Dynamical manifestations of many-body quantum chaos and the benefits of opening the system

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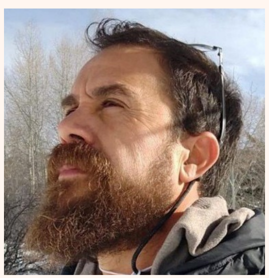
Nordita 2025

Long-Range Interactions and Dynamics in Complex Quantum Systems
(satellite meeting of Statphys29)



Jonathan Torres-Herrera

Mauro Schiulaz

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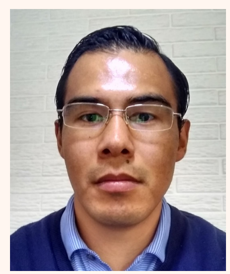


Isaías Vallejo

Adway Kumar Das

David Zarate

Sriram Lakshmanan

PRA **108**, 062201 (2023); PRB **110**, 075138 (2024); PRR **7**, 013181 (2025)

Many-Body Quantum Dynamics & Timescales

Quantum chaos = spectral correlations as in RMT.

Spectral correlations take a long time to get manifested.

Timescales?

Model, quantity, initial state.

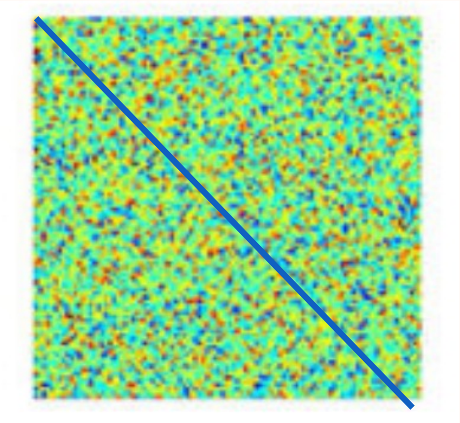
Dependence on system size.

Full Random Matrices
vs
Physical Models

Full Random Matrices

- Matrices filled with random numbers: GOE (real and symmetric)

$$\langle H_{ij}^2 \rangle = \begin{cases} 1, & i = j \\ 1/2, & i \neq j. \end{cases}$$



$$H|\alpha\rangle = E_\alpha |\alpha\rangle$$

Random vectors

$$|\alpha\rangle = \begin{pmatrix} C^{(1)} \\ C^{(2)} \\ C^{(3)} \\ C^{(4)} \\ C^{(5)} \\ \ldots \end{pmatrix}$$

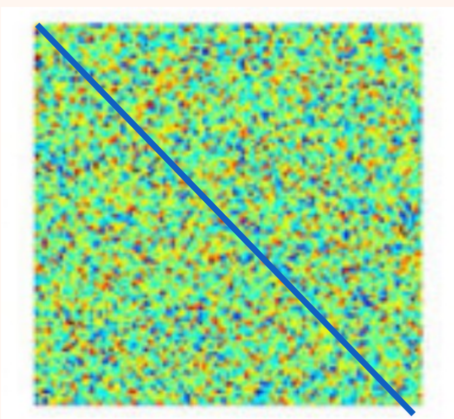
Gaussian
random numbers
(normalization)

RANDOM MATRICES:
Wigner (1950s)
Study statistically the spectra of
heavy nuclei
(atoms, molecules, quantum dots)

Full Random Matrices

- Matrices filled with random numbers: GOE

$$\langle H_{ij}^2\rangle = \begin{cases} 1, & i=j \\ 1/2, & i\neq j. \end{cases}$$



$$H|\alpha\rangle = E_\alpha |\alpha\rangle$$

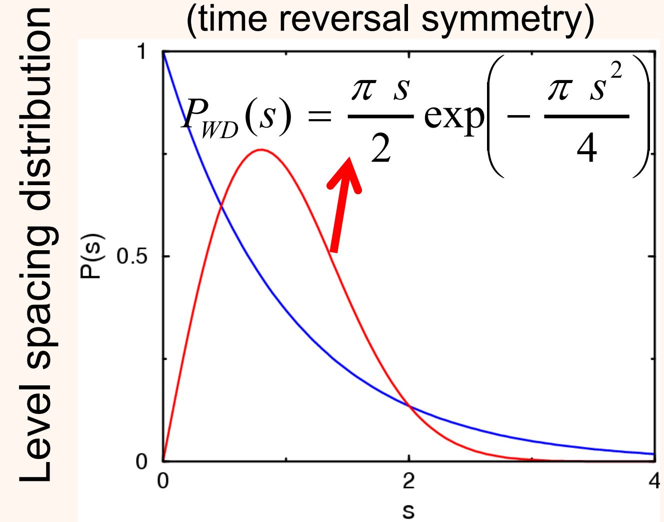
Random vectors

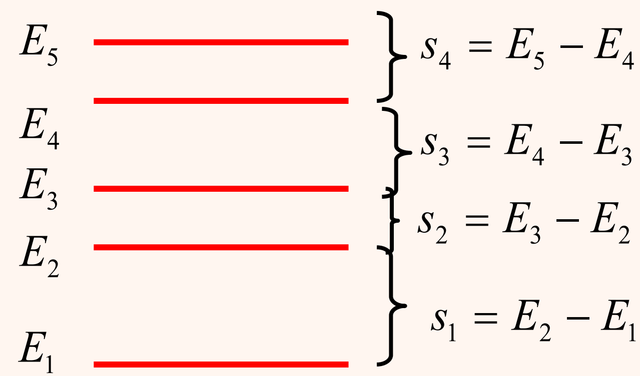
$$|\alpha\rangle = \begin{pmatrix} C^{(1)} \\ C^{(2)} \\ C^{(3)} \\ C^{(4)} \\ C^{(5)} \\ \dots \end{pmatrix}$$

Gaussian random numbers (normalization)

Level repulsion
Rigid spectrum

Wigner-Dyson distribution
(time reversal symmetry)





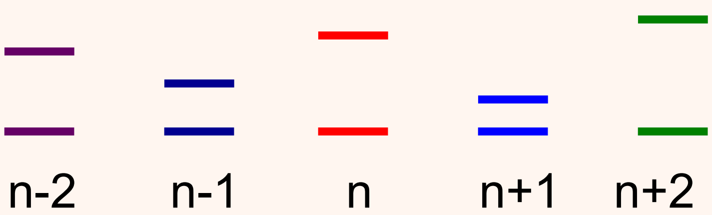
Physical Model

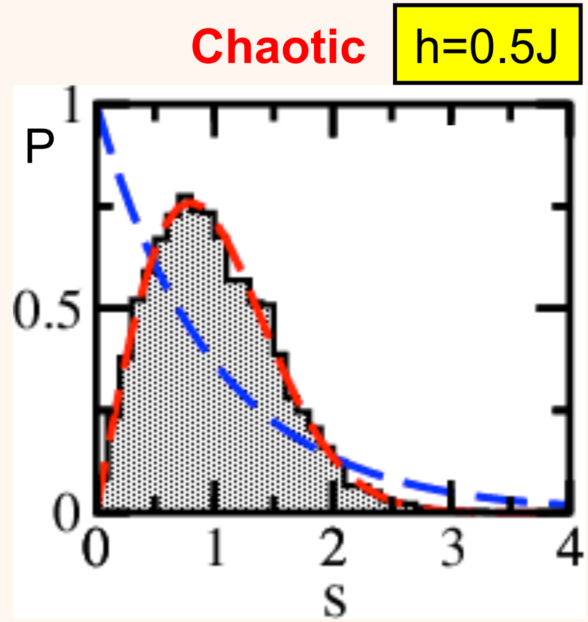
1D spin-1/2 system with nearest-neighbor couplings and onsite disorder

Cold atoms
NMR
Ion traps

$$H = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L} \frac{J}{4} \left[\sigma_n^z \sigma_{n+1}^z + \left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y \right) \right]$$

$h_n \in [-h, h]$





Many-body quantum chaos: interaction between particles

Physical Model: Short-Range Couplings

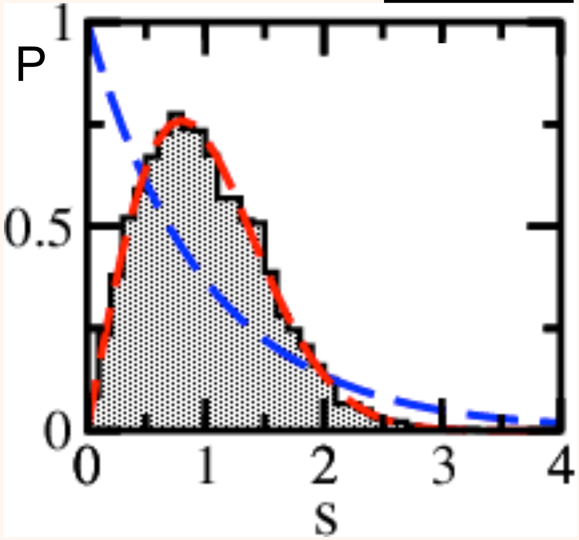
1D spin-1/2 system with nearest-neighbor couplings and onsite disorder

$$H = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L} \frac{J}{4} \left[\sigma_n^z \sigma_{n+1}^z + \left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y \right) \right]$$

$$H = \sum \frac{J}{4} \frac{\vec{S}_n . \vec{S}_m}{|n-m|^\alpha}$$

$$\alpha > 1$$

Chaotic h=0.5J

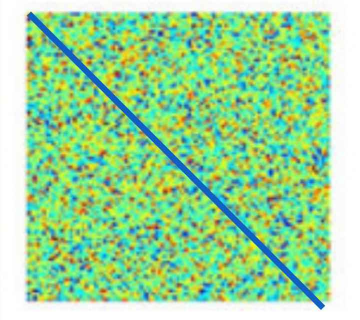


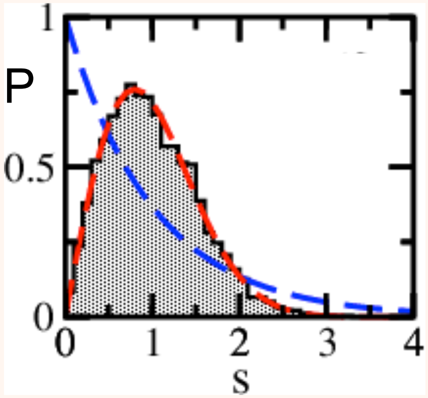
Many-body quantum chaos: interaction between particles

Random Matrices vs Physical Systems

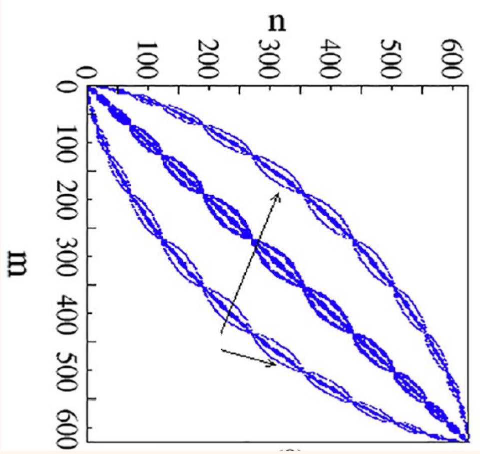
GOE (real and symmetric)

$$\langle H_{ij}^2 \rangle = \begin{cases} 1, & i = j \\ 1/2, & i \neq j. \end{cases}$$





Physical
many-body quantum systems



$$\langle \uparrow \downarrow \uparrow \downarrow | H | \uparrow \downarrow \downarrow \uparrow \rangle = J_{34}$$

$$\langle \downarrow \uparrow \uparrow \downarrow | H | \downarrow \uparrow \downarrow \uparrow \rangle = J_{34}$$

Random Matrices vs Physical Systems

GOE (real and symmetric)

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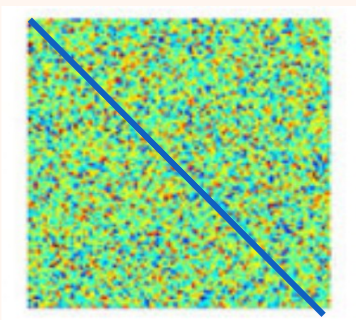
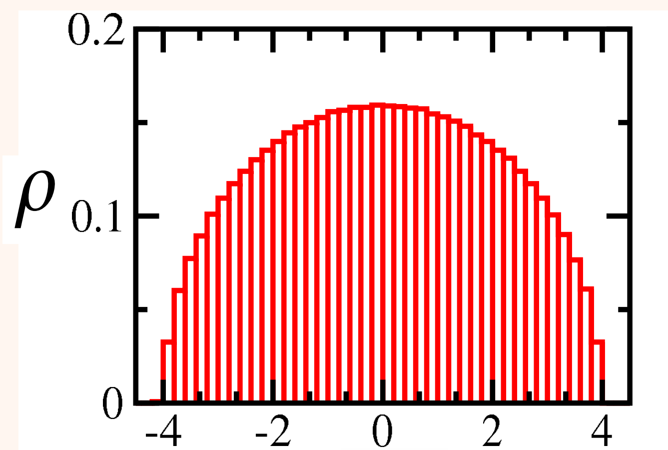
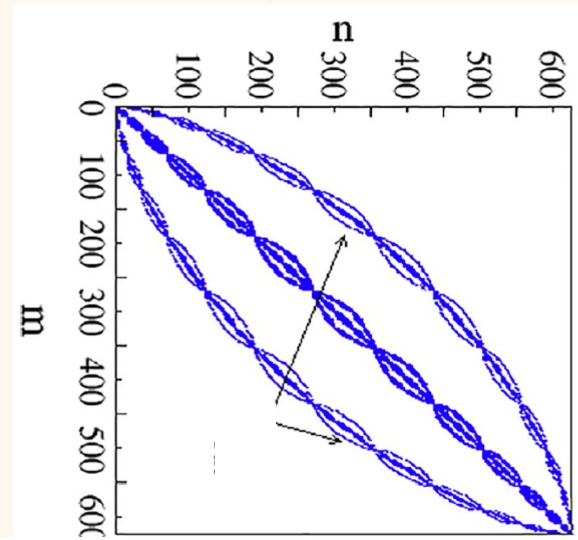


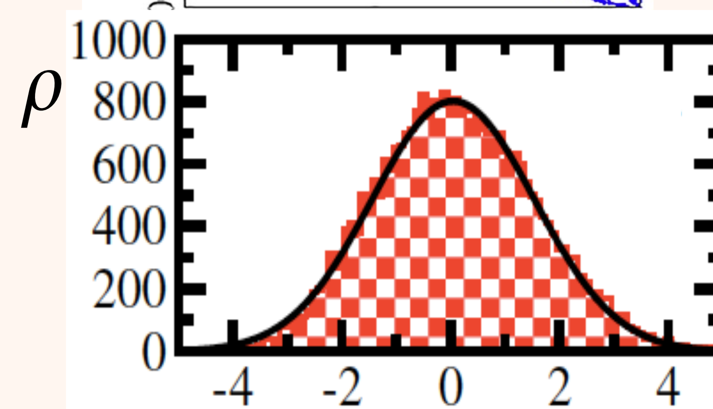


Wigner (1957)

Physical
many-body quantum systems



Density of States

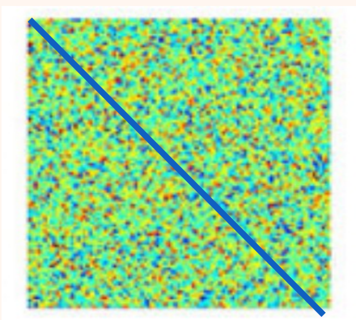


French & Wong, PLB (1970)

Random Matrices vs Physical Systems

GOE (real and symmetric)

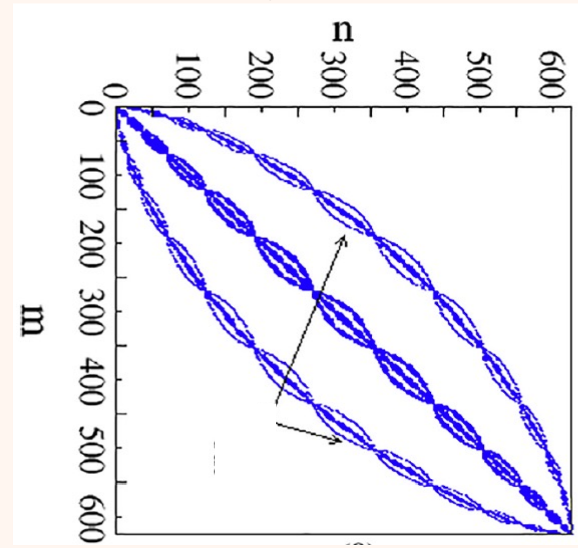
$$\langle H_{ij}^2 \rangle = \begin{cases} 1, & i = j \\ 1/2, & i \neq j. \end{cases}$$

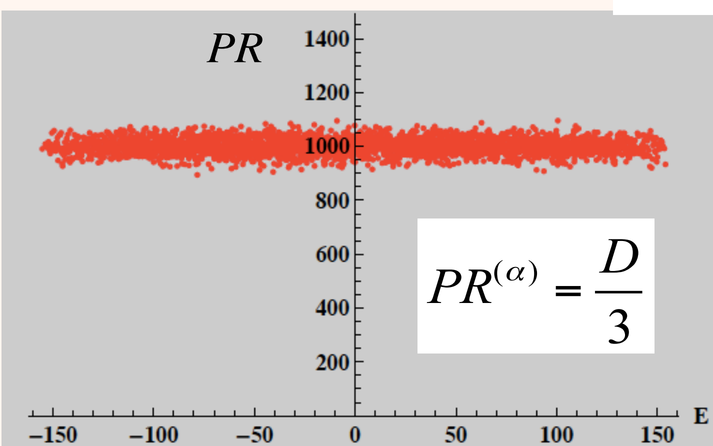


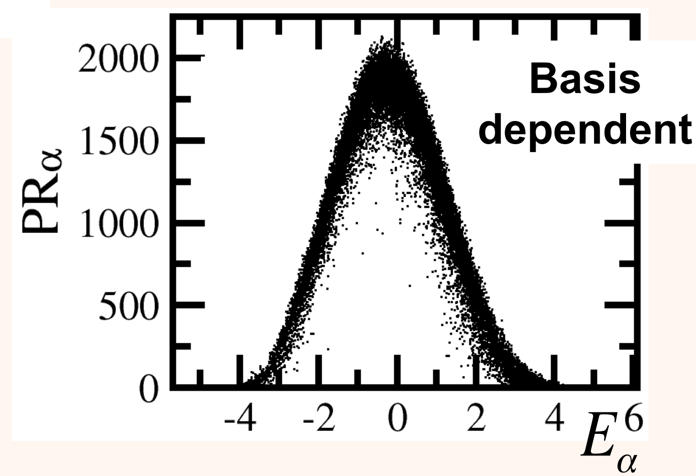
Physical many-body quantum systems

$$\psi^{(\alpha)} = \sum_{i=1}^{D} c_i^{(\alpha)} \phi_i$$

$$PR^{(\alpha)} \equiv \frac{1}{\sum_{n=1}^{D} | C_n^{(\alpha)} |^4}$$







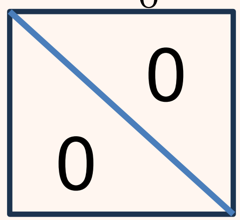
Dynamics and Timescales

Full Random Matrices vs Physical Models

Quench Dynamics: GOE

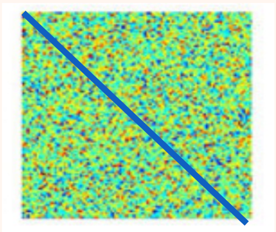
GOE

H_0



$$|\Psi(0)\rangle = |n_0\rangle$$

$H = H_0 + V$



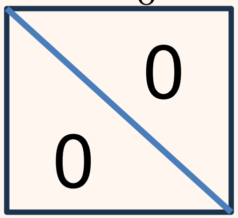
$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

Unphysical,
but analytical results

Quench Dynamics: Spin Model

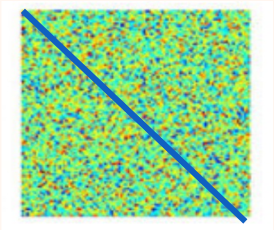
GOE

H_0



$|\Psi(0)\rangle = |n_0\rangle$

$H = H_0 + V$



$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$

Physical Model

Initial states with energy in the middle of the spectrum

$$\Psi(0) = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$$

$$H = \sum_{n=1}^{L} \frac{h_n}{2}\sigma_n^z + \sum_{n=1}^{L} \frac{J}{4}\left[\sigma_n^z\sigma_{n+1}^z + \left(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y\right)\right]$$

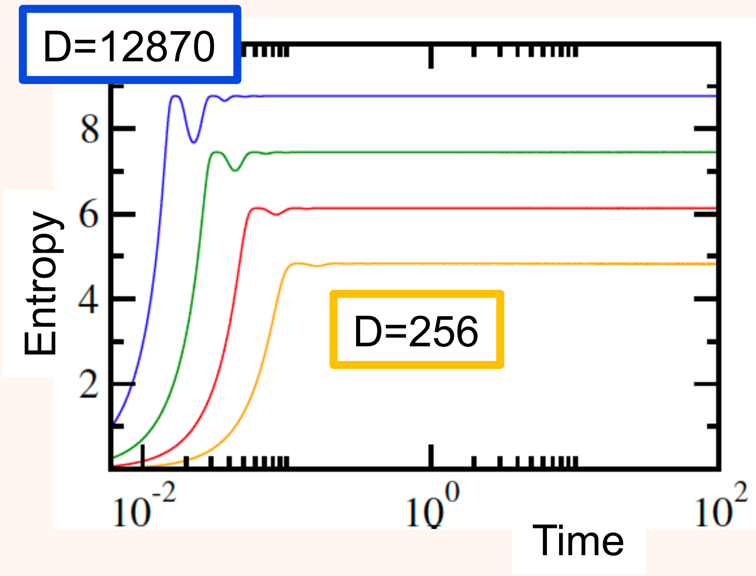
Timescales: GOE

"Participation" entropy: $-\ln(\sum_n |\langle n|\Psi(t)\rangle|^4)$

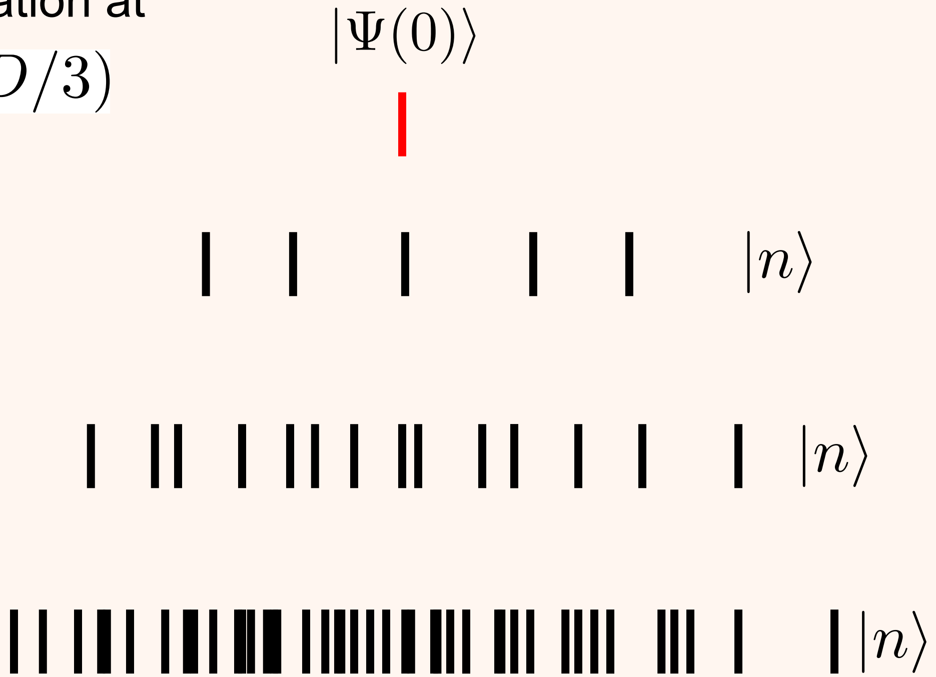
$$H_0|n\rangle = \epsilon_n|n\rangle$$

$$H = H_0 + V$$

GOE model



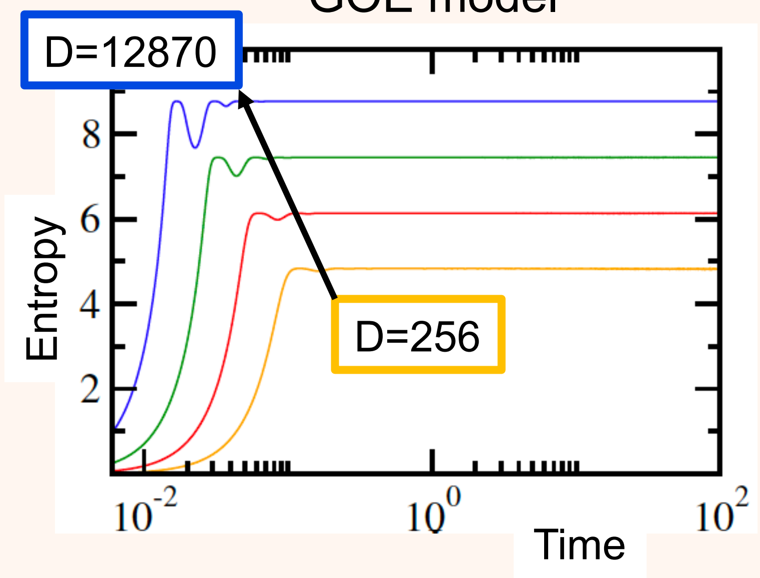
Saturation at $\ln(D/3)$



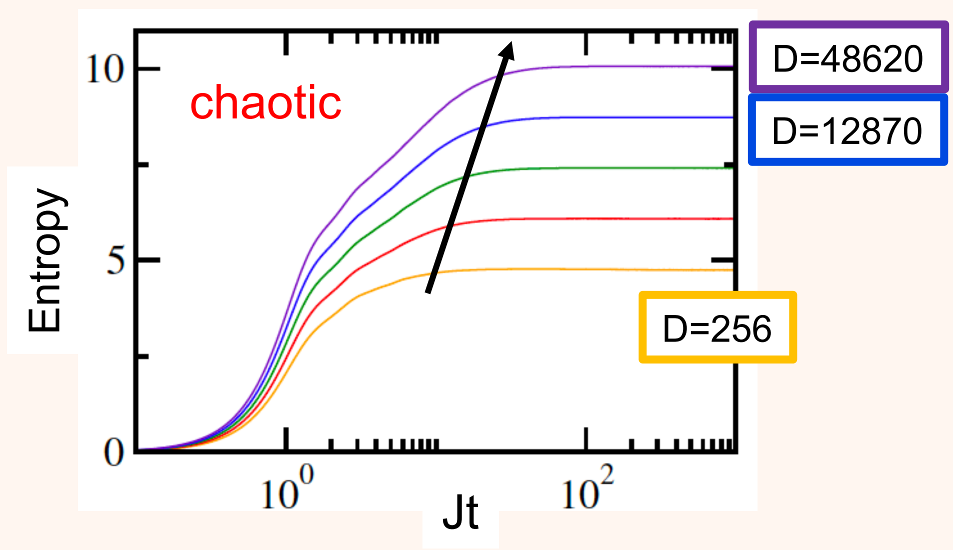
Timescales: GOE vs Physical Model

"Participation" entropy: $-\ln(\sum_n |\langle n|\Psi(t)\rangle|^4)$

GOE model



1D disordered spin-1/2 model



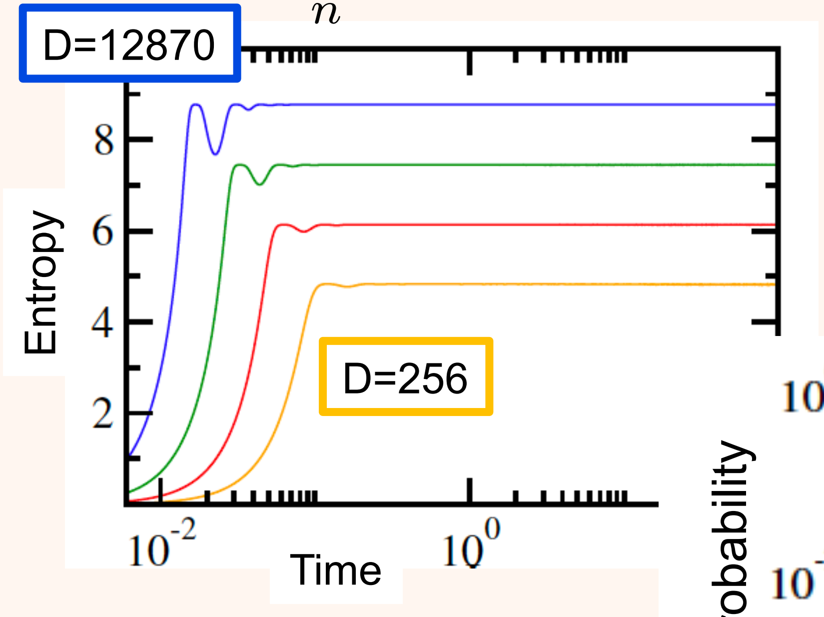
$$t^* \propto L^\gamma$$

PRB **104**, 085117 (2021)

Timescales: GOE

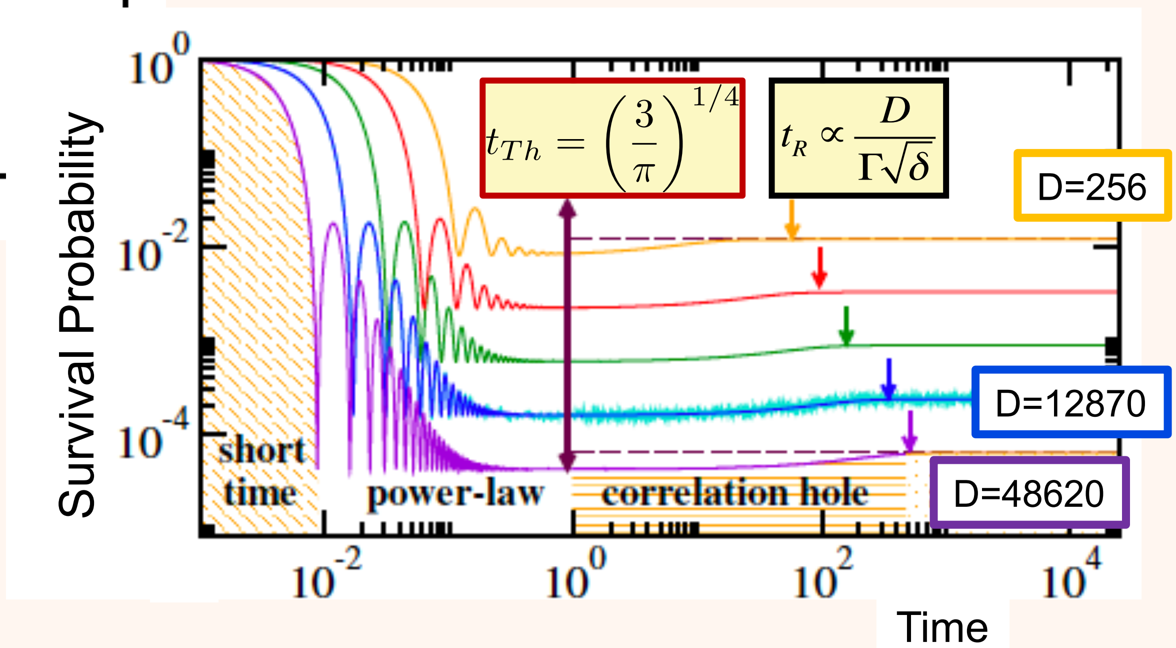
"Participation" Entropy:

$$-\ln\left(\sum_n |\langle n|\Psi(t)\rangle|^4\right)$$



Survival Probability:

$$\left|\langle\Psi(0)|\Psi(t)\rangle\right|^2$$



PRB **97**, 060303 (R) (2018)
PRB **99**, 174313 (2019)

Survival Probability and LDOS

Survival Probability
Return Probability
Fidelity

$$\left|\langle \Psi(0) \mid \Psi(t) \rangle\right|^2 = \left|\langle \Psi(0)|e^{-iHt}|\Psi(0)\rangle\right|^2$$

$$|\Psi(0)\rangle = \sum_{\alpha} C_{\alpha}^{ini} |\alpha\rangle \qquad H|\alpha\rangle = E_{\alpha}|\alpha\rangle$$

$$SP(t) = \left|\langle \Psi(0) \mid \Psi(t) \rangle\right|^2 = \left|\sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 e^{-iE_{\alpha}t}\right|^2 \cong \left|\int \rho_{ini}(E) e^{-iEt}\, dE\right|^2$$

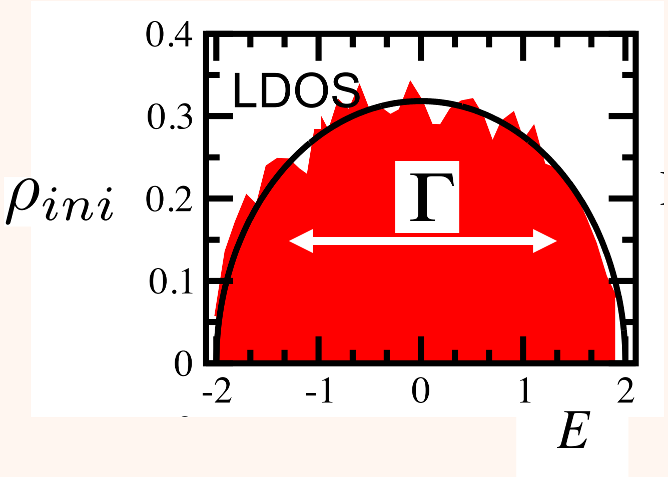
Survival probability is the Fourier transform of the LDOS

$$\rho_{ini}(E) = \sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 \delta(E - E_{\alpha})$$

Energy distribution of the initial state
LDOS = Strength function

LDOS: Semicircle

Full Random Matrices



LDOS = $\rho_{ini}(E) = \sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 \delta(E - E_{\alpha})$

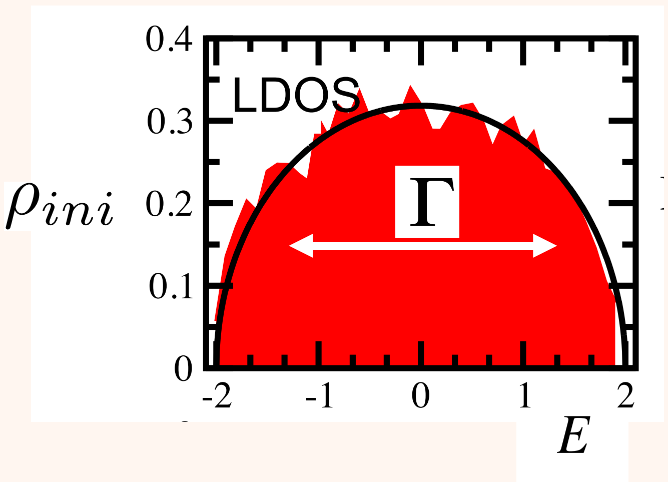
$$DOS = \sum_{\alpha} \delta(E - E_{\alpha})$$

$$\rho(E) = \frac{1}{\pi D}\sqrt{2D - E^2}$$

$$SP(t) = \left|\left\langle \Psi(0) \mid \Psi(t) \right\rangle\right|^2 = \left|\sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 e^{-iE_{\alpha}t}\right|^2 \cong \left|\int \rho_{ini}(E) e^{-iEt}\, dE\right|^2$$

Power-Law Decay

Full Random Matrices



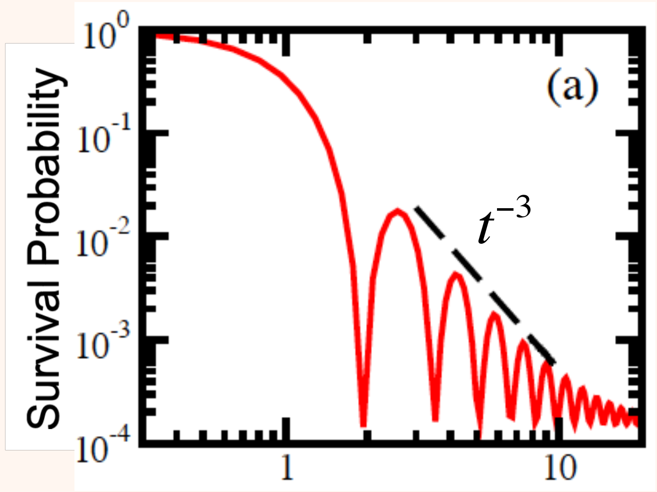
LDOS = $\boldsymbol{\rho}_{ini}(E) = \sum_{\alpha} \left| C_{\alpha}^{ini} \right|^2 \boldsymbol{\delta}(E - E_{\alpha})$

$$DOS = \sum_{\alpha} \delta(E - E_{\alpha})$$

$$\rho(E) = \frac{1}{\pi D}\sqrt{2D - E^2}$$

$$\left| \int \rho_{ini}(E) e^{-iEt}\, dE \right|^2$$

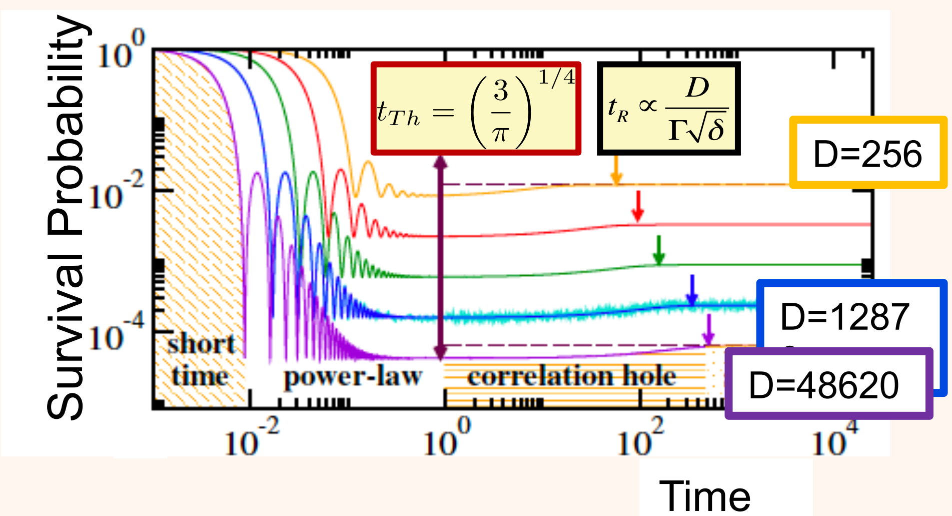
$$\frac{\left|\mathcal{J}_1(2\Gamma t)\right|^2}{\Gamma^2 t^2} \rightarrow t^{-3}$$



Survival Probability vs Discrete Spectrum

$$SP(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\sum_\alpha \left|C_\alpha^{ini}\right|^2 e^{-iE_\alpha t}\right|^2 \cong \left|\int \rho_{ini}(E) e^{-iEt}\, dE\right|^2$$

$$\langle SP(t) \rangle = \langle \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t} \rangle$$



$$\frac{1 - \overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{SP}$$

2-level form factor

correlation hole = dynamical manifestations of spectral correlations

SFF

Survival Probability and Spectral Form Factor

Survival Probability: $\left|\langle \Psi(0) \,|\, \Psi(t) \rangle\right|^2$

$$\Psi(0) = \uparrow \downarrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \uparrow$$

$$\langle SP(t) \rangle = \langle \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t} \rangle$$

$$C_\alpha^{ini} = \langle \alpha | \Psi(0) \rangle$$

Quench dynamics
(cold atoms, ion traps)

Spectral form factor

$$\mathrm{SFF}(t) = \frac{1}{D^2} \left\langle \sum_{\alpha,\beta} e^{i(E_\alpha - E_\beta)t} \right\rangle$$

Fourier transform of the
two-point spectral correlation function

Survival Probability and Spectral Form Factor

Survival Probability: $\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$

$\Psi(0) = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$

$$\langle SP(t) \rangle = \langle \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t} \rangle$$

$C_\alpha^{ini} = \langle \alpha | \Psi(0) \rangle$

Quench dynamics
(cold atoms, ion traps)

Spectral form factor

$$\mathrm{SFF}(t) = \frac{1}{D^2} \left\langle \sum_{\alpha,\beta} e^{i(E_\alpha - E_\beta)t} \right\rangle$$

Fourier transform of the
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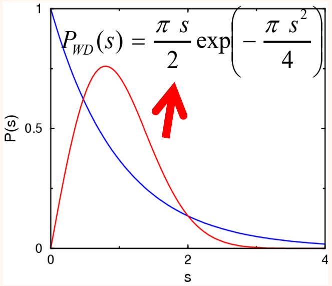
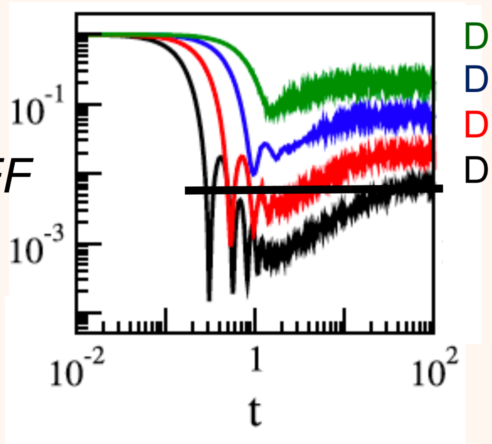


Signature of spectral correlations:

Slope-dip-ramp-plateau structure

GOE



Correlation Hole

VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

Fourier Transform: A Tool to Measure Statistical Level Properties in Very Complex Spectra

Luc Leviandier, Maurice Lombardi, Rémi Jost, and Jean Paul Pique
Laboratoire de Spectrométrie Physique, Université Scientifique et Médicale de Grenoble, 38402 Saint Martin d'Hères, France, and Service National des Champs Intenses, Centre National de la Recherche Scientifique, 38042 Grenoble Cedex, France
(Received 27 November 1985)

VOLUME 58, NUMBER 5 PHYSICAL REVIEW LETTERS 2 FEBRUARY 1987

Chaos and Dynamics on 0.5–300-ps Time Scales in Vibrationally Excited Acetylene: Fourier Transform of Stimulated-Emission Pumping Spectrum

J. P. Pique,[a] Y. Chen, R. W. Field, and J. L. Kinsey
Department of Chemistry and George Harrison Spectroscopy Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 27 October 1986)

Chemical Physics 146 (1990) 21–38
North-Holland

Correlations in anticrossing spectra and scattering theory. Analytical aspects

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Max-Planck-Institut für Kernphysik, 6900 Heidelberg, FRG

Received 12 December 1989

Experimental results of anticrossing spectroscopy in molecules, in particular the correlation hole, are discussed in a theoretical model. The laser measurements are modelled in terms of the scattering matrix formalism originally developed for compound nucleus scattering. Random matrix theory is used in the framework of this model. The correlation hole is analytically derived for small singlet–triplet coupling. In the case of the data on methylglyoxal this limit is realistic if the spectrum is indeed a superposition of several pure sequences as one can conclude from the analysis of the measurements.

PHYSICAL REVIEW E, VOLUME 65, 026214

Signatures of the correlation hole in total and partial cross sections

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(Received 3 August 2001; published 24 January 2002)

In a complex scattering system with few open channels, say a quantum dot with leads, the correlation properties of the poles of the scattering matrix are most directly related to the internal dynamics of the system. We may ask how to extract these properties from an analysis of cross sections. In general this is very difficult, if we leave the domain of isolated resonances. We propose to consider the cross correlation function of two different elastic or total cross sections. For these we can show numerically and to some extent also analytically a significant dependence on the correlations between the scattering poles. The difference between uncorrelated and strongly correlated poles is clearly visible, even for strongly overlapping resonances.

PHYSICAL REVIEW A VOLUME 46, NUMBER 8 15 OCTOBER 1992

Spectral autocorrelation function in the statistical theory of energy levels

Y. Alhassid
Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connectic and the A.W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut

R. D. Levine
The Fritz Haber Research Center for Molecular Dynamics, The Hebrew University, Jerusalem 91
(Received 11 October 1991; revised manuscript received 5 May 1992)

J. Phys. A: Math. Theor. 46 (2013) 275303 (12pp) doi:10.1088/1751-8113/46/27/275303

Fidelity under isospectral perturbations: a random matrix study

F Leyvraz[1,2], A García[1], H Kohler[3] and T H Seligman[1,2]

VOLUME 67, NUMBER 10 PHYSICAL REVIEW LETTERS 2 SEPTEMBER 1991

Time-Dependent Manifestations of Quantum Chaos

Joshua Wilkie and Paul Brumer
Chemical Physics Theory Group, Department of Chemistry, University of Toronto, Toronto, Ontario, Canada M5S 1A1
(Received 11 April 1991)

Correlation Hole

VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

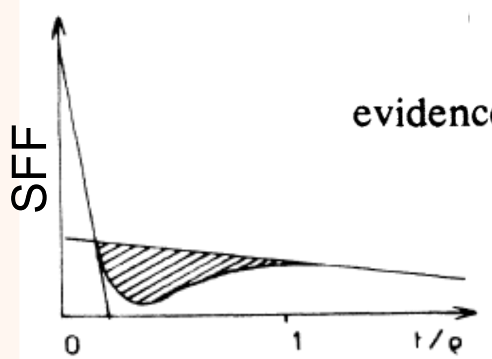
Fourier Transform: A Tool to Measure Statistical Level Properties in Very Complex Spectra

Luc Leviandier, Maurice Lombardi, Rémi Jost, and Jean Paul Pique

Laboratoire de Spectrométrie Physique, Université Scientifique et Médicale de Grenoble, 38402 Saint Martin d'Hères, France, and Service National des Champs Intenses, Centre National de la Recherche Scientifique, 38042 Grenoble Cedex, France

(Received 27 November 1985)

We show that the Fourier transform of very complex spectra gives a sound measurement of long-range statistical properties of levels even in cases of badly resolved, poorly correlated spectra. Examples of nuclear energy levels, highly excited acetylene vibrational levels, and singlet-triplet anticrossing spectra in methylglyoxal are displayed.



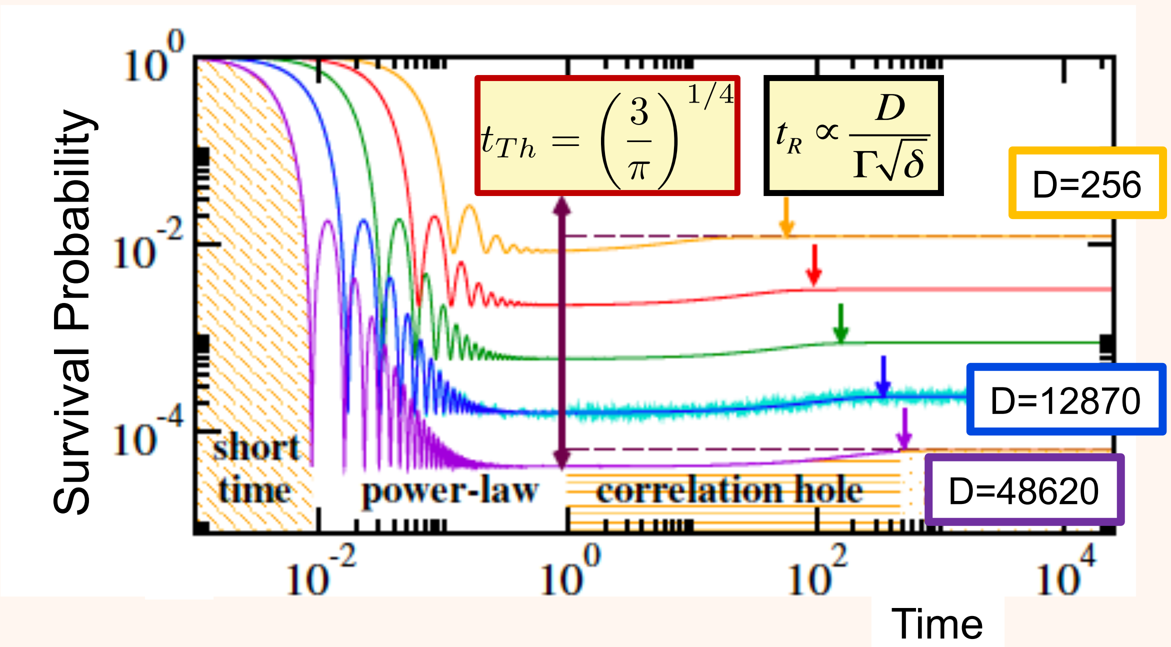
The presence of level correlation is thus evidenced by a "correlation hole"

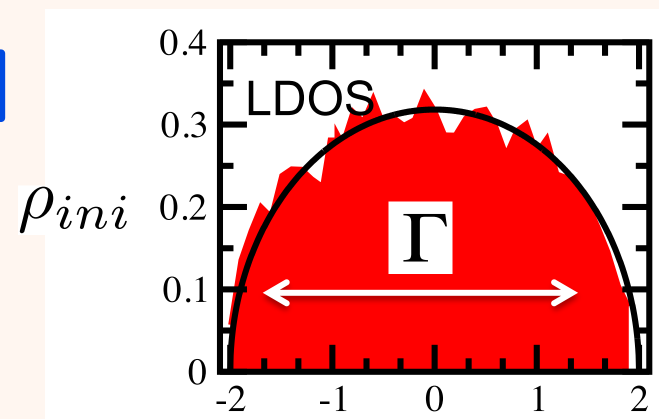
Survival Probability: GOE

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{SP}$$

2-level form factor



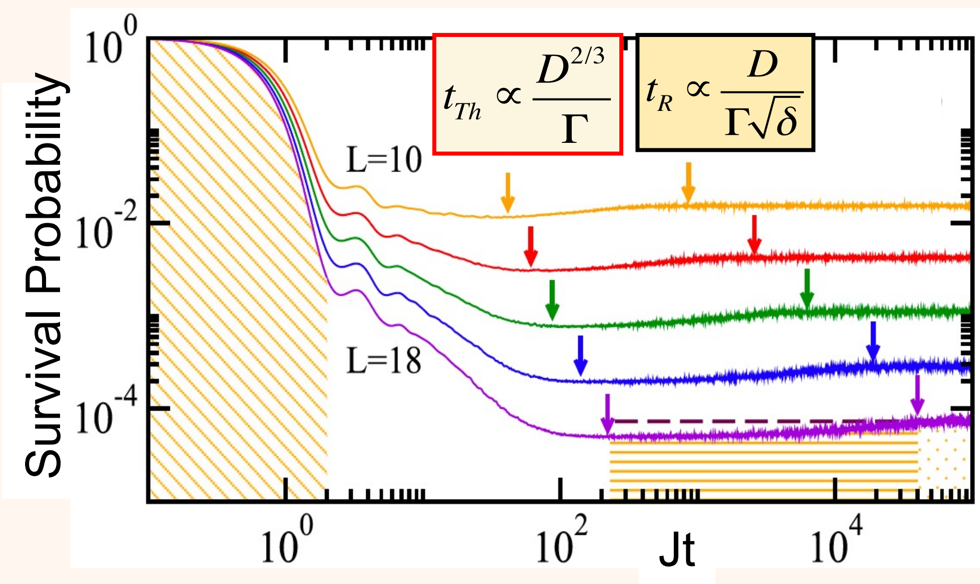


PRB **97**, 060303 (R) (2018)
PRB **99**, 174313 (2019)

Survival Probability: Physical Model

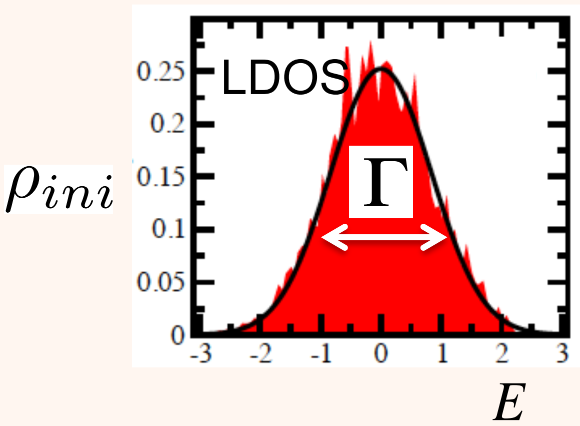
$$\Psi(0) = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$$

$$|\langle\Psi(0)|\Psi(t)\rangle|^2$$



$$\frac{1-\overline{SP}}{(D-1)}\left[\frac{De^{-\Gamma^2 t^2}}{\mathcal{N}^2}\mathcal{F}(t) - b_2\left(\frac{\Gamma t}{\sqrt{2\pi}D}\right)\right] + \overline{SP}$$

$$\mathcal{F}(t) = \left|\text{erf}\left(\frac{E_{\max} + it\Gamma^2}{\sqrt{2}\Gamma}\right) - \text{erf}\left(\frac{E_{\min} + it\Gamma^2}{\sqrt{2}\Gamma}\right)\right|^2$$



$$H = \sum_{n=1}^{L}\frac{h_n}{2}\sigma_n^z + \sum_{n=1}^{L}\frac{J}{4}\left[\sigma_n^z\sigma_{n+1}^z + \left(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y\right)\right]$$

PRB **97**, 060303 (R) (2018)
PRB **99**, 174313 (2019)

Correlation Hole

Dynamical manifestation of quantum chaos

Correlation Hole: Advantages/Disadvantages

$$\langle SP(t)\rangle = \langle \sum_{\alpha,\beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 e^{-i(E_\alpha - E_\beta)t}\rangle$$

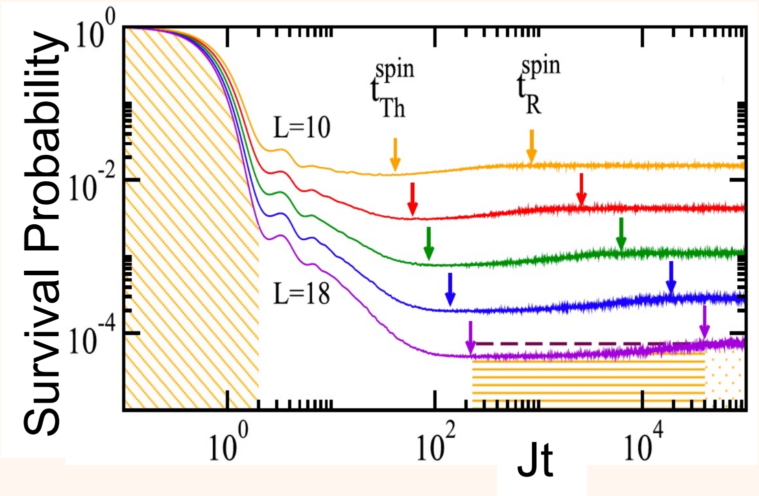
ADVANTAGES:

No unfolding

Short- and long-range correlation

Emerges despite symmetries

Dynamical quantity



DISADVANTAGES:

Long-times

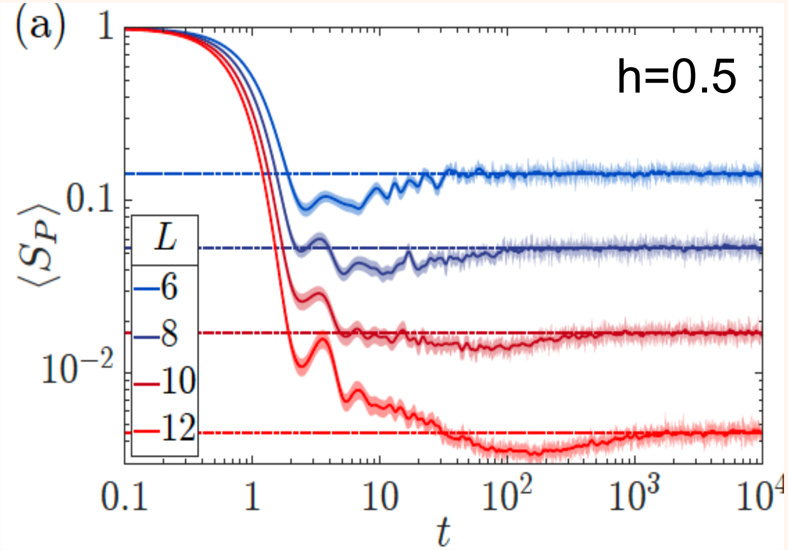
Non-local quantity

Non-self-averaging

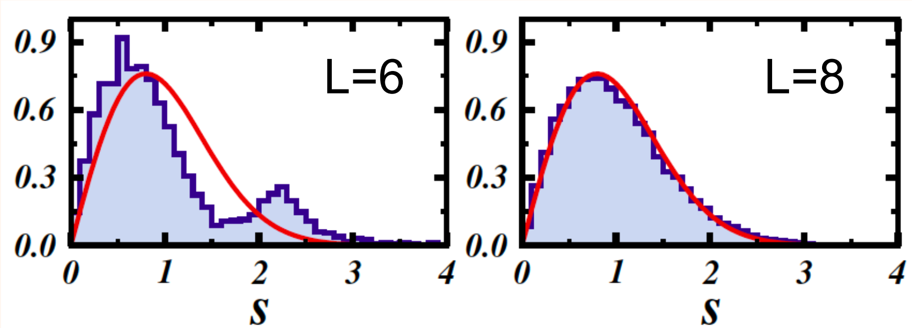
PRR **2**, 043034 (2020)

Experimental Detection of the Correlation Hole

Small Systems



Add couplings…



3 interacting particles

4 interacting particles

DISADVANTAGES:

✅ Long-times

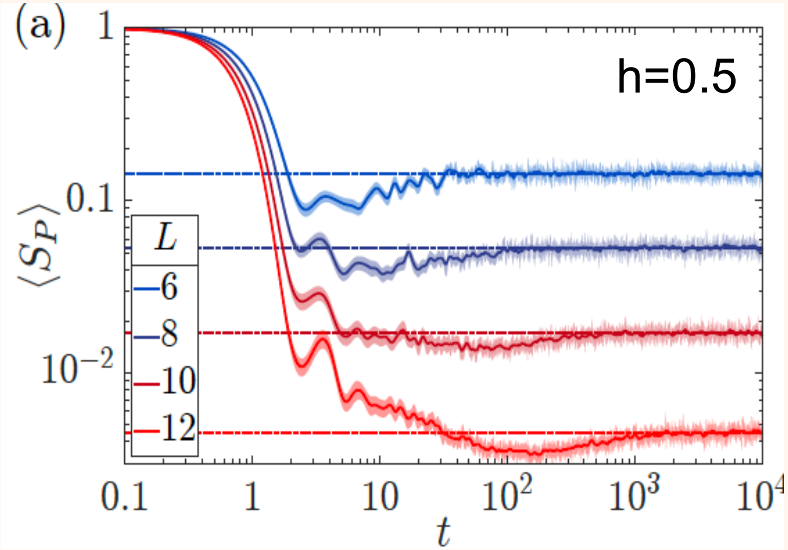
Non-local quantity

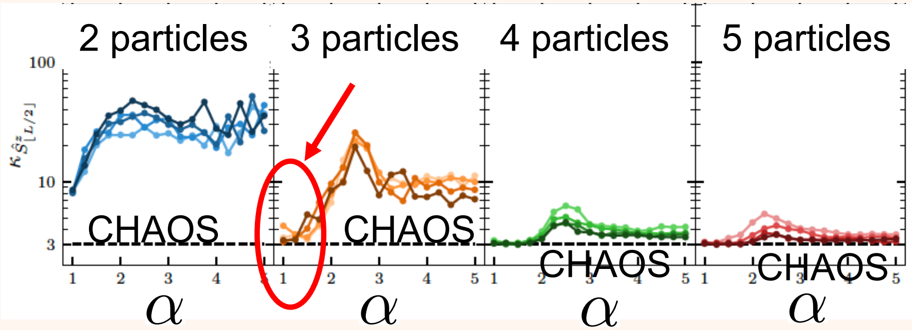
Non-self-averaging

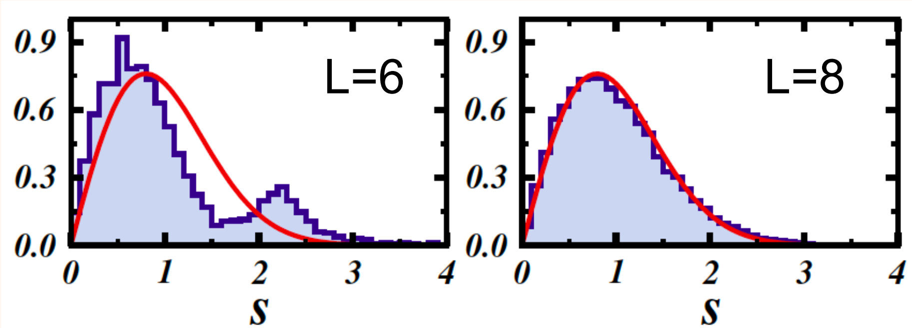
Proposal for many-body quantum chaos detection
PRR **7**, 013181 (2025)

Small Systems and Long-Range Couplings

SciPostPhys**10**, 088 (2021)







3 interacting particles

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DISADVANTAGES:

✅ Long-times

Non-local quantity

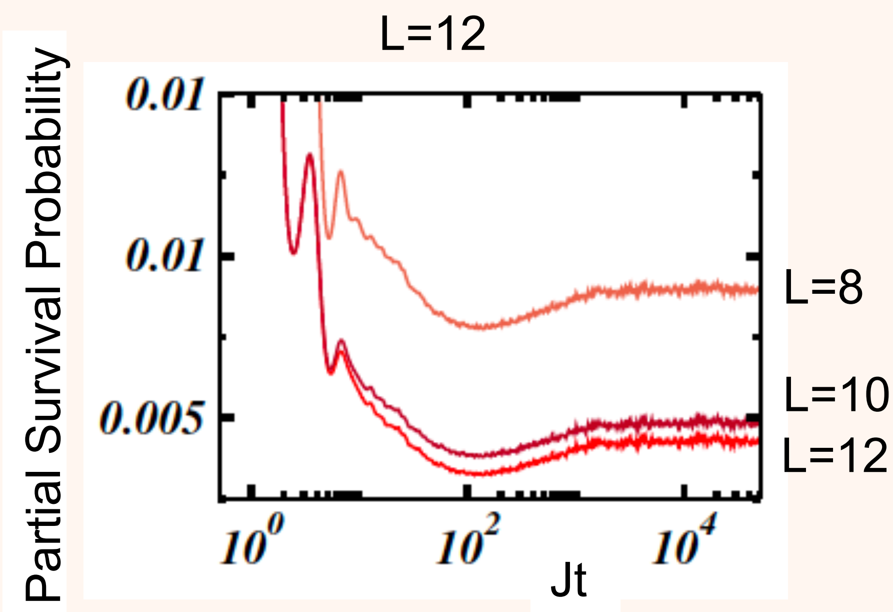
Non-self-averaging

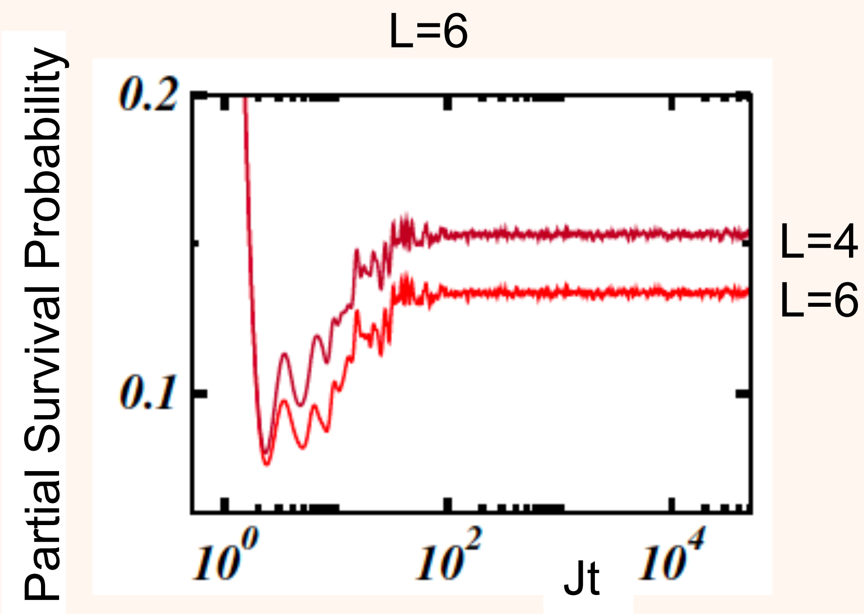
Proposal for many-body quantum chaos detection
PRR **7**, 013181 (2025)

Quasi-local Observable: Partial SP

Partial
Survival Probability

$$S_P^{(2,4)}(t) = \left| \langle \uparrow \boxed{\downarrow\uparrow} \downarrow | e^{-iHt} | \uparrow \boxed{\downarrow\uparrow} \downarrow \rangle \right|^2 + \left| \langle \downarrow \boxed{\downarrow\uparrow} \uparrow | e^{-iHt} | \uparrow \boxed{\downarrow\uparrow} \downarrow \rangle \right|^2$$





arXiv:2505.05572

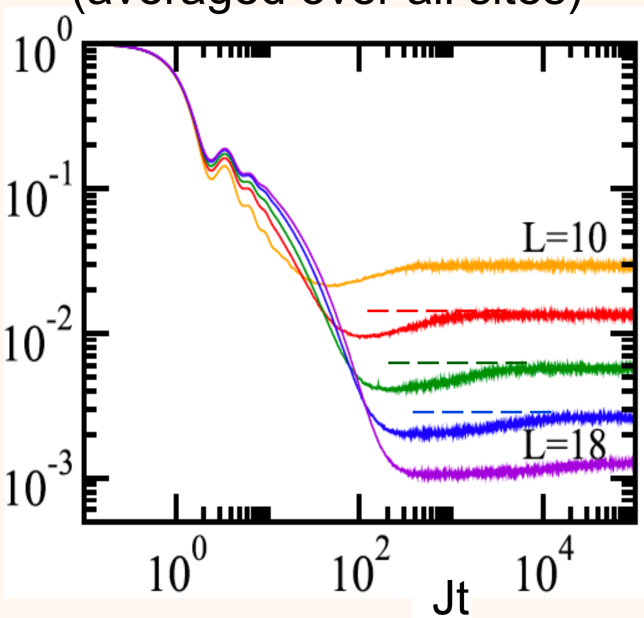
Immanuel Bloch, Monika Aidelsburger
arXiv:2501.16995

Local Observable: Spin Autocorrelation Function

Spin Autocorrelation Function:

$$\frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\hat{\sigma}_k^z\hat{\sigma}_k^z(t)|\Psi(0)\rangle$$

Spin Autocorrelation Function
(averaged over all sites)



DISADVANTAGES:

✅ Long-times

✅ Non-local quantity

Non-self-averaging

Spin Autocorrelation Function
(for a single site)

PRB **97**, 060303 (R) (2018)

PRB **104**, 085117 (2021)

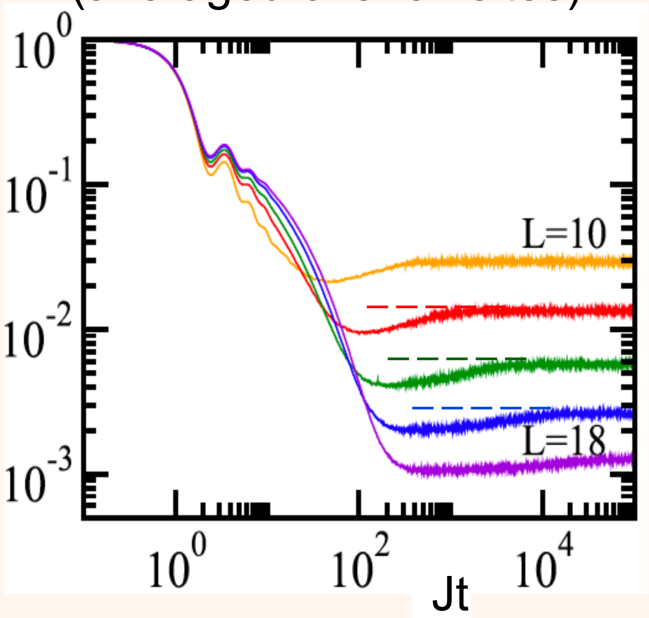
PRR **7**, 013181 (2025)

Spin Autocorrelation Function: Single Site

Spin Autocorrelation Function:

$$\frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\hat{\sigma}_k^z\hat{\sigma}_k^z(t)|\Psi(0)\rangle$$

Spin Autocorrelation Function
(averaged over all sites)



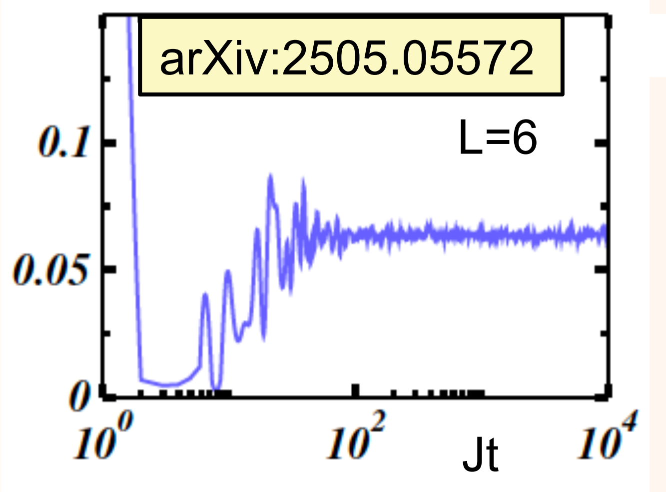
PRB **97**, 060303 (R) (2018)

PRB **104**, 085117 (2021)

PRR **7**, 013181 (2025)

Detection of many-body quantum chaos with a single site

Spin Autocorrelation Function
(for a single site)



DISADVANTAGES:

✅ Long-times

✅ Non-local quantity

Non-self-averaging

DISADVANTAGES:

✅ Long-times

✅ Non-local quantity

Non-self-averaging

Self-Averaging

A quantity O is self-averaging when its relative variance goes to zero as the system size increases

$$\mathcal{R}_O(t) = \frac{\sigma_O^2(t)}{\langle O(t) \rangle^2} = \frac{\langle O^2(t) \rangle - \langle O(t) \rangle^2}{\langle O(t) \rangle^2}$$

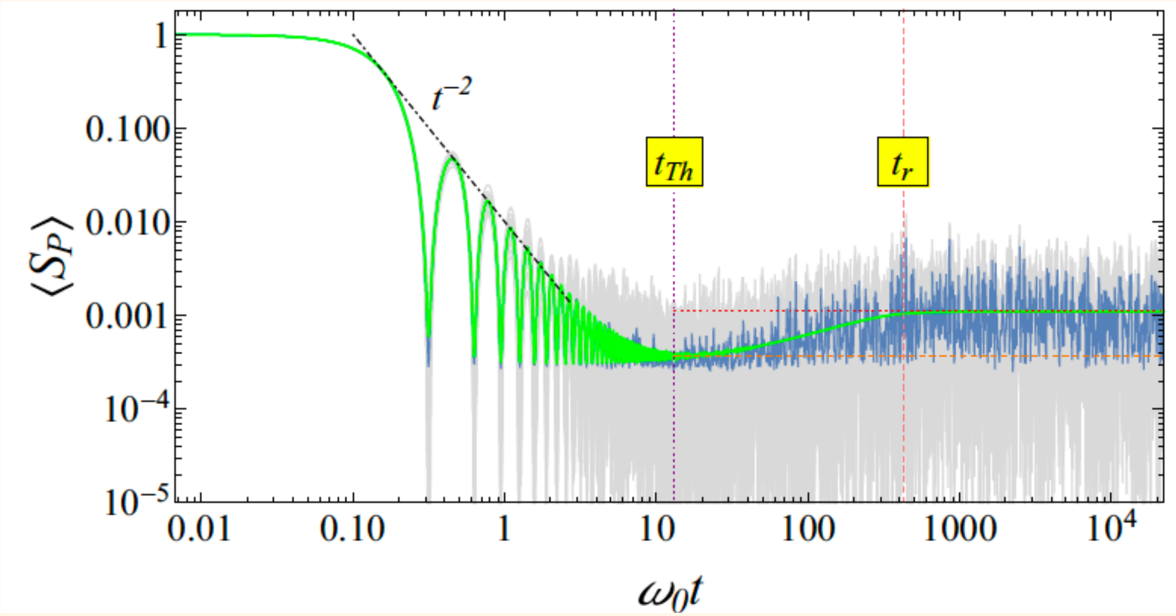
By increasing the system size, one can **reduce** the number of samples used in

- experiments
- statistical analysis.

If the system exhibits self-averaging, its physical properties are independent of the specific realization.

PRR **3,** L032030 (2021)
PRE **102**, 062126 (2020)
PRB **102**, 094310 (2020)
PRB **101**, 174312 (2020)

Lack of Self-Averaging



Dicke model
PRE **100**, 012218 (2019)

N. Argaman, F.-M. Dittes, E. Doron, J. P. Keating, A. Yu. Kitaev, M. Sieber, and U. Smilansky, Correlations in the Actions of Periodic Orbits Derived from Quantum Chaos, Phys. Rev. Lett. 71, 4326 (1993)

B. Eckhardt and J. Main, Semiclassical Form Factor of Matrix Element Fluctuations, Phys. Rev. Lett. 75, 2300 (1995)

R. E. Prange, The Spectral form Factor is Not Self-Averaging, Phys. Rev. Lett. 78, 2280 (1997)

P. Braun and F. Haake, Self-averaging characteristics of spectral fluctuations, J. Phys. A 48, 135101 (2015)

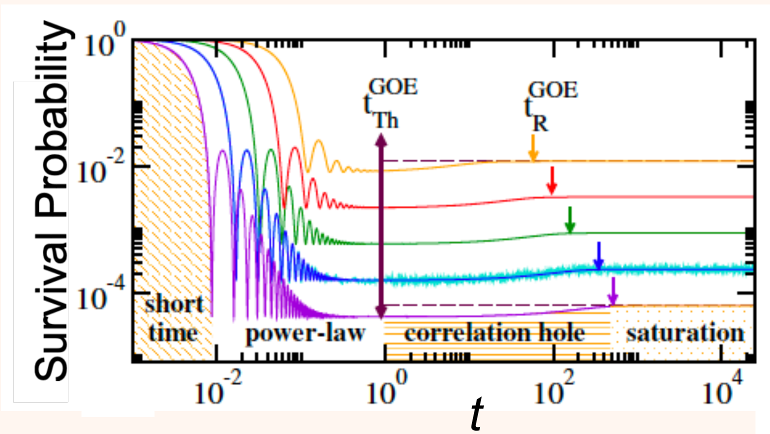
Analytical with GOE:
SP and SFF are nowhere self-averaging

PRB **101**, 174312 (2020)

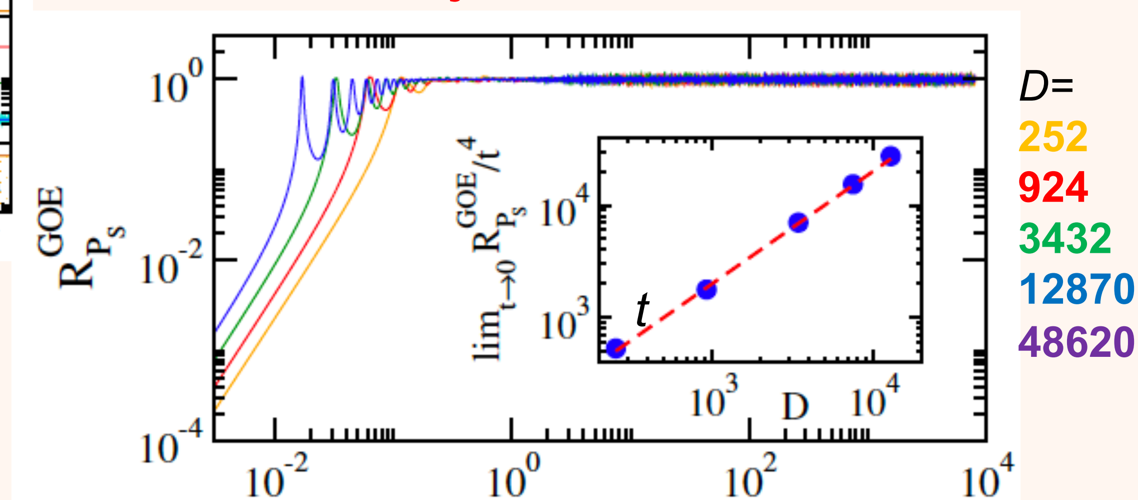
Lack of Self-Averaging: Analytical Results

$$SP(t) = \left|\langle \Psi(0)|e^{-iHt}|\Psi(0)\rangle\right|^2$$

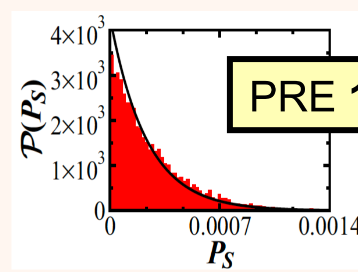
$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$



Analytical results: GOE



$$\left\langle \sum_{\alpha\neq\gamma\neq\beta\neq\delta} e^{-i(E_\alpha - E_\beta + E_\gamma - E_\delta)t} \left|c_\alpha^{(0)}\right|^2 \left|c_\beta^{(0)}\right|^2 \left|c_\gamma^{(0)}\right|^2 \left|c_\delta^{(0)}\right|^2 \right\rangle$$



PRE **102**, 062126 (2020)

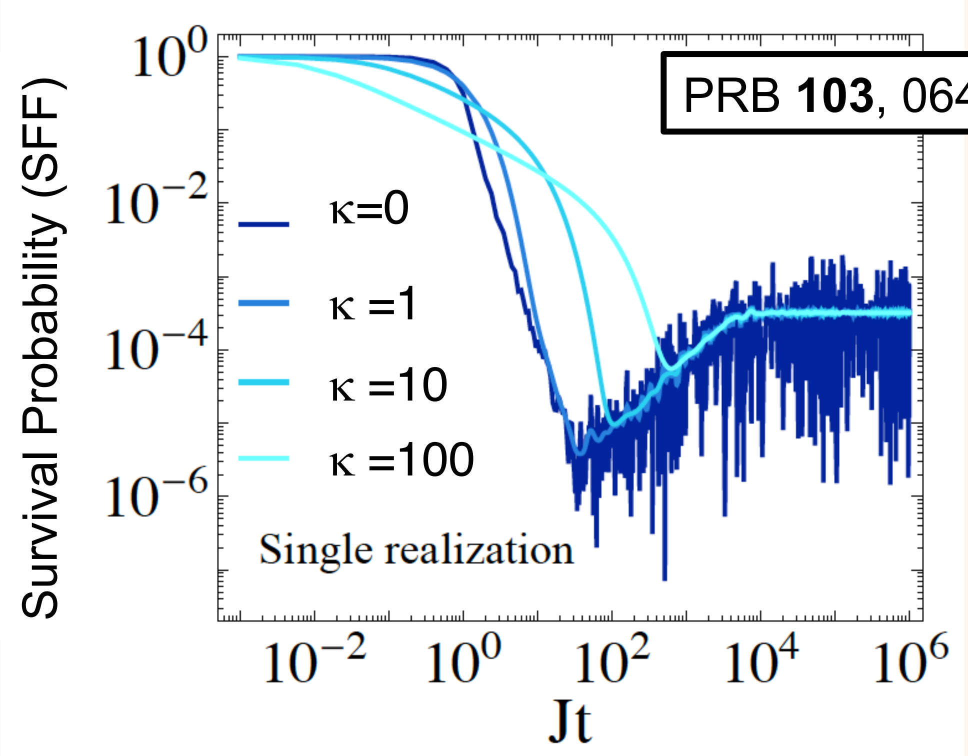
PRB **101**, 174312 (2020)

Avoiding averages with decoherence

$$\frac{d\rho}{dt} = -i[H,\rho] - \kappa[H,[H,\rho]]$$

dephasing strength

Tameshtit and Sipe,
Survival probability and chaos in an open quantum system,
PRA **45**, 8280 (1992)



PRB **103**, 064309 (2021)

Adolfo del Campo

Initial Gibbs states

$$|c_n^{(0)}|^2 = \frac{e^{-\beta E_n}}{\sum_m e^{-\beta E_m}}$$

PRA **108**, 062201 (2023)

Self-averaging in open systems: GOE

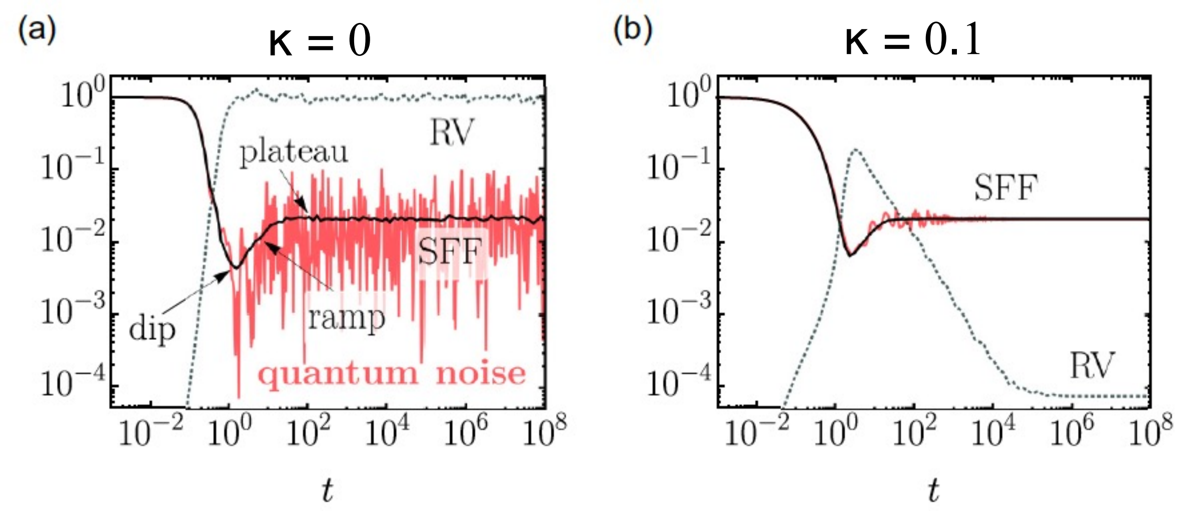
$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$

GOE

ISOLATED

OPEN

Spectral form factor for a single realization (**red**) and upon Hamiltonian average (**black**), together with the RELATIVE VARIANCE (**dotted** line).



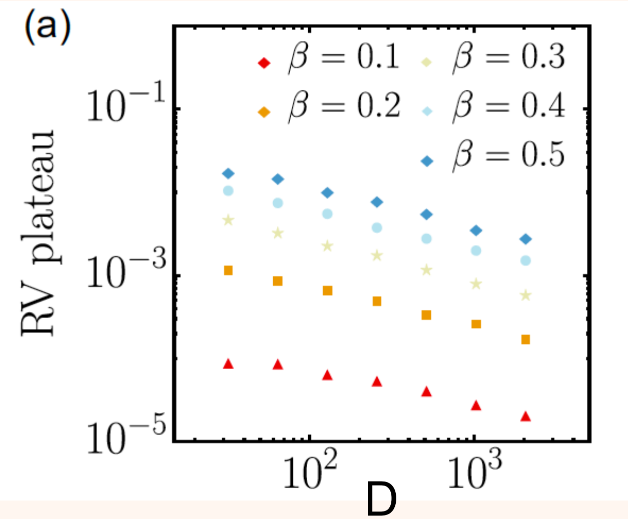
$$\frac{d\rho}{dt} = -i[H,\rho] - \kappa[H,[H,\rho]]$$

$$S_P(t) = \mathrm{Tr}[\rho(t)\rho(0)]$$

$$\rho_{\alpha\beta}(t) = \rho_{\alpha\beta}(0)e^{-i(E_\alpha - E_\beta)t - \kappa(E_\alpha - E_\beta)^2 t}$$

PRA **108**, 062201 (2023)

Self-averaging in open systems: GOE



DISADVANTAGES:

- ✅ Long-times
- ✅ Non-local quantity
- ✅ Non-self-averaging

$$\frac{d\rho}{dt} = -i[H,\rho] - \kappa[H,[H,\rho]]$$

GOE Random matrices

Initial Gibbs states

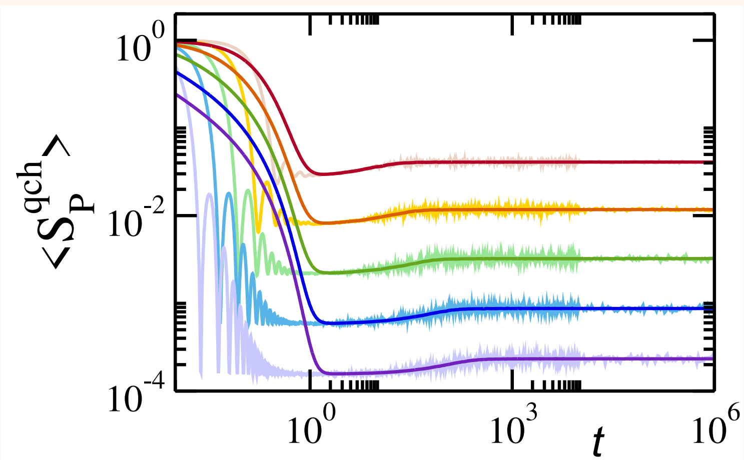
$$S_P(t) = \sum_{n,m=1}^{N} |c_n^{(0)}|^2 |c_m^{(0)}|^2 e^{-i(E_n-E_m)t-\kappa(E_n-E_m)^2 t}$$

$$|c_n^{(0)}|^2 = \frac{e^{-\beta E_n}}{\sum_m e^{-\beta E_m}}$$

Average ⇔ environment

PRA **108**, 062201 (2023)

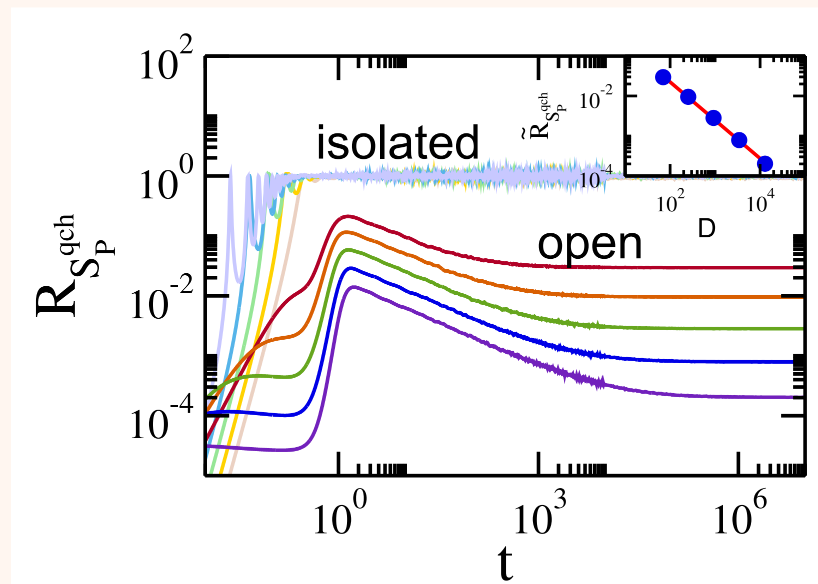
Self-averaging in GOE matrices



D=
70
252
924
3432
12870

$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$

$$\overline{R^{\kappa\neq 0}_{S_P}} = \frac{\sigma^2_{\mathrm{IPR}_0}}{\langle \mathrm{IPR}_0\rangle^2}$$



D=
70
252
924
3432
12870

PRB **110**, 075138 (2024) quench

Self-averaging:
power-law banded random matrices

$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t) \rangle^2} = \frac{\left\langle SP^2(t) \right\rangle - \langle SP(t) \rangle^2}{\langle SP(t) \rangle^2}$$

PRB **110**, 075138 (2024)

Power-law banded random matrices:

LOCALIZATION for $\alpha>1$

$$\langle H_{ij}^2 \rangle = \begin{cases} 1, & i=j, \\ \left(1+|i-j|^{2\alpha}\right)^{-1}, & i \neq j, \end{cases}$$

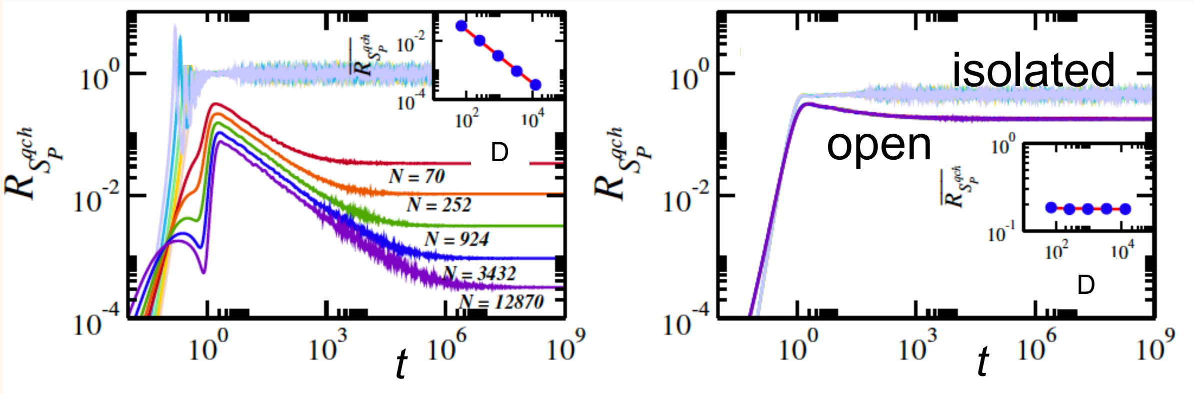
GOE | deloc | critical | localized | tridiagonal

0 | 1 | ∞ → α

Self-averaging:
power-law banded random matrices

$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$

PRB **110**, 075138 (2024)



Power-law banded random matrices:
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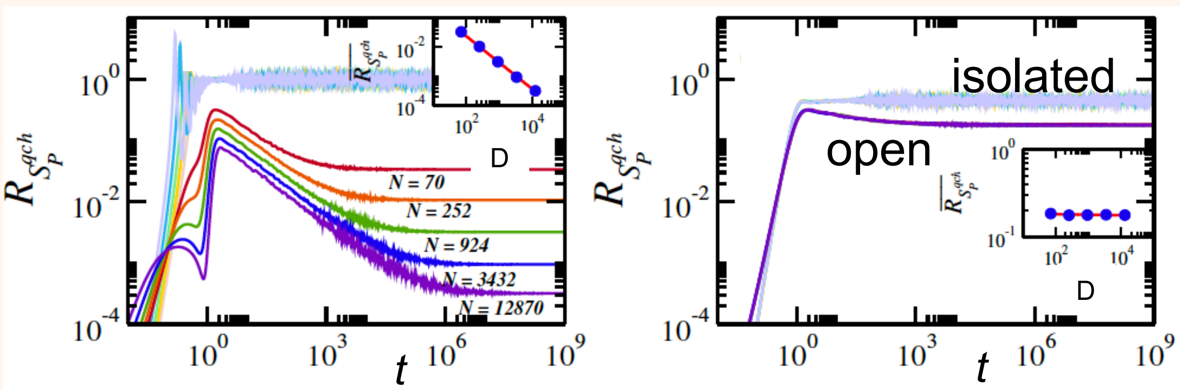
GOE deloc critical localized tridiagonal α

0 1 ∞

Self-averaging: power-law banded random matrices

$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$

PRB **110**, 075138 (2024)

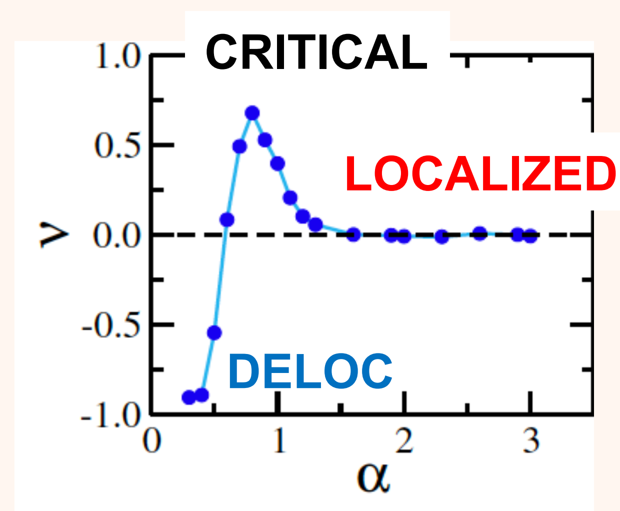


$$\overline{R^{\kappa\neq 0}_{S_P^{qch}}} \propto D^{\nu}$$

Power-law banded random matrices:
LOCALIZATION for α>1

$$\langle H^2_{ij}\rangle = \begin{cases} 1, & i = j, \\ \left(1 + |i-j|^{2\alpha}\right)^{-1}, & i \neq j, \end{cases}$$

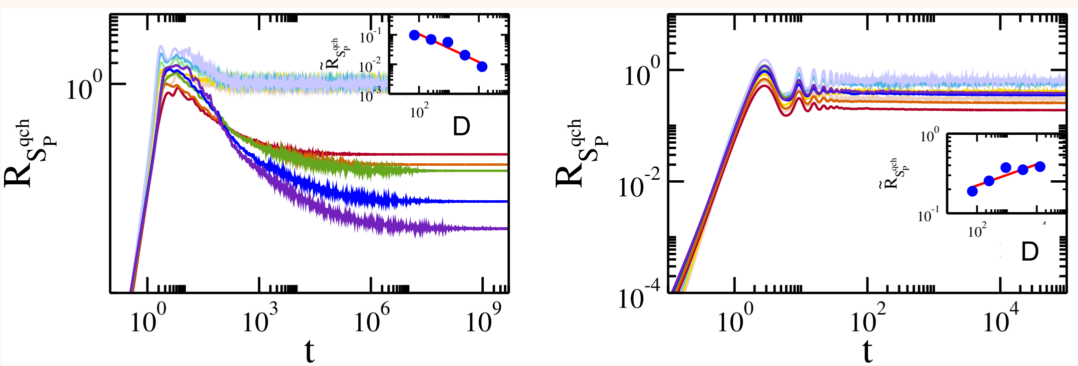
GOE	deloc	critical	localized	tridiagonal	
0		1		∞	α



Self-averaging in open physical systems

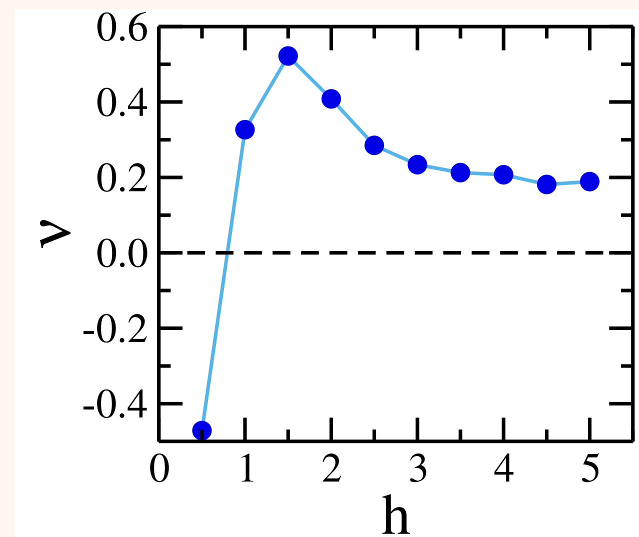
$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$

CHAOTIC (h=0.5) **NON-CHAOTIC** (h=5)



L=
8
10
12
14
16

$$\overline{R^{\kappa\neq 0}_{S_P^{qch}}} \propto D^{\nu}$$



$$H = \sum_{n=1}^{L} \frac{h_n}{2}\sigma_n^z + \sum_{n=1}^{L} \frac{J}{4}\left[\sigma_n^z\sigma_{n+1}^z + \left(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y\right)\right]$$

$$\overline{R^{\kappa\neq 0}_{S_P}} = \frac{\sigma^2_{\mathrm{IPR}_0}}{\langle \mathrm{IPR}_0\rangle^2}$$

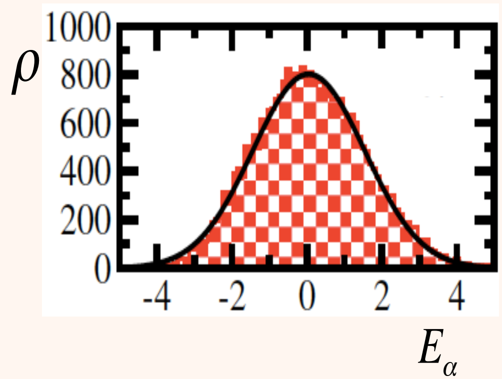
PRB **110**, 075138 (2024)

Lack of self-averaging vs initial state

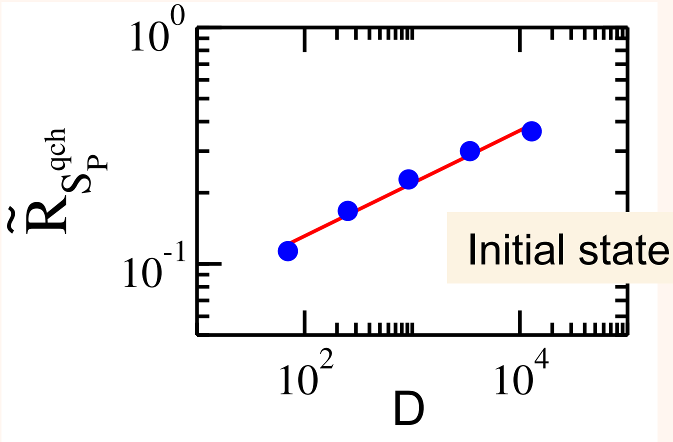
$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$

$$\overline{R^{\kappa\neq 0}_{S_P^{qch}}} \propto D^{\nu}$$

Density of states



CHAOS (h=0.5)



Initial state with low energy

PRB **110**, 075138 (2024)

Summary

- The time to reach thermal equilibrium in a chaotic system depends on the model, quantity and initial state.

- Polynomial increase with *L*.
  Quantities with **correlation hole:** Exponentially long time in *L* to equilibrate.

Summary

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- **Correlation hole:** Dynamical manifestations of spectral correlations.
  It could be **detected experimentally** (quench: SP, spin autocorrelation function).

PRR **7**, 013181 (2025)

arXiv:2505.05572

one-site

Summary

- The time to reach thermal equilibrium in a chaotic system depends on the model, quantity and initial state.

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PRR **7**, 013181 (2025)

arXiv:2505.05572

one-site

- Lack of **self-averaging**: PRB **110**, 075138 (2024)
  Avoided by **opening** the system to a dephasing environment (**chaotic** systems).

- Opening the system **reduces fluctuations**.

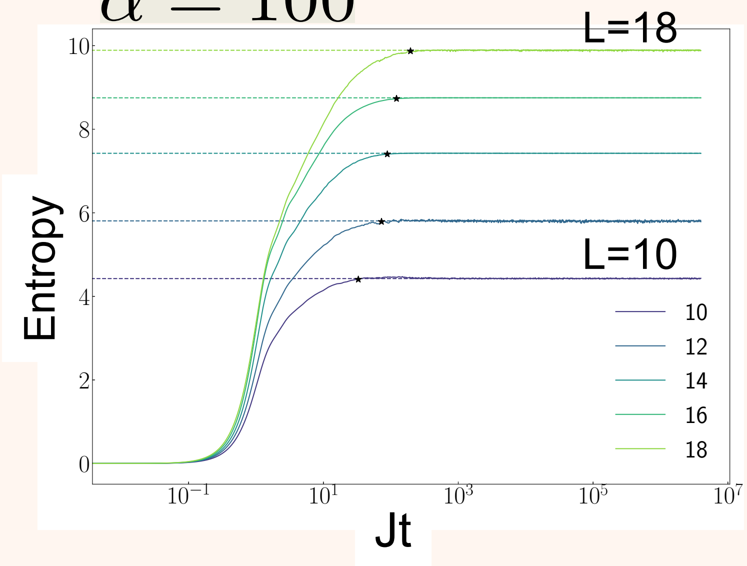
Long-range couplings

Long-range coupling (α >1)

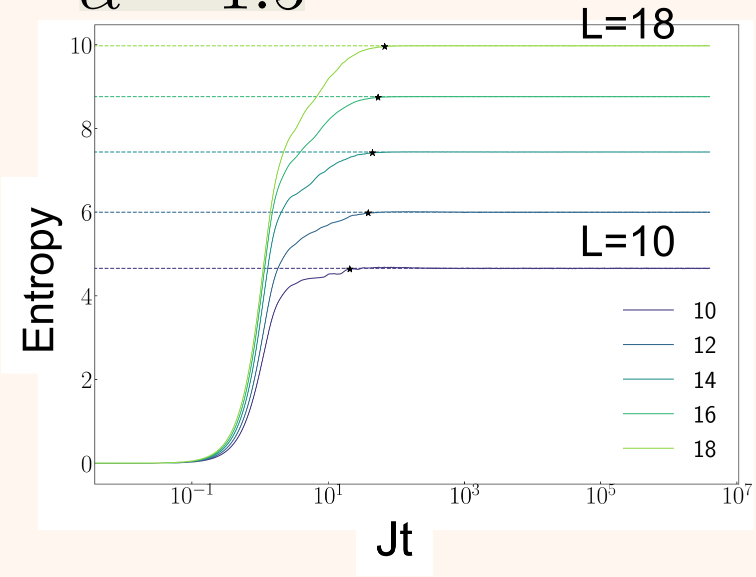
$$H = \sum_{n=1}^{L} \frac{h_n}{2}\sigma_n^z + \sum_{n<m}^{L-1} \frac{J}{4}\frac{1}{|n-m|^\alpha}[\sigma_n^x\sigma_m^x + \sigma_n^y\sigma_m^y + \sigma_n^z\sigma_m^z]$$

"Participation" Entropy: $-\ln(\sum_n |\langle n|\Psi(t)\rangle|^4)$

$\alpha = 100$



$\alpha = 1.5$

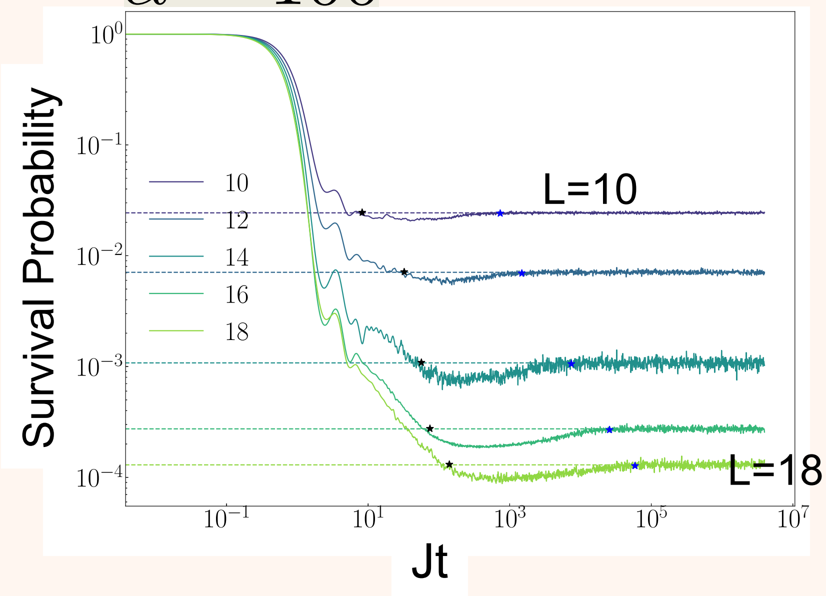


Long-range coupling (α >1)

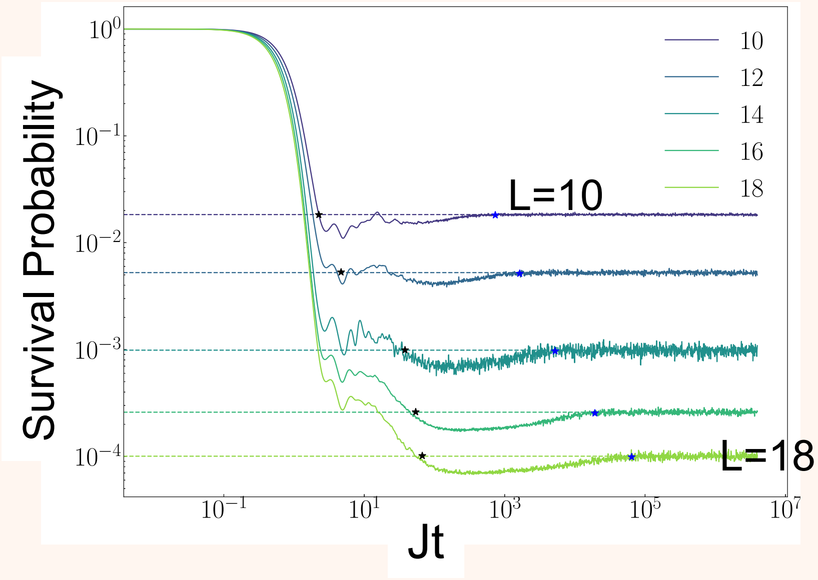
$$H = \sum_{n=1}^{L} \frac{h_n}{2}\sigma_n^z + \sum_{n<m}^{L-1} \frac{J}{4}\frac{1}{|n-m|^\alpha}\left[\sigma_n^x\sigma_m^x + \sigma_n^y\sigma_m^y + \sigma_n^z\sigma_m^z\right]$$

Survival Probability: $\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$

$\alpha = 100$



$\alpha = 1.5$

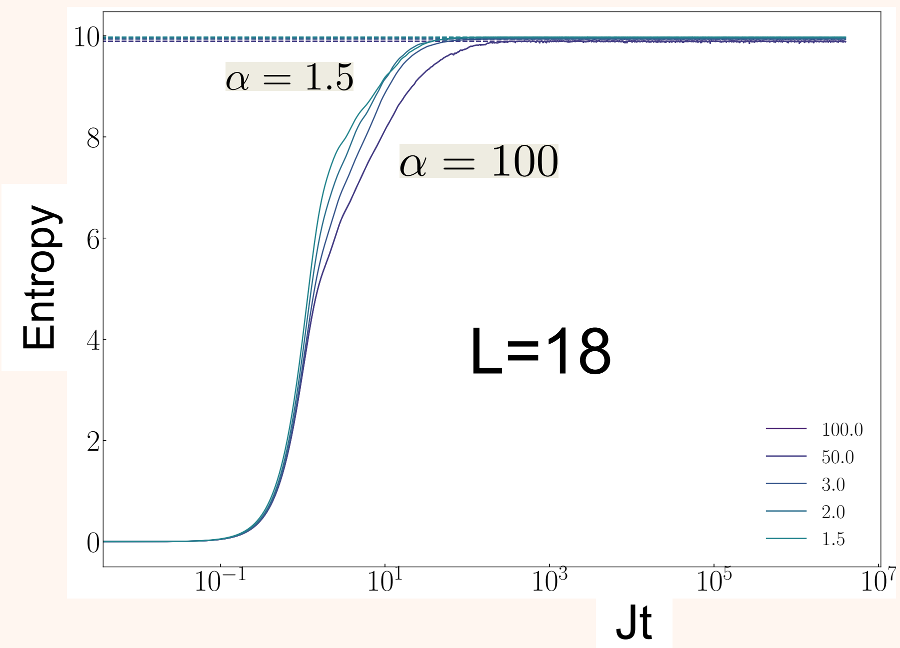


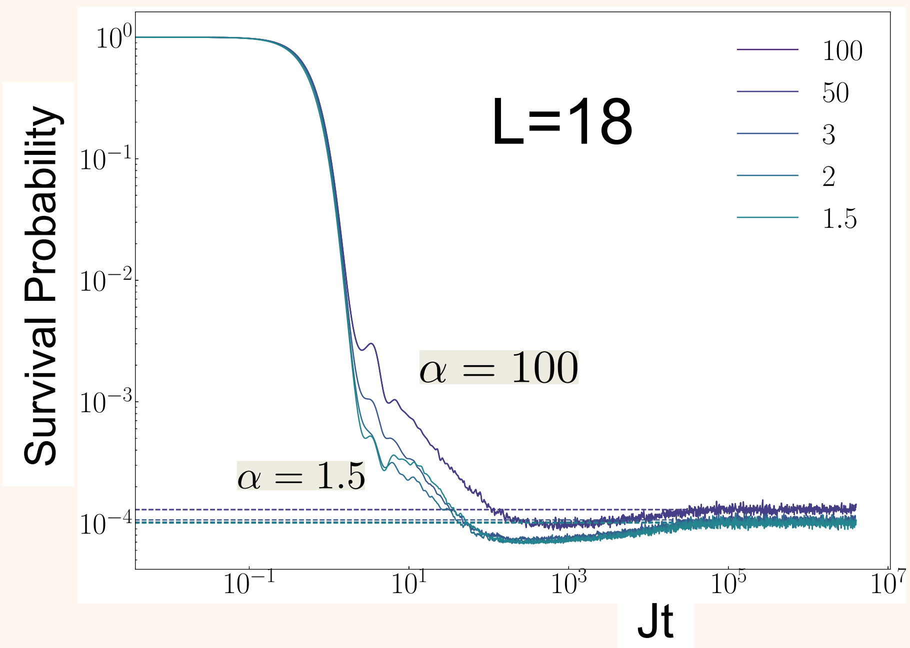
Long-range coupling (α >1)

$$H = \sum_{n=1}^{L} \frac{h_n}{2}\sigma_n^z + \sum_{n<m}^{L-1} \frac{J}{4}\frac{1}{|n-m|^\alpha}\left[\sigma_n^x\sigma_m^x + \sigma_n^y\sigma_m^y + \sigma_n^z\sigma_m^z\right]$$

"Participation" Entropy: $-\ln\left(\sum_n |\langle n|\Psi(t)\rangle|^4\right)$

Survival Probability: $\left|\langle\Psi(0)|\Psi(t)\rangle\right|^2$



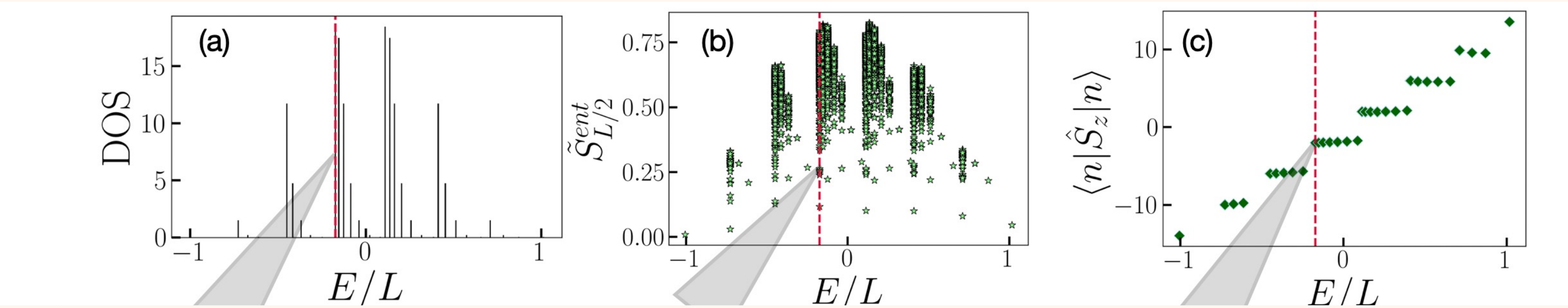


ETH
Super-long range couplings

Super long-range coupling (α<1)

Soumya Kanti Pal & Shamik Gupta

Is it ETH violated?

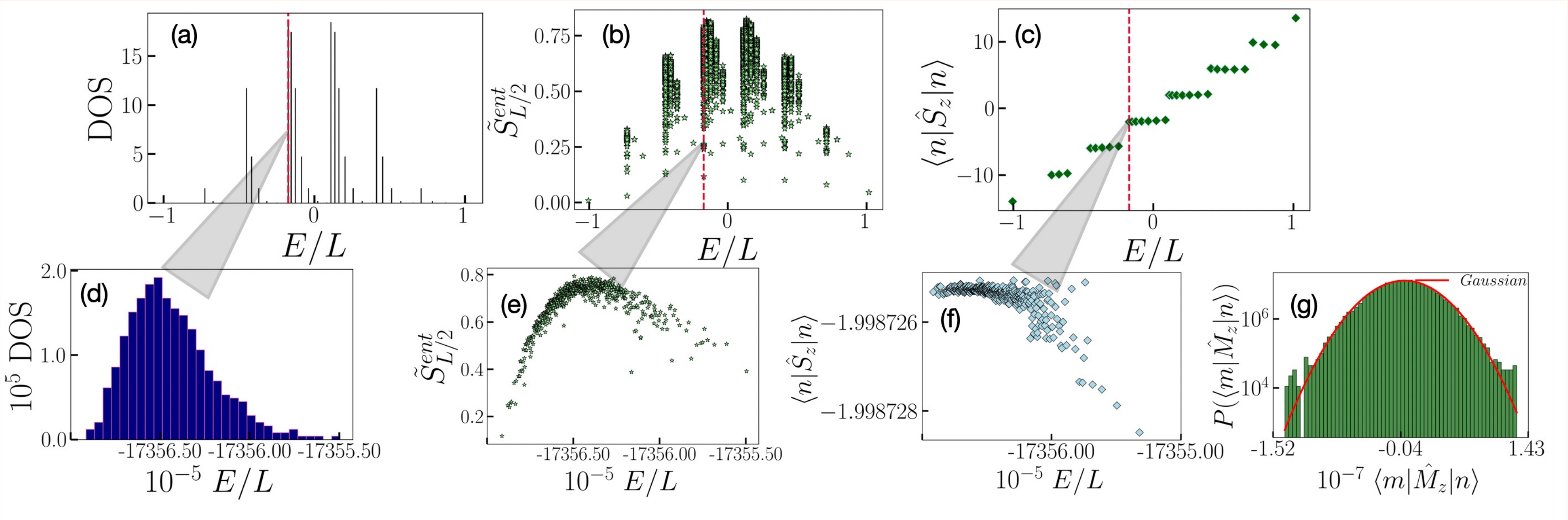


$$\hat{H} = \mathcal{N}_\alpha \sum_{i>j=1}^{L} J \frac{\hat{\sigma}_i^x \hat{\sigma}_j^x}{|i-j|^\alpha} + h \sum_{i=1}^{L} \hat{\sigma}_i^z,$$

$$\alpha = 10^{-4}$$

ETH is valid within the energy bands

Soumya Kanti Pal & Shamik Gupta

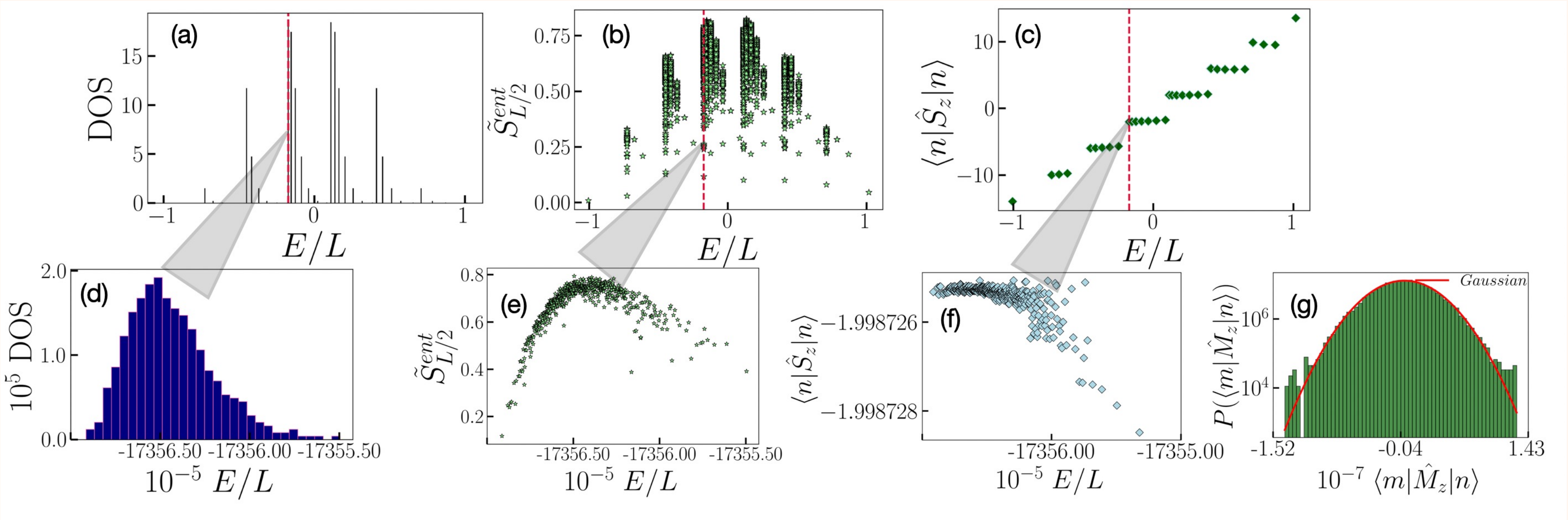


$$\hat{H} = \mathcal{N}_\alpha \sum_{i>j=1}^{L} J \frac{\hat{\sigma}_i^x \hat{\sigma}_j^x}{|i-j|^\alpha} + h \sum_{i=1}^{L} \hat{\sigma}_i^z,$$

$\alpha = 10^{-4}$

ETH is valid within the energy bands

Soumya Kanti Pal & Shamik Gupta



$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$\alpha = 10^{-4}$

$$\hat{H} = \mathcal{N}_\alpha \sum_{i>j=1}^{L} J \frac{\hat{\sigma}_i^x \hat{\sigma}_j^x}{|i-j|^\alpha} + h \sum_{i=1}^{L} \hat{\sigma}_i^z,$$

$$O_{\beta\alpha} = \langle \beta | O | \alpha \rangle$$

ETH

Thermalization

Rigid spectrum

$$\langle O(t)\rangle = \langle \Psi(t) | O | \Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha | C_\alpha^{ini} |^2 O_{\alpha\alpha}$$

$$O_{\beta\alpha} = \langle \beta | O | \alpha \rangle$$

Thermalization

Rigid spectrum

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$\langle |C_\alpha|^2\rangle = \frac{1}{D} \qquad O_{\beta\alpha} = \langle \beta|O|\alpha\rangle$$

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

$$|\alpha\rangle = \begin{pmatrix} C_\alpha^1 \\ C_\alpha^2 \\ C_\alpha^3 \\ C_\alpha^4 \\ C_\alpha^5 \\ \ldots \end{pmatrix} \Rightarrow |\Psi(0)\rangle = \begin{pmatrix} C_1^{ini} \\ C_2^{ini} \\ C_3^{ini} \\ C_4^{ini} \\ C_5^{ini} \\ \ldots \end{pmatrix}$$

Thermalization

Rigid spectrum

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$\langle |C_\alpha|^2\rangle = \frac{1}{D} \qquad O_{\beta\alpha} = \langle \beta|O|\alpha\rangle$$

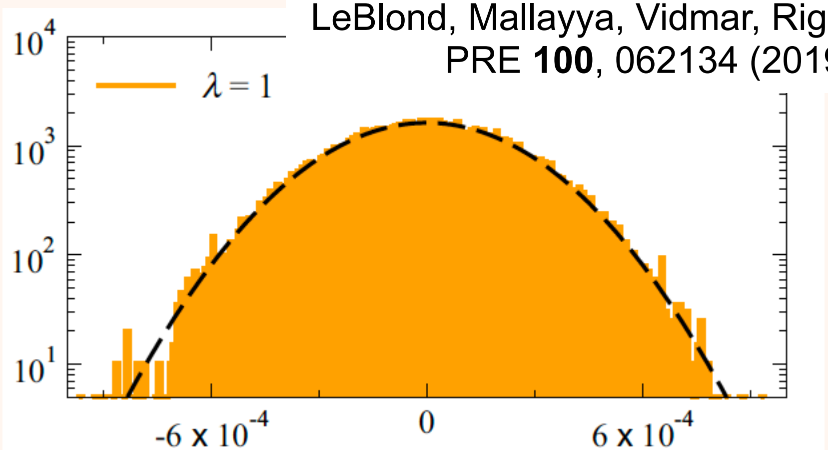
$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

$$|\alpha\rangle = \begin{pmatrix} C_\alpha^1 \\ C_\alpha^2 \\ C_\alpha^3 \\ C_\alpha^4 \\ C_\alpha^5 \\ \dots \end{pmatrix} \Rightarrow |\Psi(0)\rangle = \begin{pmatrix} C_1^{ini} \\ C_2^{ini} \\ C_3^{ini} \\ C_4^{ini} \\ C_5^{ini} \\ \dots \end{pmatrix}$$

Beugeling, Moessner, Haque
PRE **91**, 012144 (2015)
(Gaussian distribution)

Off-diagonal ETH

LeBlond, Mallayya, Vidmar, Rigol
PRE **100**, 062134 (2019)



$\langle\beta|O|\alpha\rangle$

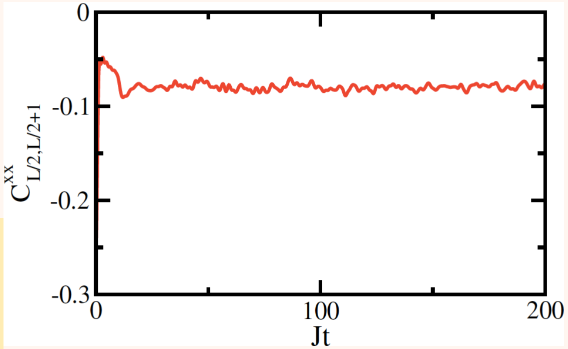
Thermalization

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$O_{\alpha\alpha} = \langle\alpha|O|\alpha\rangle$$

Equilibration:

Size of the fluctuations
PRE **88**, 032913 (2013)
SciPostPhys**15**, 244 (2023)



Thermalization

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$\langle\alpha|O|\alpha\rangle$$

Normalized random vectors

$$|\alpha\rangle = \begin{pmatrix} C_\alpha^1 \\ C_\alpha^2 \\ C_\alpha^3 \\ C_\alpha^4 \\ C_\alpha^5 \\ \dots \end{pmatrix}$$

Infinite time average | Thermodynamic average

$$\overline{\langle O(t)\rangle} \equiv \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha} \overset{=?}{\longleftrightarrow} O_{micro} \equiv \frac{1}{\mathcal{N}_{E_0,\Delta E}} \sum_{\substack{\alpha \\ |E_0 - E_\alpha| < \Delta E}} O_{\alpha\alpha}$$

depends on the initial conditions | depends only on the energy

ETH: the expectation values $O_{\alpha\alpha}$ of few-body observables do not fluctuate for eigenstates close in energy

Peres Lattice

PRL **53**, 1711 (1984)

$$PR^{(\alpha)} = 1 / \sum_{i=1}^{D} | c_i^{(\alpha)} |^4$$

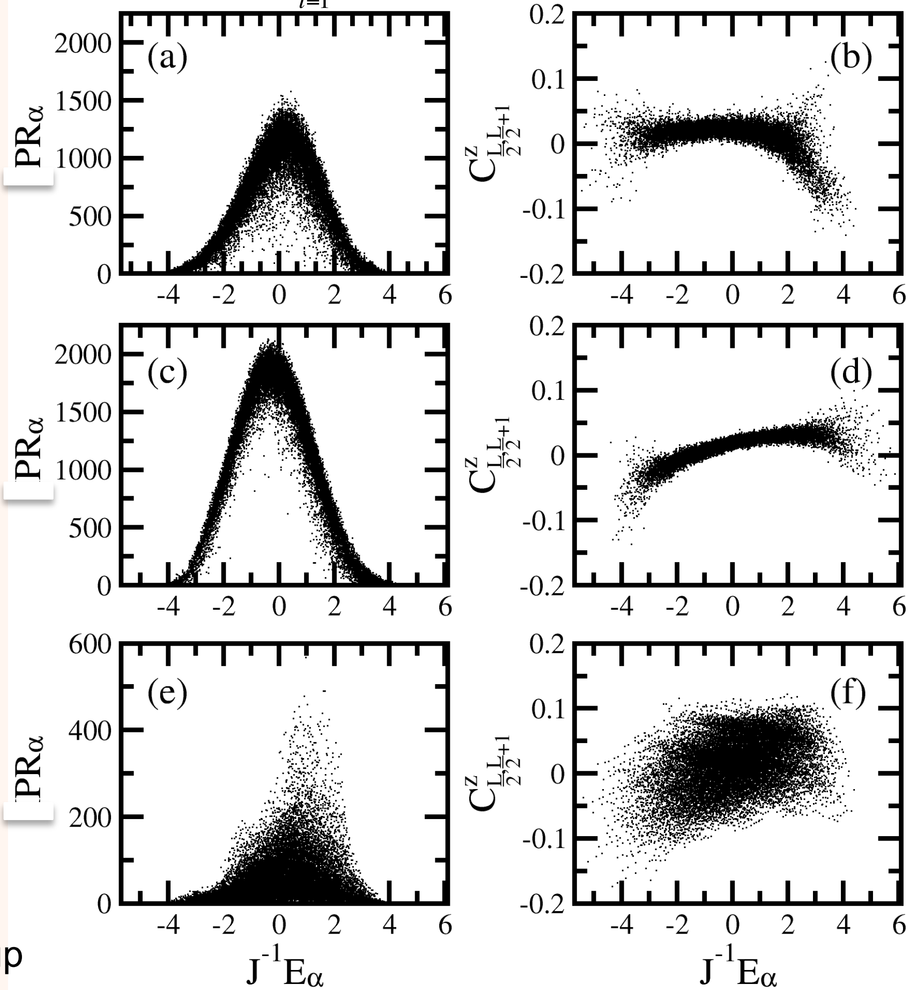
$$O_{\alpha\alpha} = \langle \alpha | O | \alpha \rangle$$

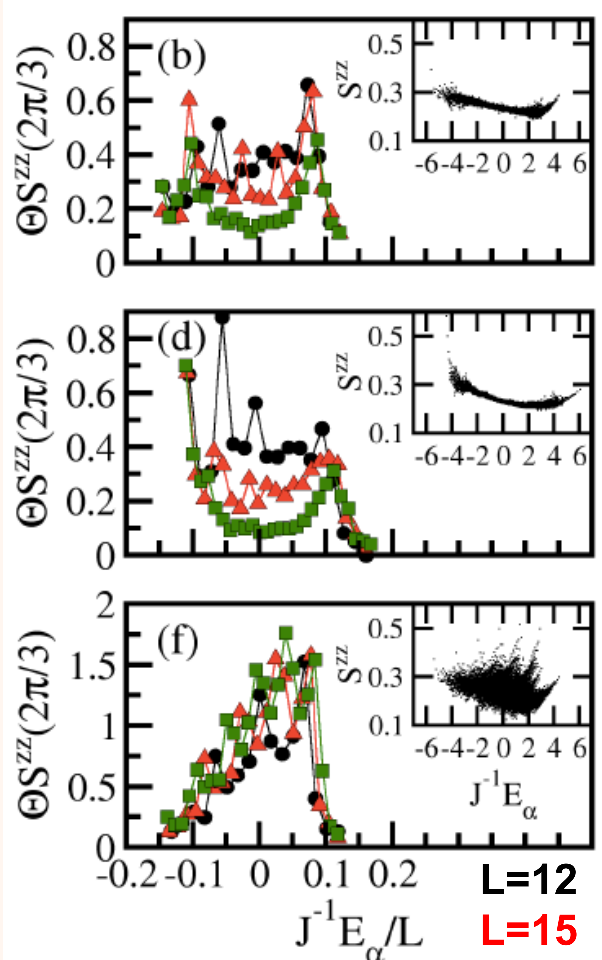
$$\Theta O = (O_{\max} - O_{\min}) / O_{micro}$$

Chaotic Single-Defect Model

Chaotic NNN Model

Integrable XXZ Model





L=12

L=15

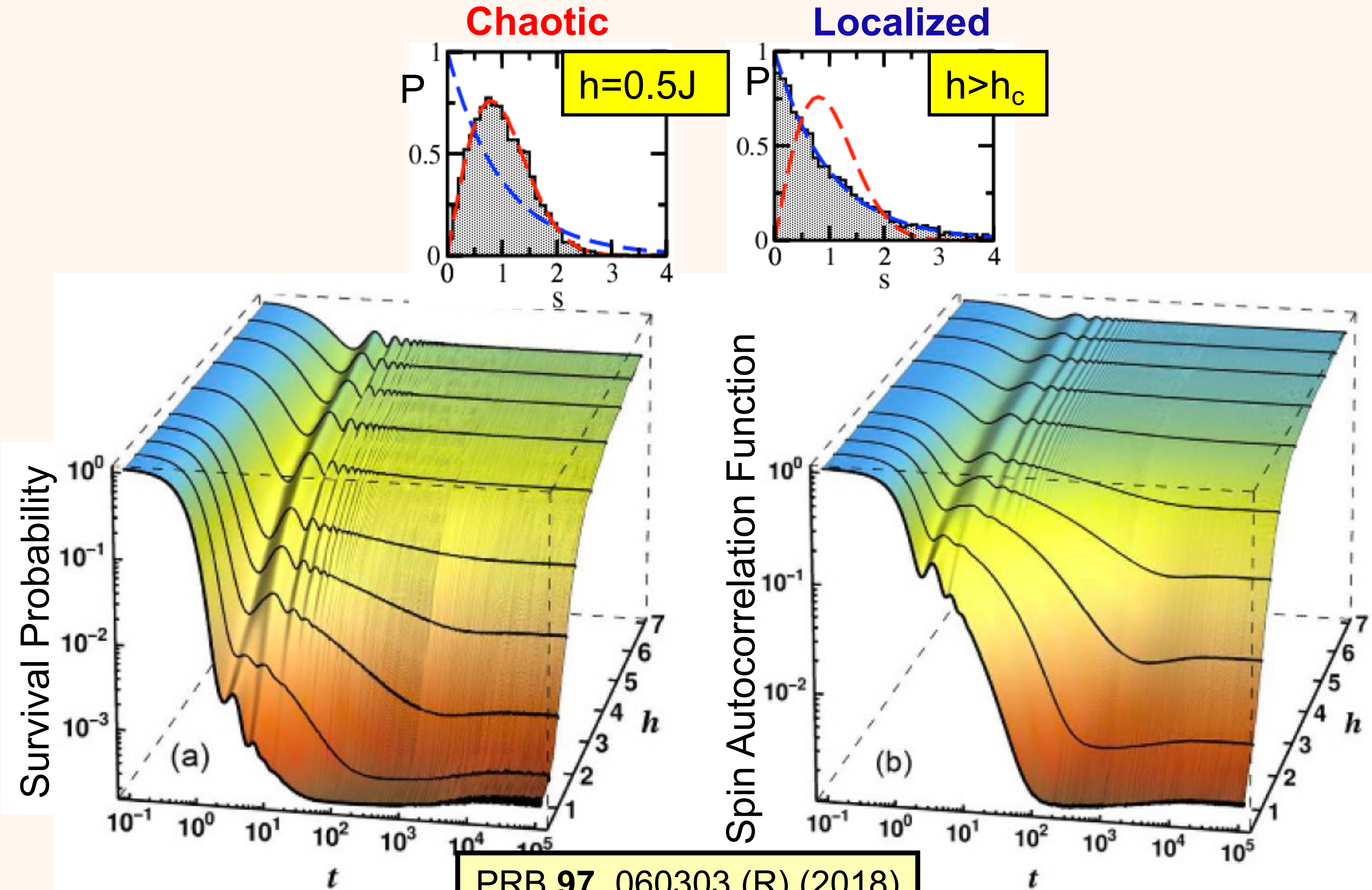
L=18

L=18, 1/3 up

PRE **89**, 062110 (2014)

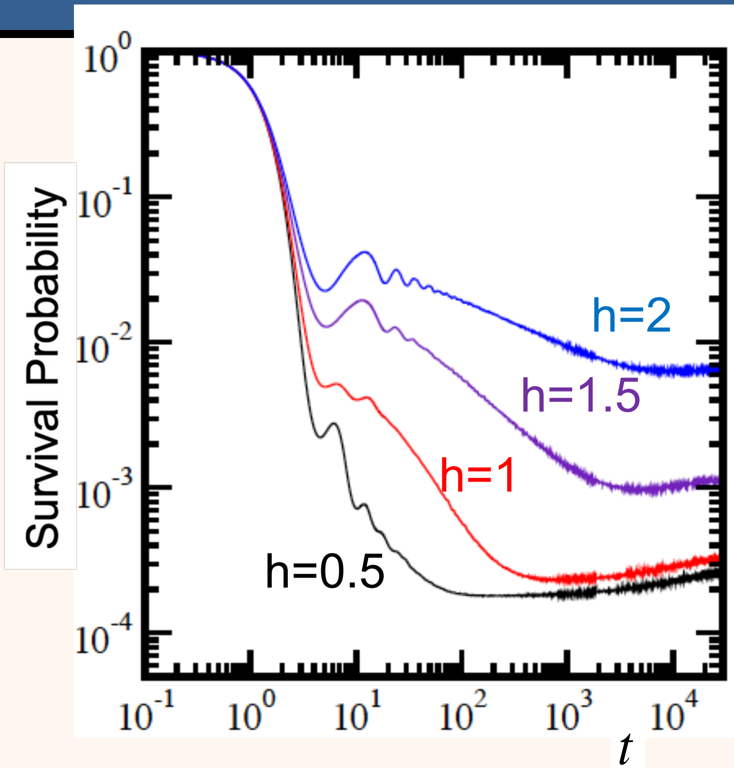
Large disorder strength

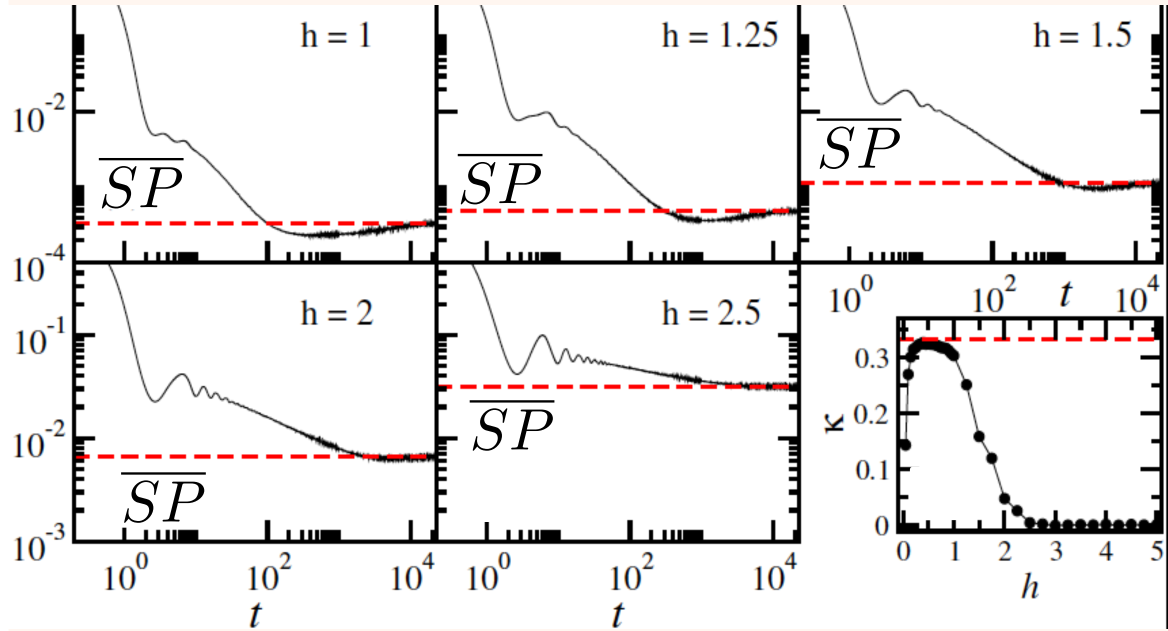
Correlation Hole and Disorder Strength



PRB **97**, 060303 (R) (2018)
PRB **99**, 174313 (2019)

Correlation Hole and Disorder Strength





Relative depth of the correlation hole:

$$\kappa = \frac{\overline{SP} - \langle SP \rangle_{min}}{\overline{SP}}$$

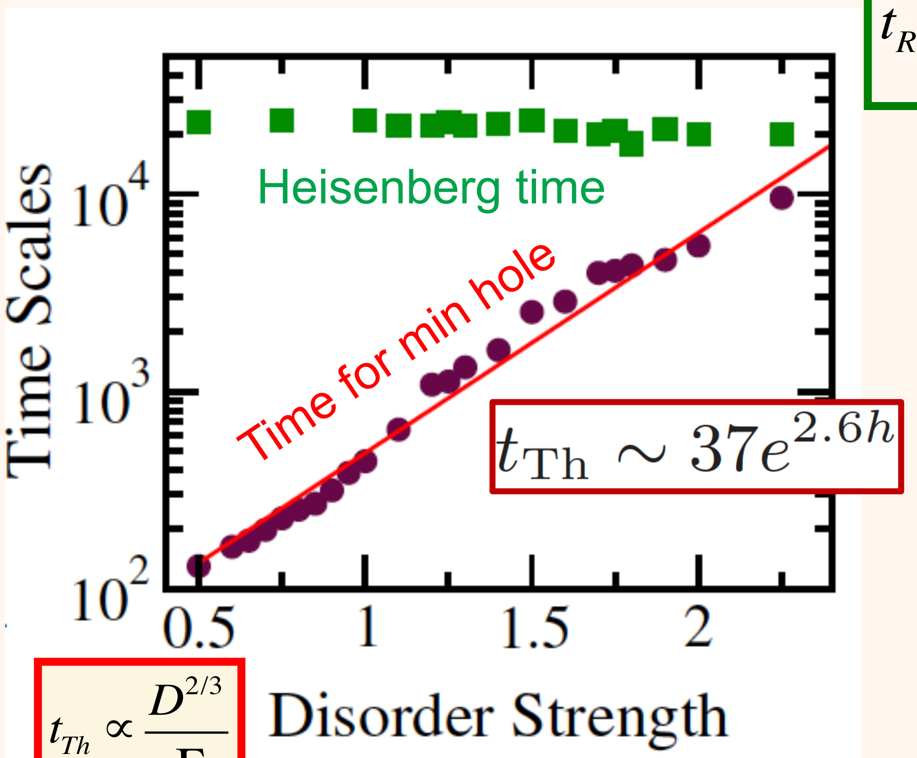
L=16

Torres & LFS
Philos. Trans. R. Soc. A **65,** 20160434 (2017)

Thouless Time and Disorder Strength

SURVIVAL PROBABILITY

$$t_R \propto \frac{D}{\Gamma\sqrt{\delta}}$$



$$t_{Th} \propto \frac{D^{2/3}}{\Gamma}$$

Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

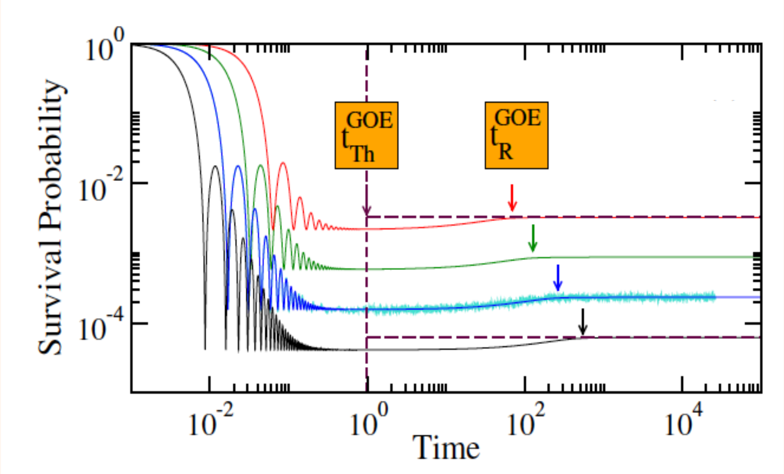
Robustness of the correlation hole

Correlation Hole and System Size

Survival probability for GOE matrices:

$$\overline{SP} = 3/D$$

$$SP_{min} = 2/D$$



Relative depth of the correlation hole:

$$\kappa = \frac{\langle\overline{O}\rangle - \langle O\rangle_{\text{min}}}{\langle\overline{O}\rangle}$$

$$SP = \left|\langle\Psi(0)|\Psi(t)\rangle\right|^2$$

$$\kappa = 1/3$$

Lezama, Torres, Bernal, Bar Lev & LFS, PRB **104**, 085117(2021)

Correlation Hole for the Survival Probability

Relative depth of the correlation hole:

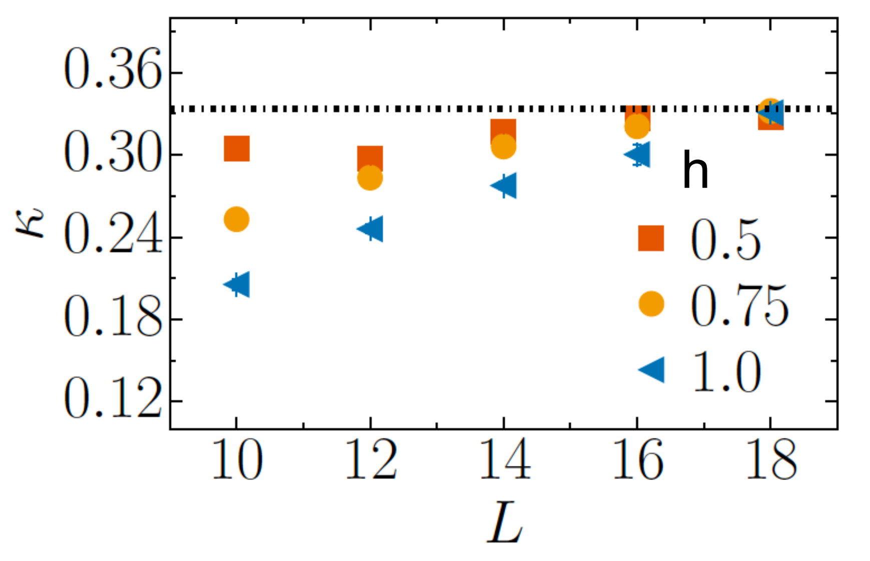
$$SP = |\langle \Psi(0)|\Psi(t)\rangle|^2$$

$$\kappa = \frac{\langle \overline{O} \rangle - \langle O \rangle_{\min}}{\langle \overline{O} \rangle}$$

<u>Survival probability</u> for realistic chaotic systems:

$$\kappa = 1/3$$

$$h \leq J = 1$$



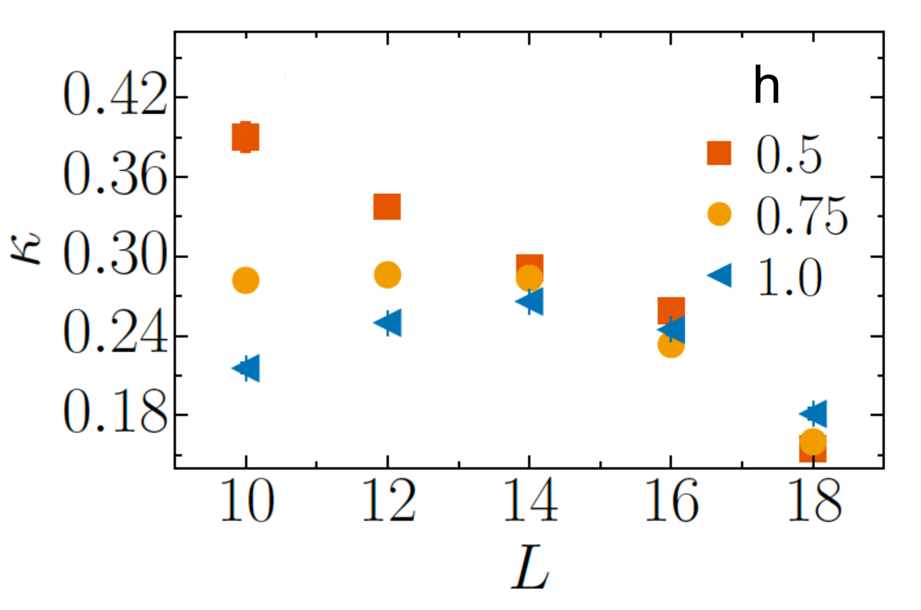
Lezama, Torres, Bernal, Bar Lev & LFS, PRB **104**, 085117(2021)

Correlation Hole for the Spin Autocorrelation Function

Relative depth of the correlation hole:

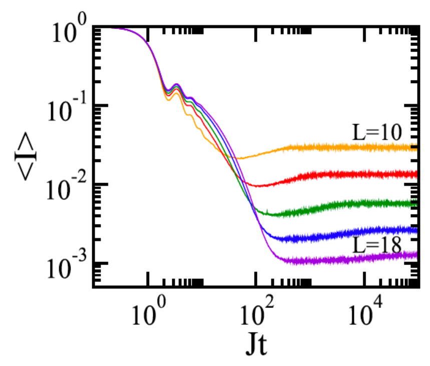
$$\kappa = \frac{\langle \overline{O} \rangle - \langle O \rangle_{\min}}{\langle \overline{O} \rangle}$$

$$I(t) = \frac{1}{L}\sum_{k=1}^{L} \langle \Psi(0)|\sigma_k^z e^{iHt} \sigma_k^z e^{-iHt}|\Psi(0)\rangle$$



Spin autocorrelation function for realistic chaotic systems:

$$h \leq J = 1$$



Lezama, Torres, Bernal, Bar Lev & LFS, PRB **104**, 085117(2021)