Nonequilibrium Phenomena in Strongly Coupled atom-photon systems: From Self-trapping to Chaos

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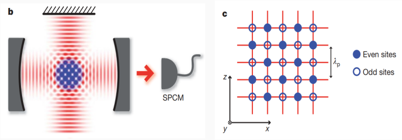
Outline

1. Preliminaries and experimental background.
2. Generalized model and setting.
3. Nonlinear Jaynes-Cummings dimer model: Mean field steady states, quantum signature and entanglement
4. Open anisotropic Dicke dimer model: persistent self-trapping, its oscillations and dissipative chaos
5. Summary and acknowledgment

Many body physics in atom photon coupled systems

Quantum gas-cavity QED setup



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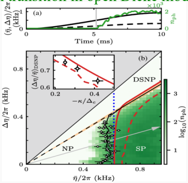
Non-equilibrium transition in open Dicke model



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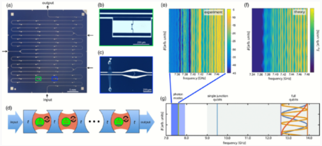
Non-equilibrium dissipative phase transition in a circuit qed lattice



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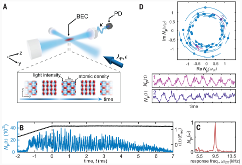
Continuous time crystal in atom-cavity system



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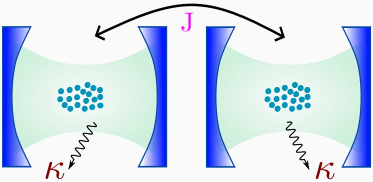
Generalized model



∗ Photon hopping between the cavities.

$$\hat{\mathcal{H}} = \sum_{i=L,R}\left[\hat{\mathcal{H}}_0^i + \hat{\mathcal{H}}_{int}^i\right] - J\left(\hat{a}_L^\dagger \hat{a}_R + h.c\right)$$

$$\hat{\mathcal{H}}_0^i = \omega \hat{a}_i^\dagger \hat{a}_i + \frac{\omega_0}{2}\sum_{j=1}^{N}\hat{\sigma}_{z,j}^i, \quad \hat{M} = \sum_{i=L,R}\hat{n}_i + \sum_{j=1}^{N}\frac{\hat{\sigma}_{z,j}^i}{2}$$

Co-rotating — Counter-rotating — Kerr-repulsion

$$\hat{\mathcal{H}}_{int}^i = \frac{\lambda_-}{\sqrt{2S}}\left(\hat{a}_i^\dagger\sum_{j=1}^{N}\hat{\sigma}_{-,j}^i + \hat{a}_i\sum_{j=1}^{N}\hat{\sigma}_{+,j}^i\right) + \frac{\lambda_+}{\sqrt{2S}}\left(\hat{a}_i^\dagger\sum_{j=1}^{N}\hat{\sigma}_{+,j}^i + \hat{a}_i\sum_{j=1}^{N}\hat{\sigma}_{-,j}^i\right) + \frac{U}{2}\left(\hat{n}_i(\hat{n}_i - 1)\right)$$

1) $N=1, \lambda_+ = 0 \Longrightarrow$ Nonlinear Jaynes-Cummings model

U(1) symmetry $[\hat{M}, \hat{\mathcal{H}}] = 0$

2) $N \gg 1, U = 0 \Longrightarrow$ Dicke model

$\mathcal{Z}_2$ symmetry $[\hat{\Pi}, \hat{\mathcal{H}}] = 0$

∗ Exchange symmetry L ↔ R

Lindblad Master Eqn:

$$\dot{\hat{\rho}} = -i[\hat{\mathcal{H}}, \hat{\rho}] + \mathcal{L}[\hat{a}]$$

$$\mathcal{L}[\hat{\mathcal{O}}] = \left(2\hat{\mathcal{O}}\rho\hat{\mathcal{O}}^\dagger - \hat{\mathcal{O}}^\dagger\hat{\mathcal{O}}\rho - \rho\hat{\mathcal{O}}^\dagger\hat{\mathcal{O}}\right)$$

N = 1, $\lambda_+ = 0$, $\kappa = 0$, Nonlinear JC Dimer

Each cavity is described by the **nonlinear Jaynes-Cummings** model

Semi-classical analysis using product state : $|\Psi\rangle = \otimes_i |\alpha\rangle_i \otimes |z\rangle_i$

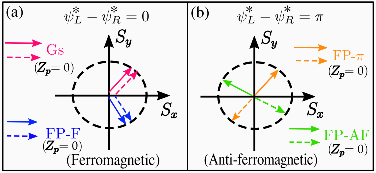
$\mathcal{L} = \imath \langle\Psi| \partial_t |\Psi\rangle - \langle\Psi| \hat{\mathcal{H}} |\Psi\rangle$

$\delta\mathcal{S} = 0 \implies$ Lagrange's EOM for the dynamical variables: $(n_i, \psi_i, \theta_i, \phi_i)$

Classification of steady states :

1) Relative phase between photons : $\psi_L - \psi_R$

2) $Z_p = n_L - n_R$



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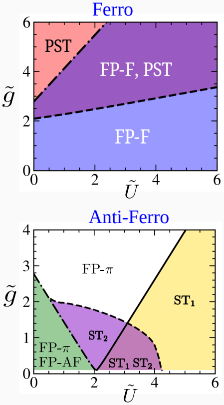
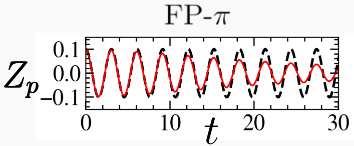


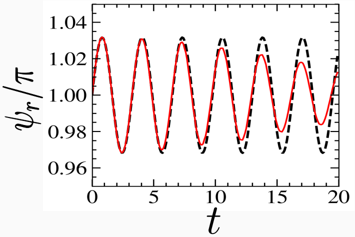
Quantum signature and decay

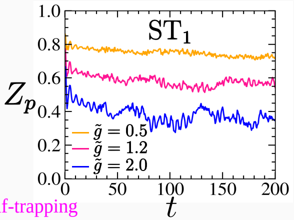
Quantum Signature of π-oscillations and self-trapping



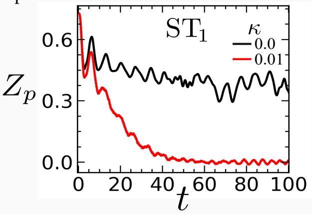
Phase states:

$$|\psi_m\rangle = \frac{1}{\sqrt{N_{\max}+1}} \sum_{n=0}^{N_{\max}} e^{in\psi_m} |n\rangle$$

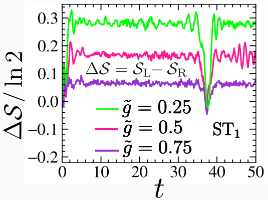


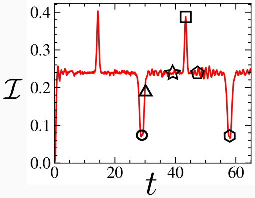


Quantum self-trapping decays in the presence of photon loss



Self-trapping and entanglement



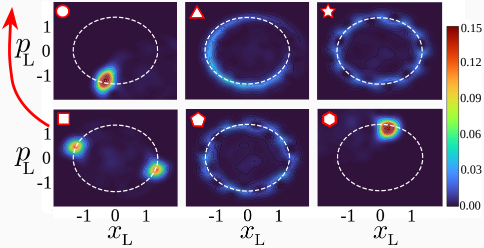


Von-Neumann Entropy $\mathcal{S}_i = -\sum_l \lambda_l^i \ \ln(\lambda_l^i)$

Mutual Information $\mathcal{I} = \mathcal{S}_{\rm L} + \mathcal{S}_{\rm R} - \mathcal{S}_{{\rm L}R}$

Schrodinger cat of photons

Evolution of quantum state of photon



Each cavity is described by the **anisotropic Dicke Hamiltonian**, with **cavity loss**

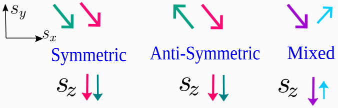
Semi-classical limit is obtained by considering the ansatz: $\hat{\rho} = \Pi_i |\alpha_i\rangle\langle\alpha_i| \otimes |z_i\rangle\langle z_i|$

in the $S \to \infty$ limit, classical equations of motion for $\chi_i = (\alpha_i, s_x^i, s_y^i, s_z^i)$

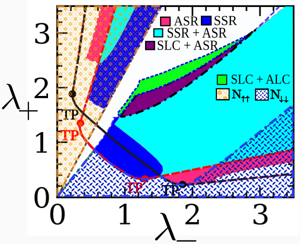
with dissipator: $\hat{L}_i = \sqrt{\kappa}\hat{a}_i$

$\dot{\chi}_i = 0 \Longrightarrow$ STEADY STATES

CLASSIFICATION: Spin orientation



Phase diagram for $|\alpha_L| = |\alpha_R|$



G.Vivek, D.Mondal, S.Chakraborty, S.Sinha
Phys. Rev. Lett. 134, 113404 (2025)

Effects of anisotropy and dissipation

Two Dicke transitions occur from the normal to two superradiant states characterised by $\tilde{\omega} = \omega \pm J$

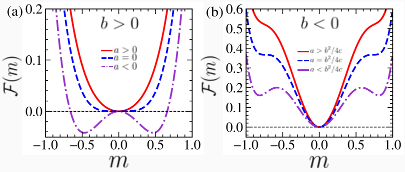
Second-order transition point

$$\lambda_+^2 = \lambda_-^2 + \tilde{\omega}\omega_0 \pm \sqrt{\omega_0(4\omega\lambda_-^2 - \kappa^2\omega_0)}$$

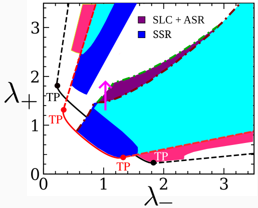
Anisotropy introduces a change in the nature of transition from second order to first order at TP

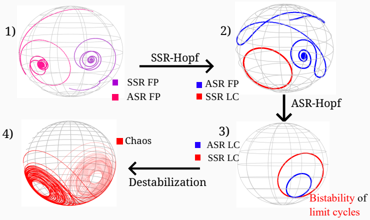
Effective Landau-Ginzberg potential

$$\mathcal{F}(m) = \frac{a}{2}m^2 + \frac{b}{4}m^4 + \frac{c}{6}m^6$$



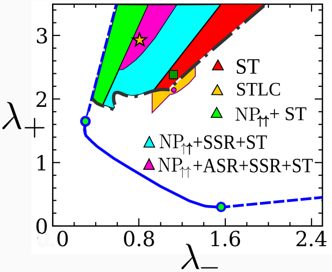
Route to dissipative chaos

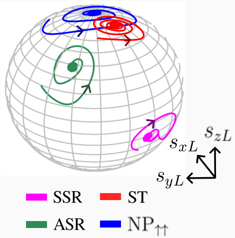




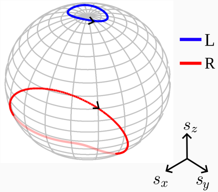
Self-trapping in a lossy cavity

Phase diagram for ST state

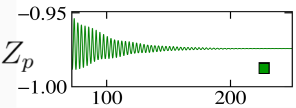




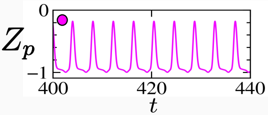
Selftrapped limit cycle



Oscillations show an imbalance in population



Self-trapped steady state



Measures of dissipative chaos

Decorrelator

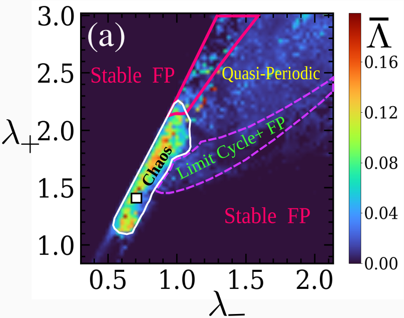
$$D(t) = 1 - \langle \boldsymbol{s}^a . \boldsymbol{s}^b \rangle$$

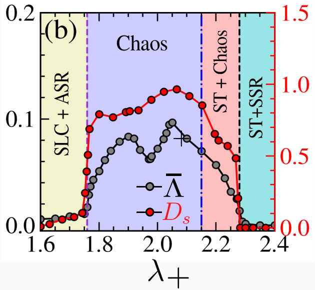
$$D_s = \frac{1}{N} \sum_{i=1}^{N} D_i \left(t = \infty\right)$$

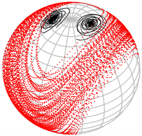
Lyapunov exponent

$$\Lambda = \lim_{t\to\infty} \frac{1}{t} \log \left(\frac{||\mathbf{X}(t)||}{||\mathbf{X}(0)||} \right)$$

$$\overline{\Lambda} = \frac{1}{N} \sum_{i=1}^{N} \Lambda_i$$







Coexistence of ST and chaos, leading to suppression of chaos in the system.

Summary and Acknowledgment

Key Takeaways:

Anisotropy and dissipation together enrich the phase diagram, enabling bistability, limit cycles, and chaos.

Quantum self-trapping and persistent oscillations even under photon loss.

Light-matter systems can mimic many-body quantum phases, despite photons being non-interacting in isolation.

Engineered cavity QED systems provide a powerful testbed for simulating nonequilibrium quantum dynamics.

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Link to my Google Scholar and papers

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