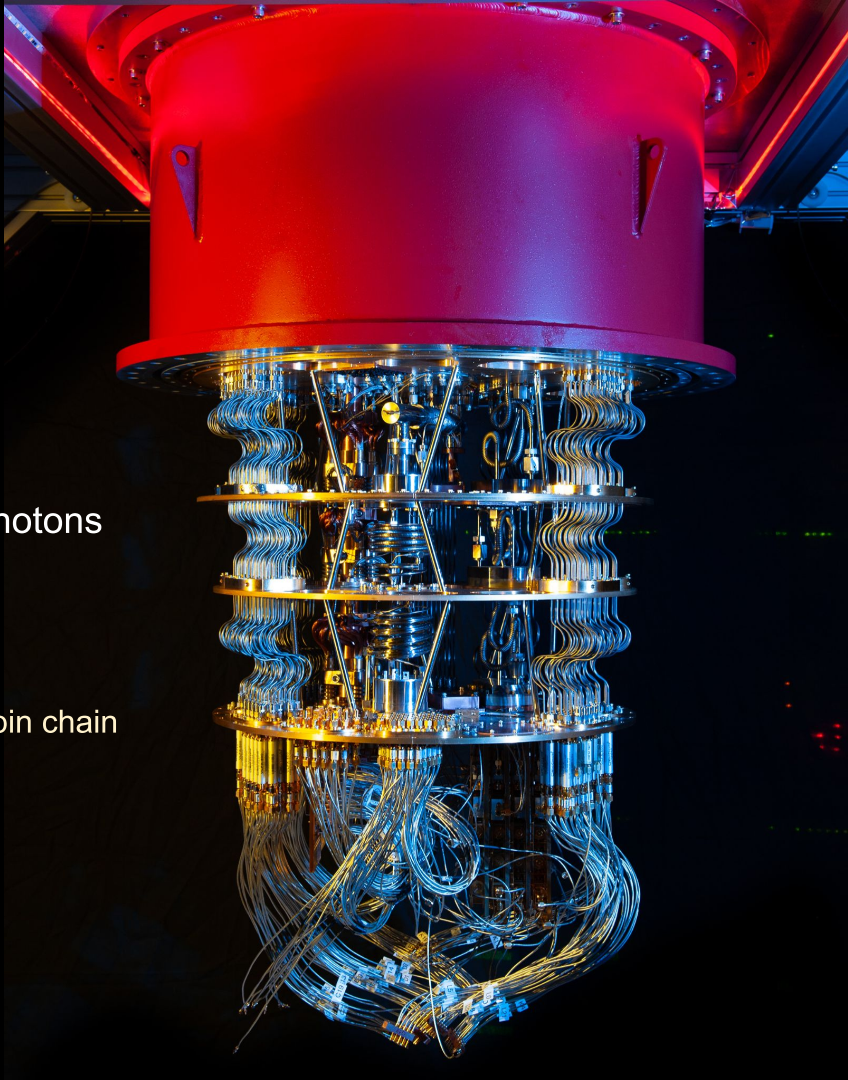
Quantum simulations with superconducting qubits

pedramr@google.com, *@PedramRoushan*

Google Quantum AI,



1. Quantum circuit model

2. Formation of robust bound states of interacting microwave photons
   Nature **612** (2022) , notes

3. Magnetization Dynamics in a 1D Heisenberg spin chain
   Dynamics of magnetization at infinite temperature in a Heisenberg spin chain (Science 384 (2024))

4. Disorder-free localization
   arXiv:2410.06557

Quantum connection , Stockholm June 2025

Quantum Dynamics through Circuit-Based Simulation on Superconducting Qubits

1. Quantum Circuit Model and Digital Quantum Simulation → Tuesday, June 17
Lec1_Digital_simulation_Spin_models

1.1,Clifford Circuits, and Stabilizer Formalism
Lecture_7_Historical_review_Quantum_Circuits_Stabilizer_formalism

1.2, Formation of Robust Bound States of Interacting Microwave Photons
Nature 612, Pages 240–245 (2022),
also Lecture_3_Integrability_and_Bound_State_of_photons

1.3, Dynamics of Magnetization at Infinite Temperature in a Heisenberg Spin Chain
Science 384, Issue 6691, Pages 48–53 (April 2024)

1.4, Observation of Disorder-Free Localization on a Quantum Processor
arXiv:2410.06557

2. Exchange statistics and dynamics of Particles → Wednesday, June 18
Lec2_Kitaev_LGT

2.1 Realizing Topologically Ordered States on a Quantum Processor
Science 374, Issue 6572, Pages 1237–1241, 2021

2.2 Non-Abelian Braiding of Graph Vertices in a Superconducting Processor
Nature 618, Pages 264–269 (2023)

2.3 Visualizing Dynamics of Charges and Strings in (2+1)D Lattice Gauge Theories
Nature, Published Online: June 4, 2025,
also Lecture_8_Lattice_Gauge_Visualization

3. Measurement-Induced Phases and Adaptive Circuits → Wednesday, June 18
Lec3_MIPT_GHZ

3.1 Measurement-induced entanglement and teleportation on a noisy quantum processor
Nature 622, pages481–486 (2023),
also Lecture_6_Measurement_Induced_Phases_and_Transitions

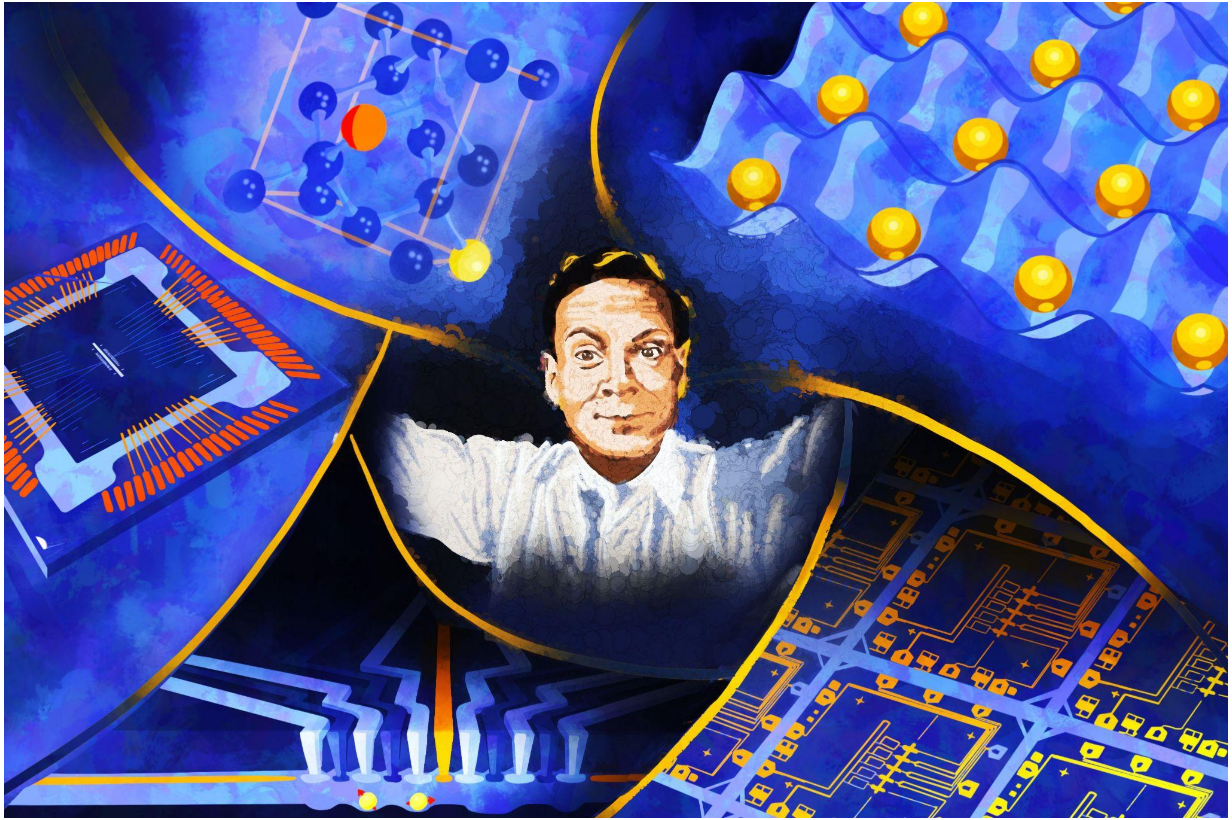
3.2 Adaptive Quantum Circuits and Break Even with GHZ state
Recent results (see slides)

Shared folder:

- Quantum Connections 2025-Roushan
- Lecture_8_Lattice_Gauge_Visualization.pdf
- Lecture_7_Historical_review_Quantum_Circuits_Stabilizer_formalis...
- Lecture_6_Measurement_Induced_Phases_and_Transitions.pdf
- Lecture_3_Integrability_and_Bound_State_of_photons.pdf
- Lec3_MIPT_GHZ
- Lec2_Kitaev_LGT
- Lec1_Digital_simulation_Spin_models

Several experimental works → *engage*
A few ~ theory tricks → *engage*

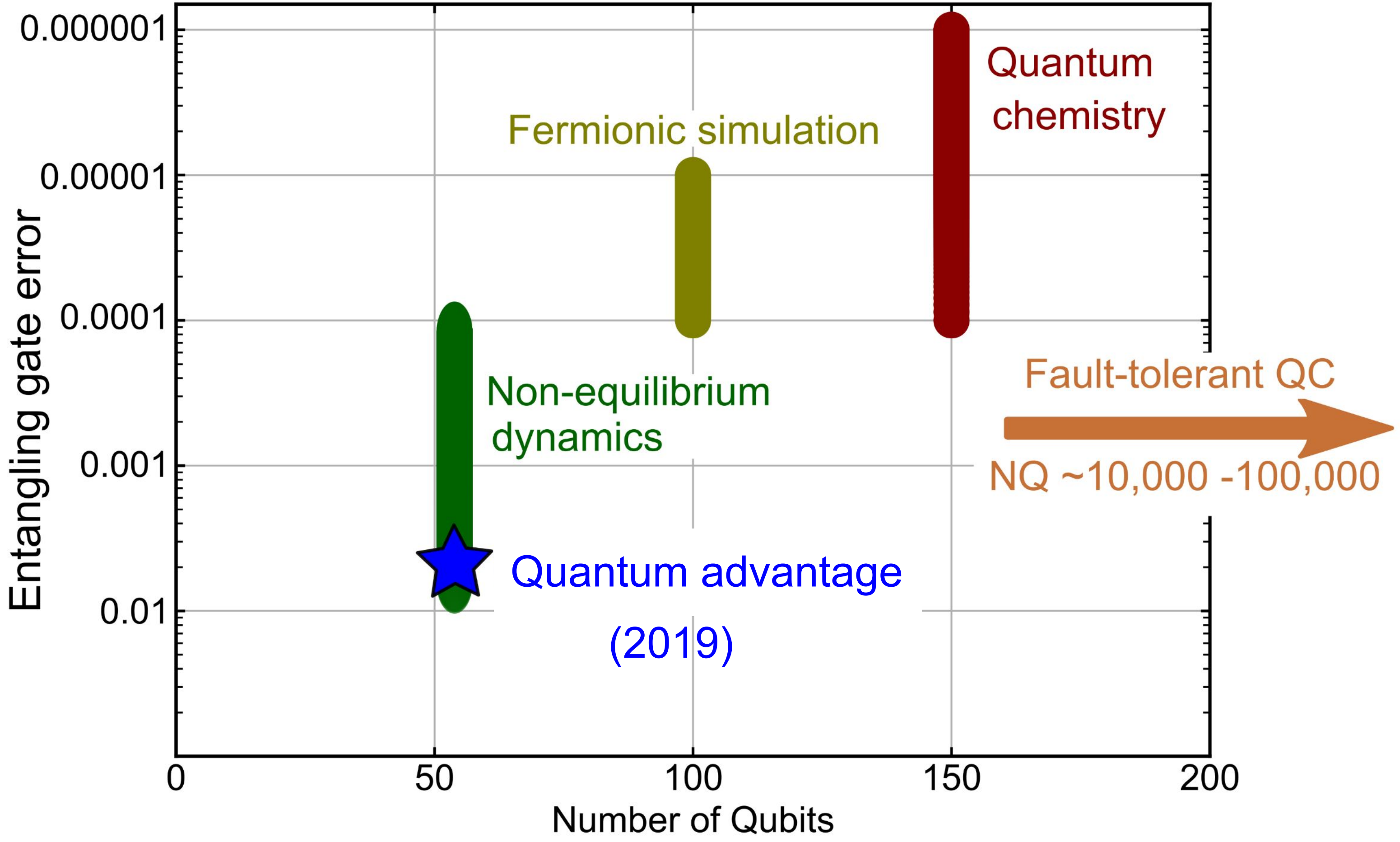
Study many-body physics with quantum processors



Feynman and others → certain quantum phenomena could not be efficiently simulated by a classical Turing machine.

“Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical..”

Two Paradigms: Error correction and NISQ

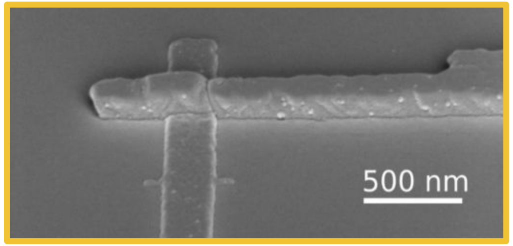


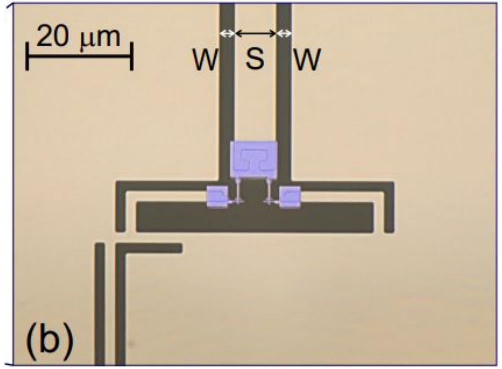
Discoverino

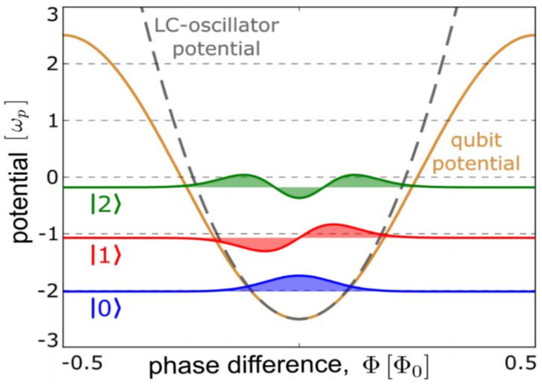
Discoveries made by NISQ processors, which could also been made in theory or by using classical computing resources.

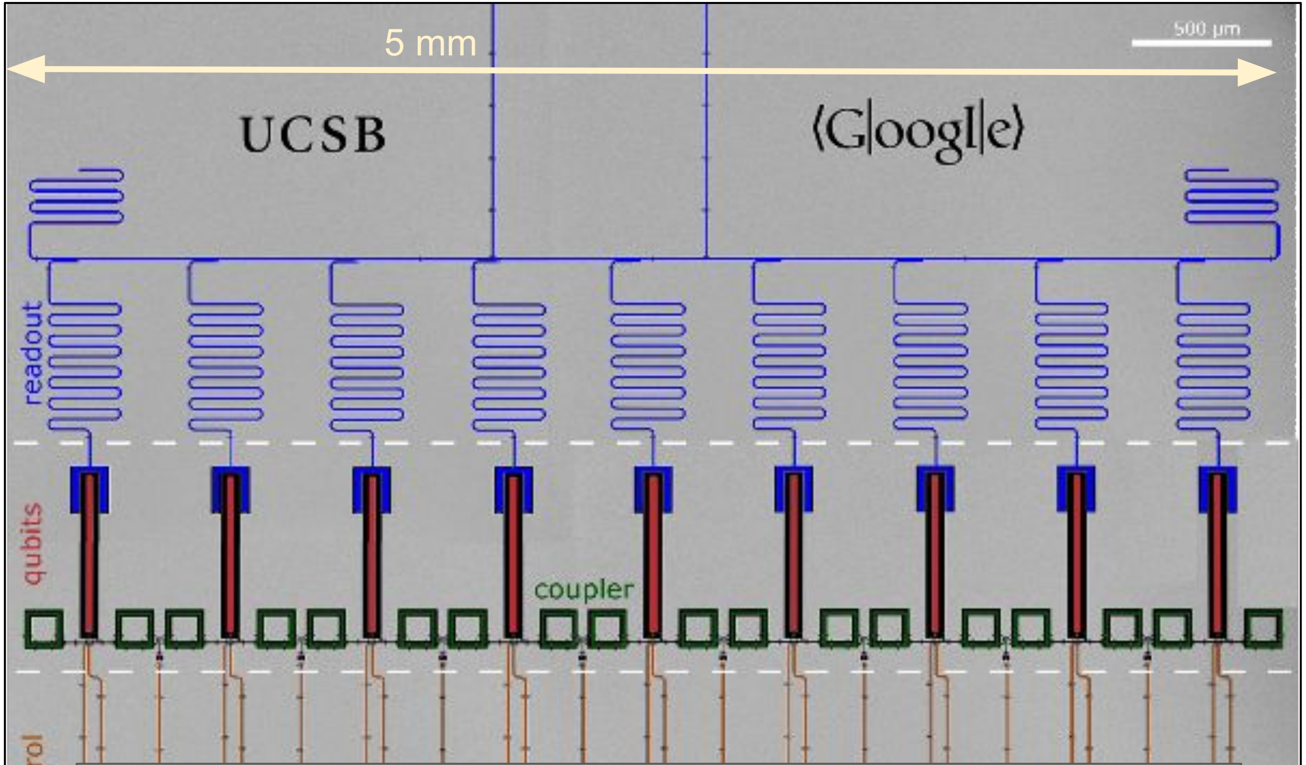
PR and L. Martin, under review (2025)

Array of coupled non-linear resonators



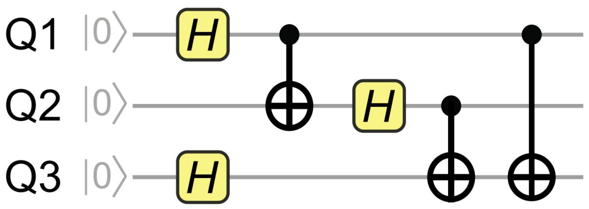






$$\hat{H}_{BH} = \sum_i h_i a_i a_i^\dagger + \sum_{<i,j>} J_{i,j}(a_i a_j^\dagger + a_i^\dagger a_j) + \frac{U}{2}\sum_i a_i a_i^\dagger (a_i a_i^\dagger - 1) + h.o.t$$

$h_i \in$ [-500, 500]MHz , $J_{i,j} \in$ [-50, +2] MHz, $U \sim$ -200MHz



Quantum Circuit Model

Clifford gates :

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

A unitary U *stabilizes* a pure state $|\psi\rangle$ if $U|\psi\rangle = |\psi\rangle$, i.e., if $|\psi\rangle$ is an eigenvector of U with eigenvalue $+1$

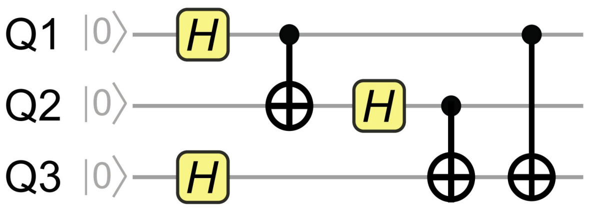
The stabilizer group of $|\psi\rangle$, Stab $(|\psi\rangle)$, is the set of all unitaries that stabilize a state $|\psi\rangle$.

Exercise 1. Show that $X_1X_2X_3$, Z_1Z_2, and Z_2Z_3 are three of the stabilizers of $|\psi\rangle = |000\rangle + |111\rangle$. The Stab$(|\psi\rangle)$ has 8 elements. These three stabilizers act as their generators. Find all 8 stabilizers of $|\psi\rangle$.

The Clifford group contains operators that conjugate Paulis into Paulis; its generating set consists of $\{H, S, CNOT\}$.

A stabilizer circuit consists solely of elements from the Clifford group.

Gottesman-Knill Theorem. Let $|\psi\rangle$ be a n-qubit stabilizer state for which the intersection of its stabilizer group with $\mathcal{P}_n$ contains 2^n elements, i.e. $|\text{Stab}(|\psi\rangle) \cap \mathcal{P}_n| = 2^n$, then $|\psi\rangle$ is reachable from the $|0\rangle^{\otimes n}$ state using only the clifford gates.



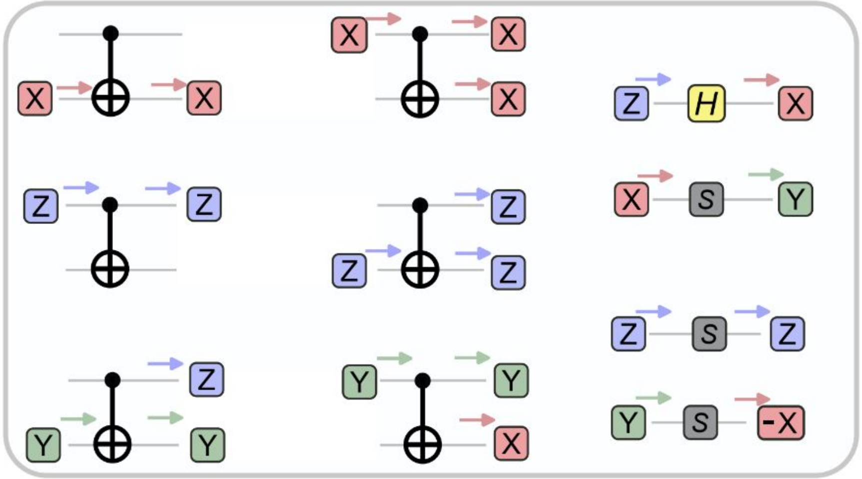
Quantum Circuit Model

Clifford gates :

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}, \quad CNOT = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}, \quad S = \begin{bmatrix}1&0\\0&i\end{bmatrix}$$

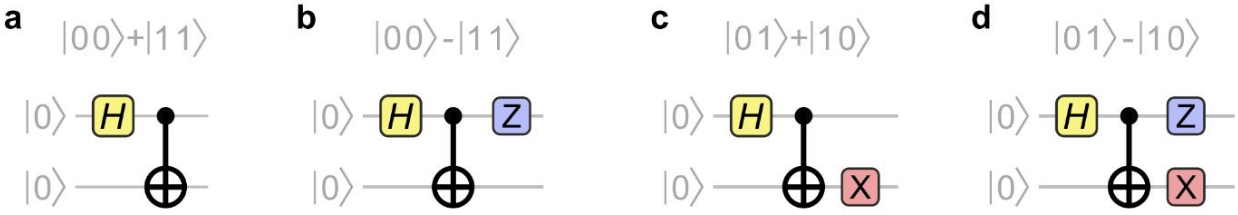
- Find n independent stabilizer for initial state
- Evolve each stabilizers to final stabilizers
- Find a reference state
- $|\psi_f\rangle = \prod^{n} \frac{I+S_j}{2}|\psi_{ref}\rangle$

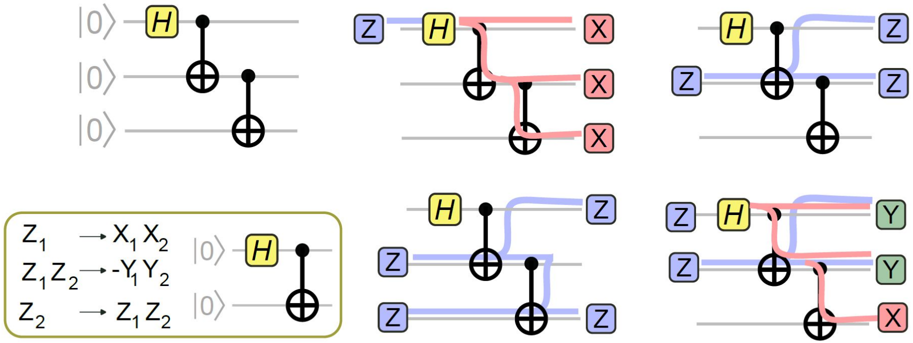
ZX calculus of stabilizers



Quantum Circuit Model

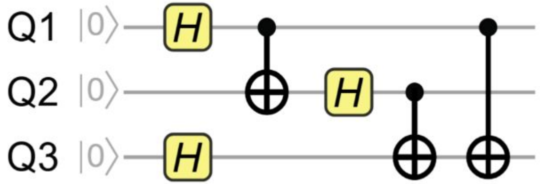
Exercise 4. Show that each circuit below gives the associated Bell pair written on top of it.



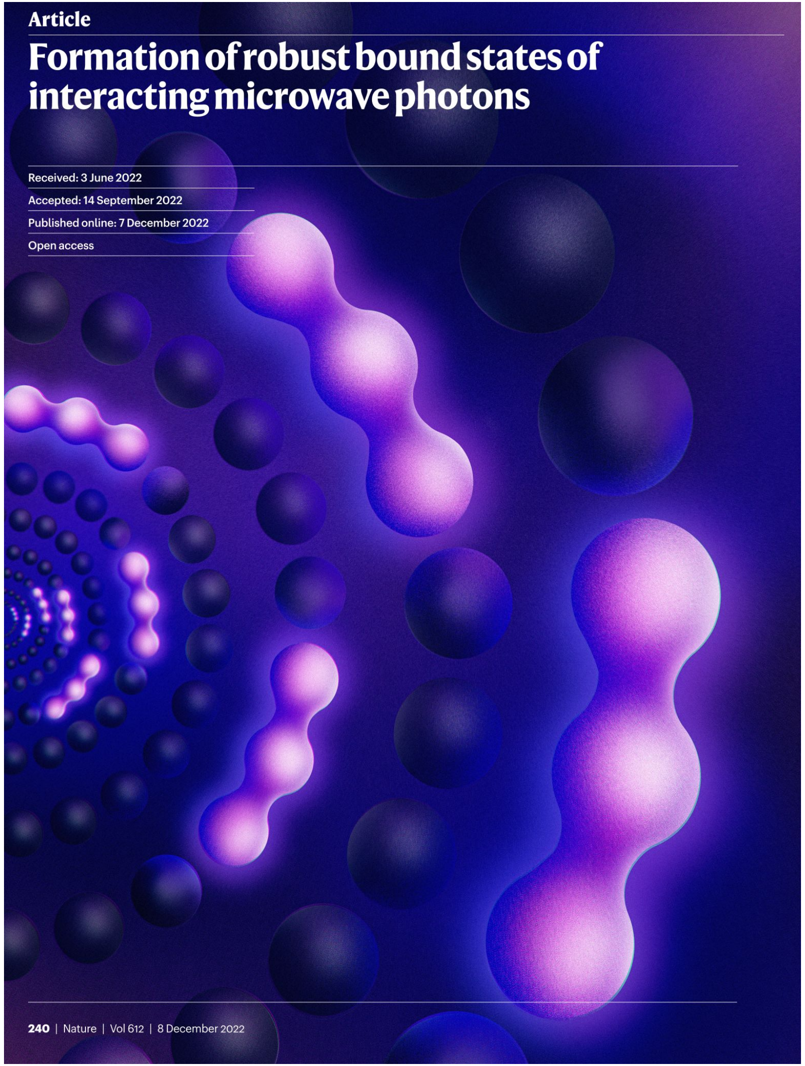


Quantum Circuit Model

Exercise 5. Given the initial state $|000\rangle$, find a generative of the final stabilizers of the circuit below.



Exercise 6. Given the initial state $|000\rangle$, find a generative of the final stabilizers of the circuits below. What could good choice of $|\psi_{ref}\rangle$ in each case.



Quantum AI


Alexis Morvan


Charles Neill


Trond Andersen


Xiao Mi


Lev Ioffe


Andre Petukhov


Kostyantyn Kechedzhi


Igor Aleiner

For a summary: Tomaž Prosen, News and Views, Nature 612, December 2022

Article

Formation of robust bound states of interacting microwave photons

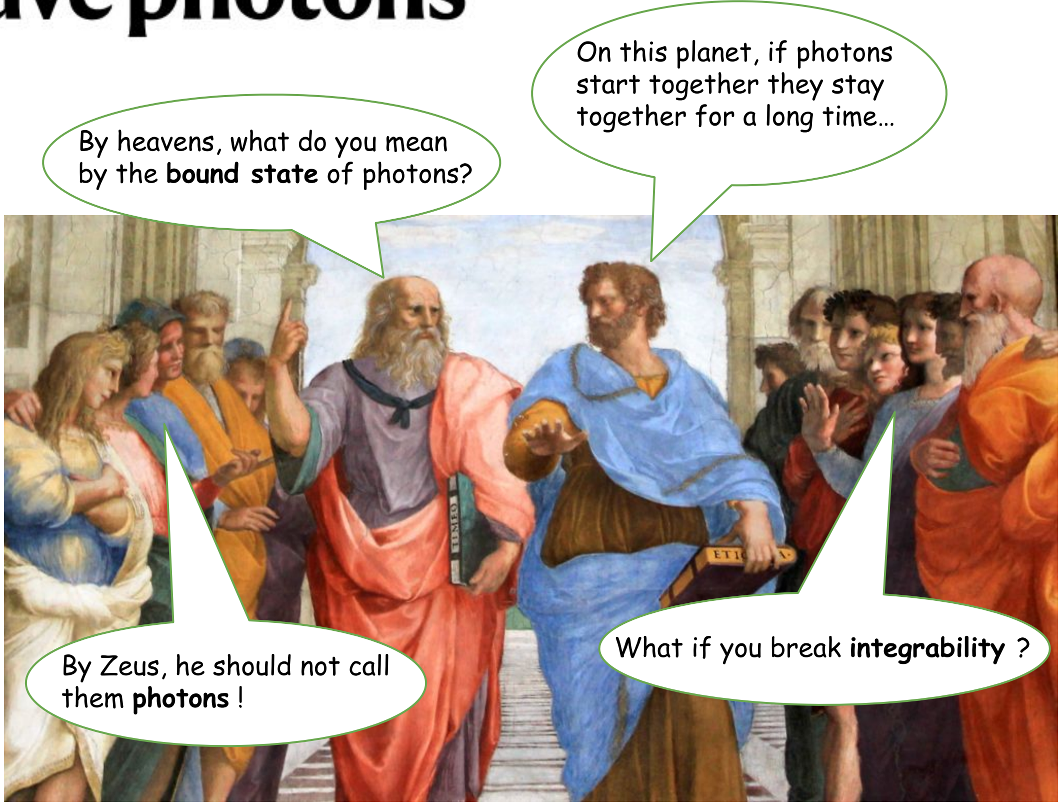
https://doi.org/10.1038/s41586-022-05

Received: 3 June 2022

Accepted: 14 September 2022

Published online: 7 December 2022

Open access

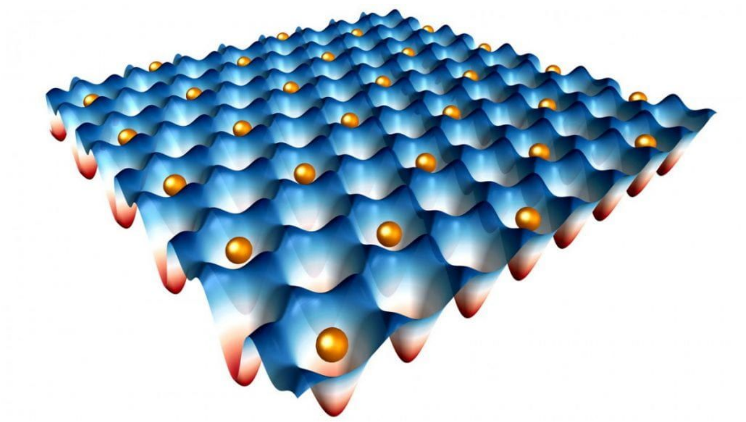


* "photons" → They are excitations of the electromagnetic field that are accompanied by waves of charge and current on the surface of the metal. This hybrid nature would make it more appropriate to call them microwave plasmons.

2-Qubit fSim gate tp study XXZ model

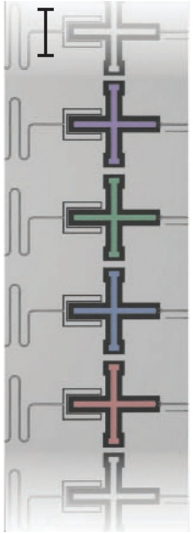
Canonical 1D interacting XXZ Hamiltonian model:

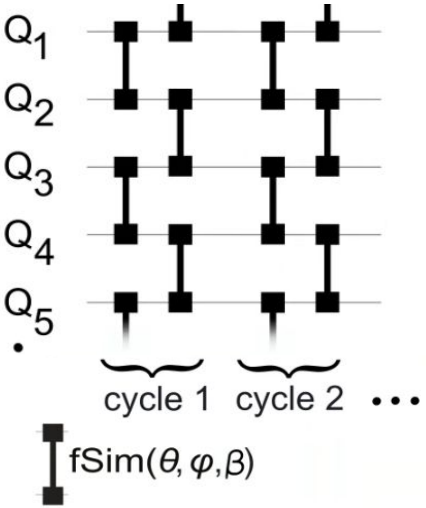
$$\mathcal{H} = \sum_i \boxed{(X_i X_{i+1} + Y_i Y_{i+1})} + \boxed{\Delta Z_i Z_{i+1}}$$



XXZ Floquet :

$$\mathrm{fSim}(\theta,\phi,\beta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & ie^{i\beta}\sin\theta & 0 \\ 0 & ie^{-i\beta}\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & \boxed{e^{i\phi}} \end{pmatrix}$$





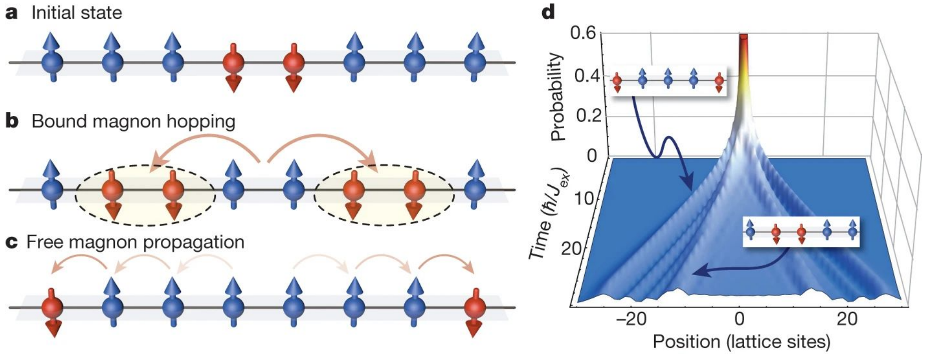
$$\hat{U}_F = \prod_{\text{even bonds}} \mathrm{fSim}(\theta,\phi,\beta) \prod_{\text{odd bonds}} \mathrm{fSim}(\theta,\phi,\beta)$$

2-Qubit fSim gate tp study XXZ model

Canonical 1D interacting XXZ Hamiltonian model:

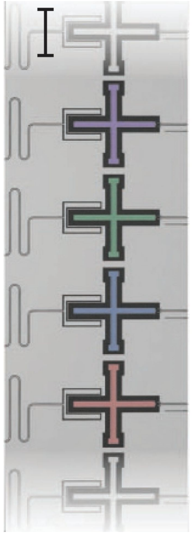
$$\mathcal{H} = \sum_i \boxed{(X_i X_{i+1} + Y_i Y_{i+1})} + \boxed{\Delta Z_i Z_{i+1}}$$

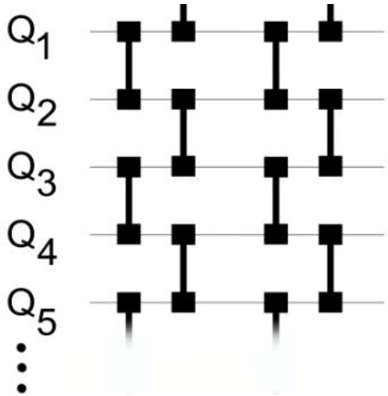
- Analytical solution and Bound States
  H. Bethe, **Zeitschrift für Physik 71**, 205 (1931)
- Observation of Bound States in XXZ
  Ganahl, Rabel, Essler, Evertz **PRL 108,** 077206 (2012)
  T. Fukuhara *et al.* **Nature 502,** 76-79 (2013)



XXZ Floquet :

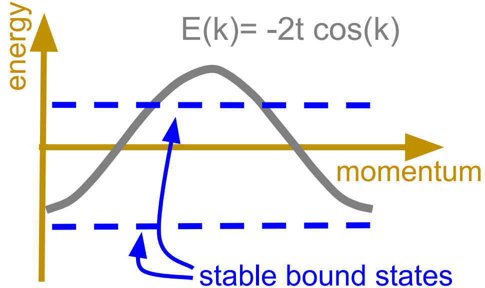
$$\mathrm{fSim}(\theta, \phi, \beta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & ie^{i\beta}\sin\theta & 0 \\ 0 & ie^{-i\beta}\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & \boxed{e^{i\phi}} \end{pmatrix}$$

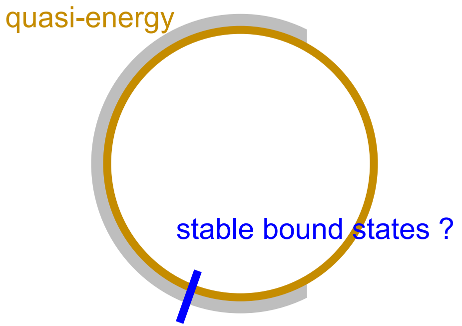




- Floquet XXZ is integrable
  M. Ljubotina *et al.* **PRL 122**, 150605 (2019)
- Analytical solution and Bound States
  I.L. Aleiner Annals of Physics **433**, 168593 (2021)

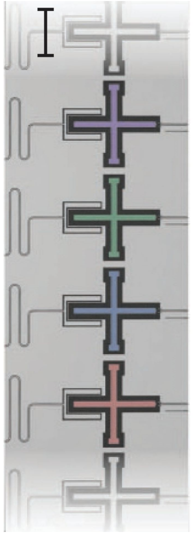
2-Qubit fSim gate tp study XXZ model

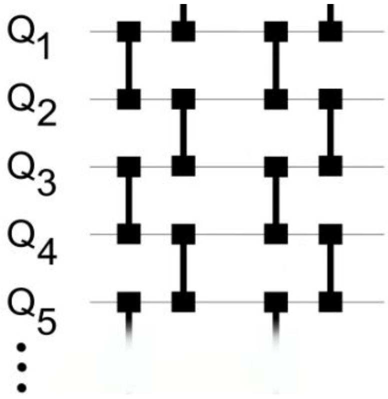




XXZ Floquet :

$$\mathrm{fSim}(\theta,\phi,\beta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & ie^{i\beta}\sin\theta & 0 \\ 0 & ie^{-i\beta}\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{pmatrix}$$





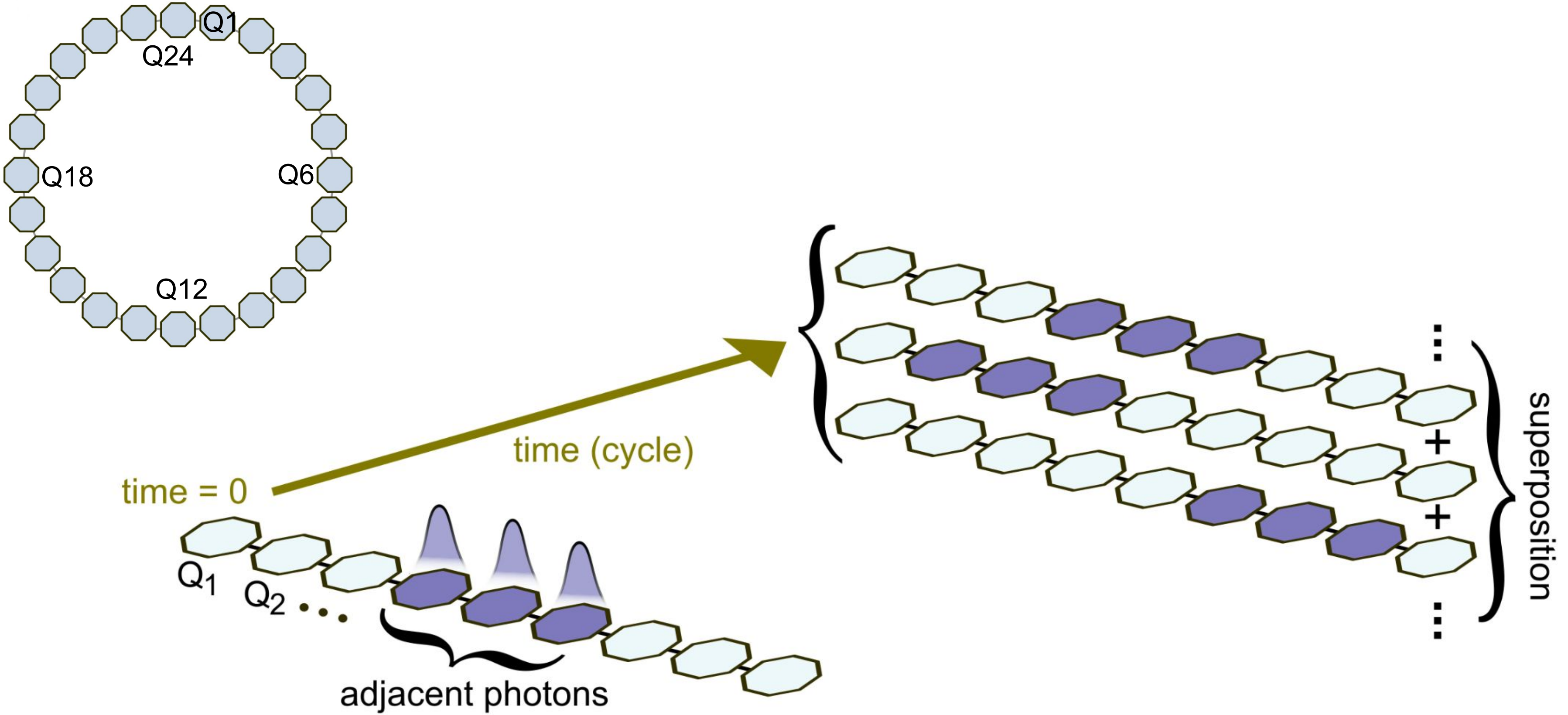
- Floquet XXZ is integrable

  M. Ljubotina *et al.* **PRL 122**, 150605 (2019)

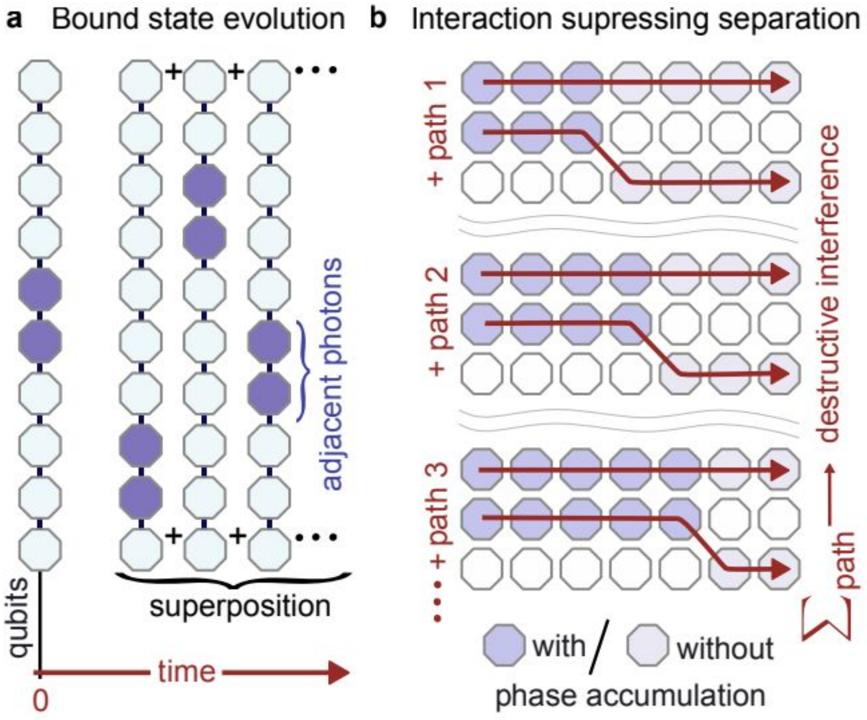
- Analytical solution and Bound States

  I.L. Aleiner Annals of Physics **433**, 168593 (2021)

Bitstring measurement → Boundstate of photons



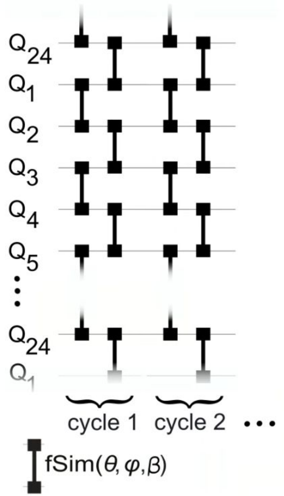
2-Qubit fSim gate tp study XXZ model



Circuit model: Floquet dynamics

$$\mathrm{fSim}(\theta,\phi,\beta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & ie^{i\beta}\sin\theta & 0 \\ 0 & ie^{-i\beta}\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{pmatrix}$$

kinetic (hopping)
interaction



$$\hat{U}_F = \prod_{\text{even bonds}} \mathrm{fSim}(\theta,\phi,\beta) \prod_{\text{odd bonds}} \mathrm{fSim}(\theta,\phi,\beta)$$

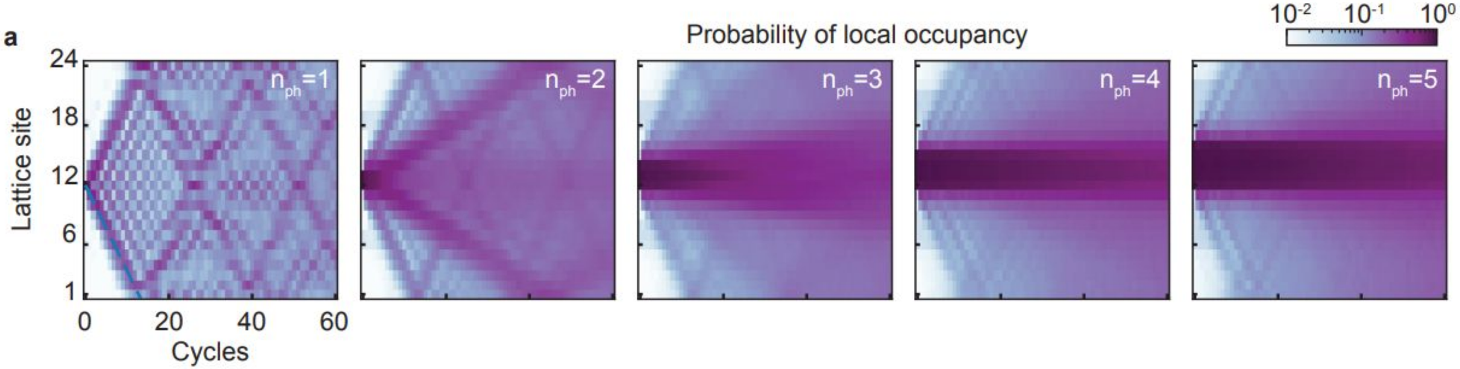
- Floquet XXZ is integrable

  M. Ljubotina *et al.* **PRL 122**, 150605 (2019)

- Analytical solution and Bound States

  I.L. Aleiner Annals of Physics **433**, 168593 (2021)

Bitstring measurement → Boundstate of photons



Band structure: few-body spectroscopy method

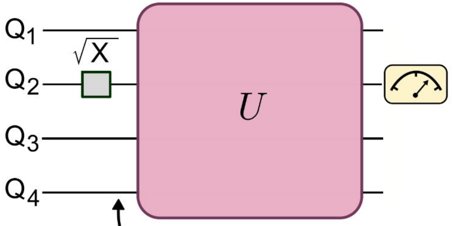
$$\hat{\mathcal{H}}\,|\varphi_\mathrm{n}\rangle = \omega_\mathrm{n}\,|\varphi_\mathrm{n}\rangle$$

Consider an initial state $|\psi_0\rangle$ and its evolution $|\psi_t\rangle$

$$|\psi_0\rangle = \sum_n c_n\,|\varphi_\mathrm{n}\rangle \longrightarrow |\psi_t\rangle = e^{-i\hat{\mathcal{H}}t}\,|\psi_0\rangle = \sum_n c_n\,e^{-i\omega_n t}\,|\varphi_\mathrm{n}\rangle$$

$$\boxed{\langle\psi_0|\,\psi_t\rangle = \sum_n c_n\,\overline{c_n}\,e^{-i\omega_n t}}$$

Green function



$$|\psi_0\rangle = |0000\rangle + |0100\rangle$$

$$\sigma_2^- = \hat{X}_2 + i\hat{Y}_2 = |0000\rangle\langle 0100| + |....\rangle\langle\text{more than single excitation}|$$

Band structure: few-body spectroscopy method

Consider a Hamiltonian $\hat{\mathcal{H}}$ with eigenenergies ω_n and eigenstates $|\varphi_n\rangle$

$$\hat{\mathcal{H}}\,|\varphi_n\rangle = \omega_n\,|\varphi_n\rangle\,. \tag{16}$$

We seek a dynamical approach to learn the spectrum of $\mathcal{H}$. Consider an initial state $|\psi_0\rangle$ and its evolution $|\psi_t\rangle$

$$|\psi_0\rangle = \sum_n c_n\,|\varphi_n\rangle\,, \qquad |\psi_t\rangle = e^{-i\hat{\mathcal{H}}t}\,|\psi_0\rangle = \sum_n c_n\,e^{-i\omega_n t}\,|\varphi_n\rangle\,, \tag{17}$$

where the complex coefficient $c_n = \langle\varphi_n|\,\psi_0\rangle$ is the overlap of $|\psi_0\rangle$ with $|\varphi_n\rangle$. The desired spectrum of $\hat{\mathcal{H}}$ can be extracted (e.g. by Fourier transform) from the overlap of $|\psi_0\rangle$ and $|\psi_t\rangle$

$$\langle\psi_0|\,\psi_t\rangle = \sum_n |c_n|^2\,e^{-i\omega_n t}\ . \tag{18}$$

Consider an initial state $|\Psi_0\rangle$, which evolves under Hamiltonian H to $|\Psi_t\rangle = e^{-iHt}|\Psi_0\rangle$, and a Hermitian operator O at site m that we like to measure it at time t,

$$\langle\Psi_t|\hat{O}_m|\Psi_t\rangle = \langle\Psi_0|\,e^{iHt}\,\hat{O}_m\,e^{-iHt}\,|\Psi_0\rangle = \langle\Psi_0|\,e^{iHt}\,\hat{O}_m\,|\Psi_t\rangle \tag{19}$$

A single excitation spectroscopy method

$$|\Psi_0\rangle = |000...\rangle + |100...\rangle = |vac\rangle + |\psi_0\rangle, \quad \rightarrow \quad |\Psi_t\rangle = e^{-iHt}|\Psi_0\rangle = |vac\rangle + e^{-iHt}|\psi_0\rangle = |vac\rangle + |\psi_t\rangle \tag{20}$$

Then

$$C_x(m,t) = \langle\Psi_0|\, e^{iHt}\, \hat{O}_m\, e^{-iHt}\, |\Psi_0\rangle = \langle vac + \psi_0|\, e^{iHt}\, \hat{O}_m\, e^{-iHt}\, |vac + \psi_0\rangle \tag{21}$$

$$= \langle vac|\, e^{iHt}\, \hat{O}_m\, e^{-iHt}\, |\psi_0\rangle + \langle\psi_0|\, e^{iHt}\, \hat{O}_m\, e^{-iHt}\, |vac\rangle \tag{22}$$

$$= \langle vac|\hat{O}_m\, e^{-iHt}\, |\psi_0\rangle + \langle\psi_0|\, e^{iHt}\, \hat{O}_m|vac\rangle, \tag{23}$$

since from 4 possible terms only two are non-zero, since $\hat{O}_m$ takes from one sector to the other, e.g $O_m = X_m$

$$= \langle\psi_0|\, e^{-iHt}\, |\psi_0\rangle + \langle\psi_0|\, e^{iHt}\, |\psi_0\rangle = \langle\psi_0|\psi_t\rangle + \langle\psi_t|\psi_0\rangle. \tag{24}$$

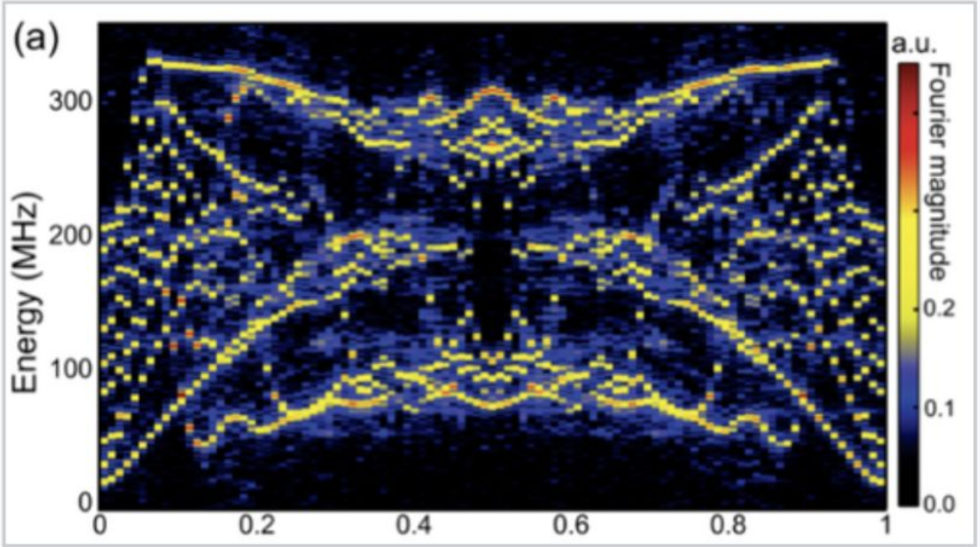
If we set $O_m = Y_m$, then

$$C_y(m,t) = -i\langle\psi_0|\psi_t\rangle + i\langle\psi_t|\psi_0\rangle. \tag{25}$$

and thus

$$C_x(m,t) + iC_y(m,t) = 2\langle\psi_0|\psi_t\rangle. \tag{26}$$

Hofstadter's butterfly

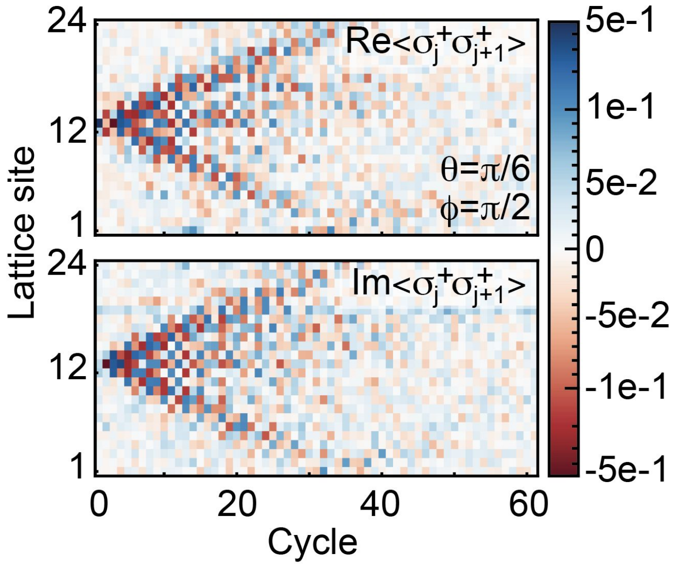


A simulation of electrons via superconducting qubits yields Hofstadter's butterfly

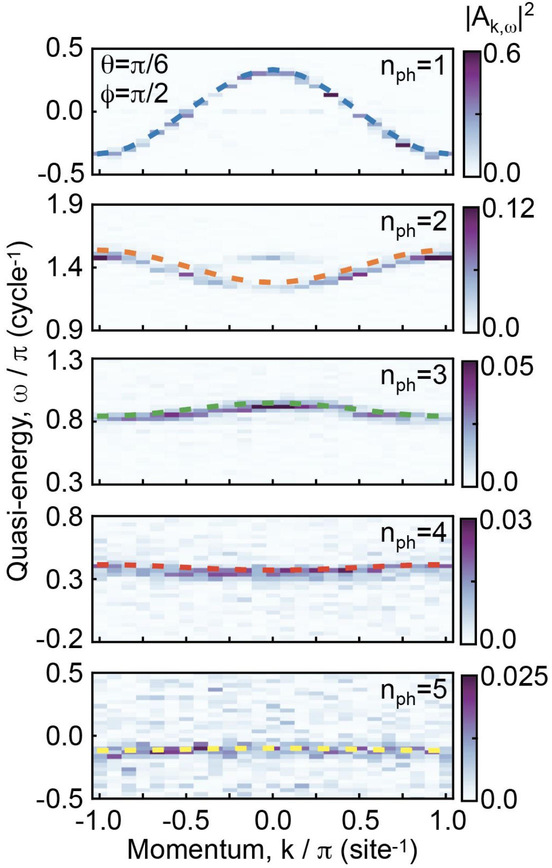
Few-body spectroscopy technique → Measuring band structure

Constructing non-Hermitian operators

$$\langle X_j X_{j+1}\rangle - \langle Y_j Y_{j+1}\rangle + i\langle X_j Y_{j+1}\rangle + i\langle Y_j X_{j+1}\rangle \text{ for } n_{\mathrm{ph}} = 2$$



Convert to energy and momentum via 2D Fourier transform



∴ Bound photons are quasiparticles with well-defined momentum, energy, and charge

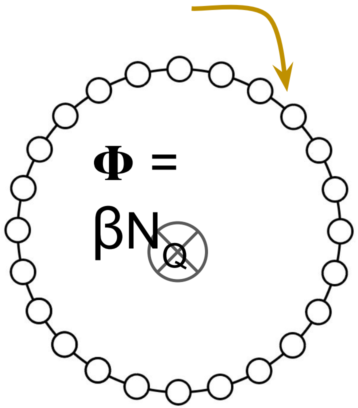
Analytical results:

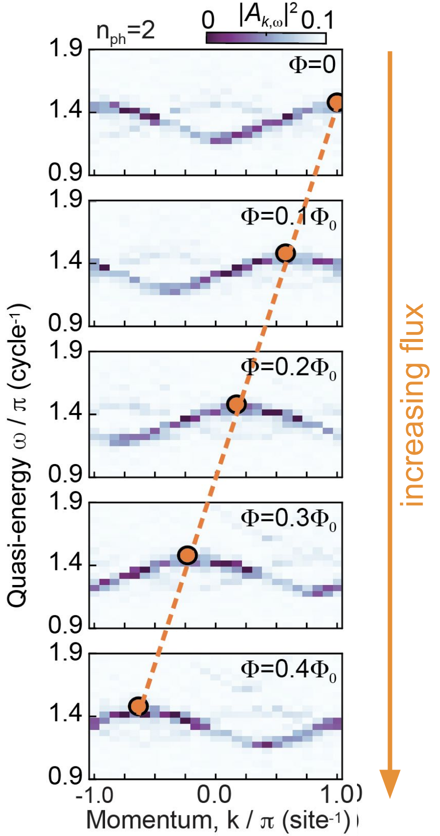
$$\cos(E(k)-\chi) = \cos^2(\alpha) - \sin^2(\alpha)\cos(k)$$

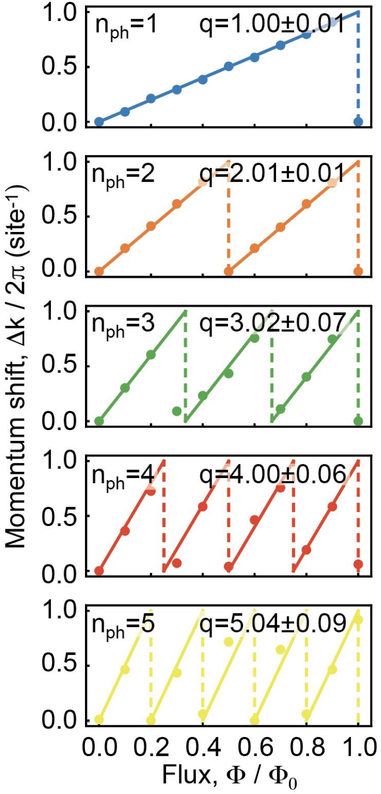
Extraction of the bound state pseudo-charge

Include SQ rotation before and after fSim:

$$\mathrm{fSim}(\theta,\phi,\beta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & ie^{i\beta}\sin\theta & 0 \\ 0 & ie^{-i\beta}\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{pmatrix}$$



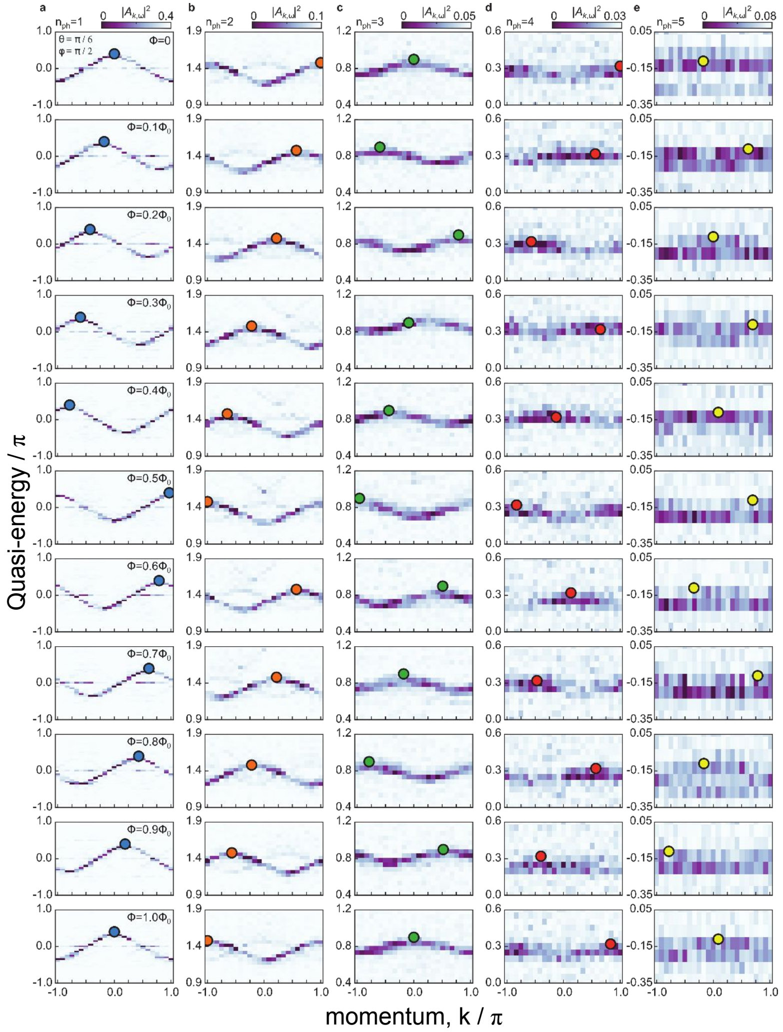


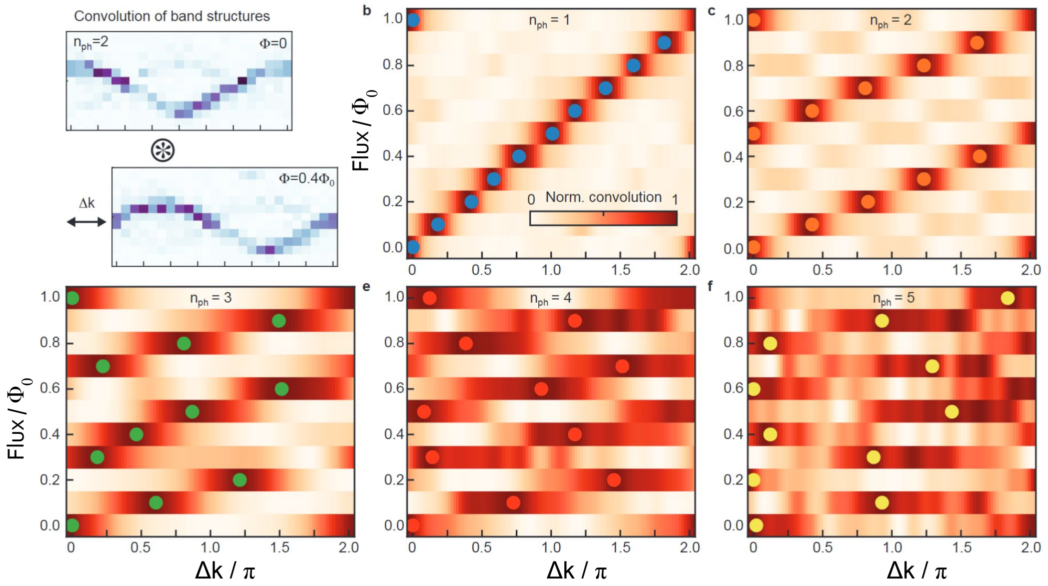


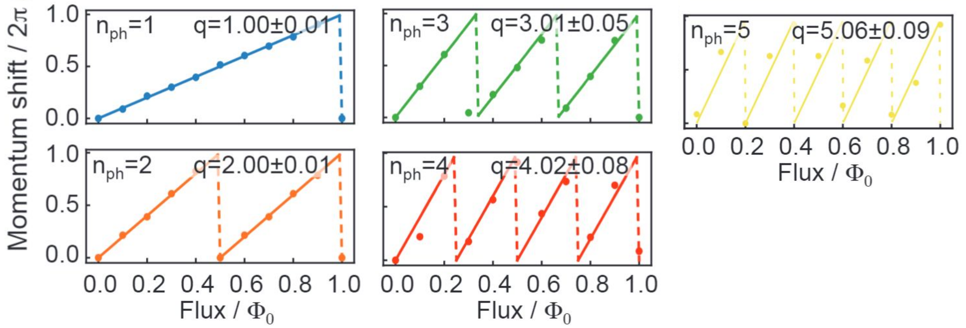
-Momentum shift!

$\exp(-i*k*j)|\, j, n_{ph}) \rightarrow \exp(i\beta n_{ph}\, j)*\exp(-i*k*j)\,|\, j, n_{ph})$

Measuring “charge”

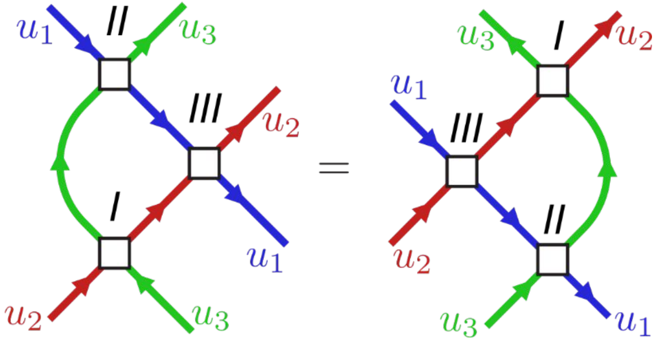






Interaction and integrability

Yang-Baxter relation:

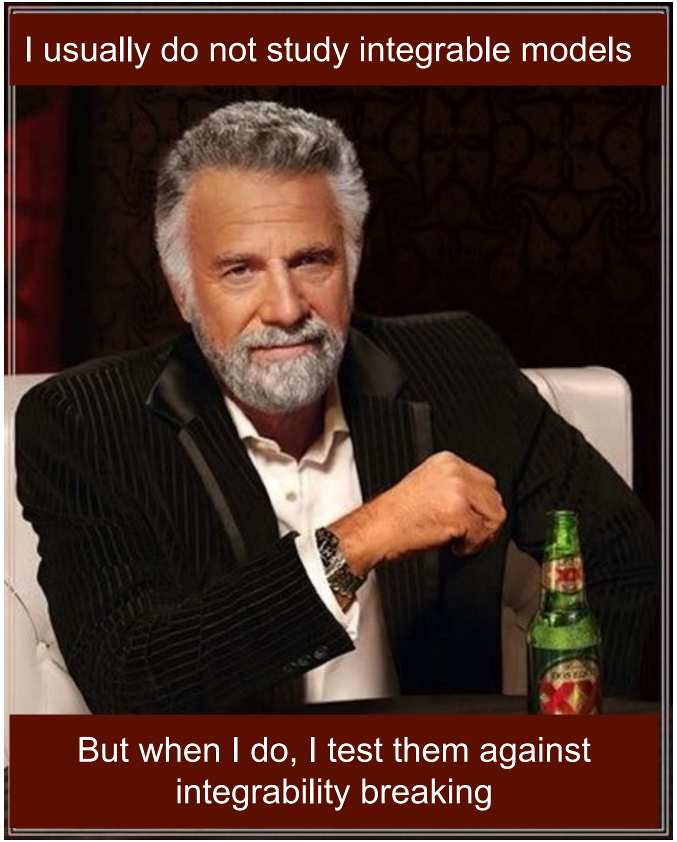


integrable : scattering is factorizable to

$2 \to$ 2 scattering processes

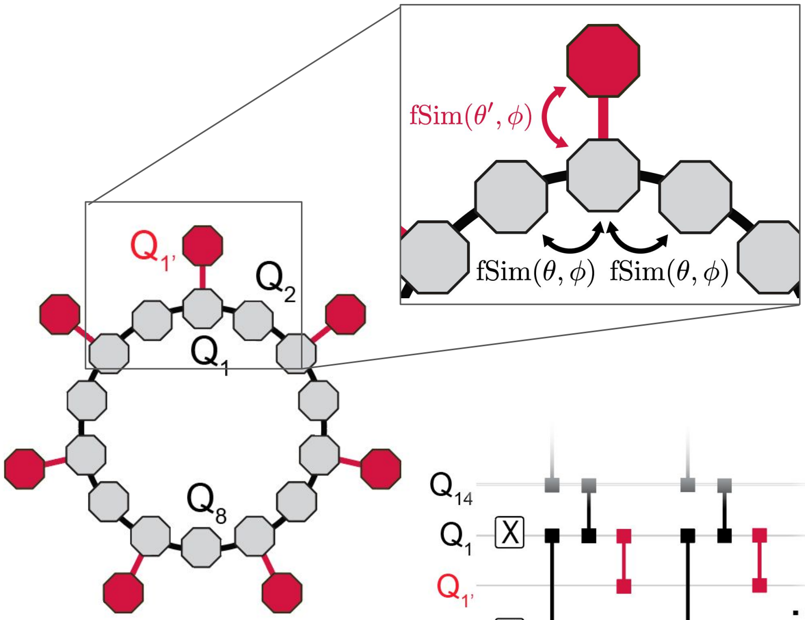
scattering order:

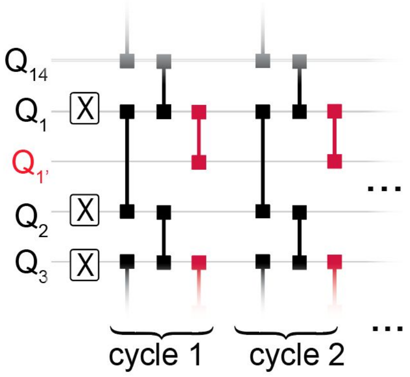
$I \to II \to III$ $\quad III \to II \to I$



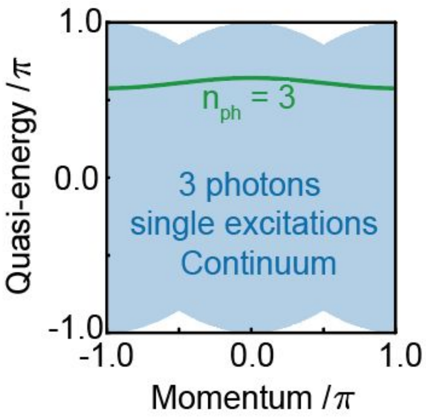
C. W. Hsu *et al.*, Nature Reviews Materials 1, 16048 (2016).
S. Groha and F. H. L. Essler, Journal of Physics A, 50, 334002 (2017).

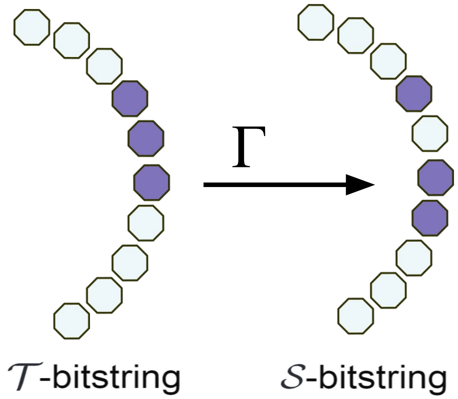
Breaking integrability continuously





$$\frac{\binom{21}{3}}{\binom{14}{3}} = 3.65$$

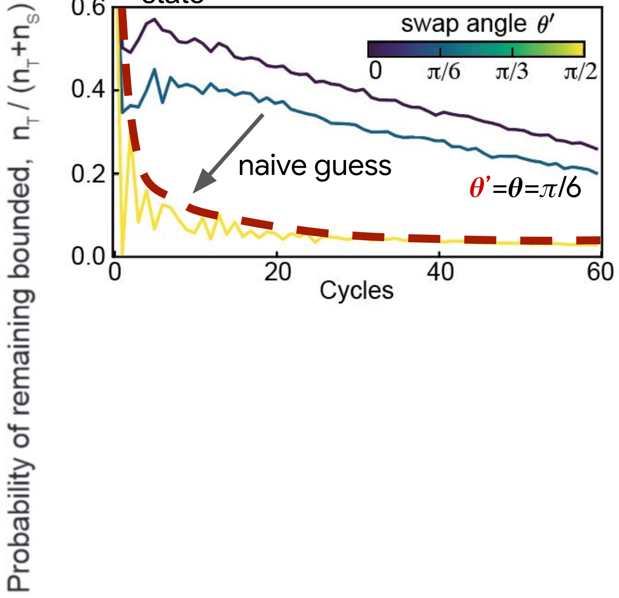


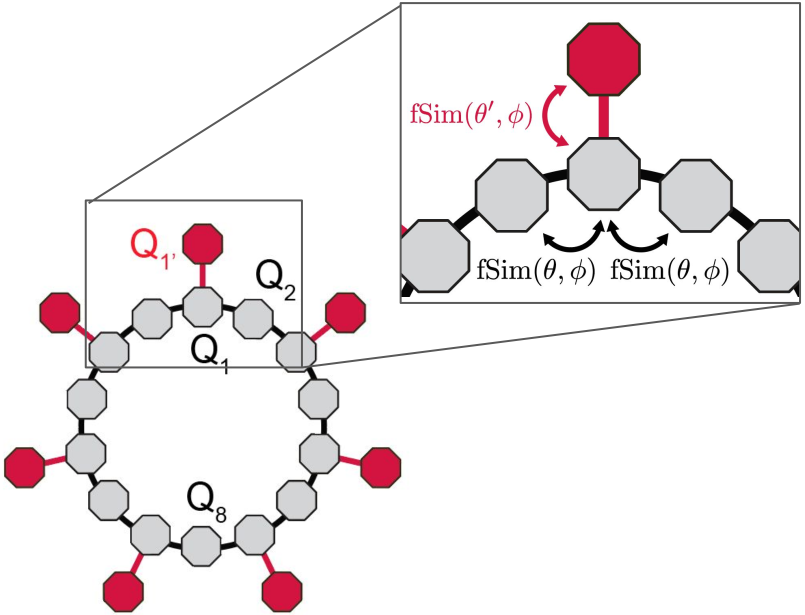


$$\boxed{\Gamma = \frac{2\pi}{\hbar} \left| \langle BS | H' | Cont. \rangle \right|^2 \rho(E_f)}$$

Breaking integrability continuously

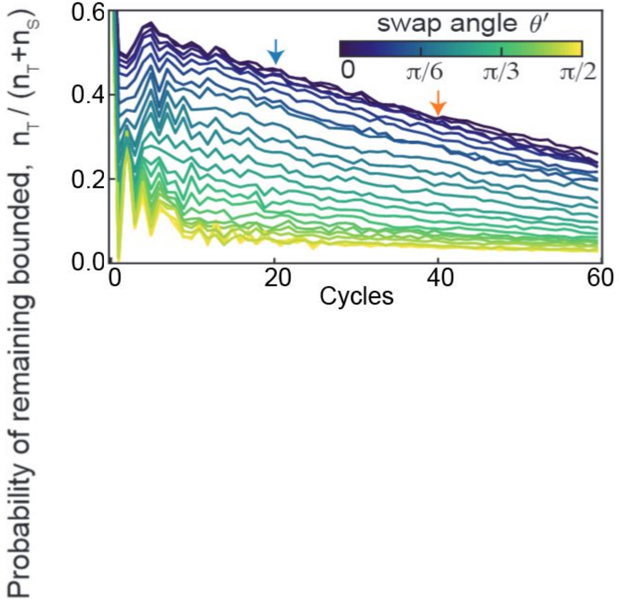
Decay of a 3-photon bound state

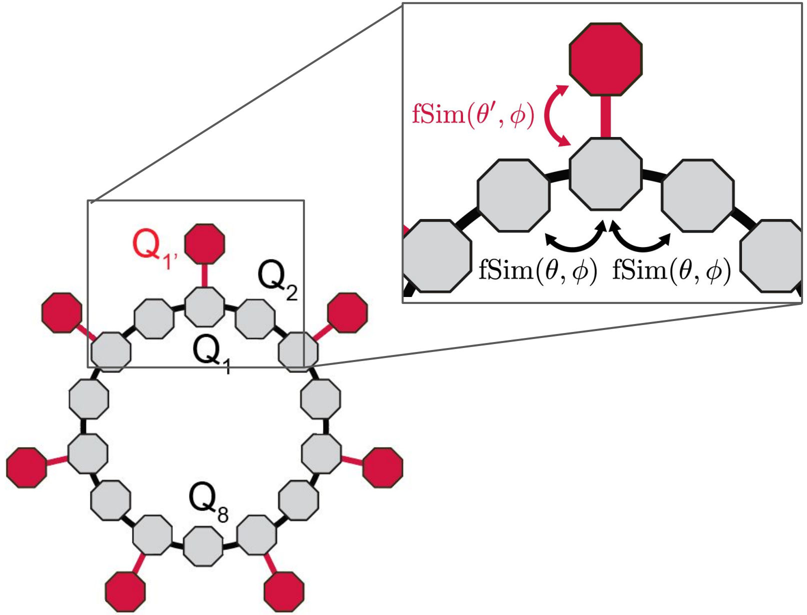




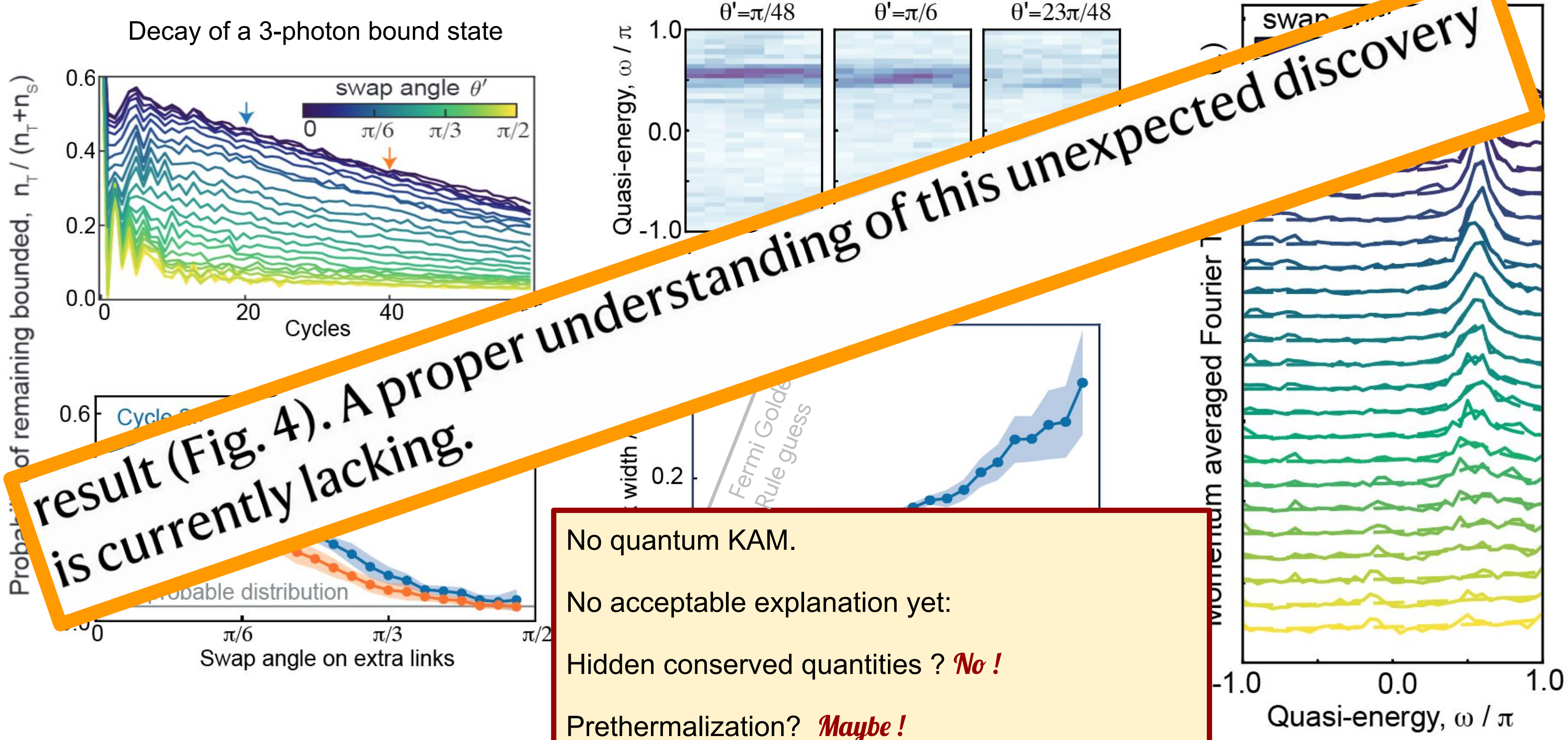
Unexpected resilience to integrability breaking

Decay of a 3-photon bound state





Breaking integrability continuously



result (Fig. 4). A proper understanding of this unexpected discovery is currently lacking.

No quantum KAM.

No acceptable explanation yet:

Hidden conserved quantities ? *No !*

Prethermalization? *Maybe !*

Emerging symmetries ? *Don't know !*

Integrability breaking and bound states in Google's decorated XXZ circuits

Ana Hudomal,[1,2] Ryan Smith,[1] Andrew Hallam,[1] and Zlatko Papić[1]

[1] *School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, United Kingdom*
[2] *Institute of Physics Belgrade, University of Belgrade, 11080 Belgrade, Serbia*

(Dated: July 26, 2023)

Recent quantum simulation by Google [Nature **612**, 240 (2022)] has demonstrated the formation of bound states of interacting photons in a quantum-circuit version of the XXZ spin chain. While such bound states are protected by integrability in a one-dimensional chain, the experiment found the

Large but dilute bound states continues to be robust.

Robustness and eventual slow decay of bound states of interacting microwave photons in the Google Quantum AI experiment

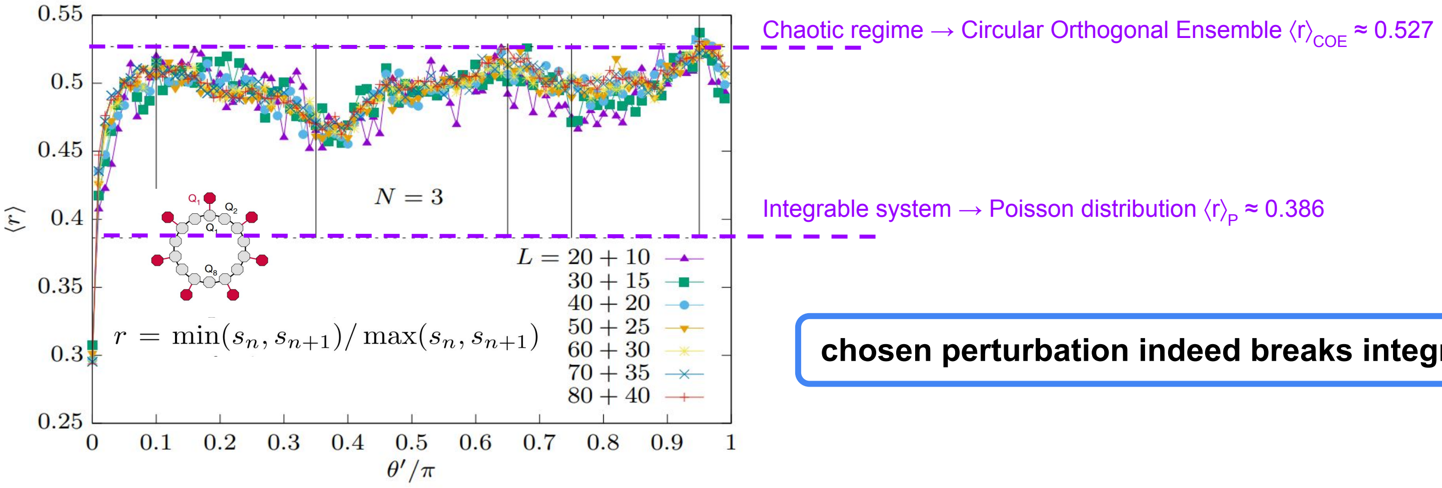
Federica Maria Surace[1] and Olexei Motrunich[1]

[1] *Department of Physics and Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, California 91125, USA*

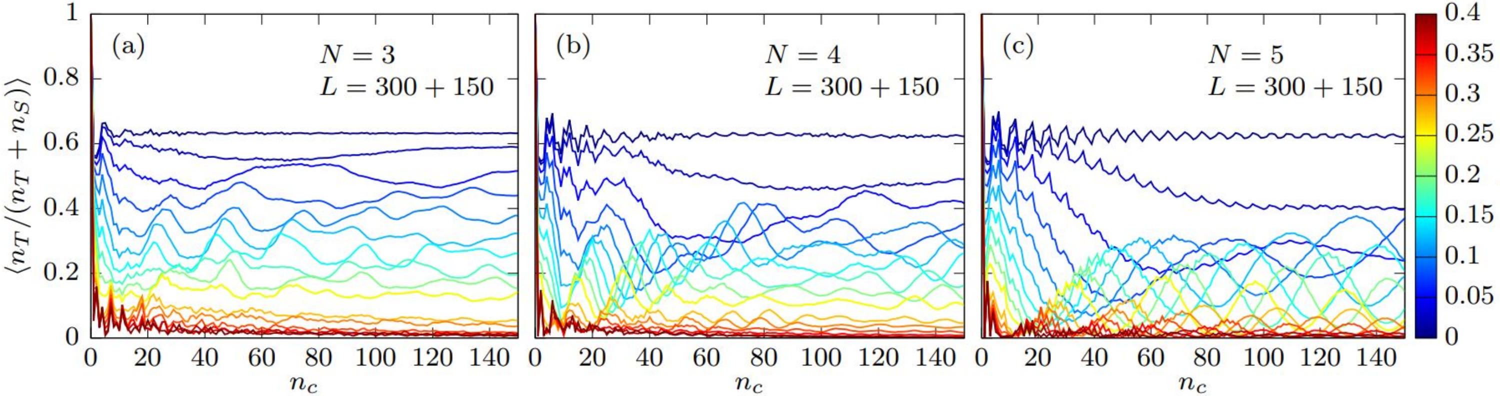
Integrable models are characterized by the existence of stable excitations that can propa indefinitely without decaying. This includes multi-magnon bound states in the celebrated spin chain model and its integrable Floquet counterpart. A recent Google Quantum AI experi [A. Morvan *et al.*, Nature **612**, 240 (2022)] realizing the Floquet model demonstrated the persistence of such collective excitations even when the integrability is broken: this observation is at odds with

It is a few-body physics and most likely will go away at larger sizes

Integrability breaking and bound states in Google's decorated XXZ circuits

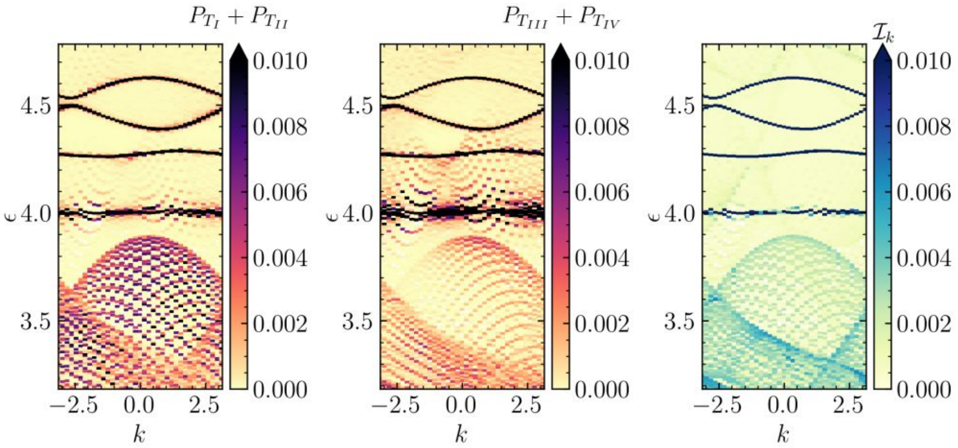


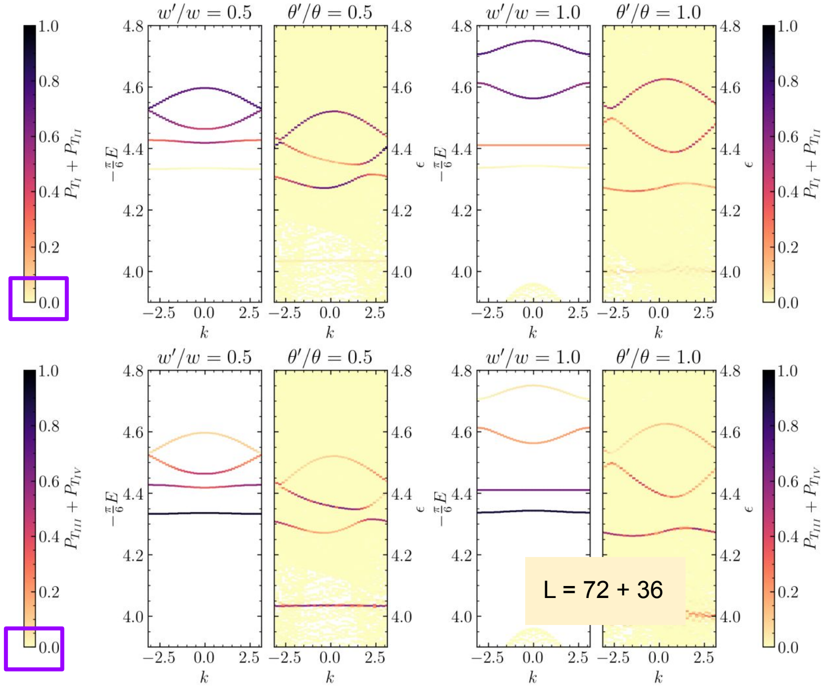
chosen perturbation indeed breaks integrability.



∴ large but dilute bound states continue to be robust.

Robustness and eventual slow decay of bound states of interacting microwave photons in the Google Quantum AI experiment



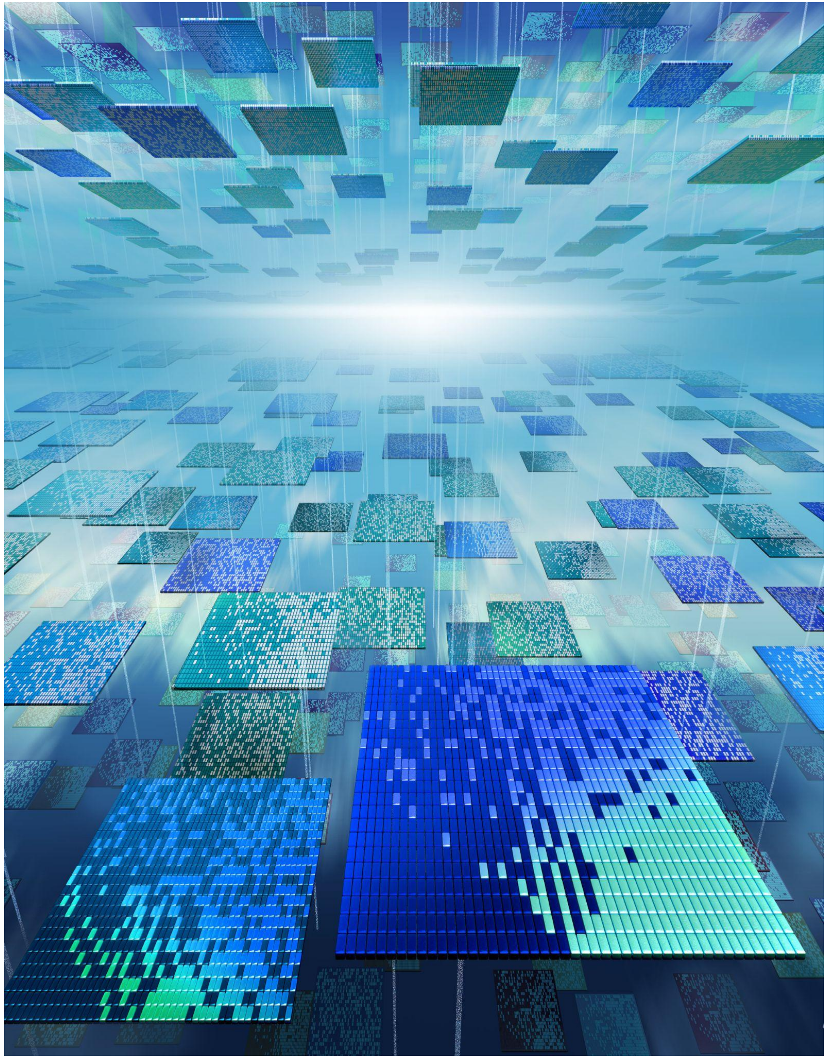


1110000 → 110010000 (no energy overlap)

1110000 → 1001010000 (small matrix element)

$$P_{T_{\mathrm{I}}} = \left| \left\langle \psi_{i,k} \right| \ldots \bullet\bullet\bullet\circ \ldots \right\rangle_k \Big|^2 \qquad P_{T_{\mathrm{III}}} = \left| \left\langle \psi_{i,k} \right| \ldots \circ\circ\bullet\bullet \ldots \right\rangle_k \Big|^2$$

$$P_{T_{\mathrm{II}}} = \left| \left\langle \psi_{i,k} \right| \ldots \circ\bullet\bullet\bullet \ldots \right\rangle_k \Big|^2 \qquad P_{T_{\mathrm{IV}}} = \left| \left\langle \psi_{i,k} \right| \ldots \circ\bullet\bullet\circ \ldots \right\rangle_k \Big|^2$$



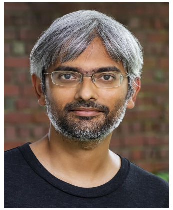
Dynamics of magnetization at infinite temperature in a Heisenberg spin chain

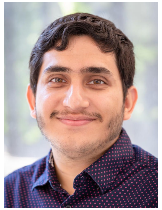

Eliott Rosenberg


Trond Andersen


Andreas Bengtsson


Andre Petukhov


Sarang Gopalakrishnan (Princeton)


Rhine Samajdar (Princeton)


Vedika Khemani (Stanford)

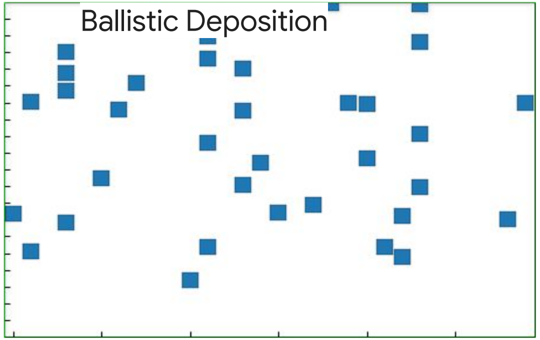

Tomaž Prosen (Ljubljana)

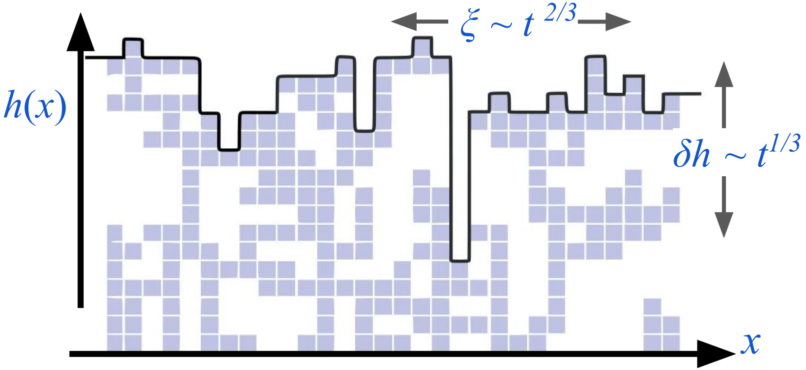
Rosenberg *et al.*, Science 384 (2024)
Arxiv 2306.09333

Kardar-Parisi-Zhang (KPZ) Universality Class

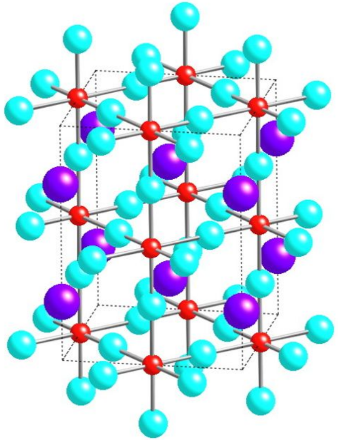
$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta(x,t)$$

diffusion growth noise





Spin dynamics of a 1D Heisenberg antiferromagnet



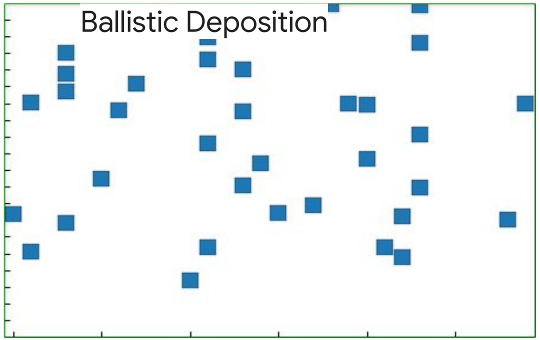
KCuF3

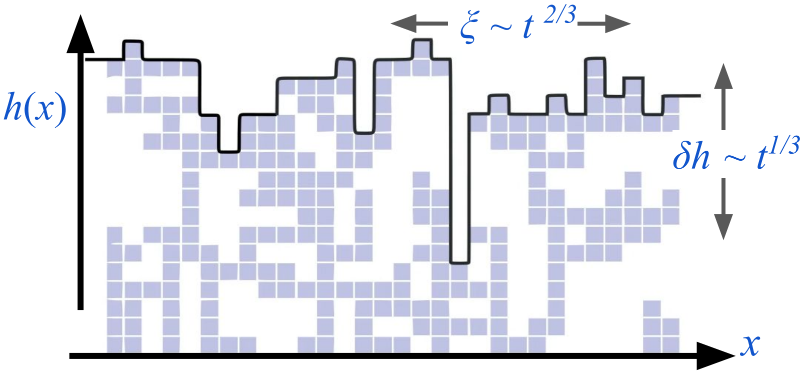
A. Scheie *et al.*, Nat. Phys. 17, 726 (2021)

Kardar-Parisi-Zhang (KPZ) Universality Class

$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta(x,t)$$

diffusion growth noise





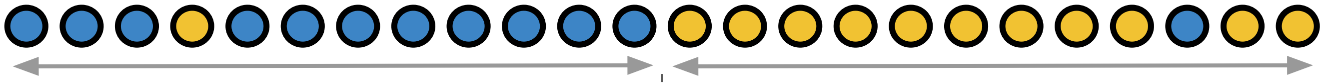
The KPZ conjecture :

KPZ		Heisenberg spin chains
$\partial h/\partial x$	$\longrightarrow$	$S^z(x,t)$ Magnetization profile
$2\,h(0,t) - h(-L/2,t) - h(L/2,t)$	$\longrightarrow$	$\mathcal{M}(t)/2 = N_{R,1}(b_t) - N_{R,1}(b_i)$
Relative height at the center		Transferred magnetization

Baik-Rains $\boldsymbol{\mu} = 0$ TW-GUE $\boldsymbol{\mu} \neq 0$

Transferred magnetization dynamics in a XXZ spin chain

Initial state:



prob. of ● $e^{\mu} / (e^{\mu} + e^{-\mu})$

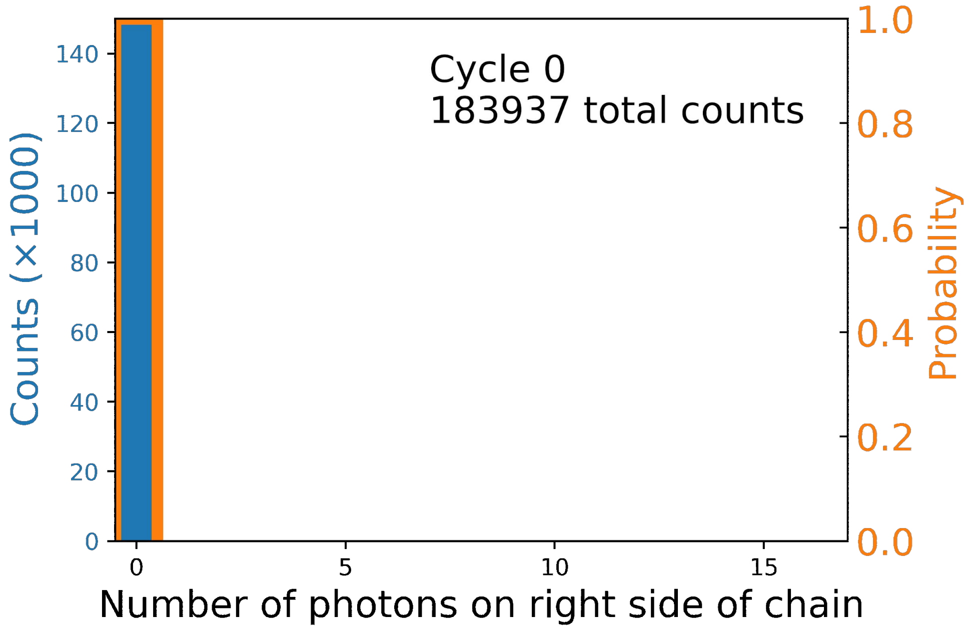
prob. of ● $e^{-\mu} / (e^{\mu} + e^{-\mu})$

Floquet Heisenberg:

$$\mathcal{H} = \sum_i (X_i X_{i+1} + Y_i Y_{i+1}) + \Delta Z_i Z_{i+1}$$

Measure transferred magnetization:

$$\mathcal{M}(t)/2 = N_{R,●}(b_t) - N_{R,●}(b_i)$$



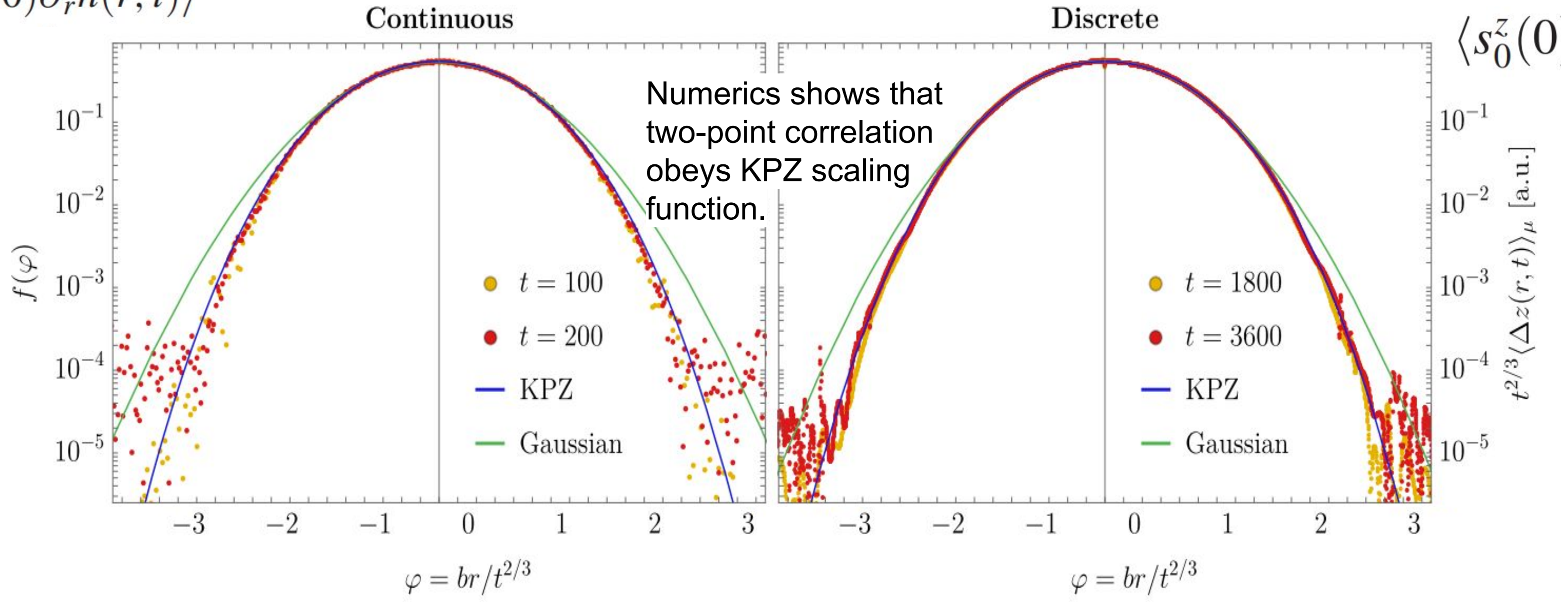
Evidence for being in the KPZ universality class

KPZ prediction

$\langle \partial_r h(0,0) \partial_r h(r,t) \rangle$

Spin chain numerics (red and yellow)

$\langle s_0^z(0) s_r^z(t) \rangle$



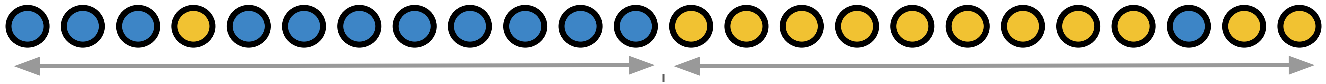
Numerics shows that two-point correlation obeys KPZ scaling function.

The KPZ conjecture :

In the long time limit : $\lim_{\mu \to 0} \mathcal{M}(t) \longleftrightarrow 2h(0,t) - h(-\infty,t) - h(\infty,t)$

Numerical: Ljubotina, Žnidaric, Prosen, PRL 122, 210602 (2019)
Experimental: D. Wei *et al.*, Quantum gas microscopy of Kardar-Parisi-Zhang superdiffusion, Science 376, 716 (2022).

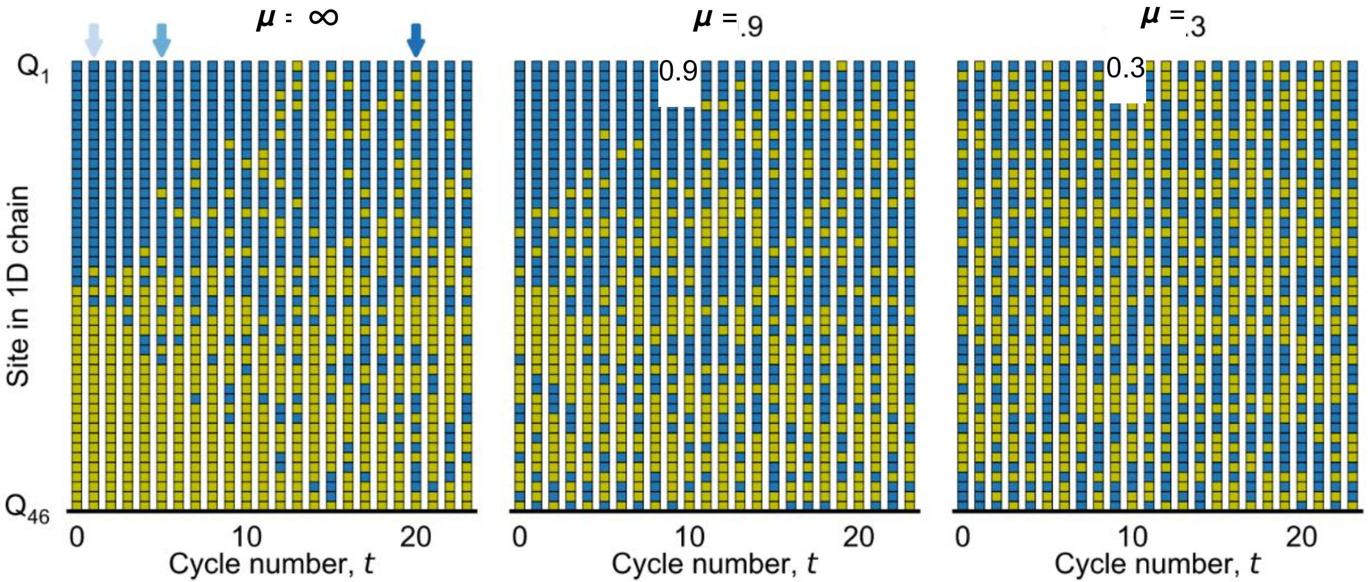
Domain wall dynamics in a Heisenberg spin chain of 46 qubits



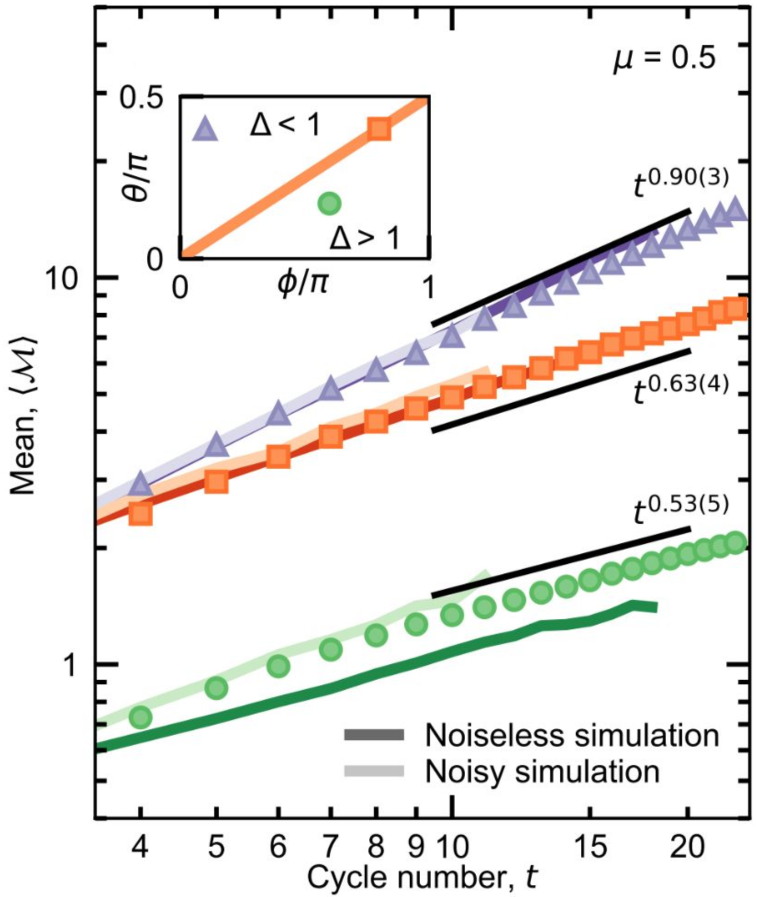
prob. of $e^{\mu} / (e^{\mu} + e^{-\mu})$

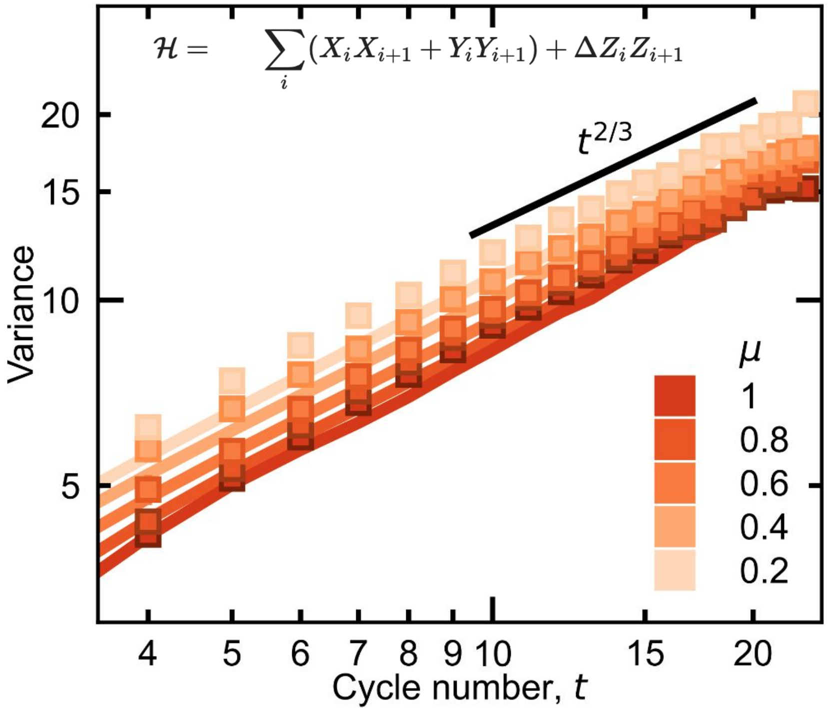
prob. of $e^{\mu} / (e^{\mu} + e^{-\mu})$

In the long time limit : $\lim_{\mu \to 0} \mathcal{M}(t) \longleftrightarrow 2h(0,t) - h(-\infty,t) - h(\infty,t)$



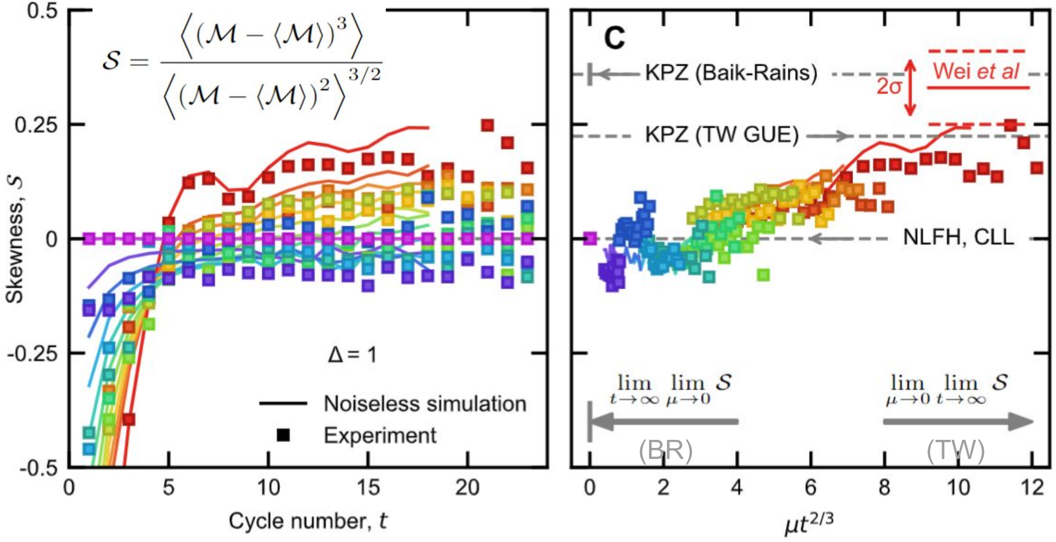
Mean and Variance of *M* → consistent with KPZ

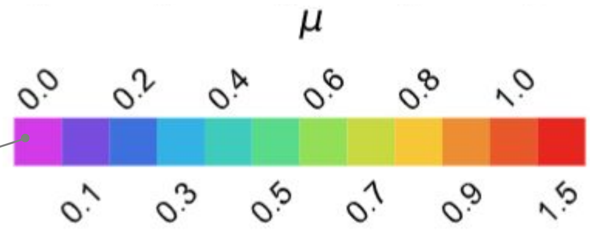




M. Znidaric PRL (2011), M. Ljubotina *et al.* Nat. Commun. (2017), J.-M. Stephan, J. Stat. Mech. (2017), R. J. Sanchez *et al.* PRL (2018), E. Ilievski *et al.* PRL (2018), S. Gopalakrishnan *et al.* PRL (2019), J. De Nardis et al. PRL (2019), S. Gopalakrishnan *et al.* Proc. Natl. Acad. Sci. (2019), J. De Nardis *et al.* PRL (2020), M. Dupont *et al.* PRB (2020), M. Dupont *et al.* PRL (2021), E. Ilievski *et al.* PRX (2021), A. Scheie *et al.* Nat. Phys. (2021), Wei *et al.*, Science 376, 2022

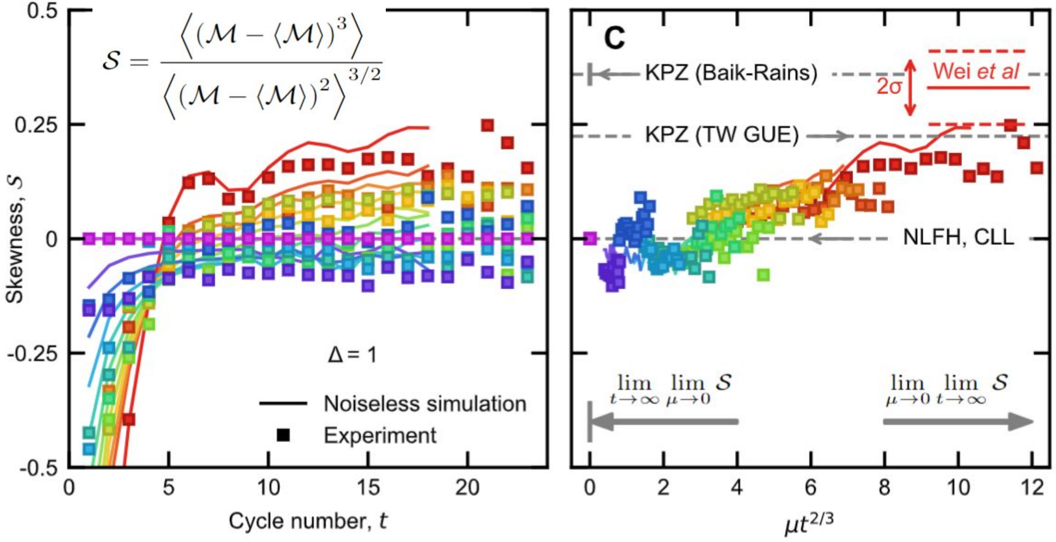
Skewness of transferred magnetization



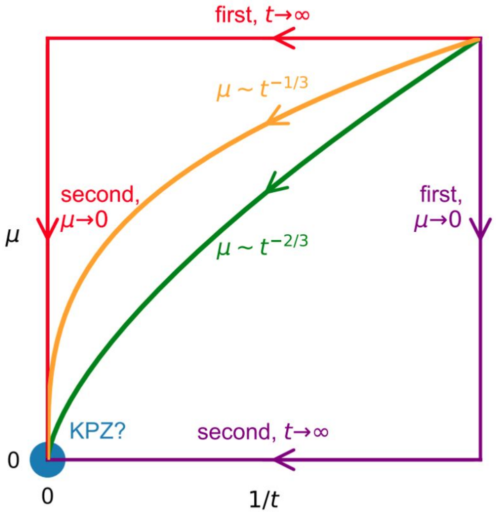


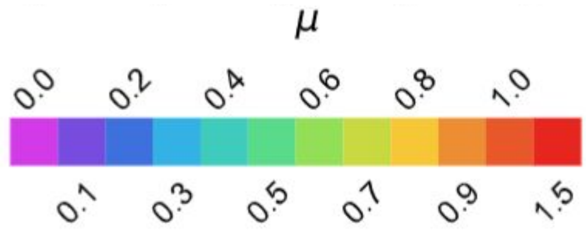
SU(2) symm initial state

Skewness of transferred magnetization



	$\langle \mathcal{M} \rangle$	σ^2	$\mathcal{S}$
Experiment	$t^{2/3}$	$t^{2/3}$	0 *
KPZ (Baik-Rains)	$t^{2/3}$	$t^{2/3}$	0.36
NLFH	$t^{2/3}$	$t^{2/3}$	0
CLL	$t^{2/3}$	$t^{2/3}$	0

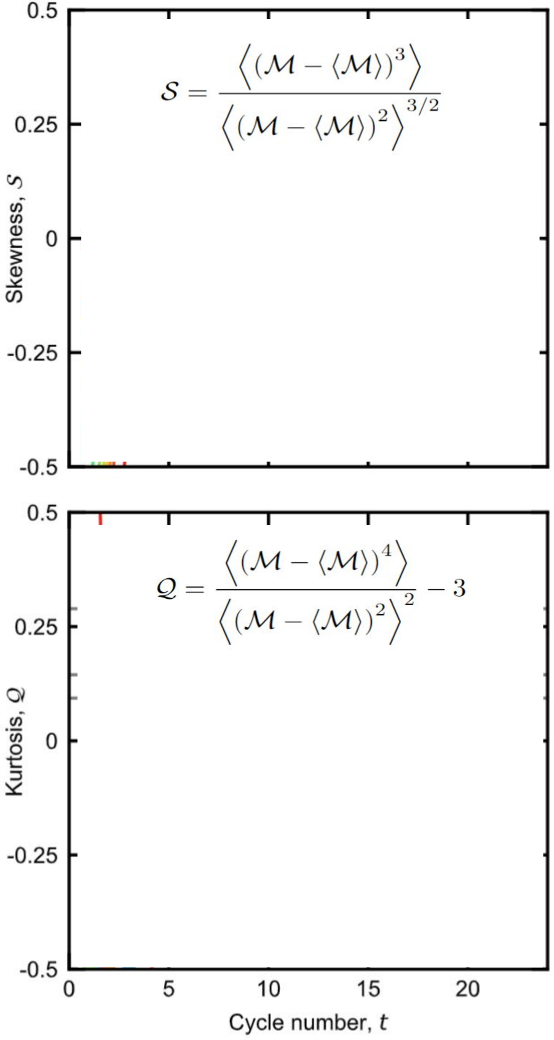


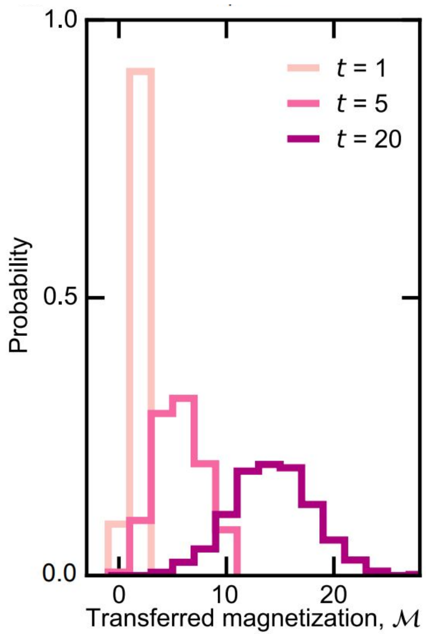


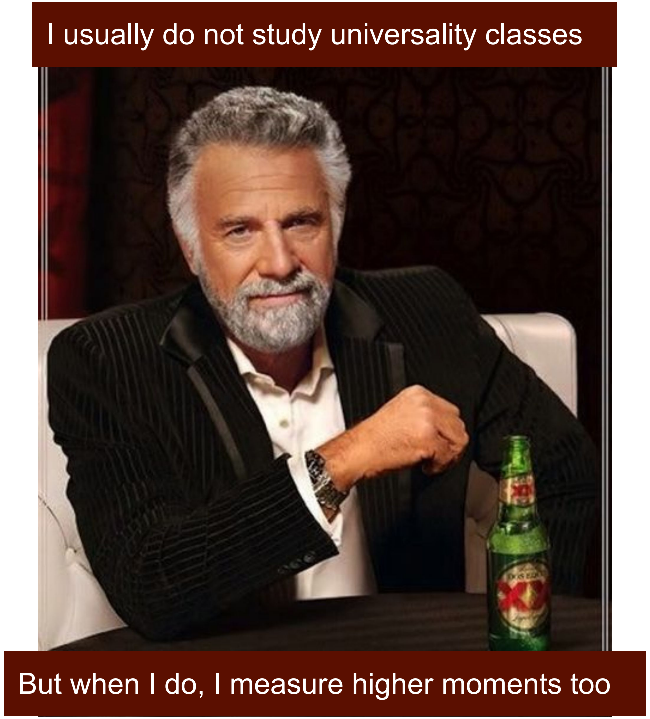
NLFH = non-linear fluctuating hydrodynamics
(De Nardis, **Gopalakrishnan**, and Vasseur, 2023)

CLL = classical Landau-Lifshitz
(Krajnik, Ilievski, and **Prosen**, 2022)

Higher moments of the transferred magnetization

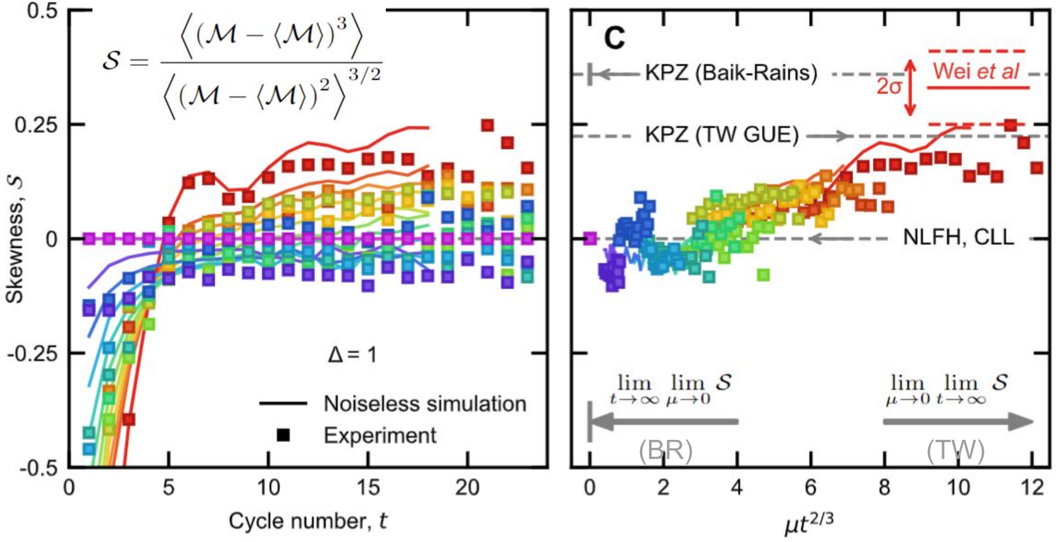


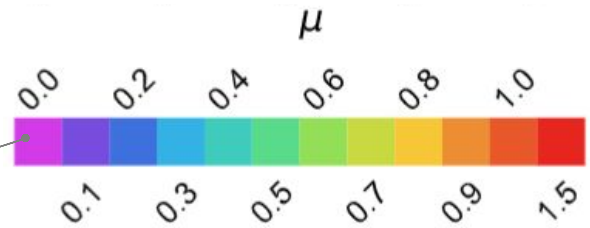




Significance of studying higher moments in determining dynamic universality classes ?

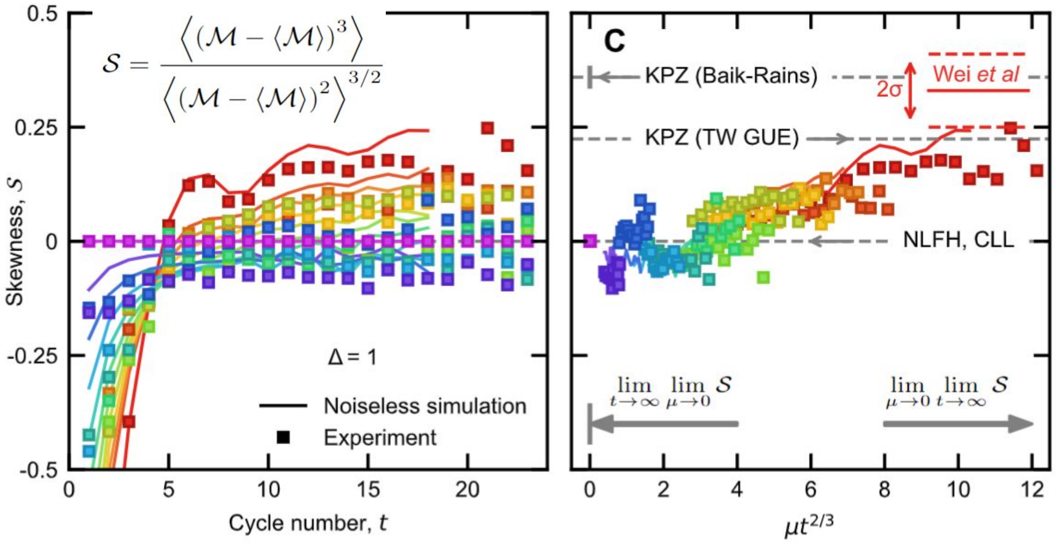
Skewness of transferred magnetization



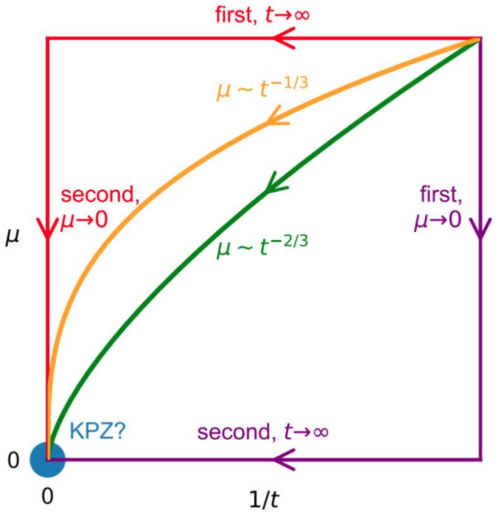


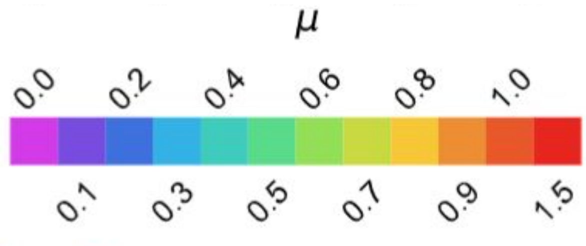
SU(2) symm initial state

Skewness of transferred magnetization



	$\langle \mathcal{M} \rangle$	σ^2	$\mathcal{S}$
Experiment	$t^{2/3}$	$t^{2/3}$	$0\,*$
KPZ (Baik-Rains)	$t^{2/3}$	$t^{2/3}$	0.36
NLFH	$t^{2/3}$	$t^{2/3}$	0
CLL	$t^{2/3}$	$t^{2/3}$	0

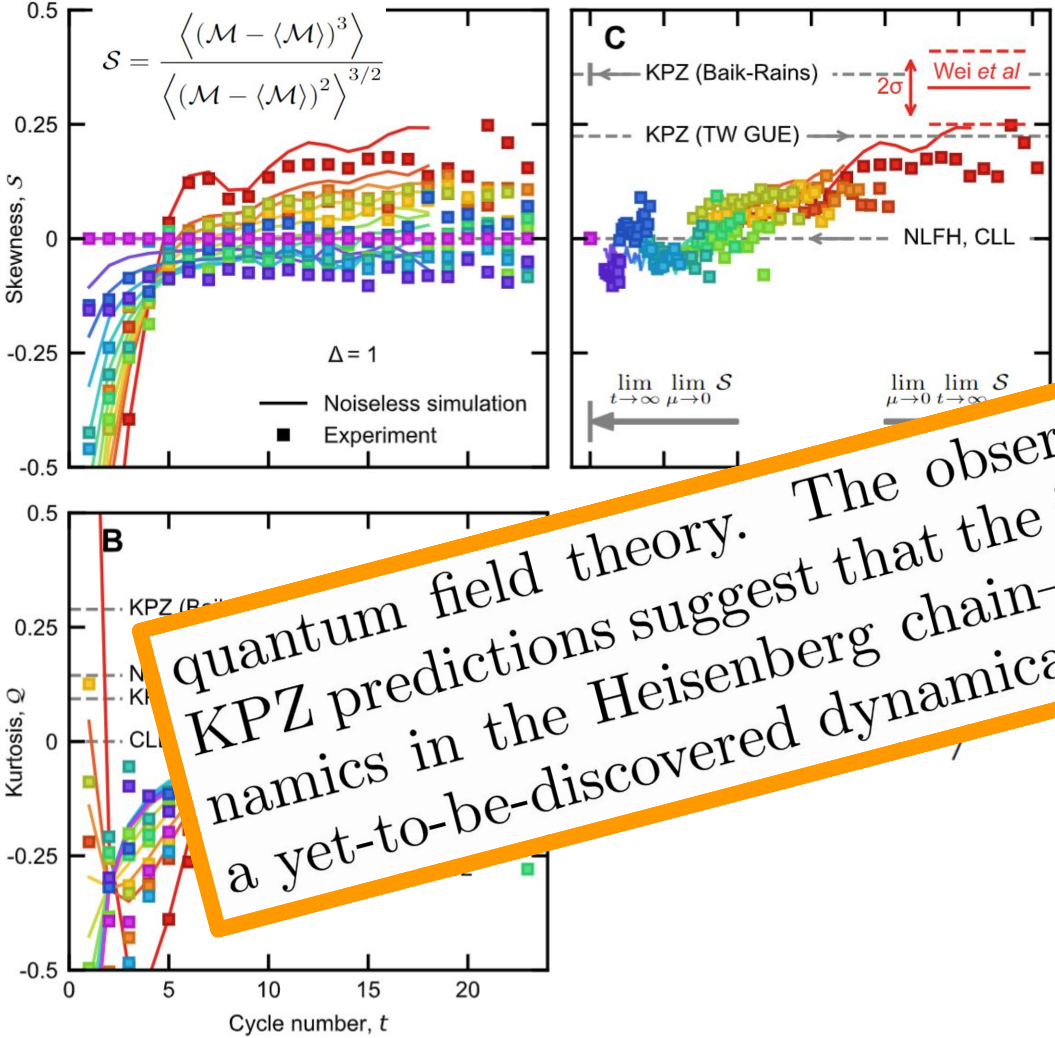




NLFH = non-linear fluctuating hydrodynamics
(De Nardis, **Gopalakrishnan**, and Vasseur, 2023)

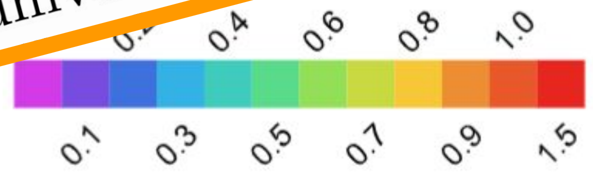
CLL = classical Landau-Lifshitz
(Krajnik, Ilievski, and **Prosen**, 2022)

Higher moments of the transferred magnetization



	$\langle \mathcal{M} \rangle$	σ^2	$\mathcal{S}$	$\mathcal{Q}$
Experiment	$t^{2/3}$	$t^{2/3}$	$0\,*$	[illegible]05 ± 0.02
KPZ (Baik-Rains)	$t^{2/3}$	[illegible]	[illegible]	[illegible]0.29
NLFH	[illegible]	[illegible]	[illegible]	[illegible]$.14$
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]$03, 0.03]$

> quantum field theory. The observed discrepancies with KPZ predictions suggest that the infinite-temperature dynamics in the Heisenberg chain—if universal—belong to a yet-to-be-discovered dynamical universality class.



NLFH = non-linear fluctuating hydrodynamics
(De Nardis, **Gopalakrishnan**, and Vasseur, 2023)

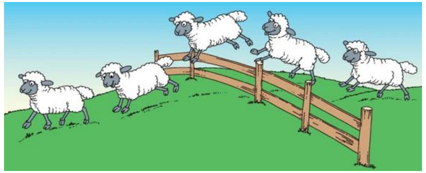
CLL = classical Landau-Lifshitz
(Krajnik, Ilievski, and **Prosen**, 2022)

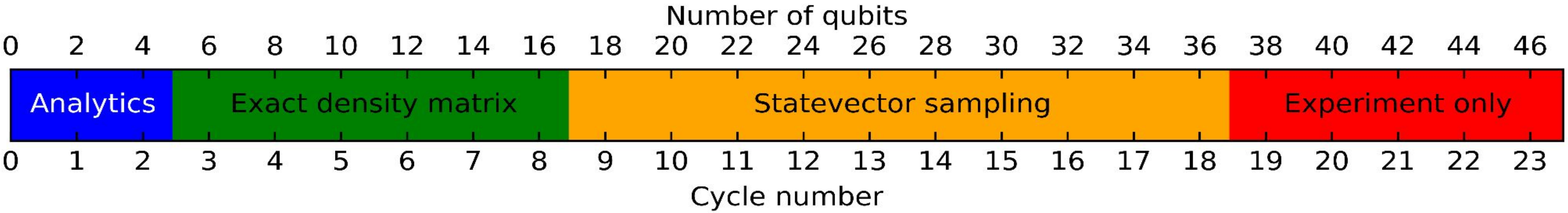
Numerical simulations

Transferred magnetization

$$\mathcal{M}(t)/2 = N_{R,1}(b_t) - N_{R,1}(b_i)$$

M at cycle T only depends on the 2T spins closest to the center
⇒ simulate smaller chains to enable comparison with experiment





Sympy- based analytics

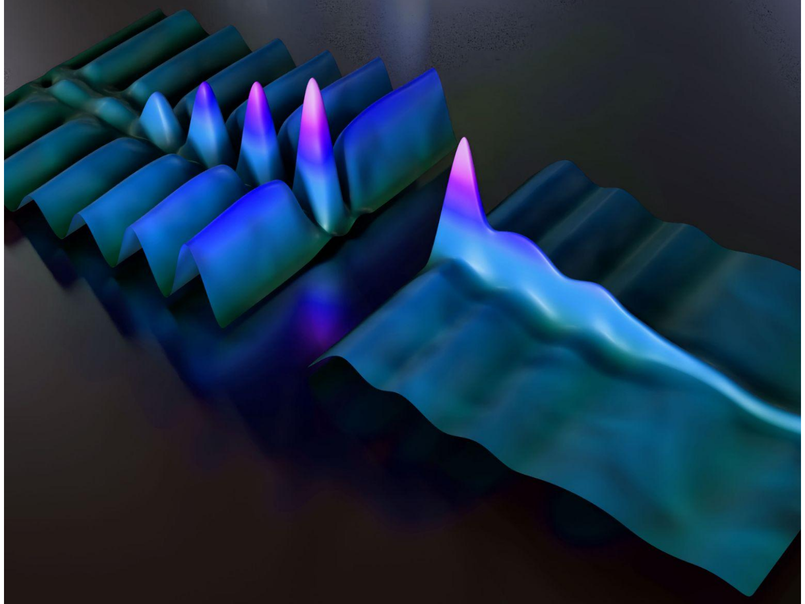
Custom numpy density matrix simulator.

Statevector sampling using qsim + NVIDIA cuQuantum on up to 8 GPUs.

e.g. Cycle 1:

$$\langle \hat{\mathcal{M}}^k \rangle = \begin{cases} 2^k \sin^2\theta \tanh\mu, & k \text{ odd} \\ 2^{k-1} \sin^2\theta \left(1 + \tanh^2\mu\right), & k \text{ even} \end{cases}$$

Observation of disorder-free localization on a quantum processor



Arxiv: 2410.06557

Residents

Quantum AI

LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
LMU

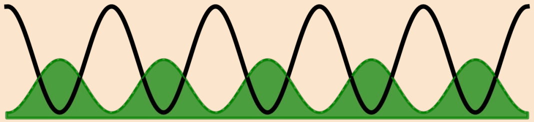
TECHNISCHE UNIVERSITÄT MÜNCHEN

CERGY PARIS UNIVERSITÉ

Imperial College London

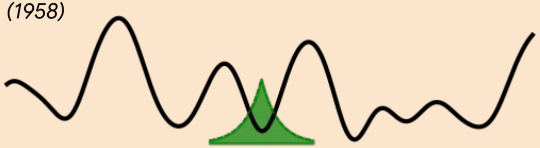
Localize excitations without breaking translational invariance ?

Ergodic dynamics



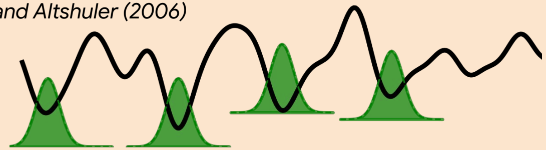
Anderson localization

P. W. Anderson (1958)



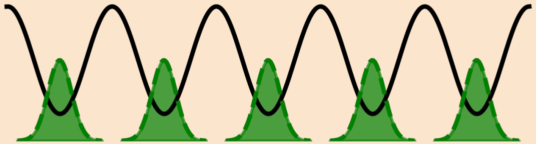
Many body localization

Basko, Aleiner, and Altshuler (2006)



Can we realize disorder free localization (?)

Smith, Knolle, Kovrizhin,Moessner (2017)



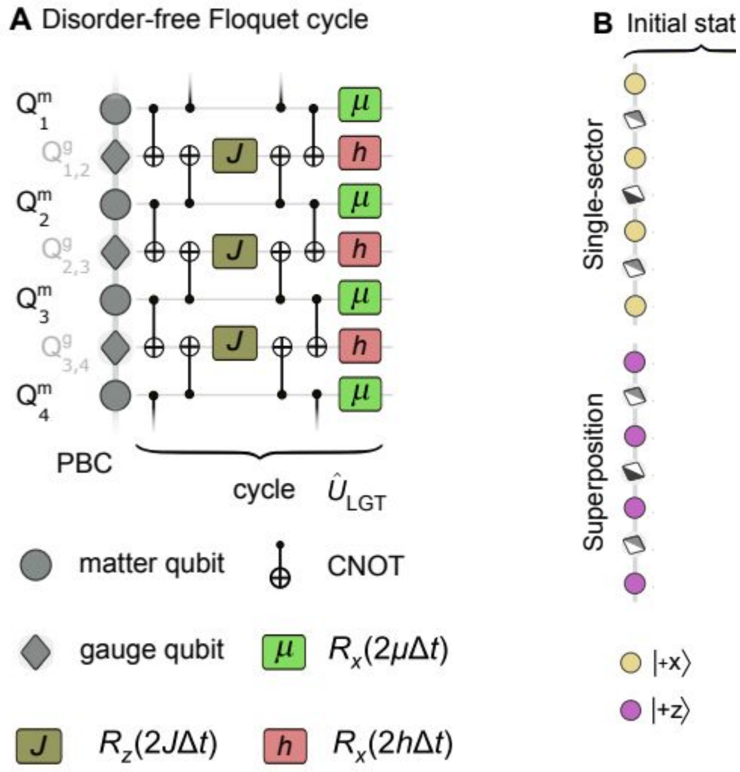
M. Grover *et al.* Journal of Statistical Mechanics: (2014)
M. Schiulaz *et al.*, PRB (2015)
N. Yao *et al.*, PRL (2016)
J. Hickey *et al.*, Journal of Statistical Mechanics: (2016)
R. Mondaini *et al.* PRB (2017)
P. P. Mazza *et al.*, PRB(2019)
H. Bernien *et al.*, Nature(2017)
A. Chandran *et al.*, Annual Review of Condensed Matter Physics (2023)

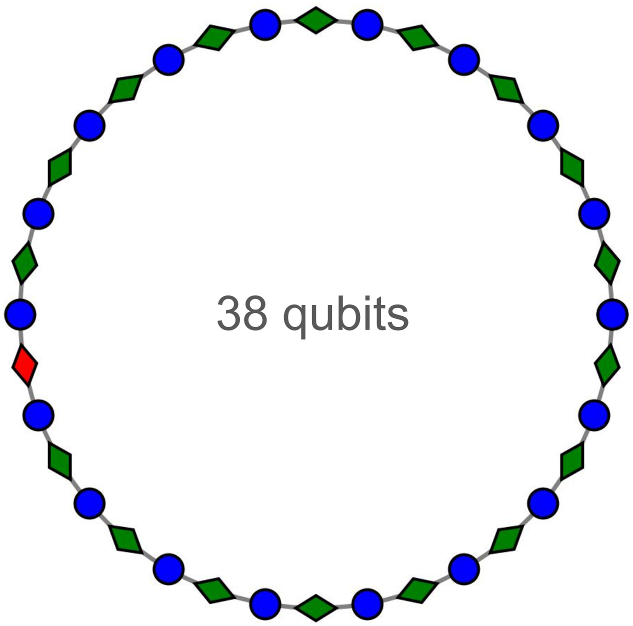
Local perturbation in a translationally invariant 1D ring

$$\mathcal{H} = \mu \sum_i \sigma_i^x + h \sum_{\langle ij \rangle} X_{ij} + J \sum_{\langle ij \rangle} \sigma_i^z Z_{ij} \sigma_j^z$$

$\hat{\sigma}_j^{X/Y/Z}$ matter qubit

$\hat{X}/\hat{Y}/\hat{Z}_{j,k}$ gauge qubit



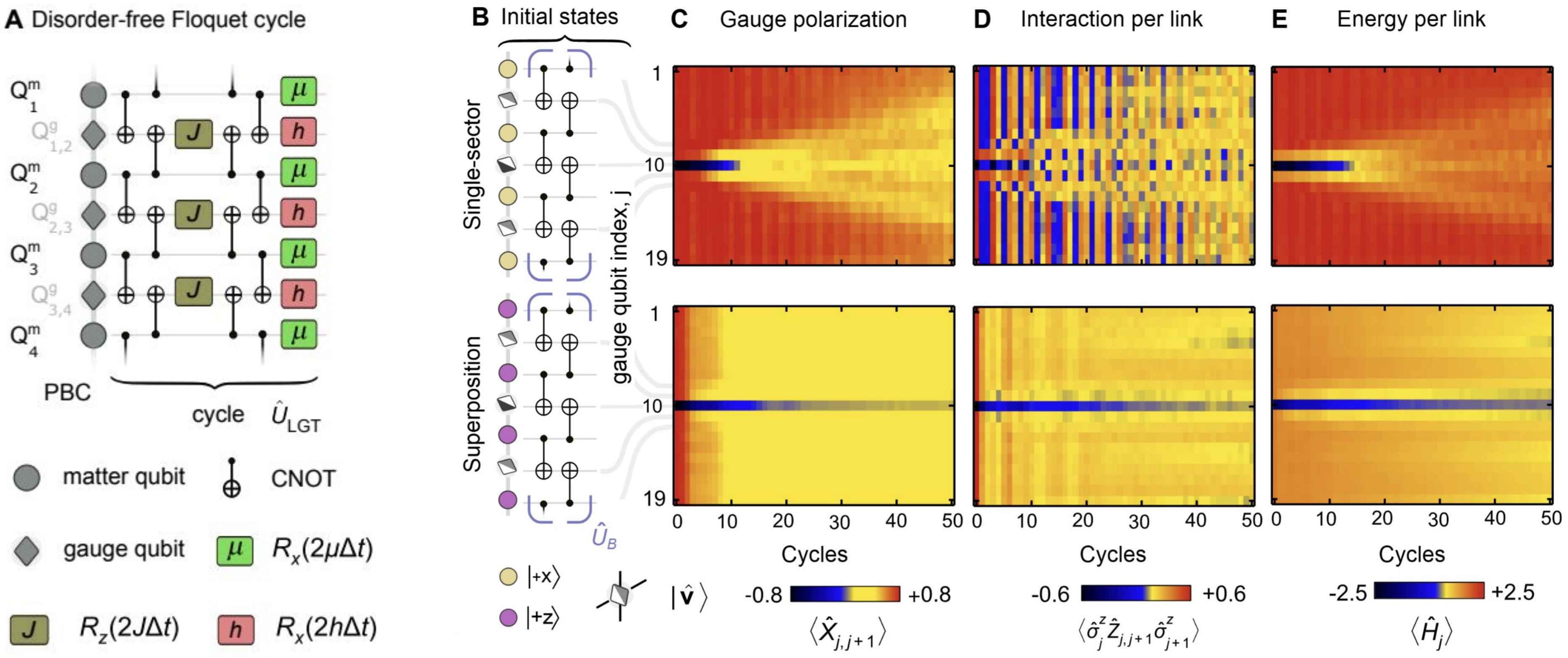


Local perturbation in a translationally invariant 1D ring

$$\mathcal{H} = \mu \sum_i \sigma_i^x + h \sum_{\langle ij \rangle} X_{ij} + J \sum_{\langle ij \rangle} \sigma_i^z Z_{ij} \sigma_j^z$$

$\hat{\sigma}_j^{X/Y/Z}$ matter qubit

$\hat{X}/\hat{Y}/\hat{Z}_{j,k}$ gauge qubit

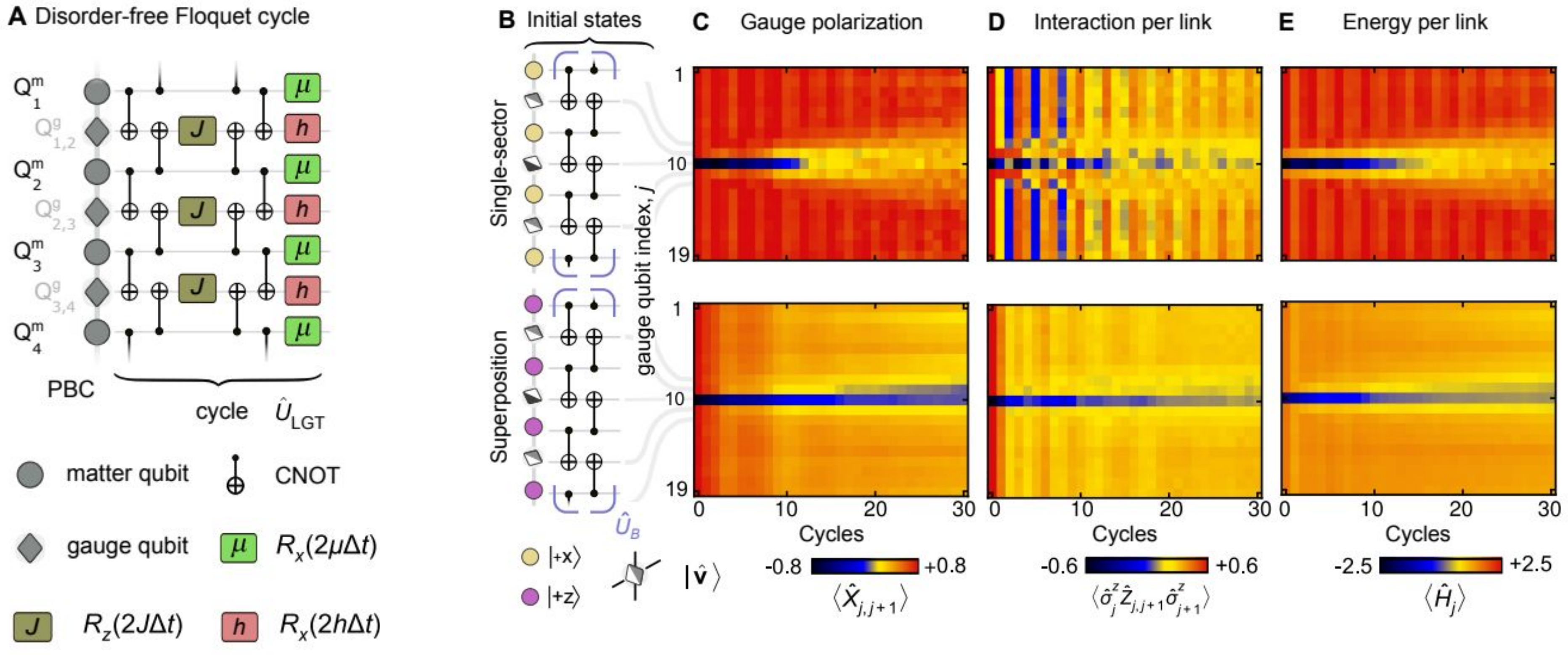


Local perturbation in a translationally invariant 1D ring

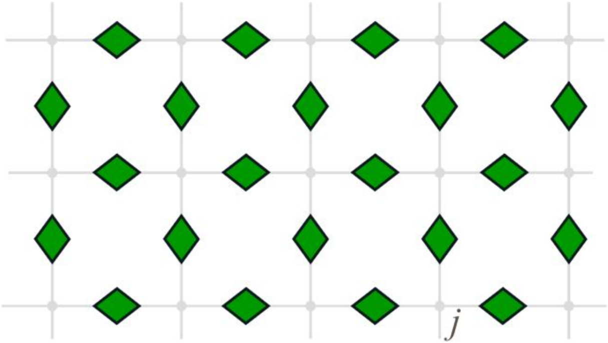
$$\mathcal{H} = \mu \sum_i \sigma_i^x + h \sum_{\langle ij \rangle} X_{ij} + J \sum_{\langle ij \rangle} \sigma_i^z Z_{ij} \sigma_j^z$$

$\hat{\sigma}_j^{X/Y/Z}$ matter qubit

$\hat{X}/\hat{Y}/\hat{Z}_{j,k}$ gauge qubit



Disorder free Hamiltonian and initial states



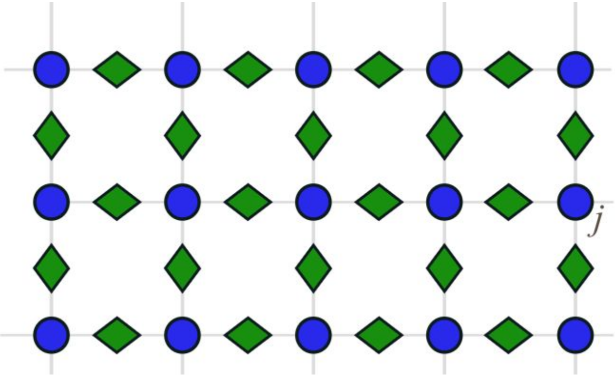
◆ qubits on links of a lattice

$$\hat{H} = \frac{J}{2d}\sum_j \sum_k \hat{Z}_{j,k} + \frac{h}{2d}\sum_j \sum_k \hat{X}_{j,k} + \mu \sum_j g_j \prod_k \hat{X}_{j,k}$$

$$\hat{H} = \hat{H}_{\mathrm{ord}} + \hat{H}_{\mathrm{dis}}$$

$g_j = \pm 1$

binary disorder



● ancilla, ◆ dynamical qubits

$$\hat{H} = \frac{J}{2d}\sum_j \sum_k \hat{Z}_{j,k} + \frac{h}{2d}\sum_j \sum_k \hat{X}_{j,k} + \mu \sum_j \hat{\sigma}^x_j \prod_k \hat{X}_{j,k}$$

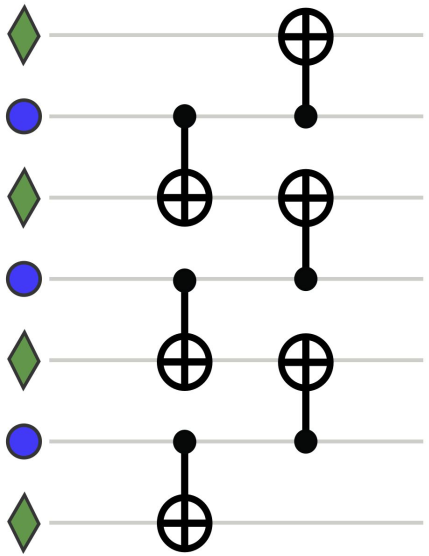
$[\hat{H}, \hat{\sigma}^x_j] = 0\,, \quad \forall j \in$ lattice sites

Unitary transoforation to dual Hamiltonian

What we discussed :

$$\hat{H}_{1D} = \mu \sum_j X_{j,1}\sigma_j^x X_{j,2} + J\sum_j \frac{Z_{j,1}+Z_{j,2}}{2} + h\sum_j \frac{X_{j,1}+X_{j,2}}{2}$$

$$[\hat{H}, \sigma_j^x] = 0$$



ancilla, dynamical qubits

What we implemented :

$$\hat{H}_{1D} = \mu \sum_j \sigma_j^x + J\sum_j Z_{j,1}\sigma_j^z Z_{j,2} + h\sum_j \frac{X_{j,1}+X_{j,2}}{2}$$

$$[\hat{H}, X_{j,1}\sigma_j^x X_{j,2}] = 0$$

Decoupled gauge (super-selection) sectors

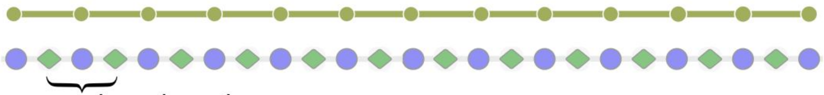
$$\hat{H} = \frac{J}{2d}\sum_j \sum_k \hat{Z}_{j,k} + \frac{h}{2d}\sum_j \sum_k \hat{X}_{j,k} + \mu \sum_j \hat{\sigma}^x_j \prod_k \hat{X}_{j,k}$$

$[\hat{H}, \hat{\sigma}^x_j] = 0\,, \quad \forall j \in$ lattice sites

sector labeled by the charges $\mathbf{g} = \{g_1, g_2, \cdots, g_{n_c}\}$

example of a disorder-free sector

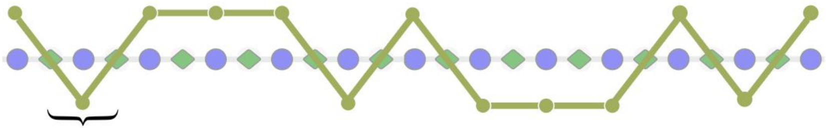
$<\sigma^x_j> = +1$



g_j = +1 +1 +1 •••

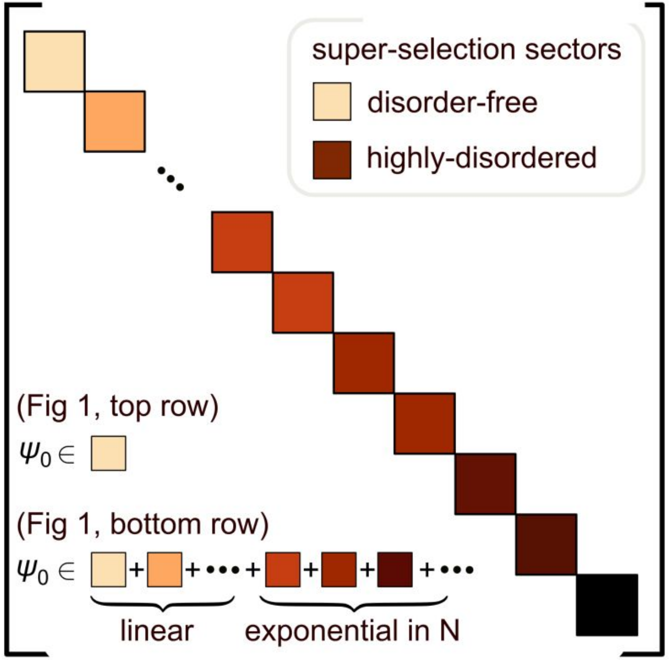
example of a disordered sector

$<\sigma^x_j> =$ random



g_j = -1 +1 ••• -1 +1 -1 •••

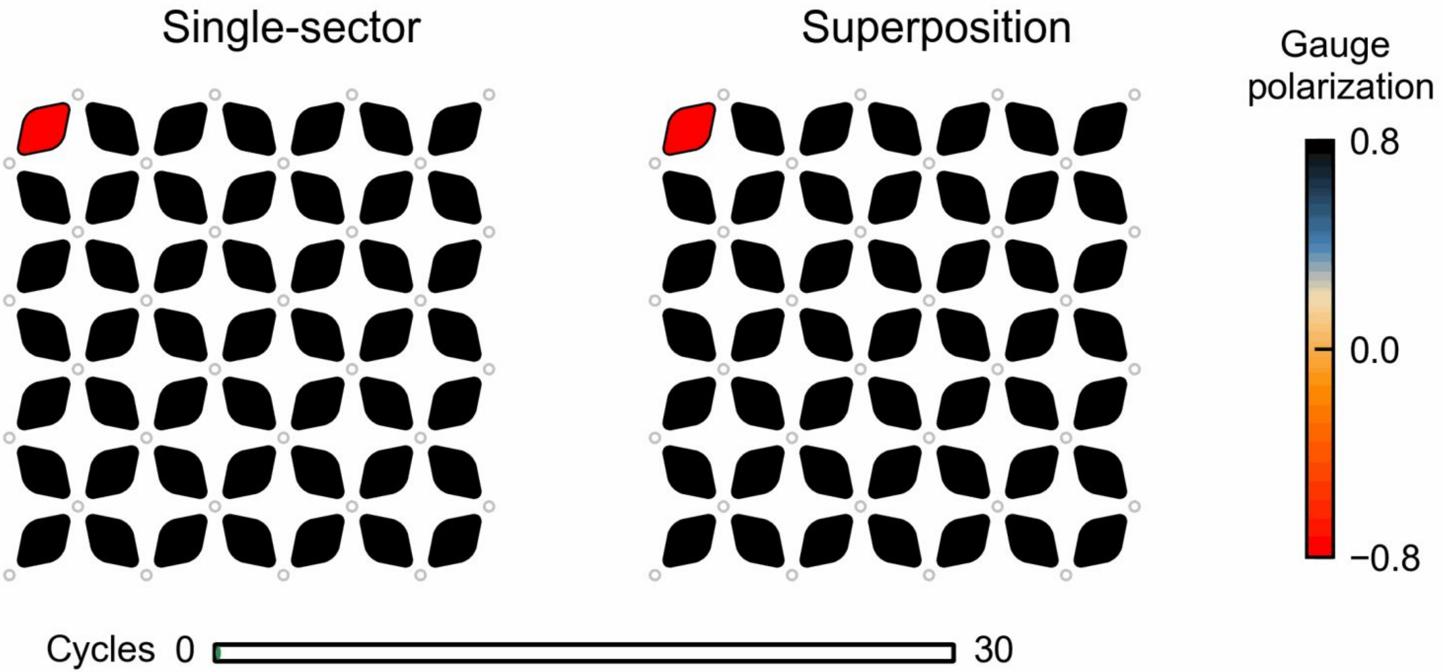
$<\sigma^z_j> = 1$ → superposition of all sectors



$$\overline{\langle \hat{O}(t) \rangle} = \sum_{\mathbf{g}} w_{\mathbf{g}} \langle \hat{O}(t) \rangle_{\mathbf{g}}$$

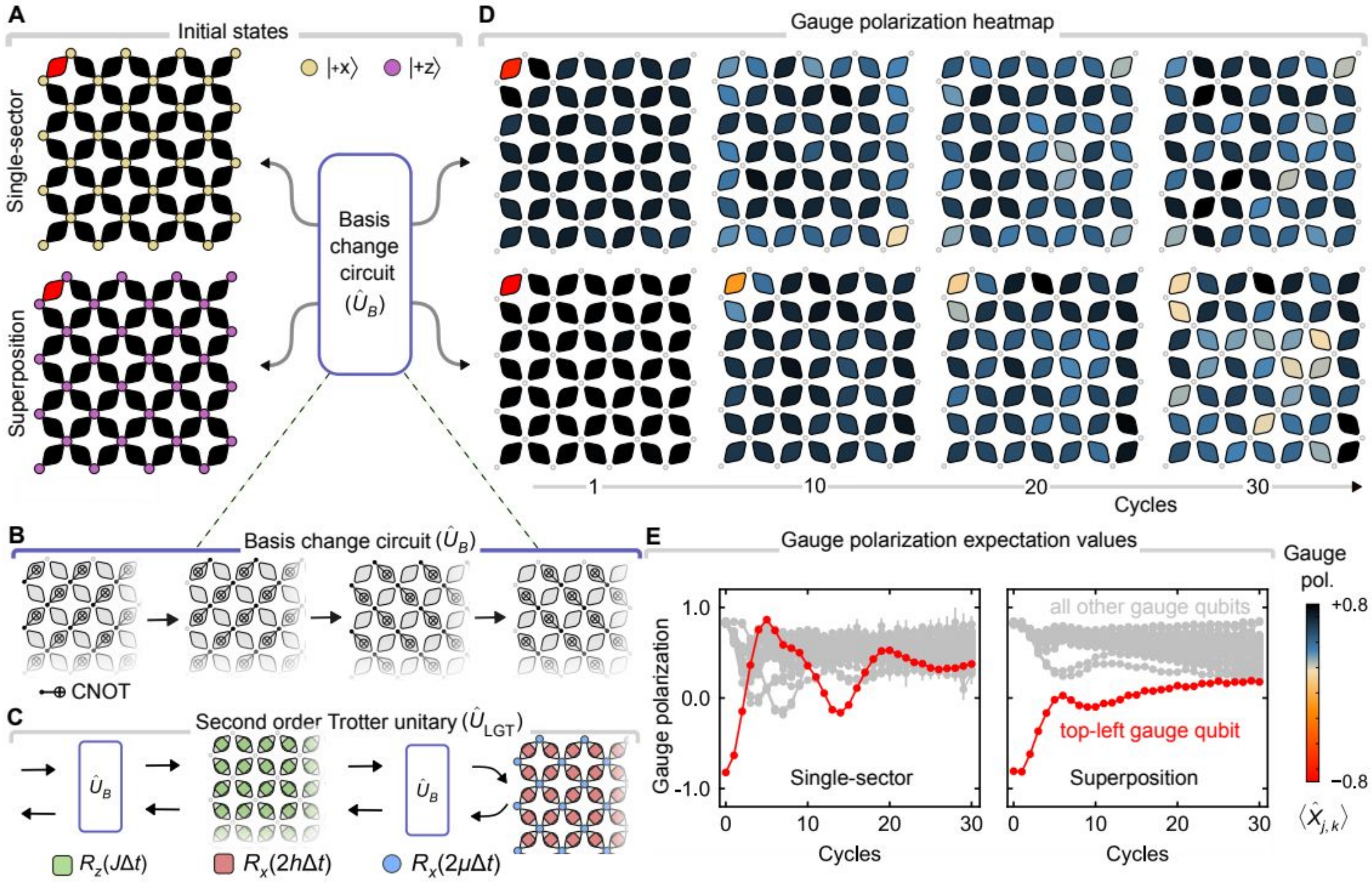
Challenge : rare events → exponential sampling

Dynamics of a local perturbation in a 2D

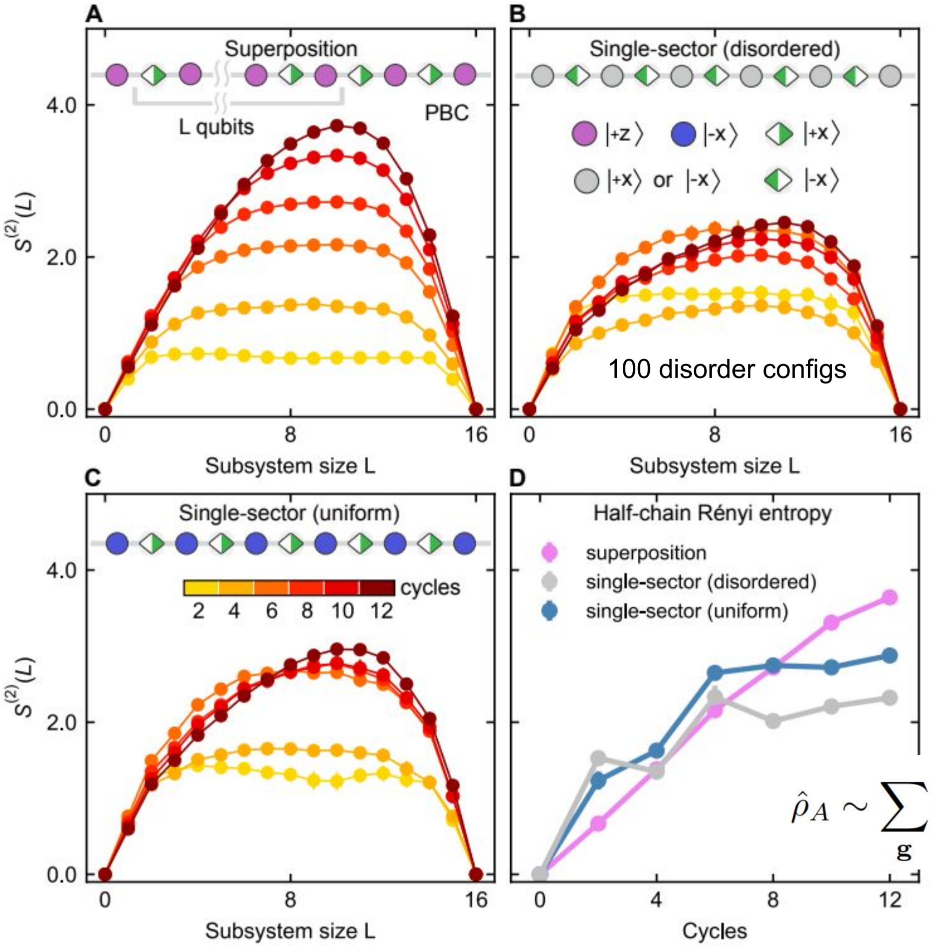


32 matter + 49 gauge = 81 qubits

Dynamics of a local perturbation in a 2D



Entanglement entropy in a 1D chain ($N_m = 8$, $N_g = 8$)



Two contributions to entropy:

dynamically generated correlations between matter and gauge qubits.

background charge disorder

$$\hat{\rho}_A \sim \sum_{\mathbf{g}} w_{\mathbf{g}}\, \hat{\rho}^{MBL}_{A,\mathbf{g}} \otimes |\mathbf{g}\rangle\langle\mathbf{g}| + \sum_{\mathbf{g_1} \neq \mathbf{g_2}} \delta\hat{\rho}_{A,\mathbf{g_1},\mathbf{g_2}} \otimes |\mathbf{g_1}\rangle\langle\mathbf{g_2}|$$