Zeros of Complexified Survival Amplitude at Different Time-Scales

Long-Range Interactions and Dynamics in Complex Quantum Systems

NORDITA

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Outline

Survival Amplitude

Motivation

Distribution of the zeros in the complex plane

- Long times
- Short times

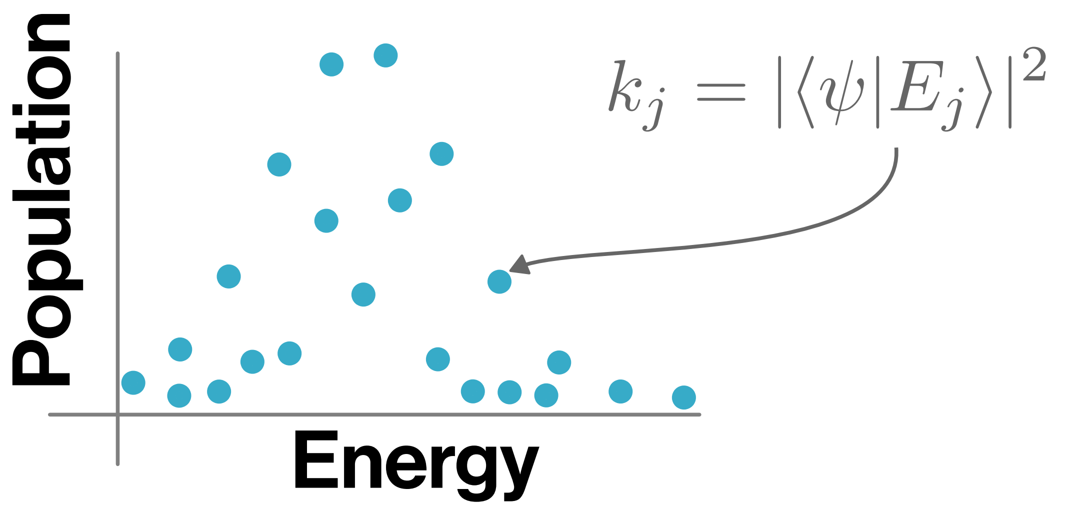
Remarks and Quenches

Survival amplitude

$$\mathcal{L}(t) = \langle\psi|\hat{U}(t)|\psi\rangle = \sum_j k_j e^{-iE_j t}$$

State

$|\psi\rangle$



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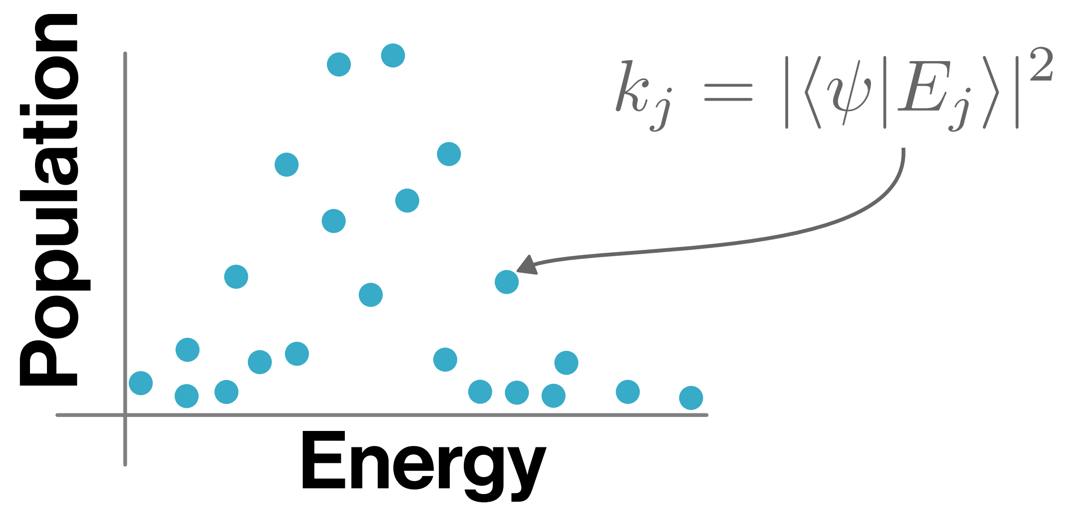

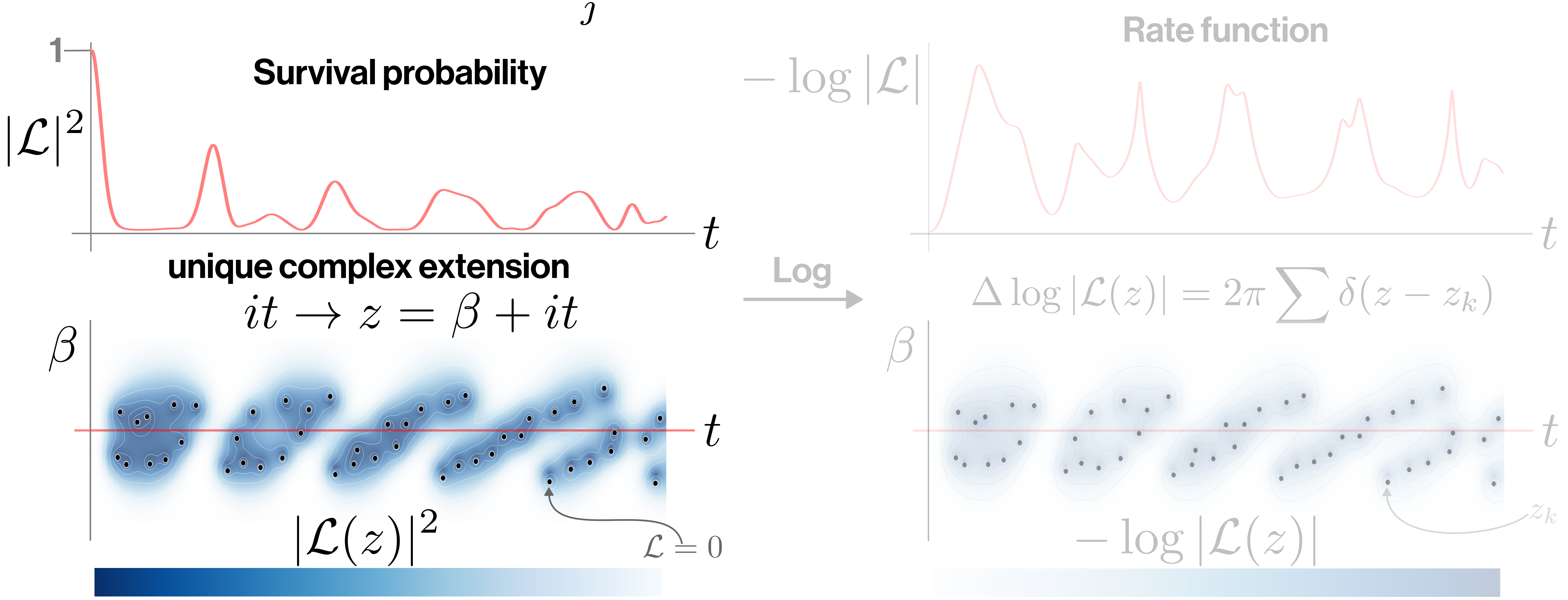

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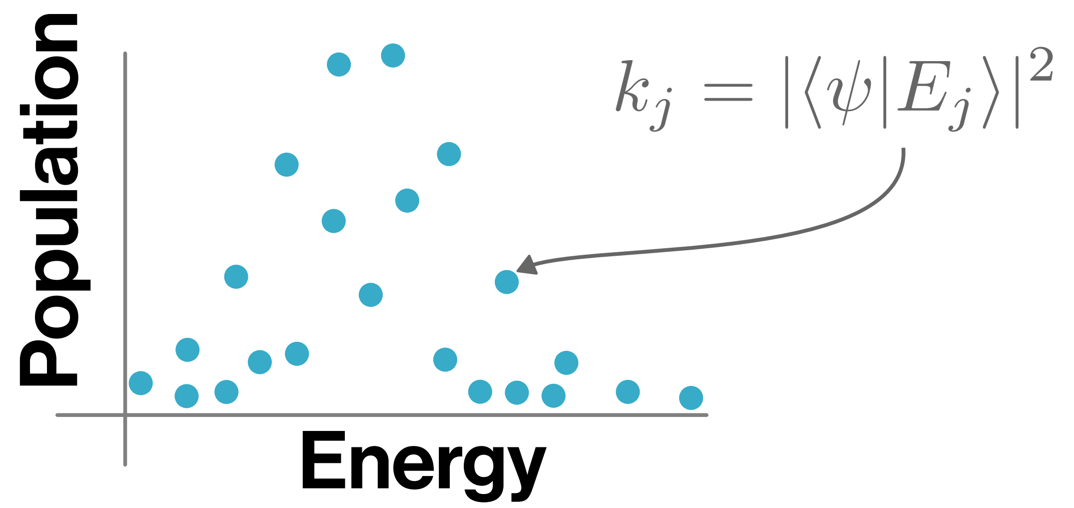

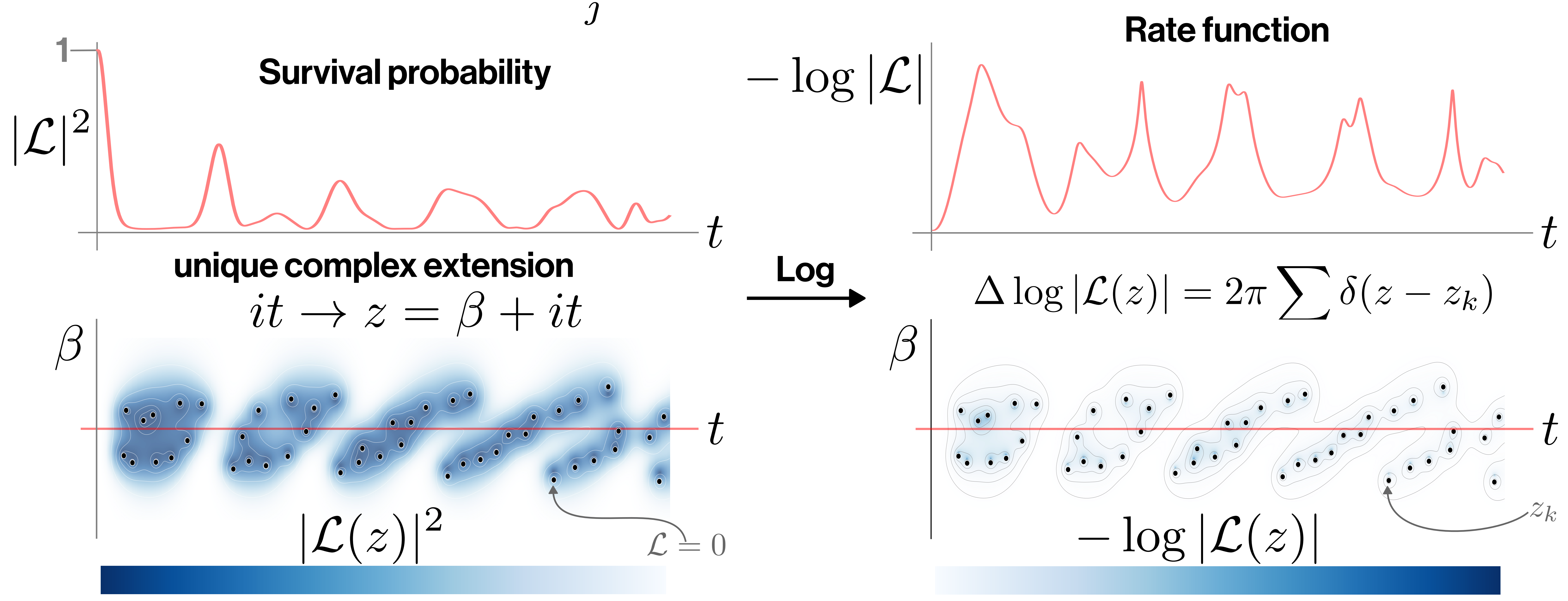

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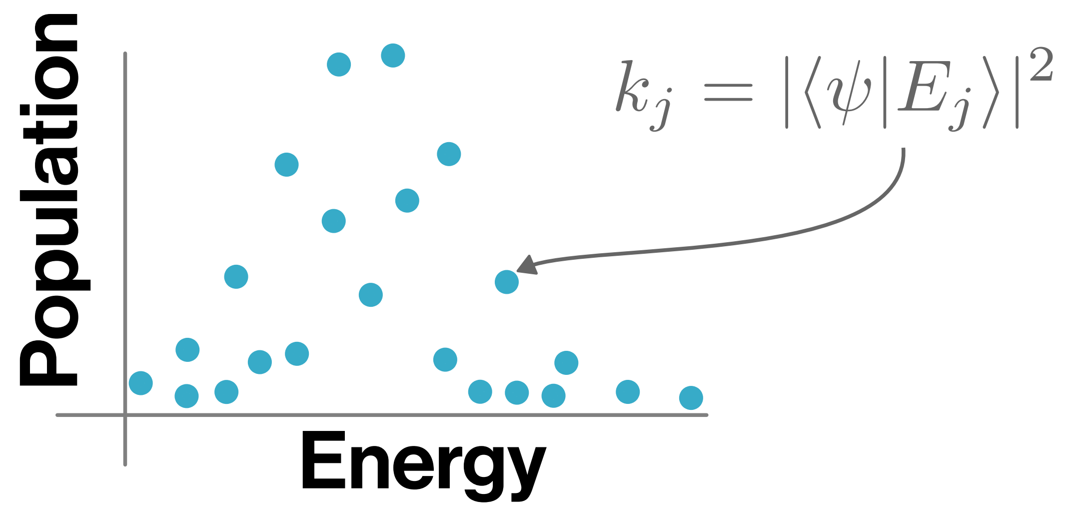
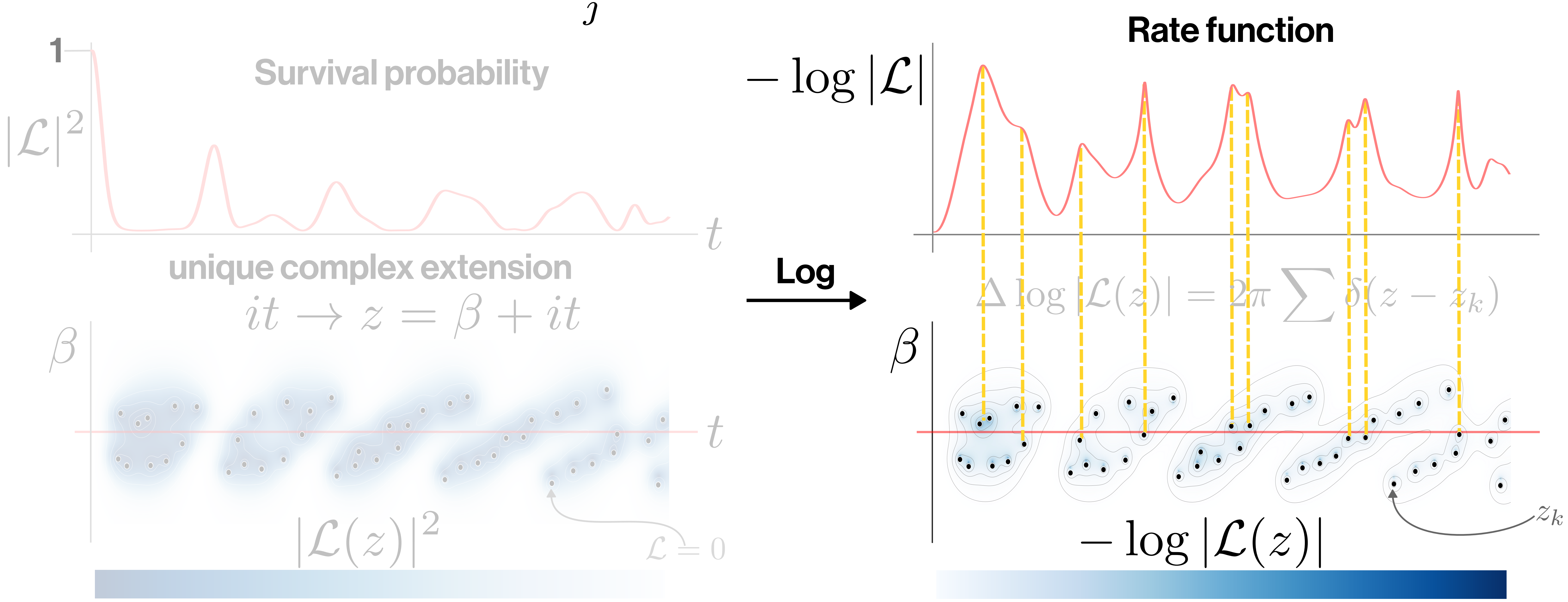


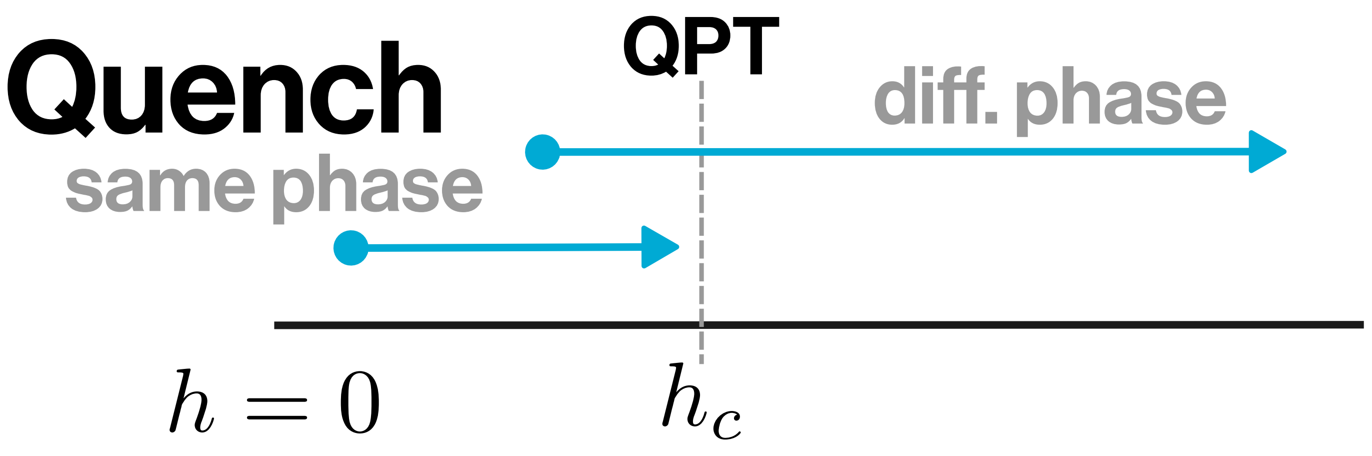


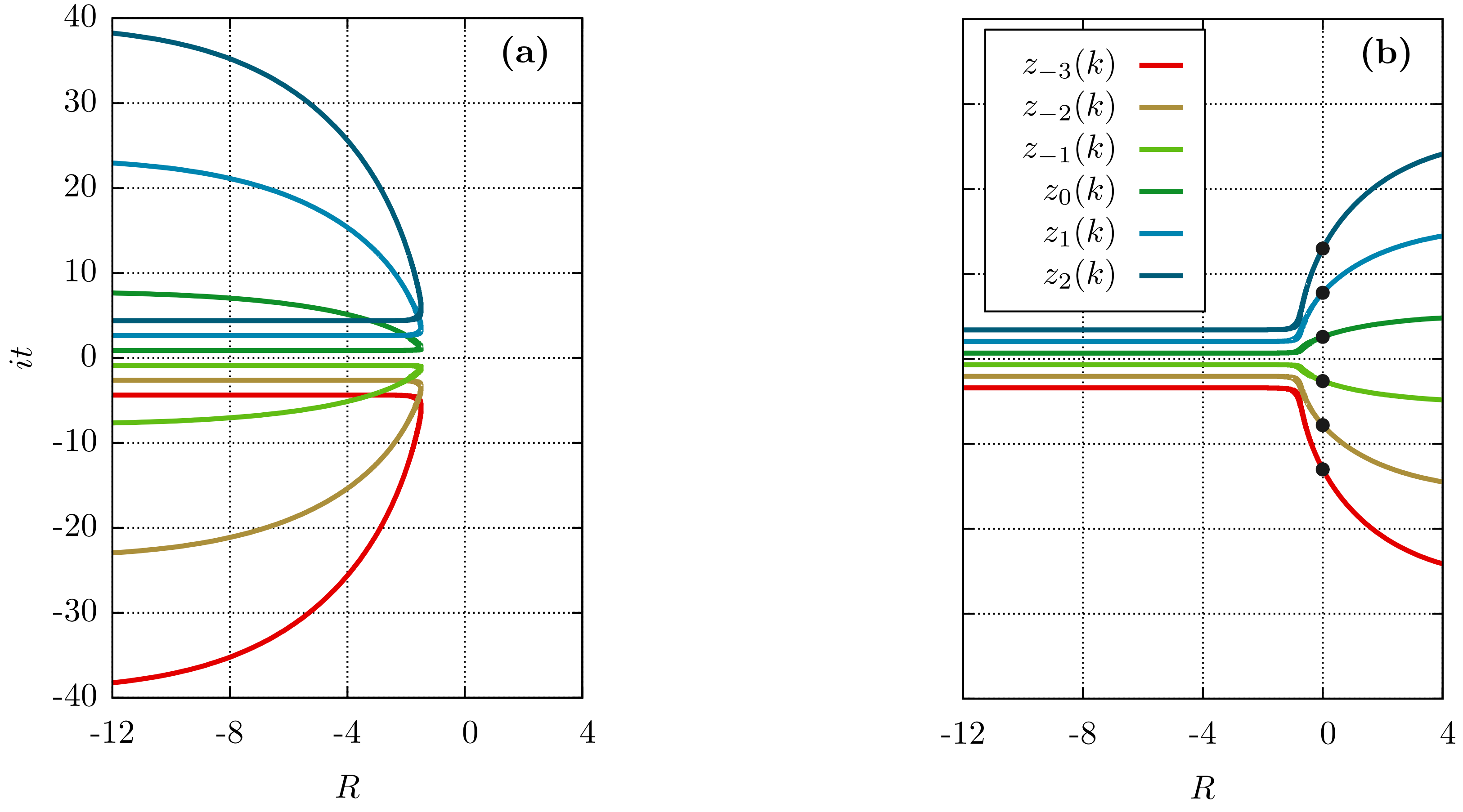
Why zeros?

Heyl et al.
PRL 110, 135704 (2013)

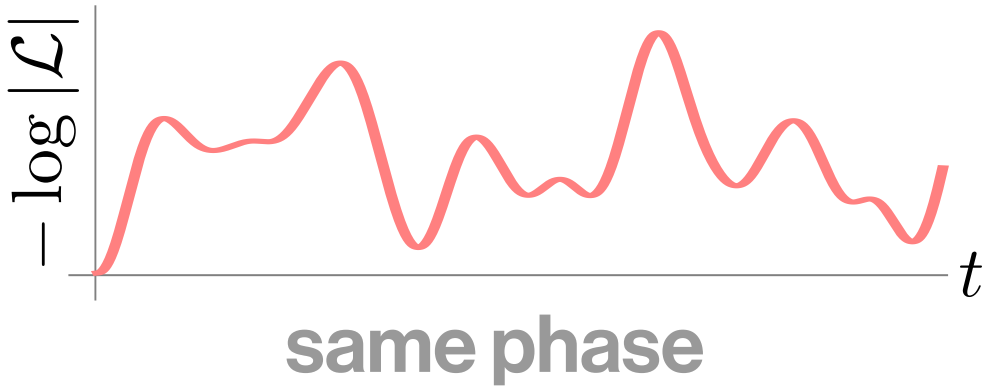
$$\hat{H} = -J\sum_{i=1}\hat{\sigma}_i^z\hat{\sigma}_{i+1}^z + h\sum_{i=1}\hat{\sigma}_i^x$$

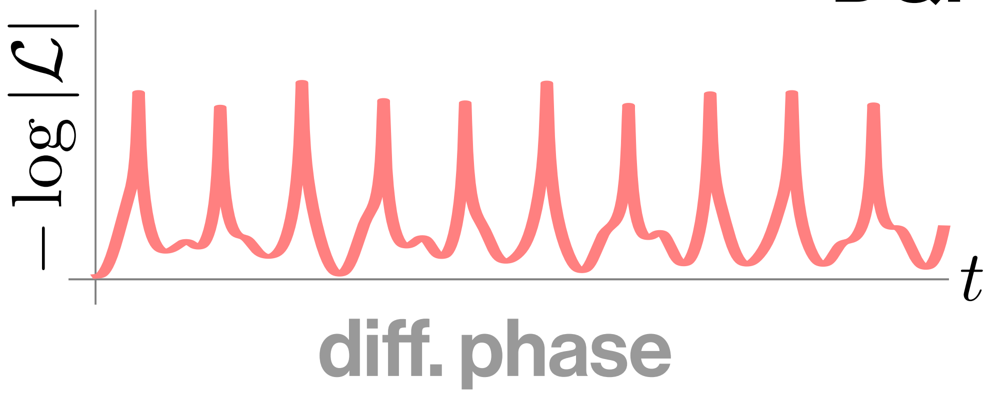
Quench





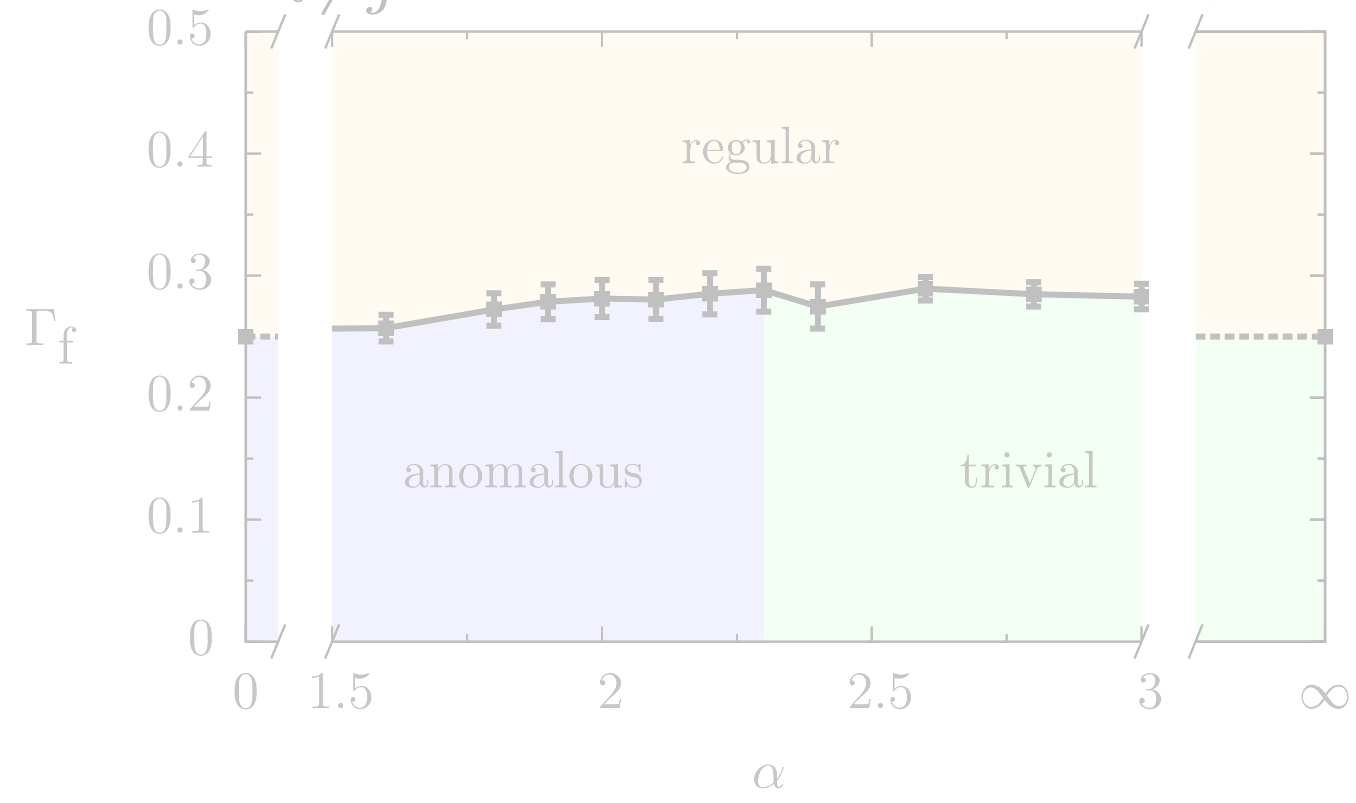
$N \to \infty$
DQPT (-II)

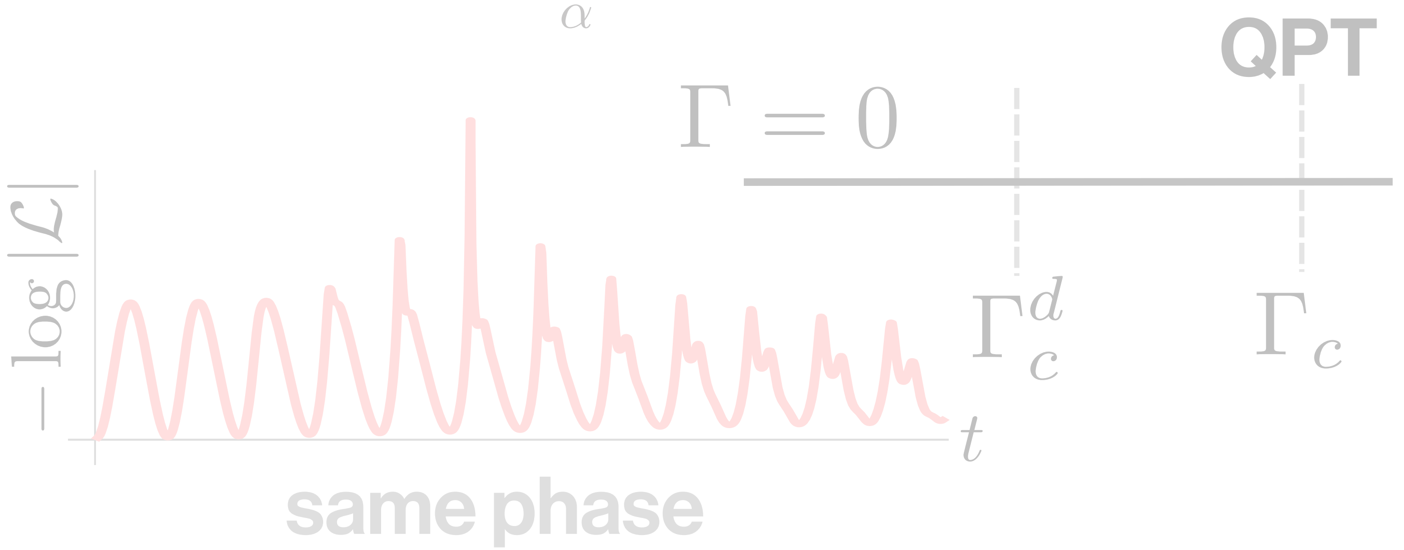




Homrighausen et al.
PRB 96, 104436 (2017)

$$\hat{H} = -\frac{J}{2N}\sum_{i\neq j}\frac{1}{|i-j^{\alpha}|}\hat{S}_i^z\hat{S}_{i+1}^z + \Gamma\sum_{i=1}\hat{S}_i^x$$





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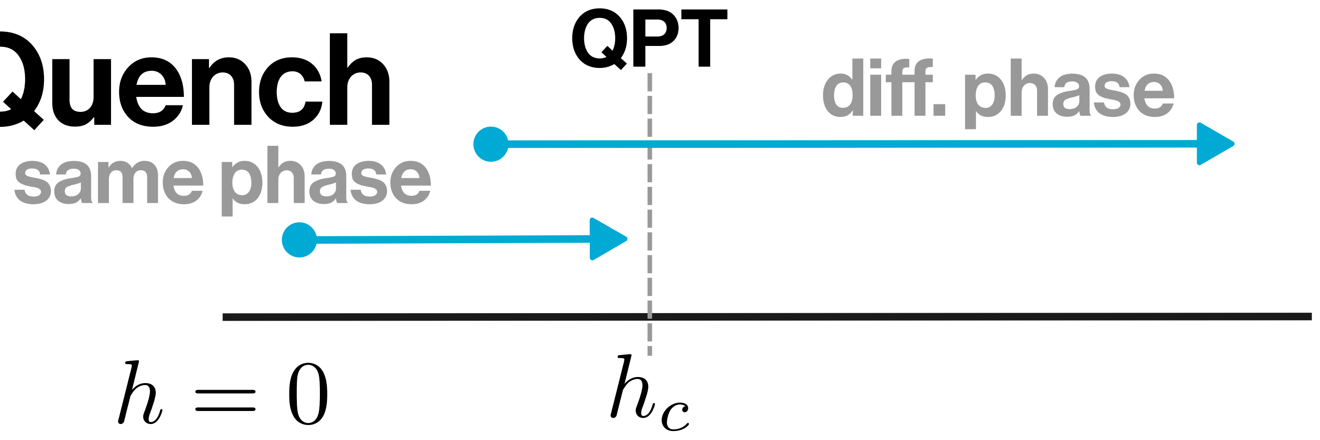

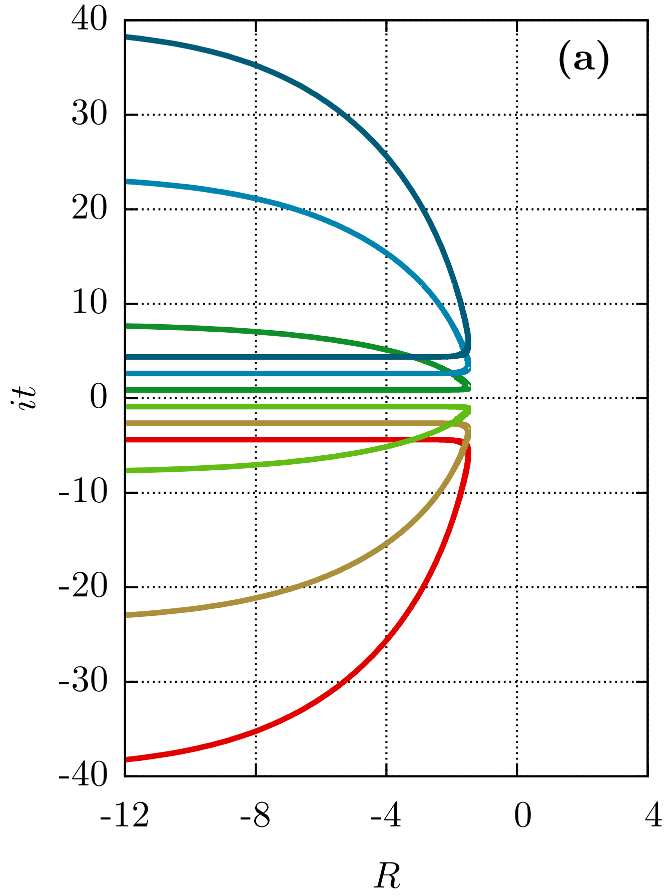

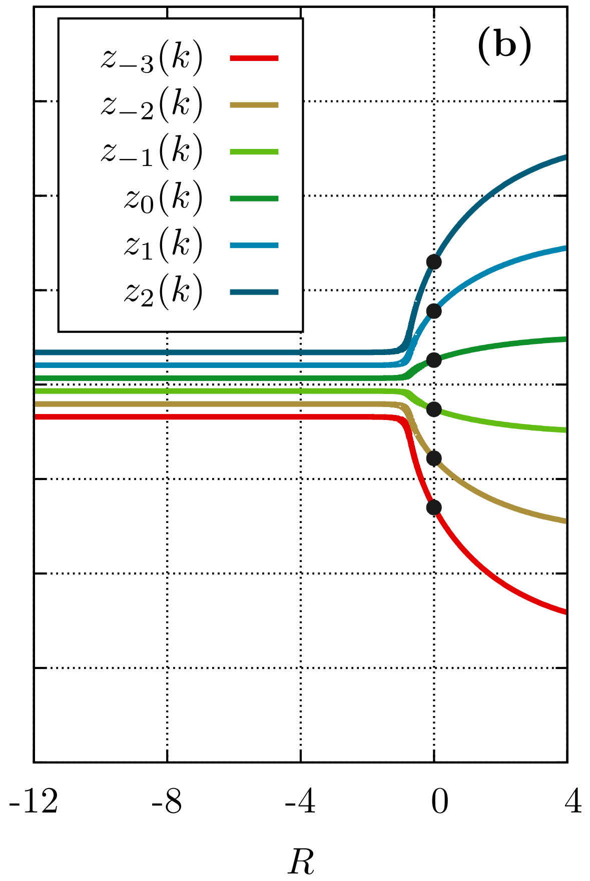

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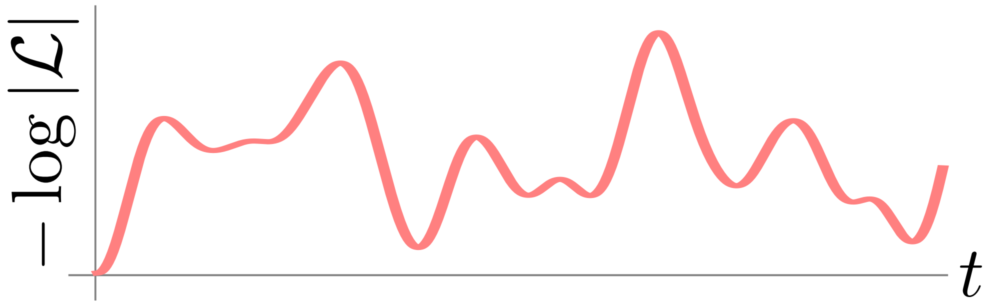
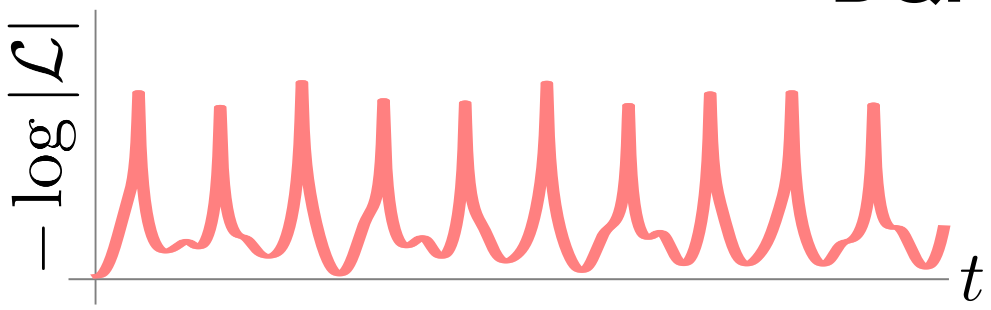


same phase



diff. phase

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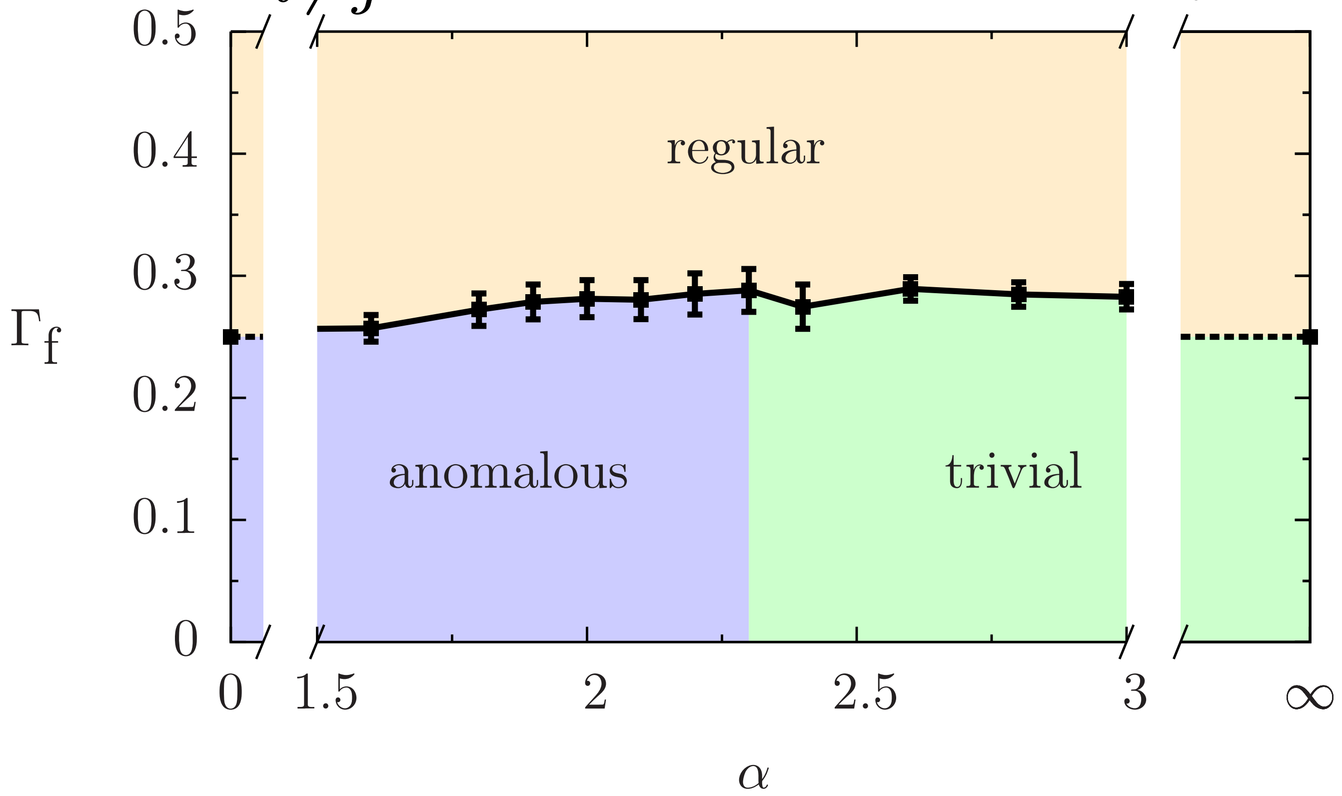

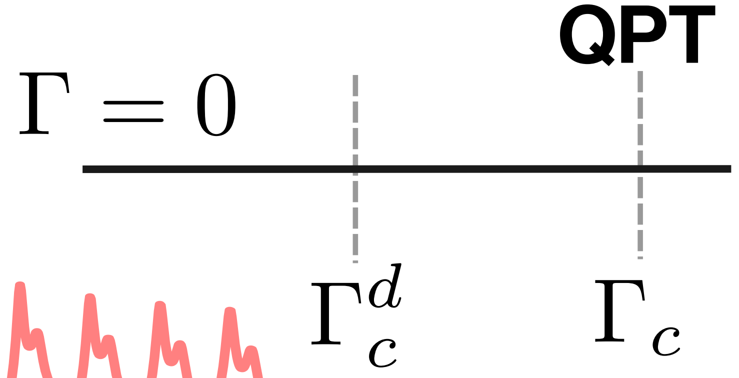

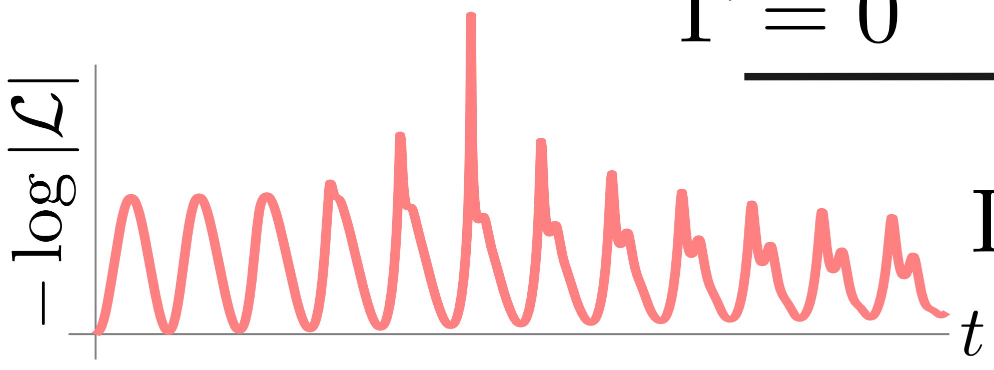

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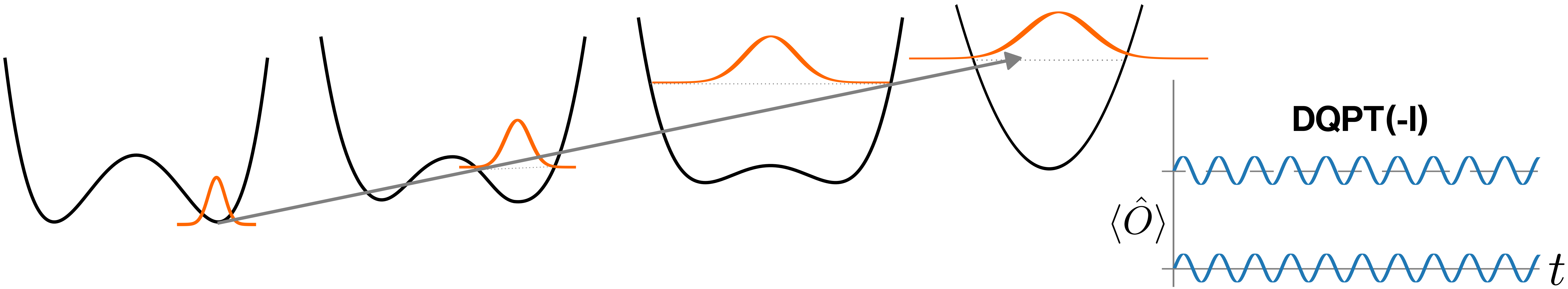
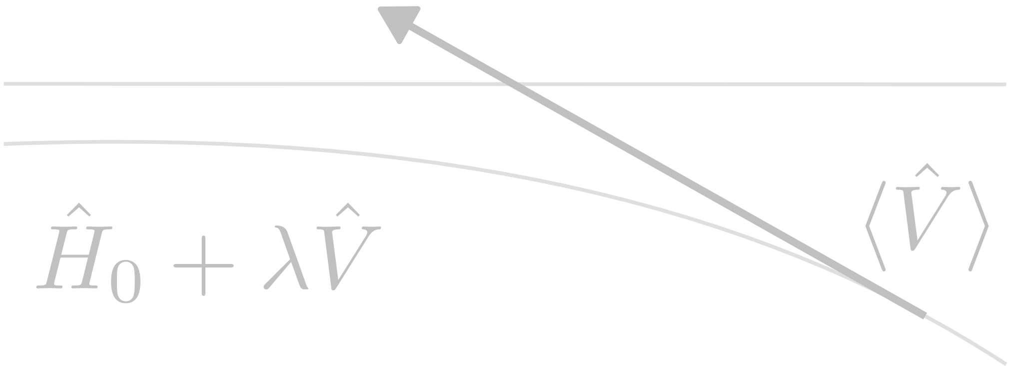
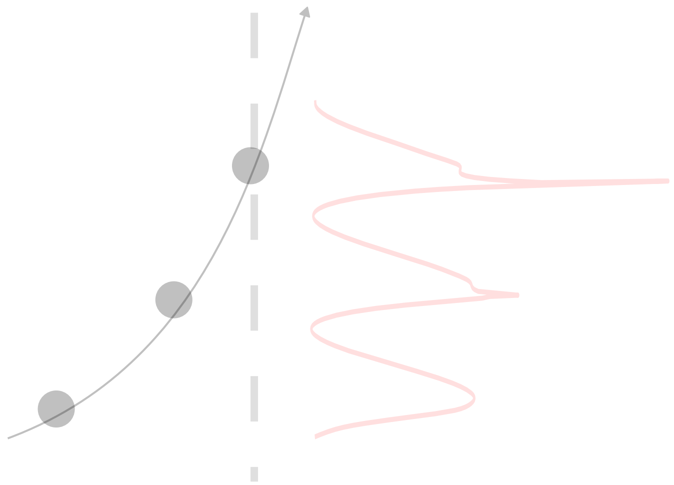
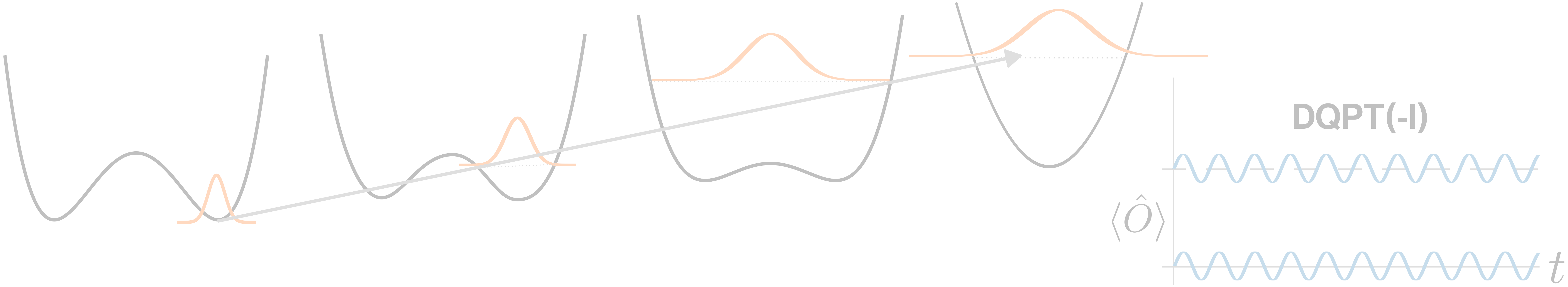


What determines the distribution of the zeros?







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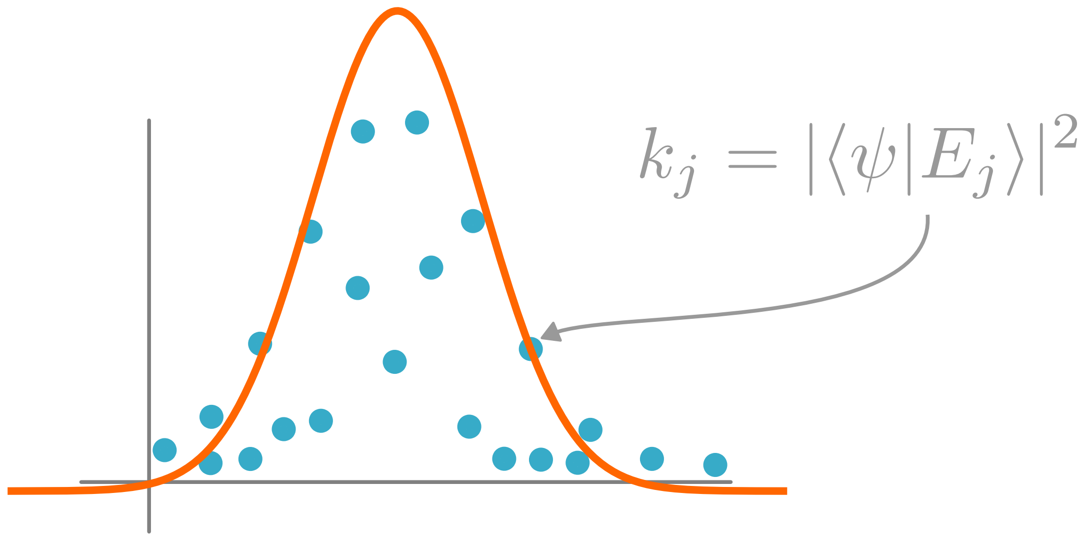
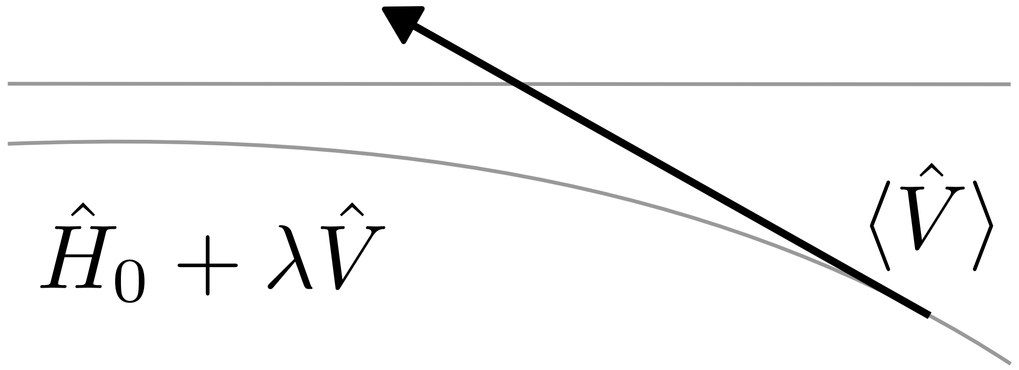
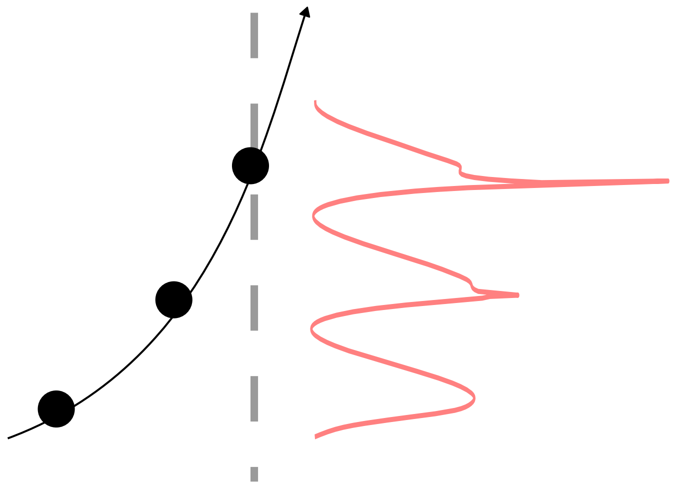


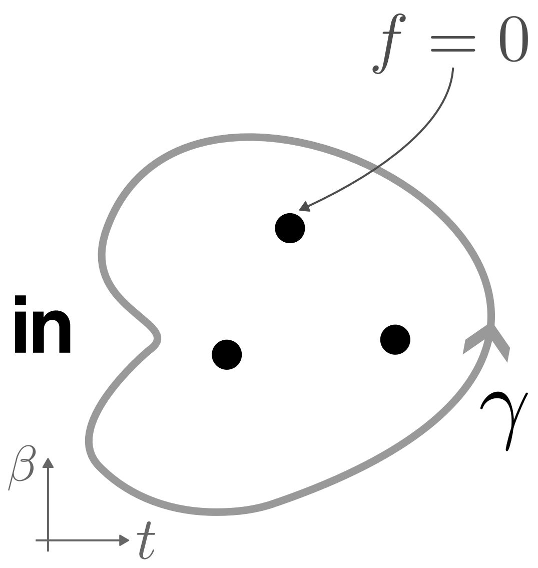




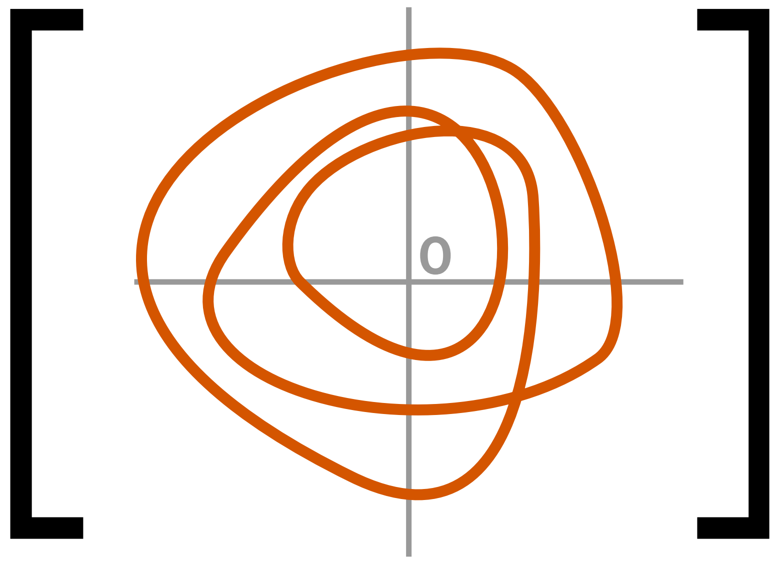
Zeros of holomorphic functions

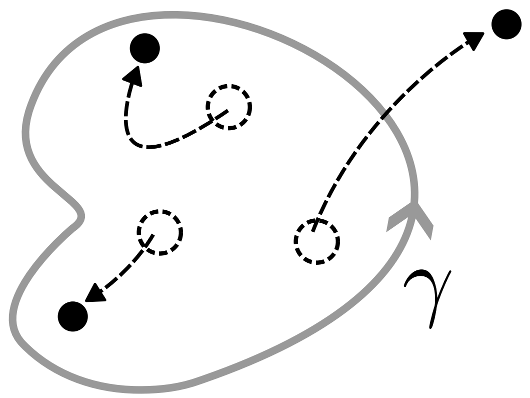
Zeros cannot be created or annihilated by a hol. perturbation

zeros of $f(z)$ in

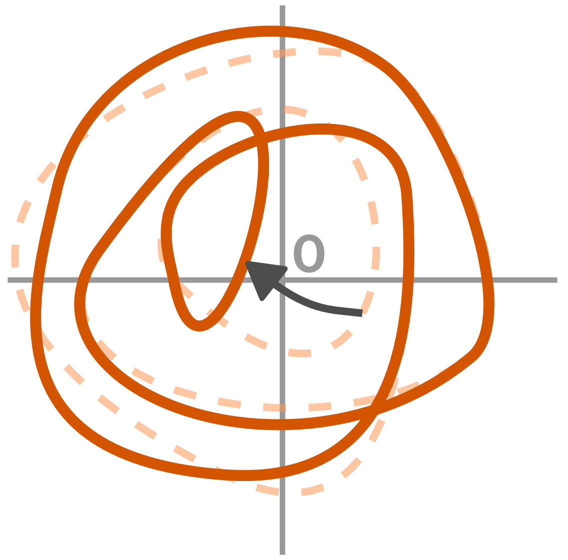


= winding number $W[f \circ \gamma]$ = W



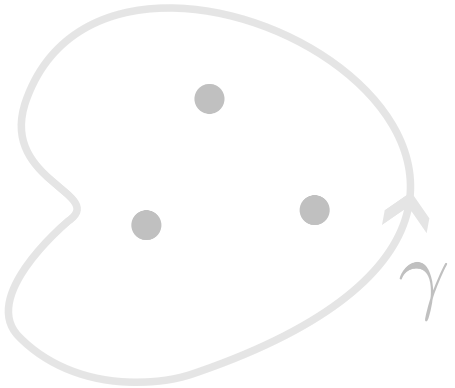


$f + g = 0$



Num. of zeros is determined by the dominant part

zeros in



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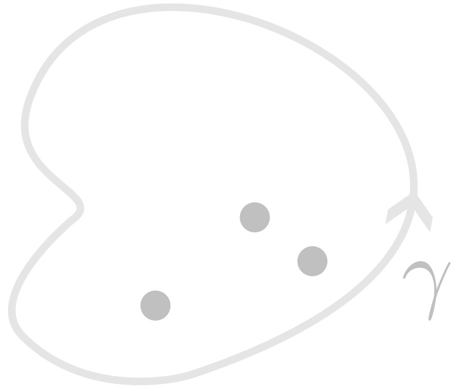

$f(z) + g(z)$

if $g < f$ on γ

Rouché's theorem

Zeros of holomorphic functions

Zeros cannot be created or annihilated by a hol. perturbation

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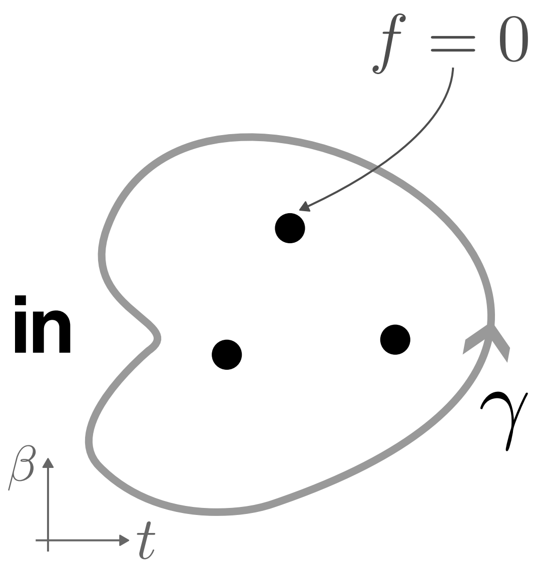

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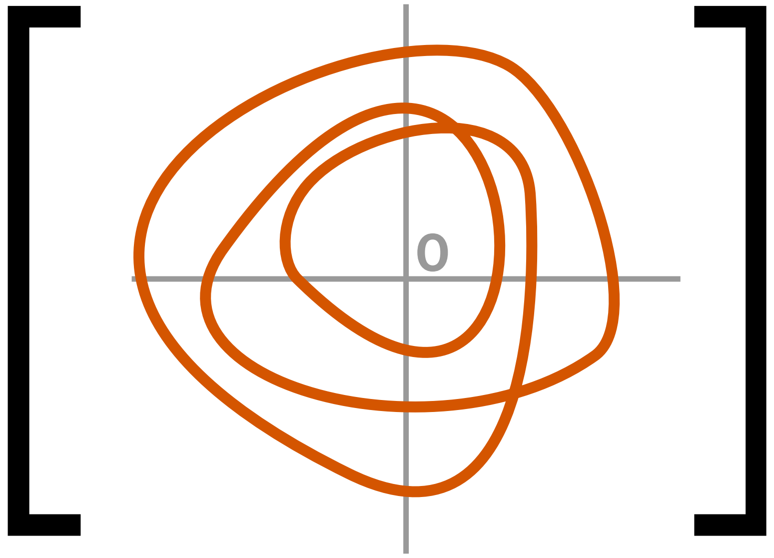

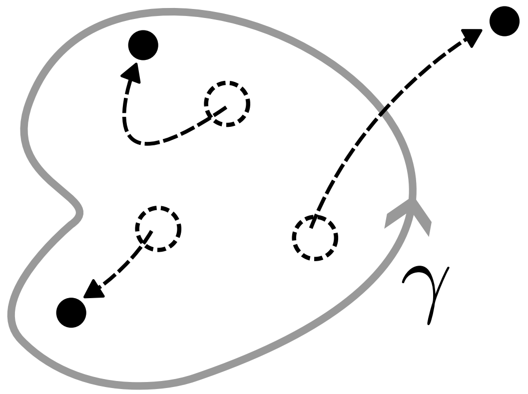

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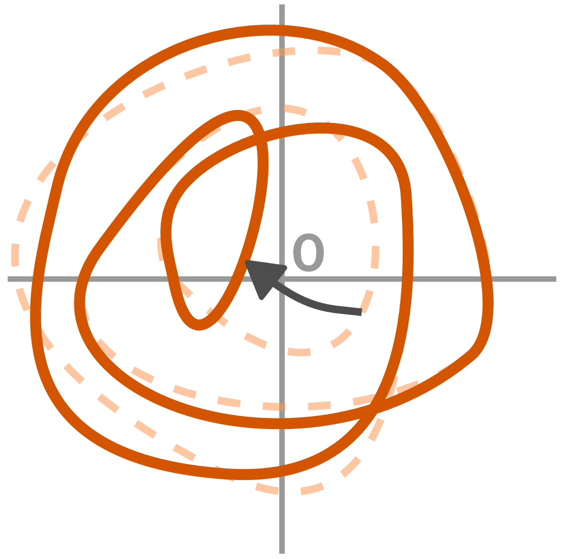

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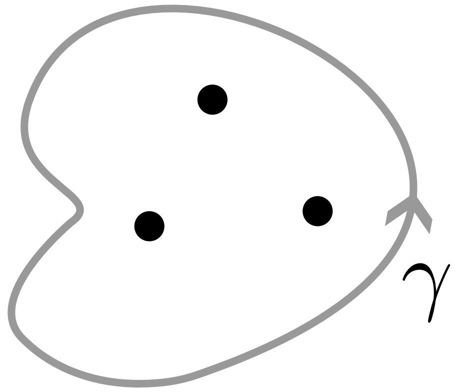

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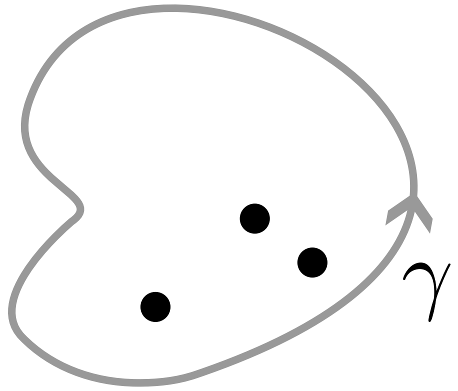


$f(z) + g(z)$

if $g < f$ on γ

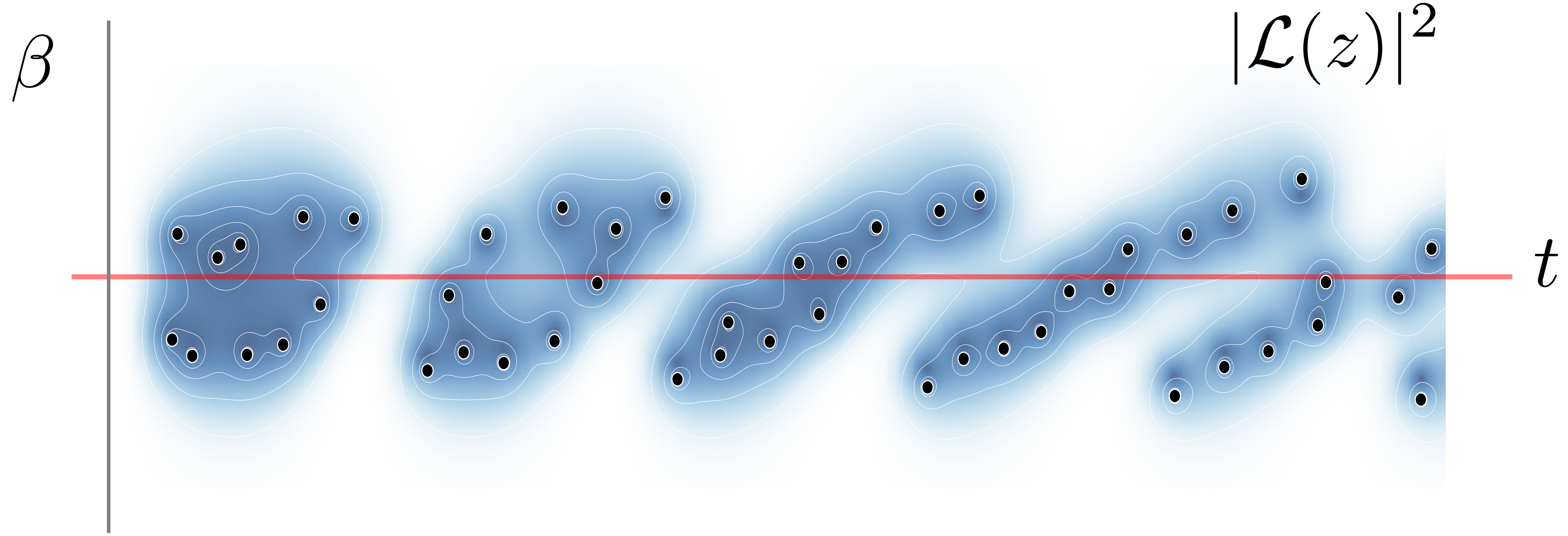
Rouché's theorem

Different time scales

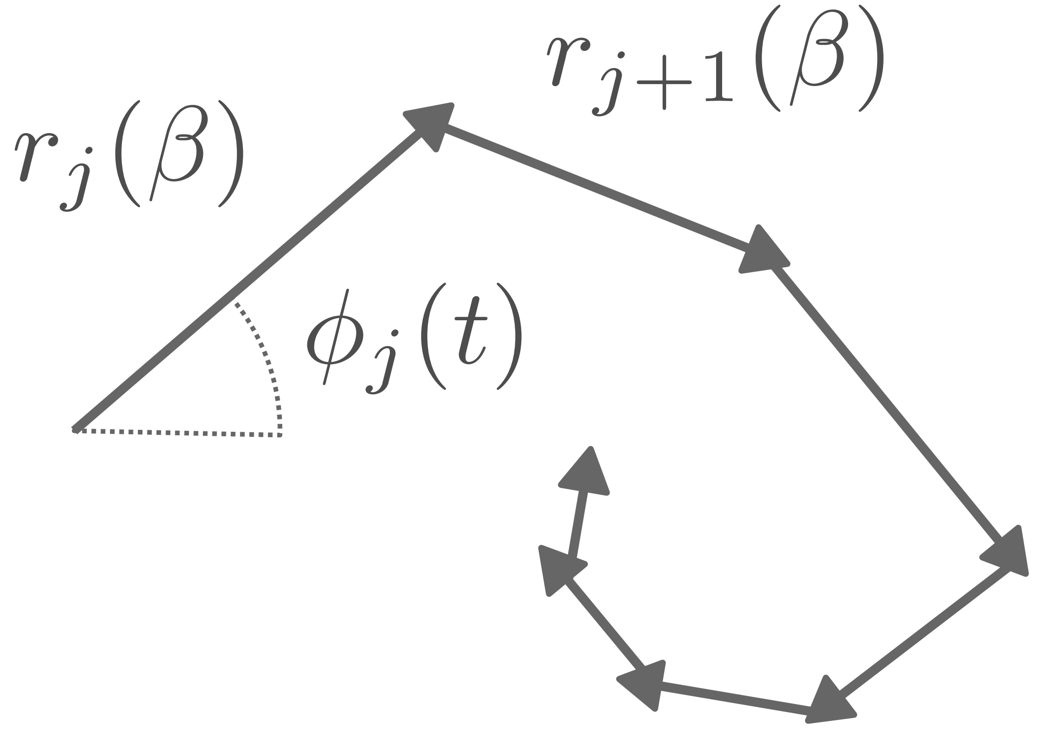
Level spacing
$\Delta_i = \Delta + \varepsilon_i$

→ **Dephasing**

$$\mathcal{L}(\beta + it) = \sum_j \underbrace{k_j e^{-\beta E_j}}_{r_j(\beta)} e^{-iE_j t}$$



Short times **Long times**

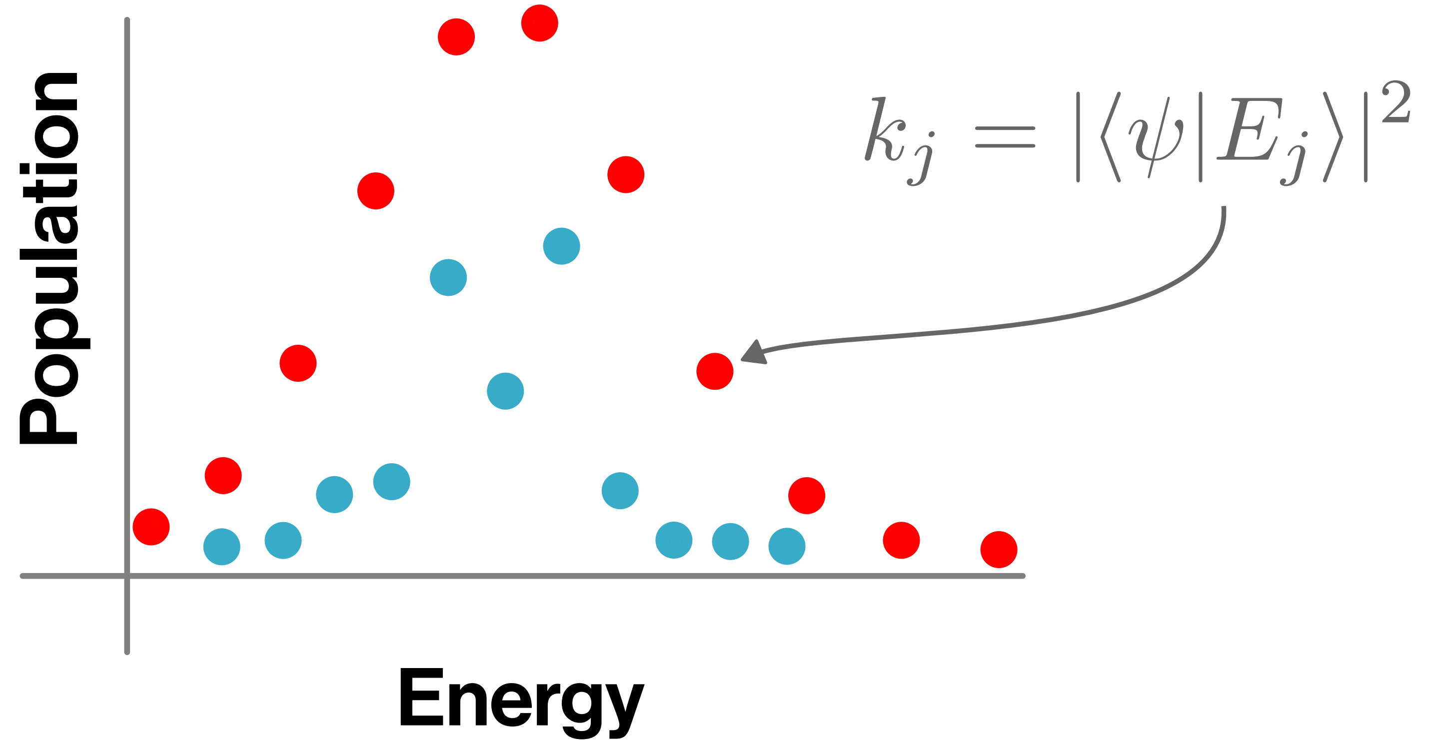


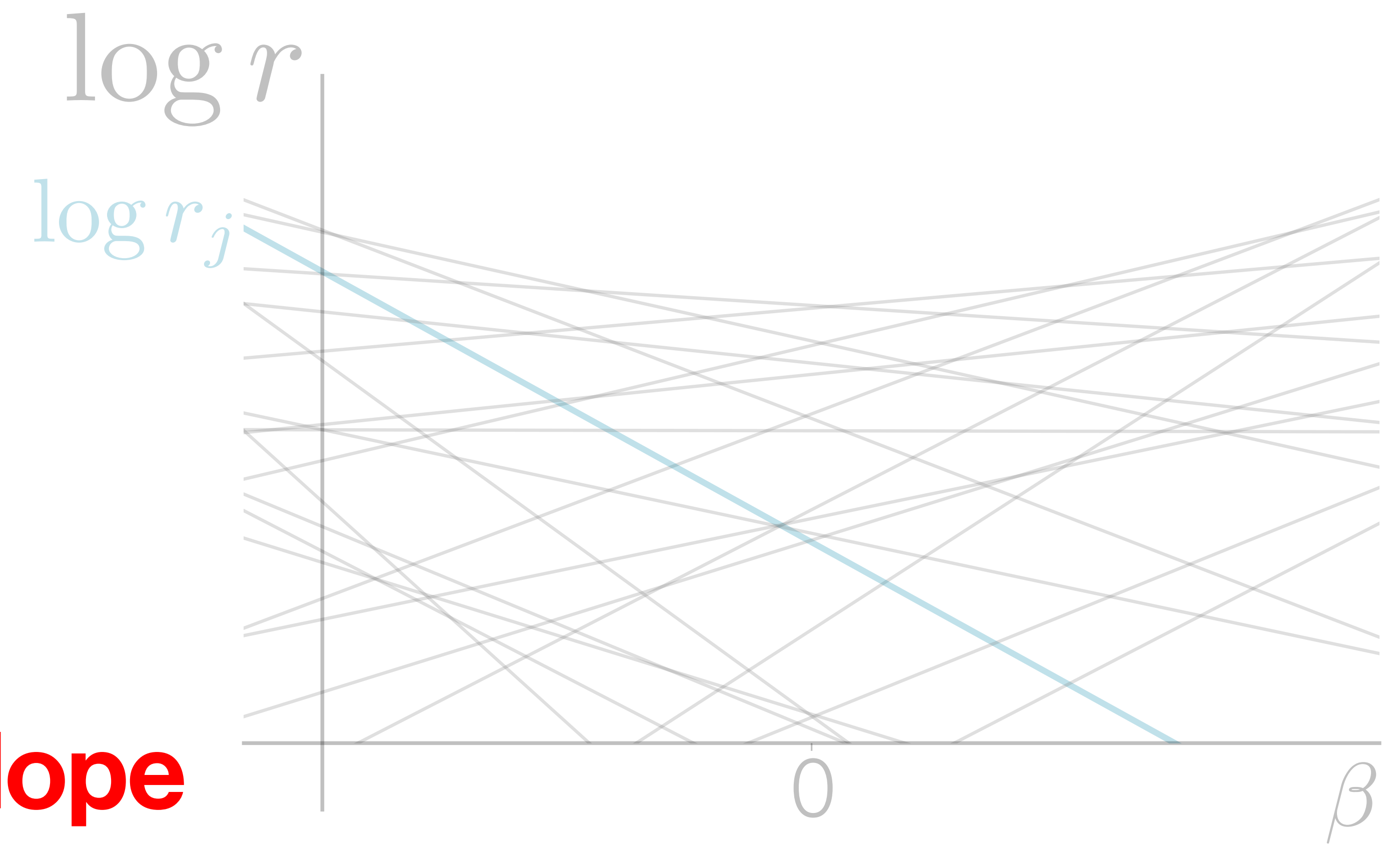
Long times

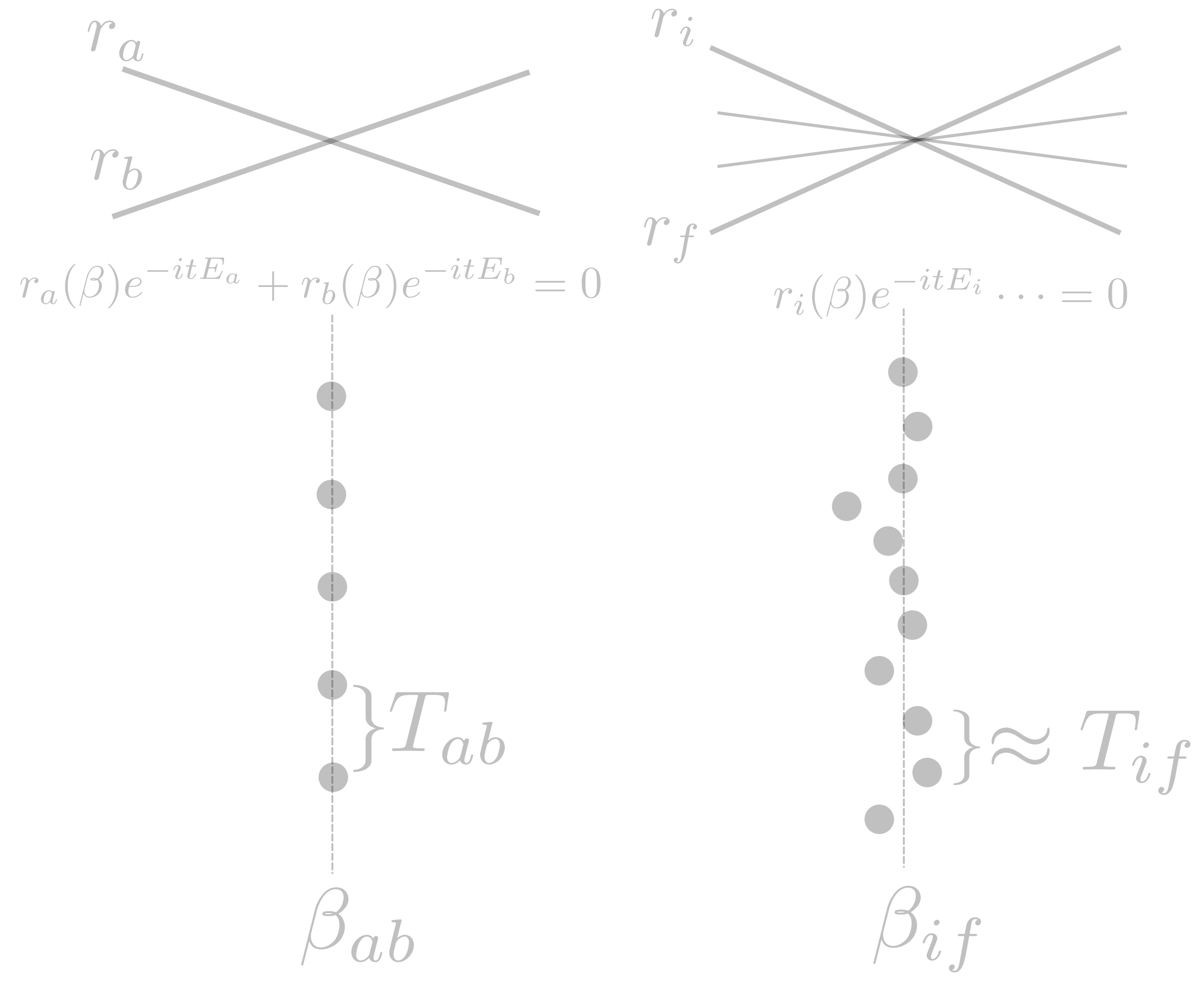
$$\mathcal{L}(\beta + it) = \sum_j r_j(\beta) e^{-iE_j t}$$

The dominant part = The Envelope

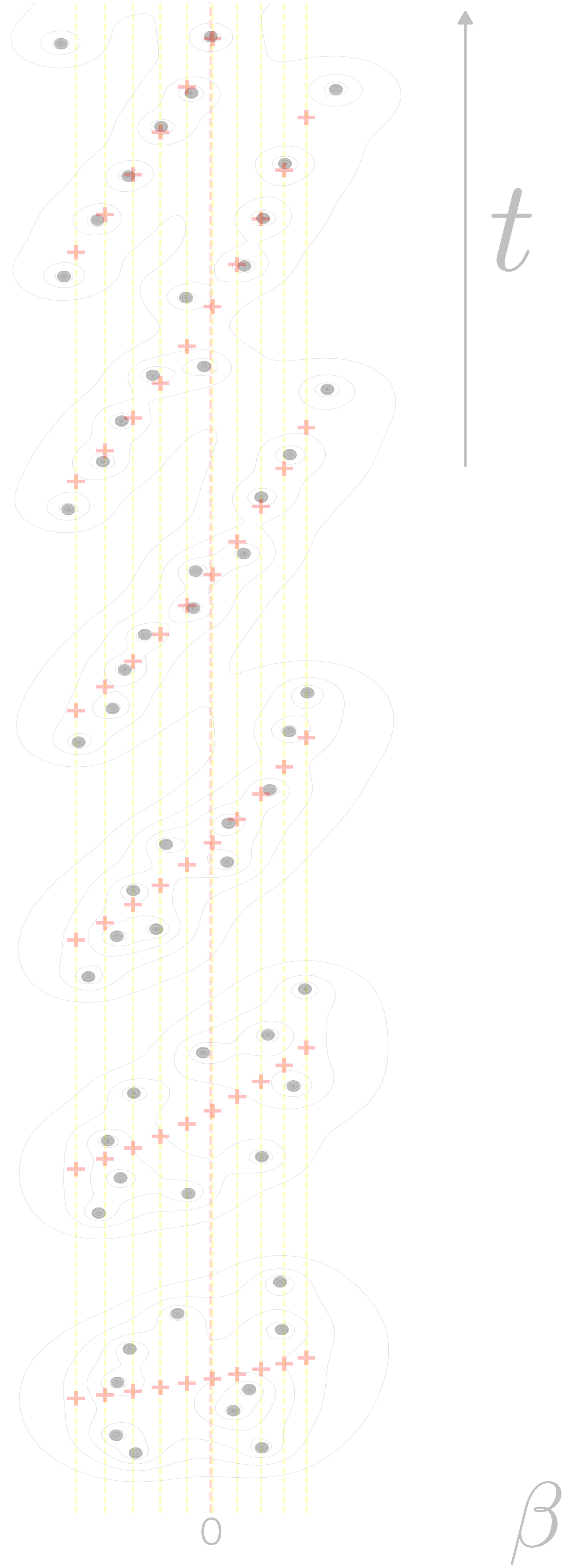
$$k_a \left(\frac{k_a}{k_b}\right)^{-\frac{\Delta_{ac}}{\Delta_{ab}}} \geq k_c, \quad \forall k_c, E_c$$







$$z^*_{ab}(n) = \frac{1}{\Delta_{ab}}\left(\log\left(\frac{k_a}{k_b}\right) + i\pi\,(2n+1)\right)$$



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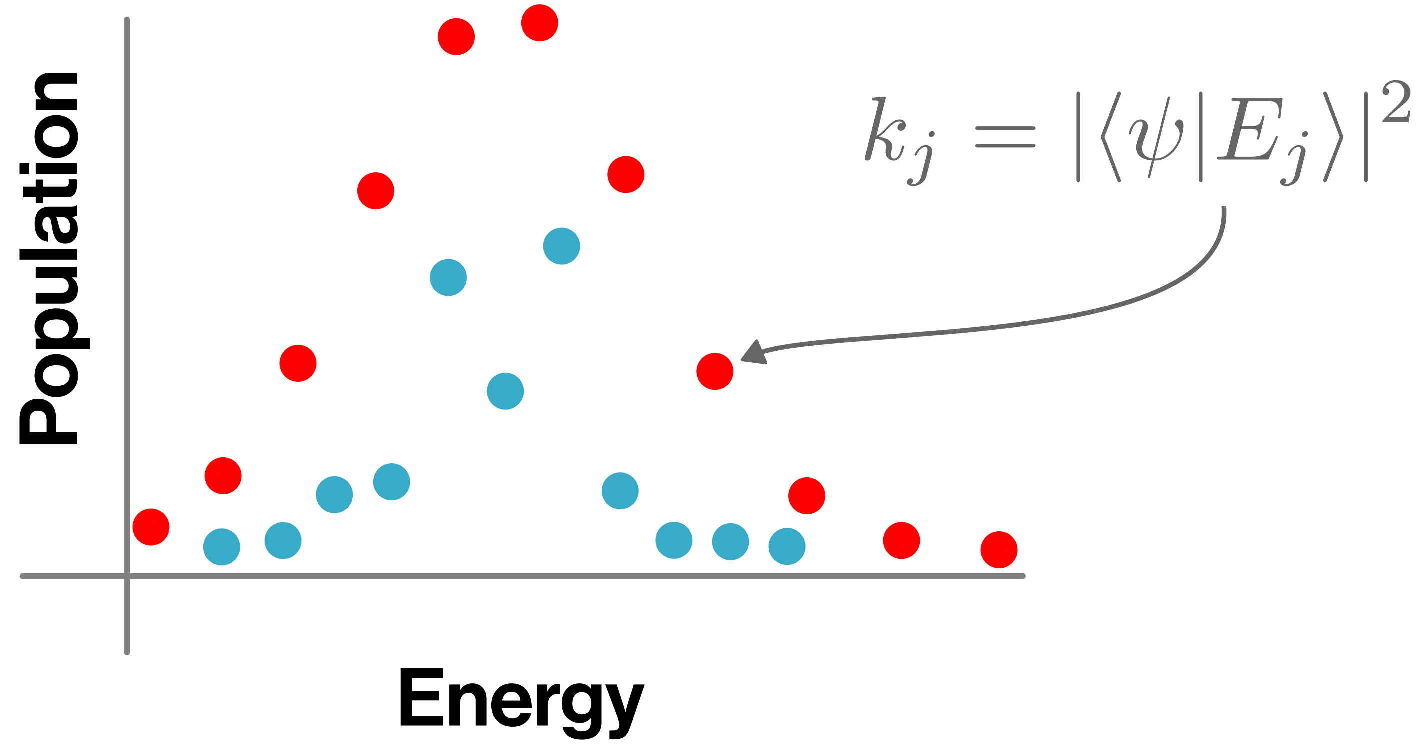

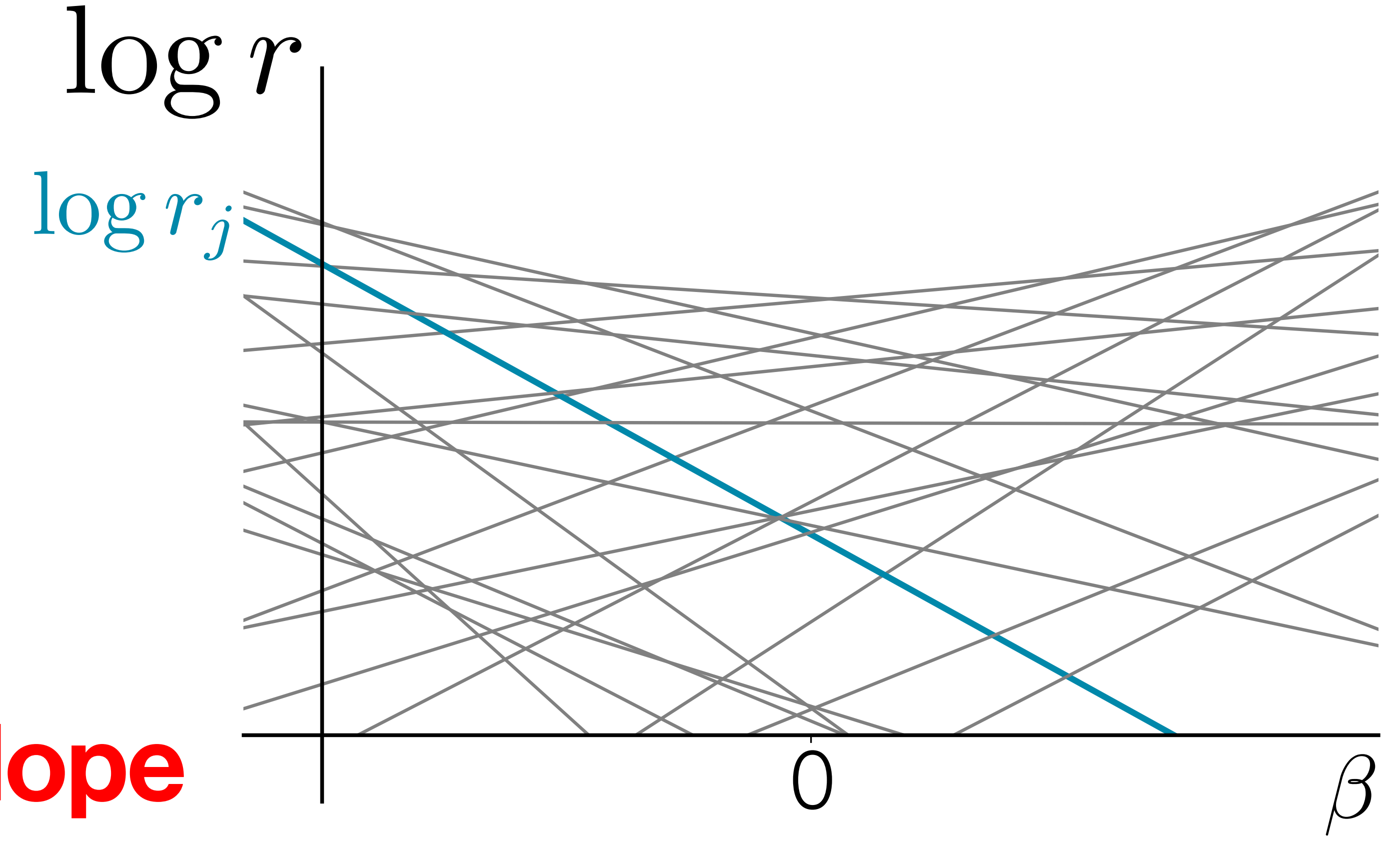

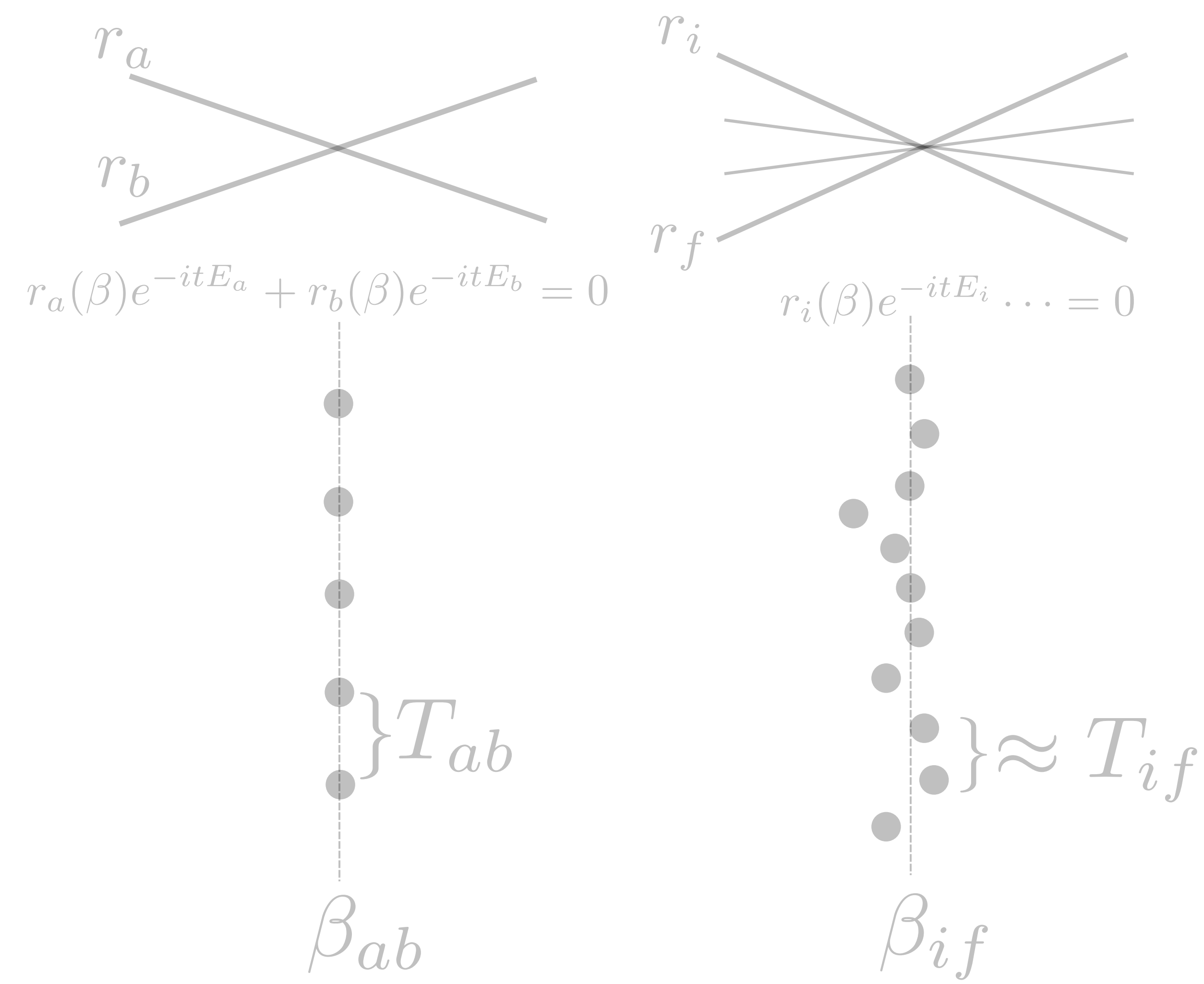

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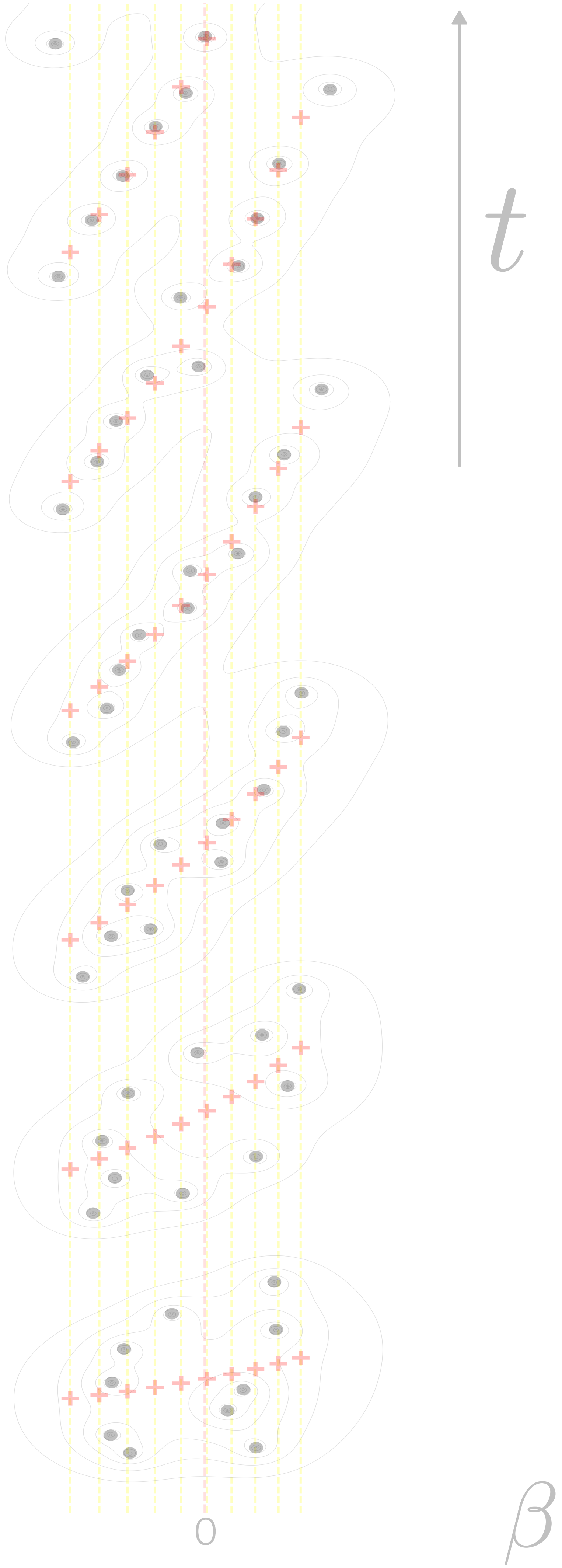

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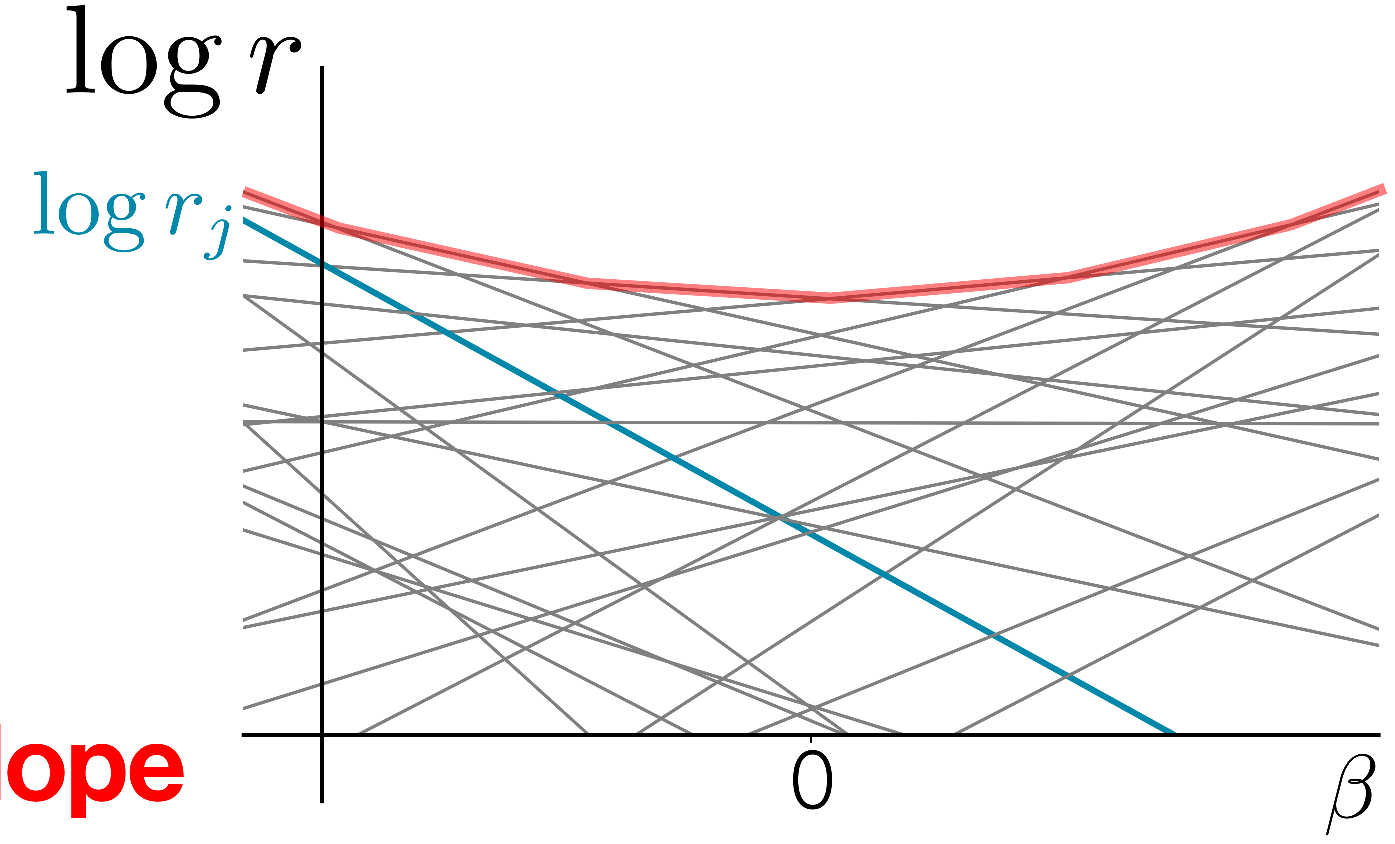

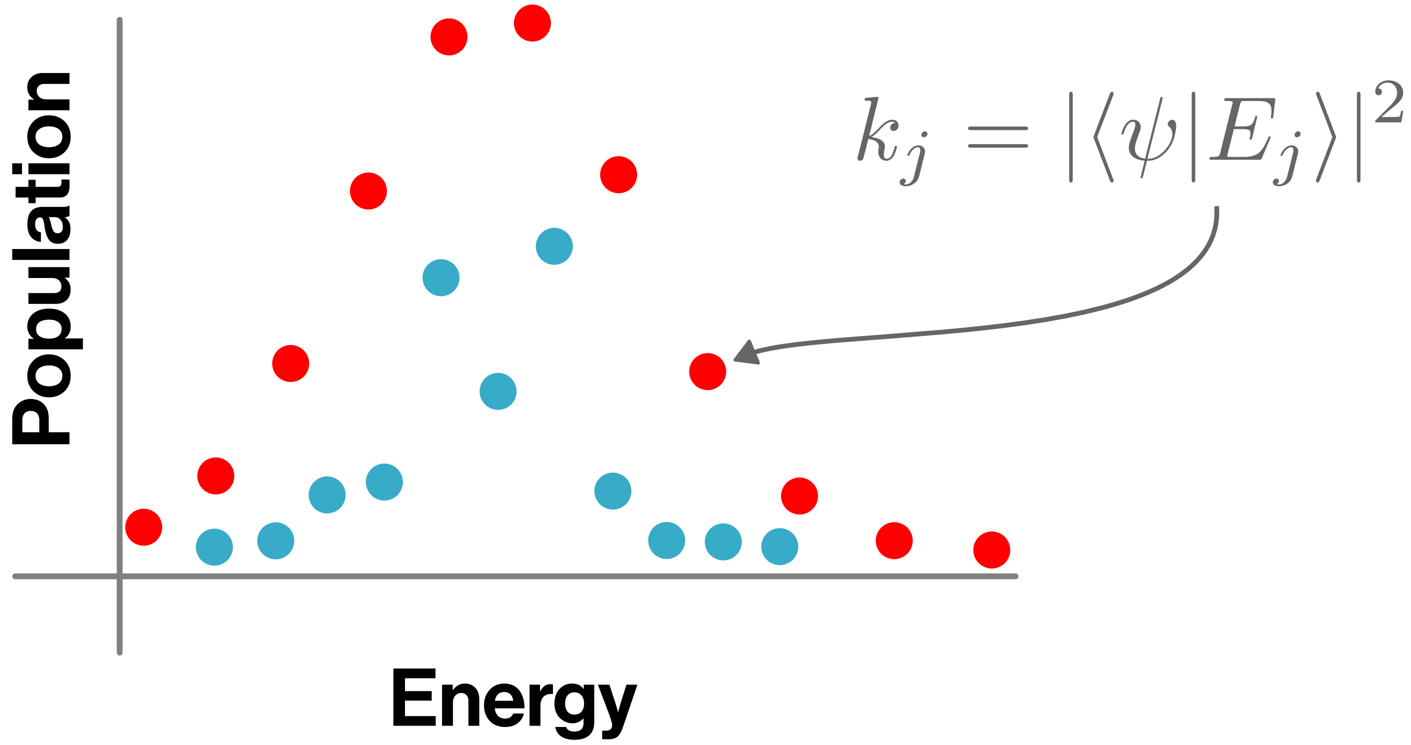

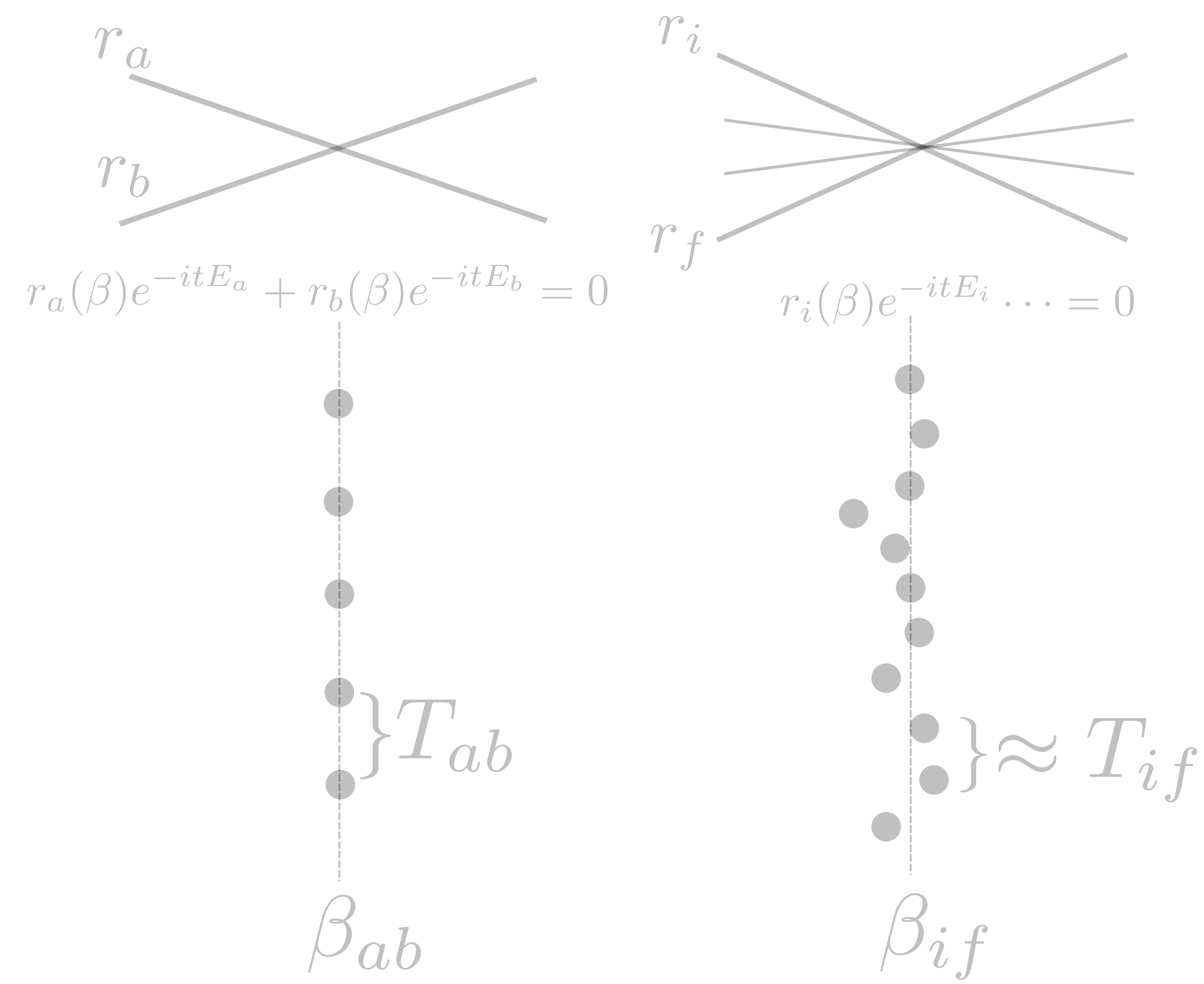

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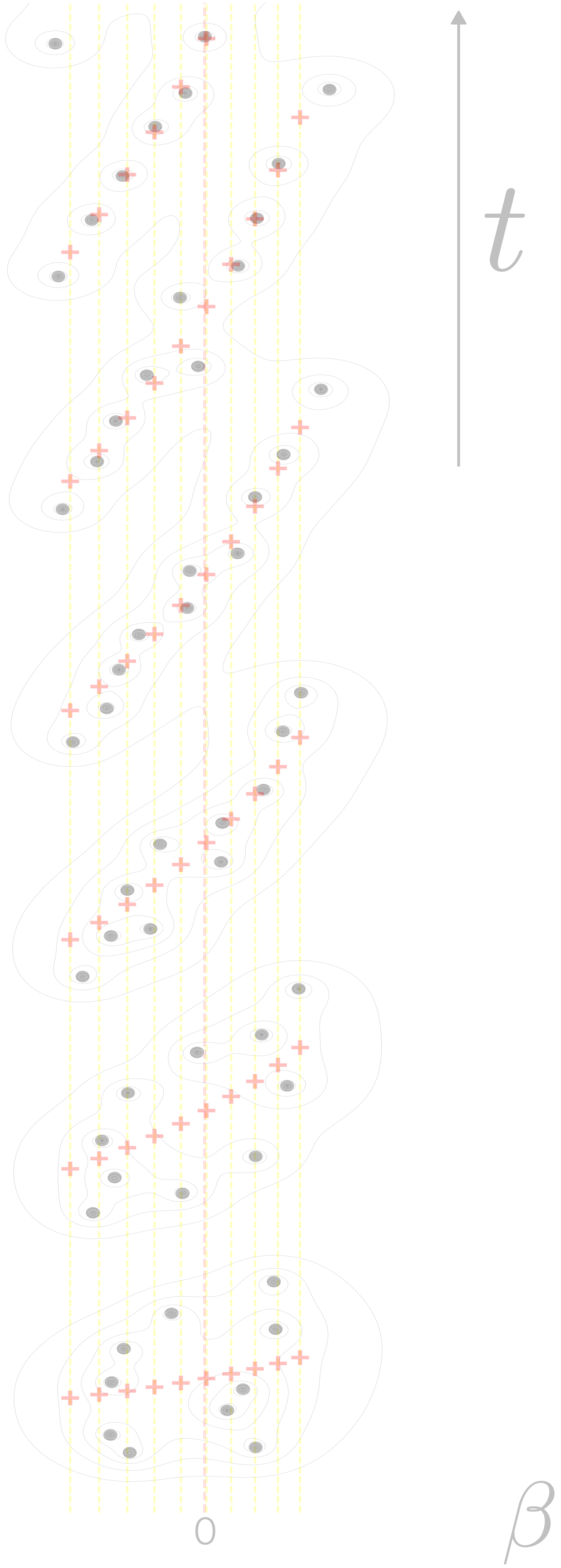

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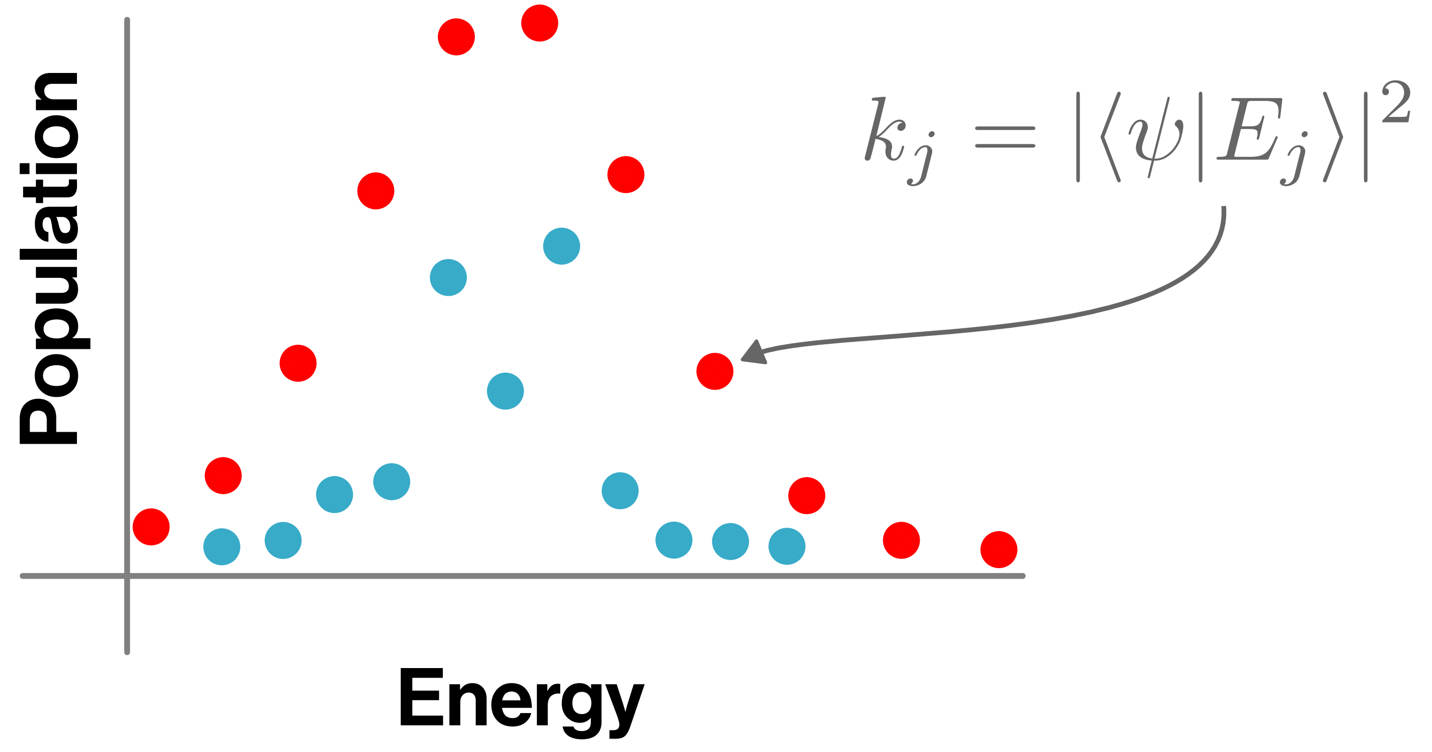

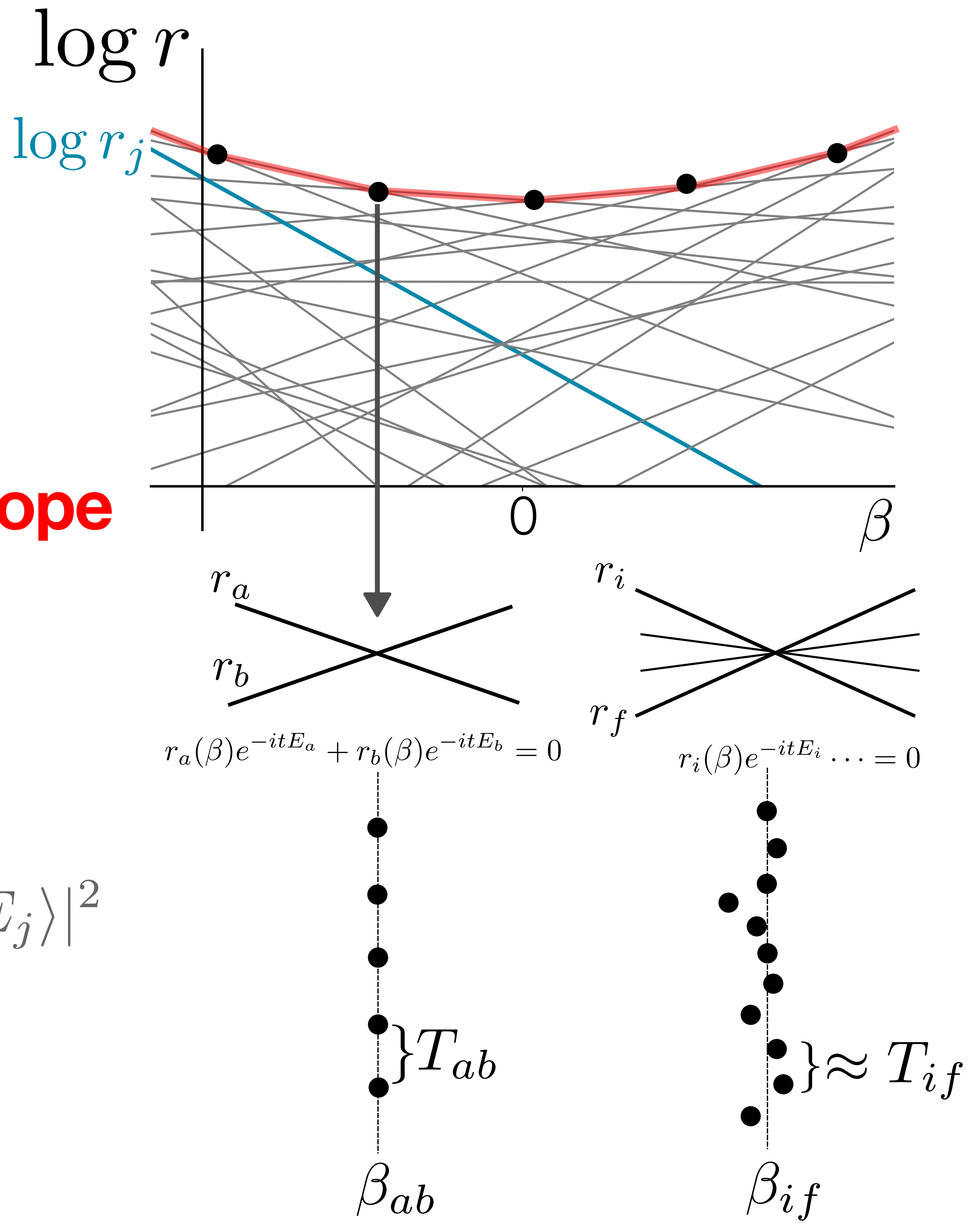

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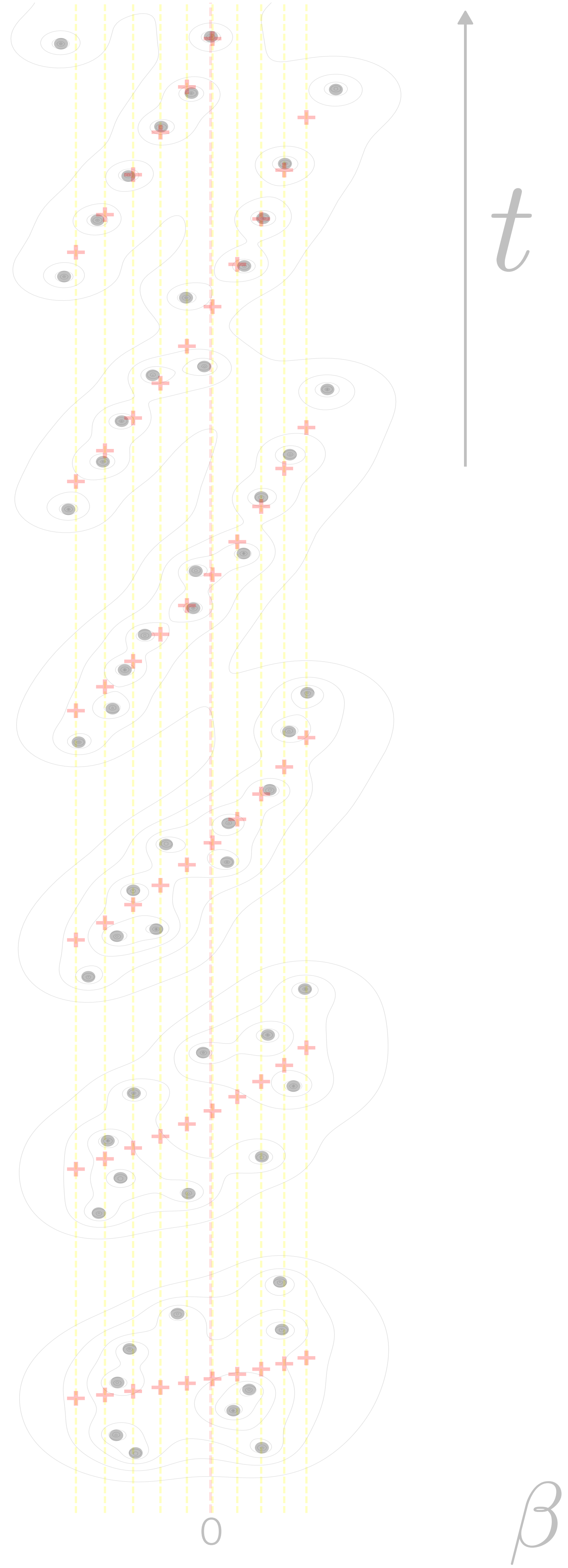

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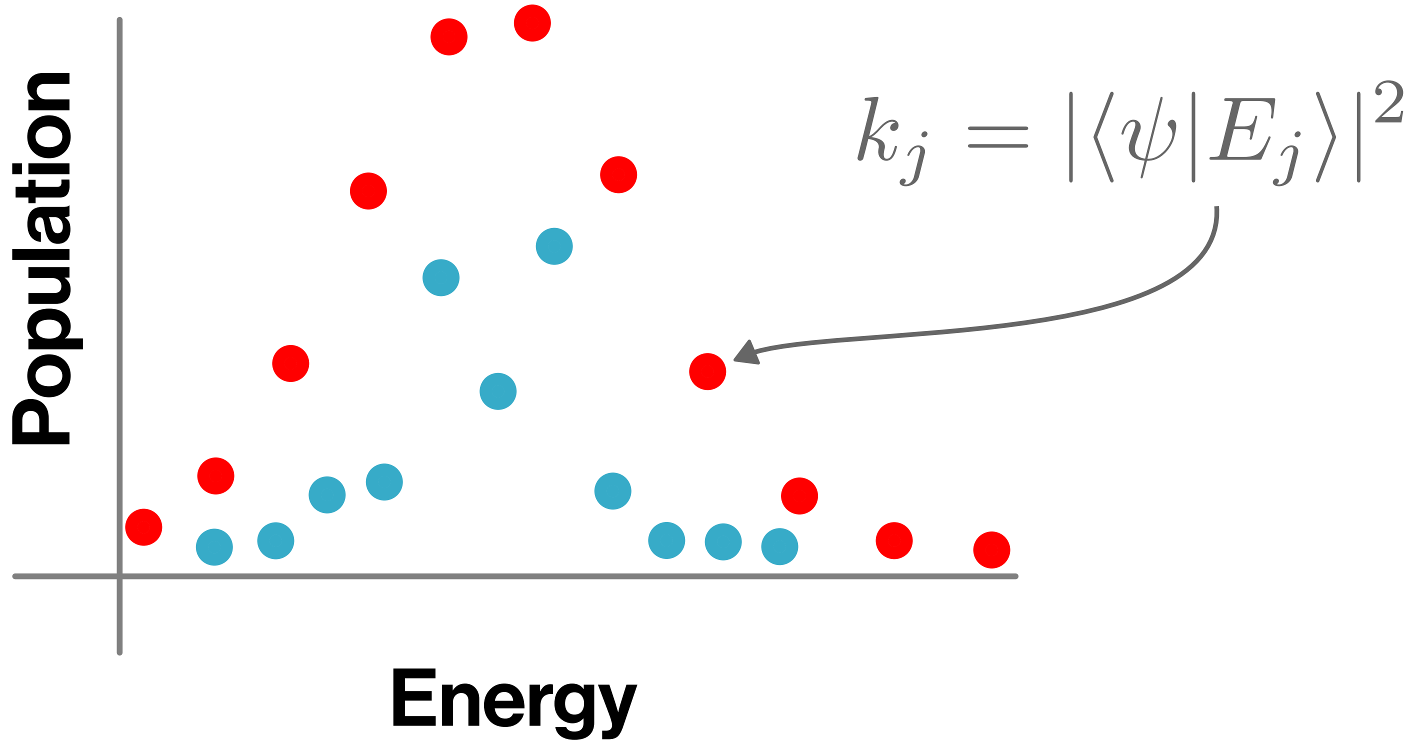

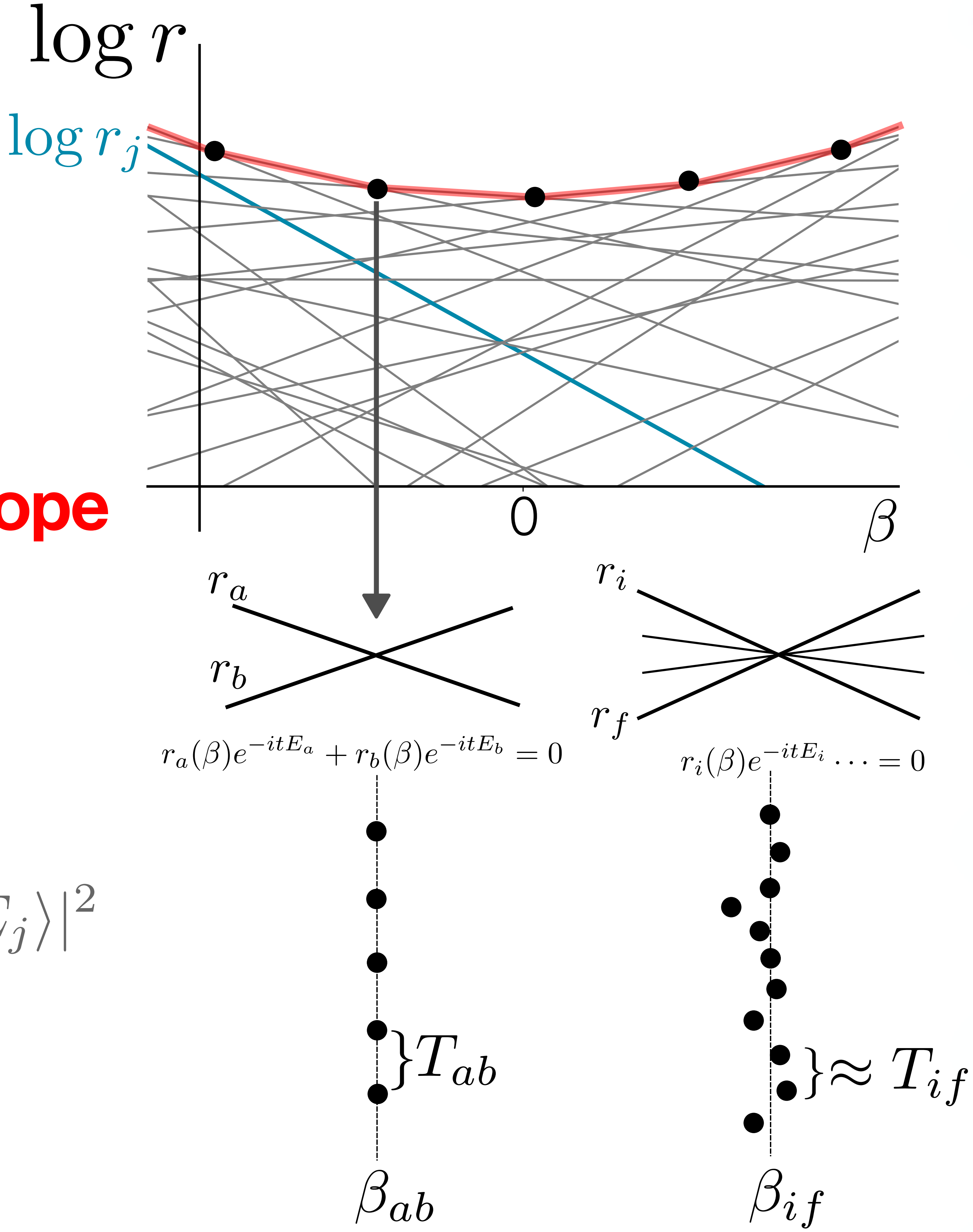

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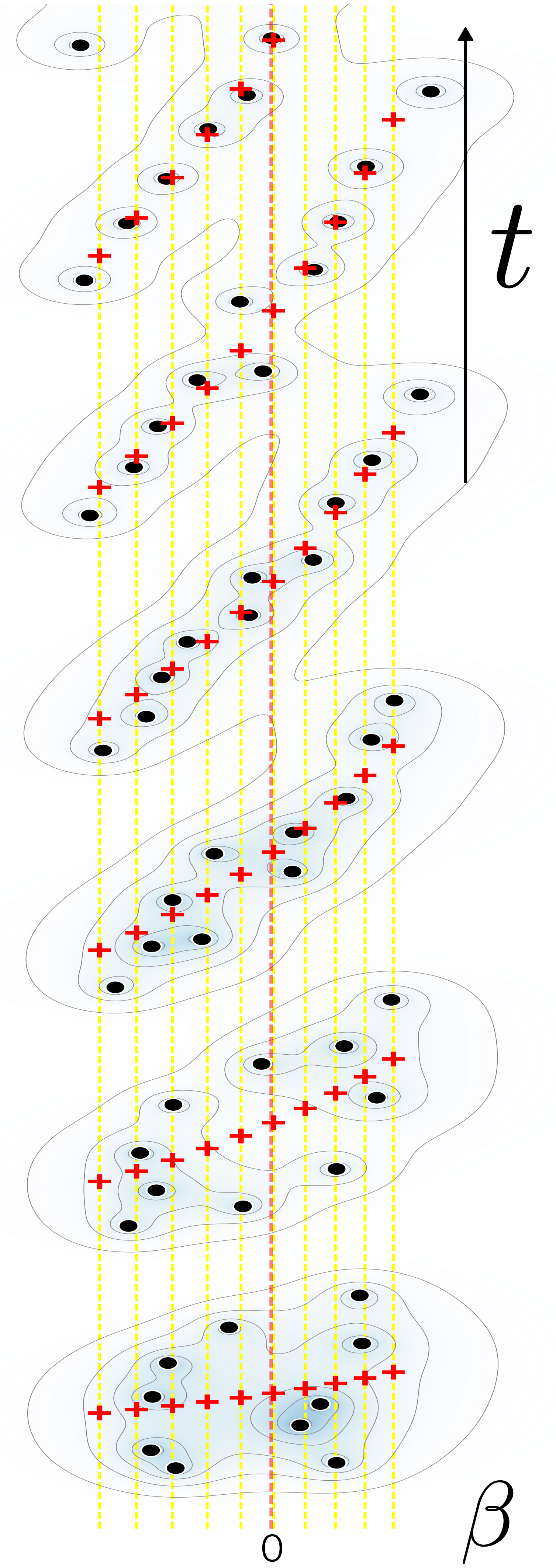
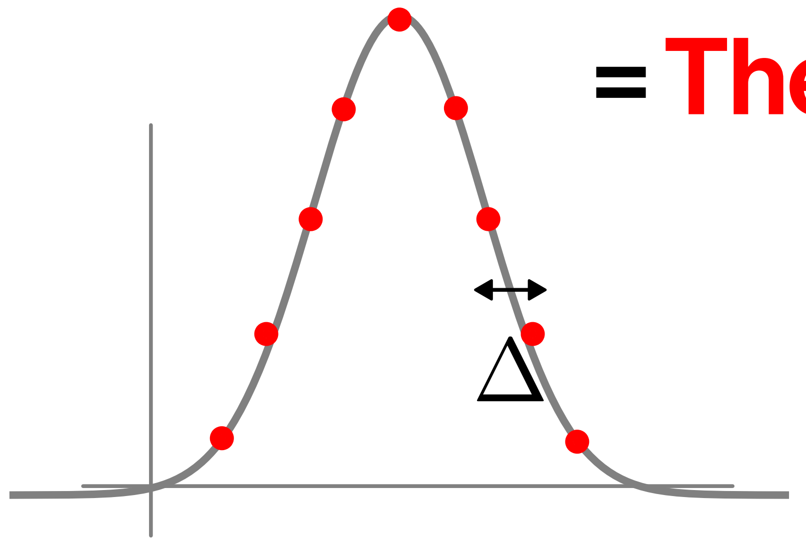


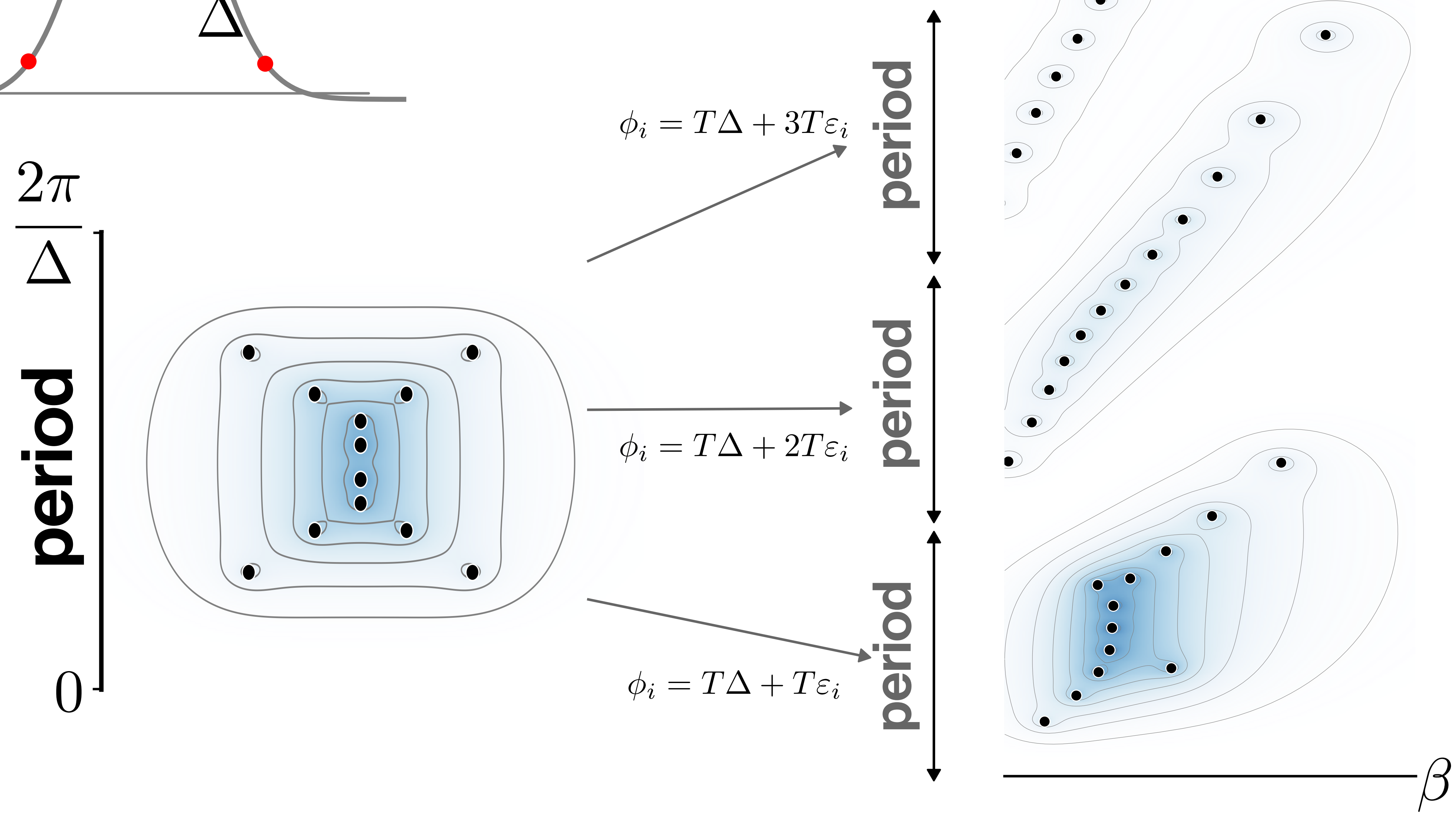
Short times

The dominant part



= **The equid. model**

$$\Delta_i = \Delta$$

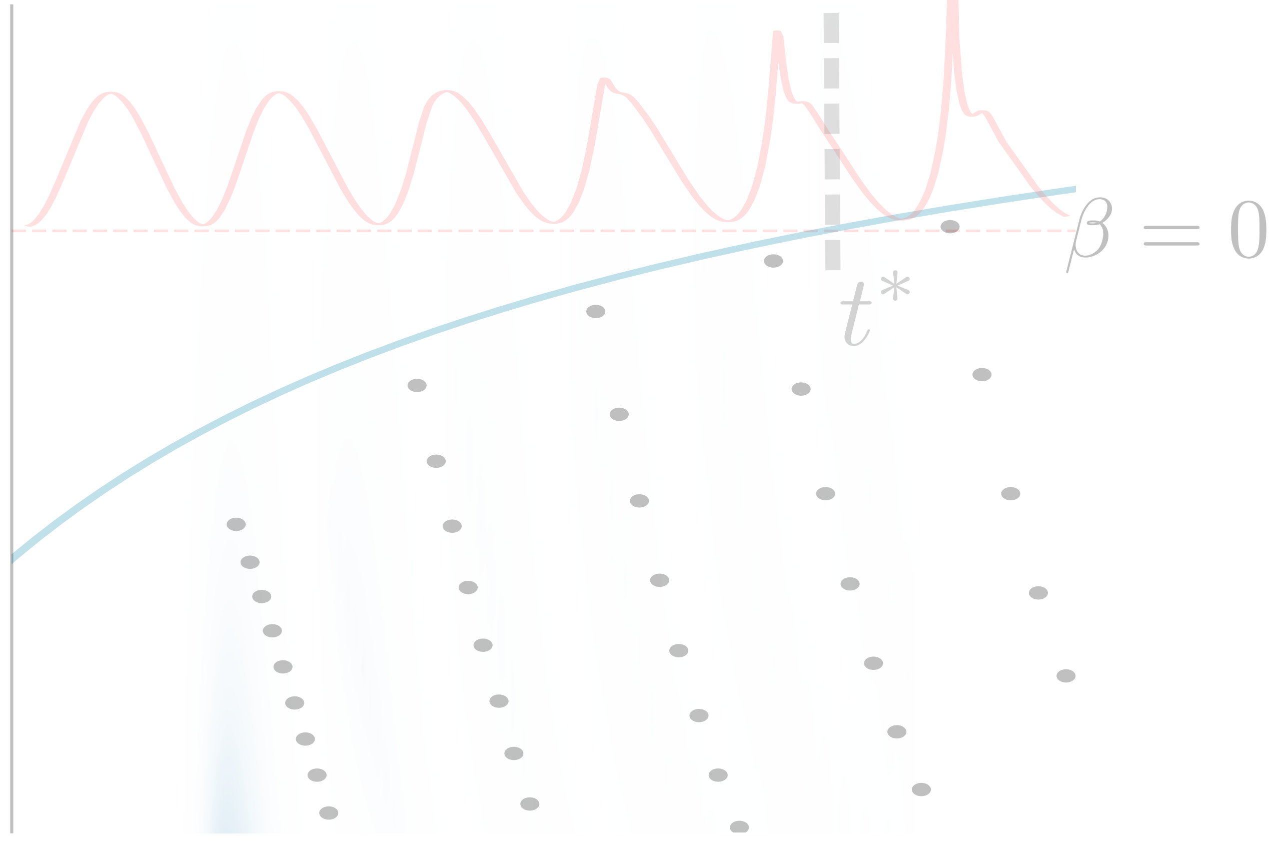


Asymmetry → running zeros

Arrival of the First zero

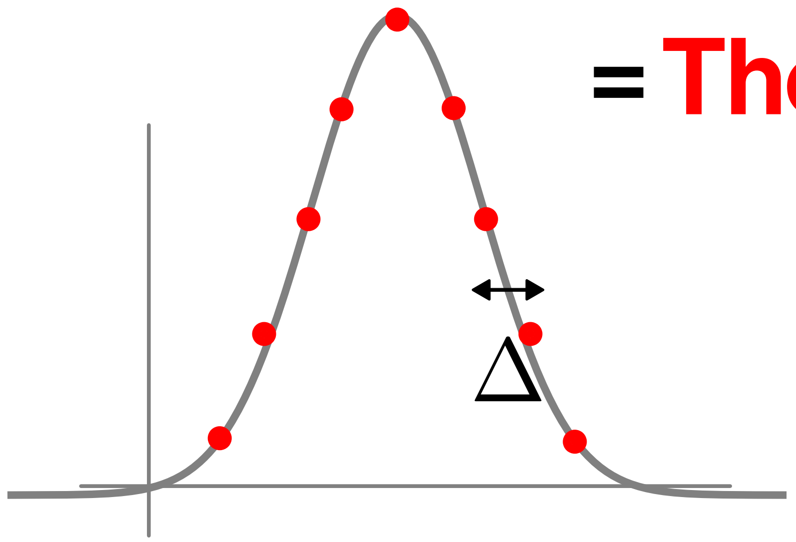
$$t^*(\beta) \approx t_0 + \frac{\beta - \beta_0}{|\omega_S(\beta) - \omega_A(\beta)|}$$

Lipkin model $\hat{H} = -\frac{J}{N}\hat{S}_x^2 + h\hat{S}_z$



Short times

The dominant part



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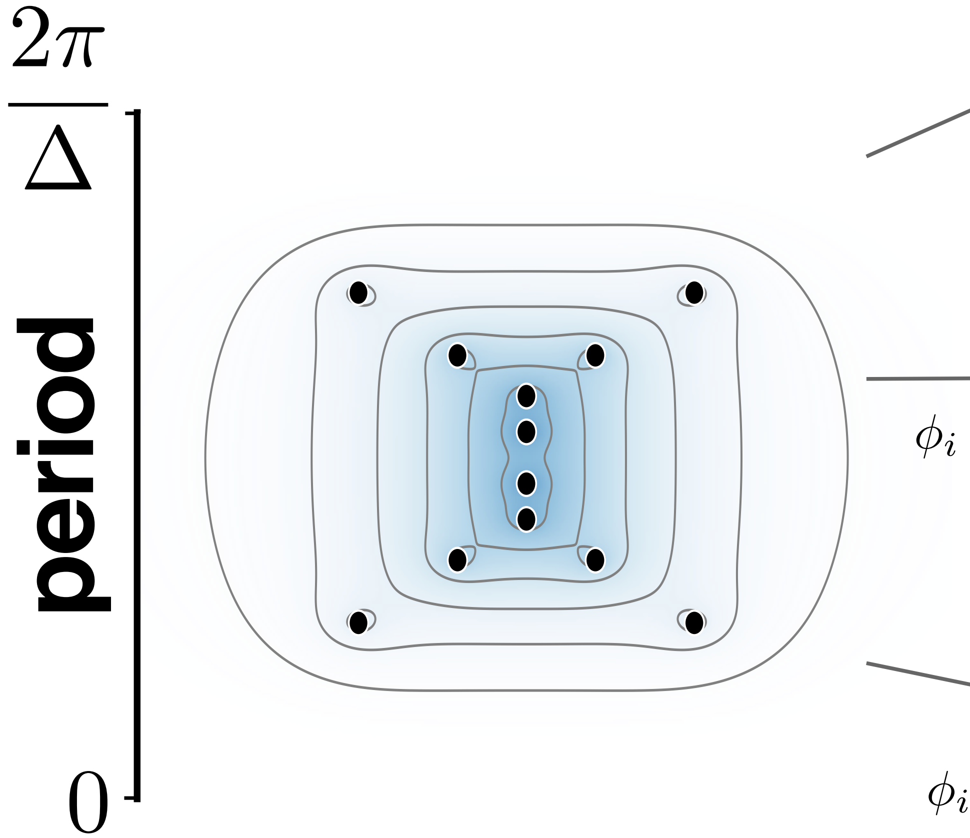

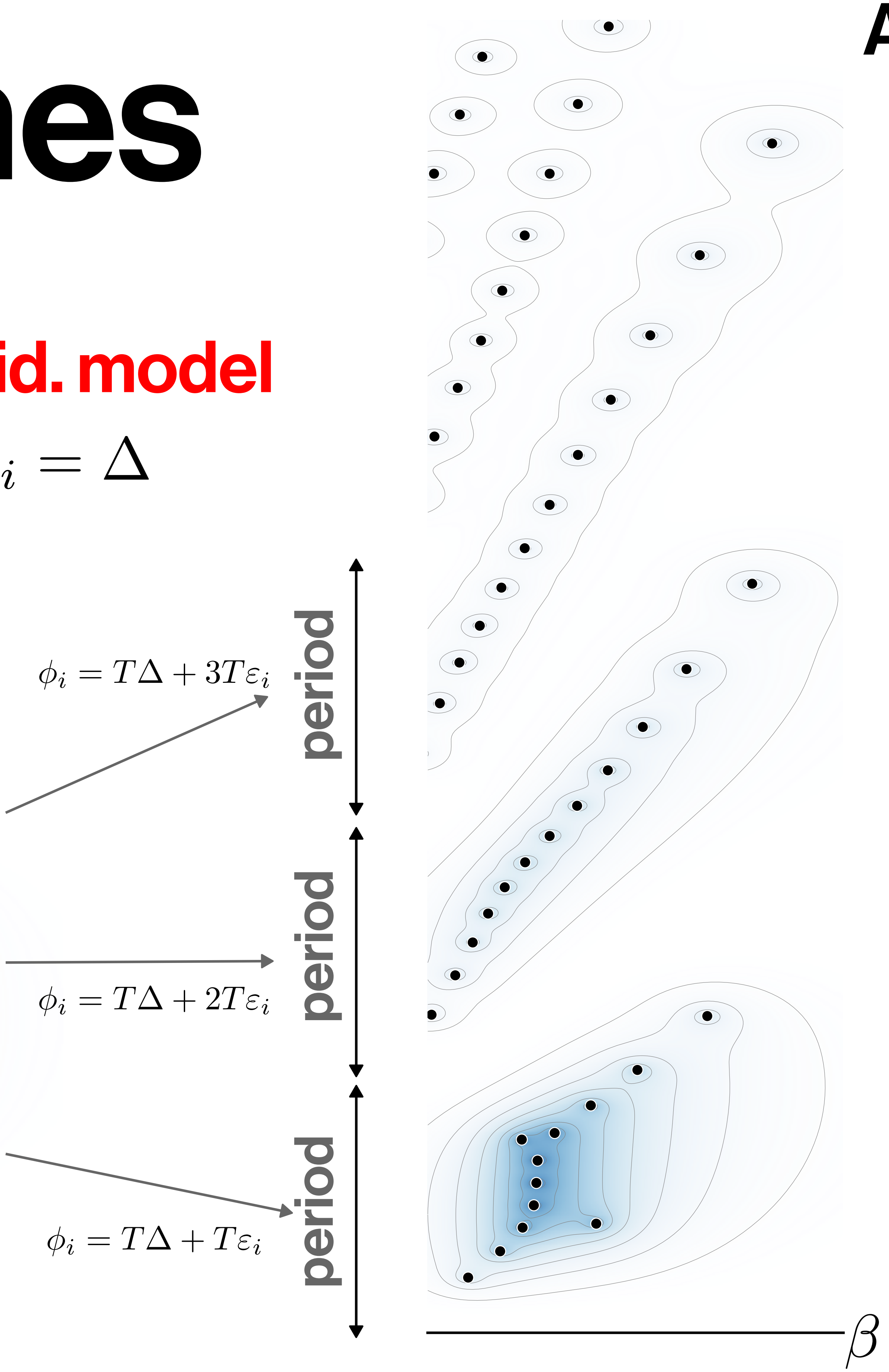

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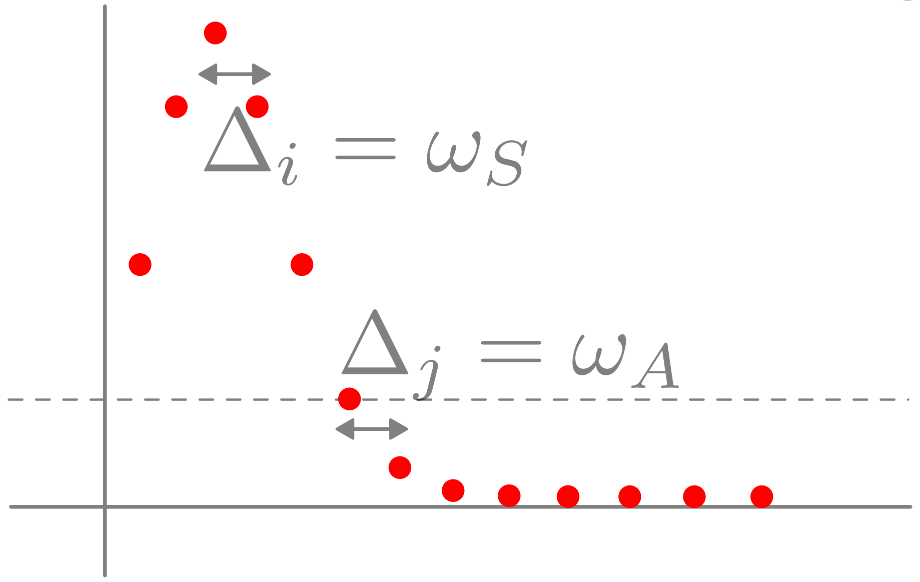

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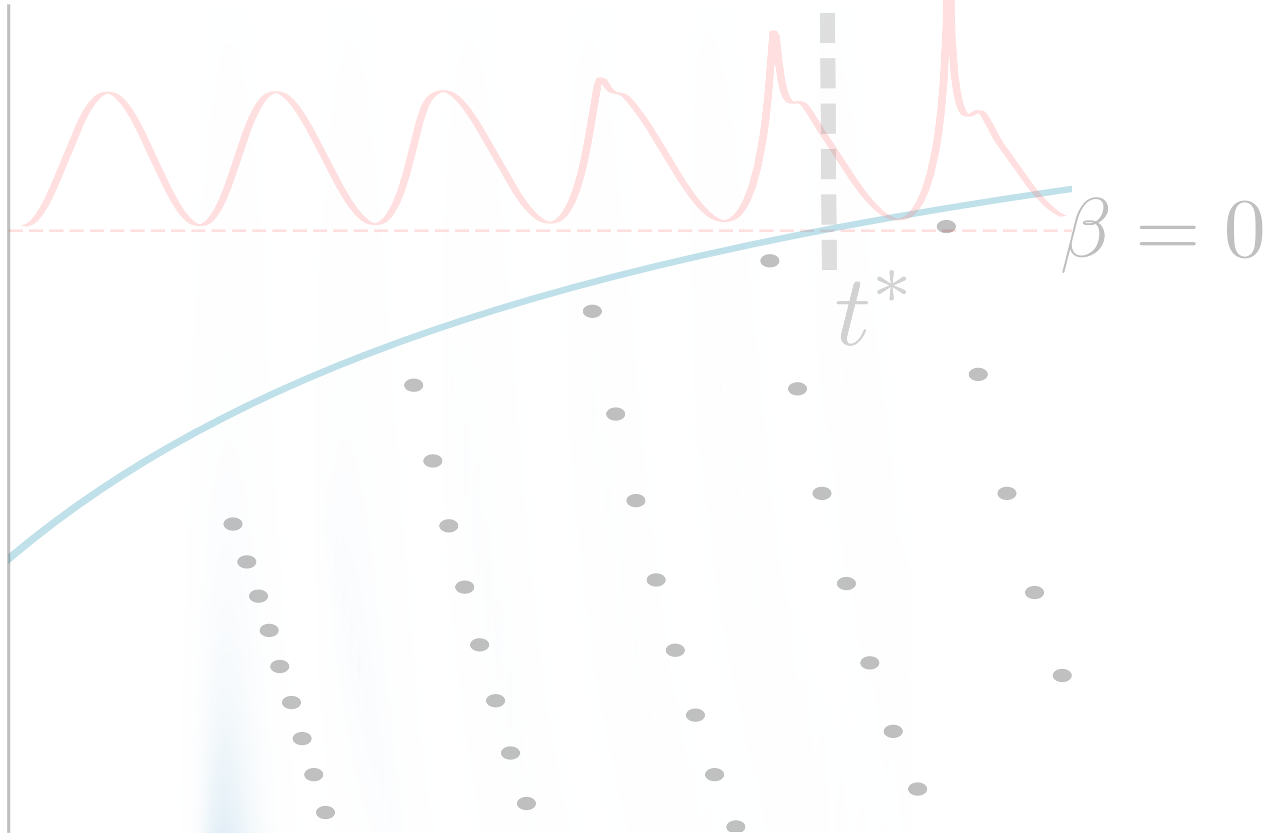

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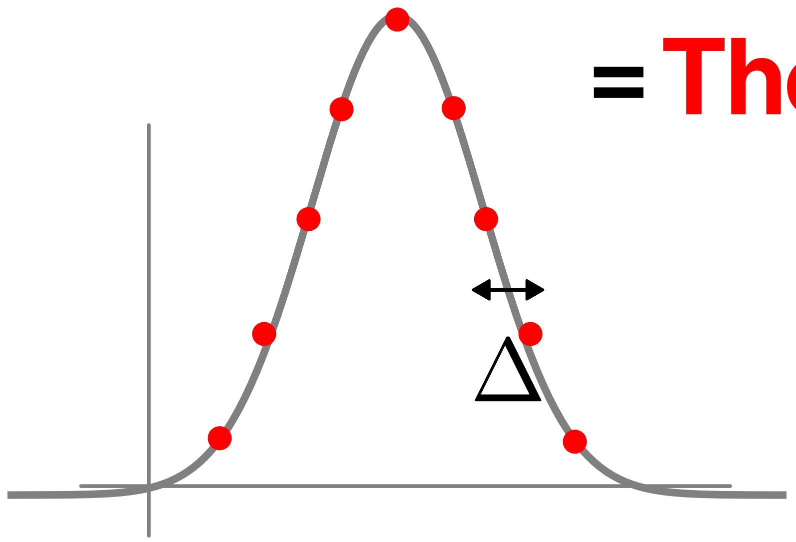

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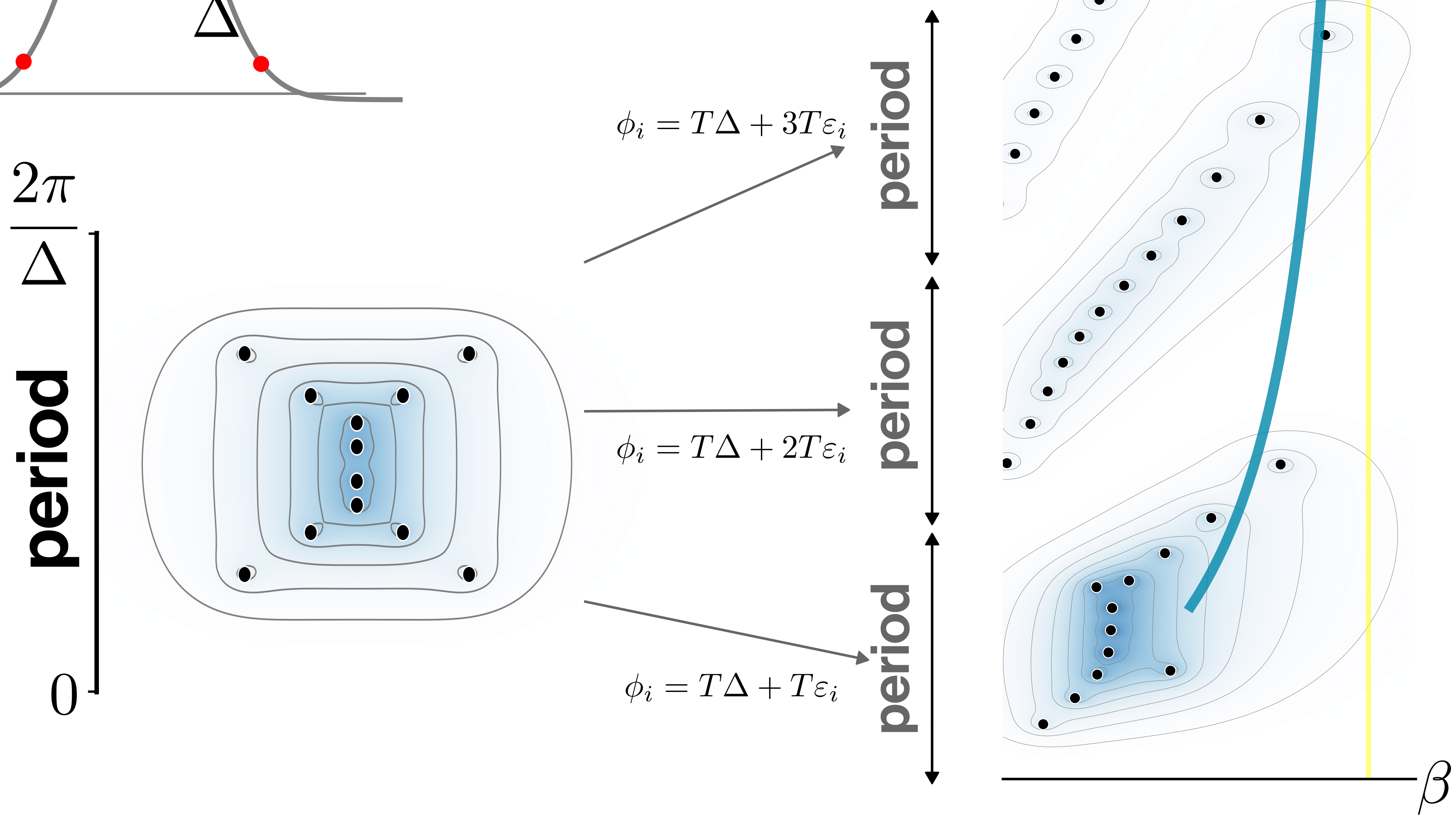

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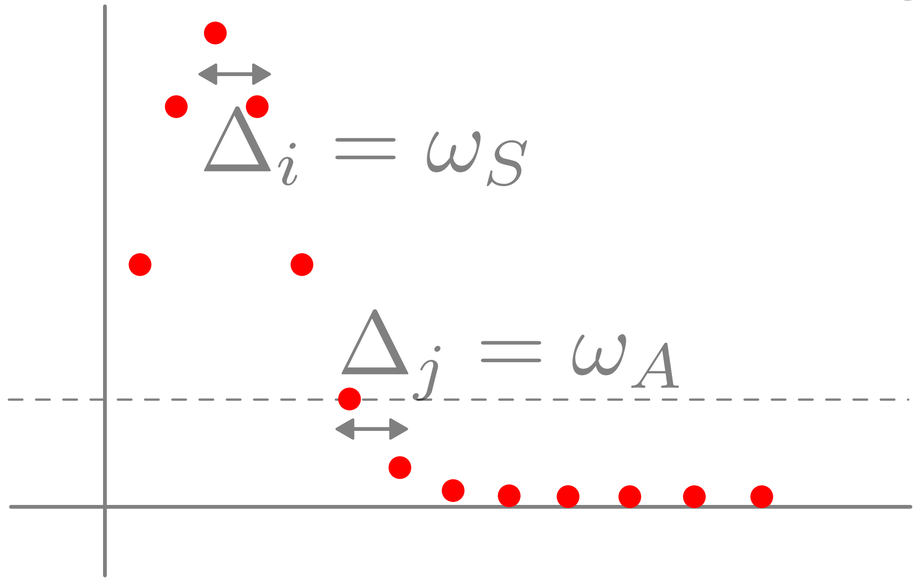

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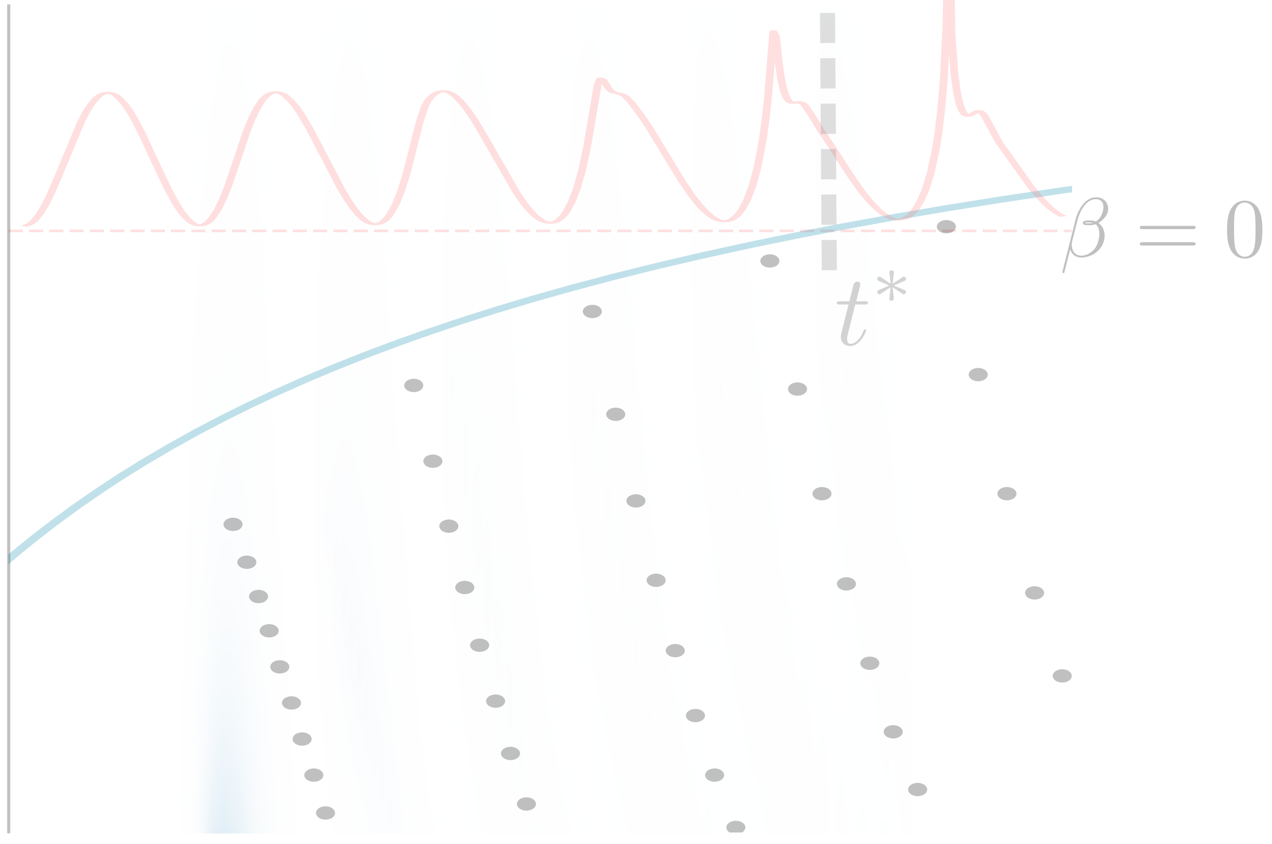

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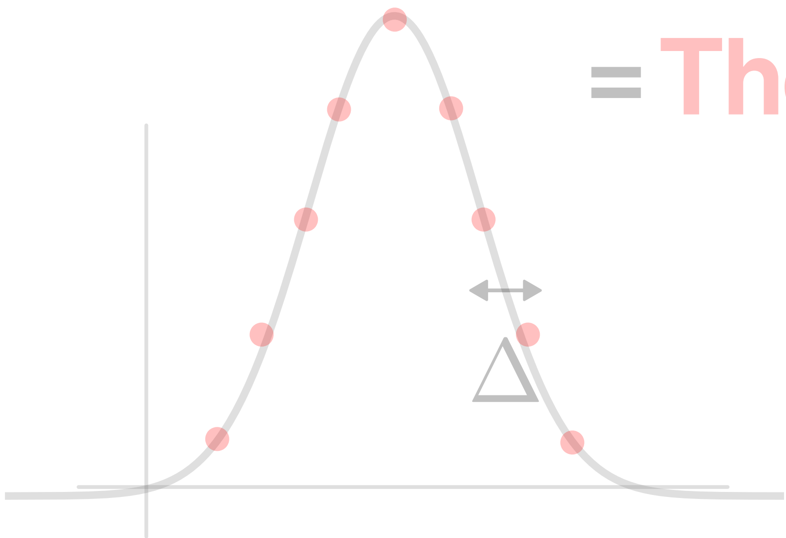

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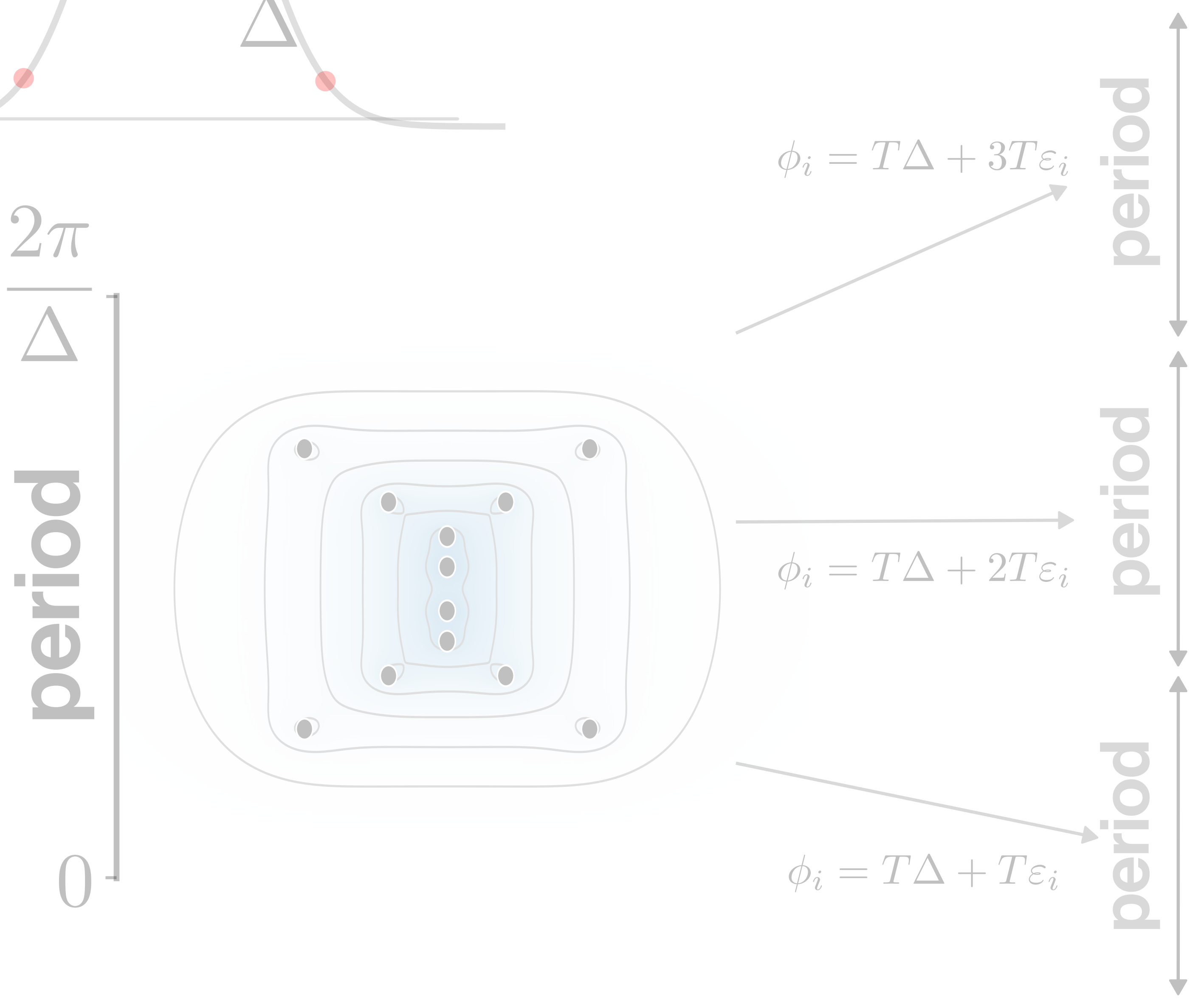

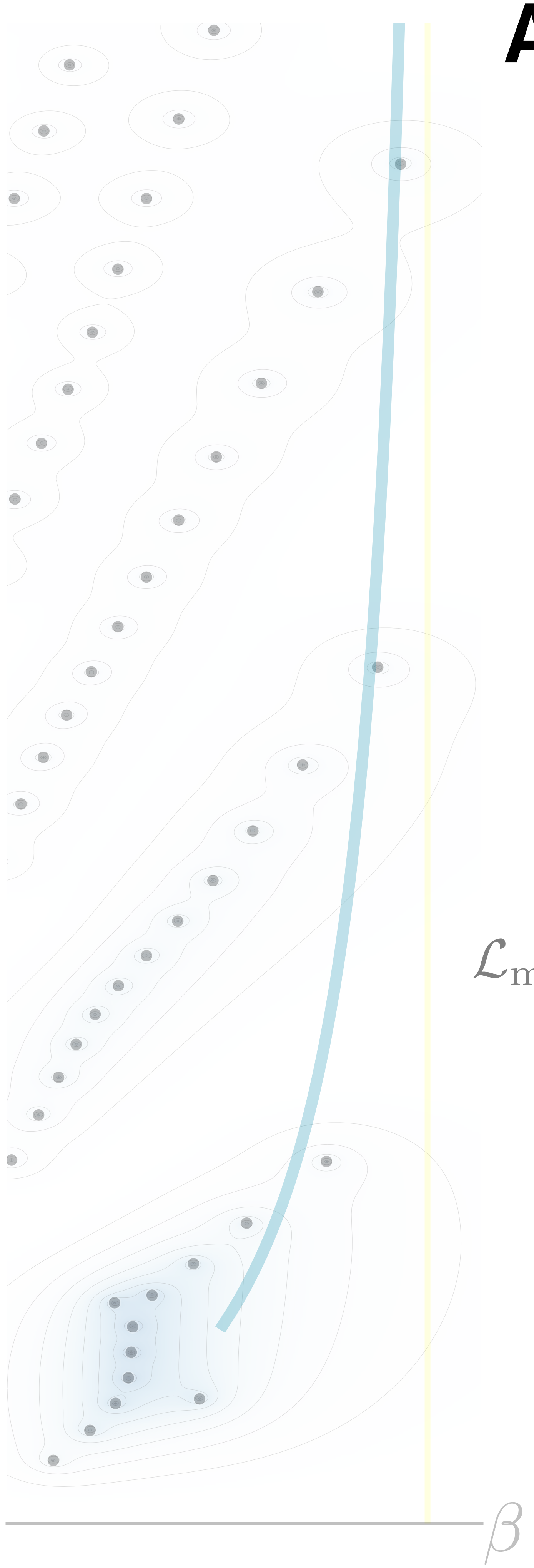

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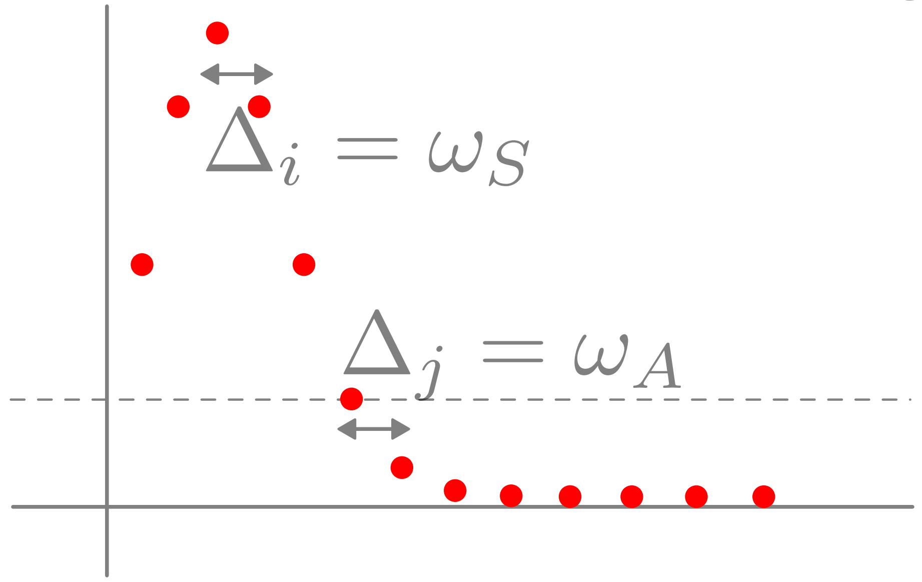

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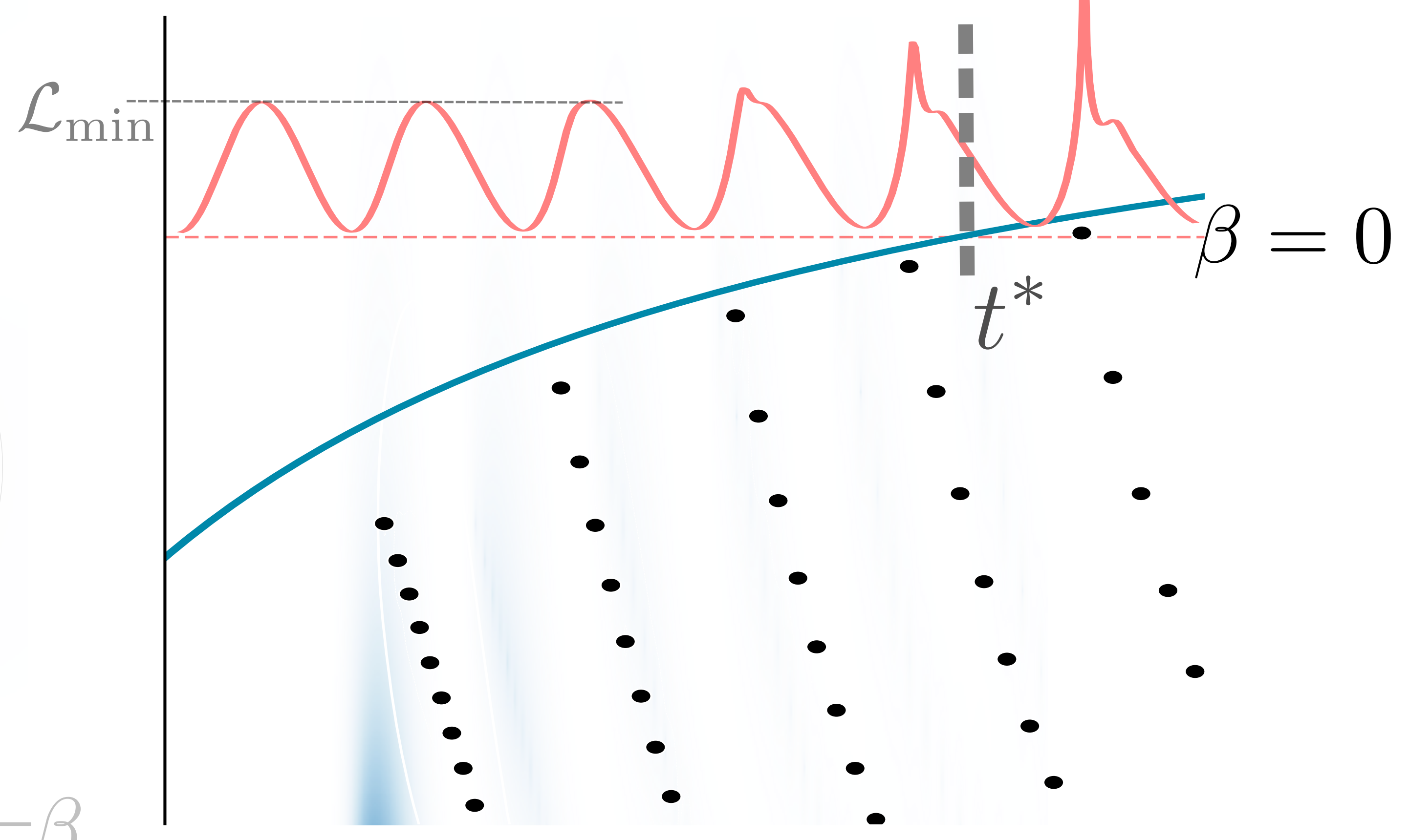
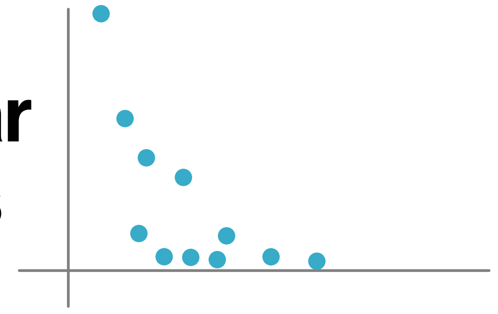


Quenches

- $\max\{k_j\} > \frac{1}{2}$ **no zeros near the time axis**



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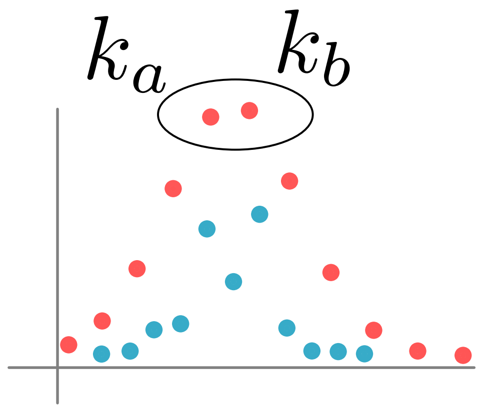
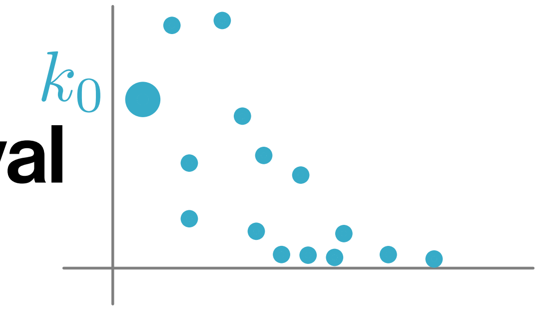


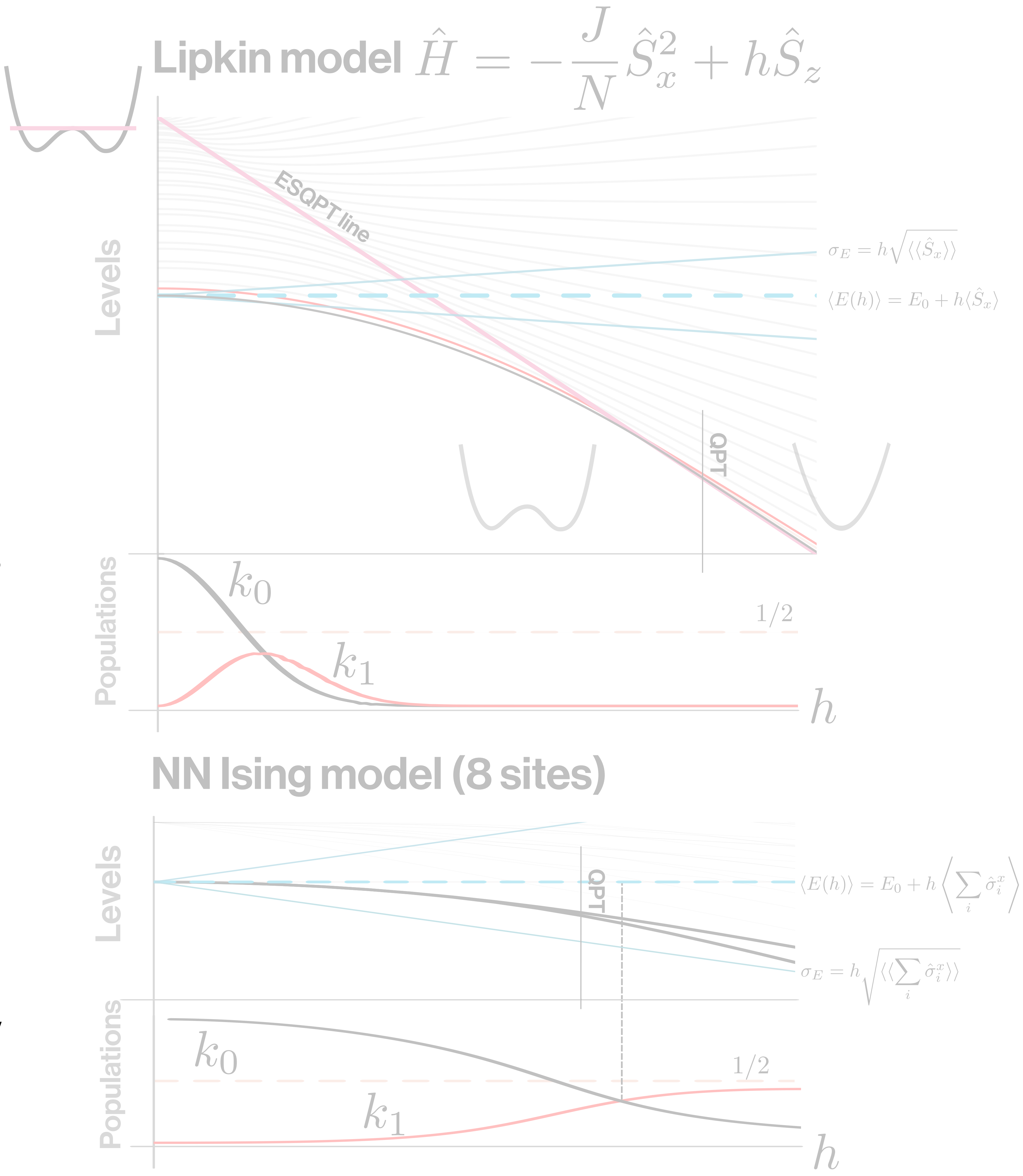
T_{ab} β_{ab}

- $k_0 > \mathcal{L}_{\min}$ **late time arrival**



Ground state quench

"zeros close to the time axis" ⟷ **"two (or more) highly populated states"**



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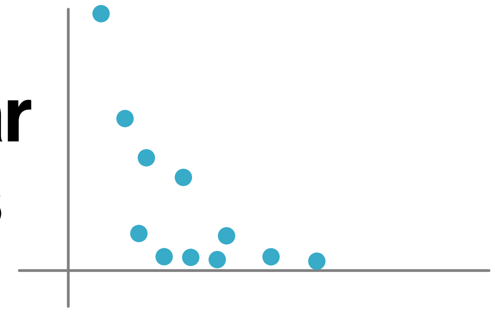

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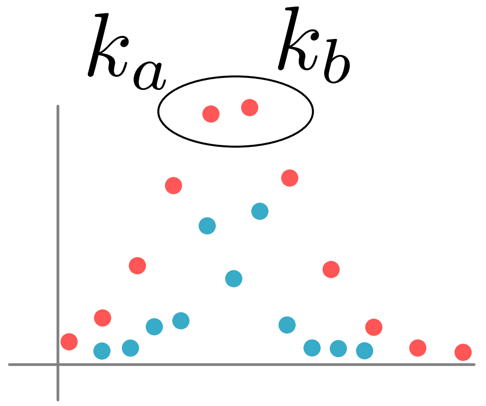

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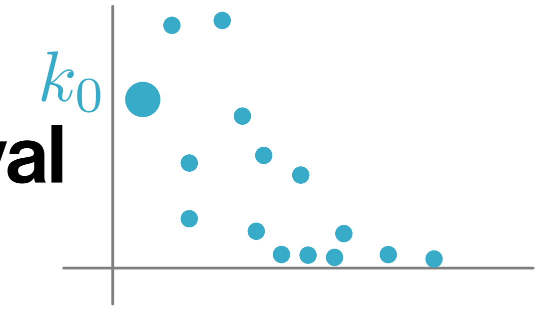

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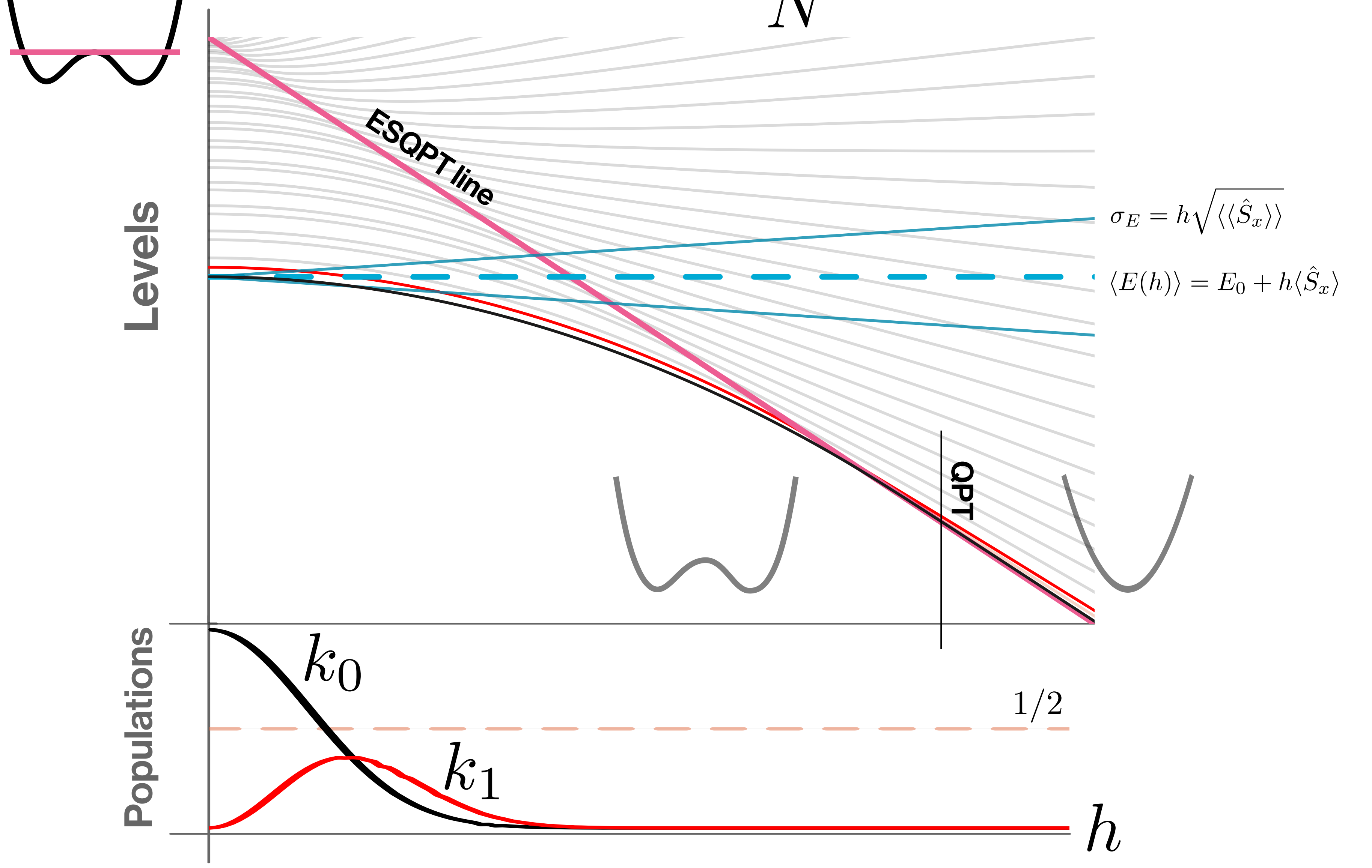
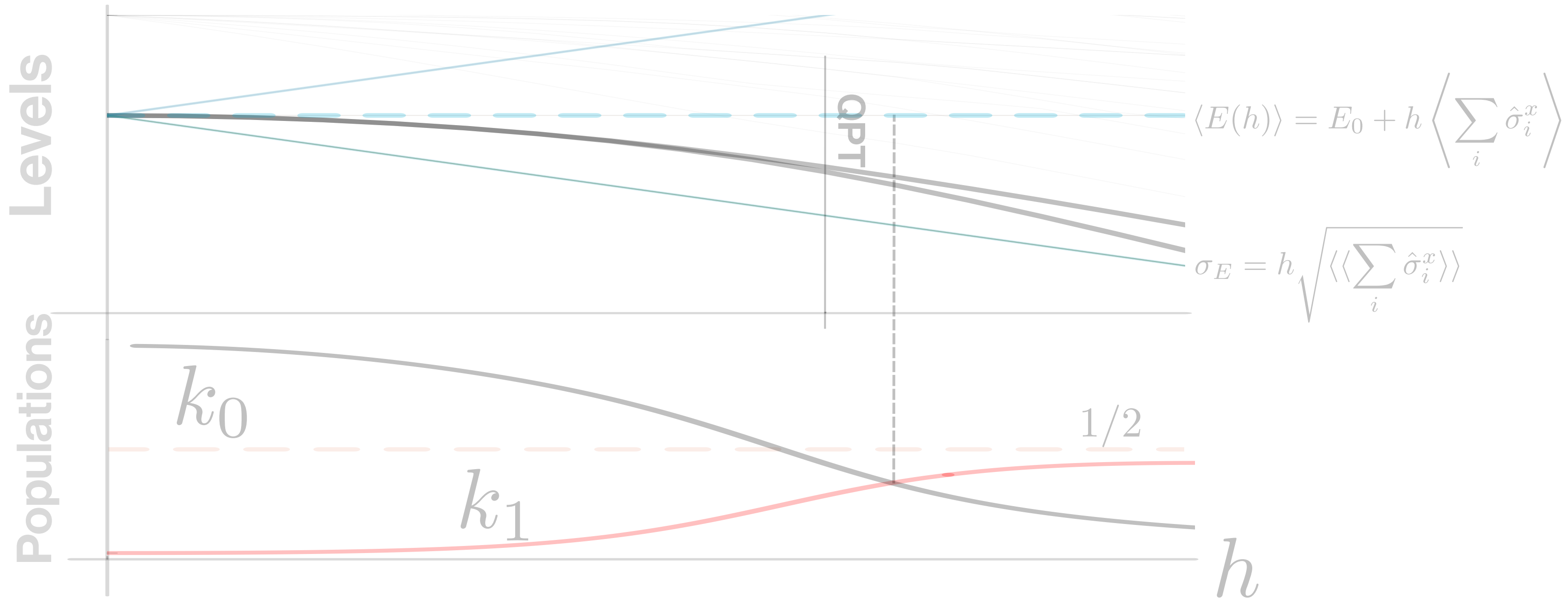


NN Ising model (8 sites)



Quenches

Cejnar et al.
JPA (2021)

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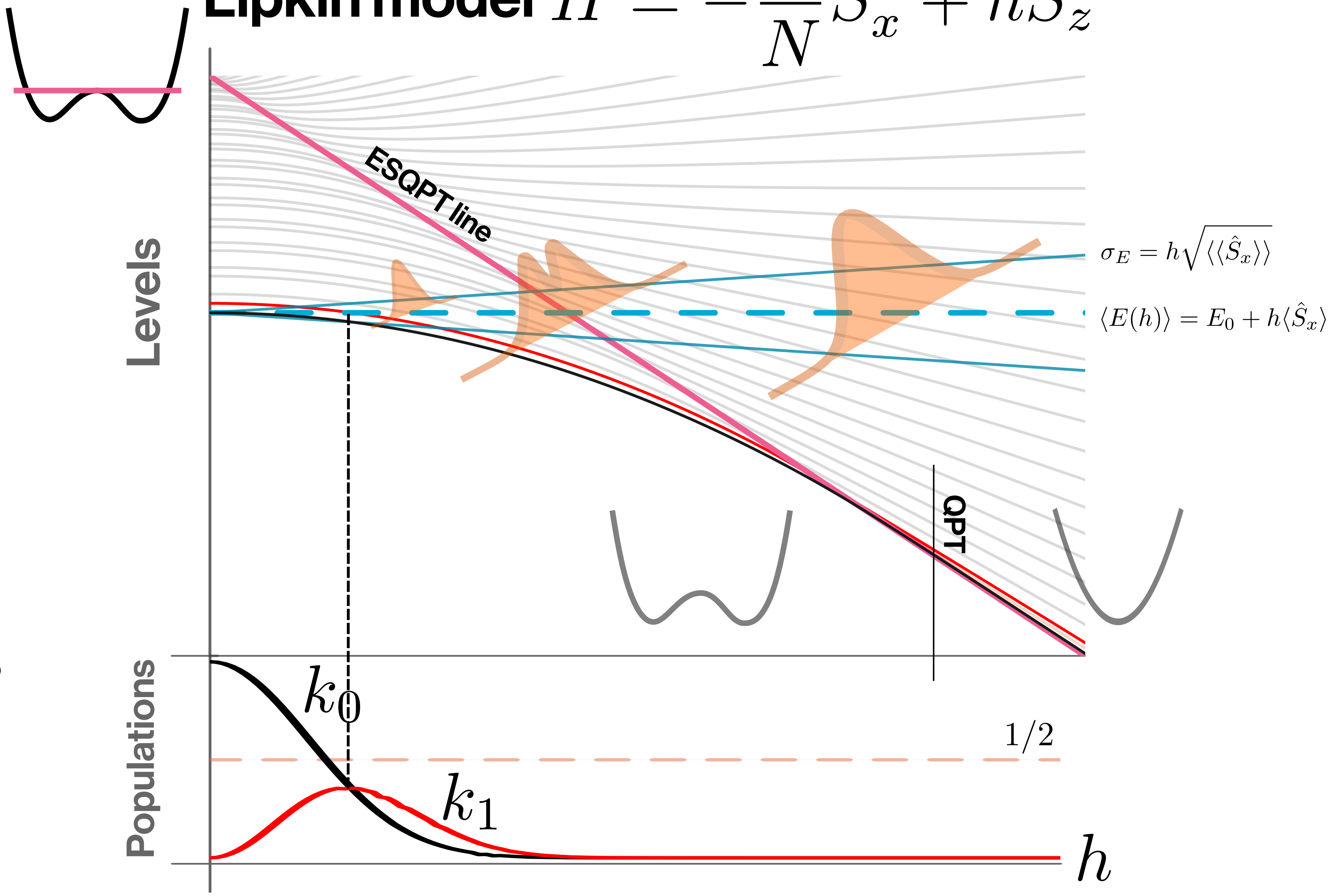

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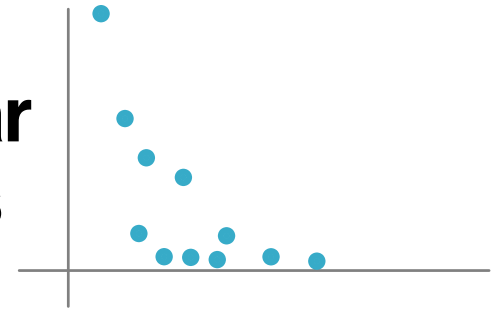

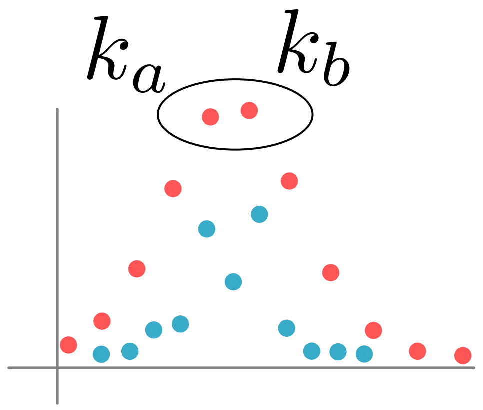

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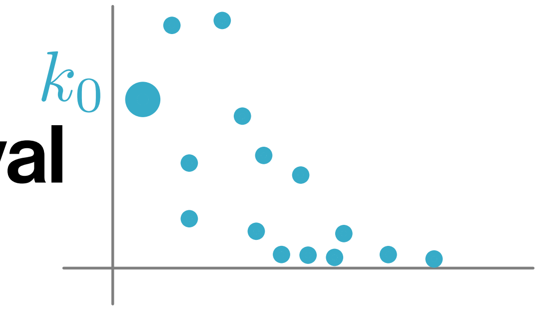

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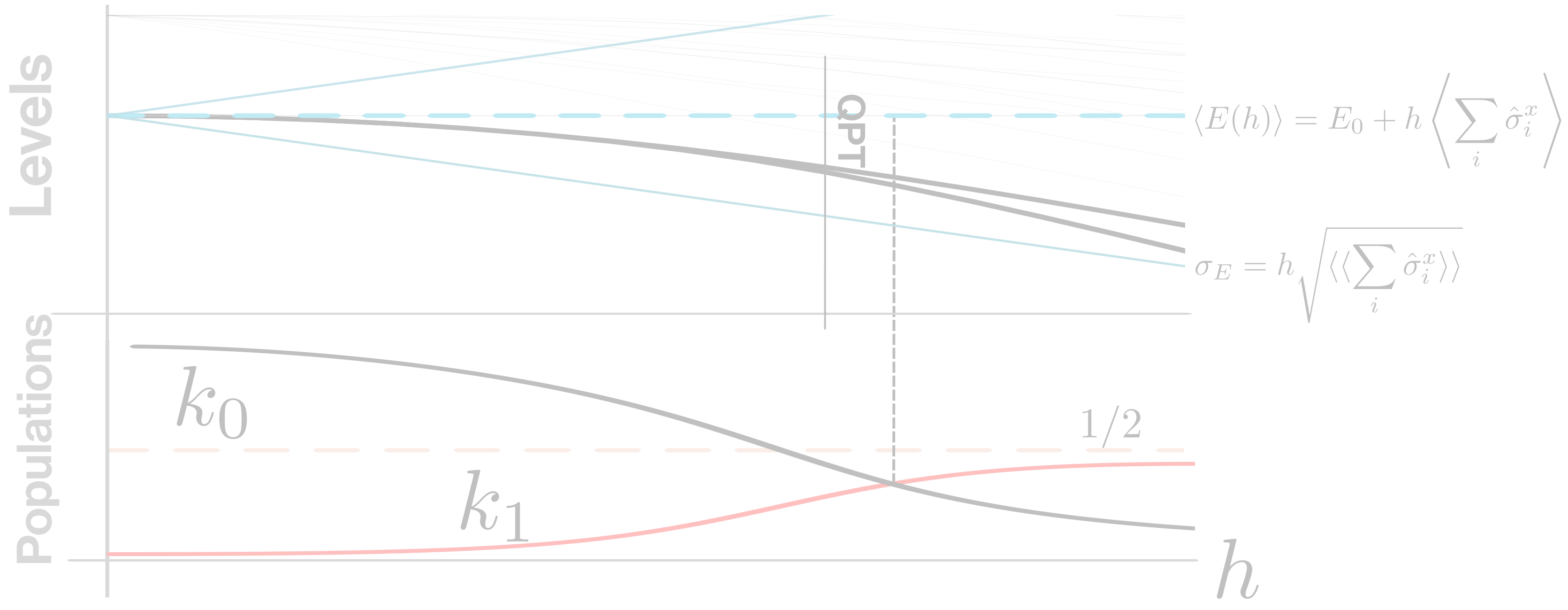

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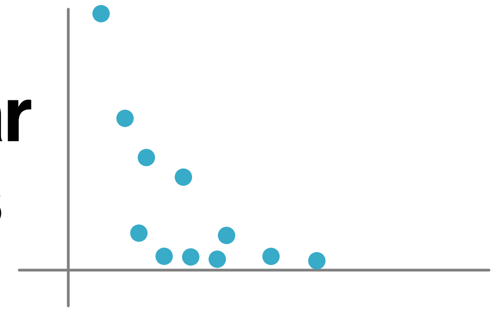

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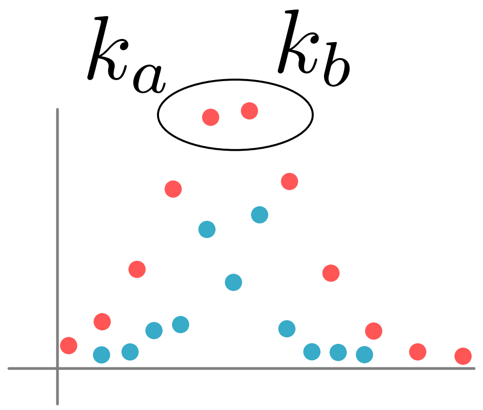

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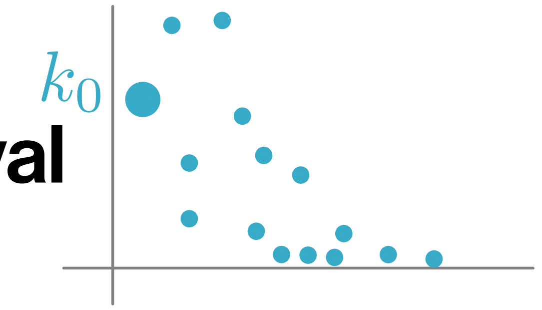

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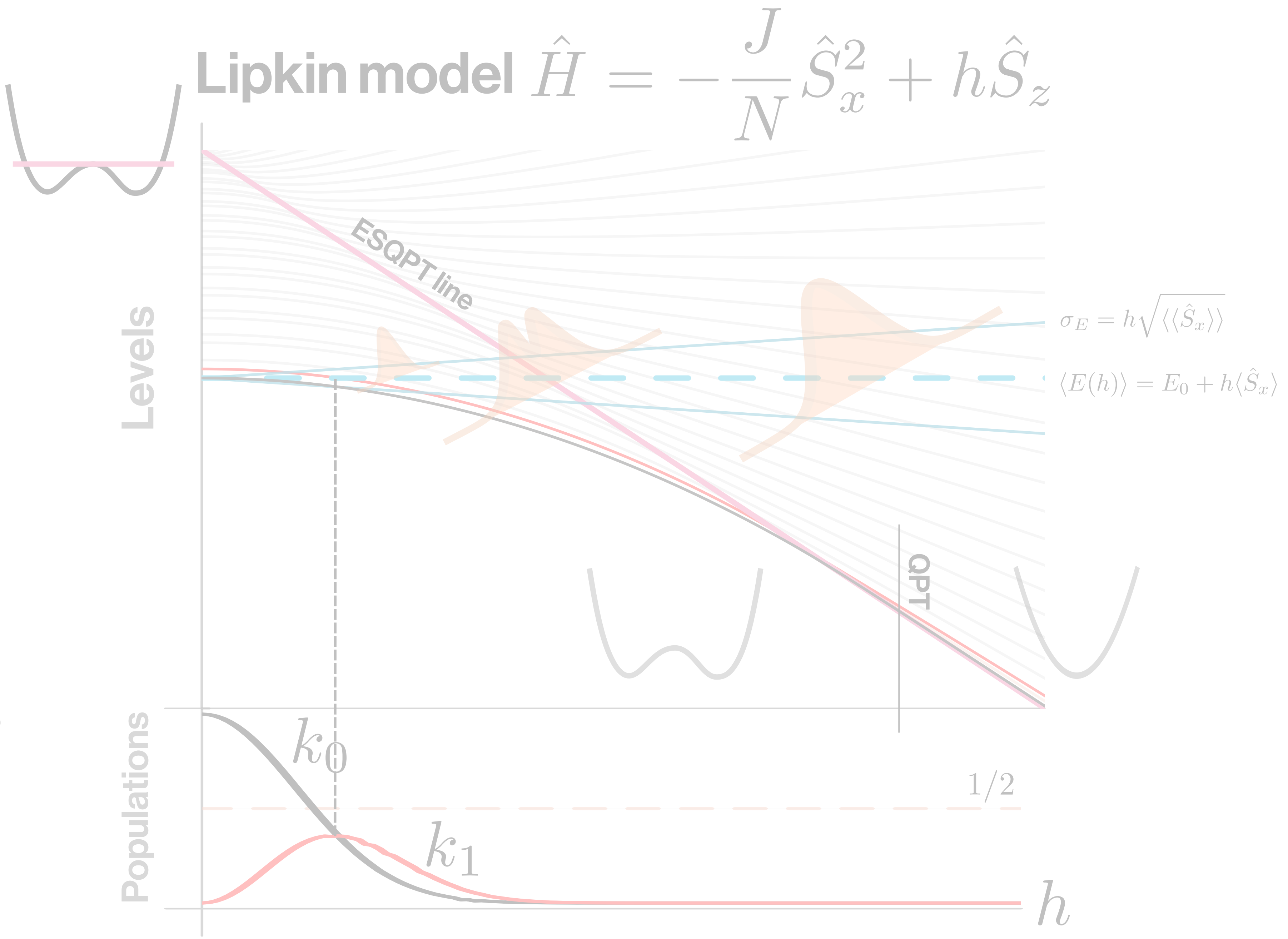

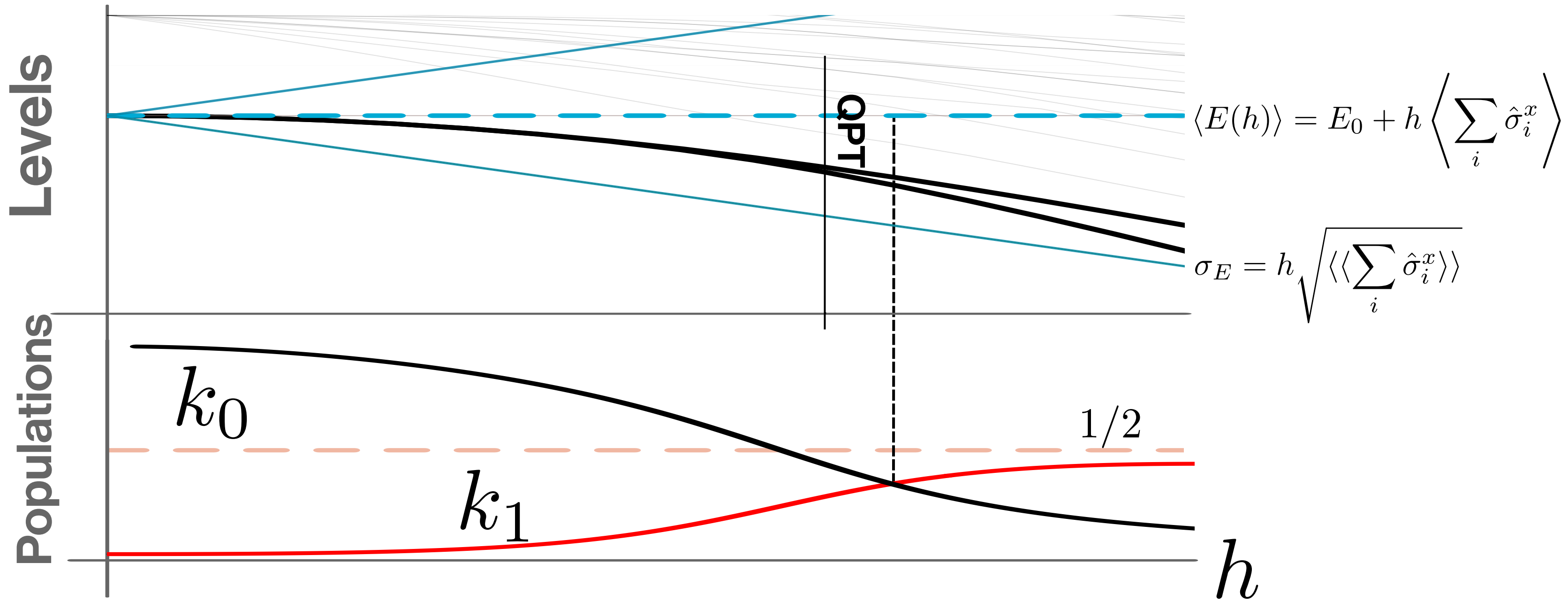

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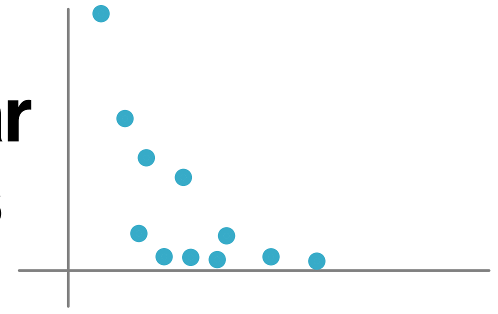

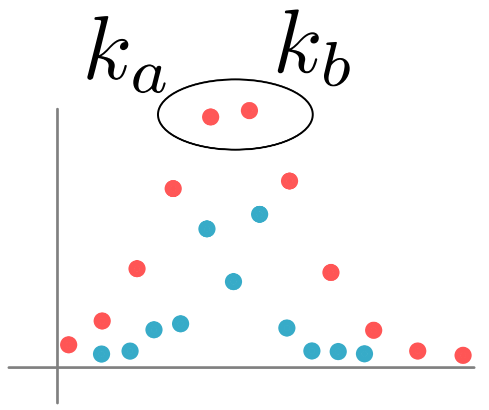

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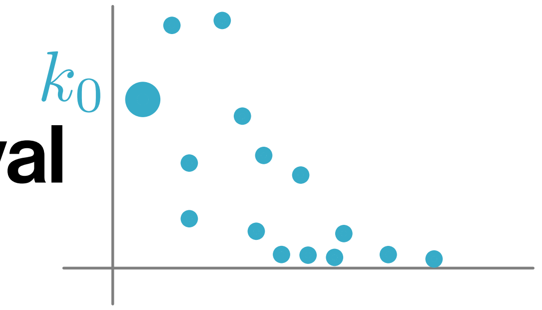

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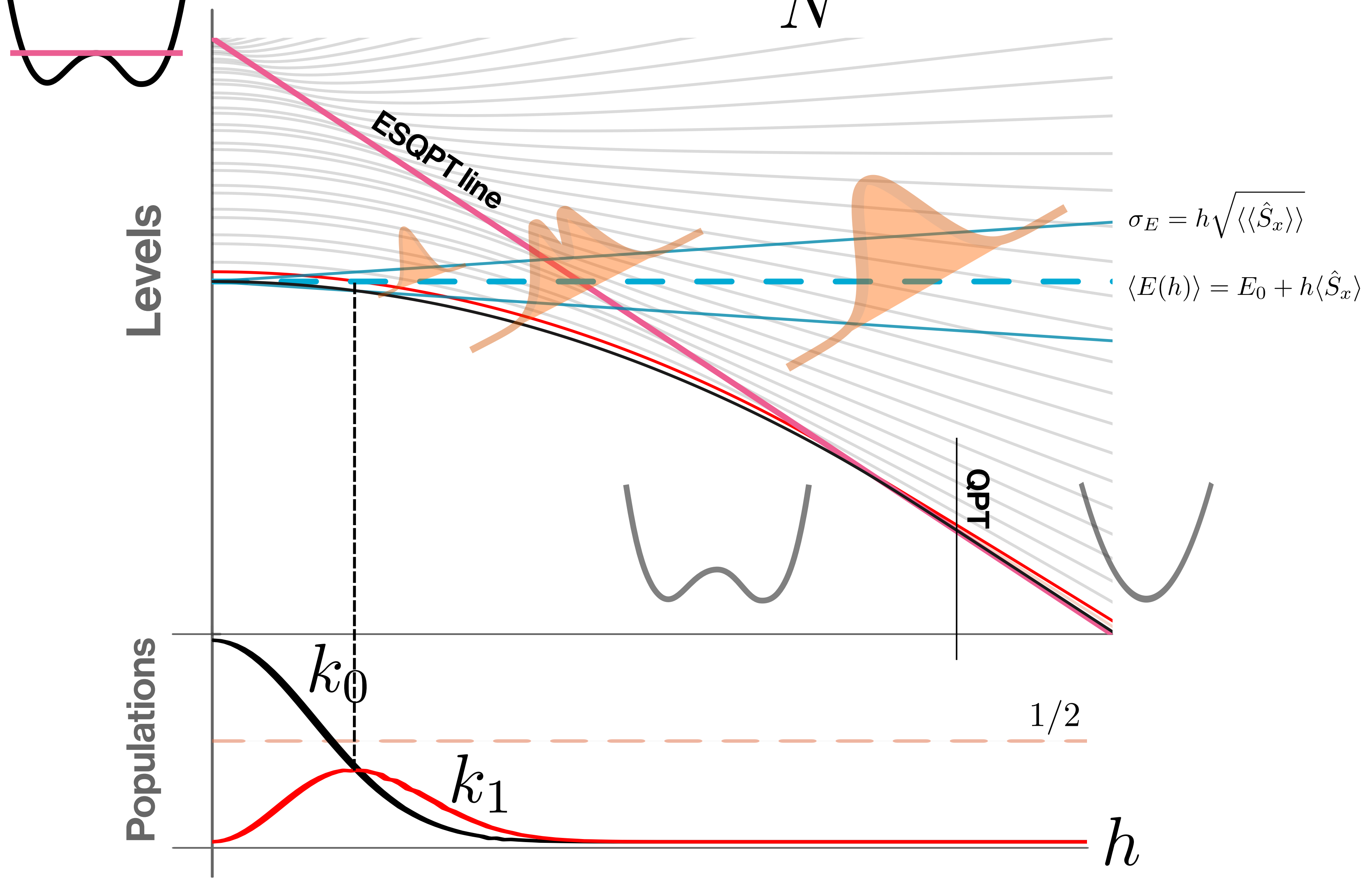

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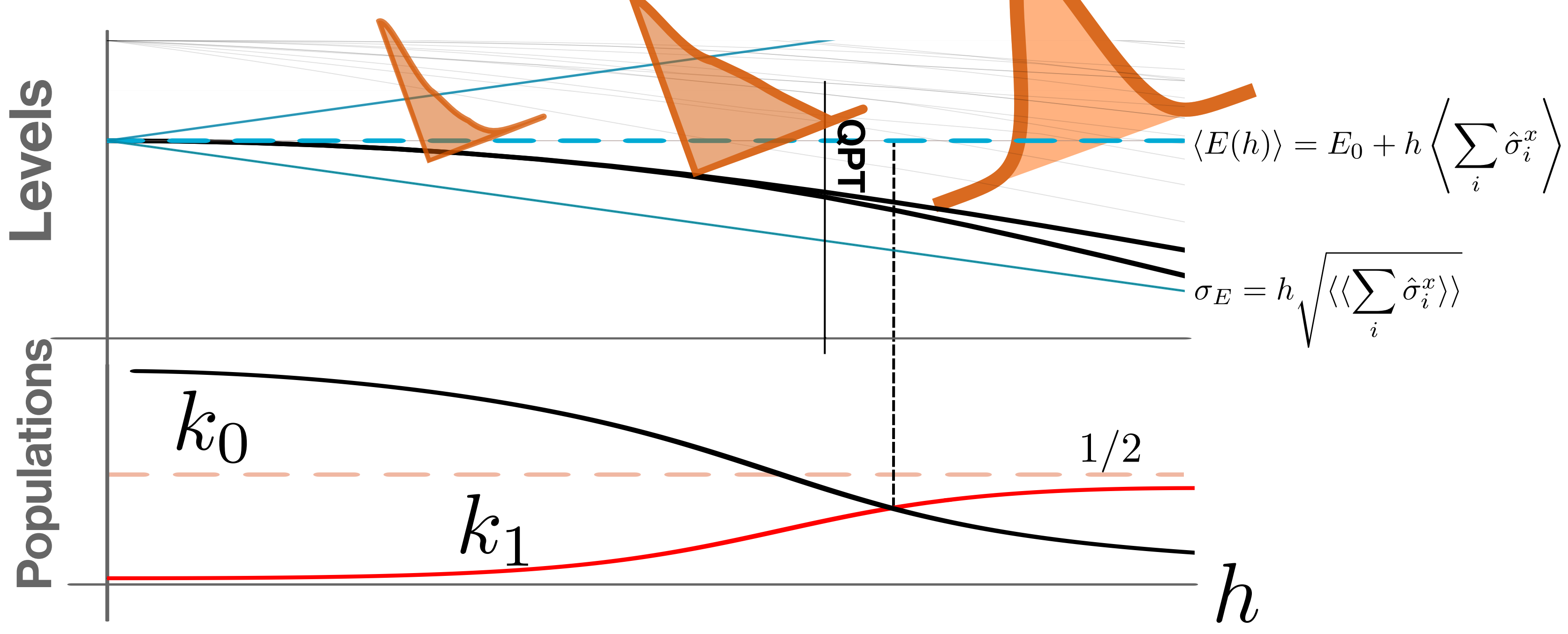

Conclusions

Zeros comes from the dominant parts of the initial state

- the envelope

Zeros form chain-like structures around β_{ab} with (quasi-) periods T_{ab}

Asymmetry of the initial state may delay the first zero

"zeros close to the time axis" ⟷ "two (or more) highly populated states"

Thank you for your attention

The work was supported by the The Charles University Grant Agency (grant no. 215323)