Local-Information Time Evolution

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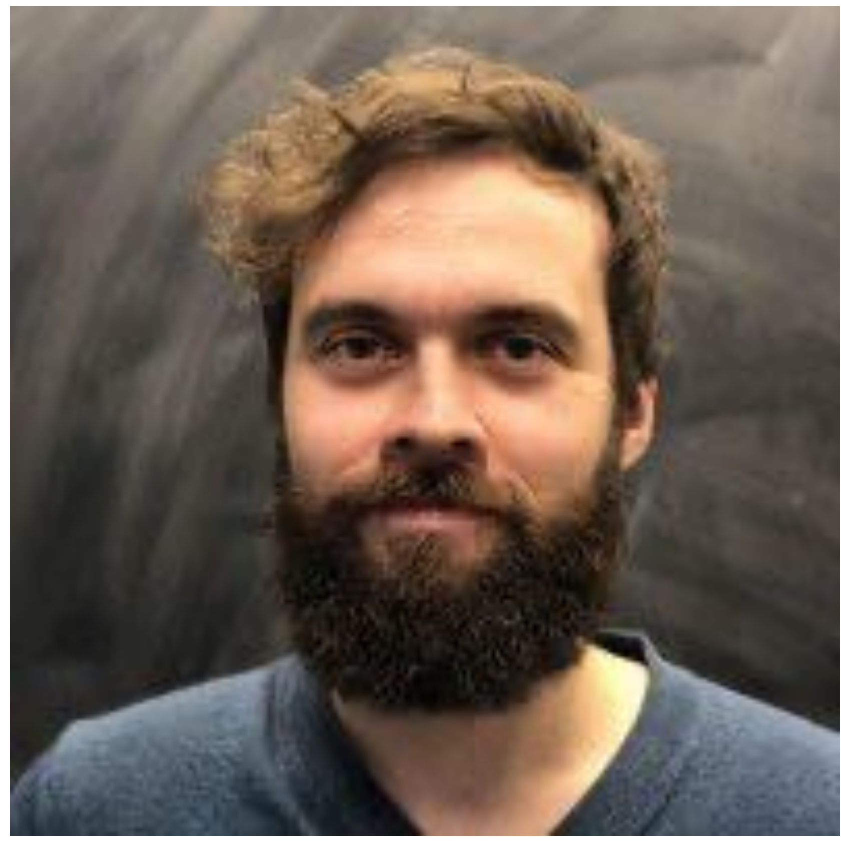


Jens H. Bárðarson
KTH

- T. Klein Kvorning, L. Herviou, J. H. Bárðarson, **SciPost Phys. 13 (4), 080 (2022)**
- C. Artiaco, C.Fleckenstein, D. Aceituno, T. Klein Kvorning, J. H. Bárðarson, **PRX Quantum 5 (2), 020352 (2024)**
- K. Harkins, C. Fleckenstein, N. D'Souza, P. M. Schindler, D. Marchiori, C. Artiaco, Q. Reynard-Feytis, U. Basumallick, W. Beatrez, A. Pillai, M. Hagn, A. Nayak, S. Breuer, X. Lv, M. McAllister, P. Reshetikhin, E. Druga, M. Bukov, A. Ajoy, **Sci. Adv., 11 (13), eadn 9021 (2025)**
- C. Artiaco, T. Klein Kvorning, D. Aceituno, L. Herviou, J. H. Bárðarson, **Phys. Rev. Lett. 134 (19), 190401 (2025)**
- N. P. Bauer, B. Trauzettel, T. Klein Kvorning, J. H. Bárðarson, C. Artiaco, **arXiv:2505.00537**

"Entanglement is […] the characteristic trait of quantum mechanics"

Erwin Schrödinger

"Information is physical"

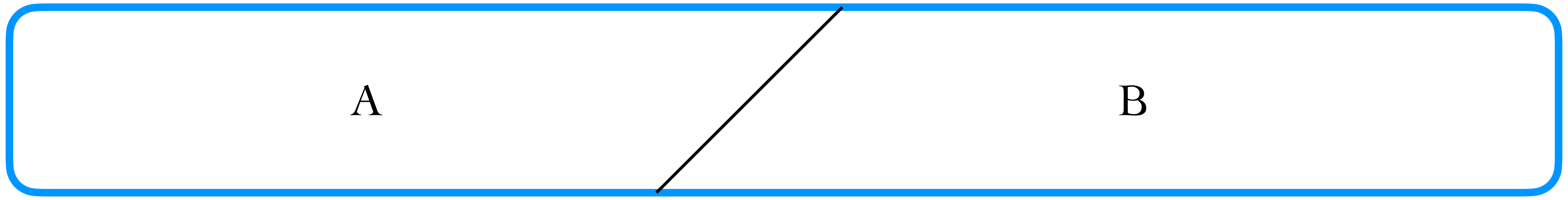
Rolf Landauer

Entanglement poses significant challenges in simulating the out-of-equilibrium dynamics of interacting quantum systems

What is entanglement? And how is it related to information?

Composite system

$$\mathscr{H} = \mathscr{H}_A \otimes \mathscr{H}_B$$



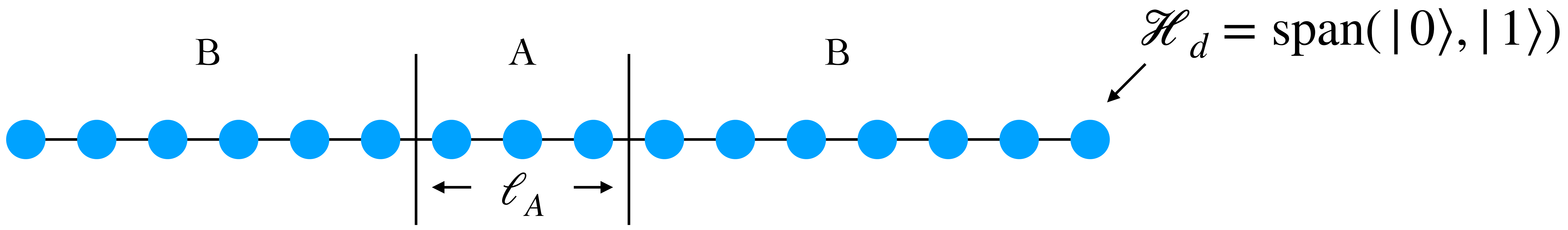
$\left.\begin{array}{l}\rho \text{ acts on } \mathscr{H} \\ \rho^2 = \rho\end{array}\right\}$ $\rho \neq \rho_A \otimes \rho_B$ in any basis $\implies$ ρ has entanglement between A and B

Information of state of subsystem A is contained in $\rho_A = \mathrm{Tr}_B(\rho)$ via $\langle O_A \rangle = \mathrm{Tr}_A(\rho_A O_A)$

If ρ is entangled, then all information in the state can **not** be accessed locally in A and B!

Entanglement is nonlocal information

The von Neumann information



If $\rho_A^2 = \rho_A$ then we have access to $\ln(2^{\ell_A})$ bits of information

If $\rho_A \propto \mathbf{1}_A$ then we have access to 0 bits of information

In between, the accessible information is the von Neumann (Shannon) information

$$I(\rho_A) = \ln 2^{\ell_A} - S(\rho_A)$$

$$S = -\,\mathrm{Tr}_A(\rho_A \ln \rho_A)$$

Q: What is the distribution of information in a given state?

Example: information in a singlet

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$\rho = \frac{1}{4}\mathbf{1}_4$$

There is **no** information in the single sites!

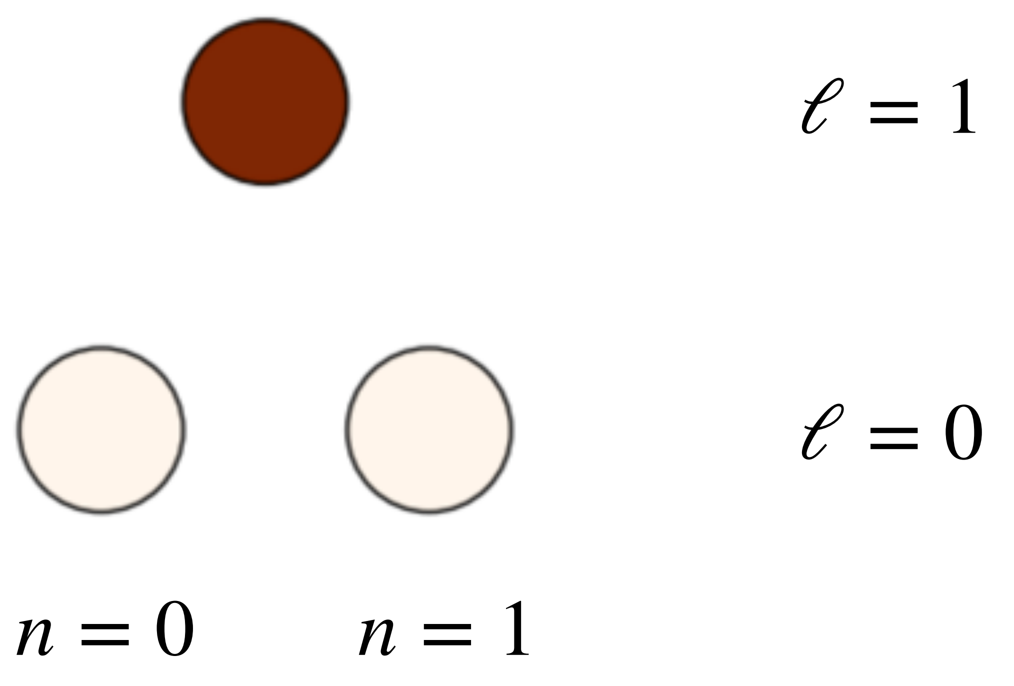
$$\rho_A = \frac{1}{2}\mathbf{1}_2 = \rho_B$$

$S_A = \ln 2 = S_B$ $\qquad$ $I_A = 0 = I_B$

Information on single site says nothing about information on two sites

$$\rho_A = \frac{1}{2}\mathbf{1}_2 = \rho_B$$

$S_A = \ln 2 = S_B$ $\qquad$ $I_A = 0 = I_B$



All information is on the two site state

$$S_{A\cup B} = 0$$

$$I_{A\cup B} = \ln 4$$

Fully mixed two site state has no information

$$S_{A\cup B} = \ln 4$$

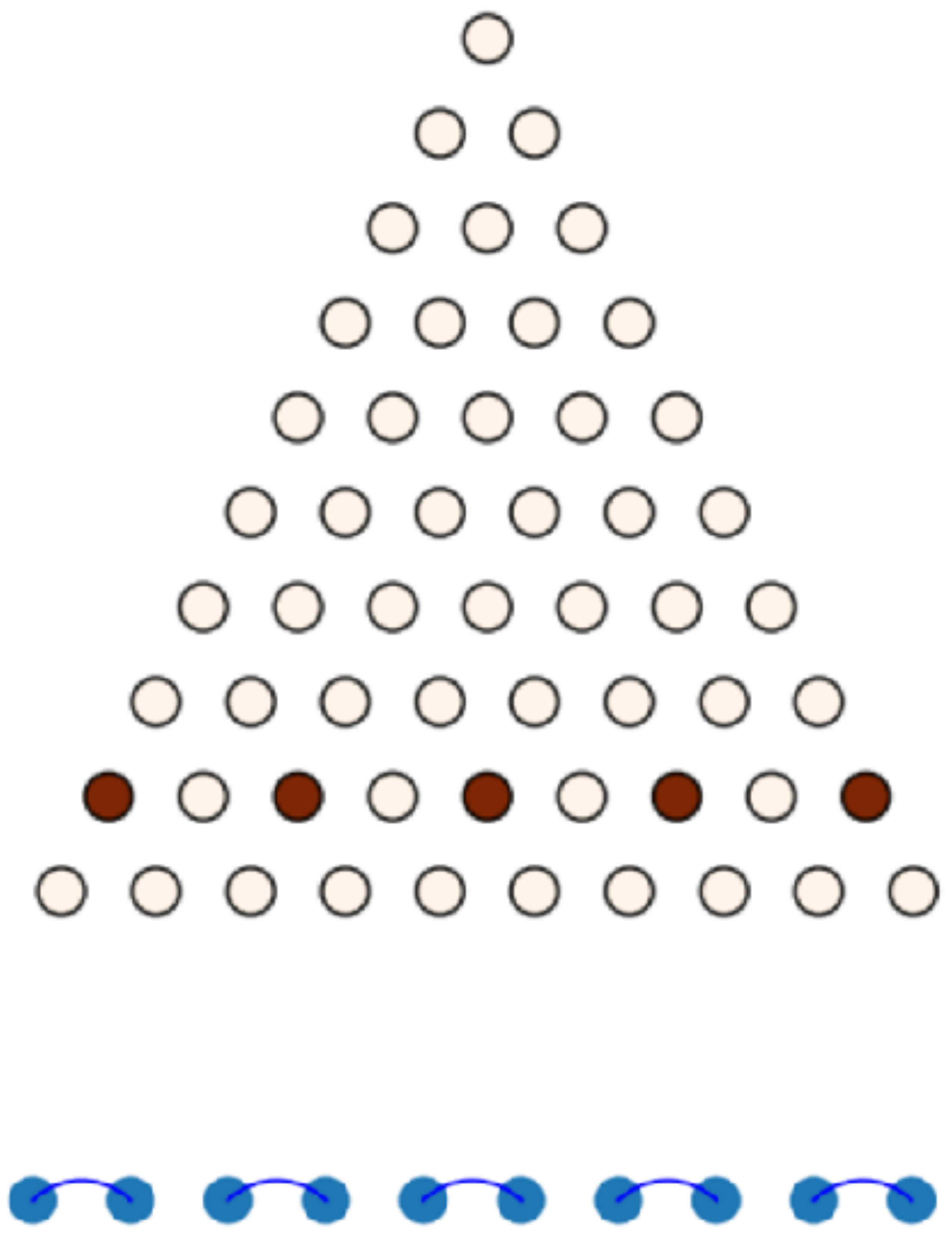
$$I_{A\cup B} = 0$$

We define **local information** as

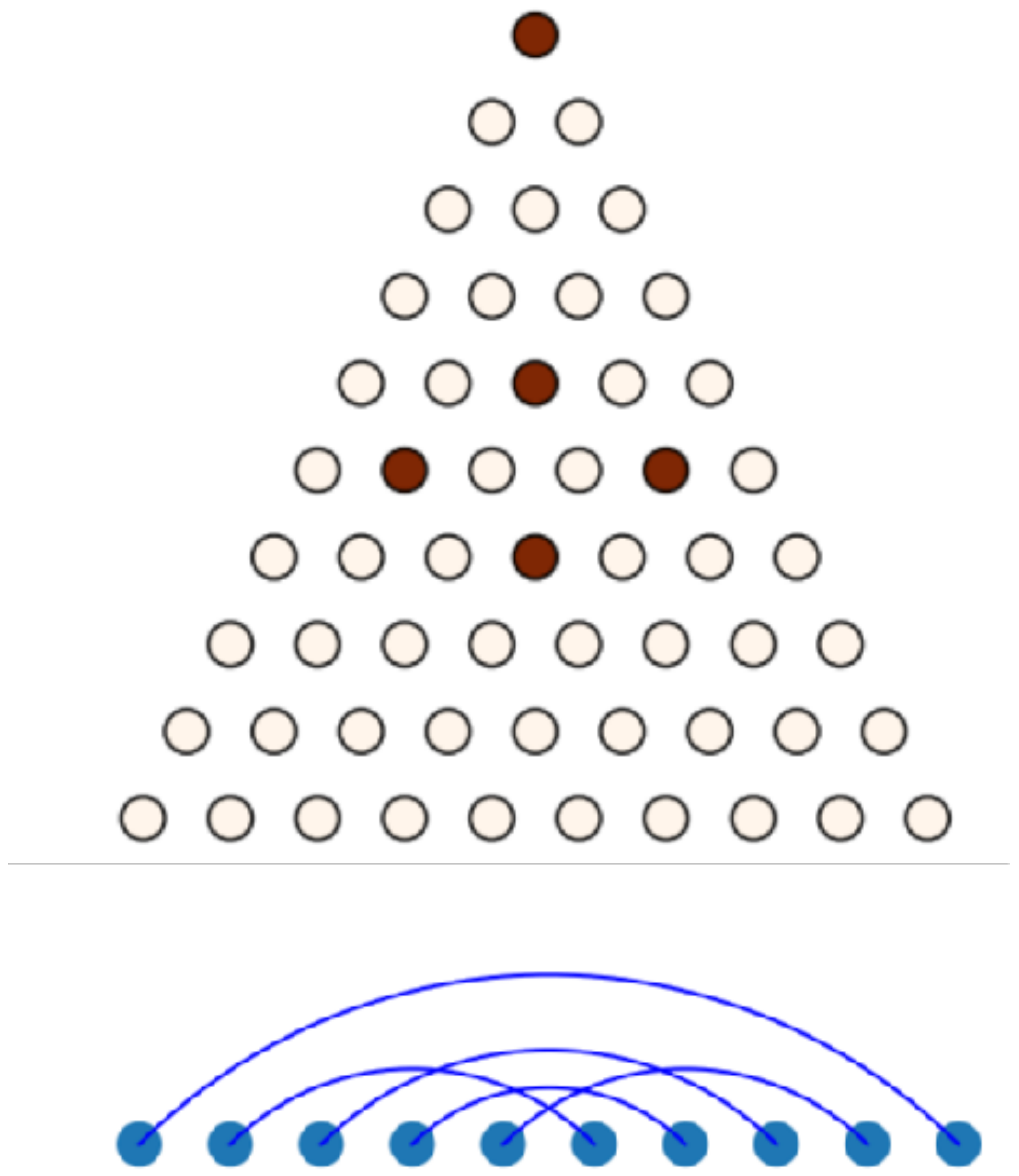
$$i_n^\ell = I_n^\ell - I_{n-1/2}^{\ell-1} - I_{n+1/2}^{\ell-1} + I_n^{\ell-2}$$

Local information within the information lattice

Product state of neighboring singlets



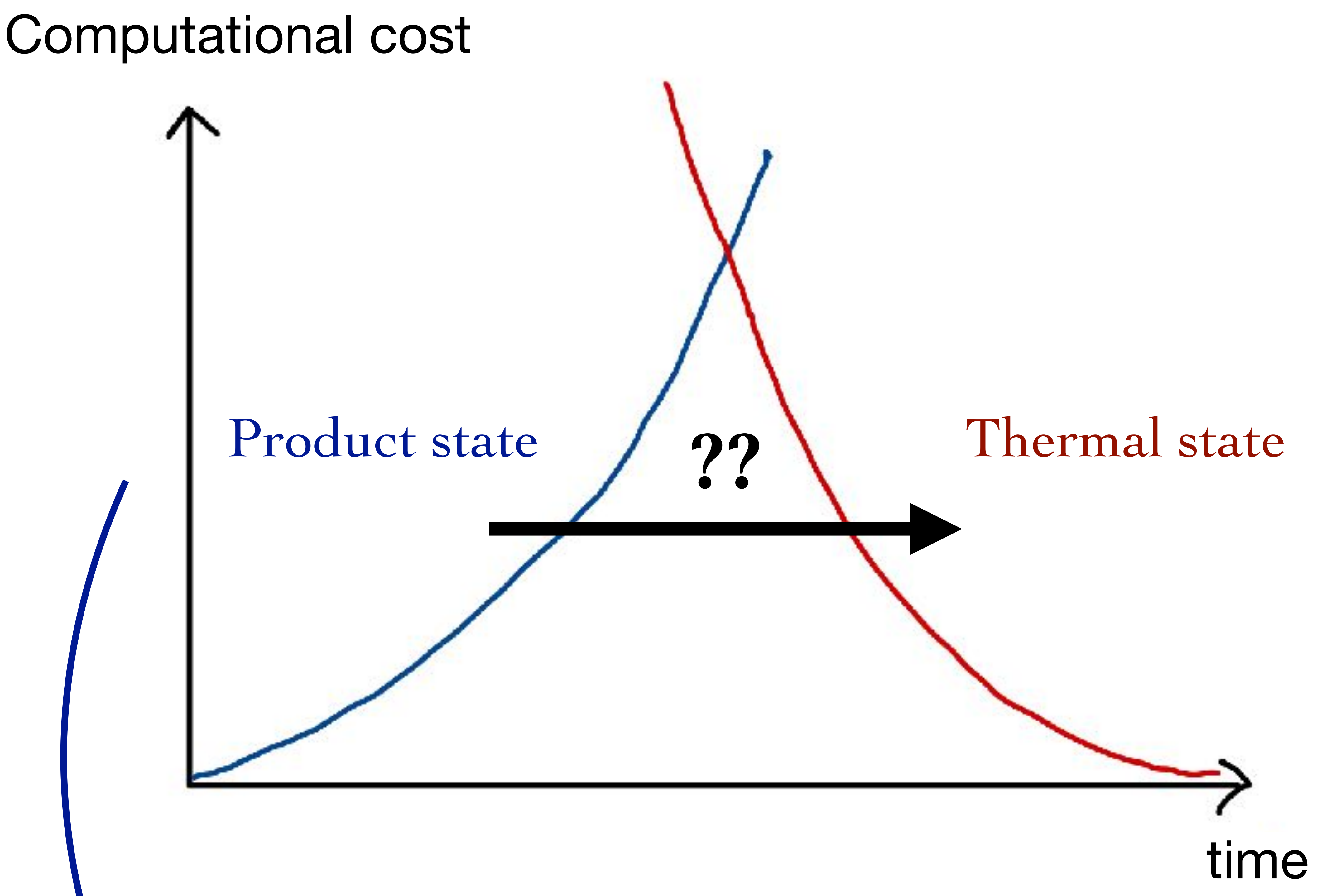
Product state of singlets

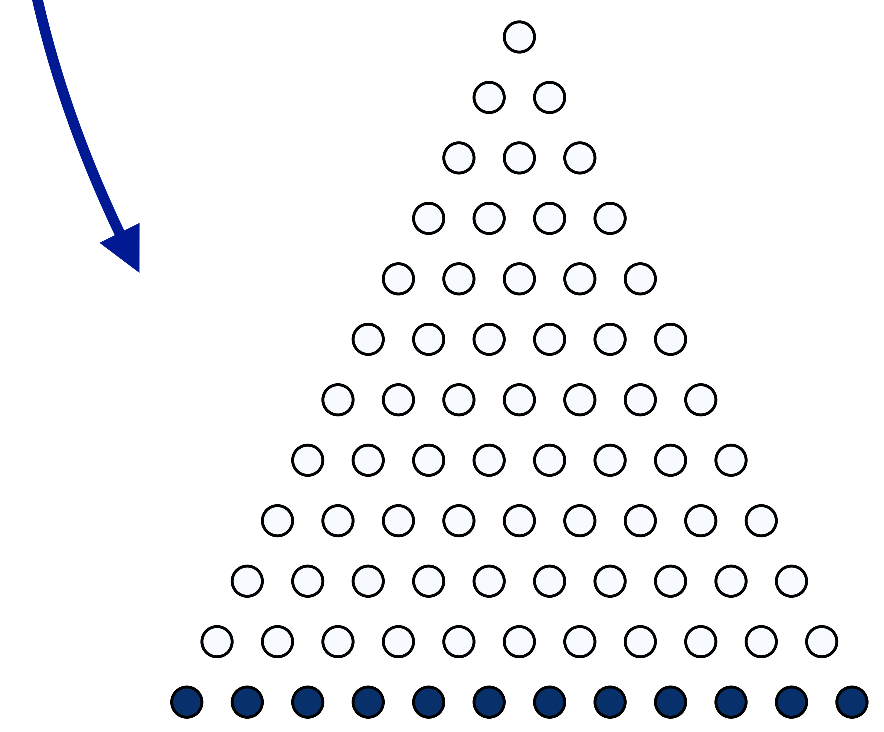


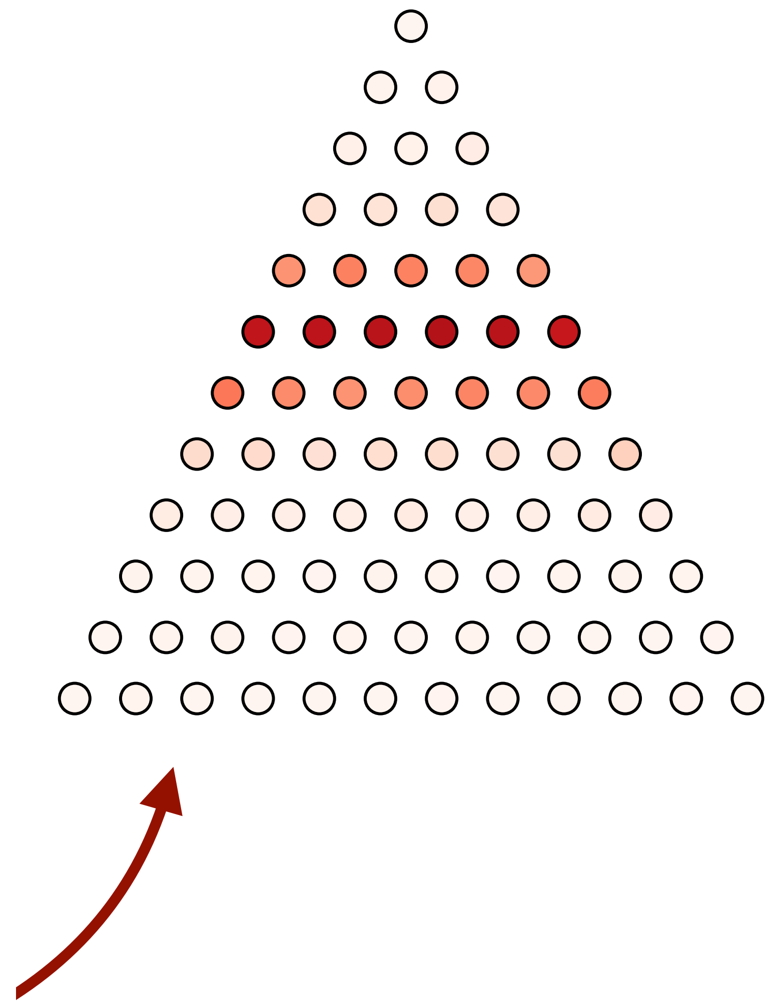
- i_n^ℓ **decomposes the total information in a region centered at** n **according to the corresponding scales**
- i_n^ℓ **is local: a local unitary on nearest neighbors can only move it from** ℓ **to** $\ell \pm 1$

Local information for the efficient time evolution of interacting quantum systems

Q: How to simulate the dynamics of thermalizing interacting quantum systems?







New approach: Local information time evolution (LITE)

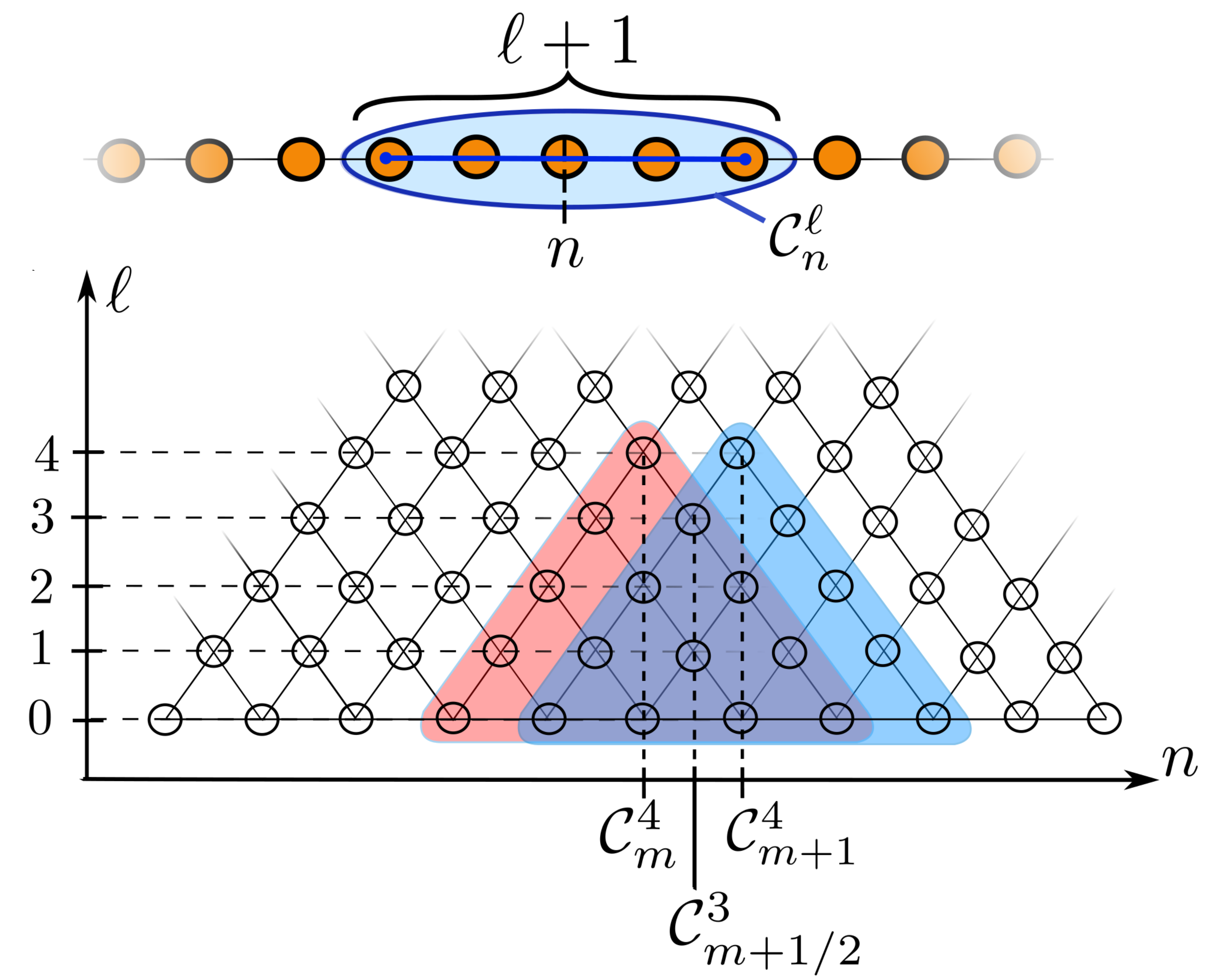
↓

Efficient simulation of the dynamics of local observables for interacting local Hamiltonians

[T.Klein Kvorning, L.Herviou, J.H.Bárðarson, SciPost Physics 2022]
[C.Artiaco, C.Fleckenstein, D.Aceituno Chávez, T.Klein Kvorning, J.H.Bárðarson, PRX Quantum 2024]

Time evolution on subsystems

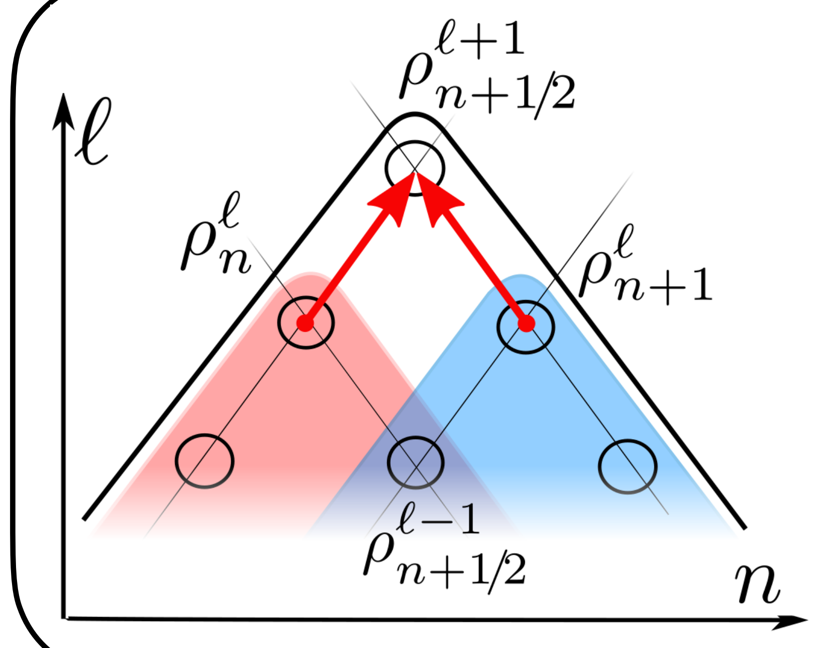
We decompose the system in smaller subsystems…



… to solve the von Neumann equation in parallel

$$\partial_t \rho_n^\ell = -i\left[H_n^\ell, \rho_n^\ell\right] - i\mathrm{Tr}_L^r\left(\left[H_{n-r/2}^{\ell+r} - H_n^\ell, \rho_{n-r/2}^{\ell+r}\right]\right) - i\mathrm{Tr}_R^r\left(\left[H_{n+r/2}^{\ell+r} - H_n^\ell, \rho_{n+r/2}^{\ell+r}\right]\right)$$

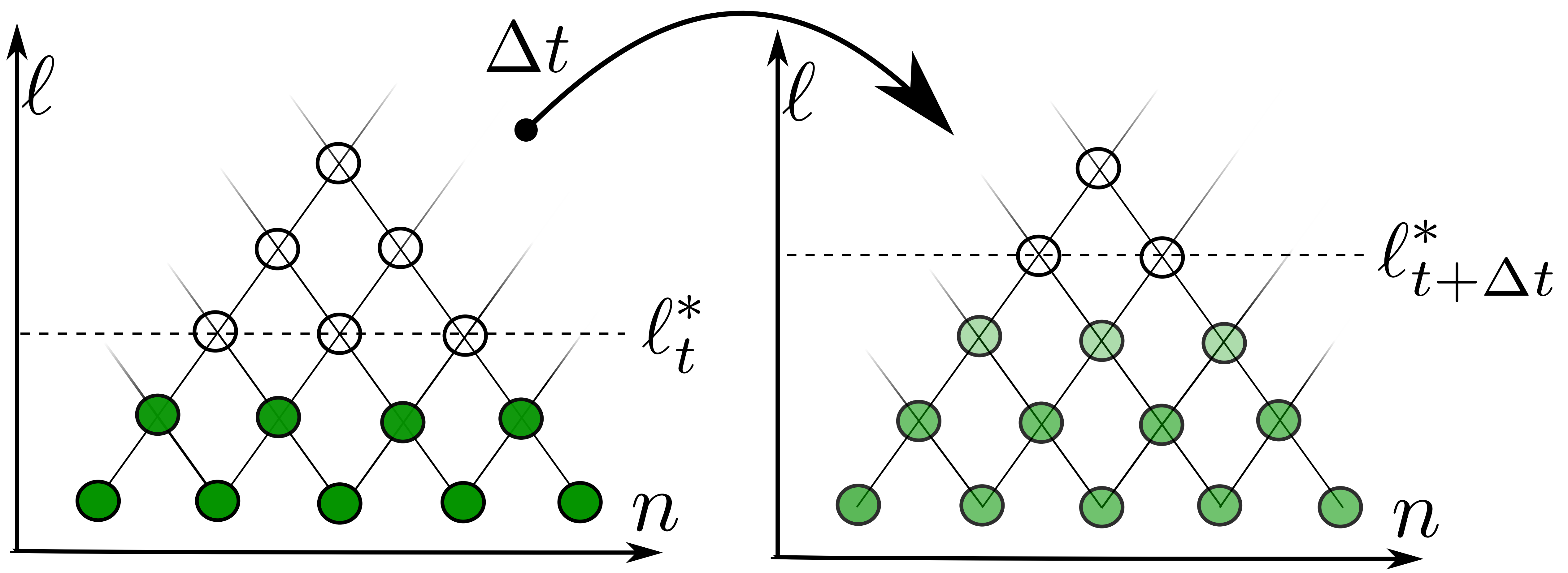
This gives rise to a non-closed hierarchy of equations!



$r = 1$: If there is **no information on scale** $\ell + 1$, we can construct $\rho^{\ell+1}$ via a **Petz recovery map**

$$\rho_{n+1/2}^{\ell+1} = \exp\left[\ln(\rho_n^\ell) + \ln(\rho_{n+1}^\ell) - \ln(\rho_{n+1/2}^{l-1})\right]$$

closing the von Neumann equation



But ℓ^* (on which local information is 0) increases with time (growth of entanglement)!

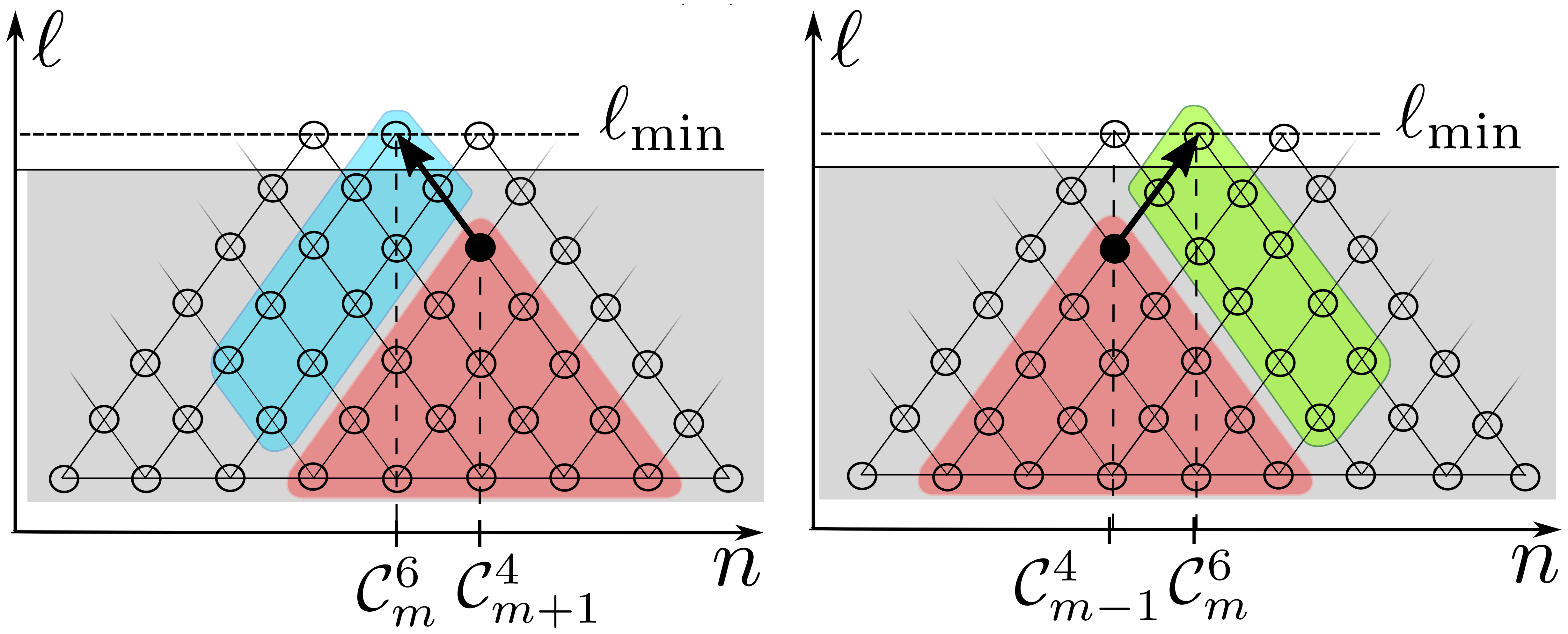
Petz recovery map at fixed scale fails

Minimize information on a large scale $\ell_{\min}$ keeping density matrices and information flow on smaller scales fixed

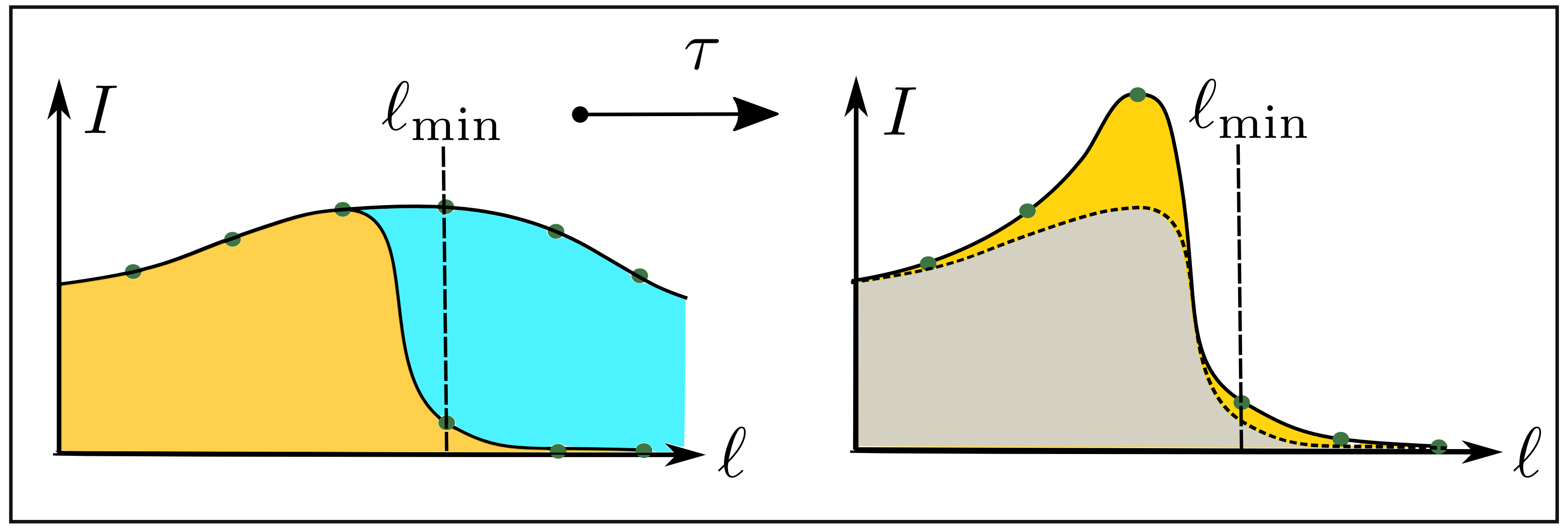
$$\bar{\rho}_n^\ell = \rho_n^\ell + \chi_n^\ell$$

$$\left.\begin{array}{ll} \text{I} & \left\{ \begin{array}{l} \mathrm{Tr}_R^1(\chi_n^\ell) = 0 \\ \mathrm{Tr}_L^1(\chi_n^\ell) = 0 \end{array} \right. \\ \text{II} & \left\{ \begin{array}{l} J_R^r(\chi_n^\ell) = 0 \\ J_L^r(\chi_n^\ell) = 0 \end{array} \right. \end{array}\right\} \text{Defines subspace to obtain } \xi = \mathbf{P}\chi$$

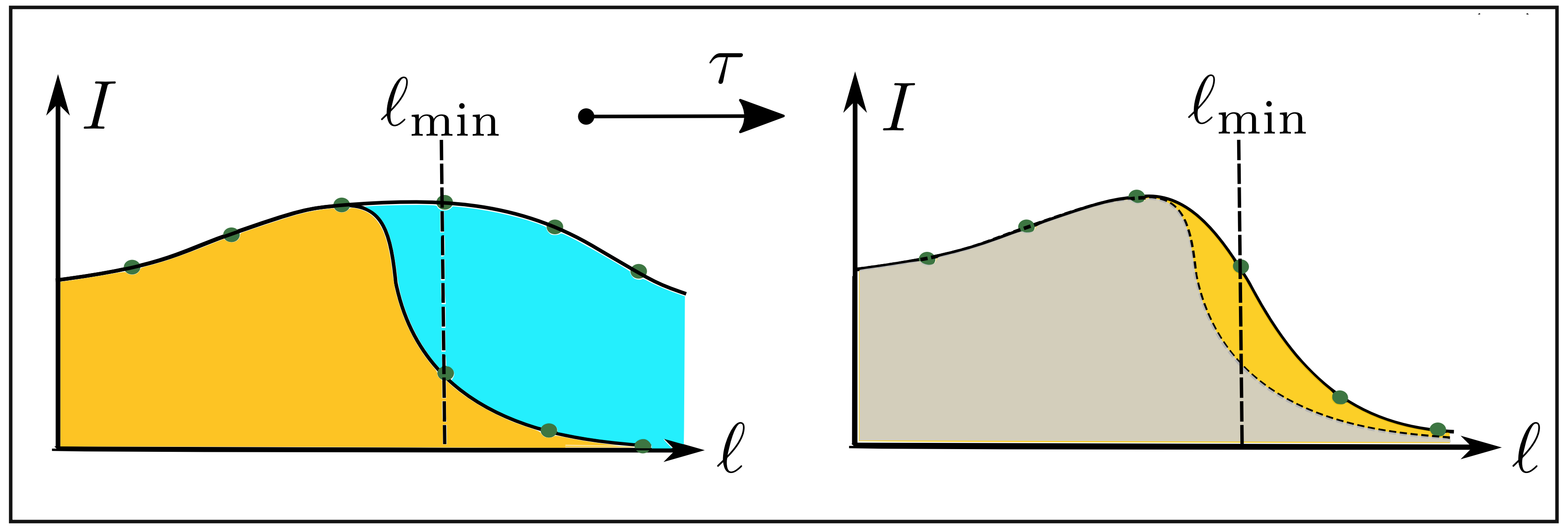
$\longrightarrow$ Maximize $S(\rho+\xi) = S(\rho) + \mathrm{Tr}\left(\mathbf{P}\nabla_\rho S \chi\right) + \frac{1}{2}\mathrm{Tr}\left(\chi\, \mathbf{P}\mathscr{H}_\rho \mathbf{P}\, \chi\right) + \mathcal{O}(\xi^3)$



Minimization under constraints I



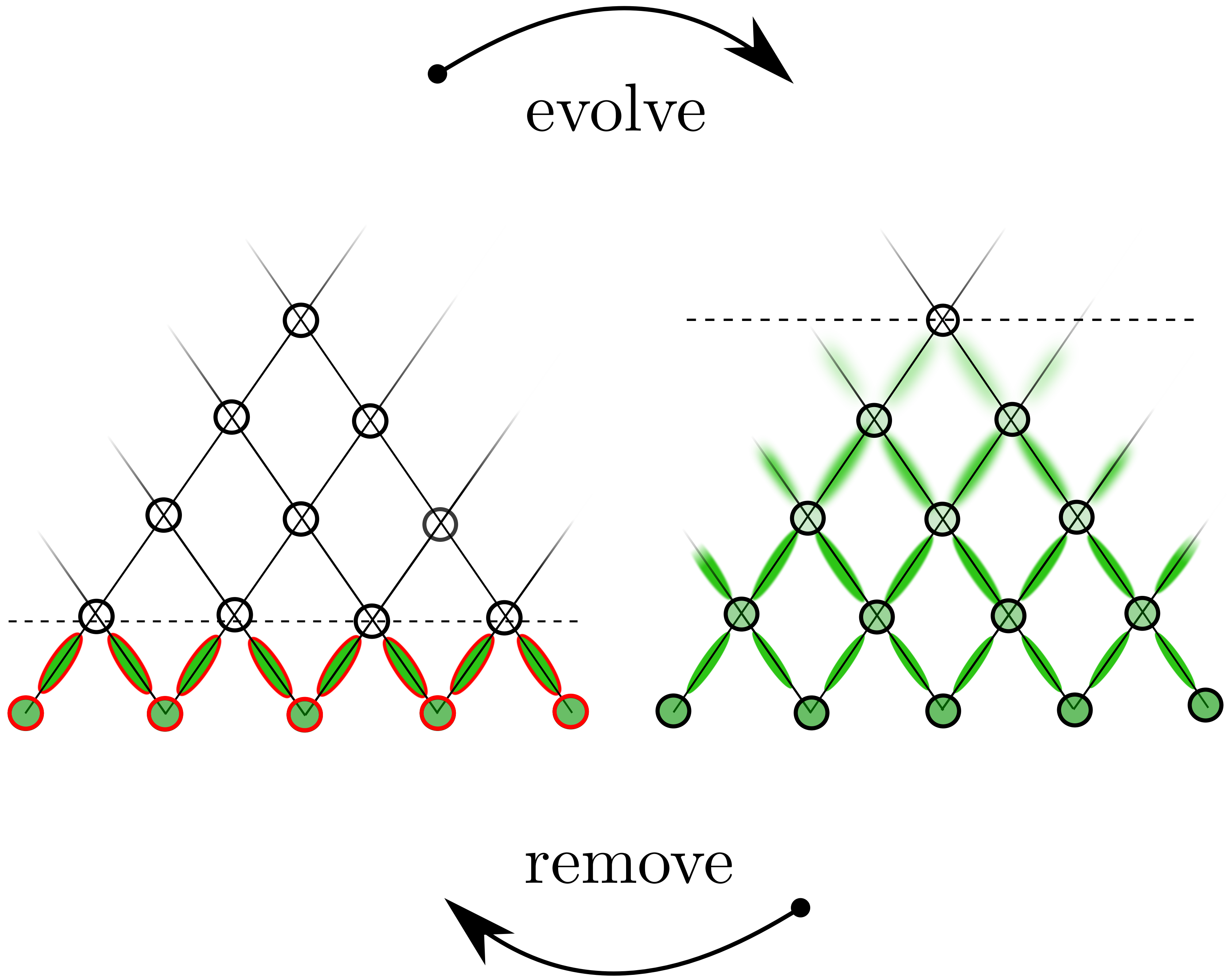
Minimization under constraints I + II



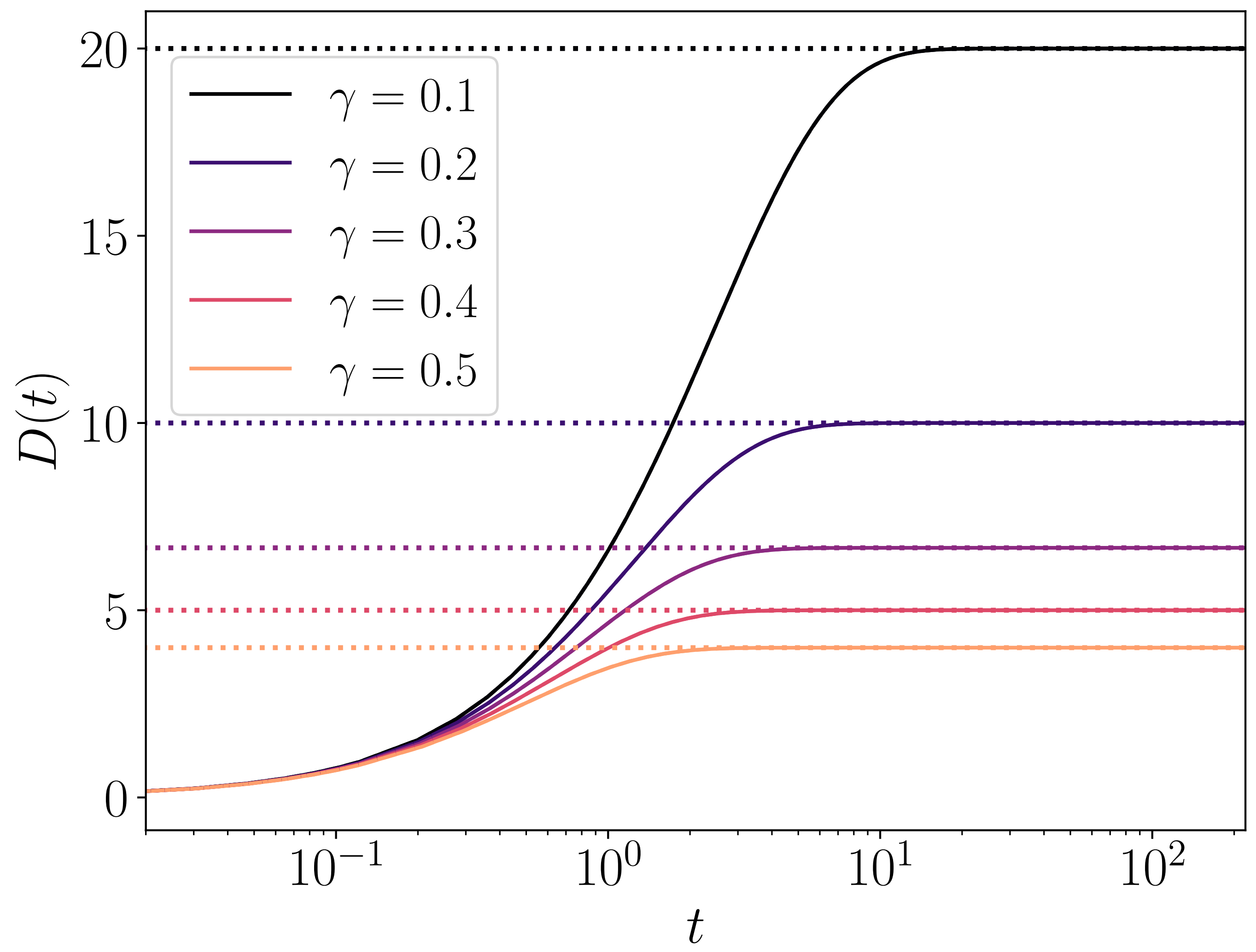
We get reliable information current evolving information to larger scale $\ell_{\max} > \ell_{\min}$

Magnetization & energy diffusion with LITE

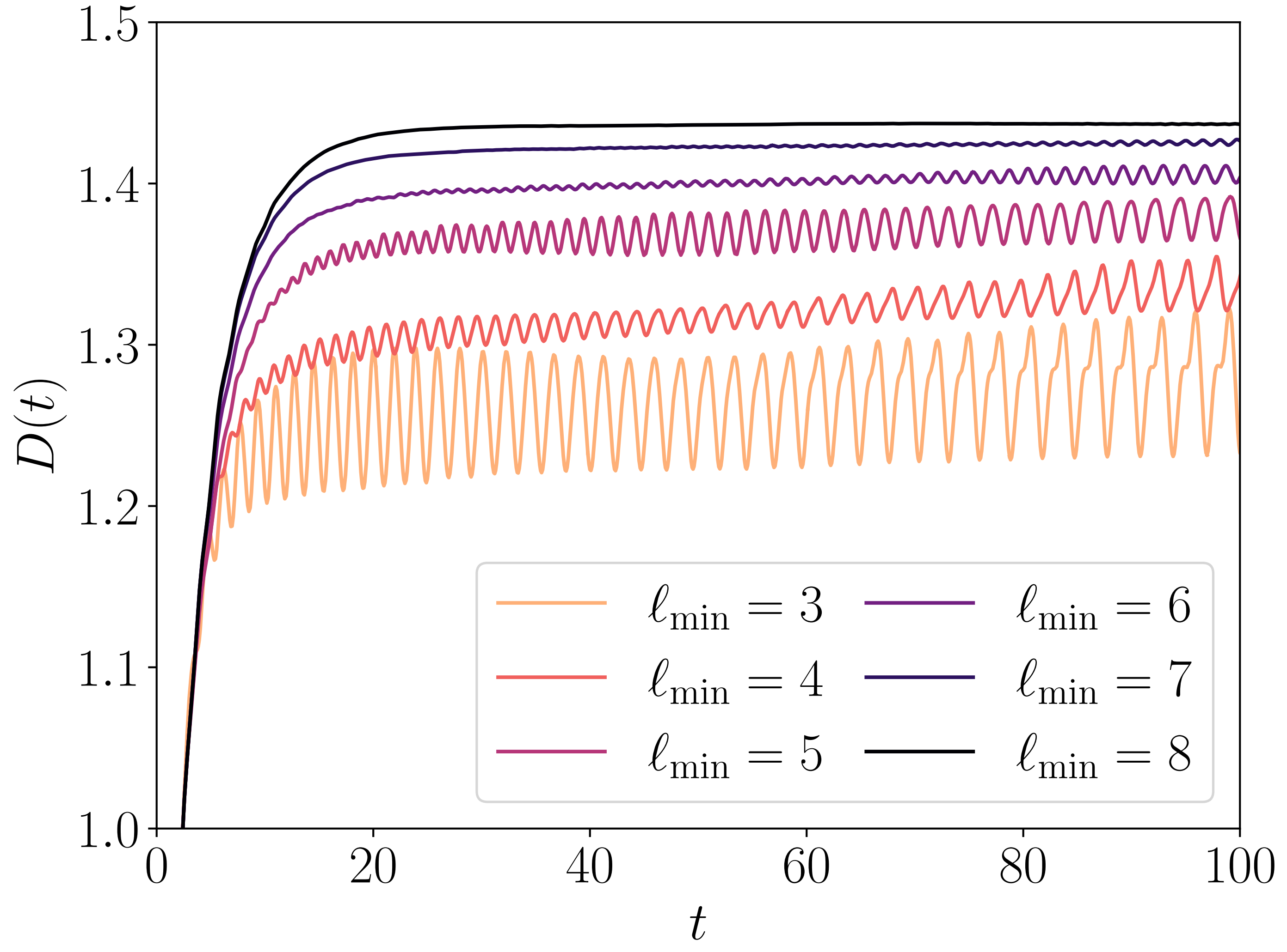
Remove information at large scales
preserving information currents



Open XX chain with dephasing



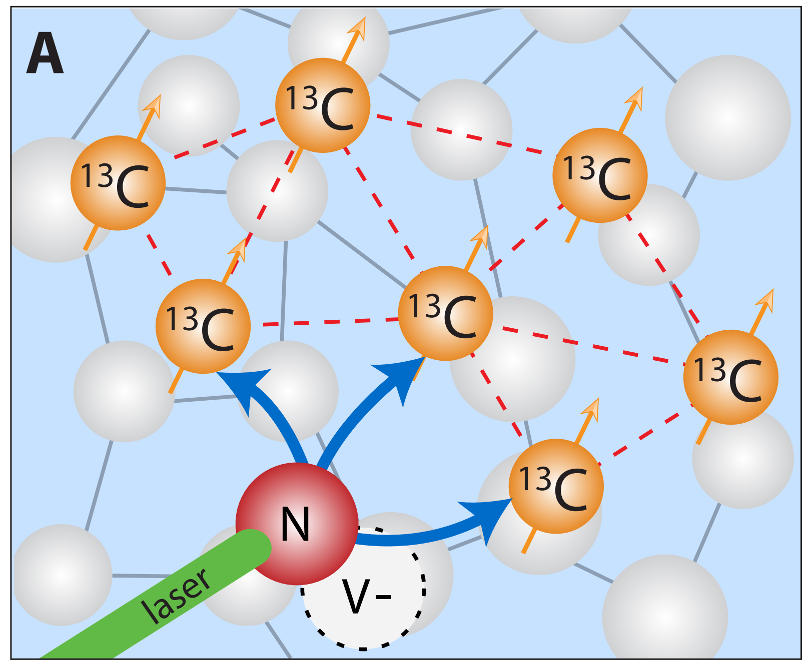
Mixed field Ising model



[C.Artiaco, C.Fleckenstein, D.Aceituno Chávez, T.Klein Kvorning, J.H.Bárðarson, PRX Quantum 2024]
[K.Harkins, C.Fleckenstein, N.D'Souza, P.M.Schindler, D.Marchiori, C.Artiaco, Q.Reynard-Feytis, U.Basumallick, W.Beatrez, A.Pillai, M.Hagn, A.Nayak, S.Breuer, X.Lv, M.McAllister, P.Reshetikhin, E.Druga, M. Bukov, A.Ajoy, Science Advances 2025]

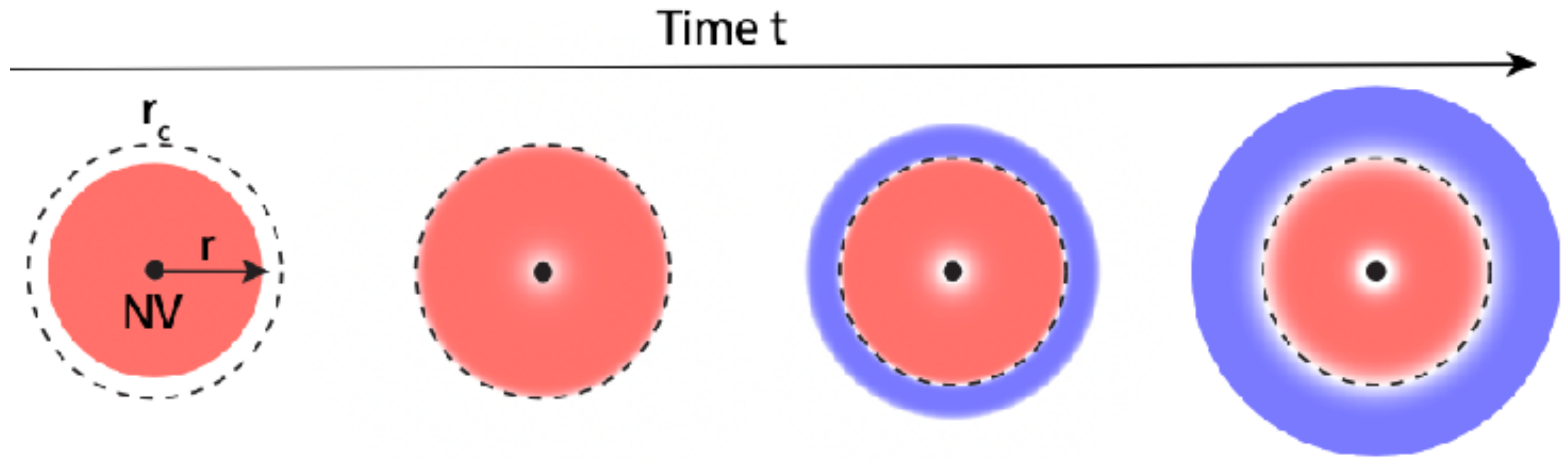
Nanoscale spin transport with LITE

Magnetization transport
in interacting Carbon-13 nuclear spins
surrounding a Nitrogen-Vacancy center
in diamond

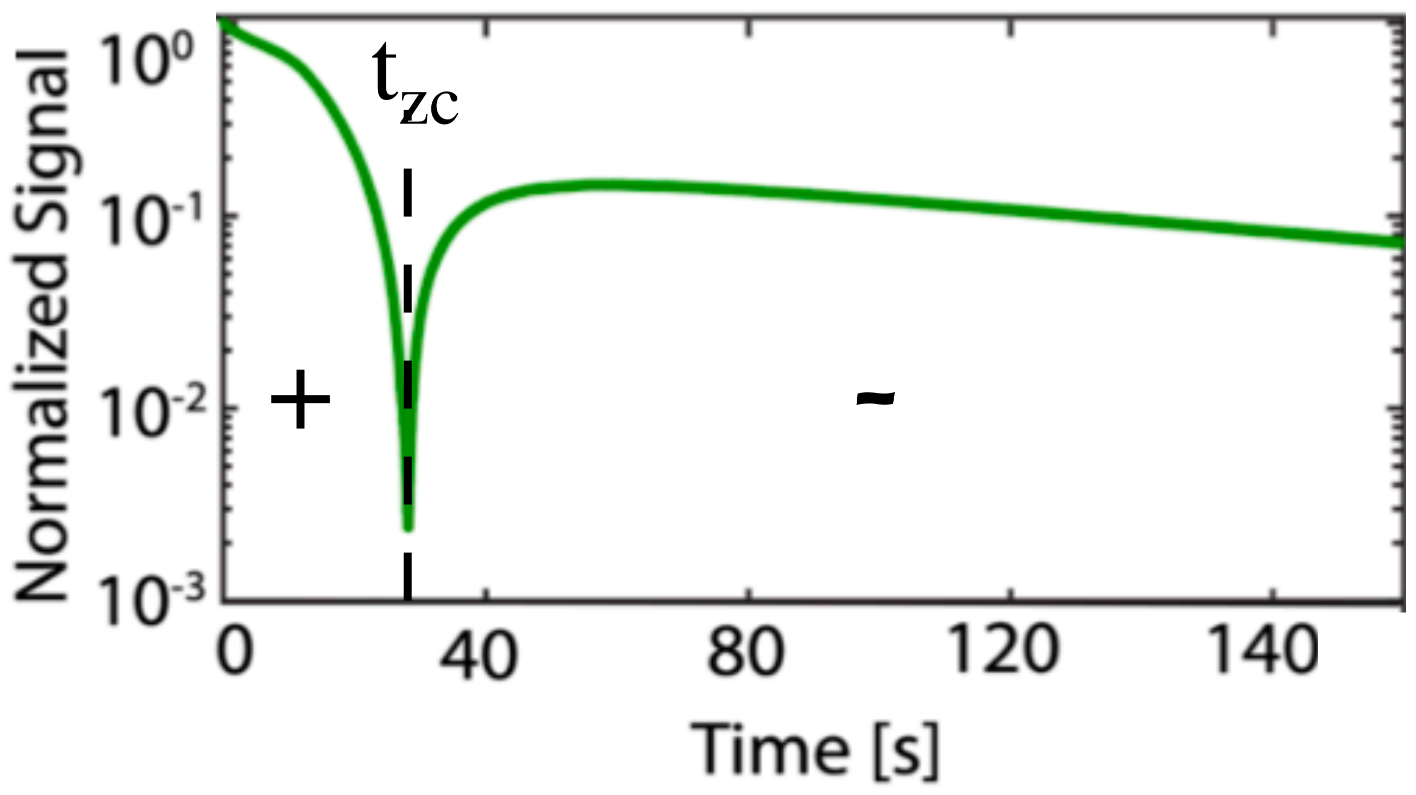


LITE is the state-of-the-art method for transport
→ **reproduced the experimental results and gave access to the spatial distribution of magnetization**

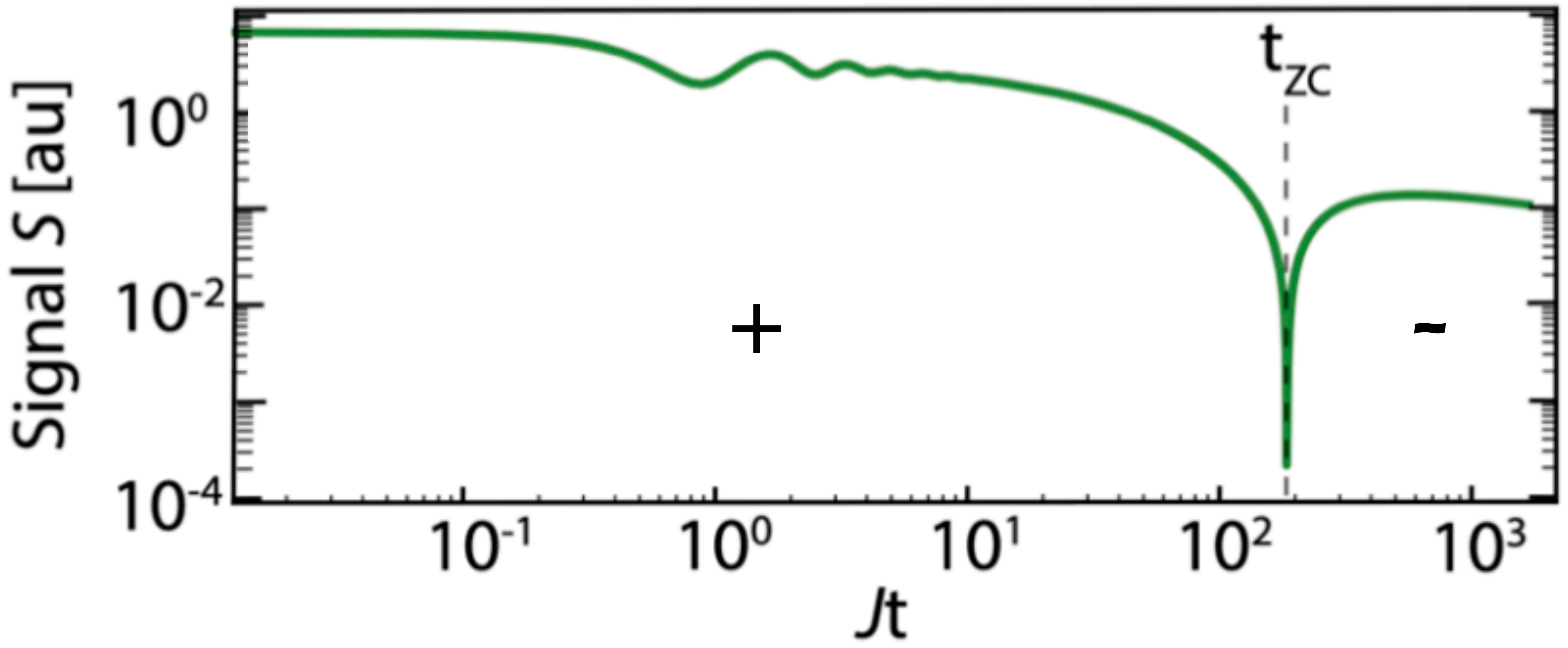
Stable spin texture



Experiment



Numerics with LITE

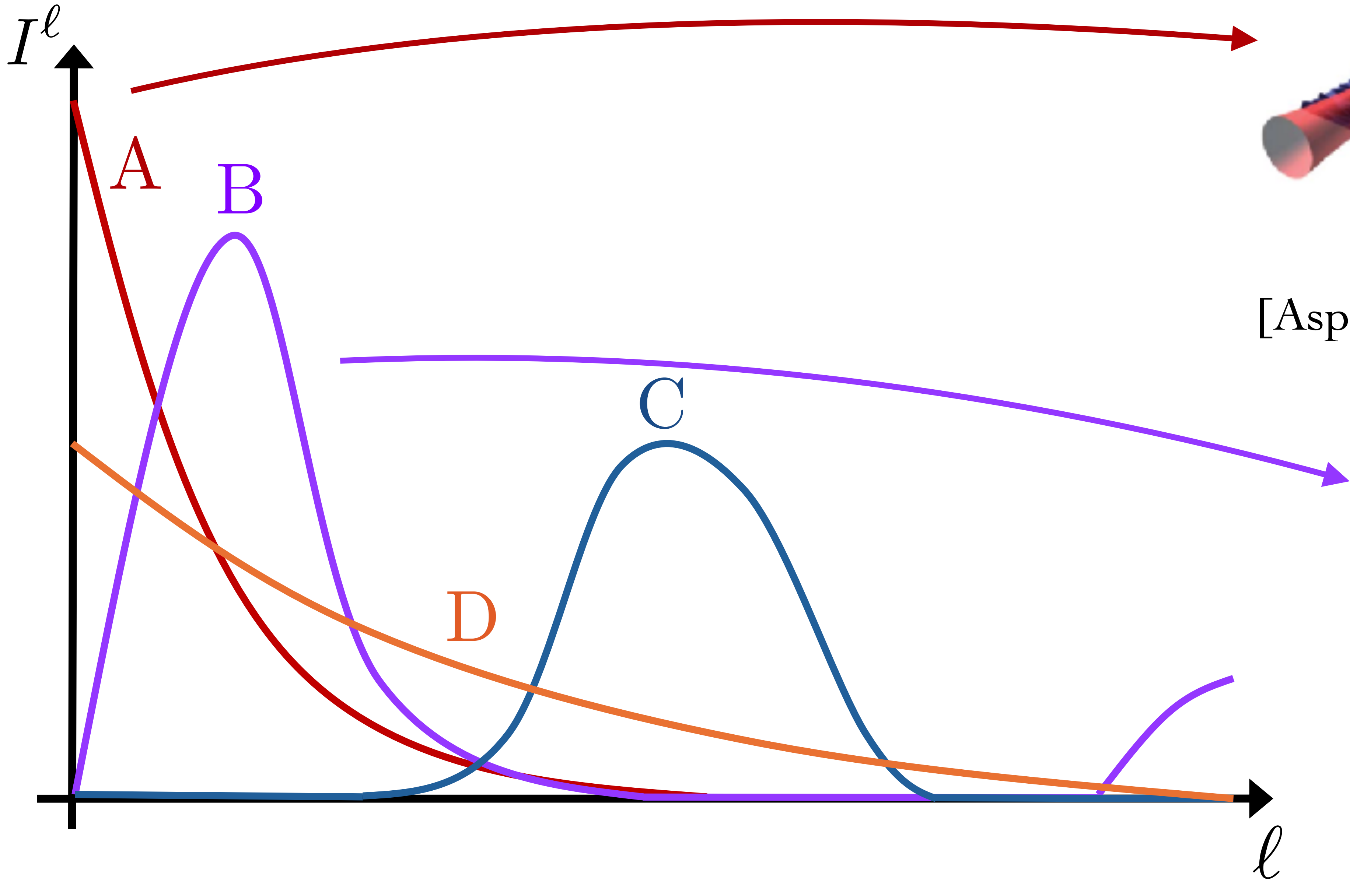


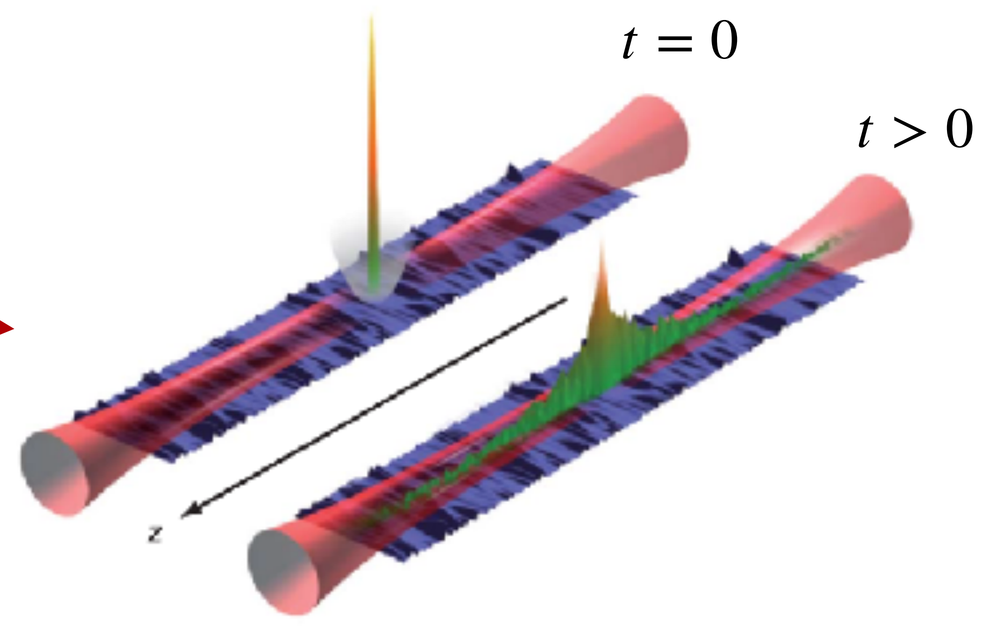
[K.Harkins, C.Fleckenstein, N.D'Souza, P.M.Schindler, D.Marchiori, **C.Artiaco**, Q.Reynard-Feytis, U.Basumallick, W.Beatrez, A.Pillai, M.Hagn, A.Nayak, S.Breuer, X.Lv, M.McAllister, P.Reshetikhin, E.Druga, M. Bukov, A.Ajoy, Science Advances 2025]

Characterizing states through local information

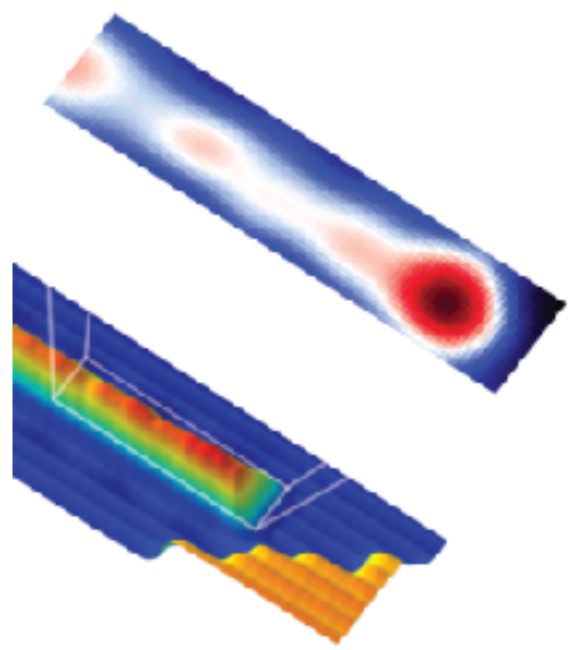
The information lattice characterizes quantum states through the distribution of local information

Information per scale $I^\ell = \sum_n i_n^\ell$





[Aspect group, Nature 2008]



[Yazdani group, Science 2014]

(A) Localized state

(B) Localized state with topological edge modes

(C) Thermal state

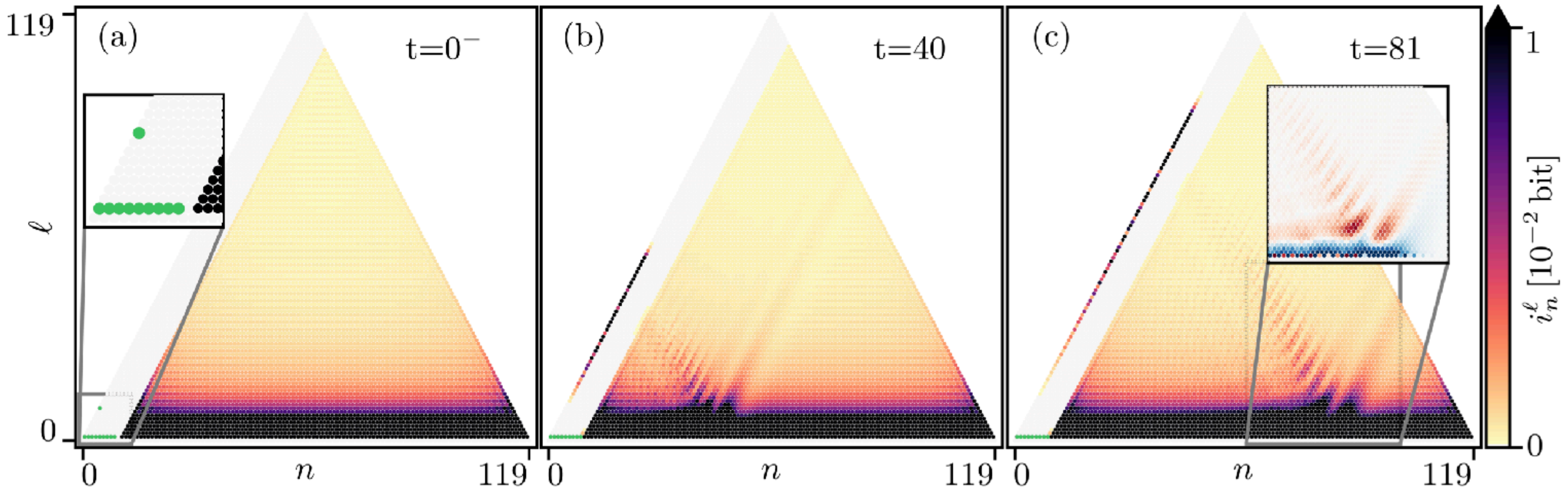
(D) Critical state

[**C.Artiaco**, T.Klein Kvorning, D.Aceituno Chávez, L.Herviou, J.H.Bárðarson, PRL 2025]

Local information flow in quantum quench dynamics

The local information flow characterizes quantum quench dynamics

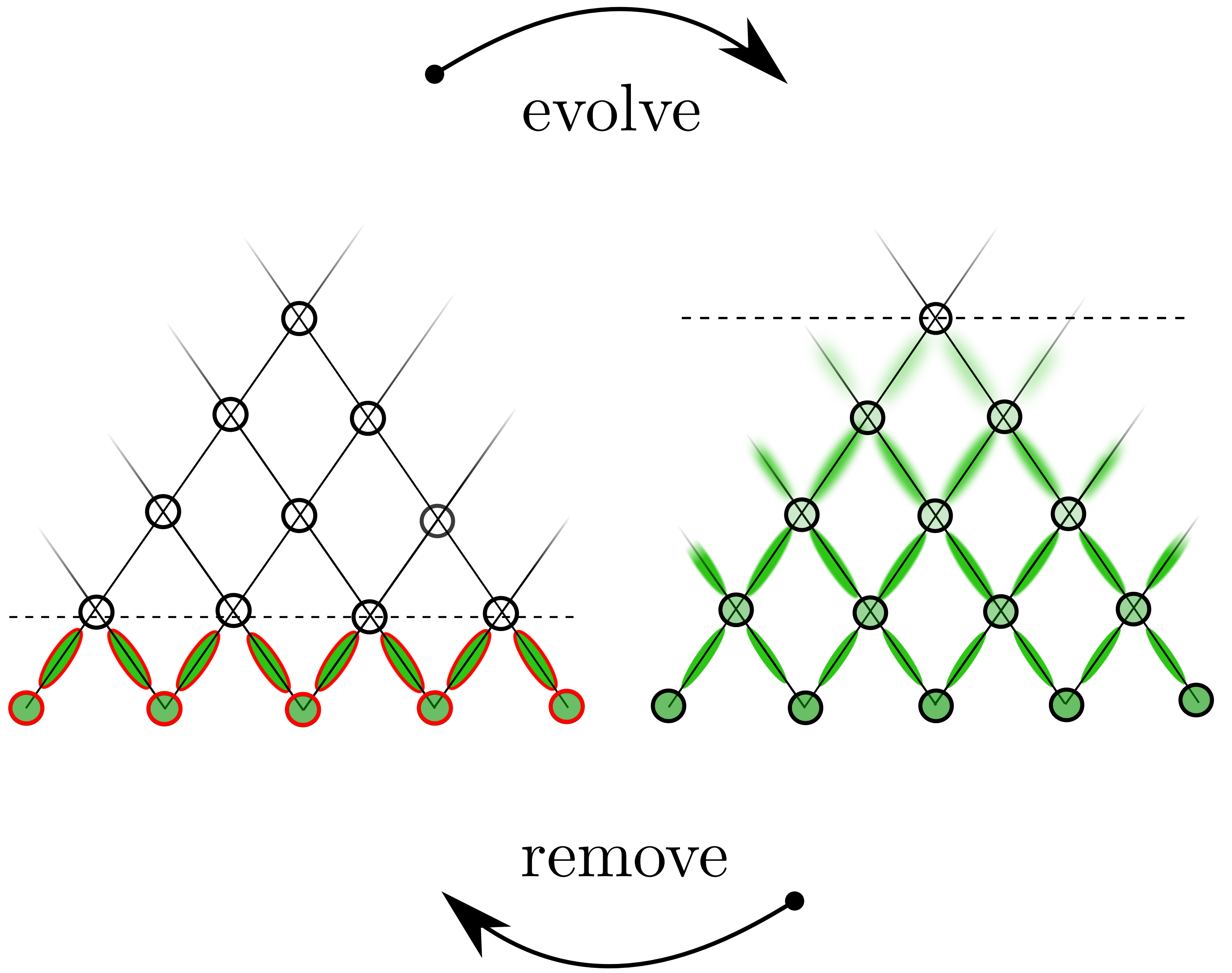
Example: Topological Kitaev chain coupled to a tight-binding chain



It explains previously observed fractional signatures of entanglement entropy

[N.P.Bauer, B.Trauzettel, T.Klein Kvorning, J.H.Bárðarson, C.Artiaco, arXiv:2505.00537]

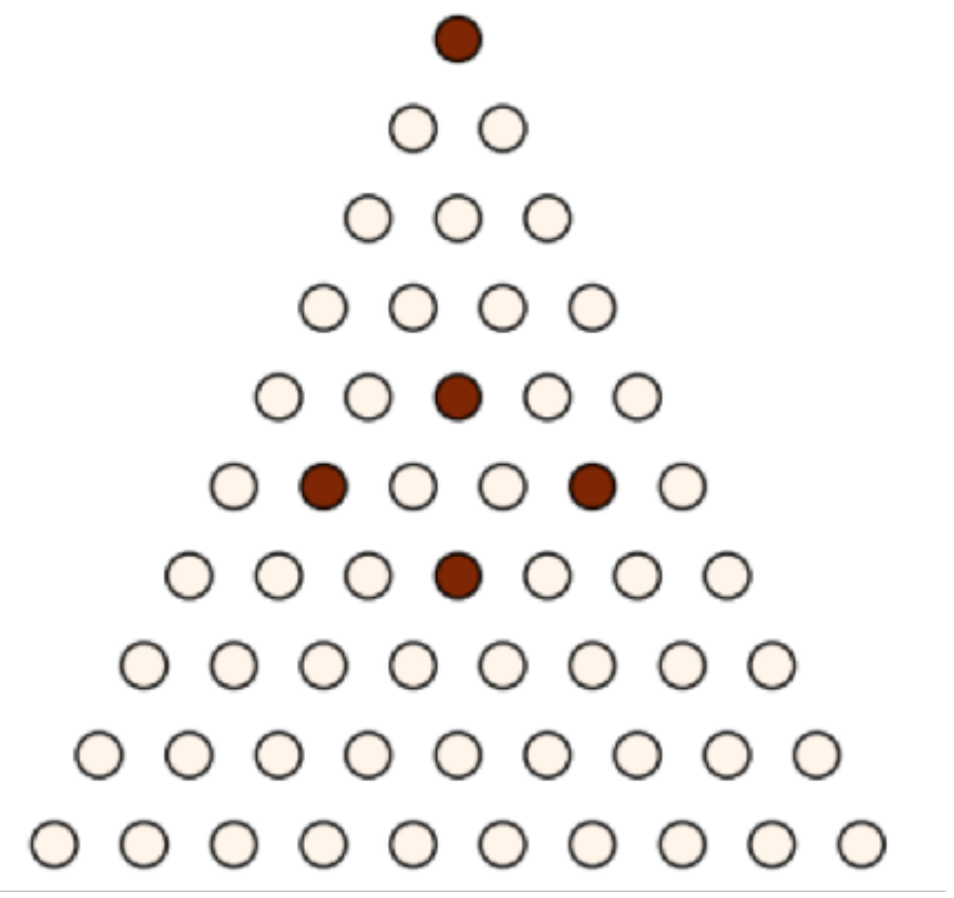
Conclusions



LITE is an efficient method for time evolution of large many-body quantum systems up to large times

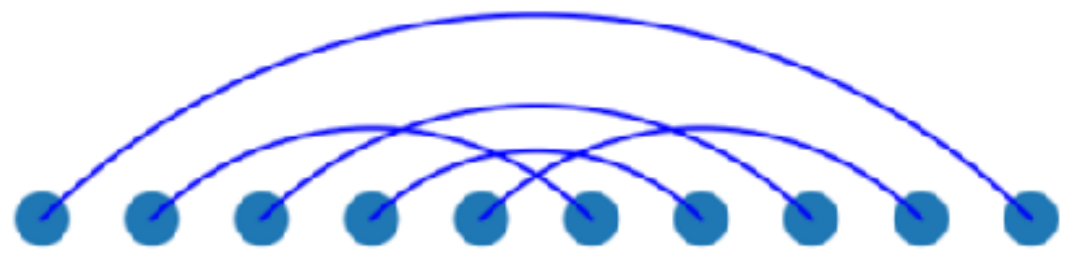
LITE applies to generic local Hamiltonians:

- Disordered and localized systems
- Subdiffusive, diffusive, superdiffusive, and ballistic hydrodynamic behaviors
- Open quantum system governed by the Lindblad equation
- Driven systems
- Higher dimensional systems



Thank you!

The code will soon be publicly available and fully documented



The information lattices a universal framework for characterizing quantum states and quantum dynamics