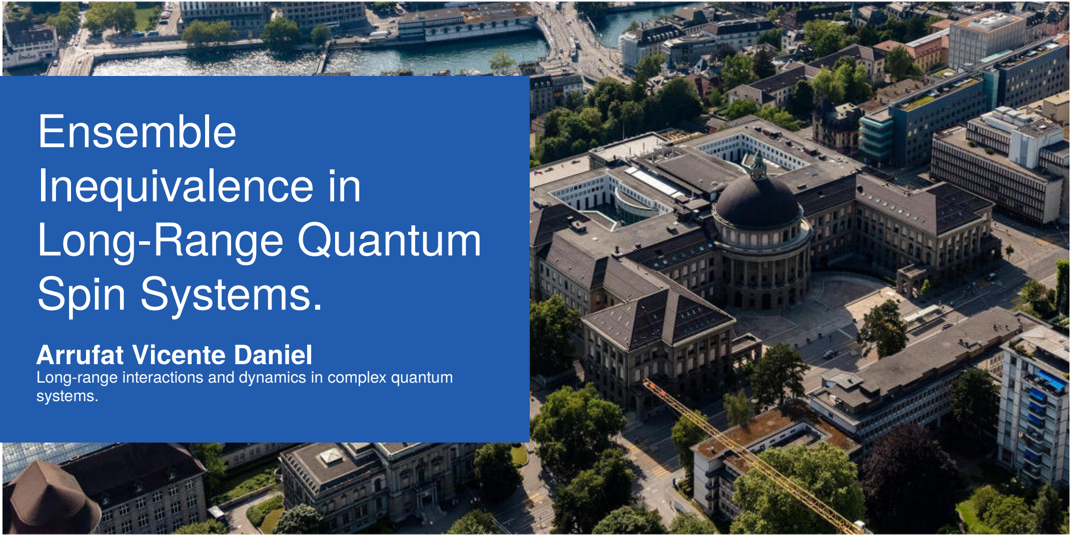
ETH zürich

Nordita



Ensemble Inequivalence in Long-Range Quantum Spin Systems.

Arrufat Vicente Daniel

Long-range interactions and dynamics in complex quantum systems.

Outline

1. Ensemble Inequivalence

2. The Model

3. Phase Diagram

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Statistical Physics

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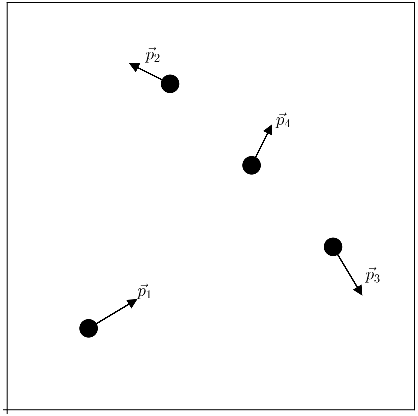
- Canonical ensemble: where we fix number of particles and average energy (temperature)

From Microcanonical to Canonical



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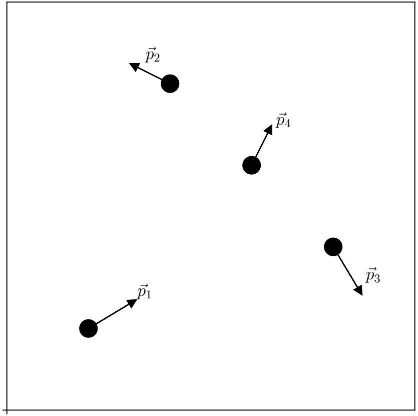

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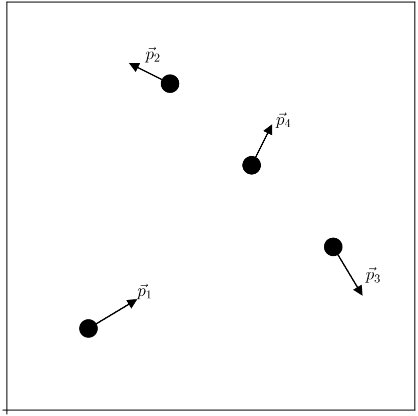

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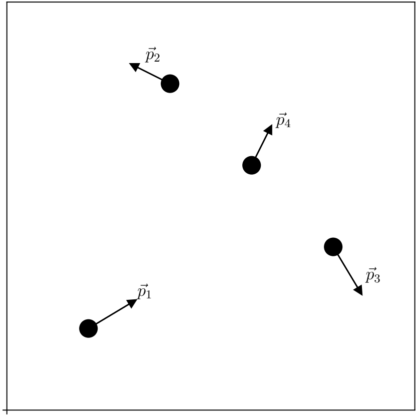

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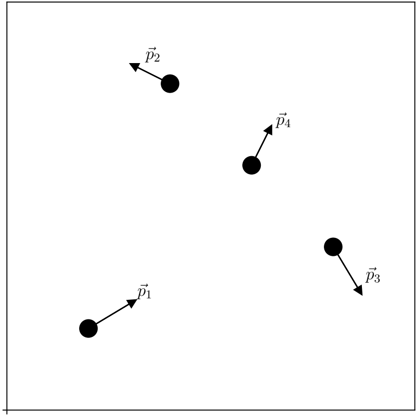

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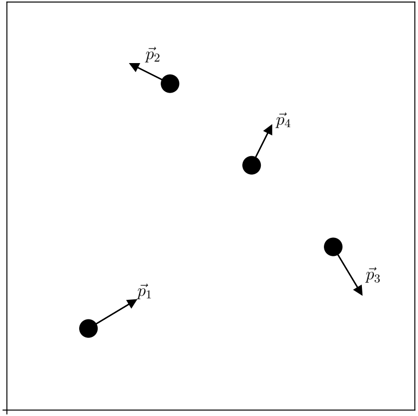

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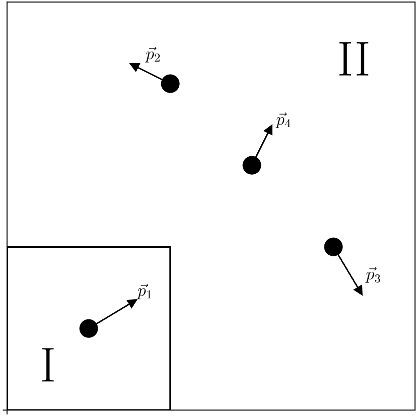

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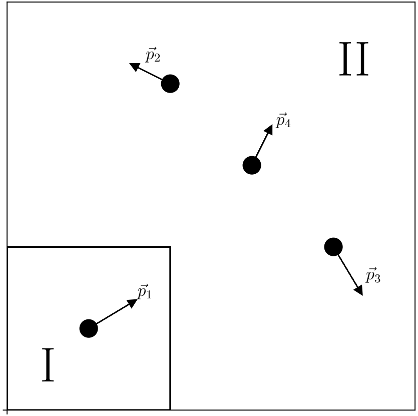

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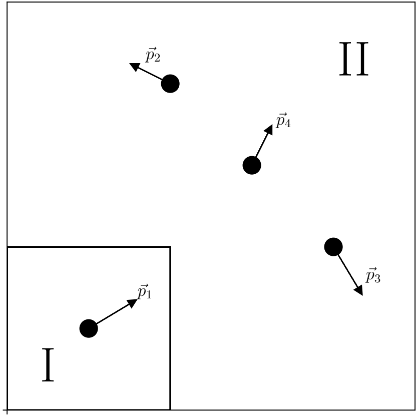

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Assumes additivity!

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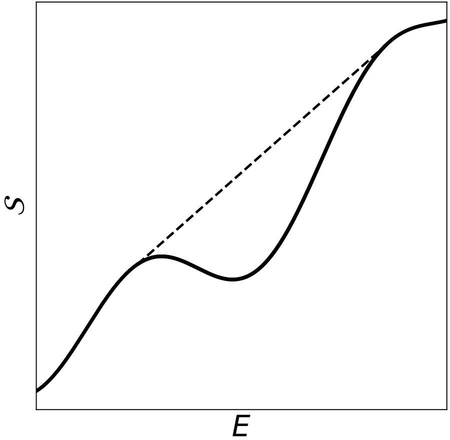

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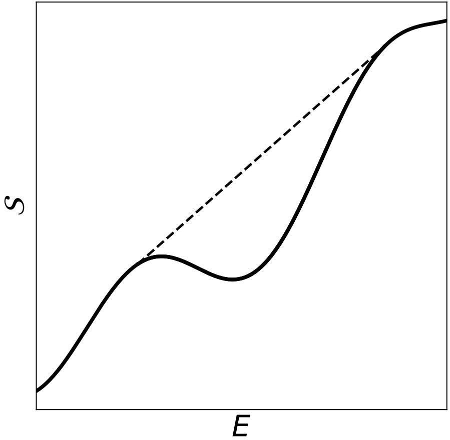

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Convex intruder

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The Model

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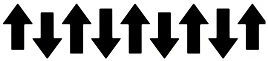

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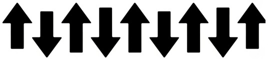

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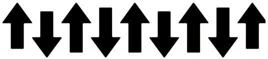

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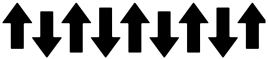

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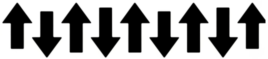

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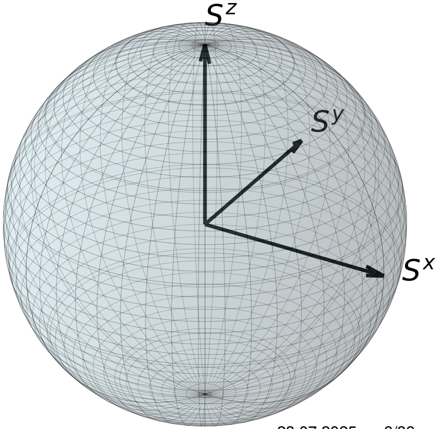

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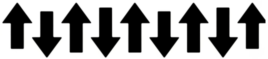

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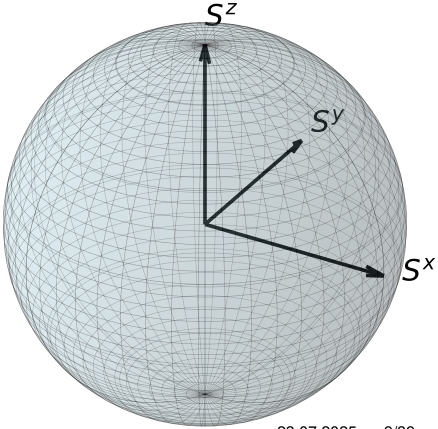

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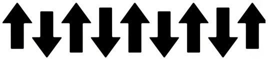

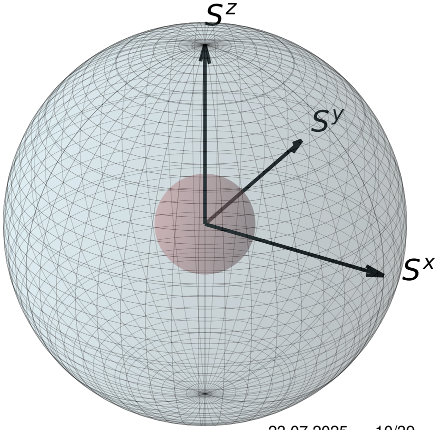

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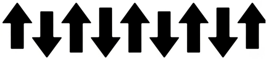

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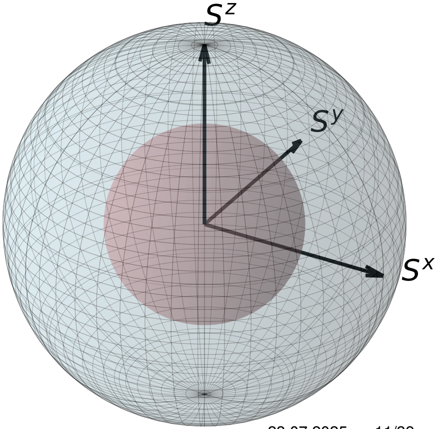

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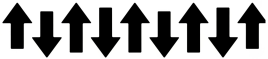

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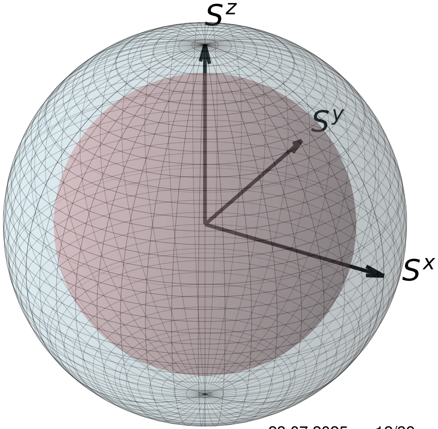

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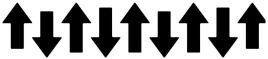

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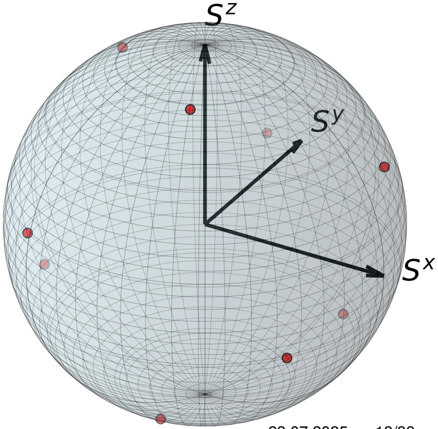

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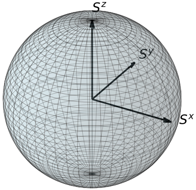

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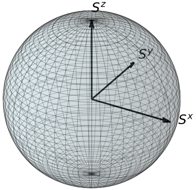

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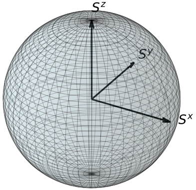

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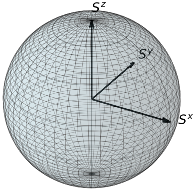

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$$f(\beta, J, h, K) = \varepsilon - \mathcal{S}/\beta = -Js^2m_z^2 - Ks^4m_z^4 - hs\sqrt{1-m_z^2} + \frac{1}{\beta}\left[\frac{1+s}{2}\ln\frac{1+s}{2} + \frac{1-s}{2}\ln\frac{1-s}{2}\right],$$

$$\mathcal{S}(s) = -\frac{1+s}{2}\log\left(\frac{1+s}{2}\right) - \frac{1-s}{2}\log\left(\frac{1-s}{2}\right).$$

[Nicolò Defenu, David Mukamel and Stefano Ruffo, Phys. Rev. Lett. 133, 050403 (2024).]

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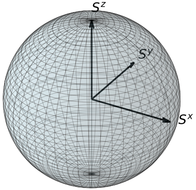

$\mathbf{S} = \frac{N}{2} s\mathbf{m}$ with $\mathbf{m} \equiv (m_x, m_y, m_z) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

[E. H. Lieb, Commun. Math. Phys 31, 327 (1973).]

$$Z(\beta, J, h, K) = \int_0^1 ds \frac{N(Ns+1)}{8\pi} g(Ms) \int e^{-N\beta e(s,\theta,\phi,J,h,K)} \sin\theta d\theta d\phi,$$

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[Nicolò Defenu, David Mukamel and Stefano Ruffo, Phys. Rev. Lett. 133, 050403 (2024).]

The Thermodynamic Potentials.

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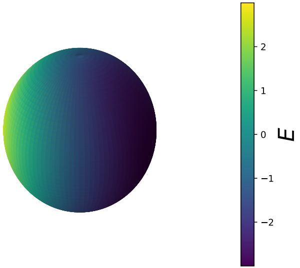

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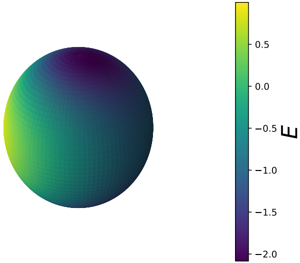

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Outline

Phase Diagram (Fixed Temperature)

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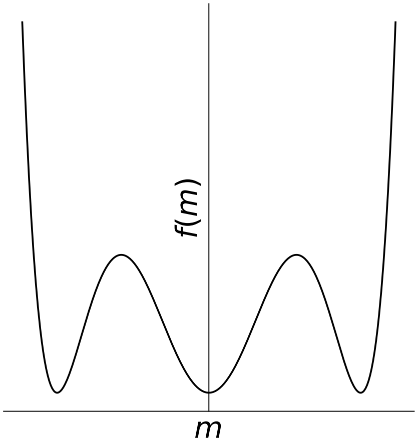

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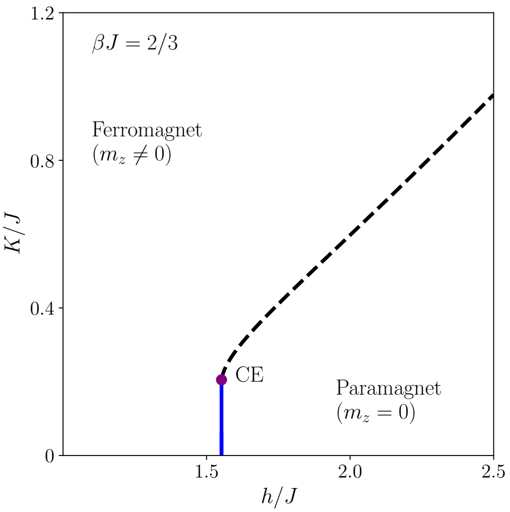

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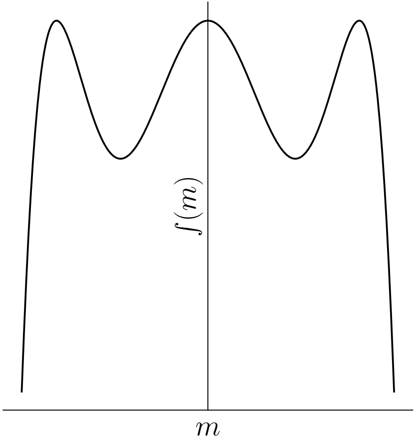

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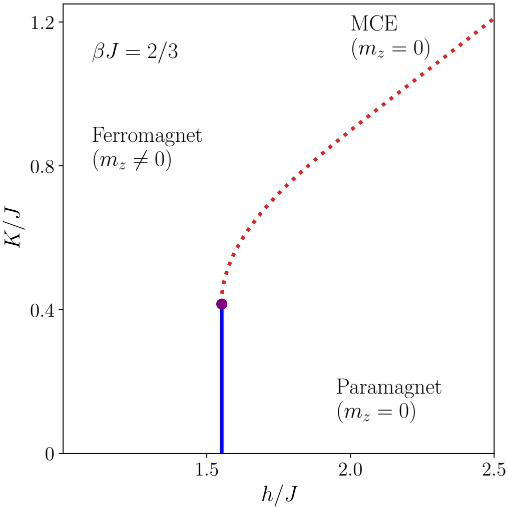

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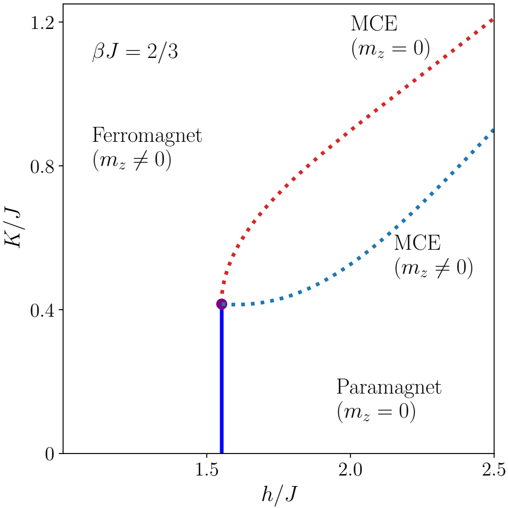

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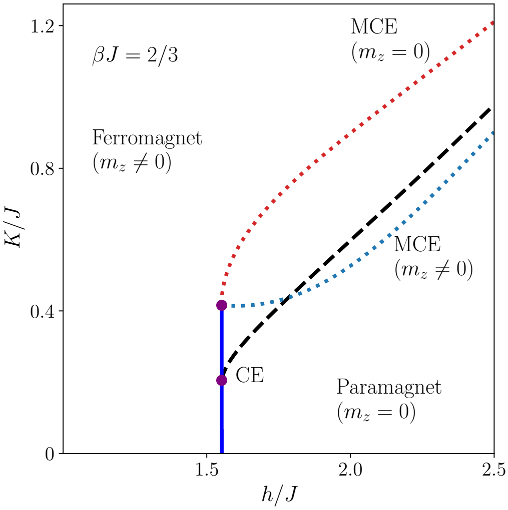
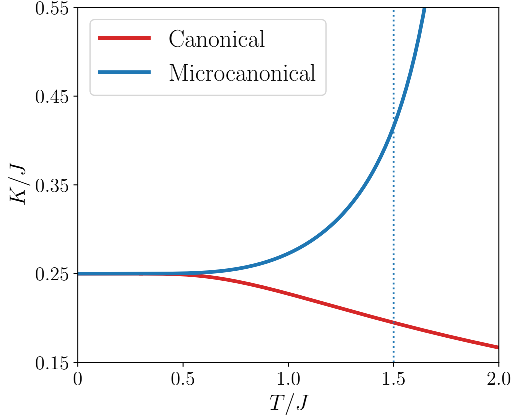
Entropy

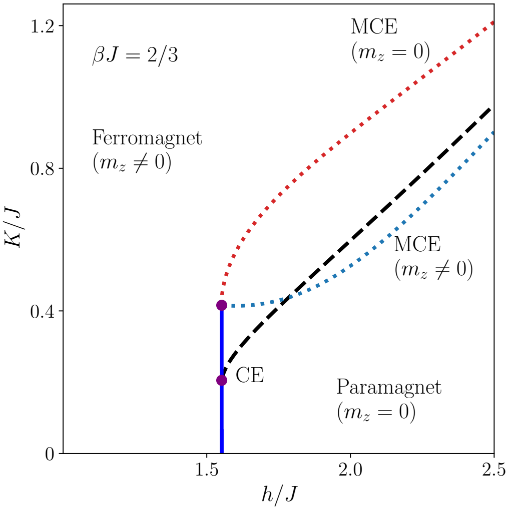
$$\begin{aligned} \int(\varepsilon, J, h, K) &= \gamma\left(Jm_z^2 + Km_z^4 + \varepsilon\right) \\ &\quad -\lambda m_z + \ln\, 2\cosh\left(\sqrt{\lambda^2+\gamma^2 h^2}\right), \end{aligned}$$



Phase Diagram (Fixed Temperature)

Tri-critical point:





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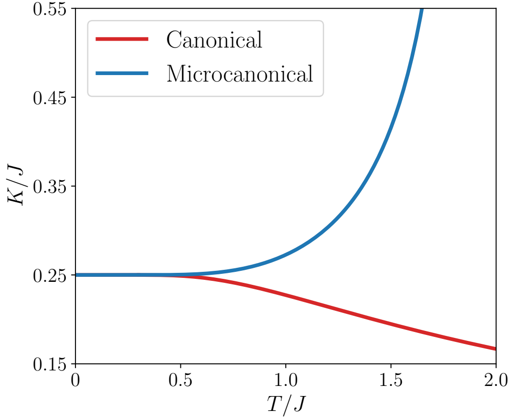

Canonical:

$$b_2 = 0: \quad \frac{h_c}{2J} = \tanh(\beta h_c) \ ,$$

$$b_4 = 0: \quad K_{tcp} = \frac{J^3}{h_c^2} + \frac{\beta J^2}{2}\left(1 - \tanh(\beta h_c)^2\right) .$$

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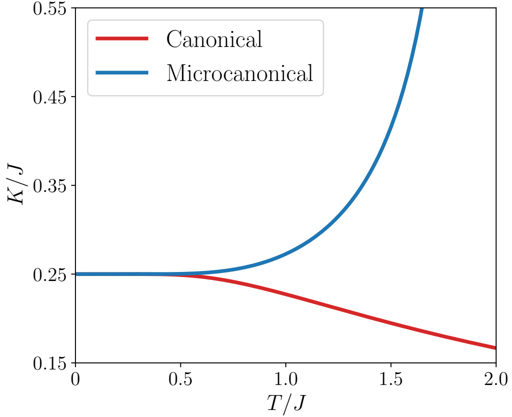
Tri-critical point:



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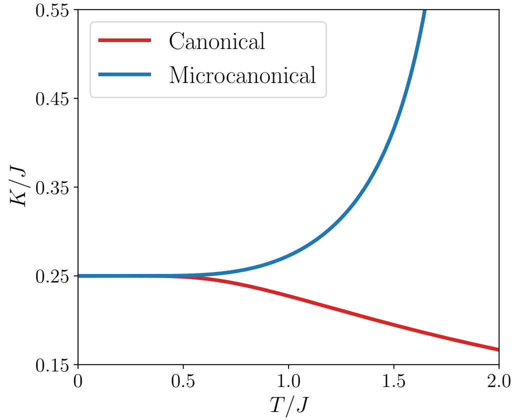
Microcanonical:

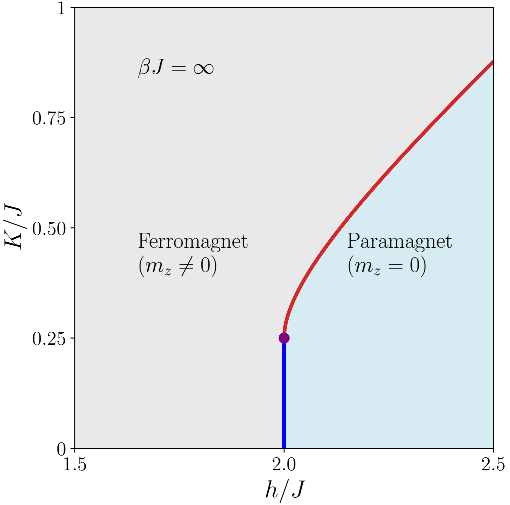
$$a_2 = 0: \quad h_c[MCE] = 2J \tanh(\beta h_c) \ ,$$

$$a_4 = 0: \quad K_{tcp}[MCE] = \frac{J}{4\tanh(\beta h_c)^2} .$$

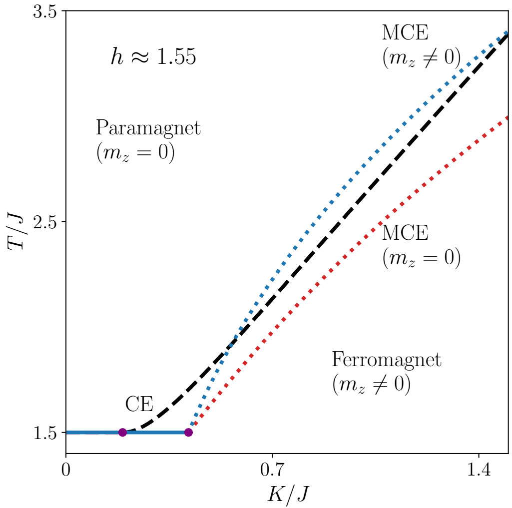
Phase Diagram ($T = 0$)

Tri-critical point:





Phase Diagram (Fixed $h \approx 1.55$)



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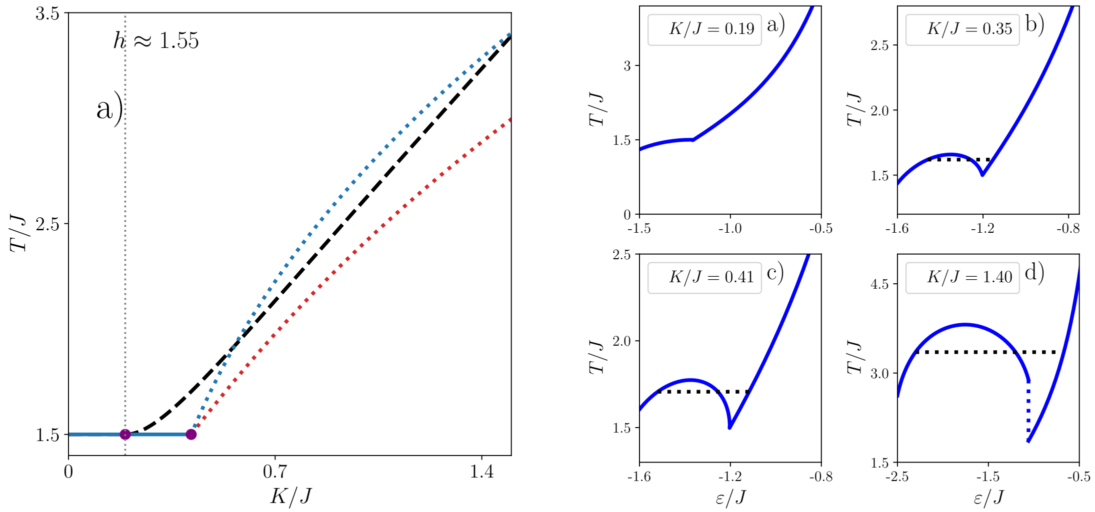

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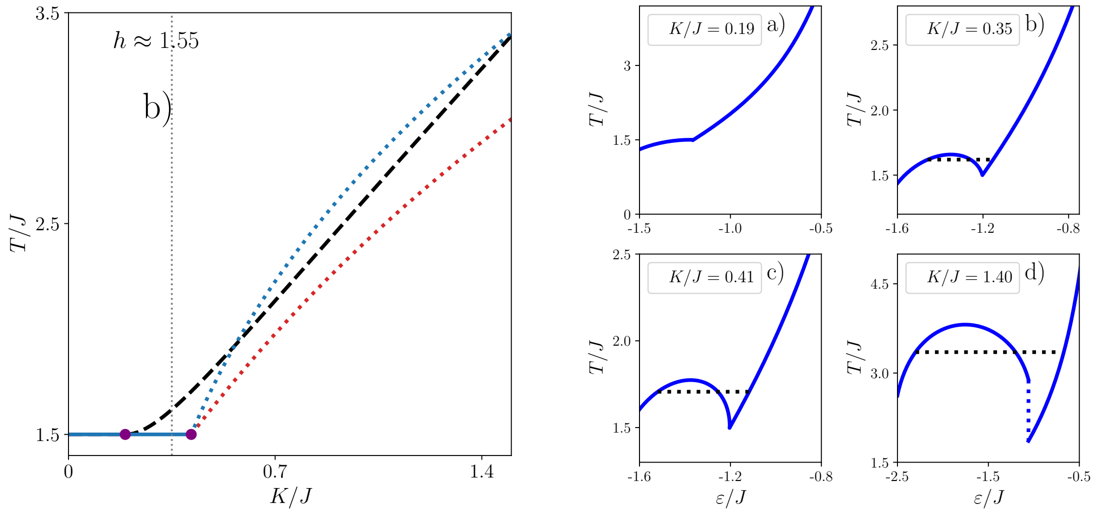
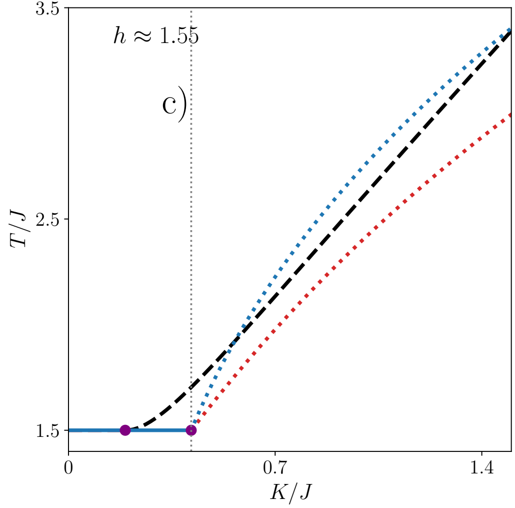
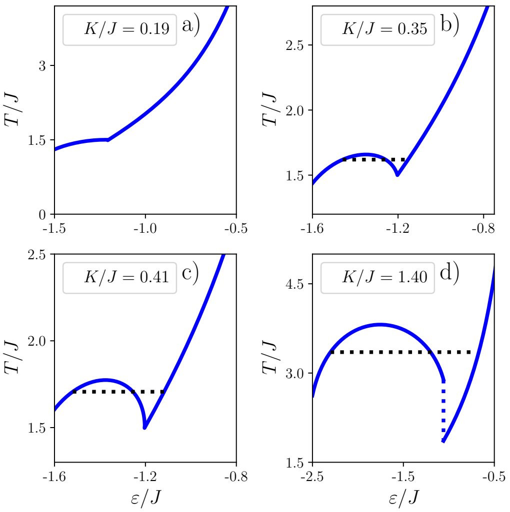
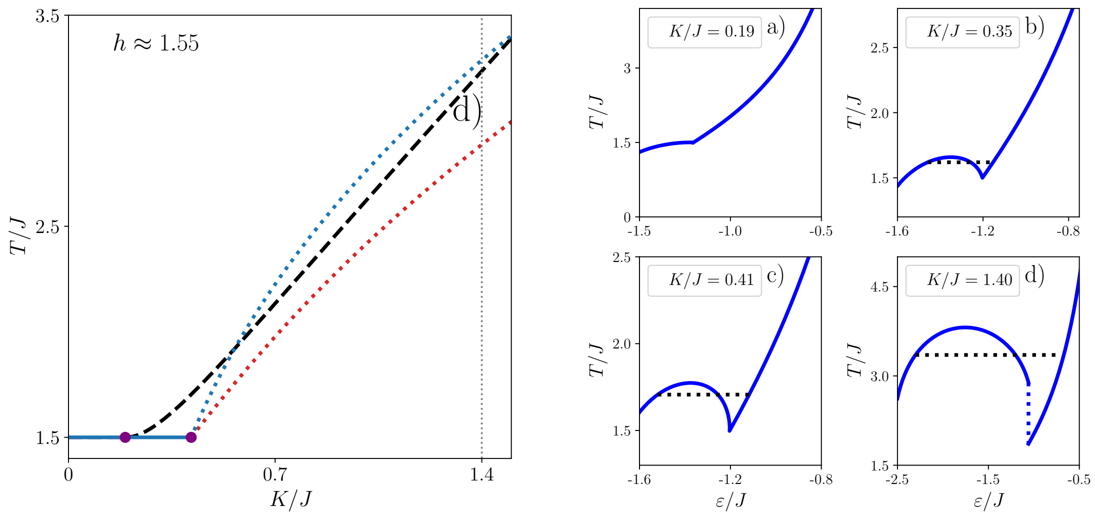


Phase Diagram (Fixed $h \approx 1.55$)

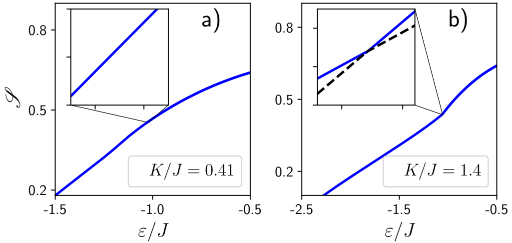


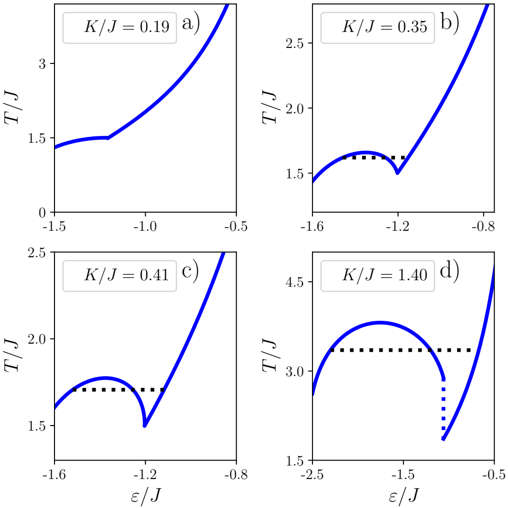


Phase Diagram (Fixed $h \approx 1.55$)



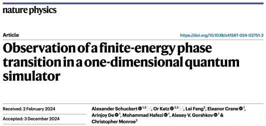
Phase Diagram (Fixed $h \approx 1.55$)





Conclusion and Outlook.

- We show and example of *ensemble inequivalence* in LR "Quantum" spin chains.
- Microcanonical entropy develops a *convex intruder*
- Relevant for current experiments
- *For more details see [arXiv:2504.14008]


nature physics

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Observation of a finite-energy phase transition in a one-dimensional quantum simulator

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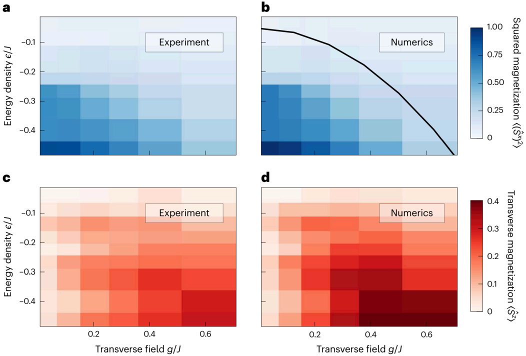

[Alexander Schuckert, et al. Nature Physics 21, 374–379 (2025).]

Conclusion and Outlook.

- We show and example of *ensemble inequivalence* in LR "Quantum" spin chains.
- Microcanonical entropy develops a *convex intruder*
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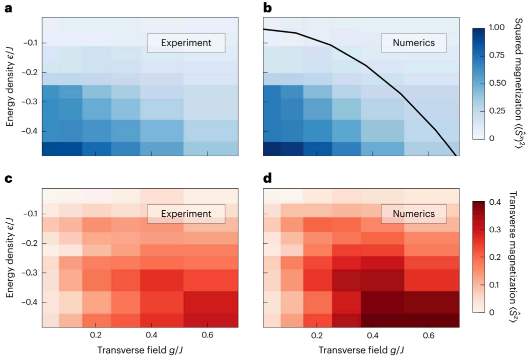
$$H = -\frac{1}{2N}\sum_{i<j} J_{ij}\hat{\sigma}_i^x\hat{\sigma}_j^x - g\sum_i \hat{\sigma}^z$$



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Thanks.