The real Ising quantum Otto engine

Giulia Piccitto
(University of Catania)

Stockholm, 25-07-2025

Outline&Purpose

The real Ising quantum Otto engine

Q1:
- Can it produce work?
- Role of many body interactions?
- Critical enhancement?

New J. Phys. 24 103023 (2022)

Q2:
- What about real transformations?
- Can real engines produce work?
- Can we optimize power?

Physical Review B 109 (22), 224309

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PAPER

The Ising critical quantum Otto engine

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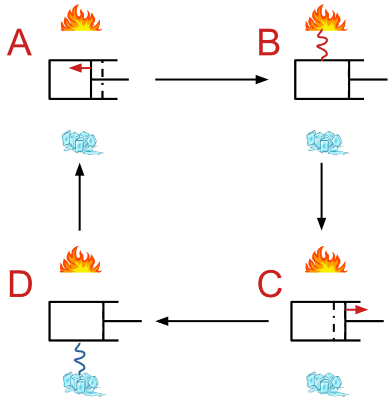
From classical to quantum Otto engine



A-B: adiabatic compression

B-C: isochoric heating up

C-D: adiabatic expansion

D-A: isochoric cooling down

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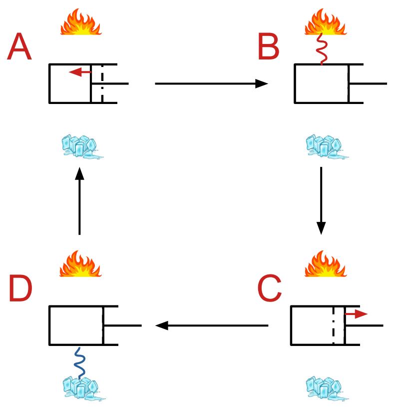

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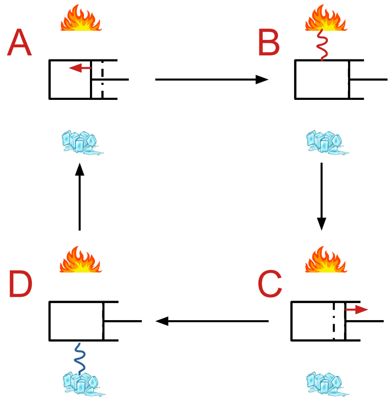
quantum $\longrightarrow$

1. Termodynamics

2. Cycle

From classical to quantum Otto engine



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quantum →

1. Termodynamics

Internal energy: $E \equiv \langle H \rangle$

1st td law: $dE = \delta Q - \delta W$

Absorbed heat	$\delta Q > 0$
Performed work	$\delta W > 0$

Work: Energy exchanged with an external source

Heat: Energy exchanged with a thermal source

2. Cycle

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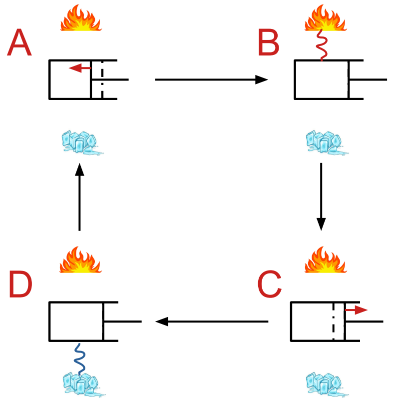

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2. Cycle

Working medium: quantum Ising chain

Bath: continuum of fermionic harmonic oscillators at finite T

Adiabatic processes: Hamiltonian evolution (only work)

Isocoric processes: Incoherent evolution (only heat)

The cycle

Working medium: quantum Ising chain

$$\hat{H}(t) = -J \sum_{j}^{N-1} \hat{\sigma}_j^x \hat{\sigma}_{j+1}^x - h(t) \sum_j \hat{\sigma}_j^z$$

- Free fermions: exactly solvable
- Gaussian eigenstates: polinomial scaling
- Critical system: gap closure $h_c = J = 1$

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$$\hat{H}_{\mathrm{env}} = \sum_{n=1}^{N_B} \int \mathrm{d}k \, \epsilon_n(k) \, \hat{c}_n^\dagger(k) \, \hat{c}_n(k)$$

We assume:

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$$dE = \cancel{\delta Q} - \delta W$$

$$h(t) = h_i + vt$$

(Which velocity? Quantum adiabaticity?)

$$\boxed{W_{i\to f} = \langle H(t_i)\rangle_{t_i} - \langle H(t_f)\rangle_{t_f}}$$

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(Which dissipator to describe thermalization?)

$$\boxed{Q = \langle H\rangle_{T_2} - \langle H\rangle_{T_1}}$$

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Thermalization

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We can write non-local Lindblad operators[1] (in the Hamiltonian eigenbasis) that simulates thermalization

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$$\rho(t) = \rho_T \left(1 - e^{-2\mathcal{J}t}\right) + \rho(t=0)\, e^{-2\mathcal{J}t}$$

- Thermal state
- Initial state
- Bath density of states

$\rho(t=0)$ ρ_T

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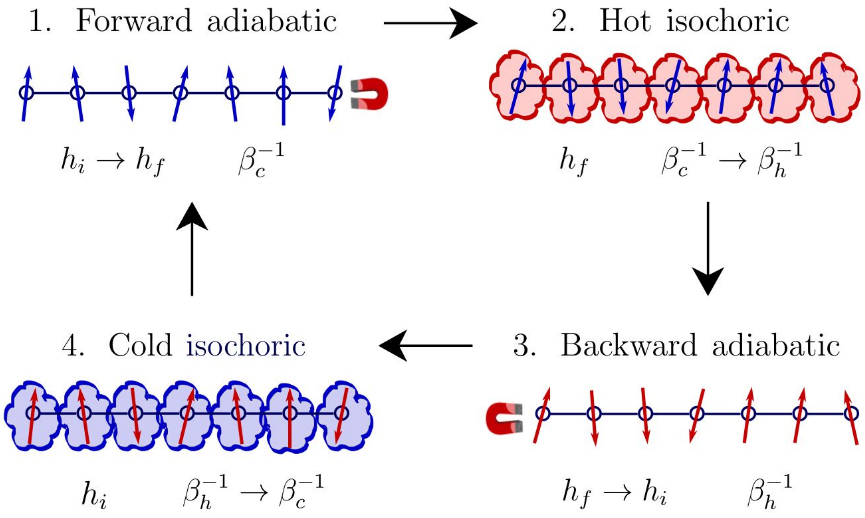
The Ising quantum Otto cycle



A-B: Forward Hamiltonian evolution

B-C: Hot isochoric

C-D: Backward Hamiltonian evolution

D-A: Cold isochoric

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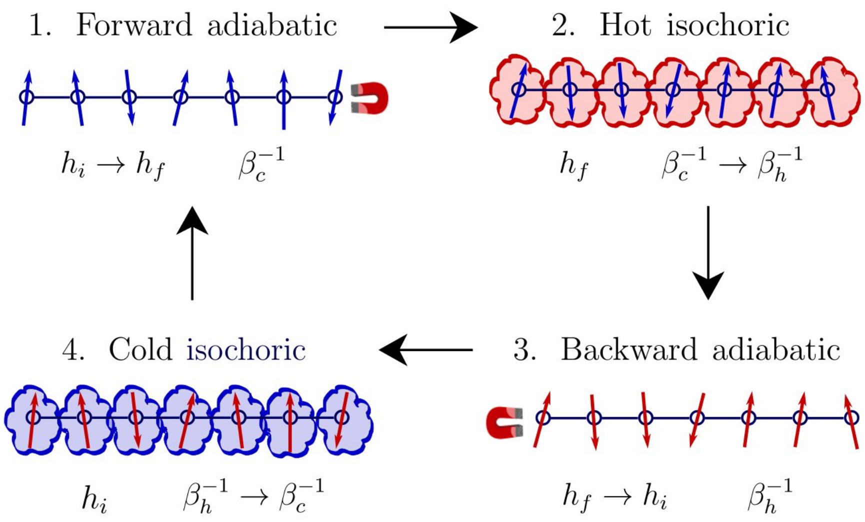
The Ising quantum Otto cycle

Lots of parameters:

- System size V
- Initial transverse field V
- Quench amplitude F
- Quench velocity F
- Hot bath temperature F
- Cold bath temperature V
- Thermalization time F

F: Fixed

V: Varying



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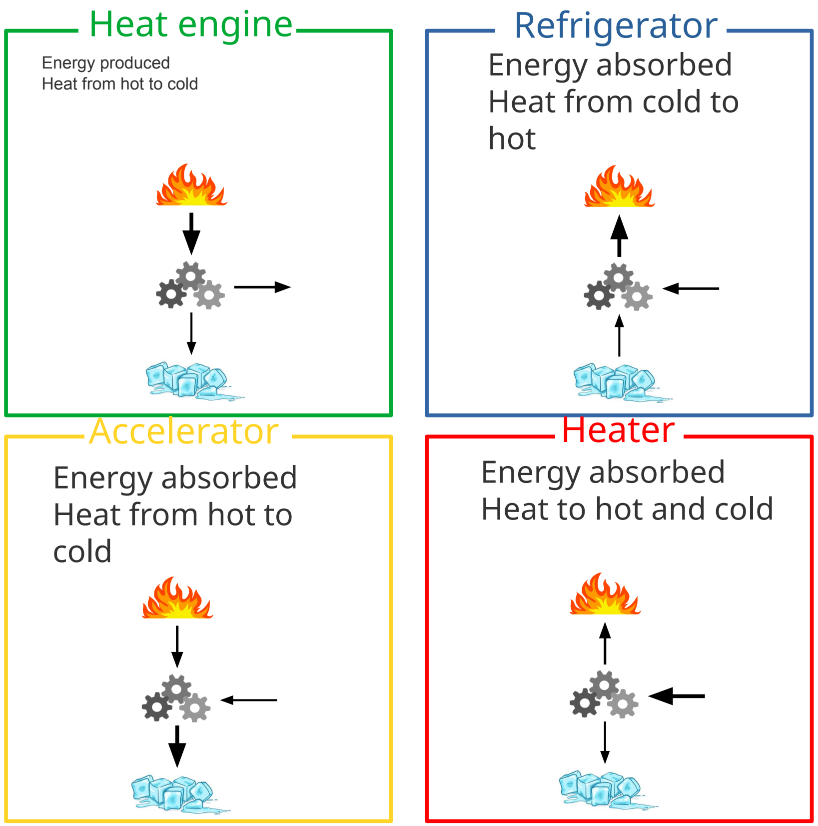
Zoology: can it be useful?



Solfanelli et al. Phys. Rev. B, 101 054513 (2020)

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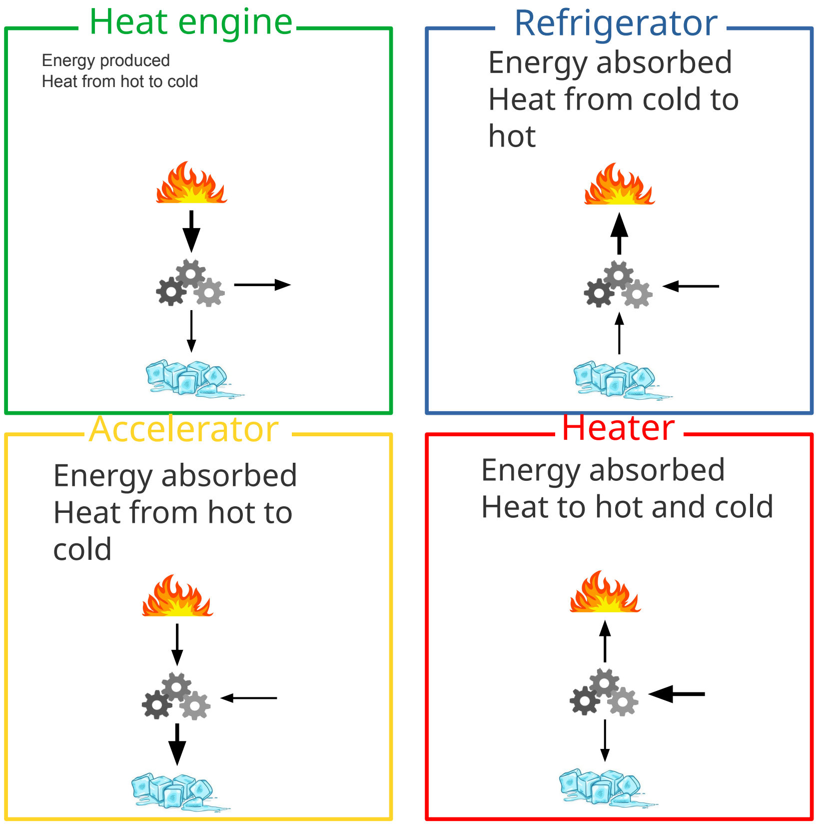

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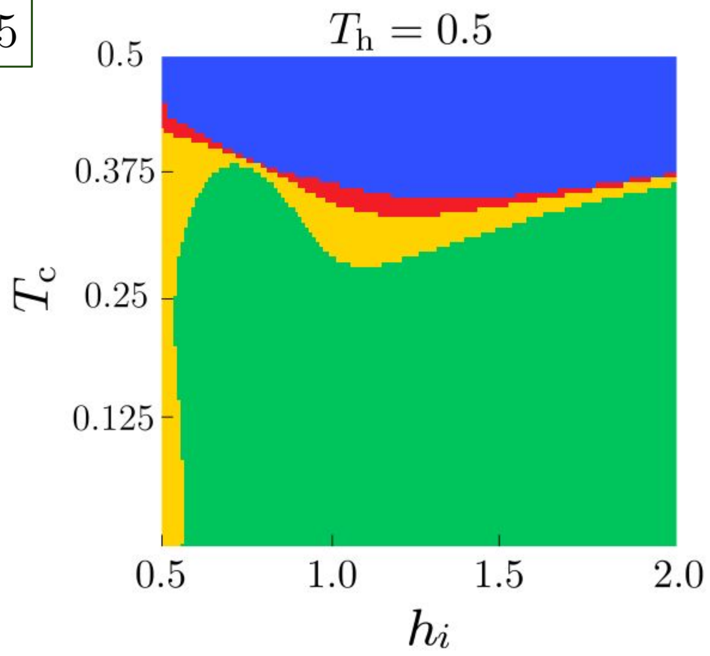
$N = 50$
$\delta h = 0.5$



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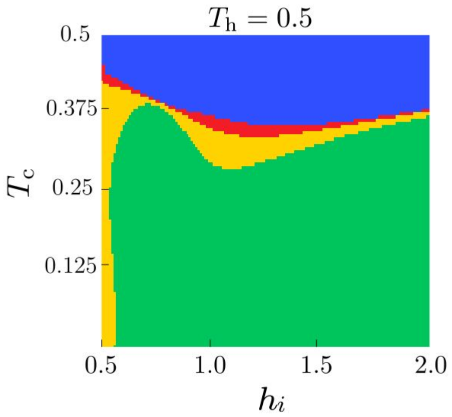
Checking parameters #1



- System size
- Quench amplitude
- Hot temperature

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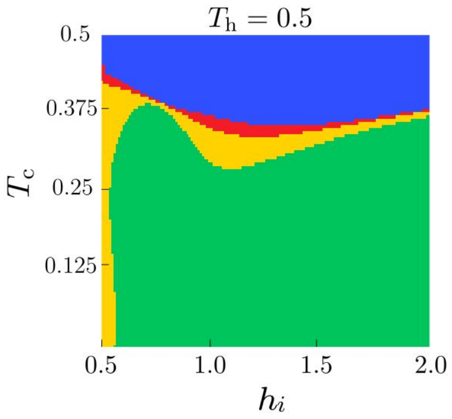

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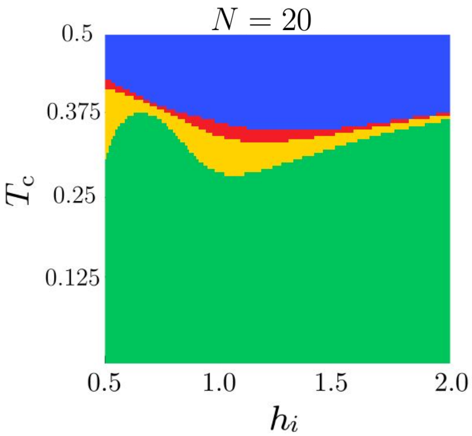

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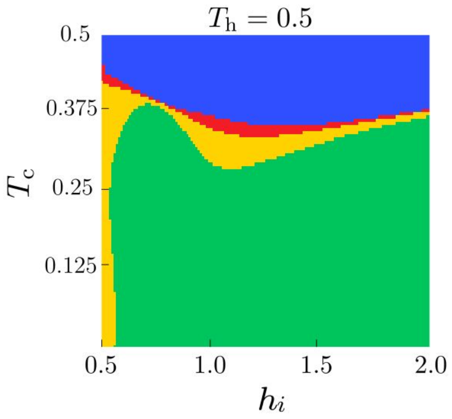

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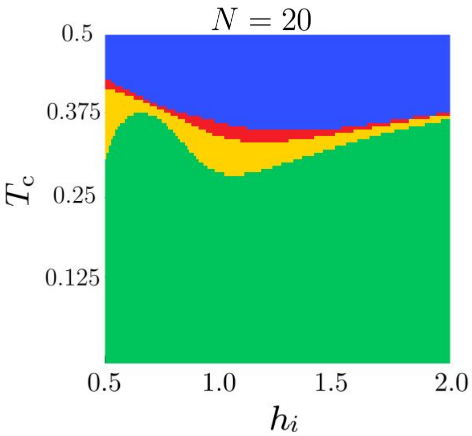

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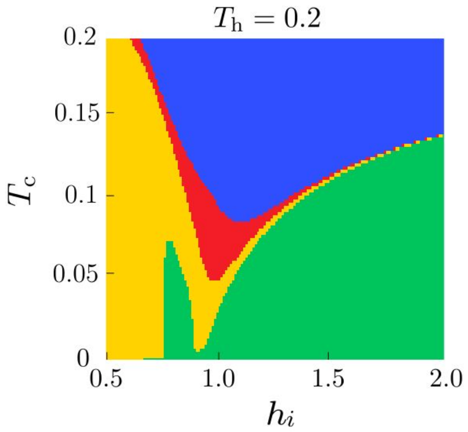

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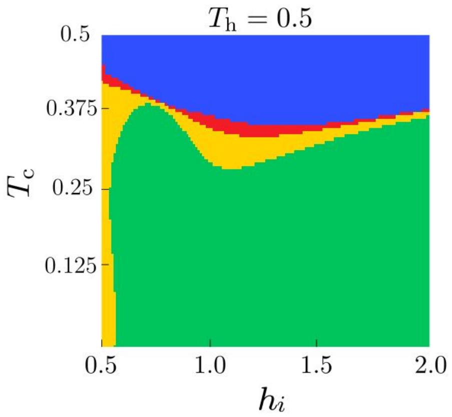

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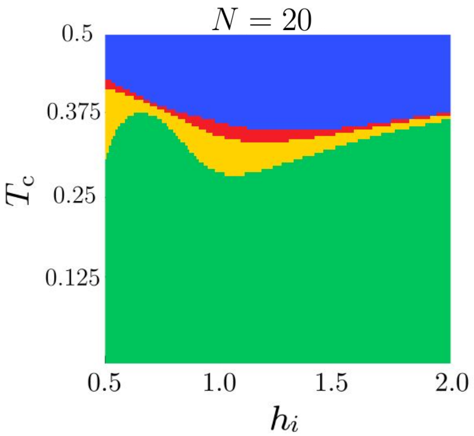

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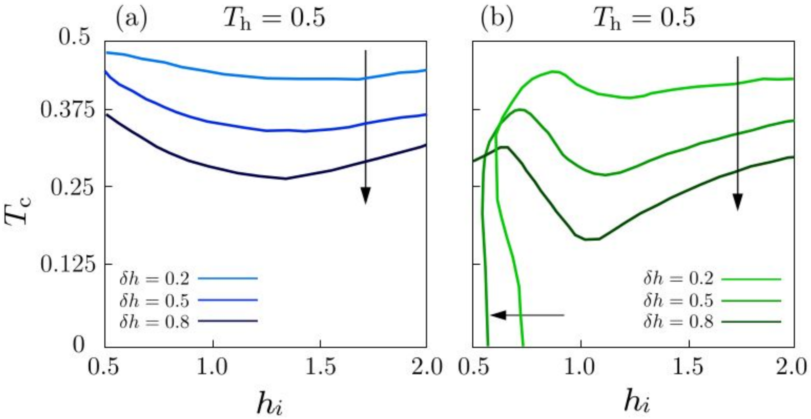

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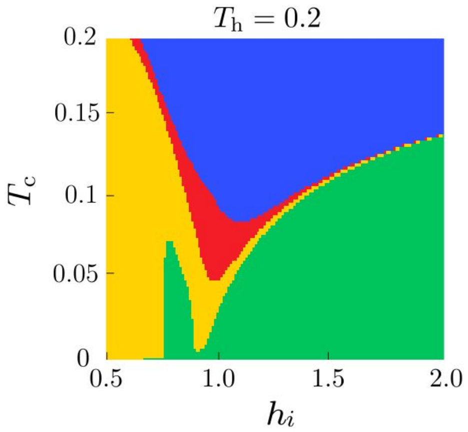

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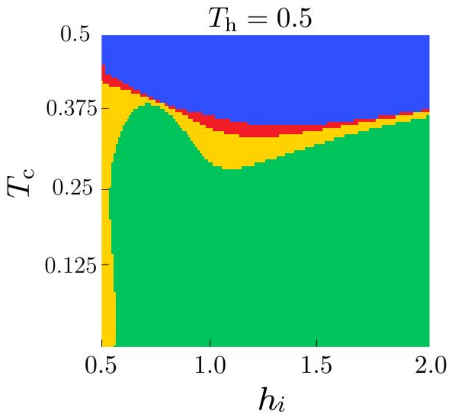

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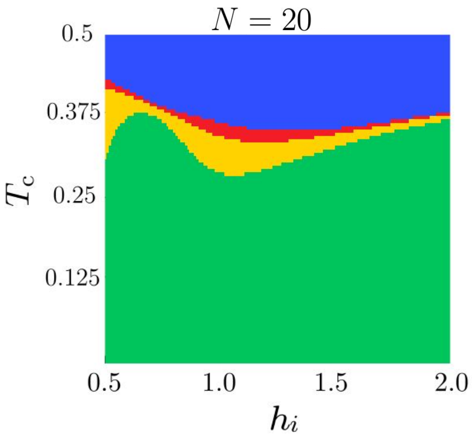

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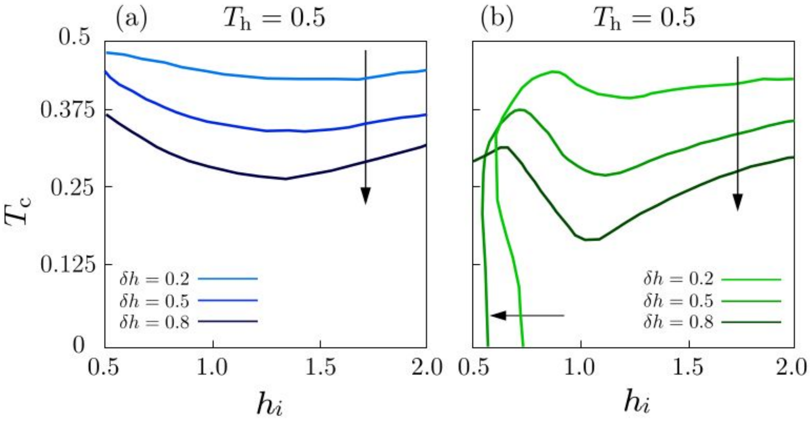

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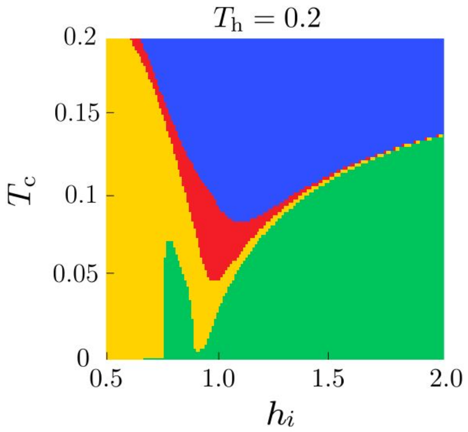

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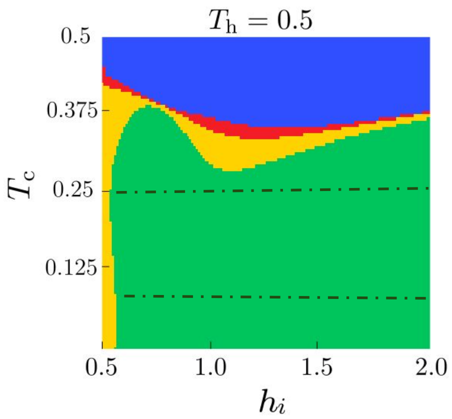
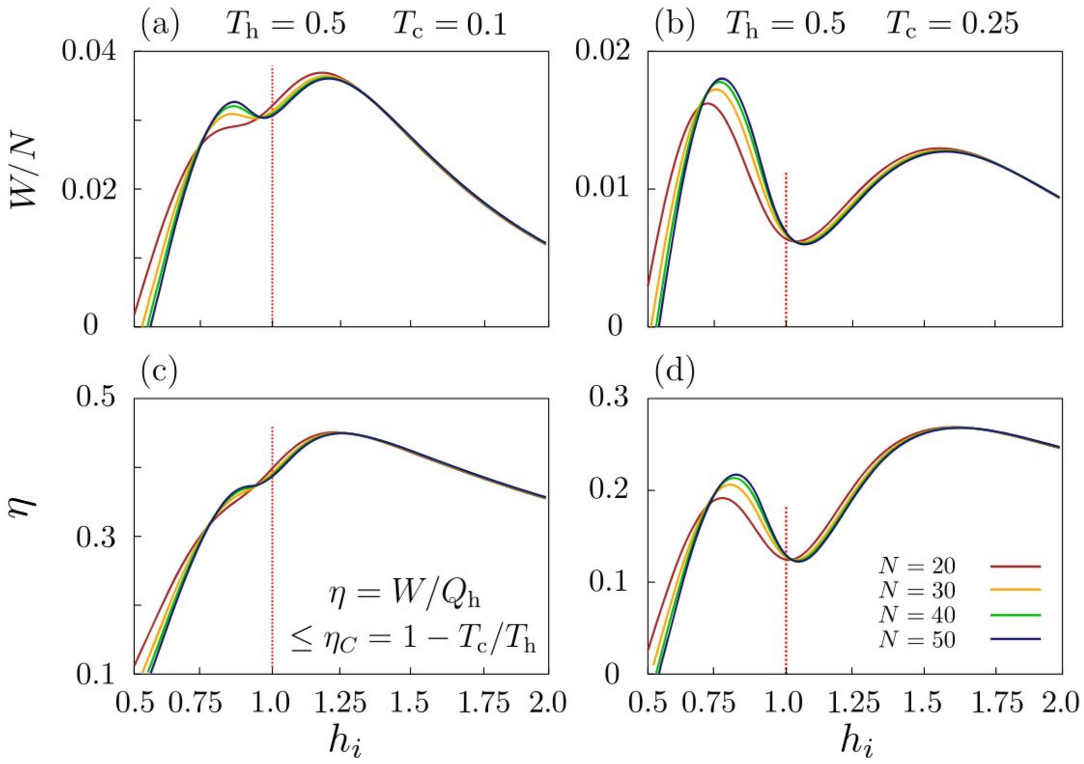
Work and efficiency





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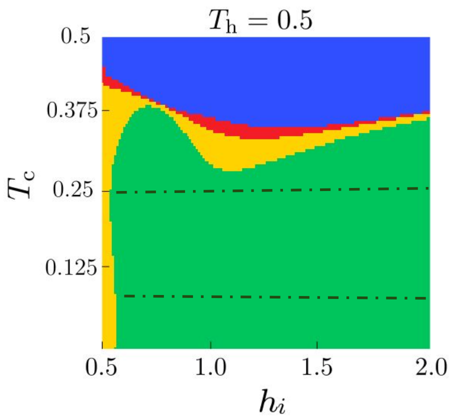

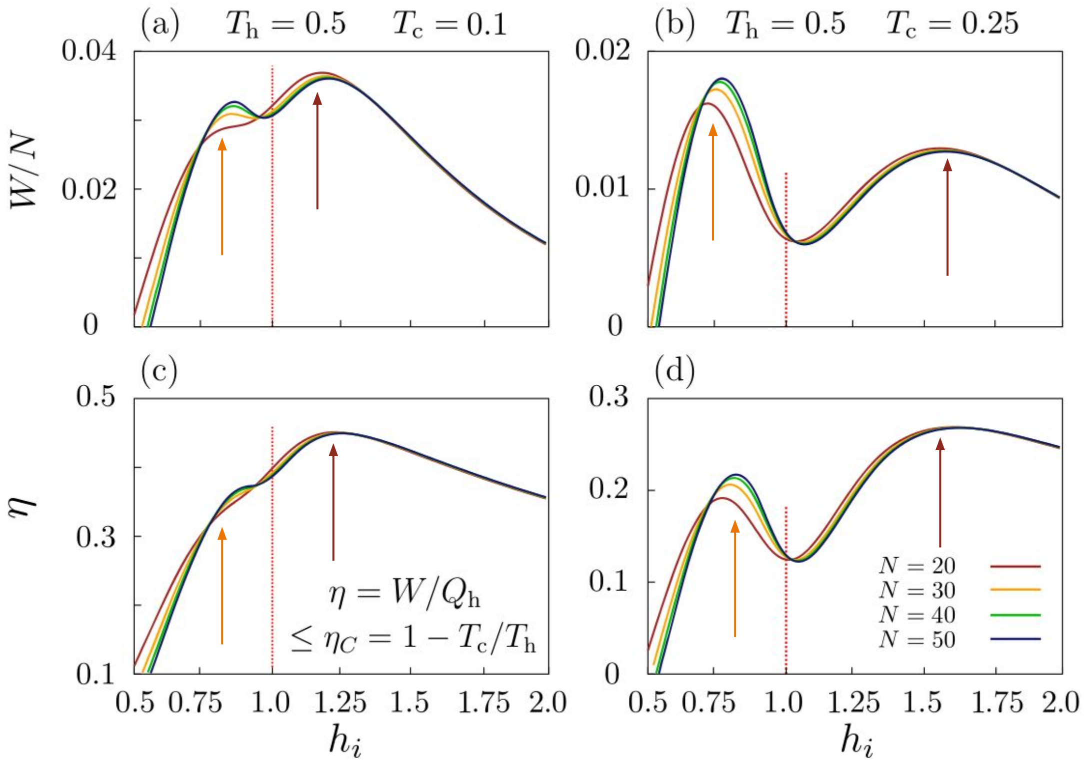

Two peaks structure:

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- Critical $h_i < 1$

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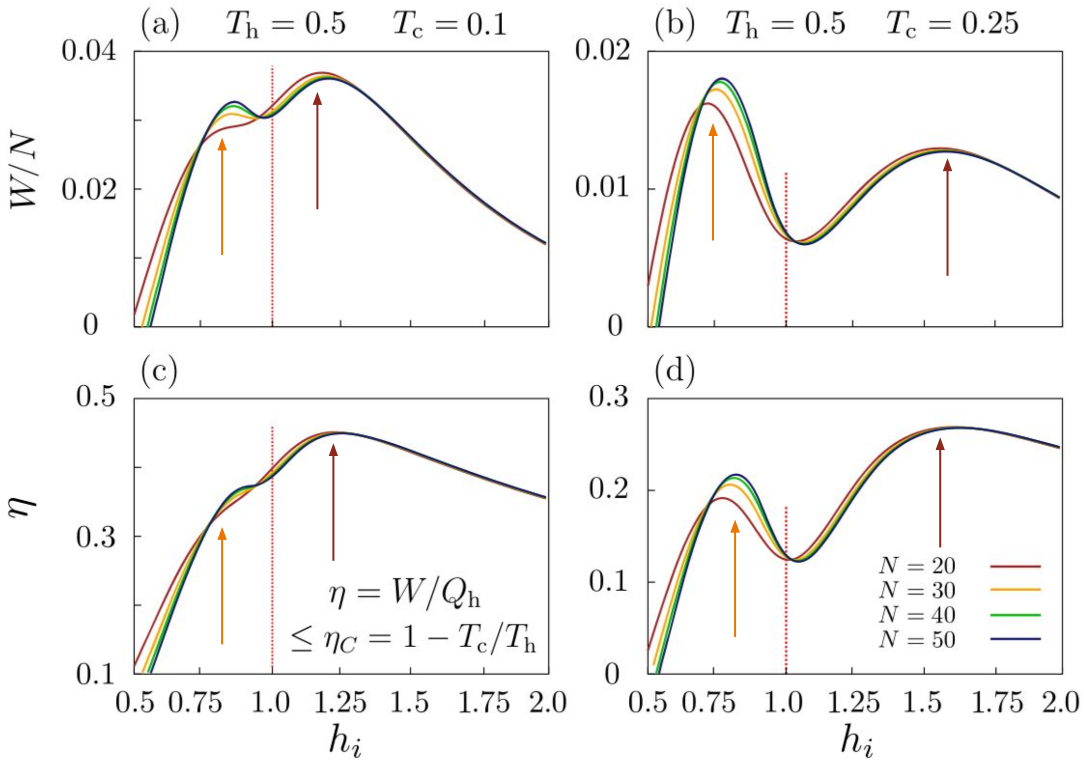

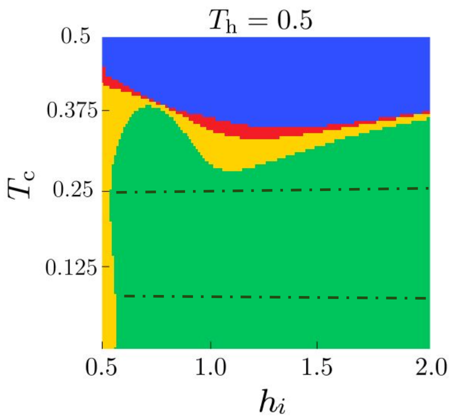

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Paramagnetic: (in general) more performant but linear with the system size

Critical: hyperscaling with the system size

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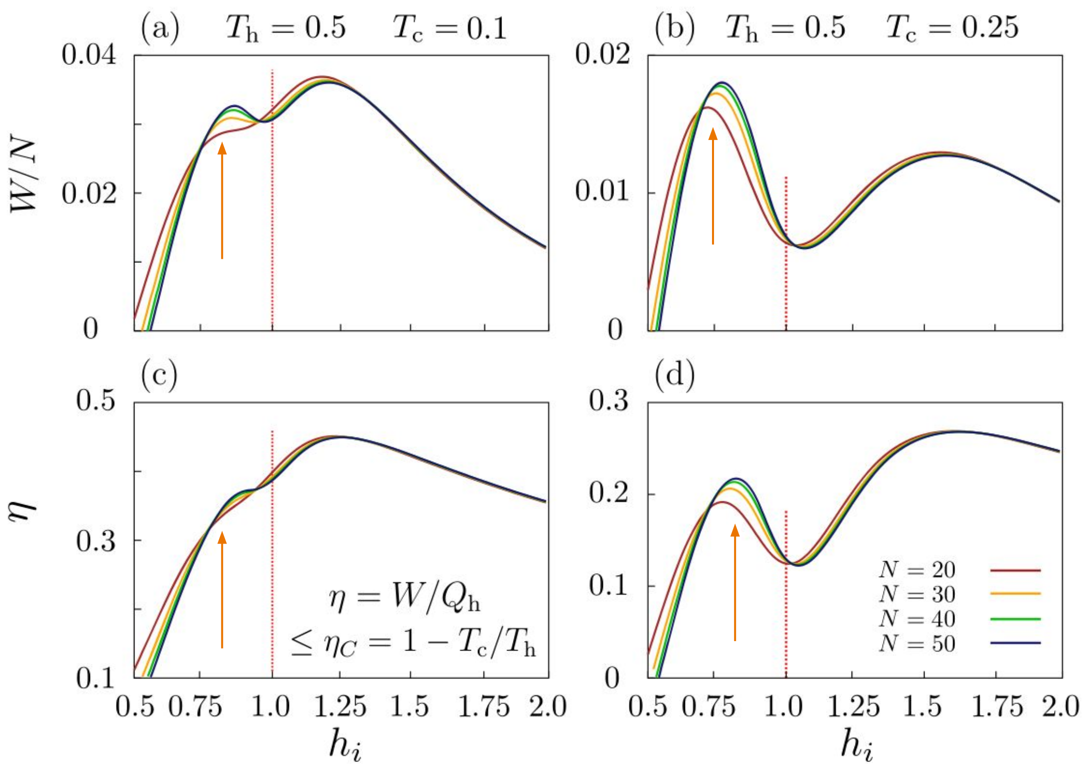
The role of criticality

Nat. Commun. 7 11895 (2016)



Critical enhancement

The divergence of the fluctuations at the critical point can lead to an enanchement of the performances

$$\Pi = W/\delta\eta$$

$$\delta\eta = \eta_C - \eta$$

(how much work with an efficiency close to the Carnot one)

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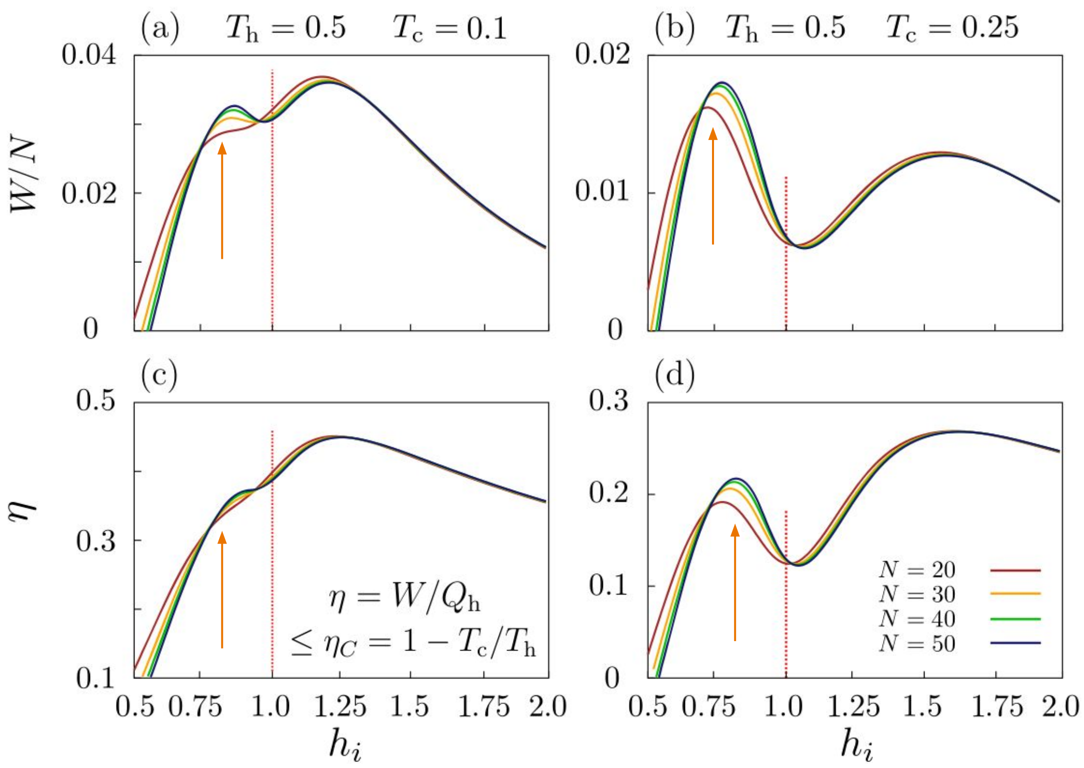
The role of criticality



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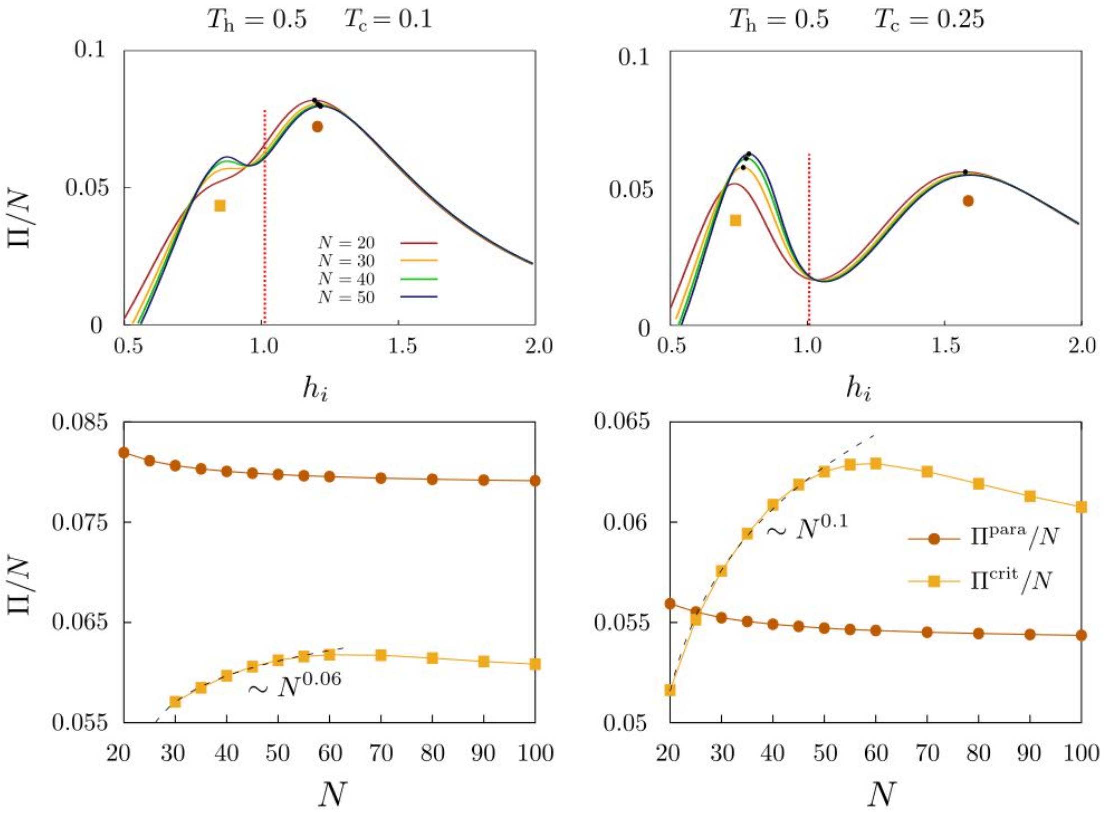

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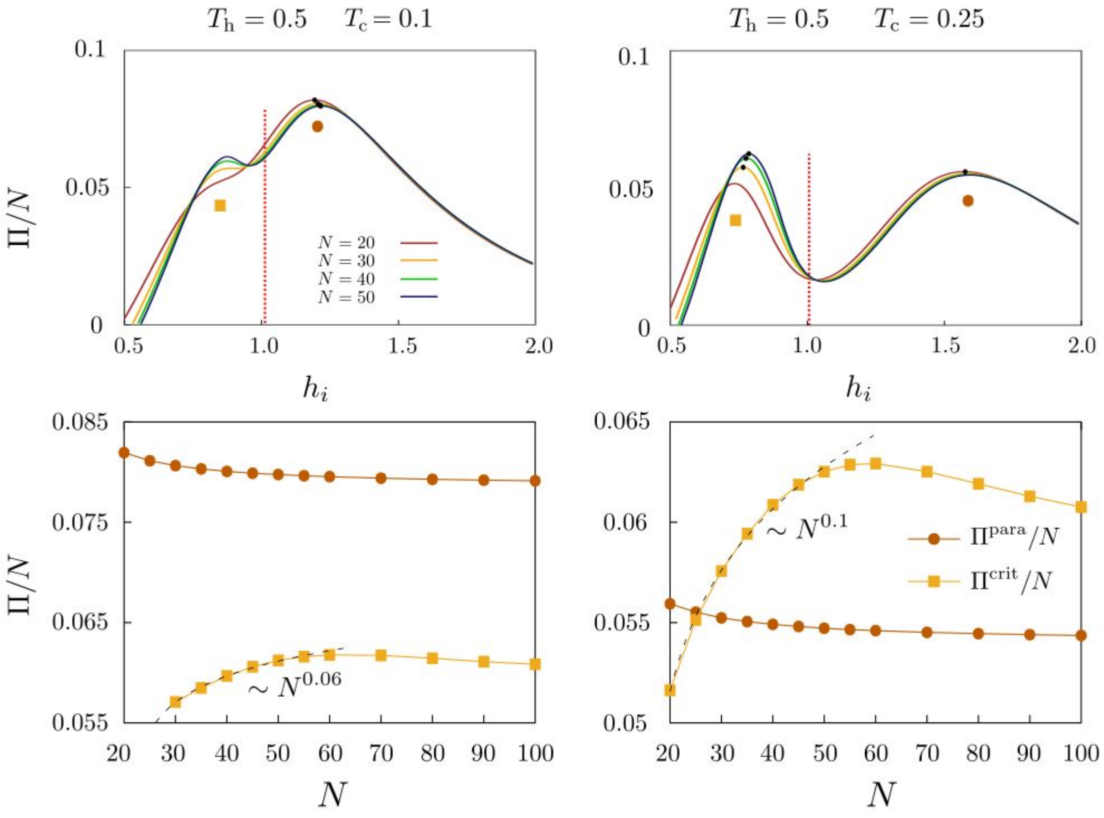

C

work ⇄ heat

The larger the temperature gradient the larger the work

Q

work ⇄ magnetization

The larger the magnetic susceptibility the larger the work

At criticality diverges!

Dorner et al. PRL 109 160601 (2012)
Fusco et al. PRX 4 031029 (2014)

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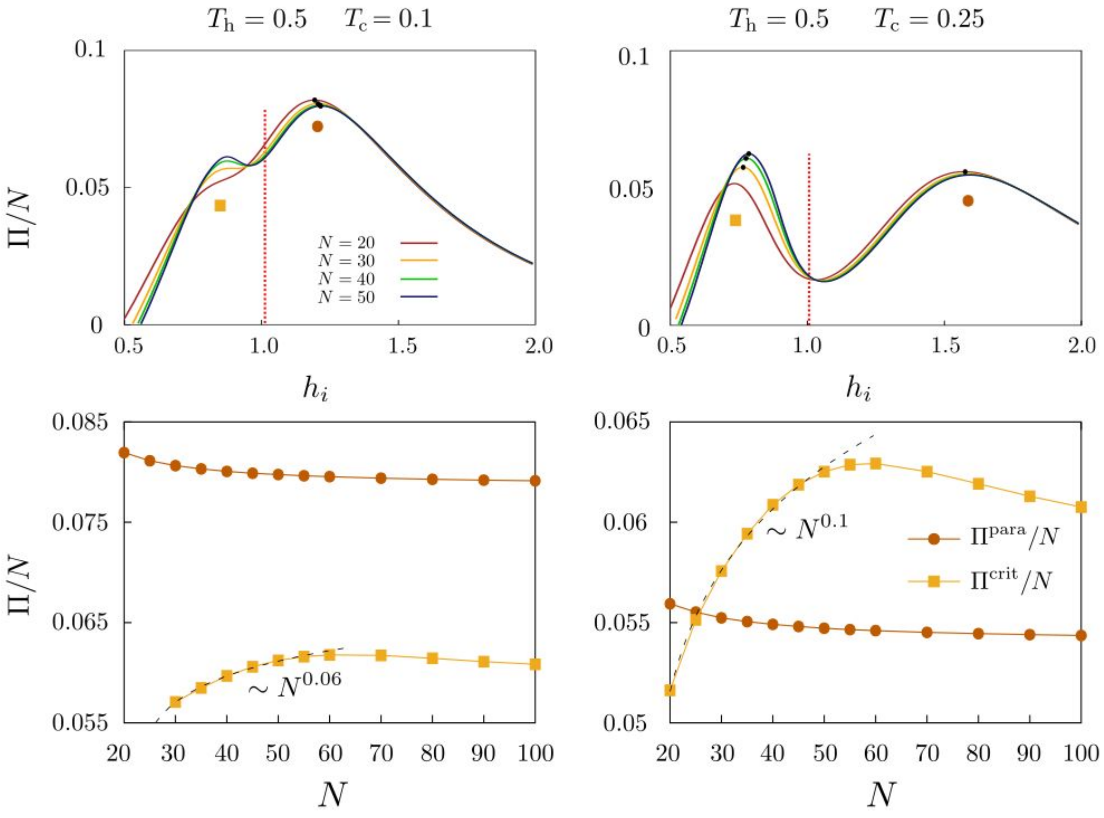
Critical enhancement



Pros:

- It actually exists
- N-body engine > N single-body engine
- Better “cold performances” (small T gradient)

Cons:

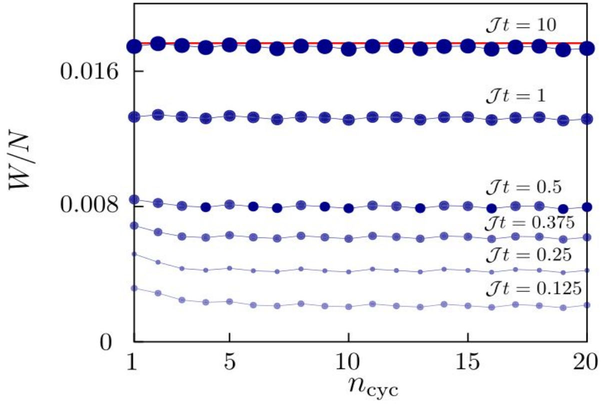
- Maybe paramagnetic is better
- Quantum adiabatic trouble
- Fluctuations?

What about real engines?

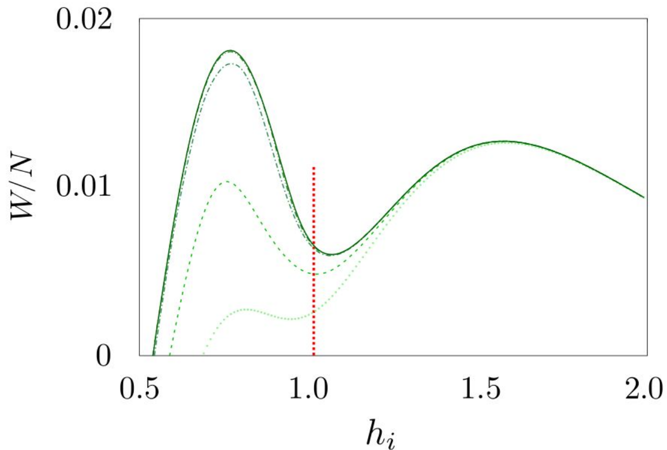
- Non perfect thermalization reduce temperature gradient
- Non perfect adiabatic dissipates energy in the excited eigenstates

Extracted work is smaller than from ideal engines!

1. Non perfect thermalization



2. Fast quenches



What about real engines?

- Non perfect thermalization reduce temperature gradient
- Non perfect adiabatic dissipates energy in the excited eigenstates

Extracted work is smaller than from ideal engines!

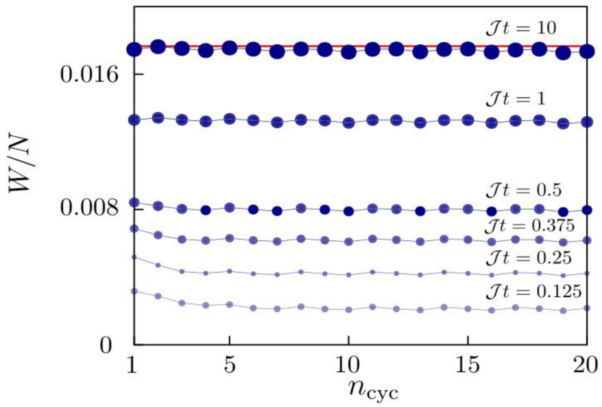
What about power?

$$\mathcal{P} = \frac{W}{T_{\text{cycle}}}$$

Total work

Cycle duration

1. Non perfect thermalization



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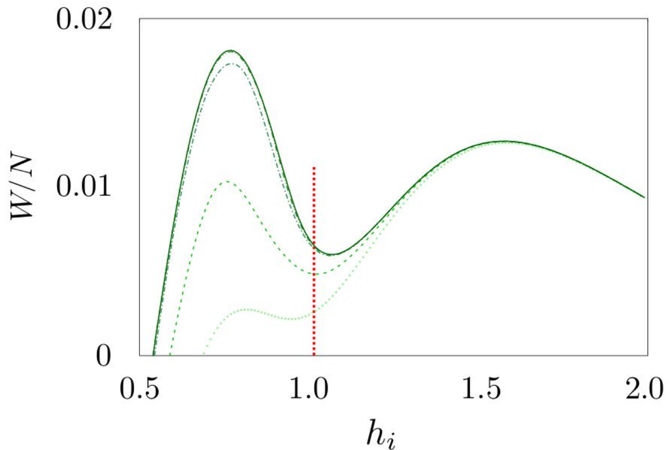

Outline&Purpose

PHYSICAL REVIEW B **109**, 224309 (2024)

Editors' Suggestion

Many-body quantum heat engines based on free fermion systems

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Q2:

- What about real transformations?
- Can real engines produce work?
- Can we optimize power?

What about real engines?

Working medium: quantum Ising chain (or any free fermions chain)

$$D_{i,j} = J\ \delta_{j,i+1} \qquad O_{i,j} = h(t)\ \delta_{ij}$$

$$\hat{H}(t) = -J \sum_{j}^{N-1} \hat{\sigma}_j^x \hat{\sigma}_{j+1}^x - h(t) \sum_j \hat{\sigma}_j^z \longrightarrow H = \sum_{i,j} D_{i,j} c_i^\dagger c_j + \frac{1}{2}(O_{i,j} c_i^\dagger c_j^\dagger + \mathrm{H.c.})$$

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Diagonalizing free fermion chains:

$$H = \Psi^\dagger \mathbb{H}\Psi + \mathrm{Tr}[D] \qquad \Psi = (c_1, \ldots, c_N, c_1^\dagger, \ldots, c_N^\dagger)^T$$

$$\mathbb{H} = \frac{1}{2}\begin{pmatrix} D & O \\ -O^* & -D^* \end{pmatrix}$$

$$\downarrow$$

$$H = \sum_{k>0}\omega_k(\lambda)\left(b_k^\dagger b_k - \frac{1}{2}\right) \qquad \Phi = (b_1, \ldots, b_N, b_1^\dagger, \ldots, b_N^\dagger)^T$$

$$\mathbb{U} = \begin{pmatrix} \mathbb{u} & \mathbb{v} \\ \mathbb{v}^* & \mathbb{u}^* \end{pmatrix}$$

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- New thermal fermions

$$\text{Tr}[b_k^\dagger b_k\, \rho] = [1 + e^{-\beta\omega_k(\lambda)}]^{-1} \equiv f[\beta, \omega_k(\lambda)]$$

- Work of the system (performed > 0)

$$W = \langle H(\lambda_i)\rangle_{\rho(\lambda_i)} - \langle H(\lambda_f)\rangle_{\tilde{\rho}(\lambda_i)}$$
$$= \sum_k [\omega_k(\lambda_i) - \omega_k(\lambda_f)]\left\{f[\beta, \omega_k(\lambda_i)] - \frac{1}{2}\right\}$$

- Heat exchanged (absorbed > 0)

$$Q = \langle H\rangle_{\rho_f} - \langle H\rangle_{\rho_i} = \sum_k \omega_k\, [f(\beta_2, \omega_k) - f(\beta_1, \omega_k)]$$

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We only need two-point correlations!

- New thermal fermions

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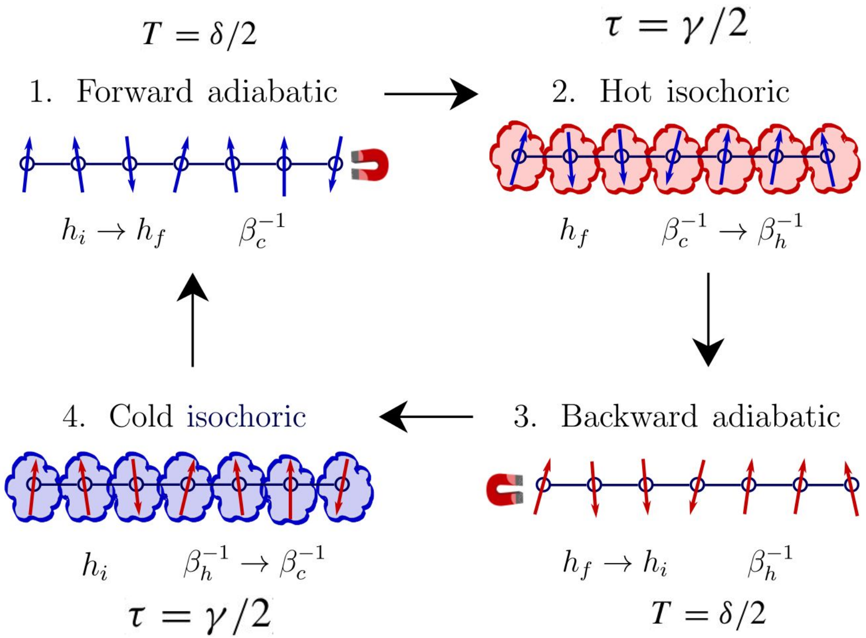

$$T_{\text{cycle}} = \delta + \gamma$$

0: Ideal $\delta, \gamma \to \infty$

1: Non perfect thermalization $\delta \to \infty, \quad \gamma < \infty$

2: Non perfect adiabatic $\gamma \to \infty, \quad \delta < \infty$

3: Nothing perfect $\delta, \gamma < \infty$

1. Non perfect thermalization

$$\delta \to \infty, \quad \gamma < \infty$$

$$\Theta_{c(h)} = \mathrm{diag}\{f[\beta_{c(h)}, \omega_k(\lambda_{i(f)})]\}_{k=1,\ldots,N}$$

$$\Gamma^{[n]}_{c(h)} = \mathrm{diag}\left\{\mathrm{Tr}\left[b_k^\dagger b_k\, \tilde{\rho}^{[n]}_{c(h)}\right]\right\}_{k=1,\ldots,N}$$

Iterative equation

$$\Gamma_c^{[n]} = (\Theta_c + e^{-\gamma}\Theta_h)(1 - e^{-\gamma}) + \Gamma_c^{[n-1]} e^{-2\gamma}$$

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whose stationary solution is

$$\Gamma_c^{\infty} = h(\gamma)(\Theta_c + e^{-\gamma}\Theta_h)$$

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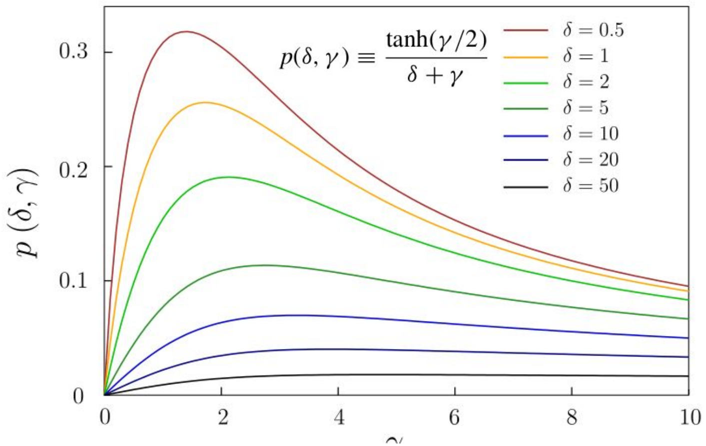
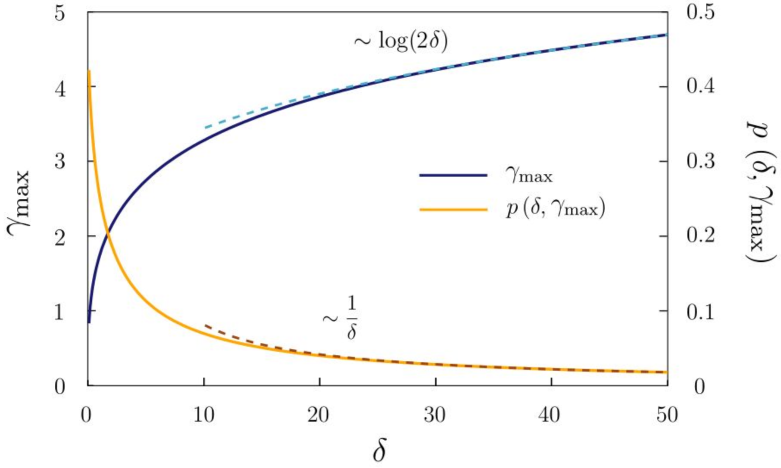
Power becomes

$$\mathcal{P}^{\text{n-th}}(\delta, \gamma) \equiv \frac{W^{\text{n-th}}}{\delta + \gamma} = p(\delta, \gamma)\, W^{\text{id}}$$





2. Non perfect adiabatic

$$\gamma \to \infty, \quad \delta < \infty$$

Work of a real adiabatic process

$$W = \sum_k [\omega_k(\lambda_i) - \tilde{\omega}_k(\lambda_f)] \left\{ f[\beta, \omega_k(\lambda_i)] - \frac{1}{2} \right\}$$

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$$\boxed{\begin{aligned} \Psi^H &= U_{\text{ev}}^\dagger(T) \Psi U_{\text{ev}}(T) = \mathbb{V}(T)\Phi \\ \tilde{\mathbb{H}} &= \mathbb{V}^\dagger(T) \mathbb{H}_f \mathbb{V}(T) \end{aligned}}$$

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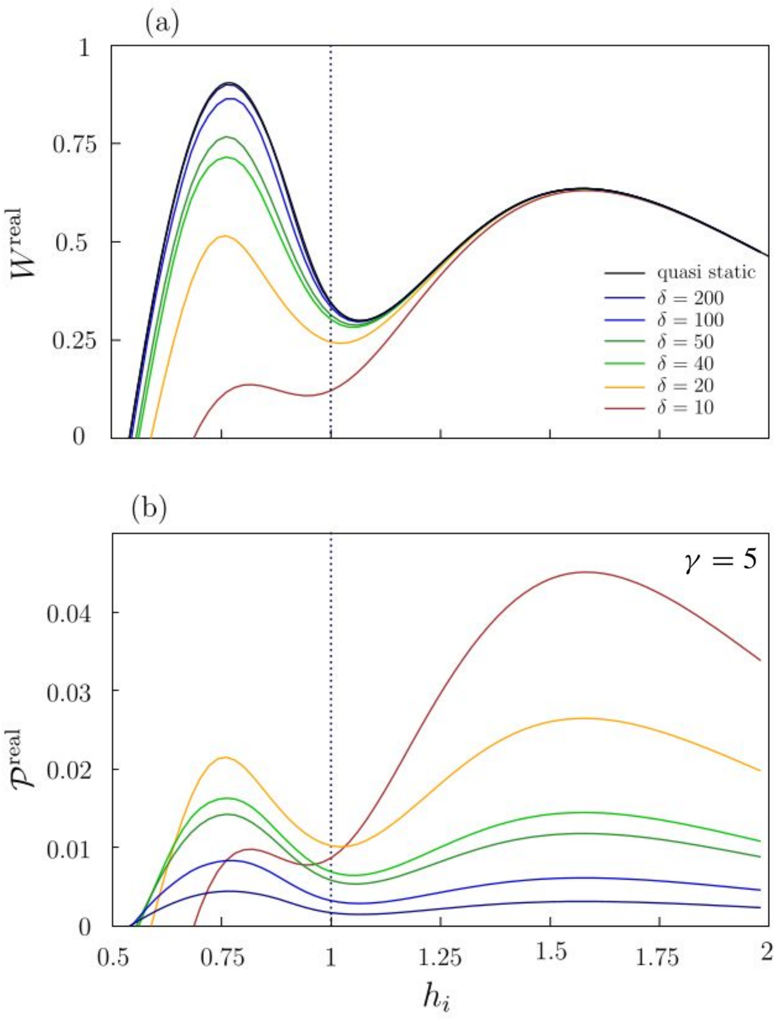

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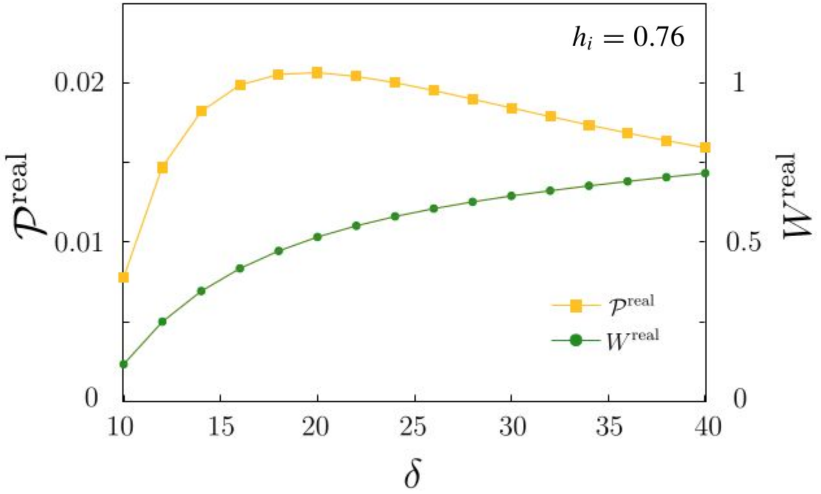

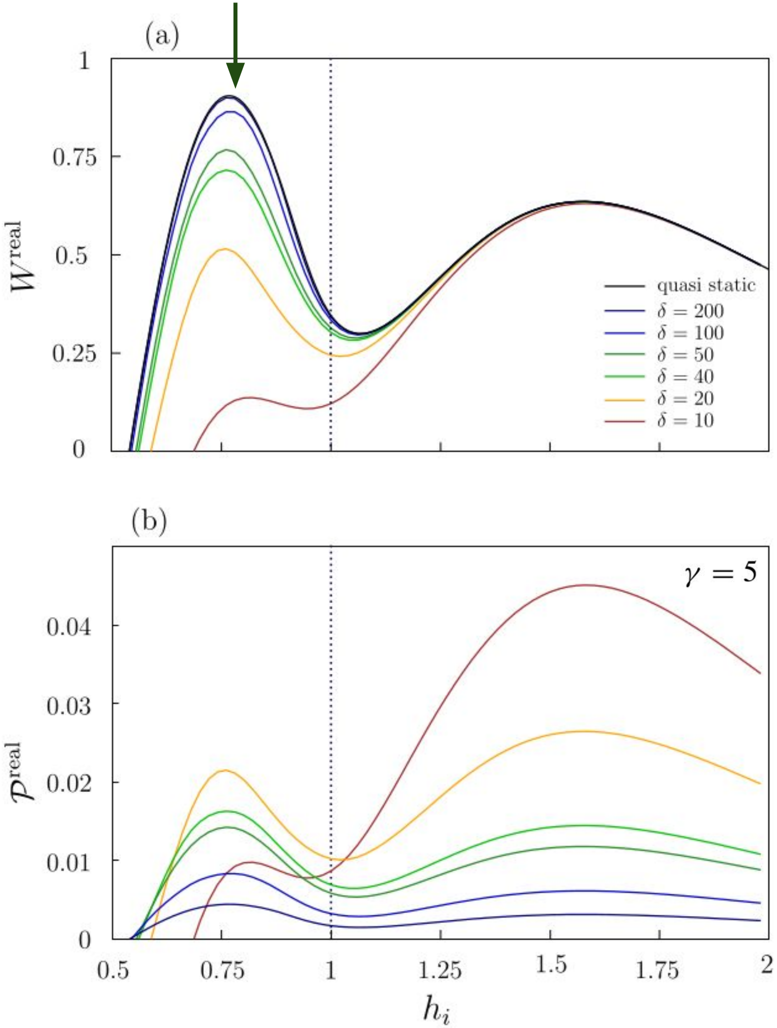

2. Nothing perfect

$\delta, \gamma < \infty$

This becomes cycle dependent

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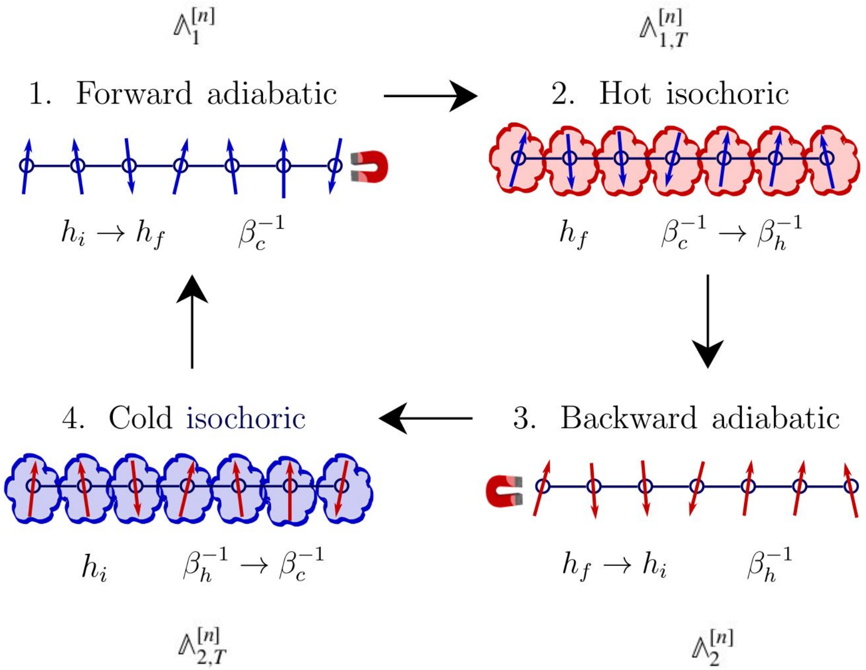
Iterative equations for the two point correlators

$$(\Lambda_i^{[n]})_{jl} = \langle \Phi_j \Phi_l^\dagger \rangle_{\rho_i^{[n]}} \qquad (\Lambda_{i,T}^{[n]})_{jl} = \langle \Phi_j \Phi_l^\dagger \rangle_{\rho_i^{[n]}(T)}$$



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$$\begin{aligned}
\mathbb{\Lambda}_{1,T}^{[n]} &= \mathbb{Q}\mathbb{\Lambda}_1^{[n-1]}\mathbb{Q}^\dagger, \\
\mathbb{\Lambda}_2^{[n]} &= \mathbb{\Omega}_h(1 - e^{-\gamma}) + \mathbb{\Lambda}_{1,T}^{[n]} e^{-\gamma}, \\
\mathbb{\Lambda}_{2,T}^{[n]} &= \mathbb{Q}'\mathbb{\Lambda}_2^{[n]}\mathbb{Q}'^\dagger, \\
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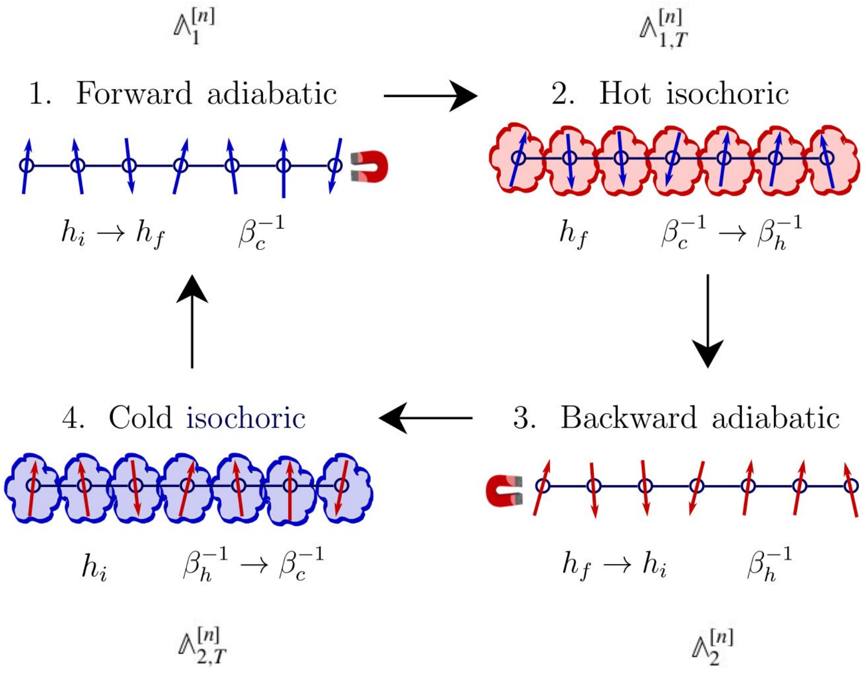

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$\delta, \gamma < \infty$

$$\mathbb{A}^{\infty}_{1,T} = \sum_{k=0}^{\infty} e^{-2k\gamma}[(\mathbb{Q}\mathbb{Q}')^k \mathbb{Q}\mathbb{K}_1\mathbb{Q}^\dagger(\mathbb{Q}'^\dagger\mathbb{Q}^\dagger)^k]$$

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We can consider the first k elements only

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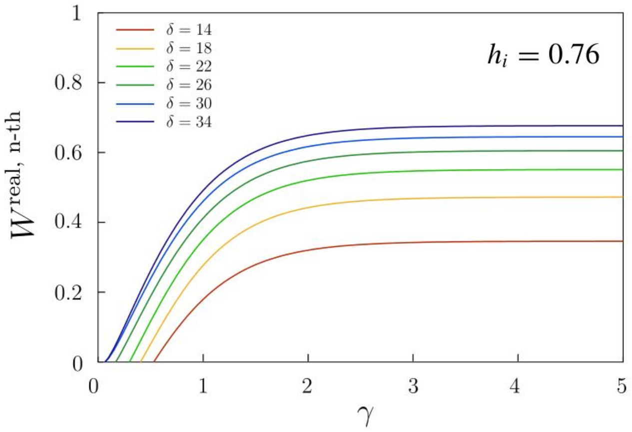

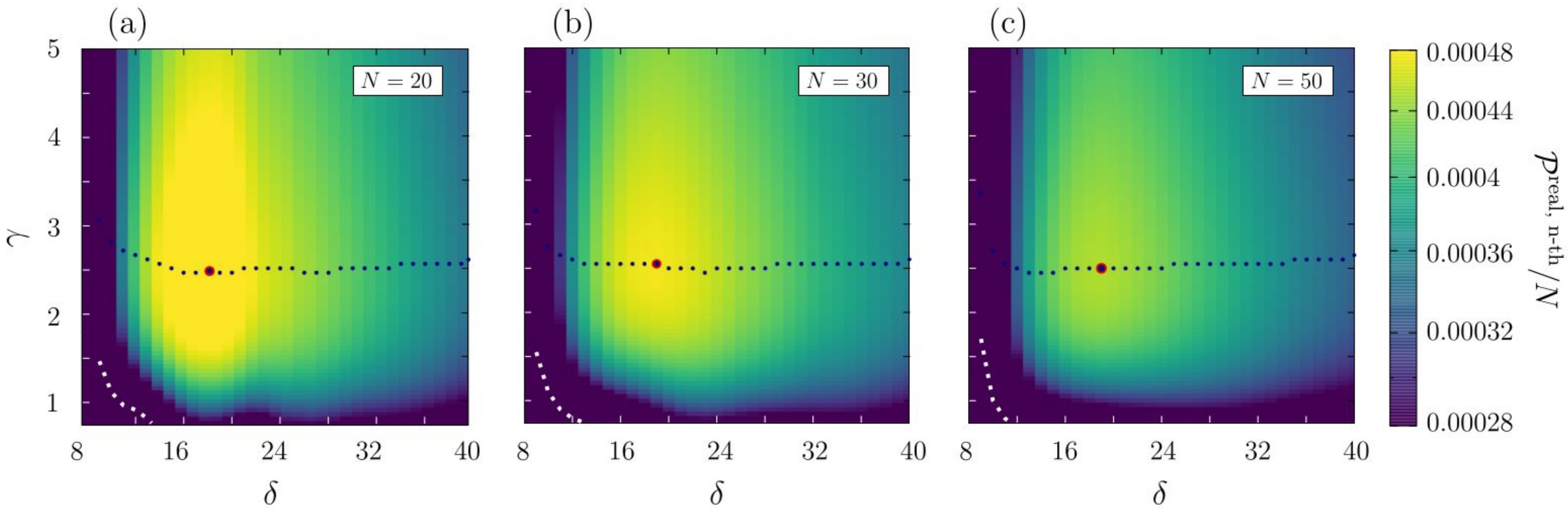

Conclusions

What we learned:

- The Ising quantum Otto engine can be useful (refrigerator, heat engine)
- The absolute performances are maximized with the “classical” work extraction mechanism
- However to have the best scaling with the system size we need to go close to criticality
- Sometimes criticality maximizes also the absolute performances
- Real engines can also be useful
- It is not easy to find the optimal working point

What we have to learn:

- Power?
- Different engines (e.g. Carnot)?
- Some shortcuts to adiabaticity?
- Is this behavior universal? If yes, can we say something more? (Spoiler: extremely non trivial)
- What about fluctuations?
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