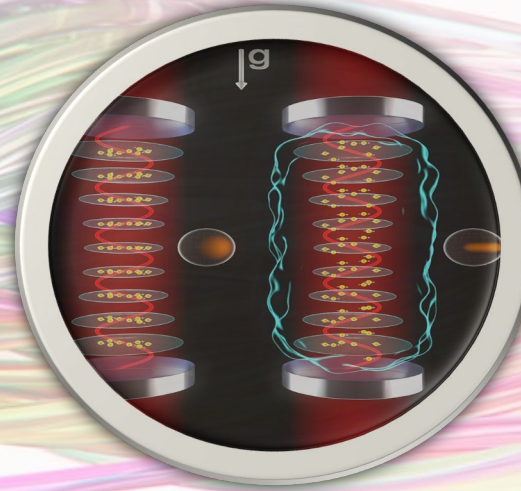
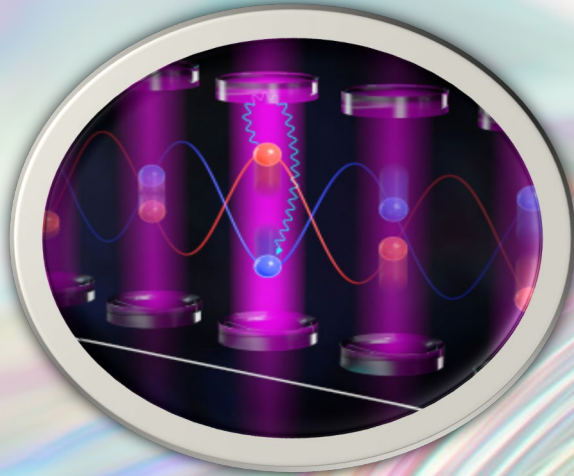


More than two can dance: Twisting, double-twisting & binding in an optical cavity

Ana Maria Rey

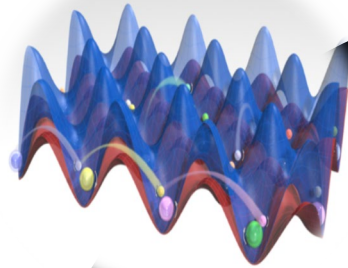


Long-Range Interactions and Dynamics in Complex Quantum Systems, July 25th (2025)

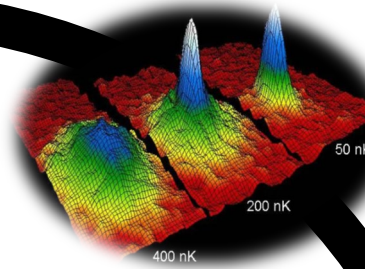
JILA
NISTCU

Controllable Long-range Interacting Systems

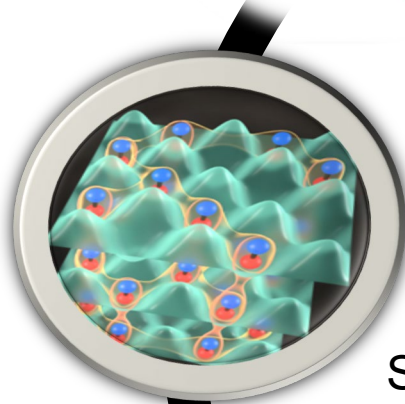
Bose-Fermi Hubbard
lattice models



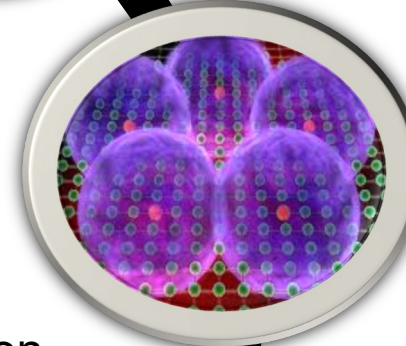
Bose and Fermi: Matter-waves



Polar Molecules:



Rydberg Atoms



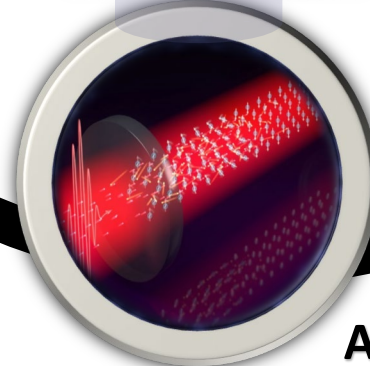
Magnetic Atoms



Trapped Ions



Atoms in Optical cavities



Spin



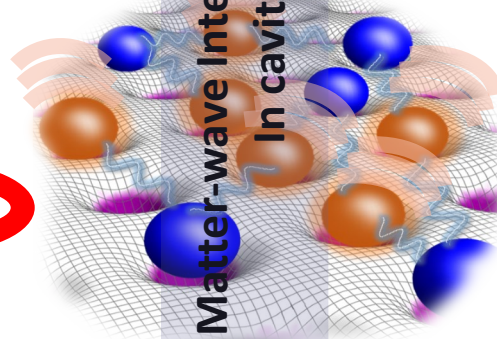
Spin Motion



Motion



Matter-wave Interferometers
In cavities



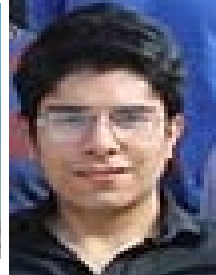
Theory:



D. Barberena R. Lewis-Swan



Chengyi Luo Anjun Chu



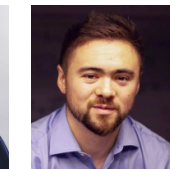
A. Piñeiro B. Sundar E. Chaparro J. Young A. Shankar B. Zhu



J Bollinger, Klaus Mølmer, M. Holland, R. Fazio

Experiment:

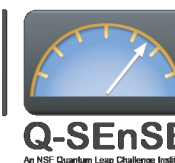
James Thompson



M. Norcia,
V. M. Schäfer

Z. Niu E. Yilun D. Young C. Maruko E. Bohr

J. Ye



Hamiltonian Engineering

Binding and Twisting

- Cavity-mediated momentum-exchange interaction
Luo, Zhang, Koh, Wilson, Chu, Holland, Rey, Thompson, *Science* 384 ,551 (2024)
- Solitons in arbitrary dimensions stabilized by photon-mediated interactions
Zhang, Chu, Luo, Thompson, Rey, arXiv:2504.1707

Double the Twisting

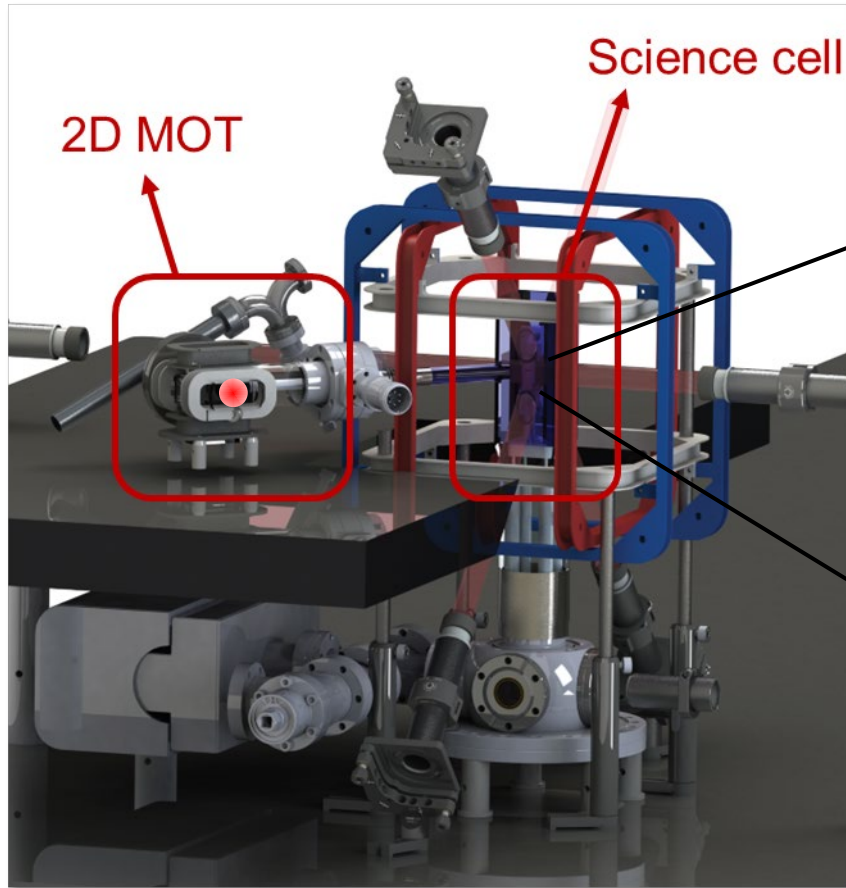
- XYZ Hamiltonian engineering of momentum states
Luo*, Zhang*, Chu, Maruko, Rey, Thompson, *Nature Physics*, 916 (2025)

Many - Buddies

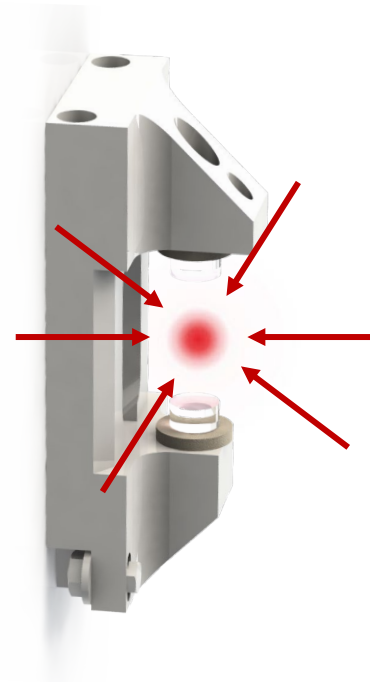
- Observation of three and four-body interactions between momentum states mediated by a high-finesse cavity
Luo*, Zhang*, Chu, Maruko, Rey, Thompson, arXiv:2410.12132, In press *Science*



JILA Cavity QED Rb System

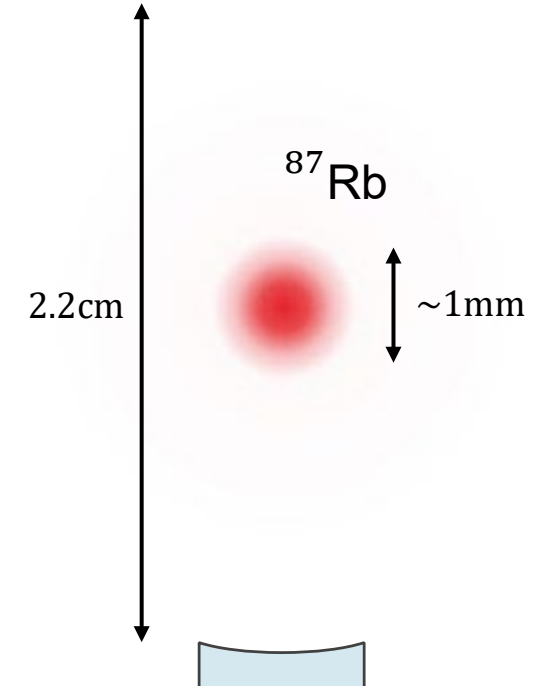


$$F = 130000$$
$$C \sim 0.5$$



$$FSR = 6.79 \text{ GHz}$$

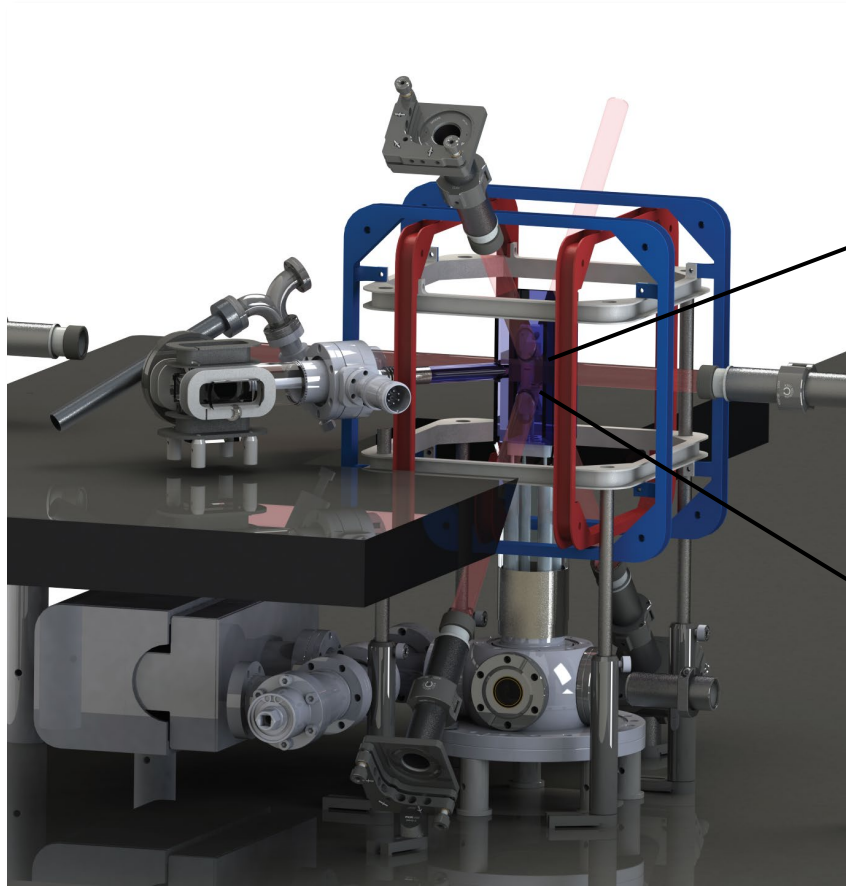
$$\kappa = 52 \text{ kHz}$$



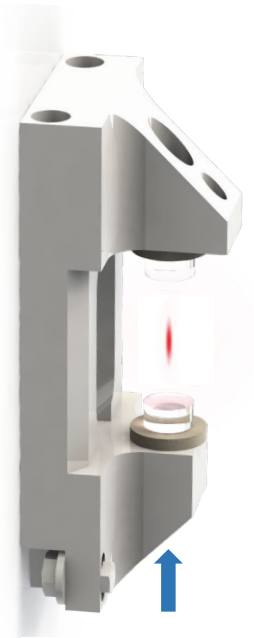
Vertical high-finesse cavity

Atoms Doppler Cooled to few μK

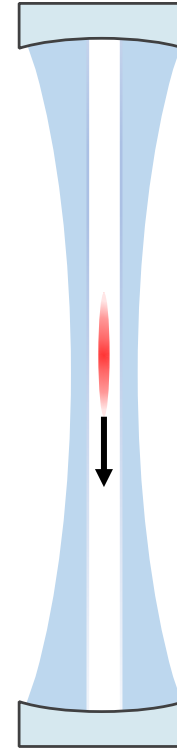
JILA Cavity QED Rb System



$$F = 130000$$
$$C \sim 0.5$$

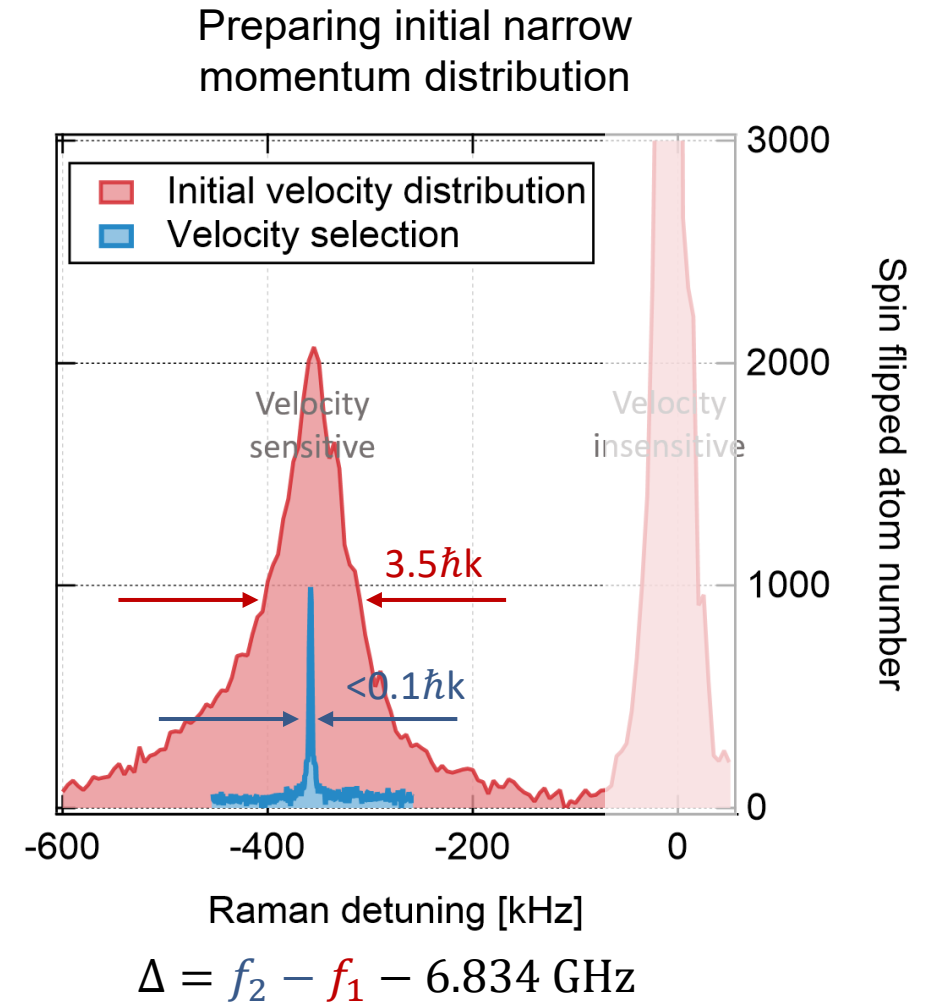
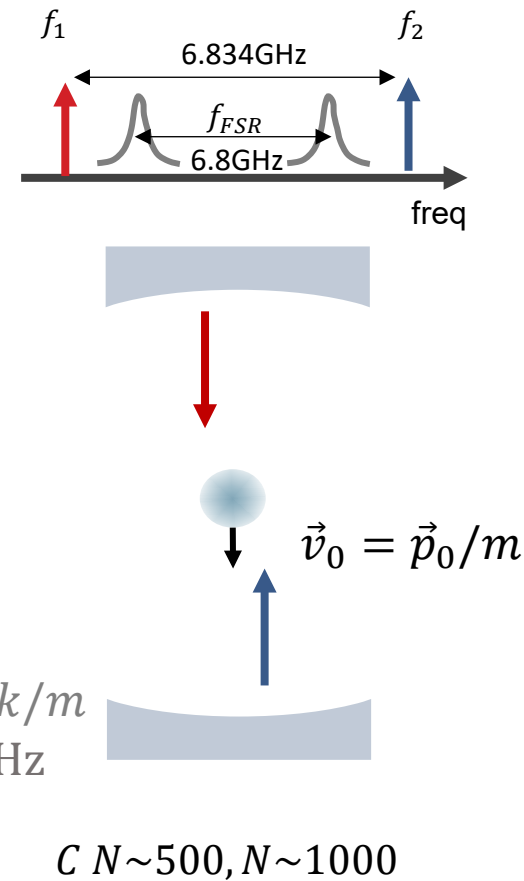
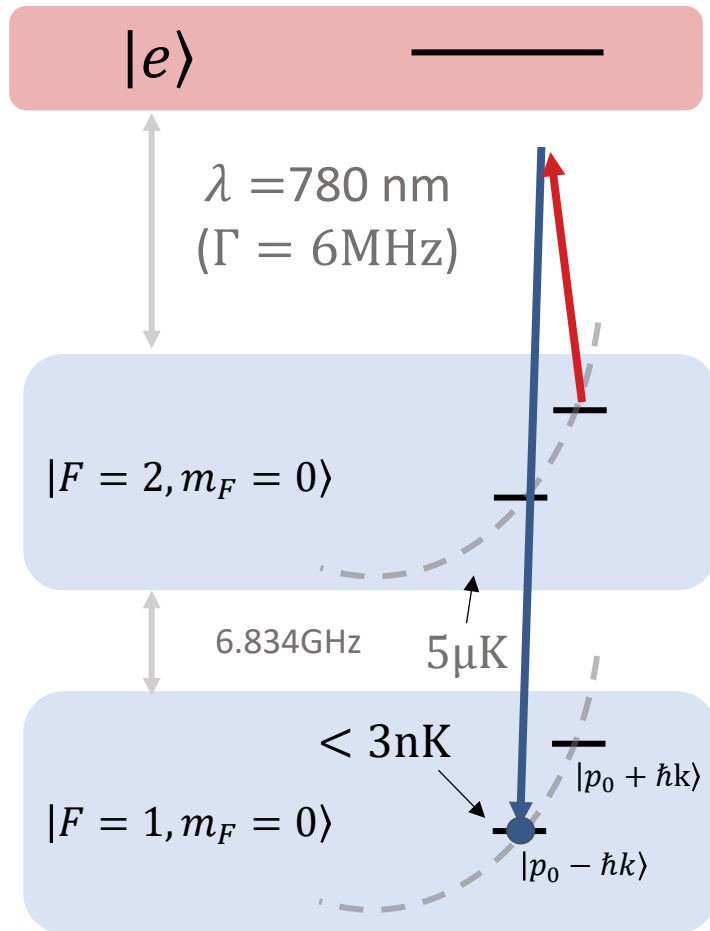


Repulsive



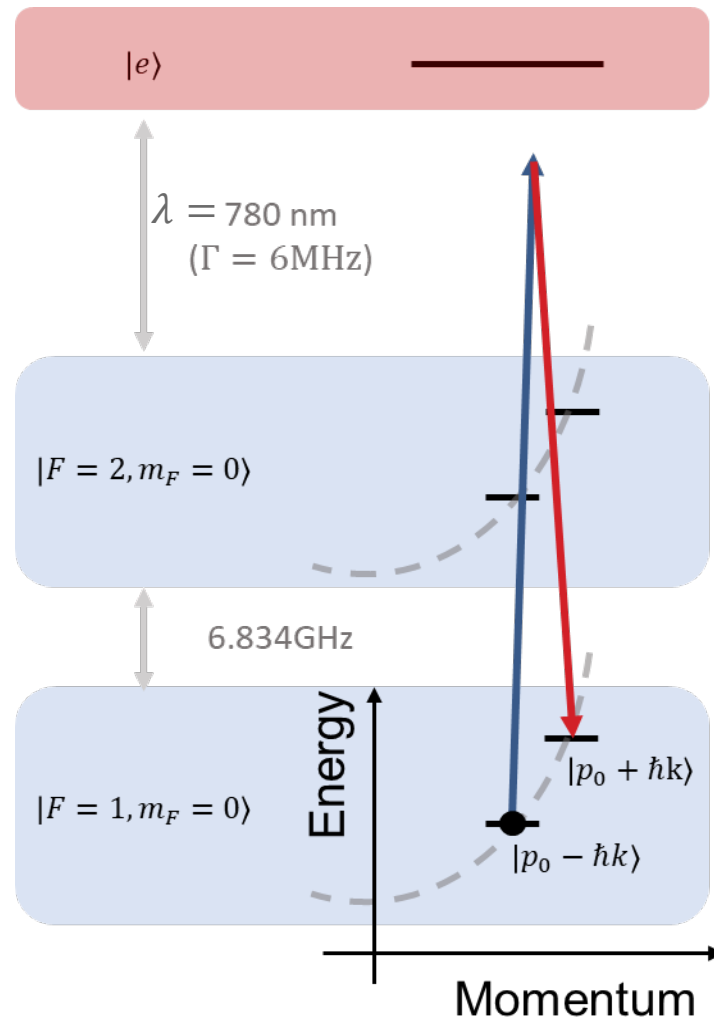
760nm dipole trap
(exciting multiple FSR)

Velocity Selection: Narrow momentum distribution

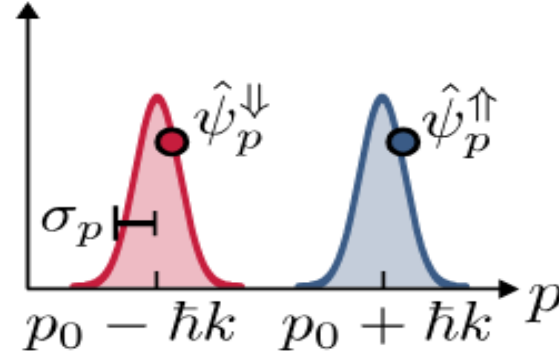


Bragg- Interferometer: Only momentum states

Luo, Zhang, ..., Rey, Thompson, *Science* 384 (6695), 551 (2024)



Momentum distribution



$$\sigma_p \sim 0.1\hbar k \ll 2\hbar k$$

$$p \in [-\hbar k, \hbar k]$$

$$|\psi(p)\rangle = \sin\left(\frac{\theta}{2}\right) |p + p_0 + \hbar k\rangle + \cos\left(\frac{\theta}{2}\right) e^{i\phi} |p + p_0 - \hbar k\rangle$$

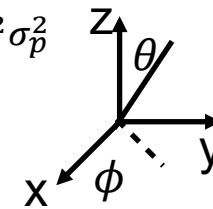
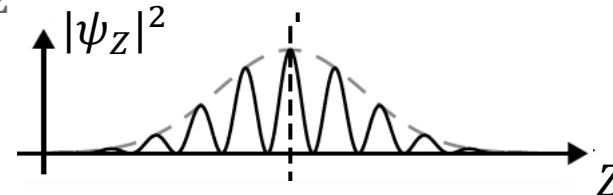
Density grating

$$|\psi_z|^2 = [1 + \sin \theta \cos(2kZ - \phi)] e^{-Z^2 \sigma_p^2}$$

$$\theta = \pi/2$$

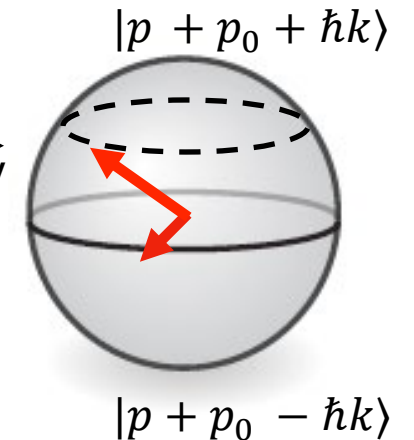
$$\theta = \pi/4$$

$$\omega_z = 2p_0 \hbar k / m = 500 \text{ kHz}$$



$$\theta = \pi/2$$

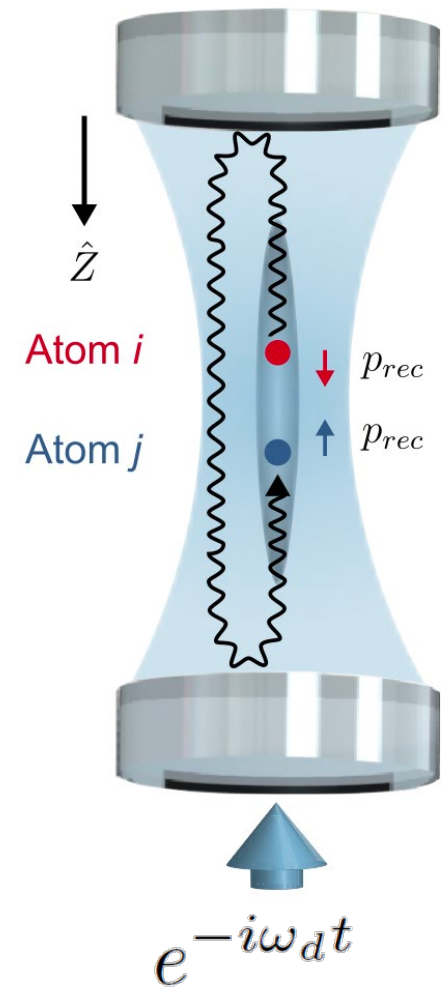
$$\theta = \pi/4$$



Shankar *et al* *QS&T*. **4** 045010 (2019)

Wilson *et al*, *PRA* **106**, 043711 (2022)

Atom/Light Hamiltonian



$$\hat{H}_{\text{lab}} = \hat{H}_{\text{atom}} + \hat{H}_{\text{light}} + \hat{H}_{\text{int}}$$

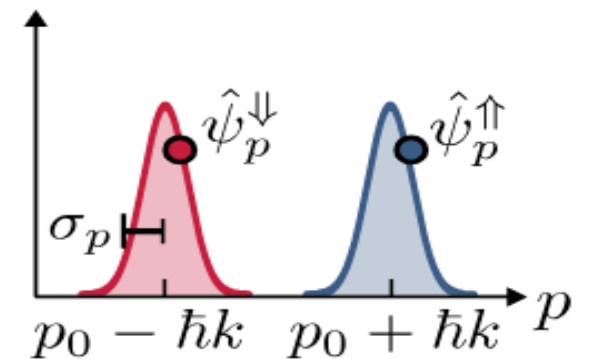
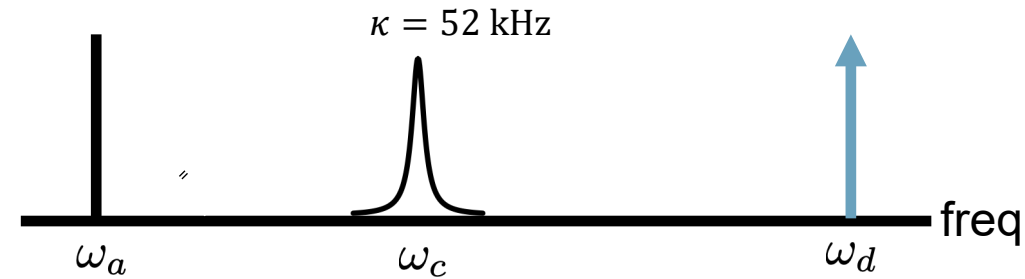
$$\hat{H}_{\text{light}} = \hbar (\eta \hat{a}^\dagger e^{-i\omega_d t} + \eta^* \hat{a} e^{i\omega_d t}) + \hbar \omega_c \hat{a}^\dagger \hat{a}$$

$$\hat{H}_{\text{atom}}^e = \hbar \omega_a \int \hat{\psi}_e^\dagger(Z) \hat{\psi}_e(Z) dZ$$

$$\hat{H}_{\text{atom}}^g = \int \hat{\psi}_g^\dagger(Z) \frac{\hat{p}^2}{2m} \hat{\psi}_g(Z) dZ$$

Kinetic energy

$$\hat{H}_{\text{atom}}^g = \omega_z \hat{S}_z + \underbrace{\frac{2\hbar k}{m}}_{\text{Doppler dephasing } \vec{v}_{\text{rec}}} \sum_p p \hat{S}_z(p) + \sum_p \frac{p^2}{2m} \hat{I}$$



$$\omega_z = \frac{2kp_0}{m}$$

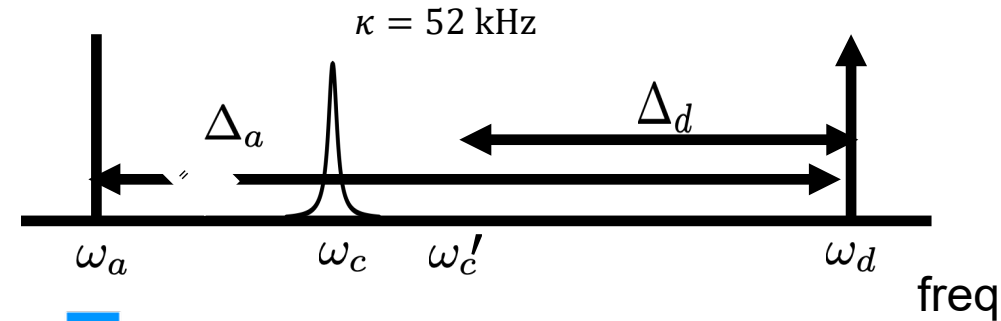
$$\hat{S}_\alpha = \sum \hat{s}_\alpha(p)$$

Atom-Light Interaction

Tavis-Cuming Interaction

$$\hat{H}_{\text{int}} = \hbar g_0 \int \cos(kZ) \left[\hat{a} \hat{\psi}_e^\dagger(Z) \hat{\psi}_g(Z) + \hat{a}^\dagger \hat{\psi}_g^\dagger(Z) \hat{\psi}_e(Z) \right] dZ$$

$$\Delta_a = \omega_a - \omega_d \sim 500 \text{ MHz} \quad \Delta_a \gg \sqrt{N_{\text{ph}}} g_0$$



Elimination of the excited state

Rotating frame of the pump

$$\Delta_d = \omega_d - \omega_c - \frac{N g_0^2}{2 \Delta_a}$$

Detuning between pump field and dressed cavity resonance

$$\hat{H}_{\text{light}} = -\hbar \Delta_d \hat{a}^\dagger \hat{a} + \hbar (\eta \hat{a}^\dagger + \eta^* \hat{a})$$

$$H_{a-c} = G \hat{a}^\dagger \hat{a} \int dZ \left[\underbrace{\hat{\psi}_g(Z)^\dagger e^{i2kZ} \hat{\psi}_g(Z)}_{\hat{S}_+} + \underbrace{\hat{\psi}_g(Z)^\dagger e^{-i2kZ} \hat{\psi}_g(Z)}_{\hat{S}_-} \right] = G \hat{a}^\dagger \hat{a} (\hat{S}_- + \hat{S}_+) = 2G \hat{a}^\dagger \hat{a} \hat{S}_x$$

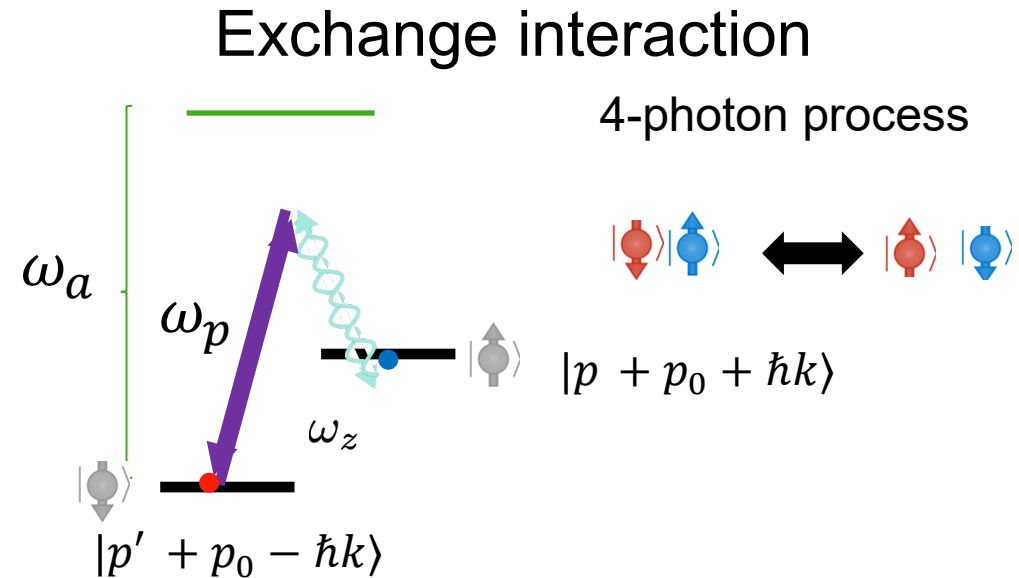
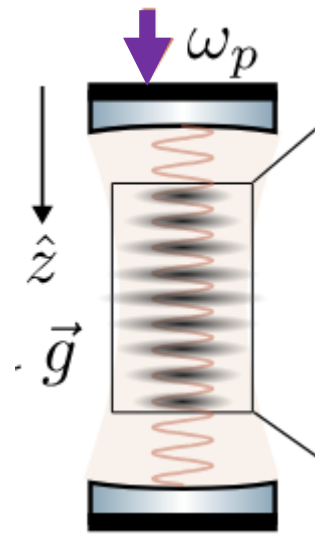
$$G = \frac{\hbar g_0^2}{4 \Delta_a}$$

Effective Atom-Atom Interaction

After adiabatic elimination of the cavity mode ($G\alpha_0^2\sqrt{N} \ll \Delta_d \pm \omega_z$)

$$\hat{a} = \alpha_0 + \hat{b}, \quad \alpha_0 = \frac{\eta}{\Delta_d + i\kappa/2} \quad \hat{b}^\dagger \hat{b} \ll 1$$

$$\begin{aligned} \hat{H}_{\text{coll}} &\approx \chi(\hat{S}_- e^{-i\omega_z t} + \hat{S}_+ e^{i\omega_z t})(\hat{S}_- e^{-i\omega_z t} + \hat{S}_+ e^{i\omega_z t}) & \chi N \sim 5\text{kHz} \\ &\sim \chi(\hat{S}_- \hat{S}_+ + \hat{S}_- \hat{S}_+) \end{aligned}$$



Richerme, ..., Monroe, *Nature*, 511, 198-201 (2014)

Jurcevic, ..., Roos, *Nature*, 511, 202-205 (2014)

Franke, ..., Rey, Roos, *Nature* 621, 740-745 (2023)

Twisting and Binding in the Cavity

$$H_{gg} = H_{sp} + H_{\text{coll}}$$

$$\hat{H}_{\text{coll}} \approx \chi(\hat{S}_- \hat{S}_+ + \hat{S}_+ \hat{S}_-) = \chi(\hat{S}^2 - \hat{S}_z^2) \quad H_{\text{sp}} = \underbrace{2 \frac{\hbar k}{m}}_{\vec{v}_{\text{rec}}} \sum_p p \hat{S}_z(p)$$

Many-body gap protection
One-axis twisting
Doppler dephasing



One Axis Twisting shift of Brag Interferometer

OAT → Invesrion dependent phase shift

$$\hat{H}_{OAT} \approx -\chi(\hat{S}_z\hat{S}_z) \approx -2\chi\langle\hat{S}_z\rangle\hat{S}_z$$

$$\rightarrow \Delta\phi = -2\chi\langle\hat{S}_z\rangle t$$

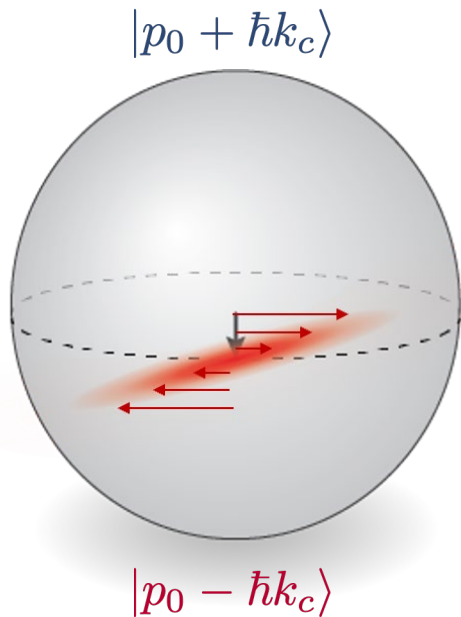
Kitagawa, Ueda PRA 47, 5138 (1993)

Wineland *et al* PRA 50, 67 (1994)

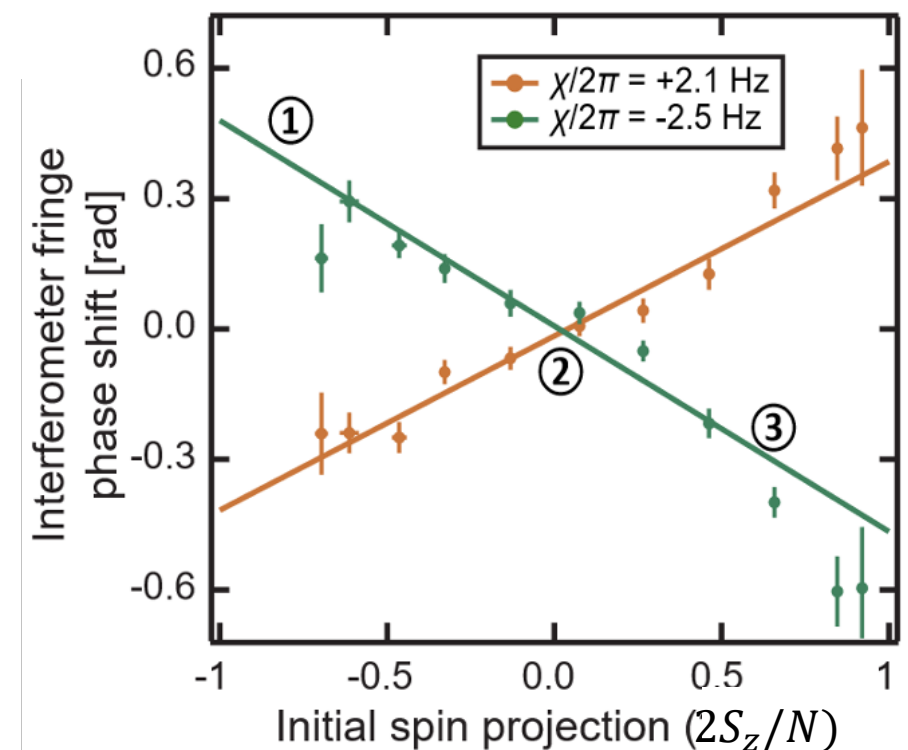
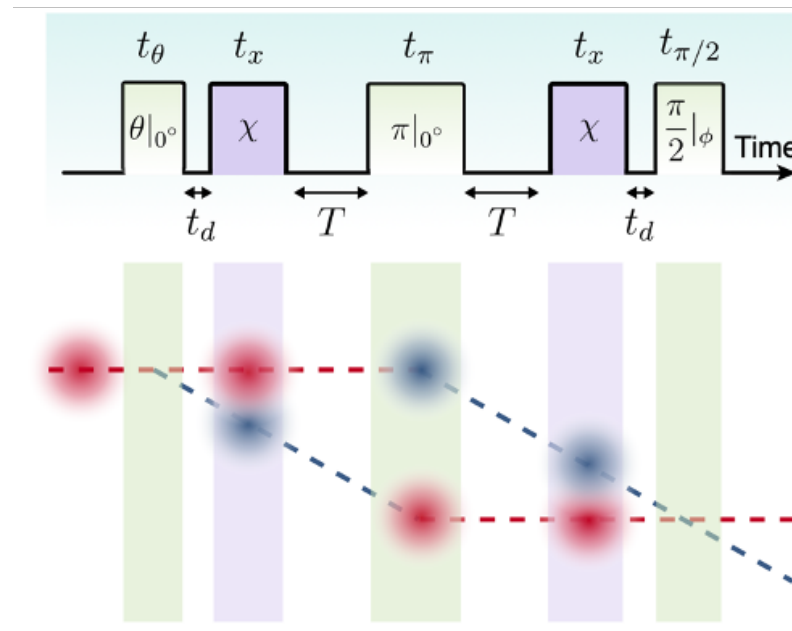
Sorensen *et al* Nature 409, 63 (2001)

Pezze *et al* RMP 90, 035005 (2018)

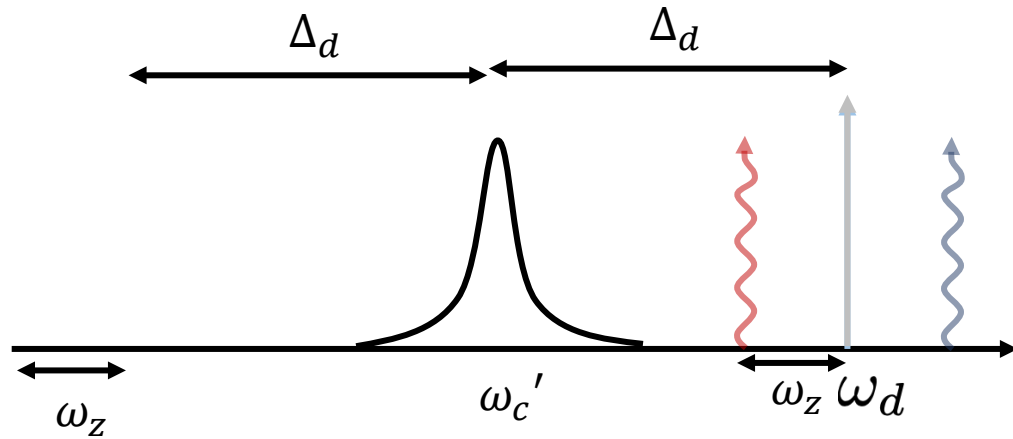
...



Interferometer



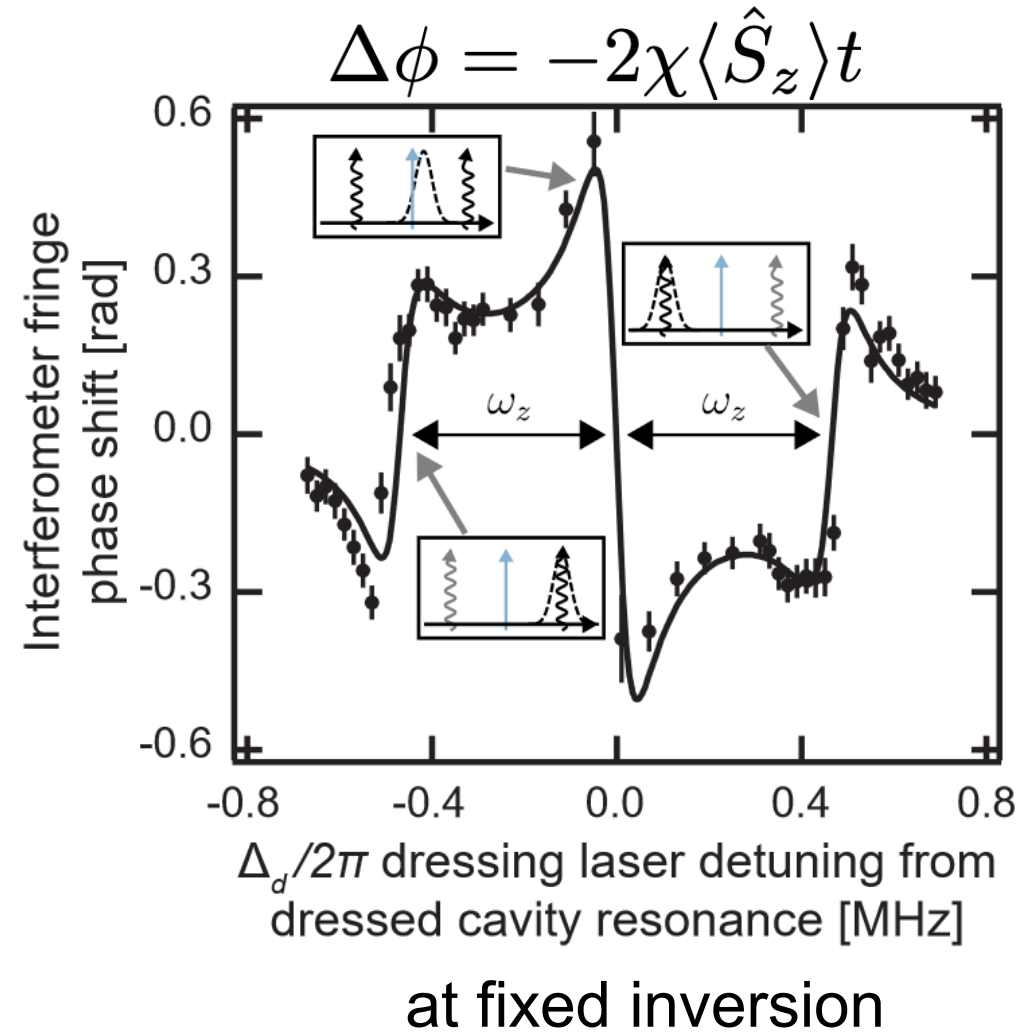
Changing χ : Scanning pump tone through resonance



$$\chi \propto \frac{|\eta|^2}{\Delta_d^2 + \frac{\kappa^2}{4}} \left(\frac{\Delta_d + \omega_z}{(\Delta_d + \omega_z)^2 + \frac{\kappa^2}{4}} + \frac{\Delta_d - \omega_z}{(\Delta_d - \omega_z)^2 + \frac{\kappa^2}{4}} \right)$$

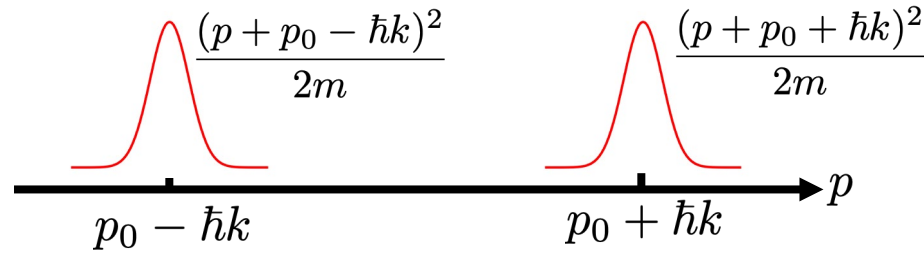
Carrier

Sidebands

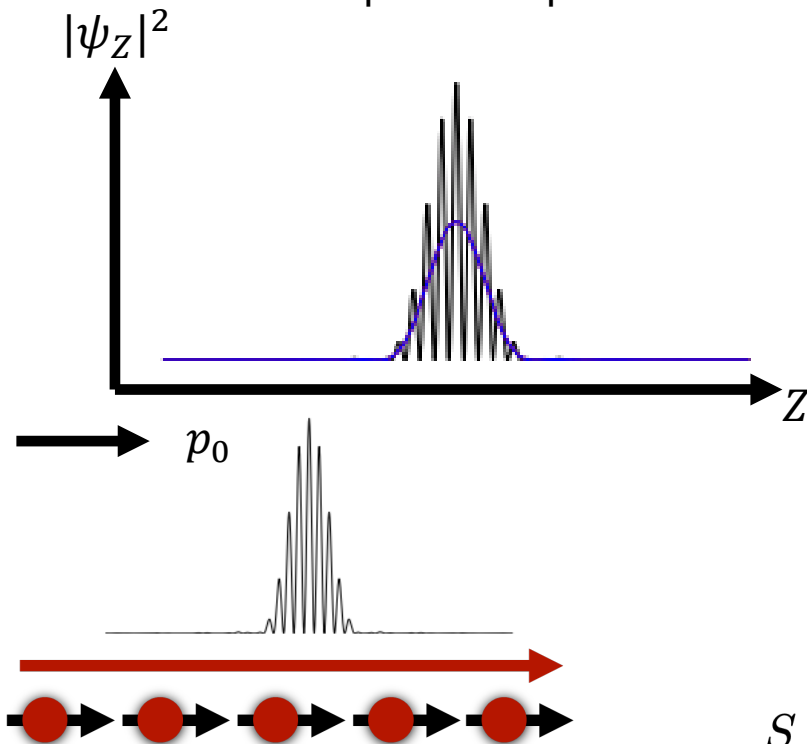


Non-Interacting Atoms

$$H_{\text{sp}} = 2 \frac{\hbar k}{m} \sum_p p \hat{s}_z(p) dp$$

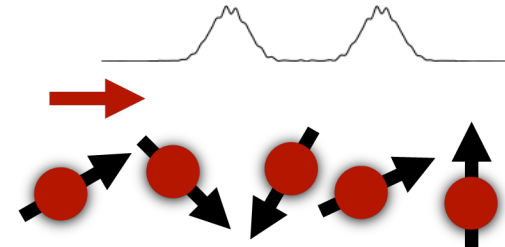
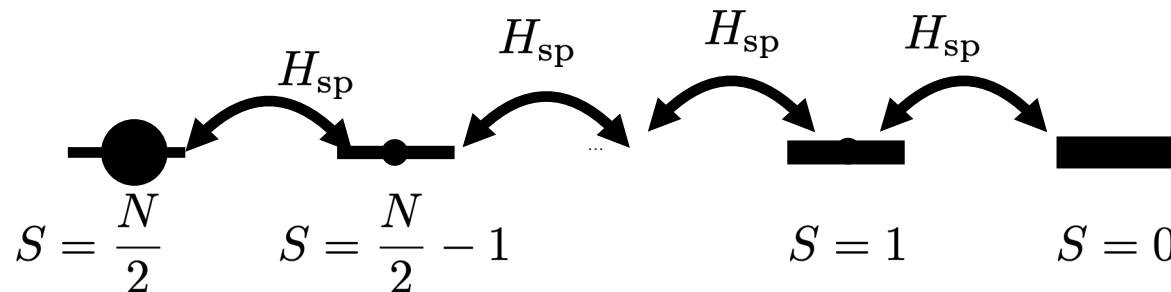
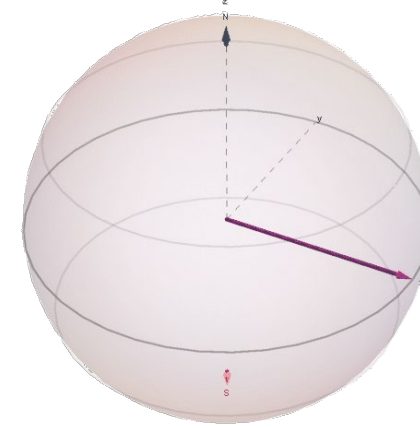


Wave packet separation



H_{sp} doesn't conserve S
 $\hat{S}^2 |S, S_z\rangle = S(S+1) |S, S_z\rangle$

Shrink of Bloch vector length

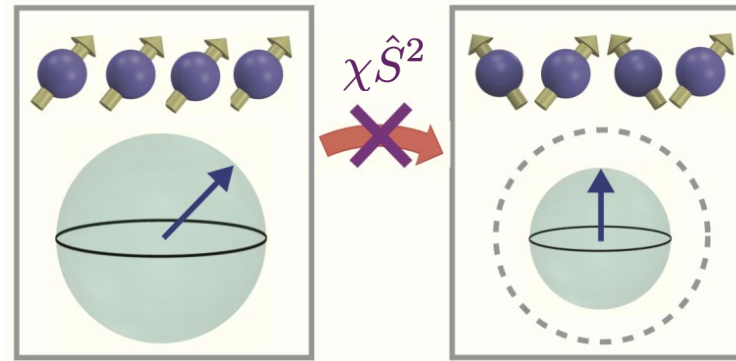


Many-body gap protection

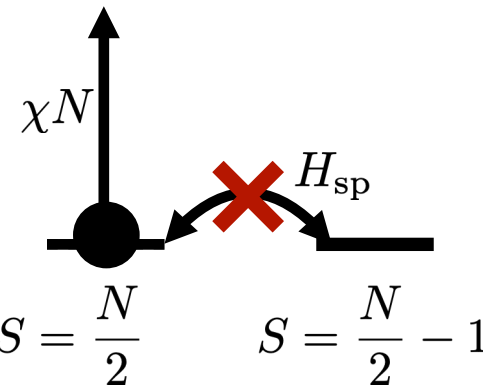
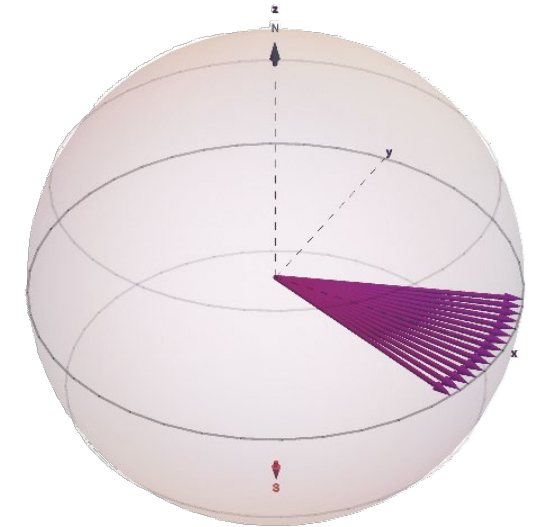
$$\hat{H}_{\text{coll}} \approx \chi(\hat{\mathbf{S}} \cdot \hat{\mathbf{S}} - \hat{S}_z^2)$$

Binding wave packets together!

$$H_{\text{sp}} = 2 \frac{\hbar k}{m} \sum_p p \hat{s}_z(p)$$



Locking spin together



Rey, ..., Lukin, Phys. Rev. A **77**, 052305 (2008)

Norcia, ..., Rey, Thompson, *Science* **361** 259 (2018) $S = \frac{N}{2}$ $S = \frac{N}{2} - 1$

Smale, He ..., Rey, Thywissen *Sci. Adv* (2019)

Davis,...Schleier-Smith PRL125, 060402 (2020)

Franke, ..., Rey, Roos, *Nature* 621, 740-745 (2023)

Binding wave-packets

Experiment sees momentum exchange interaction slows down dephasing!!.

Ideal experiment: No-binding



$$\vec{v}_{rec} = \hbar k / m$$

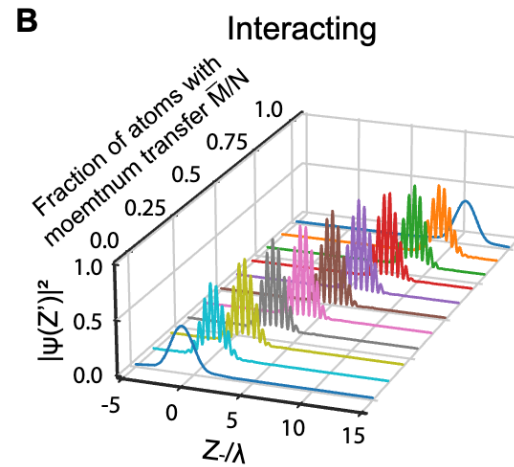
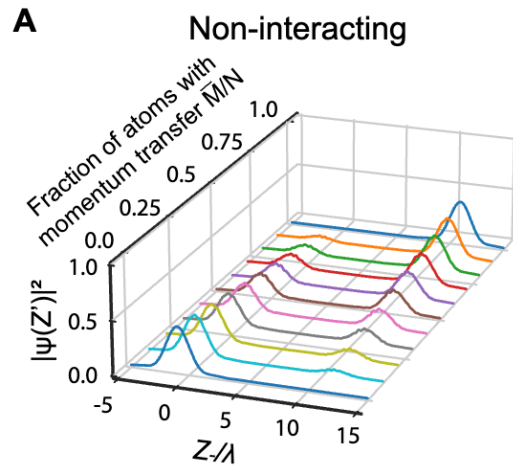
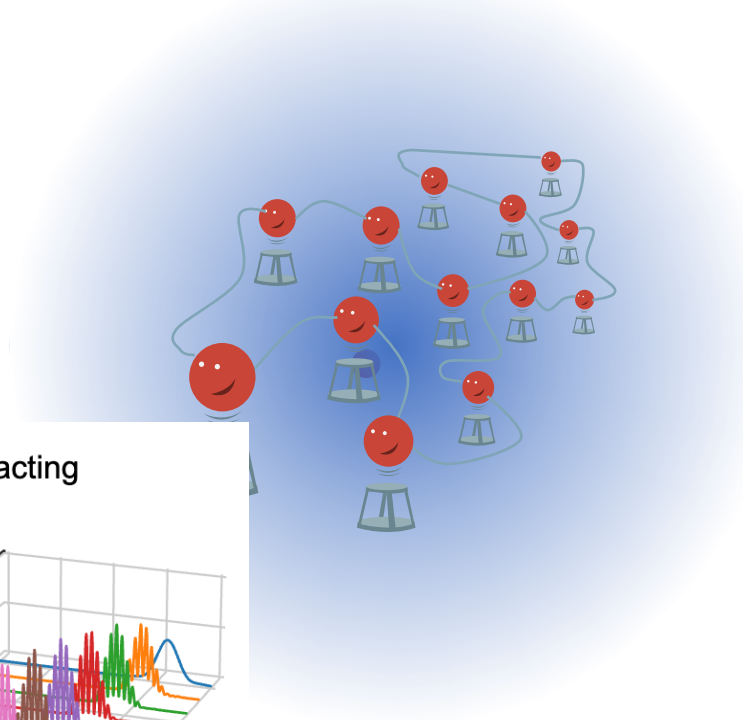
Ideal experiment: Binding

Suppressing Doppler dephasing with collective recoil!

A la Mossbauer!

A la Lamb-Dicke

$$\vec{v}_{rec}/N$$



Cavity QED implementation of BCS model

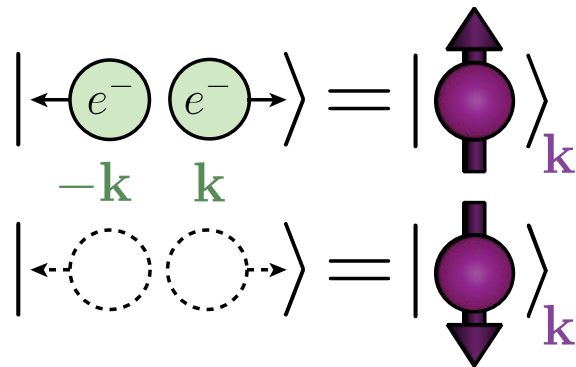
BCS model

$$\hat{H} = \sum_{\mathbf{k}, \sigma} \varepsilon_{\mathbf{k}} \hat{c}_{\mathbf{k}, \sigma}^{\dagger} \hat{c}_{\mathbf{k}, \sigma} - \chi \sum_{\mathbf{k}, \mathbf{k}'} \hat{c}_{\mathbf{k}, \uparrow}^{\dagger} \hat{c}_{-\mathbf{k}, \downarrow}^{\dagger} \hat{c}_{\mathbf{k}', \uparrow} \hat{c}_{-\mathbf{k}', \downarrow}$$

Kinetic energy of
electrons

Scattering of
Cooper pairs

Anderson
pseudospin
mapping



$$\Delta_{\text{BCS}} \equiv \frac{\chi}{2} \langle \hat{S}_{-} \rangle$$

Cavity QED
implementation

$$\hat{H} = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} \hat{S}_{\mathbf{k}}^z - \chi \sum_{\mathbf{k}, \mathbf{k}'} \hat{S}_{\mathbf{k}}^{+} \hat{S}_{\mathbf{k}'}^{-}$$

Doppler dephasing

Spin-exchanged
interactions

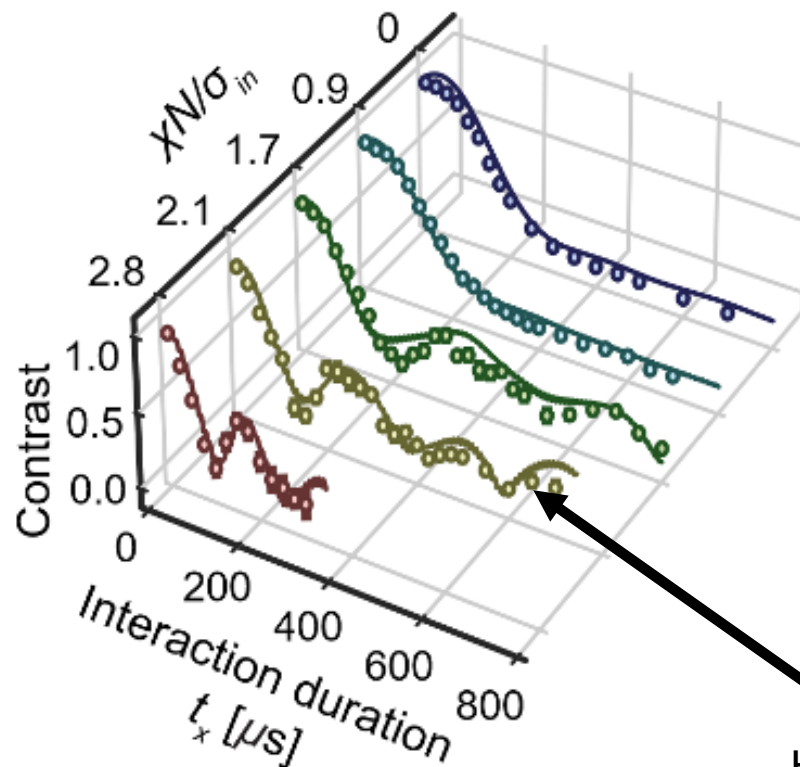
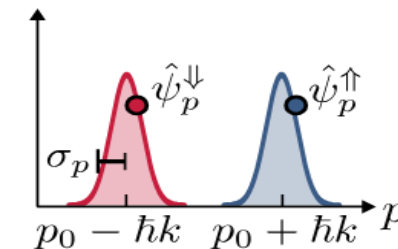
Many-body gap protection with momentum spins

Luo, Zhang, ..., Rey, Thompson, *Science* 384 (6695), 551 (2024)

Protection against contrast loss

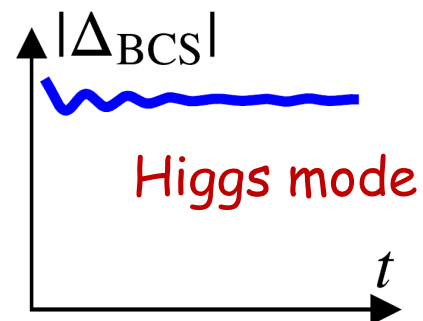
$$H_{\text{sp}} = 2 \frac{\hbar k}{m} \sum_p p \hat{s}_z(p) \quad \text{Kinetic energy of electrons}$$

$$\hat{H}_{\text{coll}} \approx \chi (\hat{\mathbf{S}} \cdot \hat{\mathbf{S}} - \hat{S}_z^2) \quad \text{Interaction Energy}$$

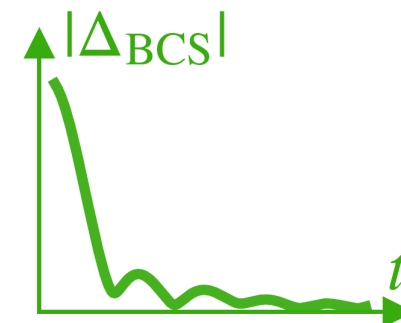


Higgs oscillations

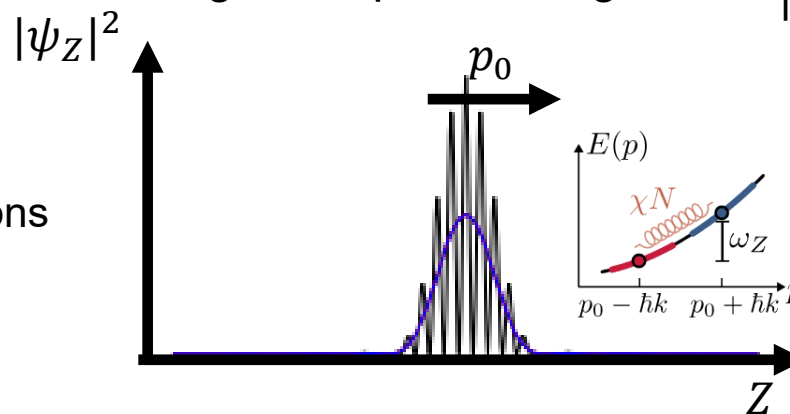
Dynamical Phase II



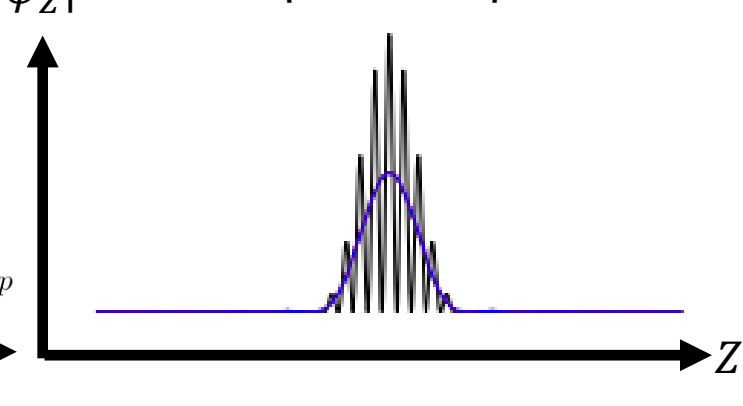
Dynamical Phase I



Binding wave packets together!



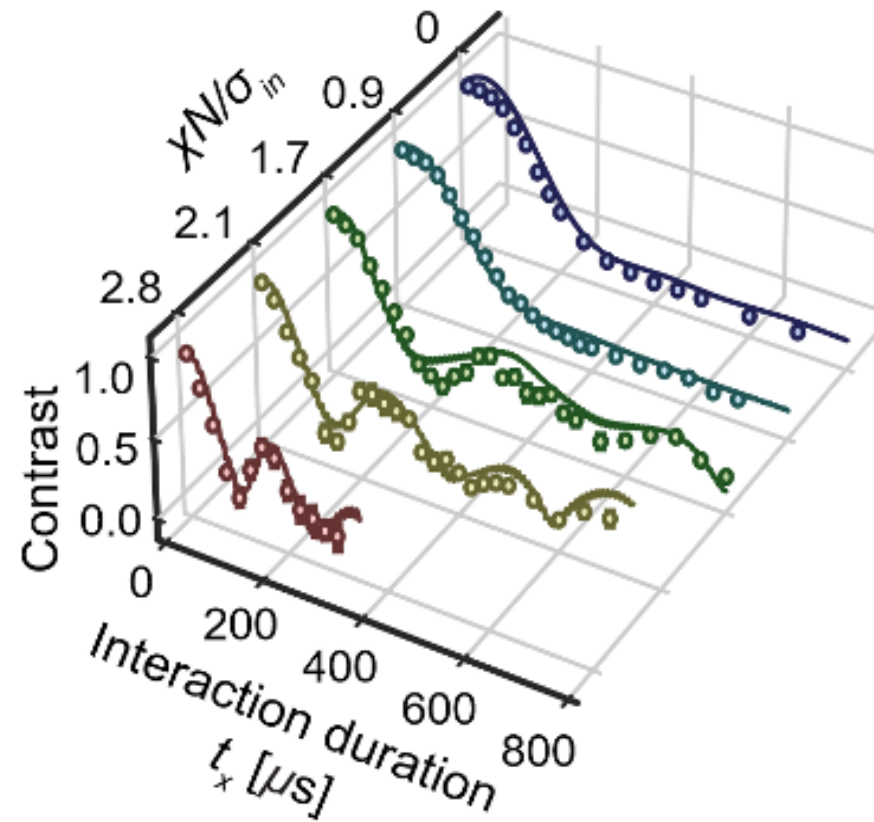
Wave packet separation



Also fermionic gases at unitarity, C. Vale,
PRL 132, 223402 (2024)

Many-body gap protection

Protection against contrast loss in Rb Bragg matter-wave interferometer



From Binding to soliton Formation

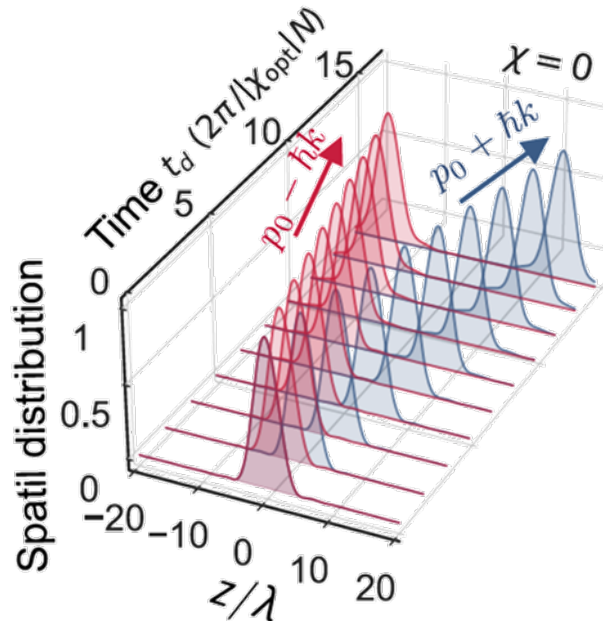
Can exchange interaction prevent the wave packet broadening?

H. Zhang, A. Chu, C. Luo, J. K. Thompson, A. M. Rey, arXiv:2504.1707

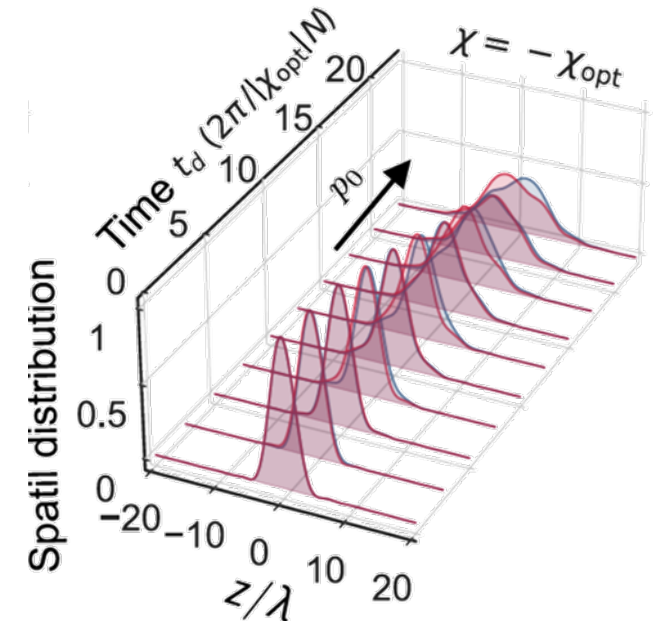


- Solitons in matter waves: 1D (stable, Salomon and Hulet's groups (2002))
2D (Townes solitons), 3D (instable, Cornell-Wieman group (2001))
 - 2D Purdue : Phys. Rev. Lett. 127, 023604 (2021), use of a Feshbach resonance with ^{133}Cs
 - 2D Paris : Phys. Rev. Lett. 127, 023603 (2021), use of a two-component gas with ^{87}Rb
- Soliton in thermal gas (no BEC necessary) and arbitrary dimension;
- New opportunities for dispersion engineerir

1D Case

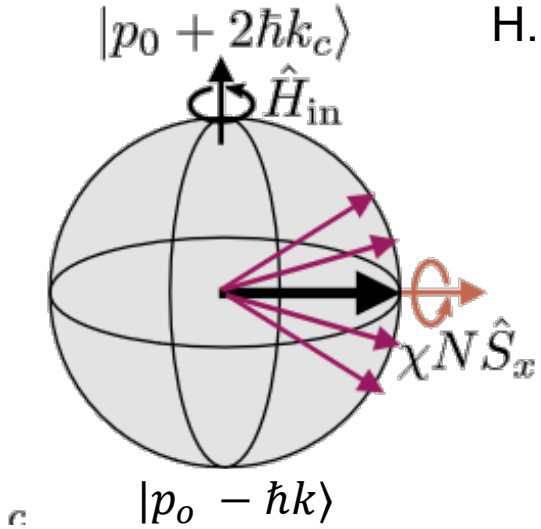


Binding with dispersion



From Binding to soliton Formation

H. Zhang, A. Chu, C. Luo, J. K. Thompson, A. M. Rey, arXiv:2504.1707

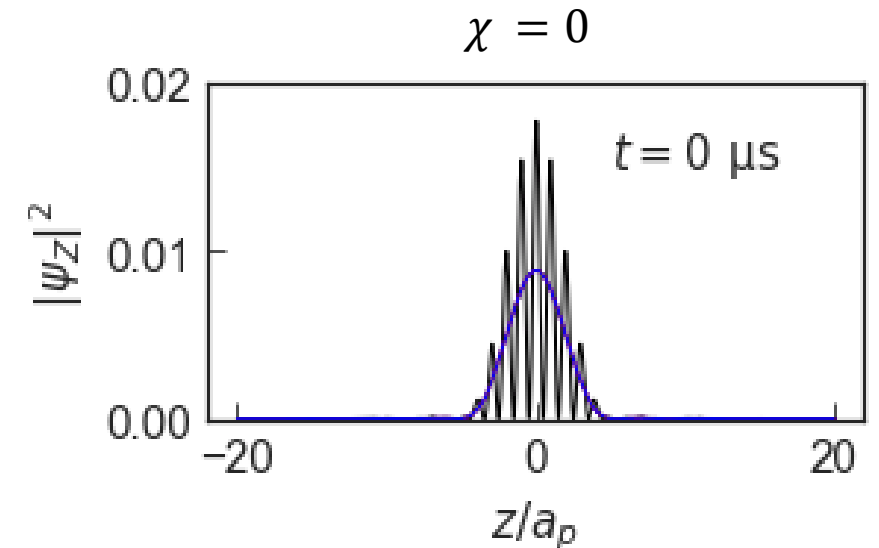
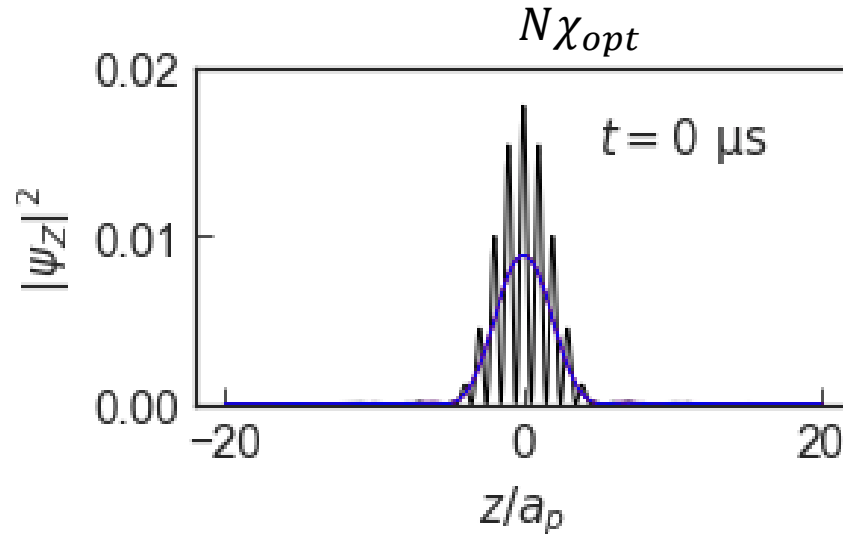
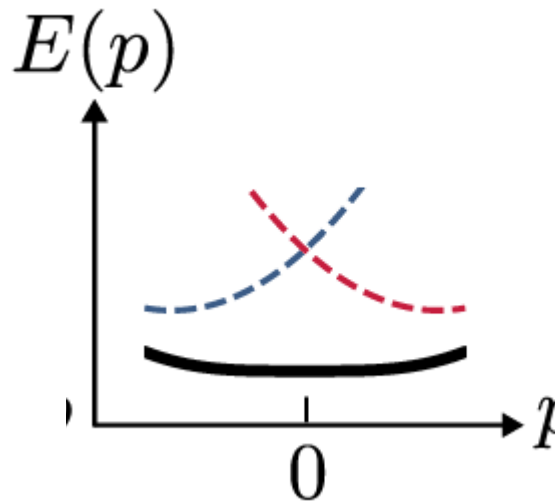


$$\hat{H}_{ex} = \chi(\hat{S}_x^2 + \hat{S}_y^2) \approx 2\chi(\langle \hat{S}_x \rangle \hat{S}_x + \langle \hat{S}_y \rangle \hat{S}_y) \approx N\chi \hat{S}_x$$

$$\hat{H}_{kin} = \sum_p \frac{p^2}{2m} \hat{I} + \frac{2\hbar k}{m} \sum_p p \hat{S}_z(p) \quad E_p = \sqrt{|N\chi|^2 + 4E_R^2 p^2} + \frac{p^2}{2m}$$

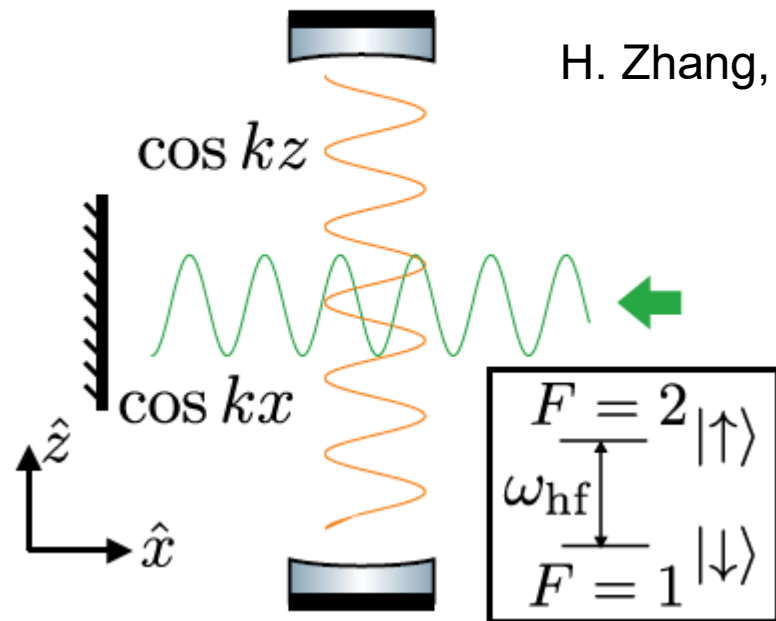
$$E_p \approx N\chi + \left(1 + \frac{4E_R^2}{N\chi}\right) \frac{p^2}{2m}$$

$$N\chi_{opt} = -4E_R^2 \quad m^* \rightarrow \infty$$



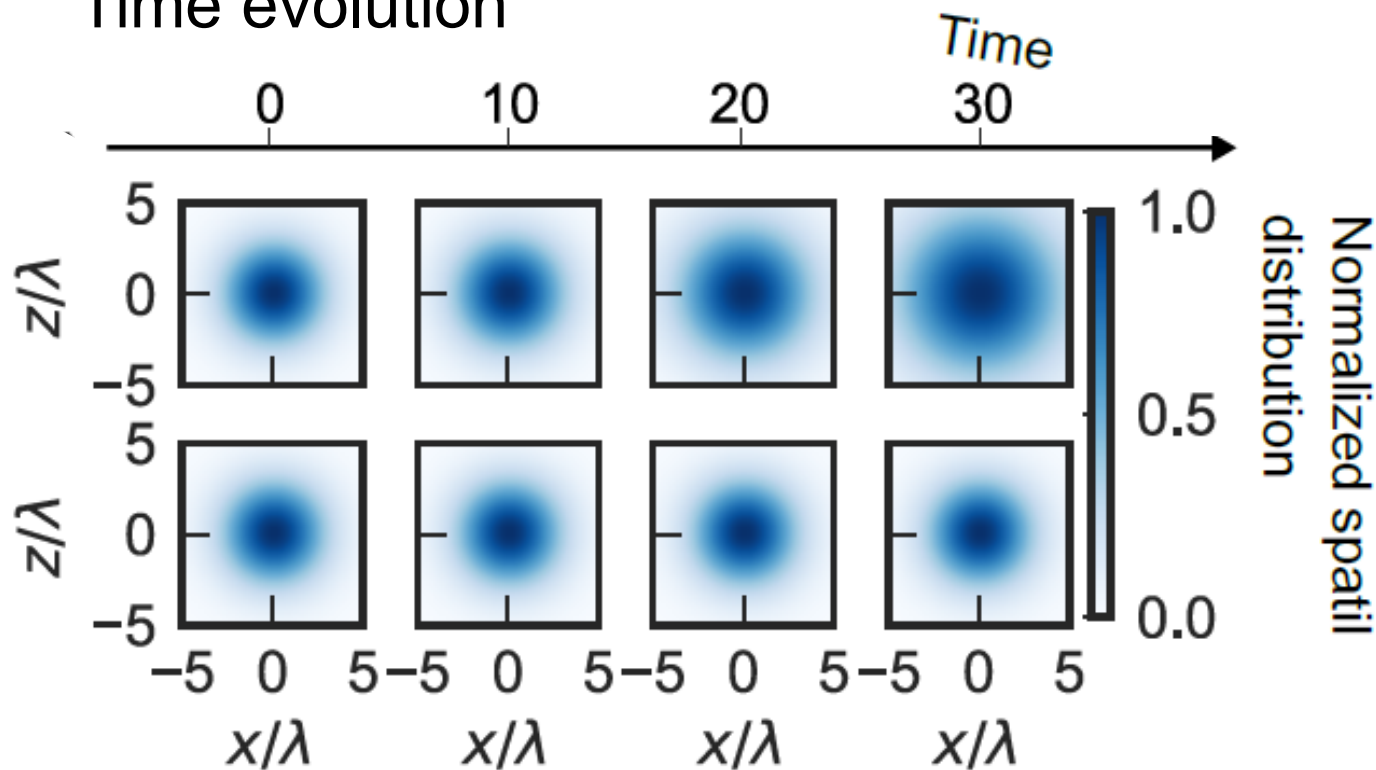
From 1D to 2D

H. Zhang, A. Chu, C. Luo, J. K. Thompson, A. M. Rey, arXiv:2504.1707

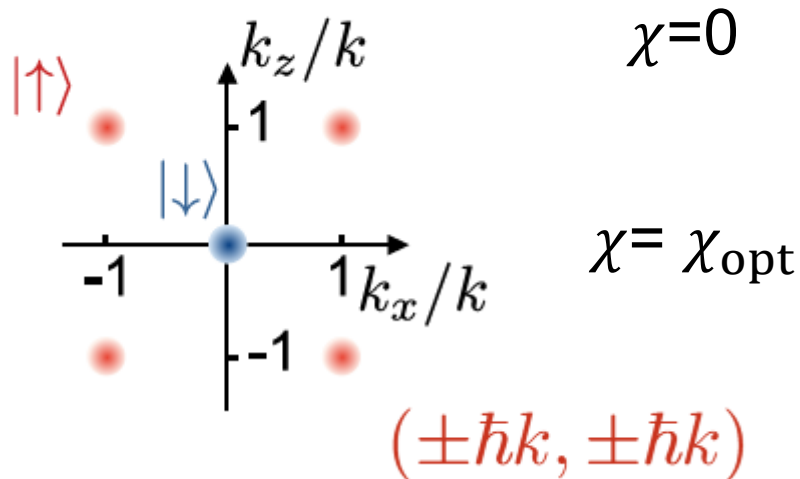


1. Add a drive perpendicular to the cavity
2. Velocity select atoms at $\mathbf{p}_0 = \{0,0\}$
3. Use an additional hyperfine level to generate a finite $\omega_z = \omega_{\text{hf}}$

Time evolution



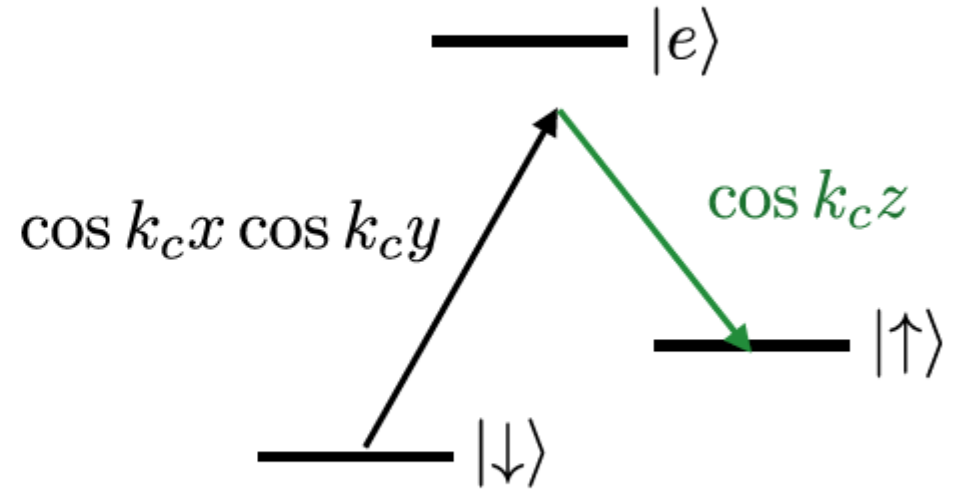
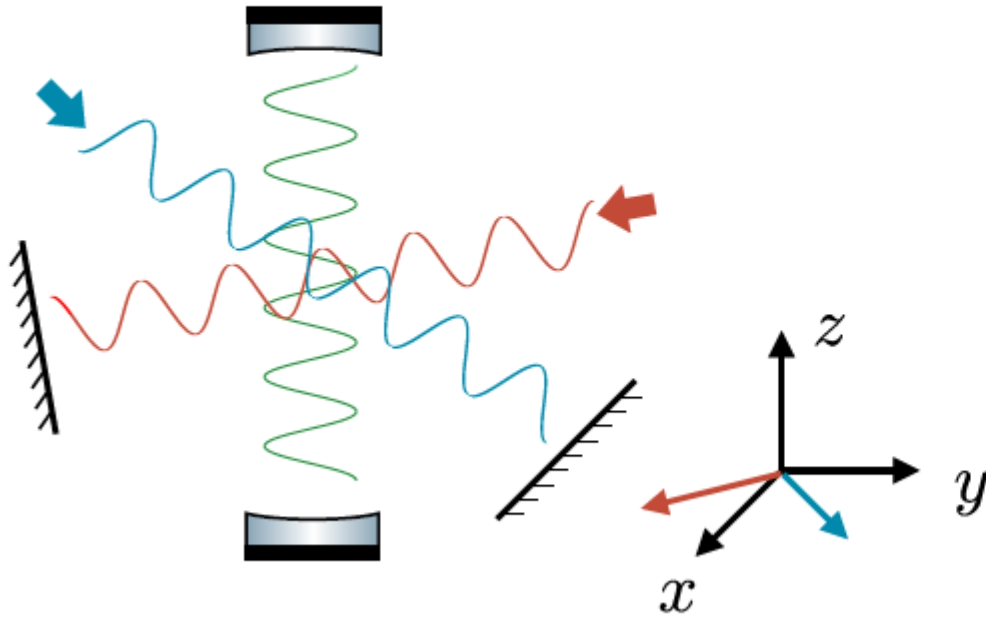
Momentum distribution



From 2D to 3D Solitons

H. Zhang, A. Chu, C. Luo, J. K. Thompson, A. M. Rey, arXiv:2504.1707

Add another perpendicular drive



No dispersion in all directions!!

Two-Axis- Counter-Twisting (TACT)

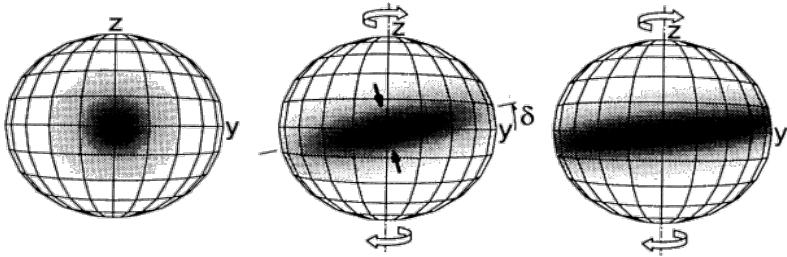
Squeezed spin states

Masahiro Kitagawa and Masahito Ueda

Nippon Telegraph and Telephone Corporation Basic Research Laboratories, Musashino, Tokyo 180, Japan
(Received 12 February 1991; revised manuscript received 3 December 1992)

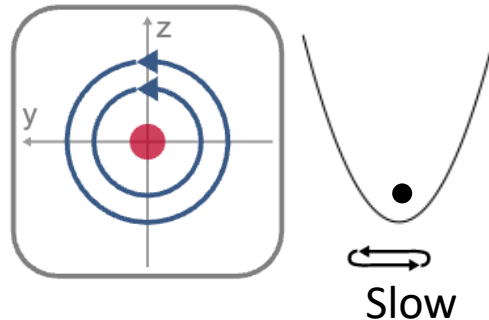
Missing for 30 years!

One-axis twisting $H_{\text{OAT}}/\hbar = \chi \hat{S}_z^2$

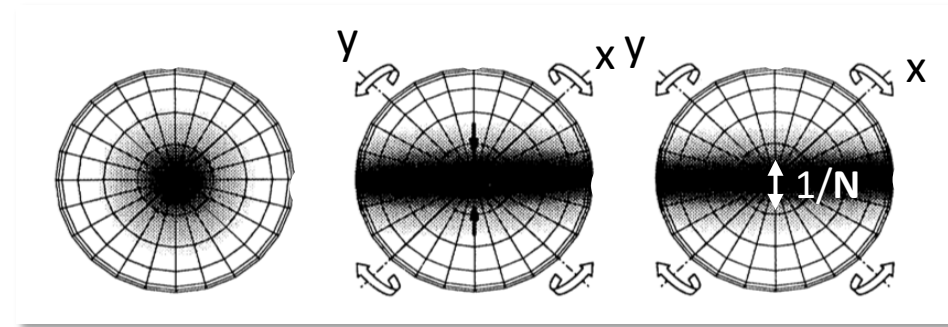


$$\hat{H}_{\text{OAT}} \sim B_{\text{ef}} \hat{S}^z \quad B_{\text{ef}} = 2\chi \langle \hat{S}^z \rangle$$

Stable point

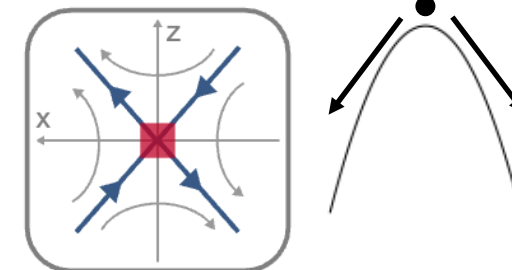


Two-Axis twisting $\hat{H}_{\text{TAT}}/\hbar = \chi(\hat{S}_x^2 - \hat{S}_y^2)$



$$\xi^2 \sim \frac{1}{N}$$

Saddle point



Exponentially fast!!

Theory proposal with four applied tones:

Borregaard, ..., Schleier-Smith, Sørensen NJP 19 093021 (2017)

Luo*, Zhang*, Rey, Thompson, Nat Phys 21, 916(2025)

Miller, ..., Lukin, Ye, Nature 633, 332 (2024)

$$\xi = \min \frac{\sqrt{N} \langle \Delta \hat{S}^\psi \rangle}{|\langle \hat{S} \rangle|} \quad \xi^2 \sim \frac{1}{N^{2/3}}$$

- Entanglement witness, $\xi^2 < 1$
- Enhanced sensitivity

Ions: Wineland, Monore, Leibfried, Bollinger, Roos, Blatt

Cavities: Vuletić, Kasevich, Thompson, Schleier-Smith, Reichel

Collisions: Oberthaler, Treutlein, Klempt, Schmeidmayer, Chapman, You

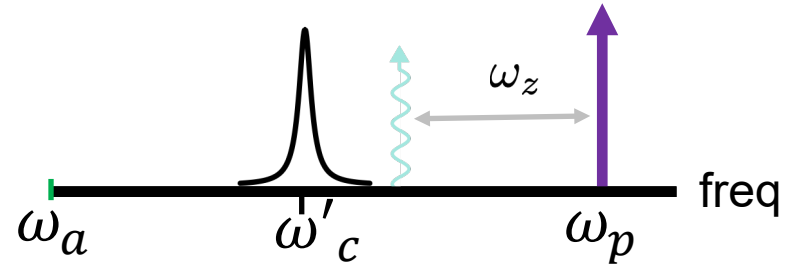
Rydberg: Kaufman, Schleier-Smith, Broeeways

Hamiltonian Engineering with two pumps

Luo*, Zhang*, Chu, ..., Rey, Thompson, Nat. Phys. **21**, 916 (2025)

Exchange interaction

$$H_{\text{ex}} = \chi_e \hat{S}_- \hat{S}_+$$



XYZ Hamiltonian Engineering

Luo*, Zhang*, Chu, ..., Rey, Thompson, Nat. Phys. **21**, 916 (2025)

General Hamiltonian

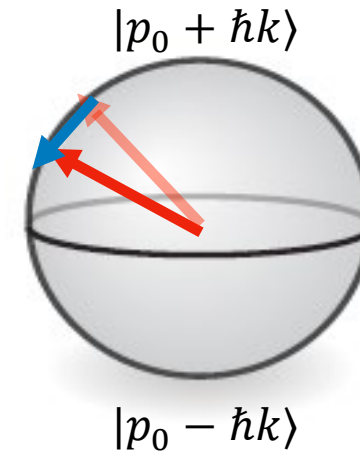
$$H_{XYZ} = \chi_e \overbrace{\hat{S}_+ \hat{S}_-}^{(\hat{S}^2 - \hat{S}_z^2)} + \chi_{\text{pair}} \overbrace{(\hat{S}_+^2 + \hat{S}_-^2)}^{(\hat{S}_x^2 - \hat{S}_y^2)} = \chi_x \hat{S}_x^2 + \chi_y \hat{S}_y^2 + \chi_z \hat{S}_z^2$$

Probe the collective dynamics

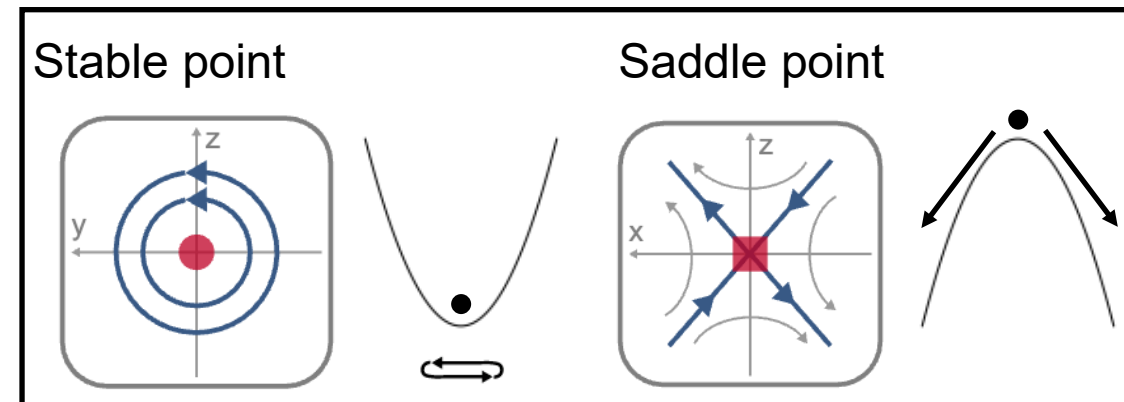
Probe the global flow line on the Bloch Sphere

1. Prepare an arbitrary spin coherent state with $\langle \hat{S}_{x,y,z} \rangle$
2. Evolve the system under H_{XYZ} for a short time Δt

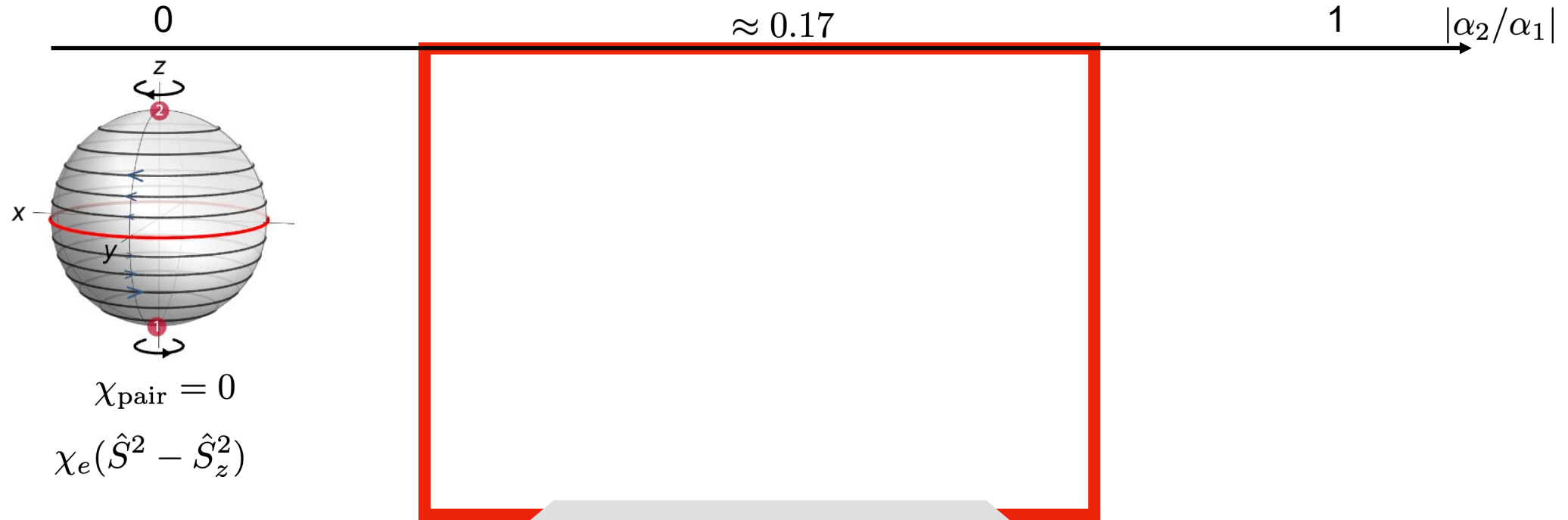
$$e^{-iH_{XYZ}\Delta t} \sim 1 - i H_{XYZ} \Delta t$$
3. Measure the changes in the spin observables $\Delta \langle \hat{S}_{x,y,z} \rangle$
4. Measure the changes with different initial states and obtain the flow line



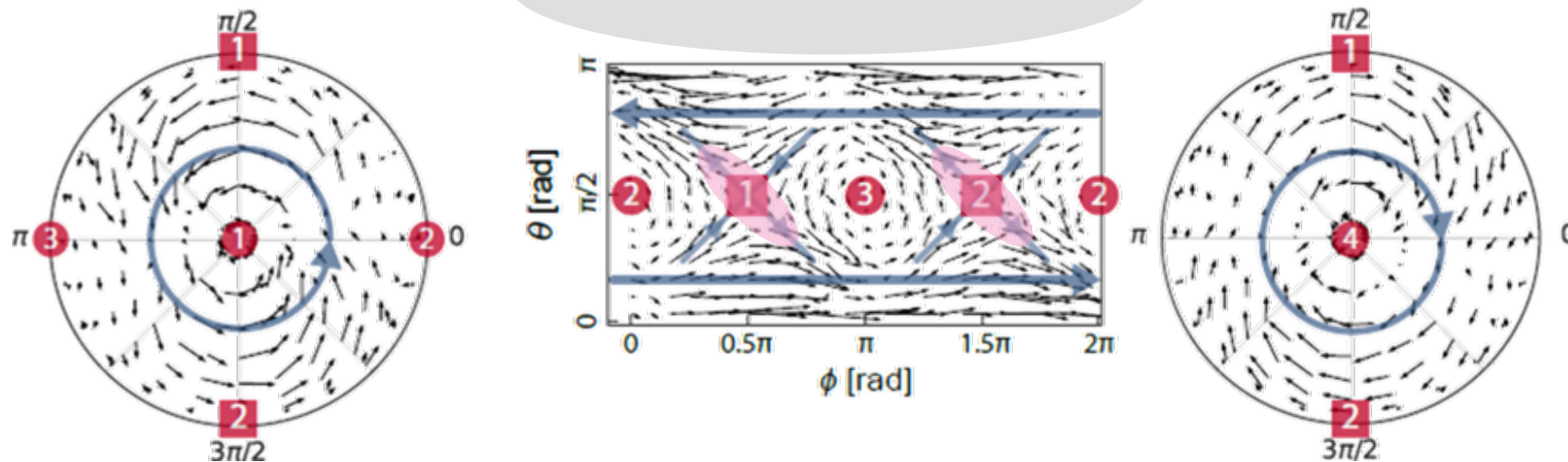
Muñoz-Arias, Deutsch, Poggi PRX Quantum **4**, 020314 (2023)



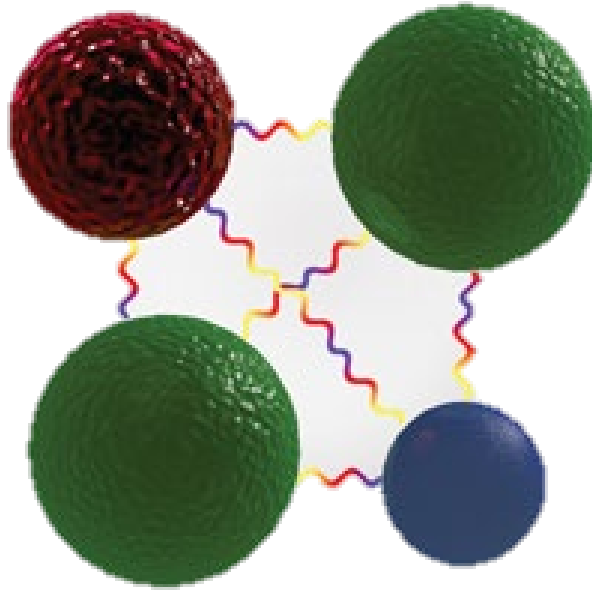
XYZ Hamiltonian Engineering



Data!!



Multi-body Interactions



Not natural in our systems. Need to be engineered !!

Multi-body Interactions

Fundamental for

❑ Quantum Information: error correction

S. Girvin *et al* review: arXiv:2407.10381....

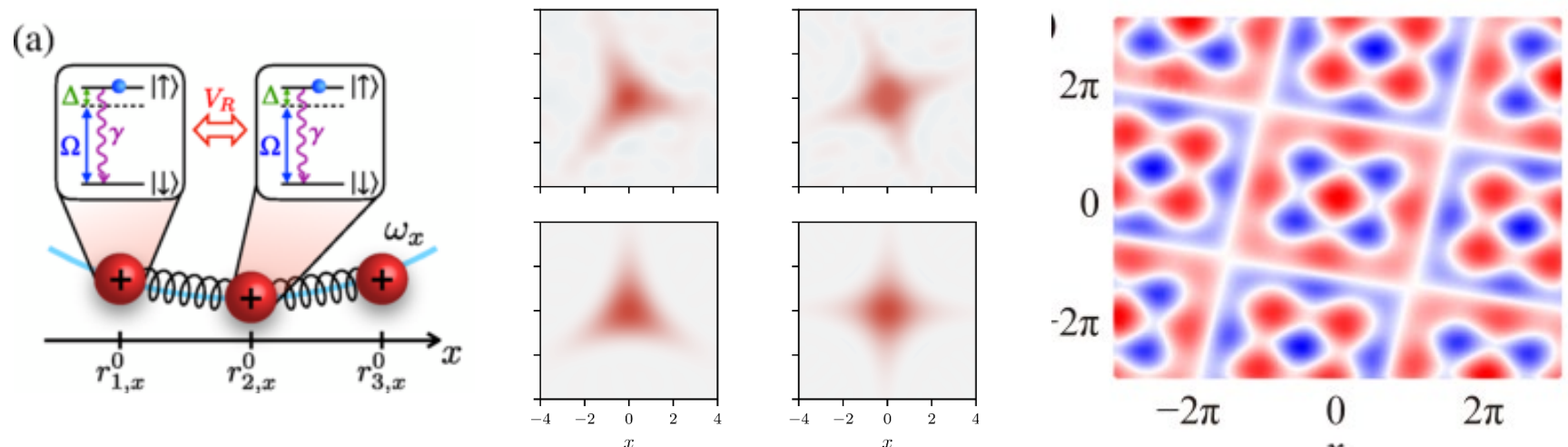
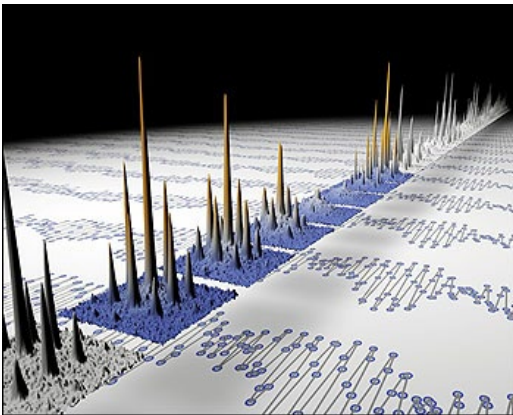
❑ Quantum information: qubits vs qudits

O. Katz *et al* Nature Physics 19 1452 (2024), PRX Quantum 4, 030311(2023), PRL 129,063603 (2023)

O. Bazavan et al arXiv:2403.05471 (2024)

❑ Quantum Simulation:

A. Daley *et al* PRL 102, 040402 2009; S. Will et al Nature 465, 197 (2010); A. Goban *et al*/Nature 563, 369 (2018), F. M. Gambetta *et al*/PRL 125, 133602 (2020)...

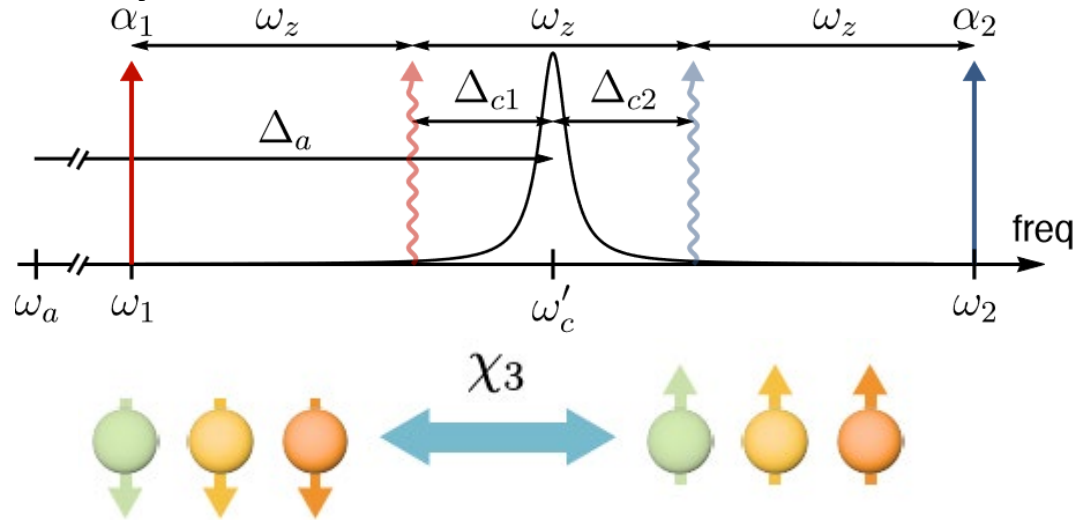


N>2-body Interactions in our Cavity

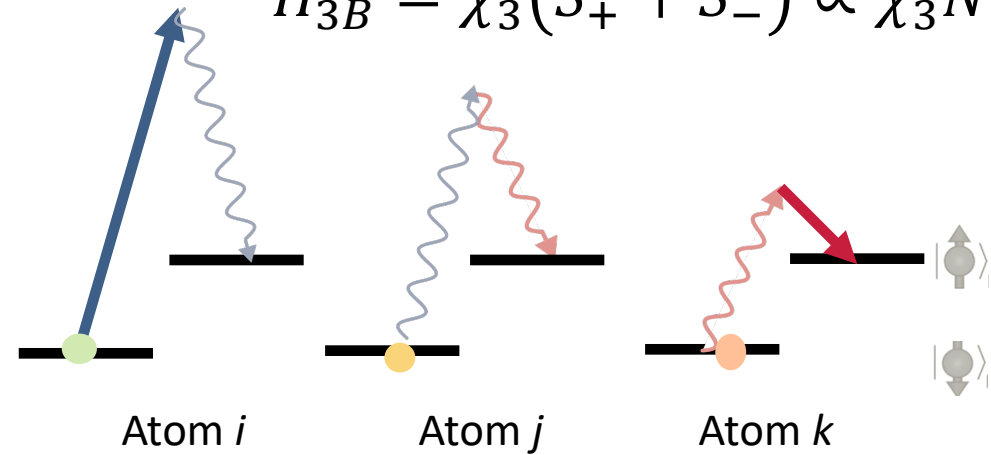
Realization of three and four-body interactions between momentum states in a cavity through optical dressing

C. Luo, H.g Zhang, ...Rey, Thompson, arXiv.2410.12132, Science, In press

Three-body



$$H_{3B} = \chi_3 (\hat{S}_+^3 + \hat{S}_-^3) \propto \chi_3 N^3$$

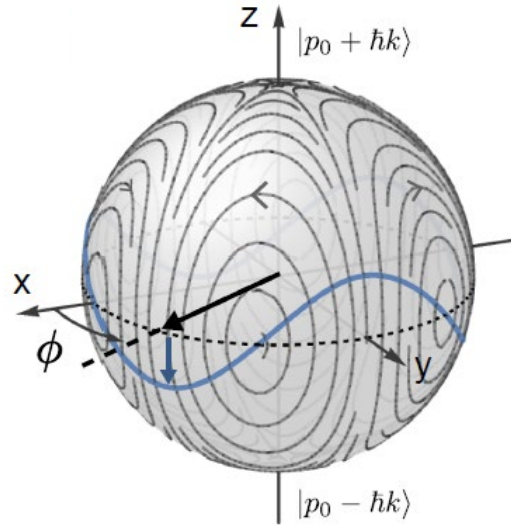


- Cancellation of two-body by construction
- Balance superradiance

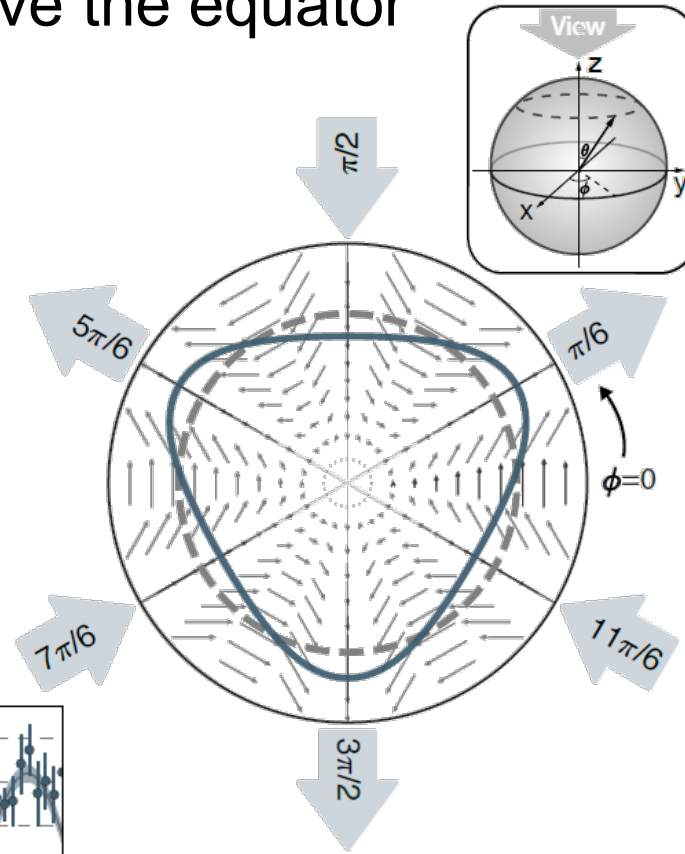
$$L_+ = \sqrt{\Gamma} \hat{S}_+ \quad L_- = \sqrt{\Gamma} \hat{S}_-$$

Three-body Interactions in our Cavity

At the equator

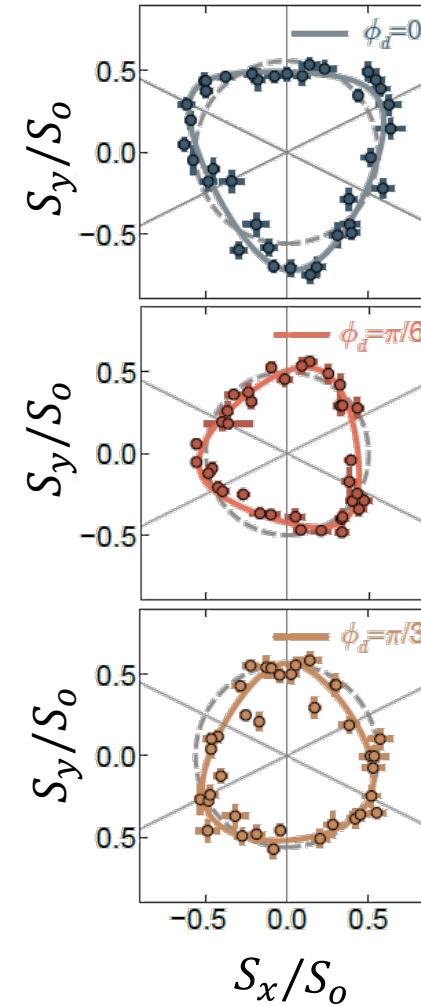
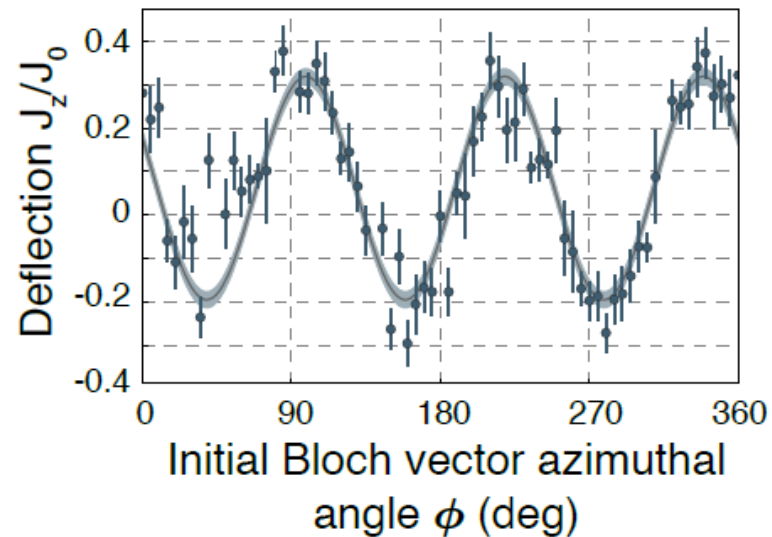


Above the equator



Mean-field dynamics
viewed from the north pole

Data



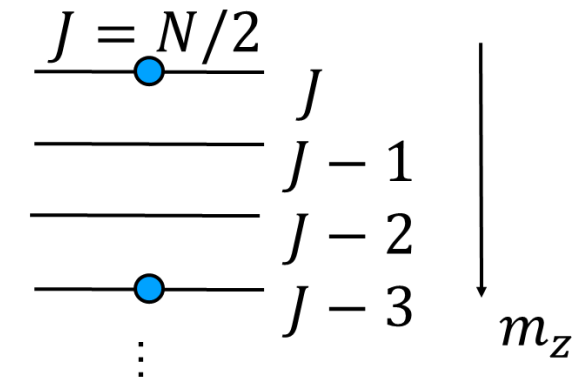
$$H_{3B} = \chi_3 (e^{i3\phi} \hat{S}_+^3 + e^{-i3\phi} \hat{S}_-^3)$$

Unitary time evolution under $H = \chi_3 (S_+^3 + S_-^3)$

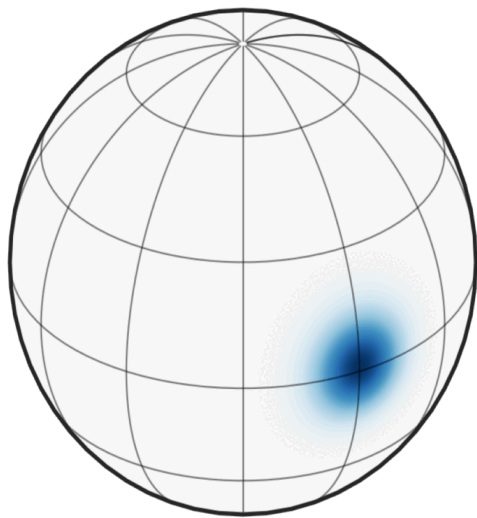
- Start with $|\uparrow, \dots, \uparrow\rangle$ — the fixed point for mean-field dynamics

$$|\psi_t\rangle = \sum_m c_{J-3m}(t) |J, m_z = J - 3m\rangle, J = N/2$$

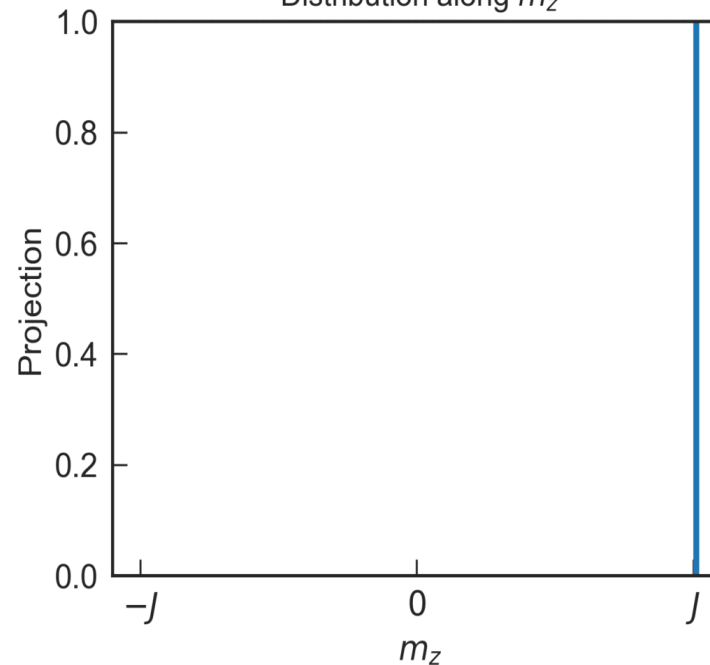
Dicke ladder



Spin Wigner Function



Distribution along m_z



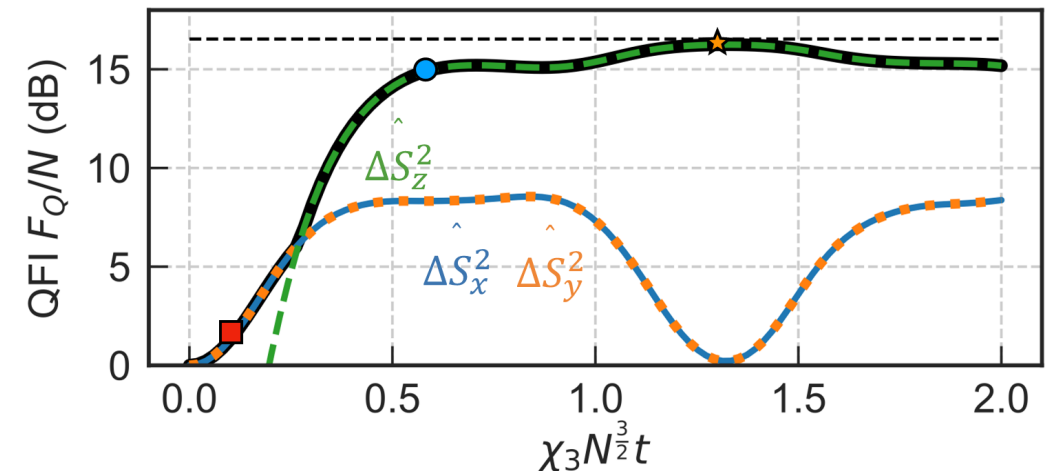
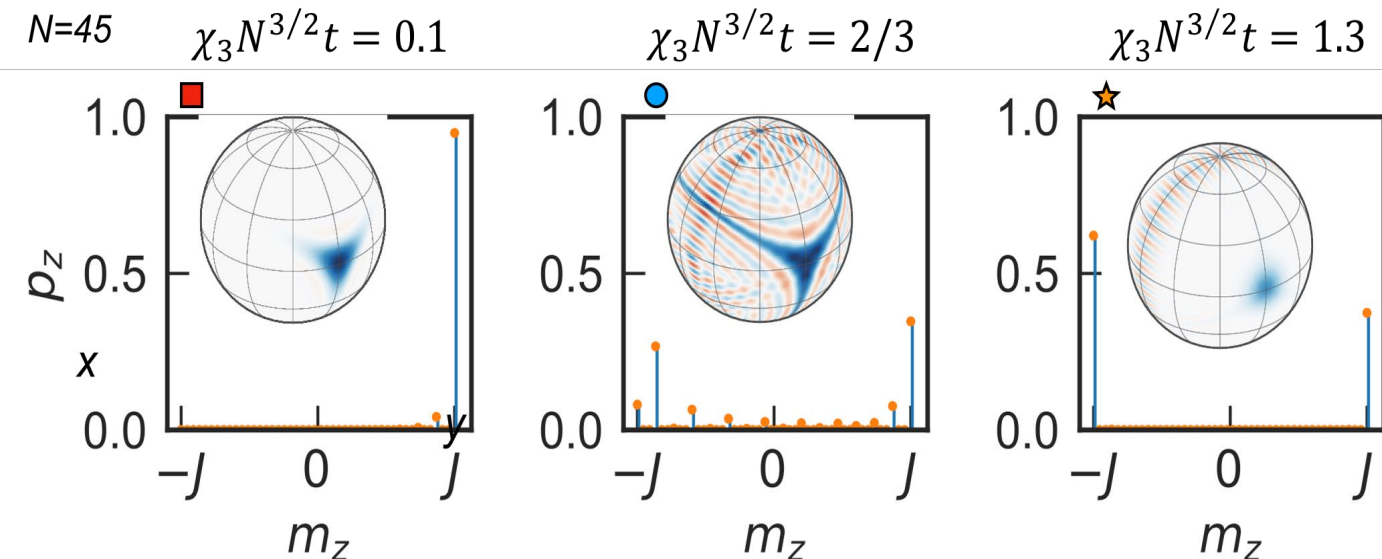
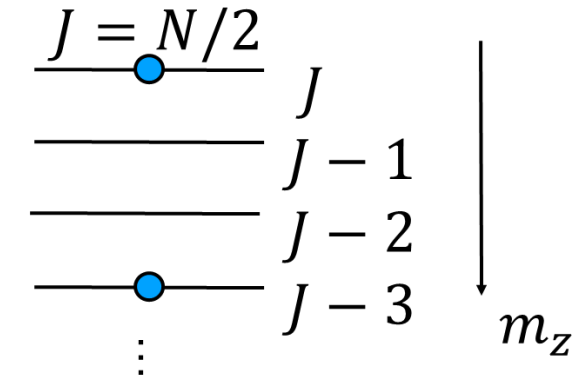
Unitary time evolution under $H = \chi_3 (S_+^3 + S_-^3)$

- Start with $|\uparrow, \dots, \uparrow\rangle$ — the fixed point for mean-field dynamics

$$|\psi_t\rangle = \sum_m c_{J-3m}(t) |J, m_z = J - 3m\rangle, J = N/2$$

- Vanishing spin-squeezing parameters
- GHZ state generation for $N = 6n + 3$ atoms at $\chi_3 N^{3/2} t = 1.3$

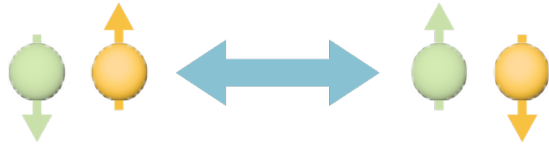
Dicke ladder



Note that a final rotation along the x-axis is performed to visualize the GHZ state.

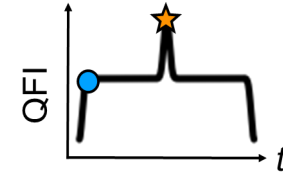
Comparing relevant timescales

Exchange interaction



$$H_{\text{ex}} = \chi_{\text{ex}} \hat{S}_+ \hat{S}_- = \chi_{\text{ex}} (\hat{S}^2 - \hat{S}_z^2)$$

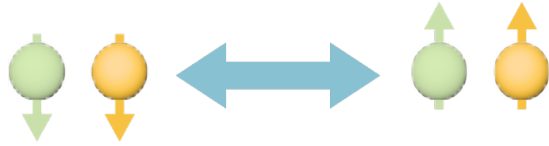
Luo, Zhang et al., *Science* **384** (6695), 551-556 (2024)



• $\chi_{\text{ex}} t_{\text{plat,ex}} = 1/\sqrt{N}$

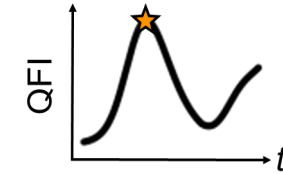
★ $\chi_{\text{ex}} t_{\text{GHZ,ex}} = \pi/2$

Two-axis twisting



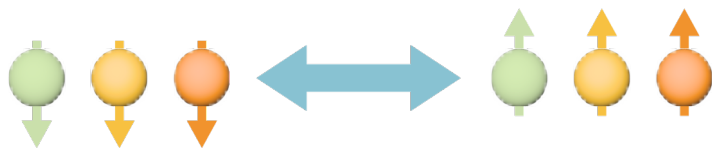
$$H_2 = \chi_2 (\hat{S}_+^2 + \hat{S}_-^2)$$

Luo, Zhang et al., *Nature Physics*, **1** (2025)



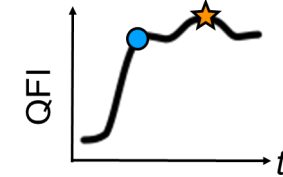
★ $\chi_2 t_{\text{peak},2} \approx \ln 4N/2N$

Three-body interactions



$$H_3 = \chi_3 (\hat{S}_+^3 + \hat{S}_-^3)$$

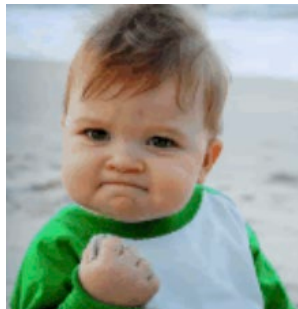
Luo, Zhang et al., arXiv:2410.12132 (2024)



• $\chi_3 t_{\text{peak},3} \approx 0.66/N^{3/2}$

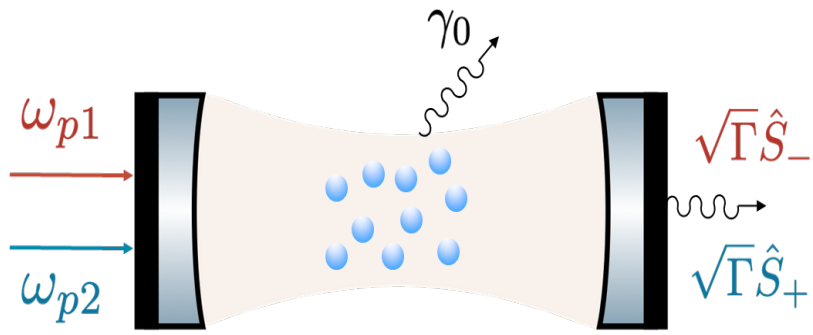
★ $\chi_3 t_{\text{GHZ},3} \approx 1.3/N^{3/2}$

$$\frac{\sqrt{N}\chi_3}{\chi_{2,\text{ex}}} \sim \frac{\sqrt{N}\mathcal{G}}{\Delta_c} \equiv \eta \ll 1 \longrightarrow \text{Requirement to eliminate cavity photons}$$



$$\frac{t_{\text{GHZ,ex}}}{t_{\text{GHZ},3}} \sim N\eta \quad \frac{t_{\text{plat,ex}}}{t_{\text{peak},3}} \sim \sqrt{N}\eta \quad \frac{t_{\text{peak},2}}{t_{\text{peak},3}} \sim \eta \ln N$$

Realistic dissipative dynamics



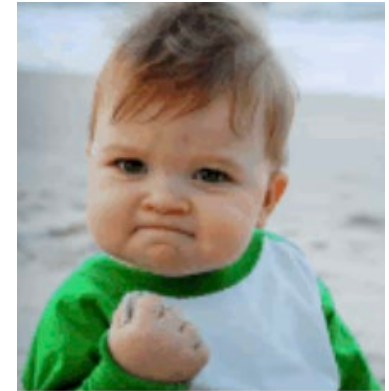
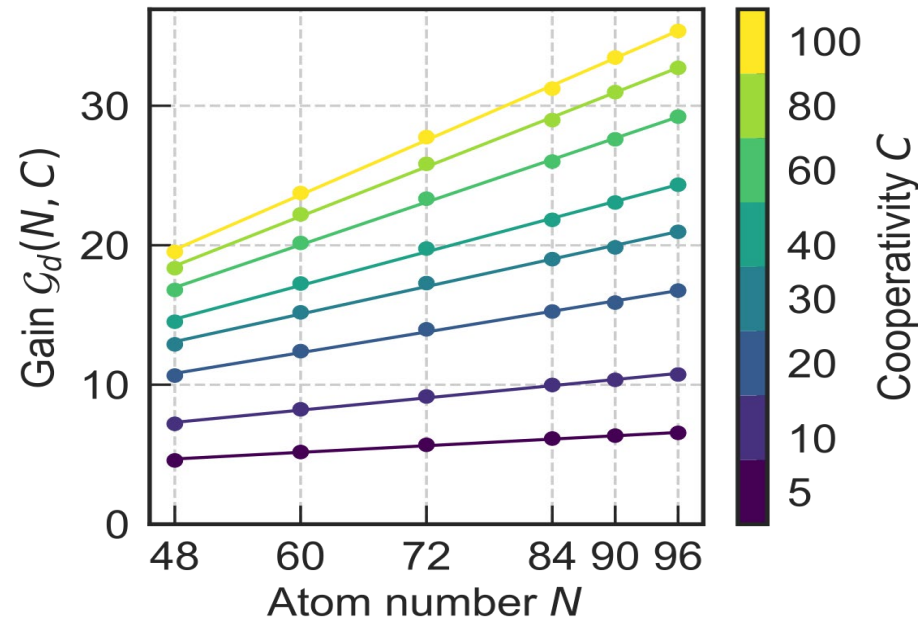
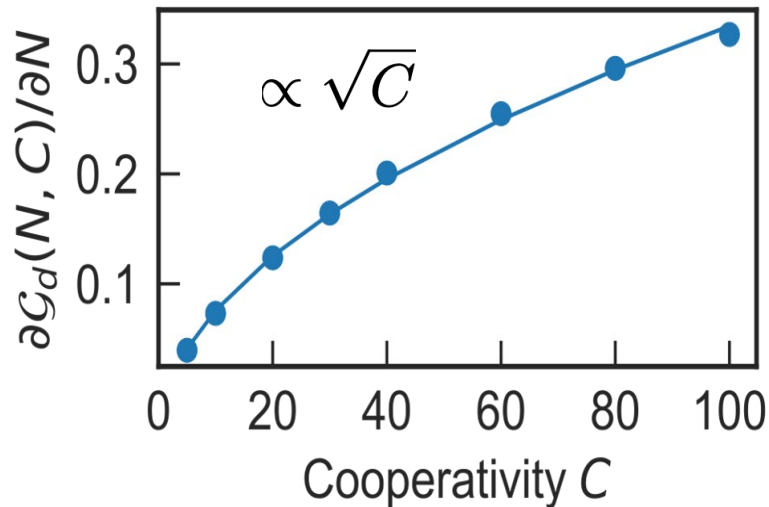
$$L_+ = \sqrt{\Gamma}\hat{S}_+ \quad L_- = \sqrt{\Gamma}\hat{S}_- \quad L_{\text{sp},i} = \sqrt{\gamma_0}\hat{s}_{i,-}$$

- Typically, spin flips lead to $\sqrt{NC}$ scaling

- With balanced superradiance, **no collective enhancement!**

$$\langle \dot{\hat{S}}_x \rangle = \dots - \Gamma \langle \hat{S}_x \rangle \quad \langle \dot{\hat{S}}_y \rangle = \dots - \Gamma \langle \hat{S}_y \rangle \quad \langle \dot{\hat{S}}_z \rangle = \dots - 2\Gamma \langle \hat{S}_z \rangle$$

- Here, we obtain a new scaling $N\sqrt{C}$ for the metrological gains!



Leroux et al, PRL 104, 073602 (2010)
 Davis et al. PRL 116, 053601 (2016)
 Borregaard. et al.,NJP 19, 093021 (2017)
 Chu et al., PRL 127, 210401 (2021)
 Colombo et al, Nat. Phys.18, 925 (2022)

Only the beginning: Great vista ahead!!

Ultracold atoms in an optical cavity

*Lumos, Obliviate, Accio,
Stupefy, Incendio, ...*



Thank YOU!!

Use All of Quantum Mechanics:

- Unitary Dynamics, Quantum Measurement, Dissipation
- Entanglement and correlations