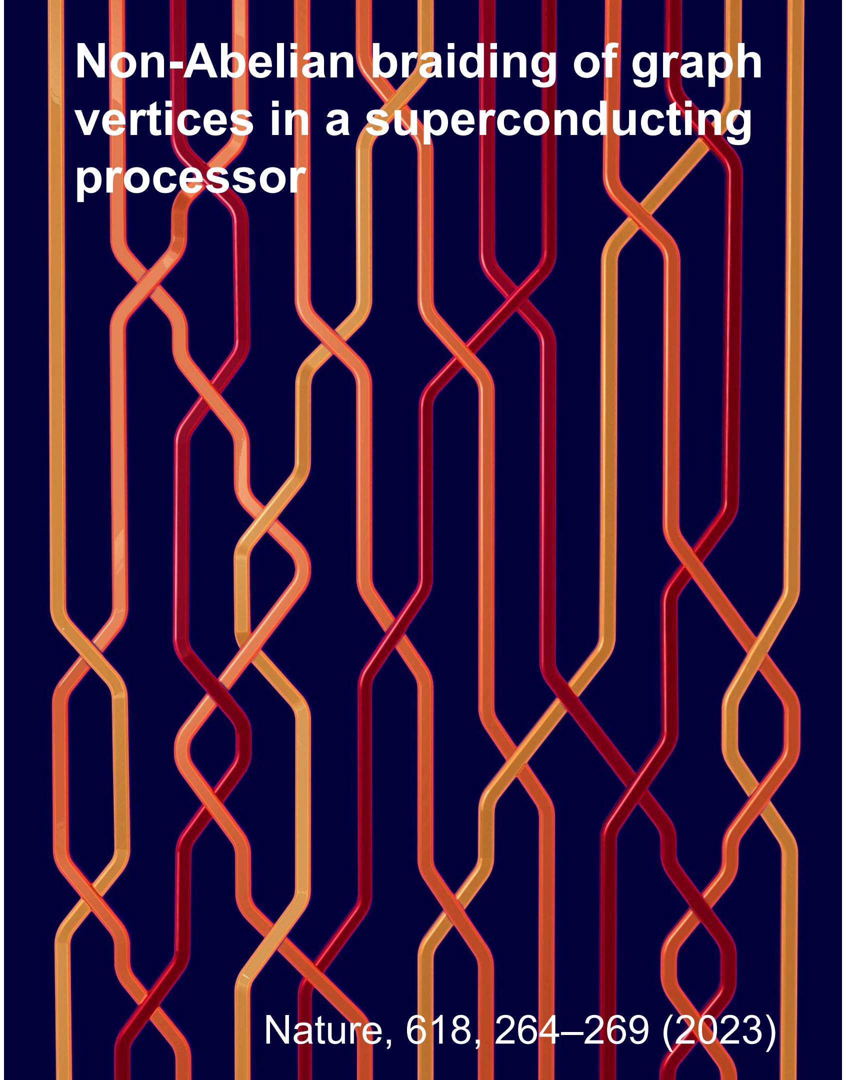




logical qubit tomography immediately after state injection



Non-Abelian braiding of graph vertices in a superconducting processor



Nature, 618, 264–269 (2023)

Trond Andersen

Yuri Lensky

Andreas Bengtsson

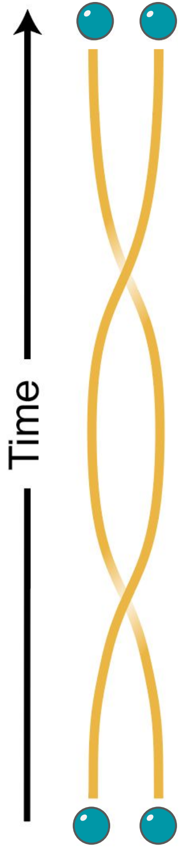
Alexis Morvan

Igor Aleiner

Eunah Kim
Cornell U.

Exchange statistics of indistinguishable particles

Indistinguishability of particles : a fundamental principle of quantum mechanics

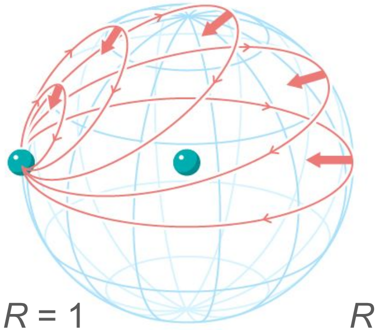


$R^2\,|\Psi\rangle$

$R\,|\Psi\rangle$

$|\Psi\rangle$ *R:* exchange operator

3 spatial dimensions

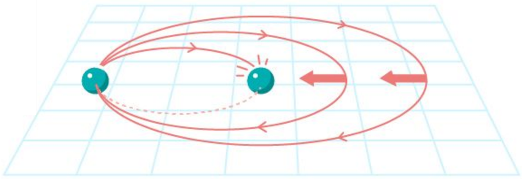


$R = 1$
e.g. photons

$R = -1$
e.g. electrons

Exchanging particles twice → $R^2 = 1$

2 spatial dimensions



$R = e^{i\theta}$
e.g. plaquette violations
in the surface code

R is a matrix
Non-Abelian anyons

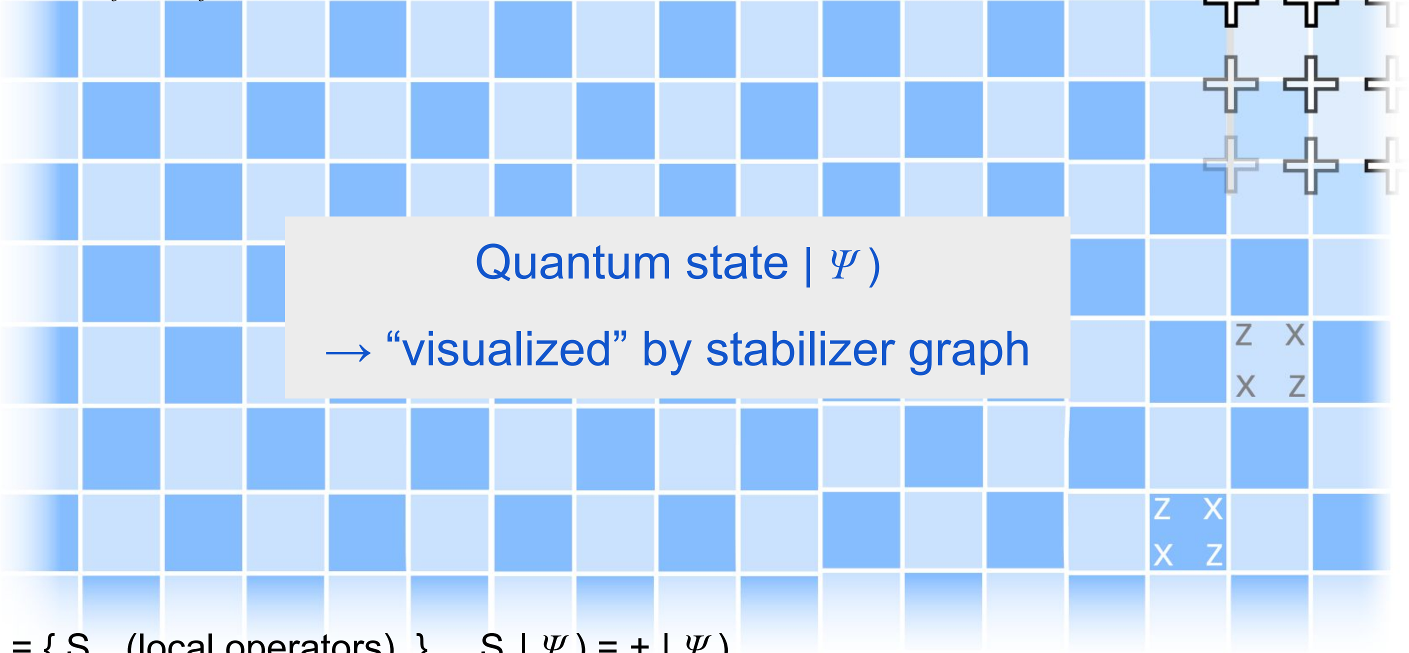
R^2 no longer has to be 1

non-Abelian quasiparticles candidates:
5/2 FQH states
vortices in topological SC
Majorana zero modes

Non-Abelian braiding of graph vertices in a superconducting processor

$$H = \sum Z_{i,j} X_{i,j+1} X_{i+1,j} Z_{i+1,j+1} \rightarrow U_G$$

$$|\Psi_G\rangle = U_G |0\rangle^{\otimes N}$$



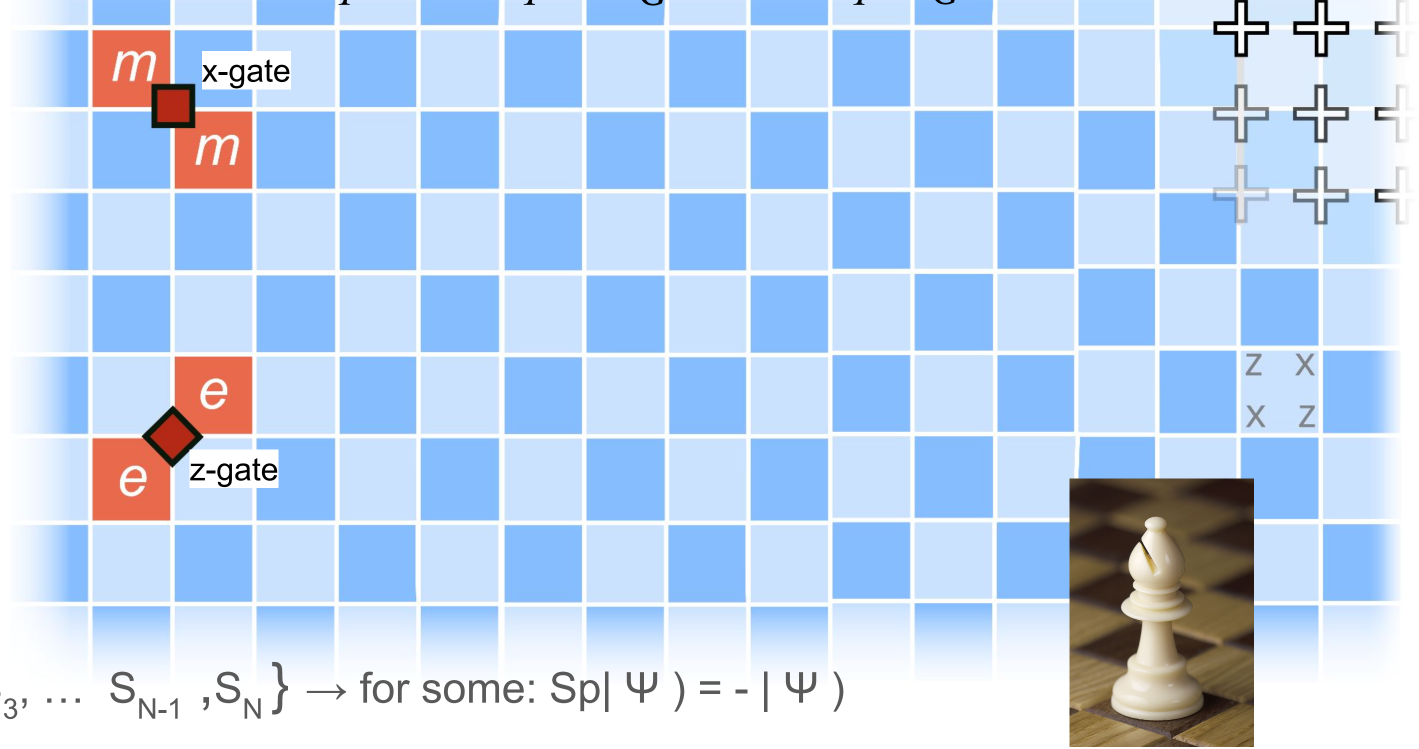
Stabilizer set = { S_p (local operators) }, $S_p|\Psi\rangle = +|\Psi\rangle$

{ $S_1, S_2, S_3, \ldots, S_{N-1}, S_N$ } → Stabilizer set

Andersen *et al.* , nature **618**, 264–269 (2023)

“quasi-particles” → Plaquette violations

$$| \Psi_1) = U_1 | \Psi_G) = U_1 U_G | 0)^{\otimes N}$$

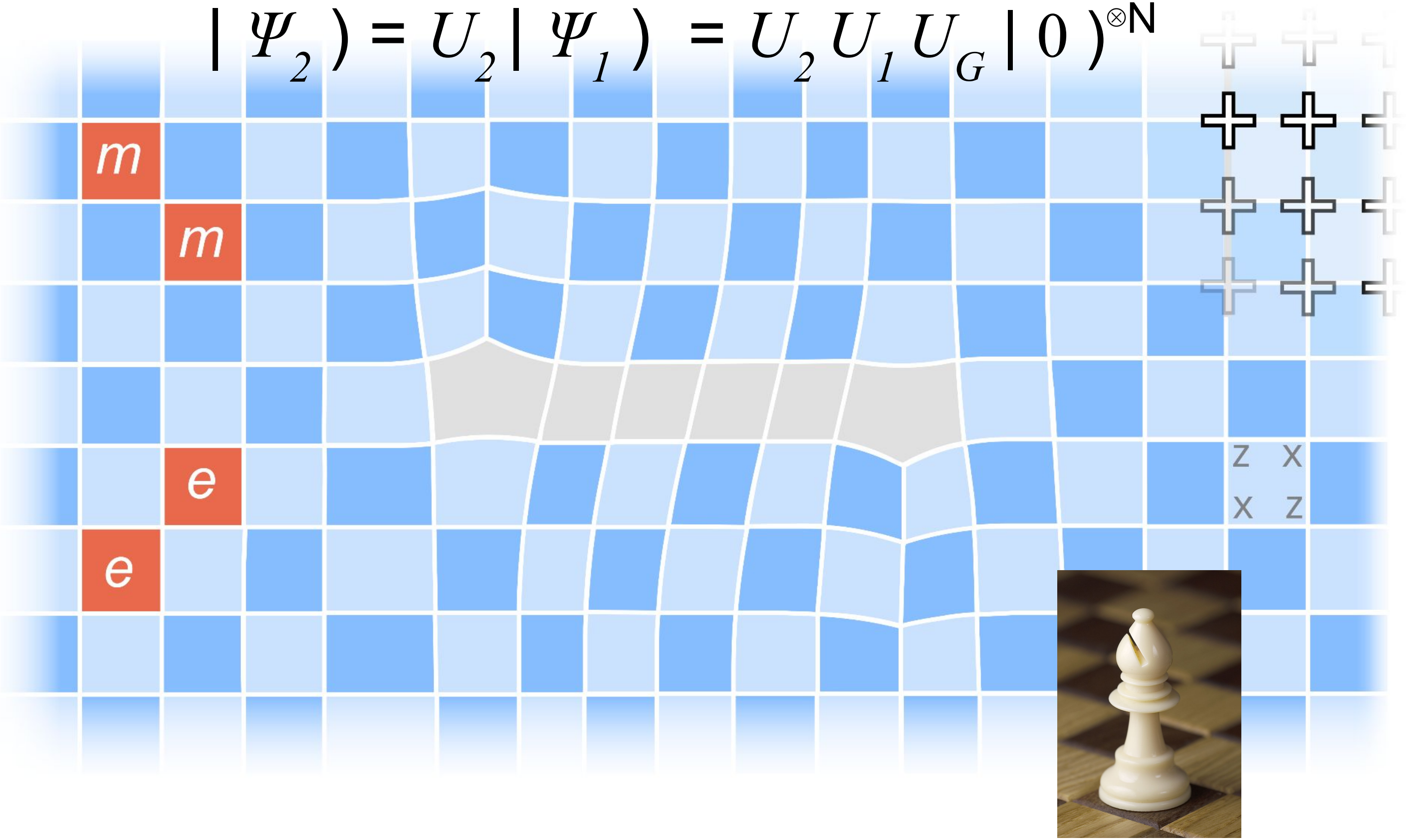


$\{ S_1, S_2, S_3, \ldots\ S_{N-1}, S_N \}$ → for some: $S_p | \Psi) = - | \Psi)$

e and *m* on different sublattices→ can never “meet”

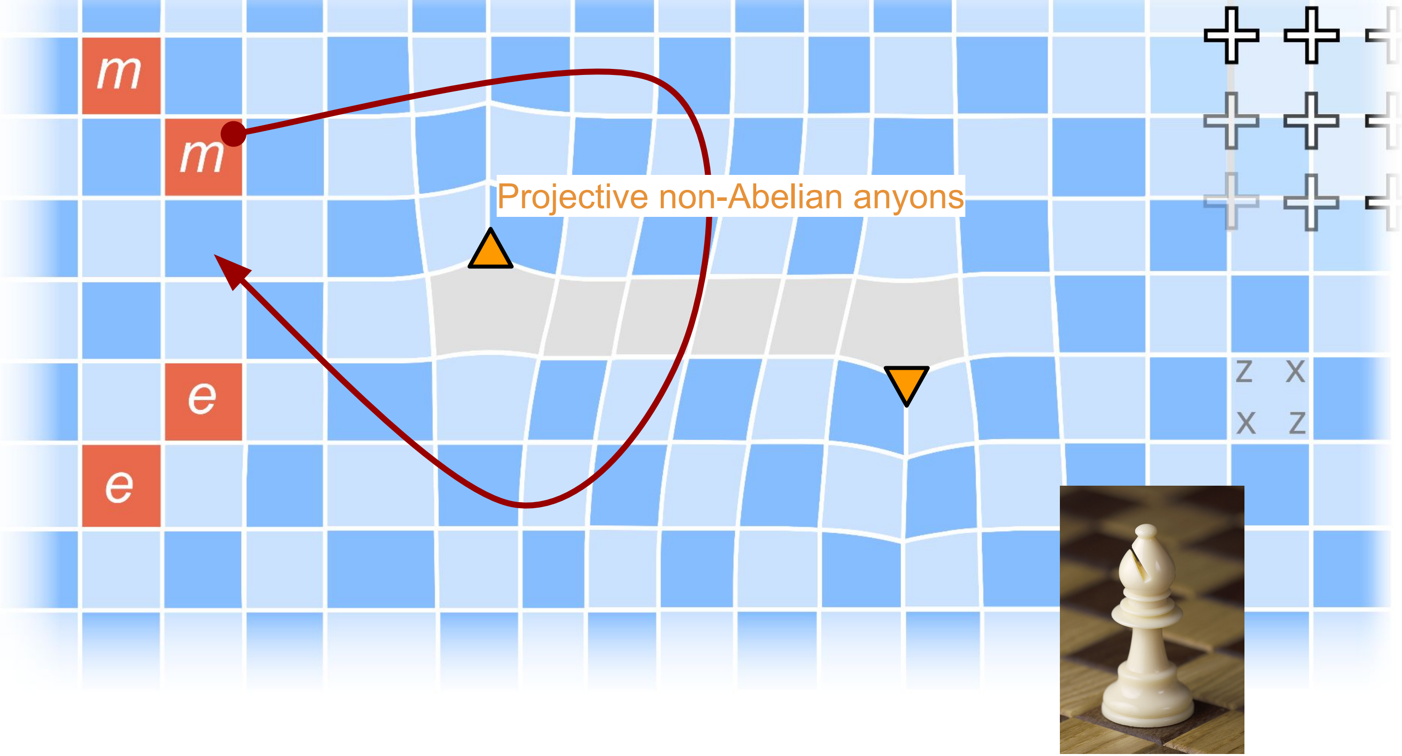
Unitarily modifying wavefunctions to have "defects"

$$|\Psi_2\rangle = U_2|\Psi_1\rangle = U_2 U_1 U_G |0\rangle^{\otimes N}$$



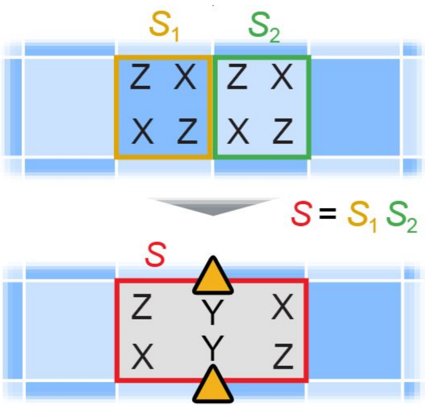
Unitarily modifying wavefunctions to have “defects”

$$|\Psi_2\rangle = U_2|\Psi_1\rangle = U_2 U_1 U_G |0\rangle^{\otimes N}$$

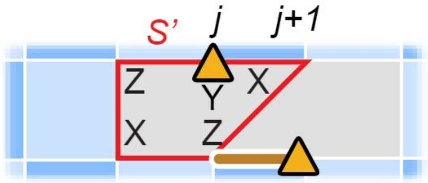


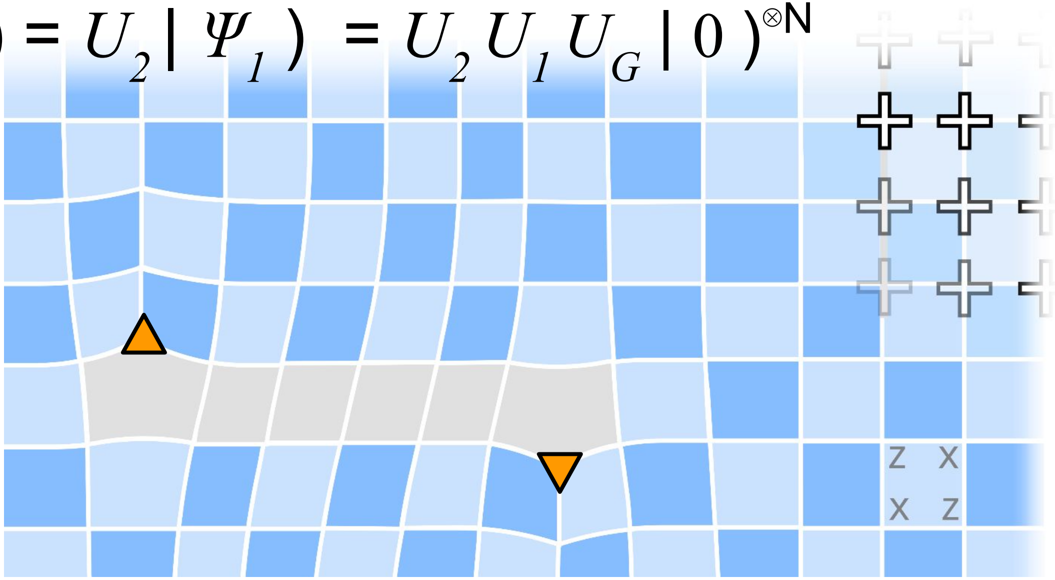
Recipe to modifying wavefunctions to have Degree-3 vertices

$$|\Psi_2\rangle = U_2 |\Psi_1\rangle = U_2 U_1 U_G |0\rangle^{\otimes N}$$



$U = \exp(\pi/8\,[S', S]) = \exp(i\,\pi/4\, X_{i,j} Z_{i,j+1})$

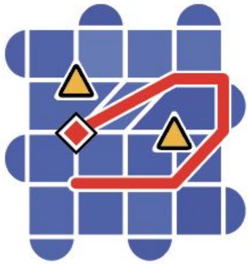




Move the D3Vs with 2-qubit gates
→ deform the stabilizer graph !

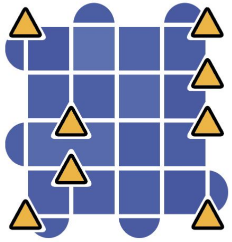
Experiments outline

- Verify fundamental fusion rules of D3Vs
  - What happens when D3Vs “collide” with each other and with fermions?
  - Do they behave as non-Abelian Ising anyons should?

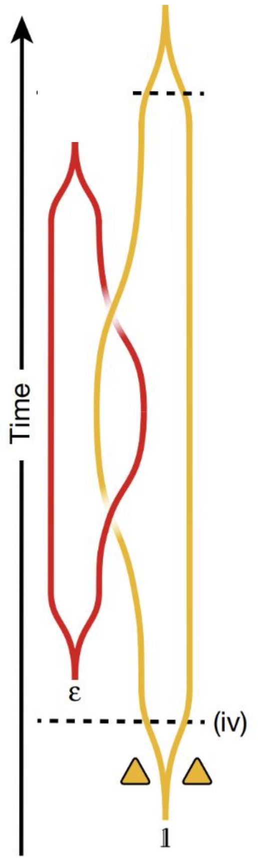


- Braid the D3Vs to realize non-Abelian exchange statistics
  - Does braiding lead to a change in observables for the first time?

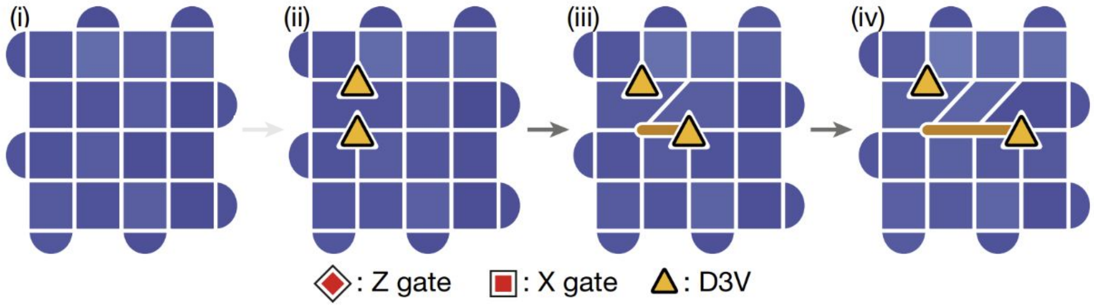
- Use braiding to entangle anyons encoding logical qubits
  - How can braiding be used in quantum computing operations?



Experimentally verifying the fusion rules

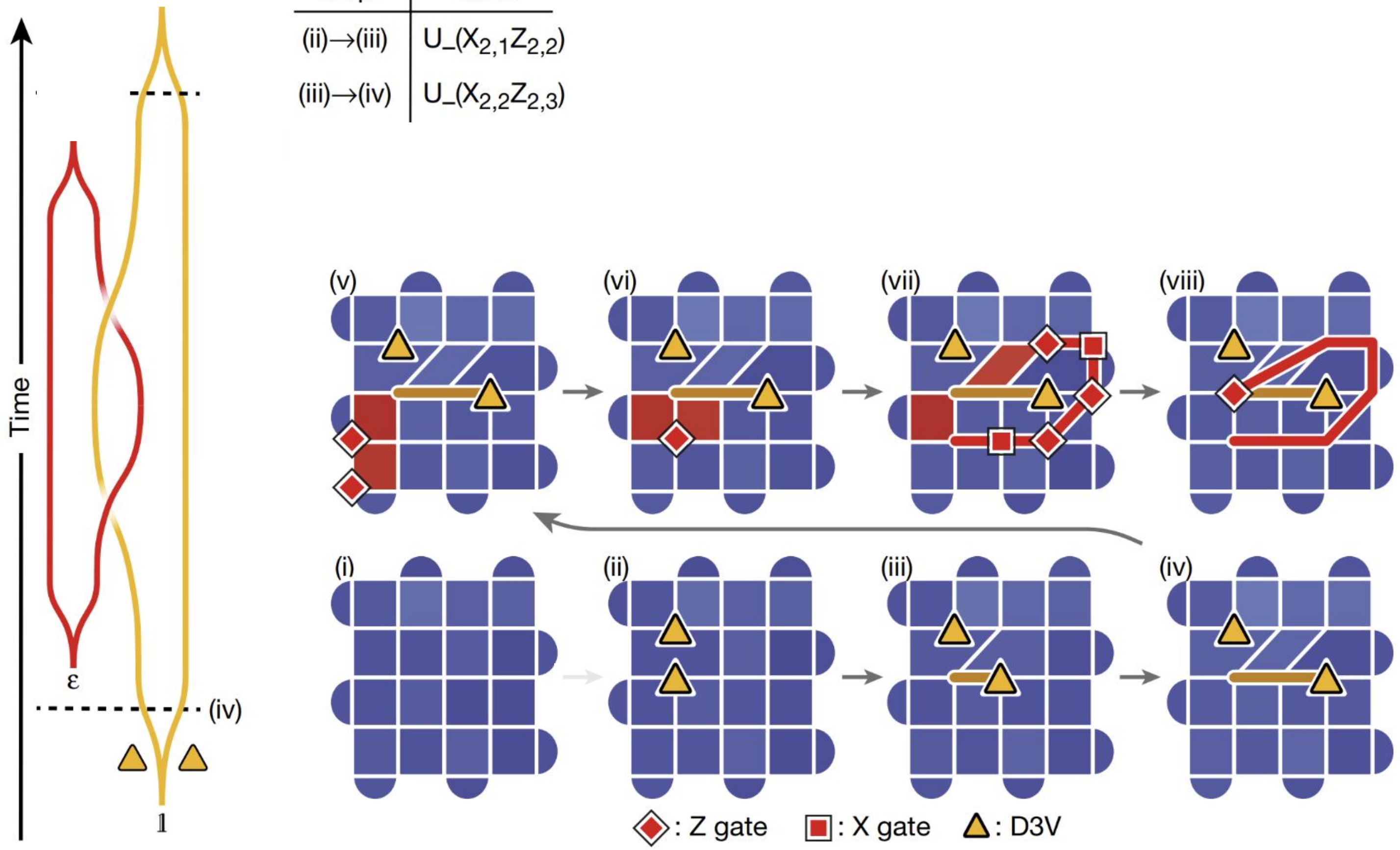


Step	Gate
(ii)→(iii)	$U_{-}(X_{2,1}Z_{2,2})$
(iii)→(iv)	$U_{-}(X_{2,2}Z_{2,3})$



Experimentally verifying the fusion rules

Step	Gate
(ii)→(iii)	$U\_(X_{2,1}Z_{2,2})$
(iii)→(iv)	$U\_(X_{2,2}Z_{2,3})$

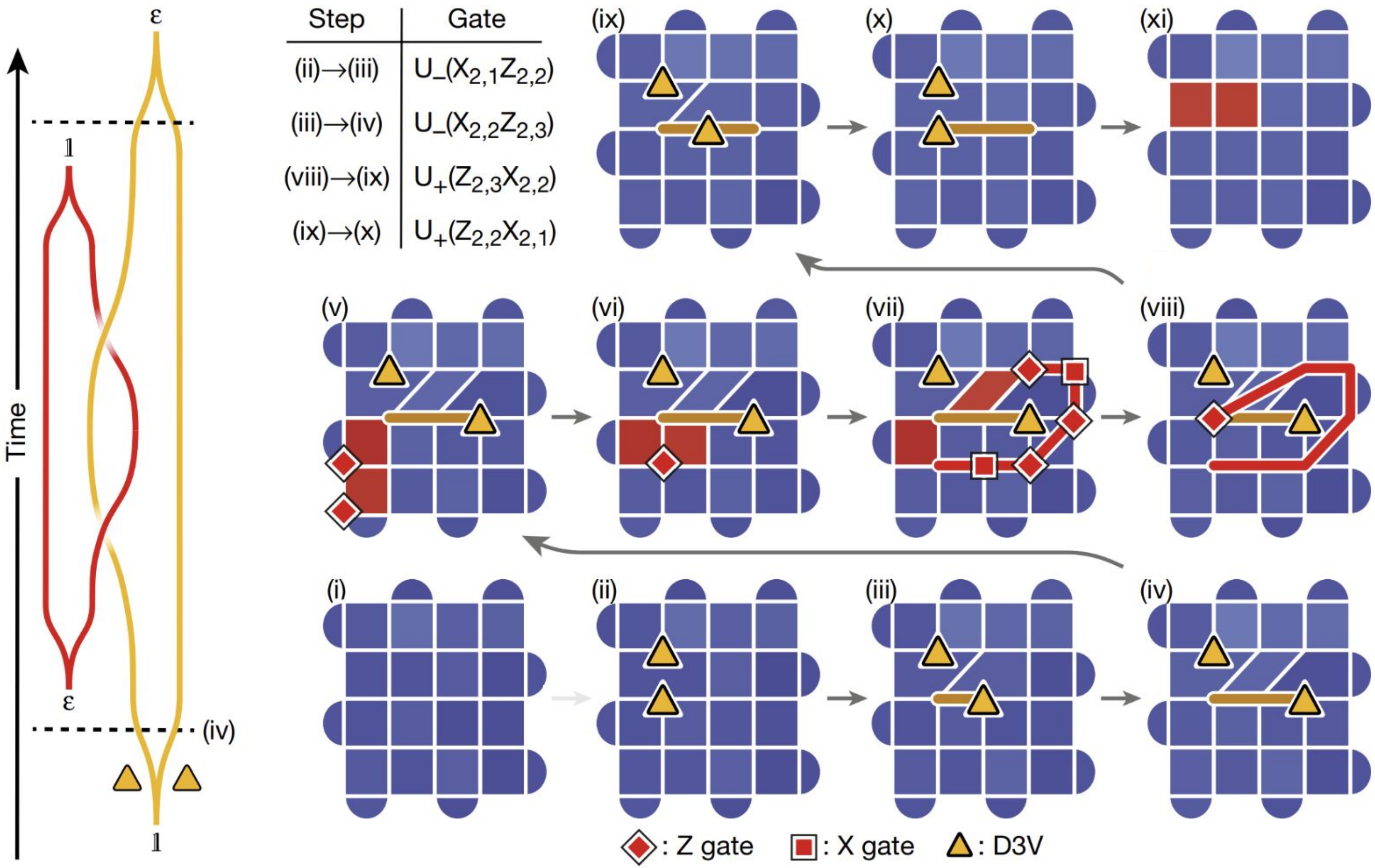


Experimentally verifying the fusion rules

Fermion can fuse into a D3V

$$\sigma \times \epsilon = \sigma$$

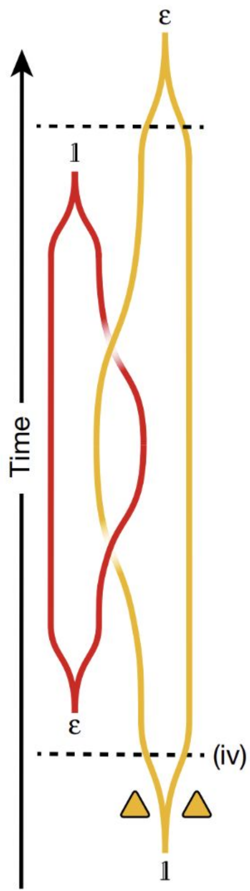
Step	Gate
(ii)→(iii)	$U_{-}(X_{2,1}Z_{2,2})$
(iii)→(iv)	$U_{-}(X_{2,2}Z_{2,3})$
(viii)→(ix)	$U_{+}(Z_{2,3}X_{2,2})$
(ix)→(x)	$U_{+}(Z_{2,2}X_{2,1})$



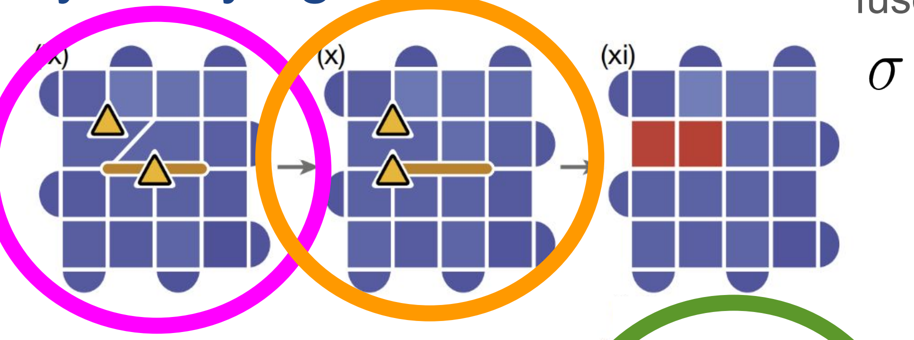
Experimentally verifying the fusion rules

Fermion can fuse into a D3V

$$\sigma \times \epsilon = \sigma$$

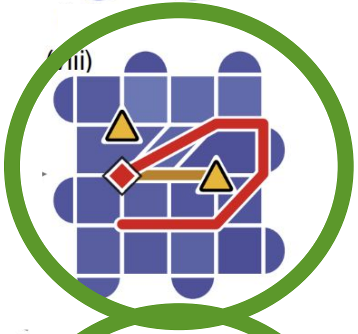


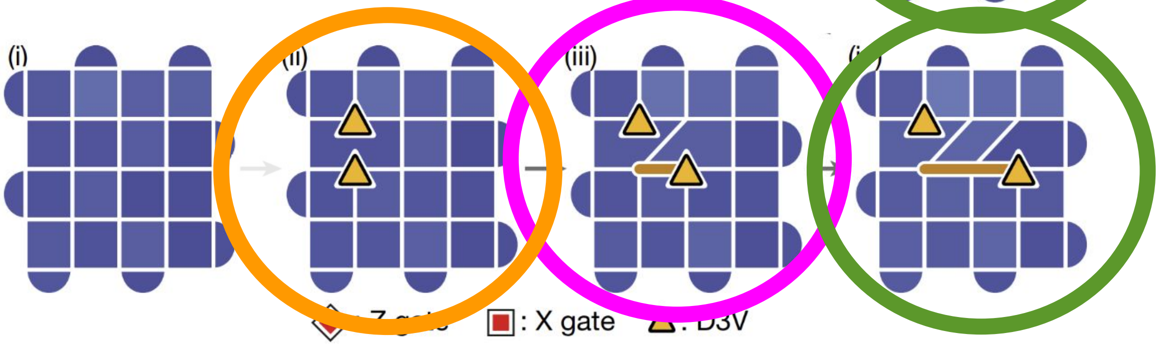
Step	Gate
(ii)→(iii)	$U_-(X_{2,1}Z_{2,2})$
(iii)→(iv)	$U_-(X_{2,2}Z_{2,3}$
(viii)→(ix)	$U_+(Z_{2,3}X_{2,2}$
(ix)→(x)	$U_+(Z_{2,2}X_{2,1})$



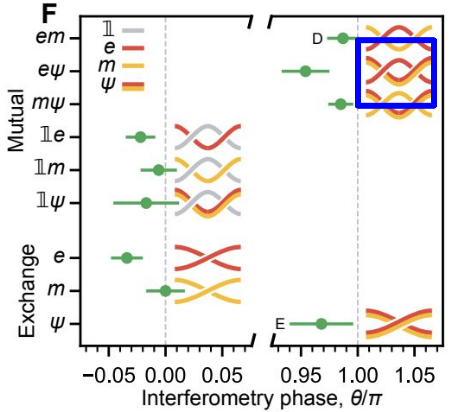
Two D3Vs can store (and later reveal through fusion) either zero or one fermion

$$\sigma \times \sigma = 1 + \varepsilon$$



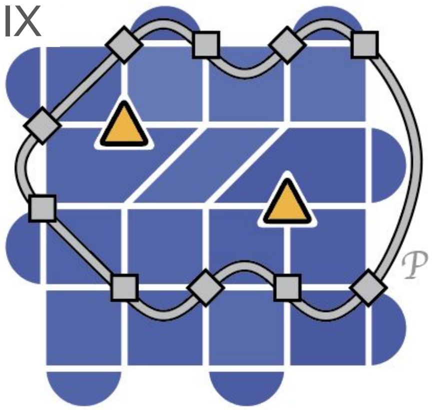


Non-local fermion measurements



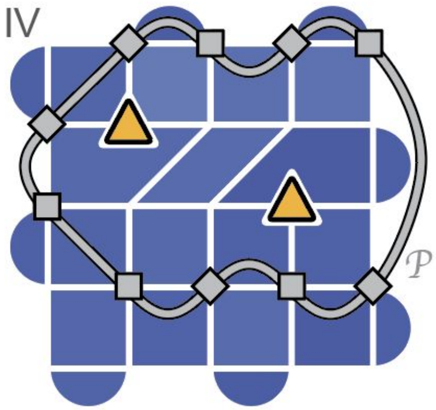
When a plaquette violation is brought around a fermion, it gains a π-phase

→Let's measure the Pauli string that corresponds to bringing a plaquette violation around the pair of D3Vs. If there's a fermion, we should get -1.



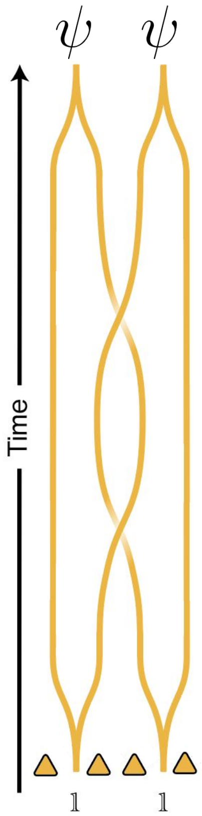
$\langle \mathcal{P} \rangle = \langle \mathcal{P}' \rangle = \langle ZX..Z \rangle$

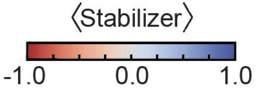
= -0.85 ± 0.02



-Information (fermion number) encoded non-locally

Braiding D3Vs to realize non-Abelian exchange statistics





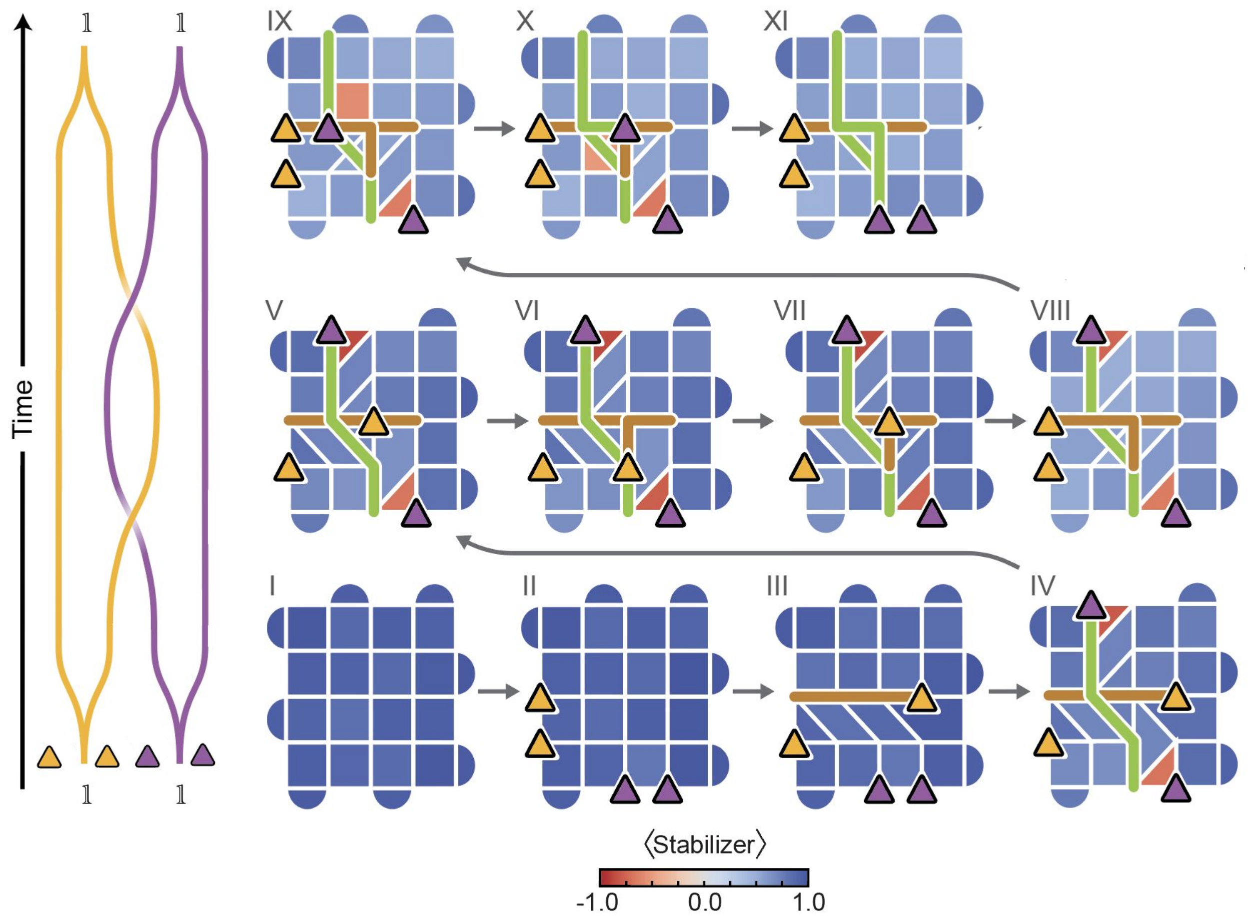
-Braiding led to a change in local observables.

Two D3Vs can store (and later reveal through fusion) either zero or one fermion

$$\sigma \times \sigma = 1 + \psi$$

QC perspective:
Braid acts as X-gate on the space spanned by $|00\rangle$ and $|11\rangle$

Control experiment: Braiding distinguishable D3Vs

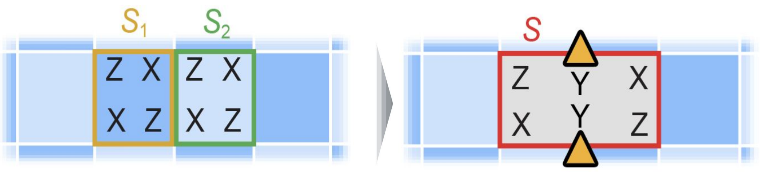


D3Vs made distinguishable by attaching a plaquette violation ($U_{\pm} \rightarrow U_{\mp}$).

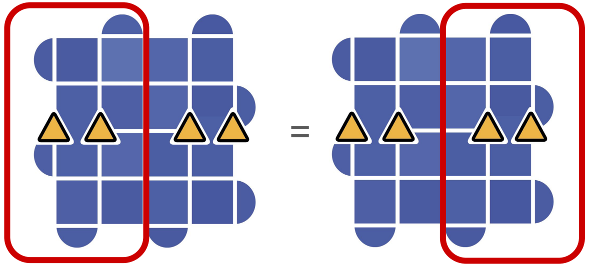
When using distinguishable particles, no fermions appear, thus a successful control experiment

Encoding logical qubits in anyon pairs

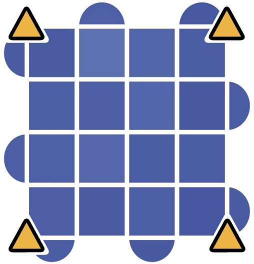
Creation of D3V pair doubles degeneracy
→ adds one logical qubit!



All pairs are not independent
→ *N* D3Vs encode *N/2-1* logical qubits

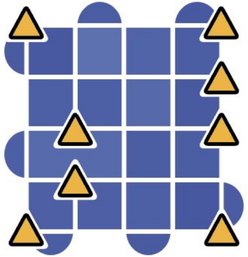


We "forgot" the D3Vs in the corners!



4/2-1=1
logical qubit

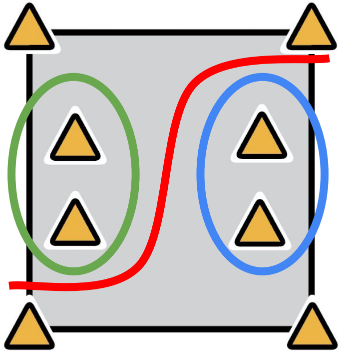
Let's use 4 bulk D3Vs+4 corner D3Vs
→ 3 logical qubits



8/2-1=3
logical qubits

Entangling anyon-encoded logical qubits

Our choice:



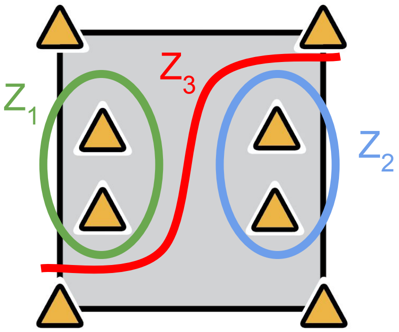
Basis choice

GHZ-state: $|\Psi_{\mathrm{GHZ}}\rangle = (|111\rangle + |000\rangle)/\sqrt{2}$ - depends on basis!

Separated anyons: non-local operators, many possible choices of basis

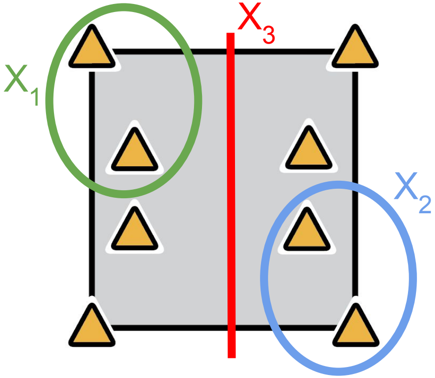
Logical operators: (non-local) Pauli strings

Logical-Z:



Loop around all D3Vs in logical qubit

Logical-X



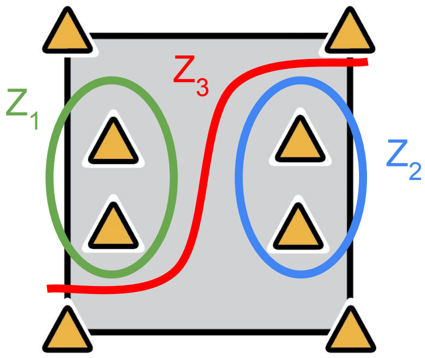
Loop around odd number of D3Vs in logical qubit

Logical-Y

$Y_i = iZ_iX_i$

Entangling anyon-encoded logical qubits

Our choice:



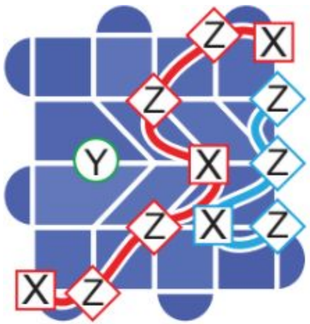
Basis choice

GHZ-state: $|\Psi_{\mathrm{GHZ}}\rangle = (|111\rangle + |000\rangle)/\sqrt{2}$ - depends on basis!

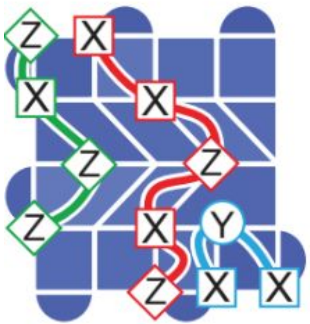
Separated anyons: non-local operators, many possible choices of basis

Logical operators: (non-local) Pauli strings

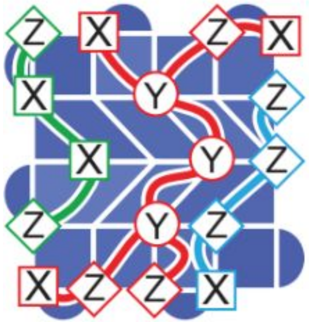
Logical-Z:



Logical-X



Logical-Y



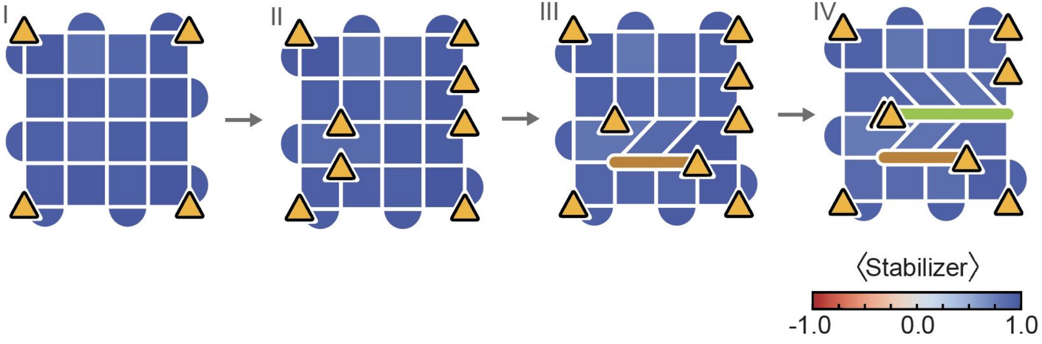
Entanglement through braiding

-*Double* exchange acted as X-gate:

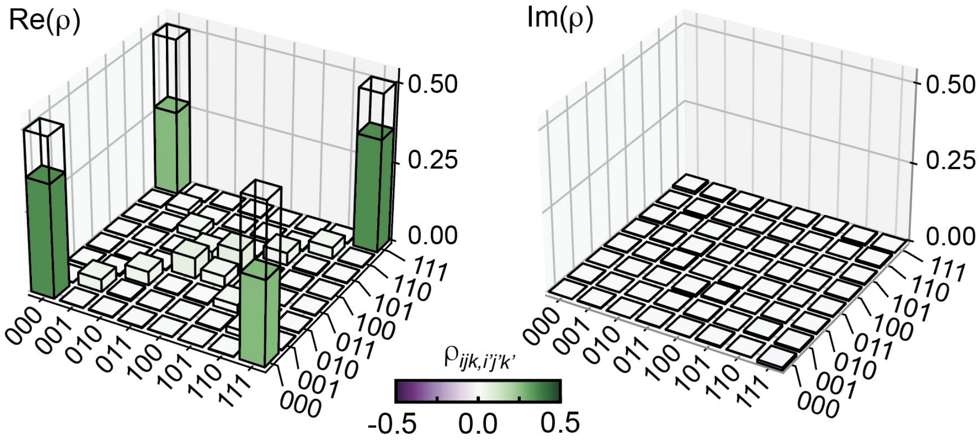
$|00\rangle \rightarrow |11\rangle$

-Use *single* exchange to achieve √X-gate:

$|000\rangle \rightarrow (|000\rangle + |111\rangle)/\sqrt{2}$



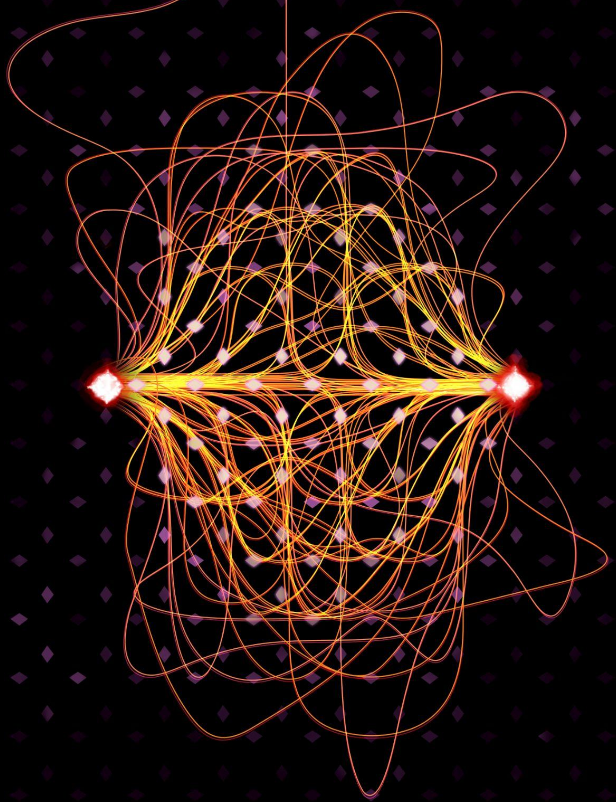
Quantum state tomography of final state:



Fidelity: $\langle \psi_{\mathrm{GHZ}} | \rho | \psi_{\mathrm{GHZ}} \rangle = 0.62$

Purity: $\sqrt{\mathrm{Tr}\{\rho^2\}} = 0.65$

Visualizing Dynamics of Charges and Strings in (2+1)D Lattice Gauge Theories



T. Cochran *et al.*, Nature 642, June 2025

Quantum AI

Residents

Tyler Cochran

Gaurav Gyawali

External Collaborators

TECHNISCHE UNIVERSITÄT MÜNCHEN

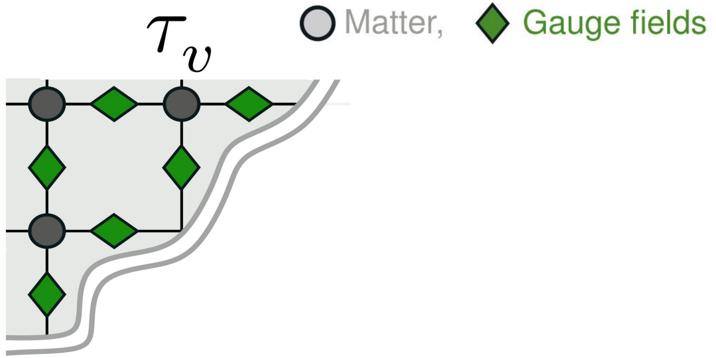
The University of Nottingham

Visualizing Dynamics of Charges and Strings in (2+1)D Lattice Gauge Theories

$$\mathcal{H}_{\mathsf{LGT}} = -\underbrace{\sum_{plaq.} B_p}_{\text{magnetic flux}} - \underbrace{\sum_{\text{links}} Z_l}_{\text{electric field}} \underbrace{-\sum_{\text{links}} \tau_v^Z\, X_i\, \tau_{v'}^Z}_{\text{matter-field coupling}} \underbrace{-\sum_{\text{vert.}} \tau_v^X}_{\text{mass / charge}}$$

$$A_v = \prod_{i \in v} Z_i$$

$$B_p = \prod_{i \in p} X_i$$



$$G_v = A_v \tau_v^X \;\; \rightarrow [\mathcal{H}_{\mathsf{LGT}}, G_v] = 0, \forall v$$

Visualizing Dynamics of Charges and Strings in (2+1)D Lattice Gauge Theories

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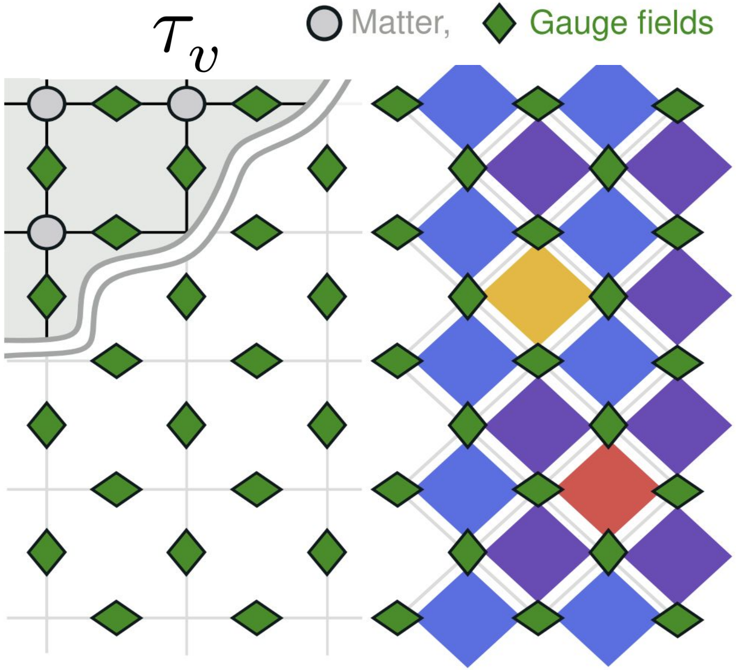
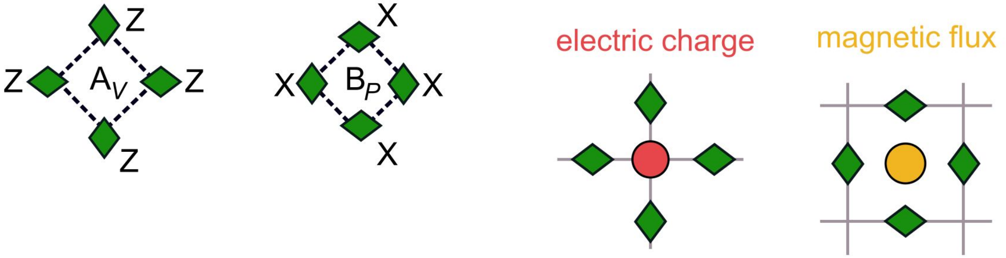


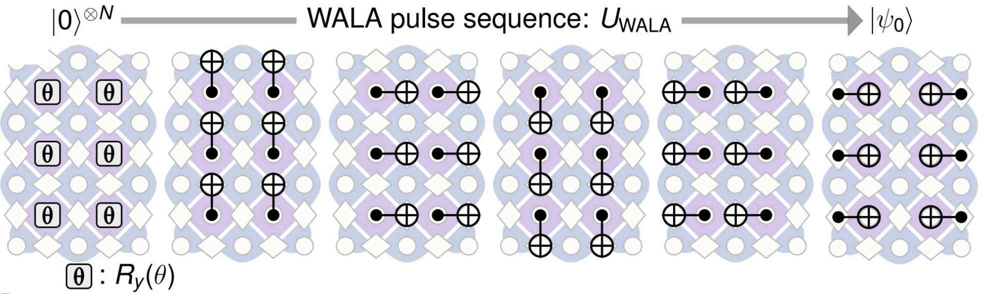
$$G_v = A_v \tau_v^X \;\rightarrow [\mathcal{H}_{\text{LGT}}, G_v] = 0, \forall v$$

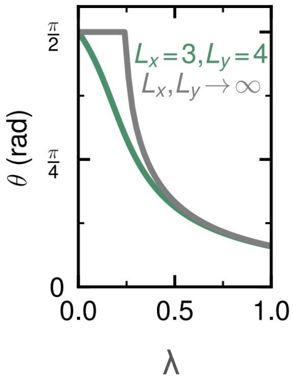
$$\mathcal{H} = -J_E \sum_v A_v - J_M \sum_p B_p - h_E \sum_{\text{links}} Z_l - \lambda \sum_{\text{links}} X_l$$

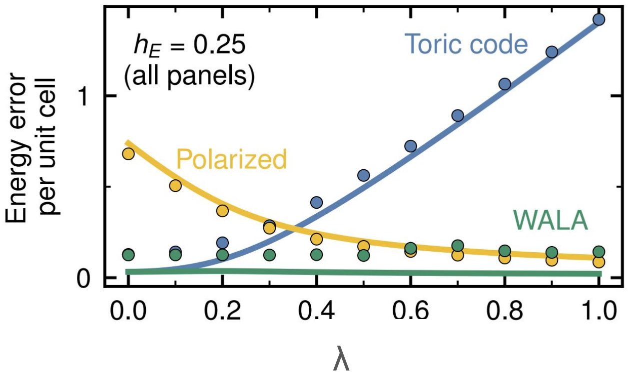


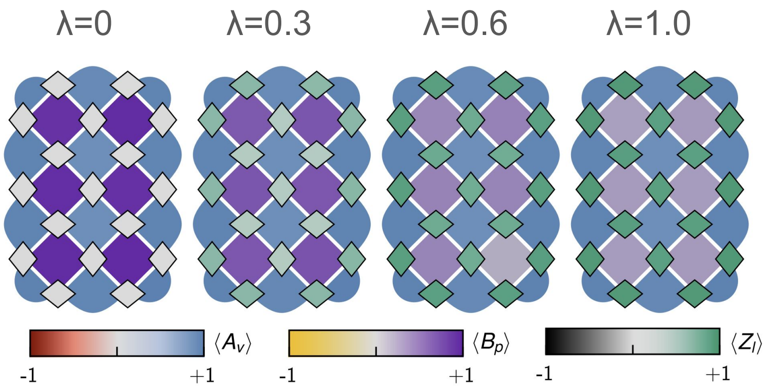
Weight Adjustable Loop Ansatz (WALA) ground state

$$\mathcal{H} = -J_E \sum_v A_v - J_M \sum_p B_p - h_E \sum_{\text{links}} Z_l - \lambda \sum_{\text{links}} X_l$$





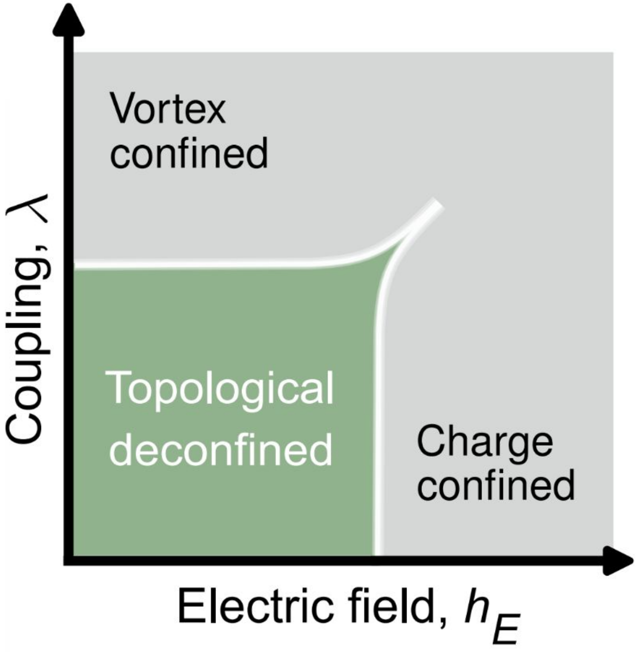


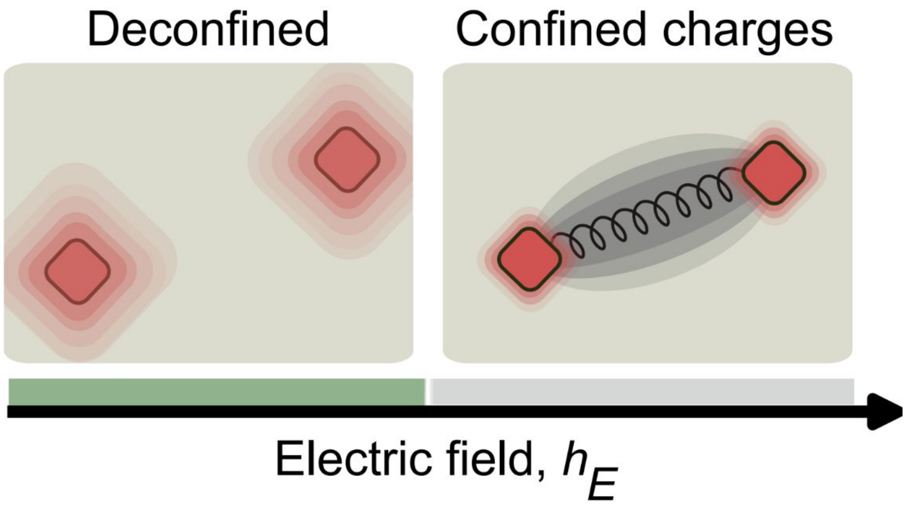


$$J_E = J_M = 1$$

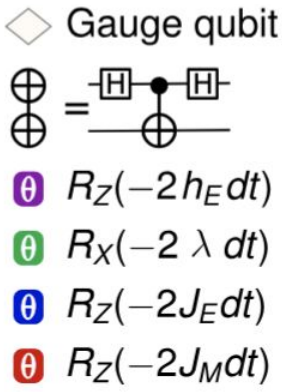
Phase diagram of the LGT

$$\mathcal{H} = -J_E \sum_v A_v - J_M \sum_p B_p - h_E \sum_{\text{links}} Z_l - \lambda \sum_{\text{links}} X_l$$

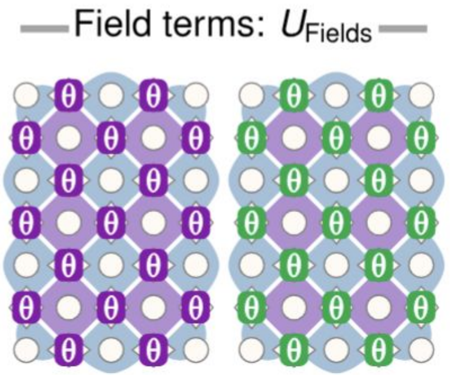




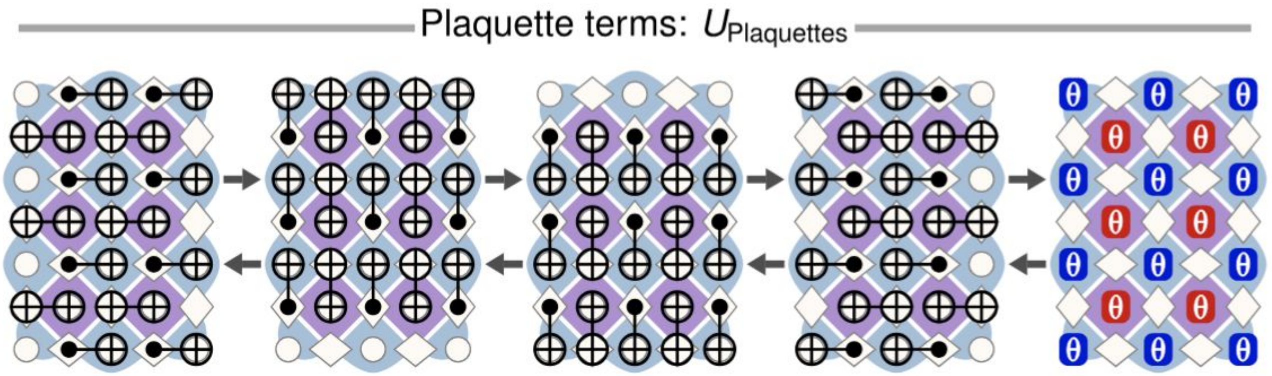
E. Fradkin and S. H. Shenker, PRD (1979), S. Trebst *et al.*, PRL(2007), J. Vidal *et al.*, PRB (2009).



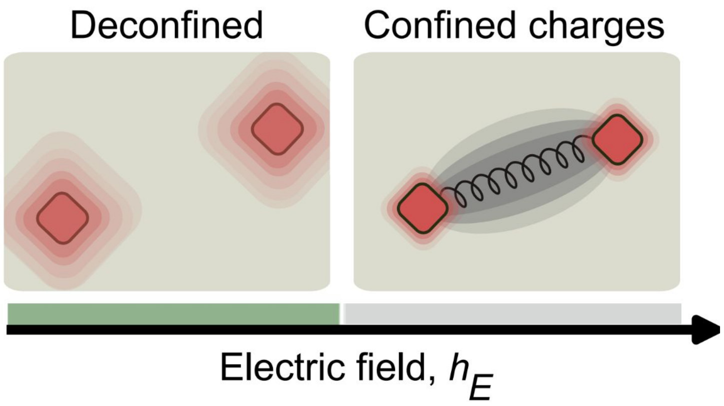
Dynamics: Trotterization, $U = \exp(iH\,dt)$



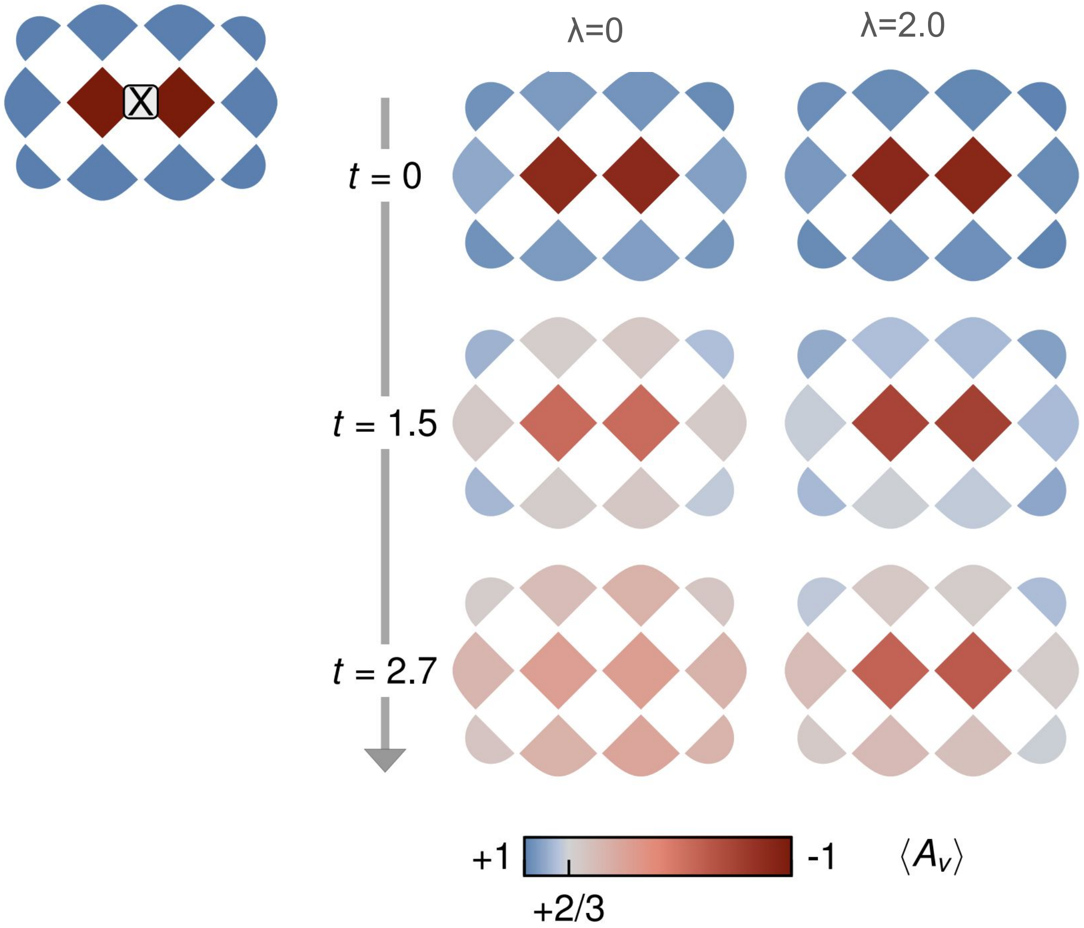
38 superconducting qubits,
85 entangling gate per layer.
1,115 total CZ gates
331 non-Clifford single qubit gates
5 microseconds total time per circuit
45 million runs



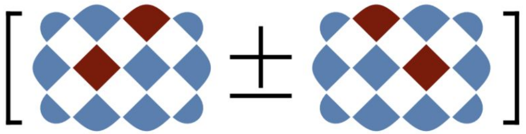
Confinement of electric excitations

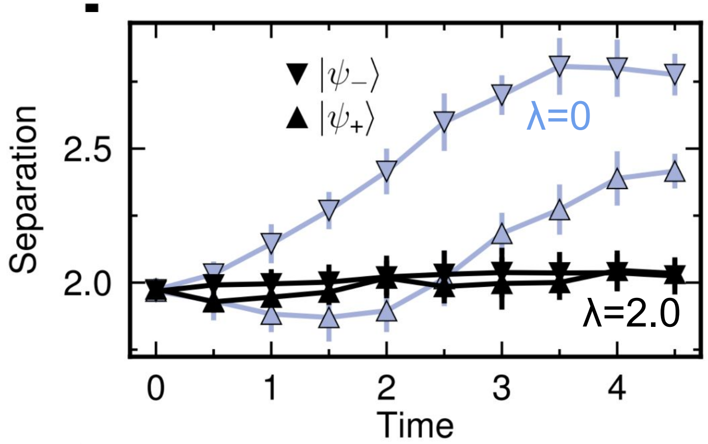


$$\mathcal{H} = -J_E \sum_v A_v - J_M \sum_p B_p - h_E \sum_{\text{links}} Z_l - \lambda \sum_{\text{links}} X_l$$



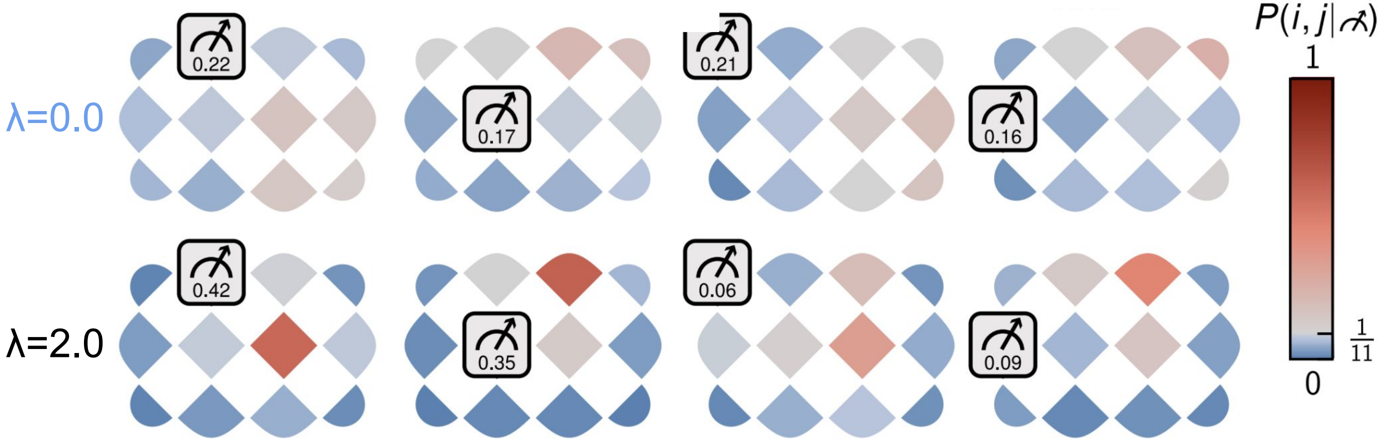
Confinement of electric excitations



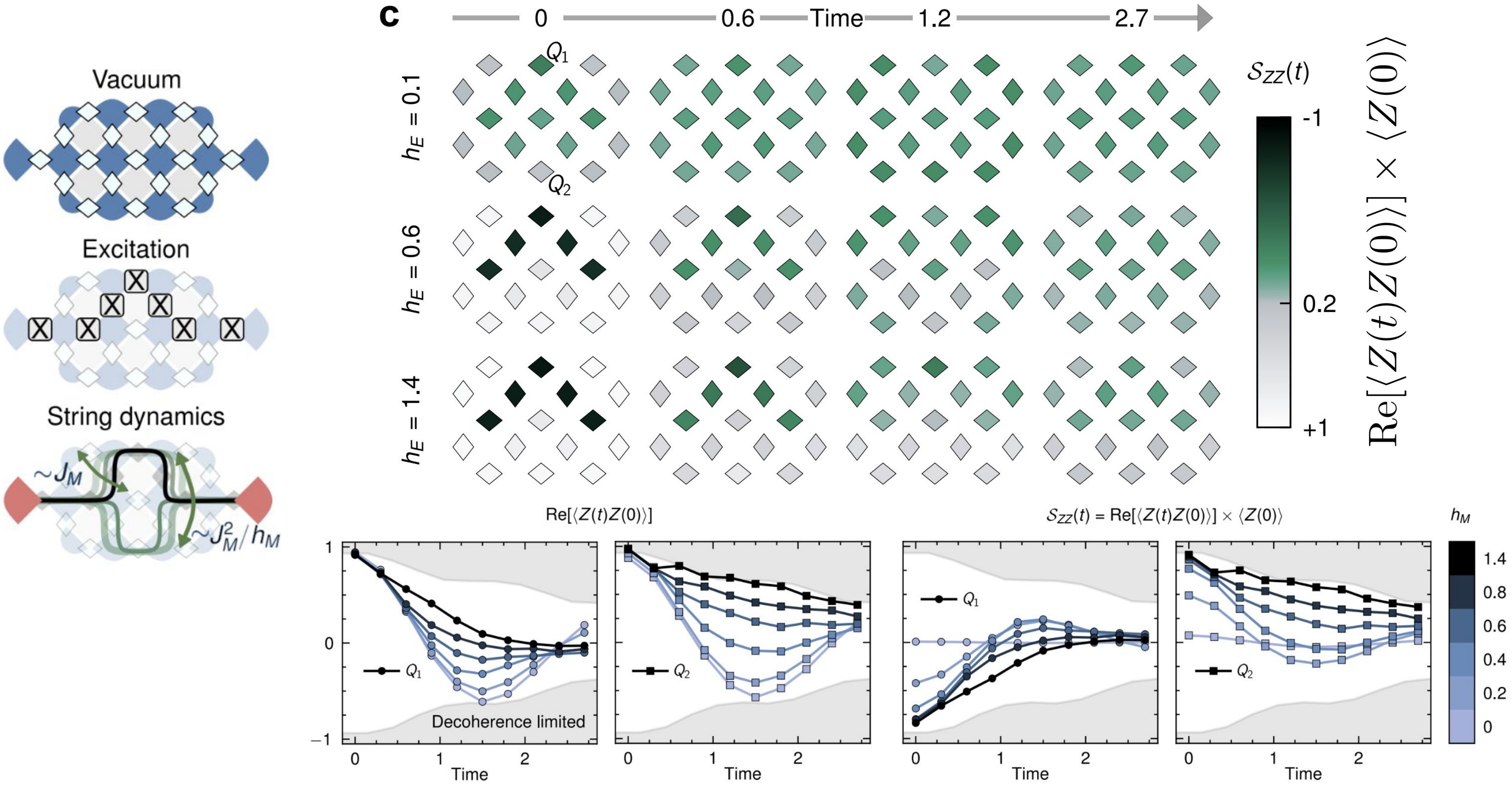


Conditional particle locations

$|\psi_-\rangle$ Initial state



Dynamics of the string connecting two fixed electric particles



Dynamics of the string connecting two fixed electric particles