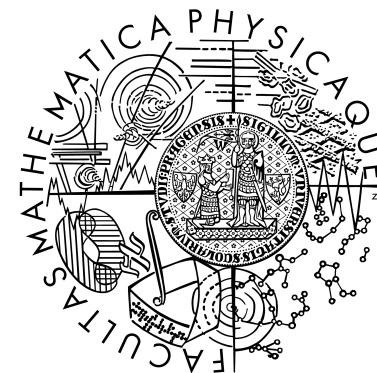


Zeros of Complexified Survival Amplitude at Different Time-Scales

**Long-Range Interactions and Dynamics
in Complex Quantum Systems**



NORDITA



Jakub Novotný

**Faculty
of Mathematics and Physics
Charles University**

Outline

Survival Amplitude

Motivation

Distribution of the zeros in the complex plane

Long times

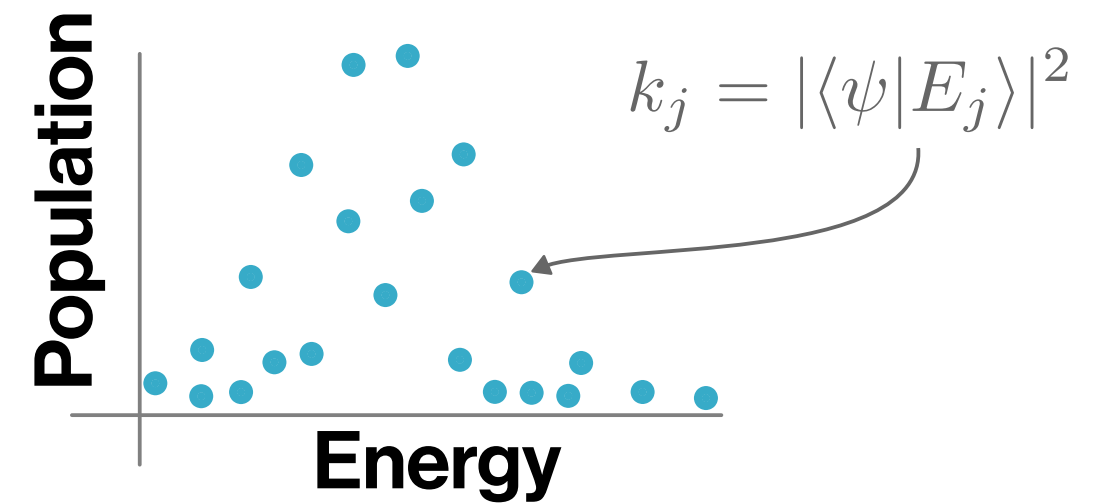
Short times

Remarks and Quenches

Survival amplitude

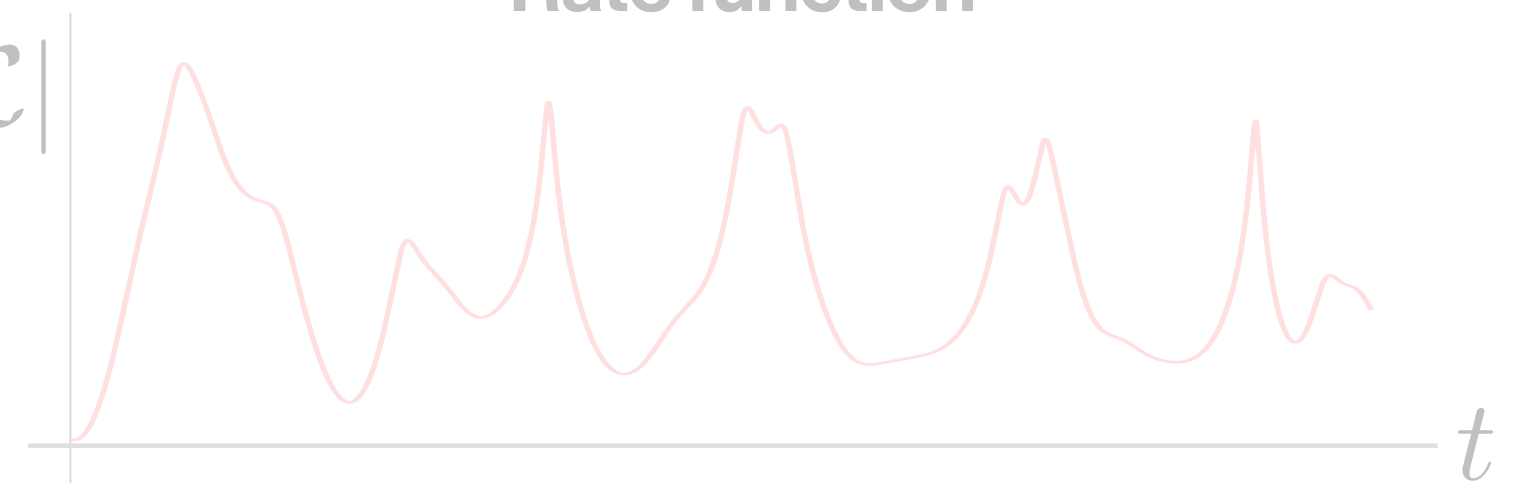
$$\mathcal{L}(t) = \langle \psi | \hat{U}(t) | \psi \rangle = \sum_j k_j e^{-iE_j t}$$

State
 $|\psi\rangle$



$-\log |\mathcal{L}|$

Rate function

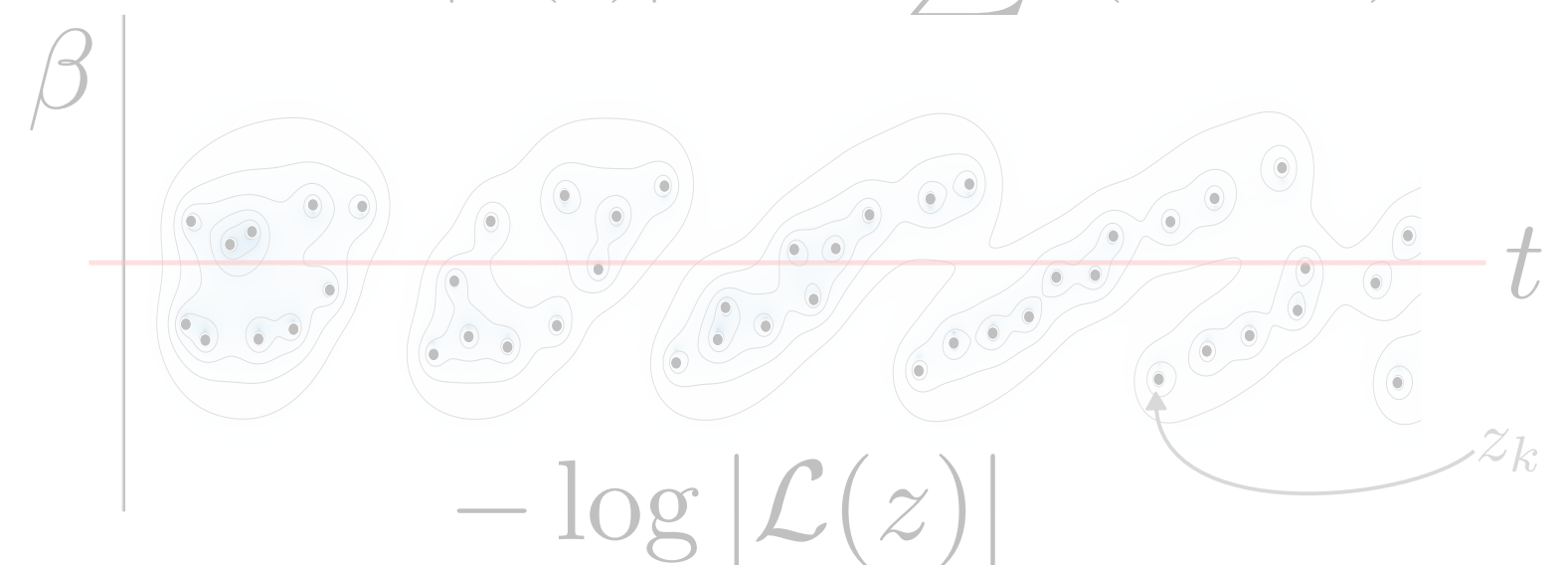
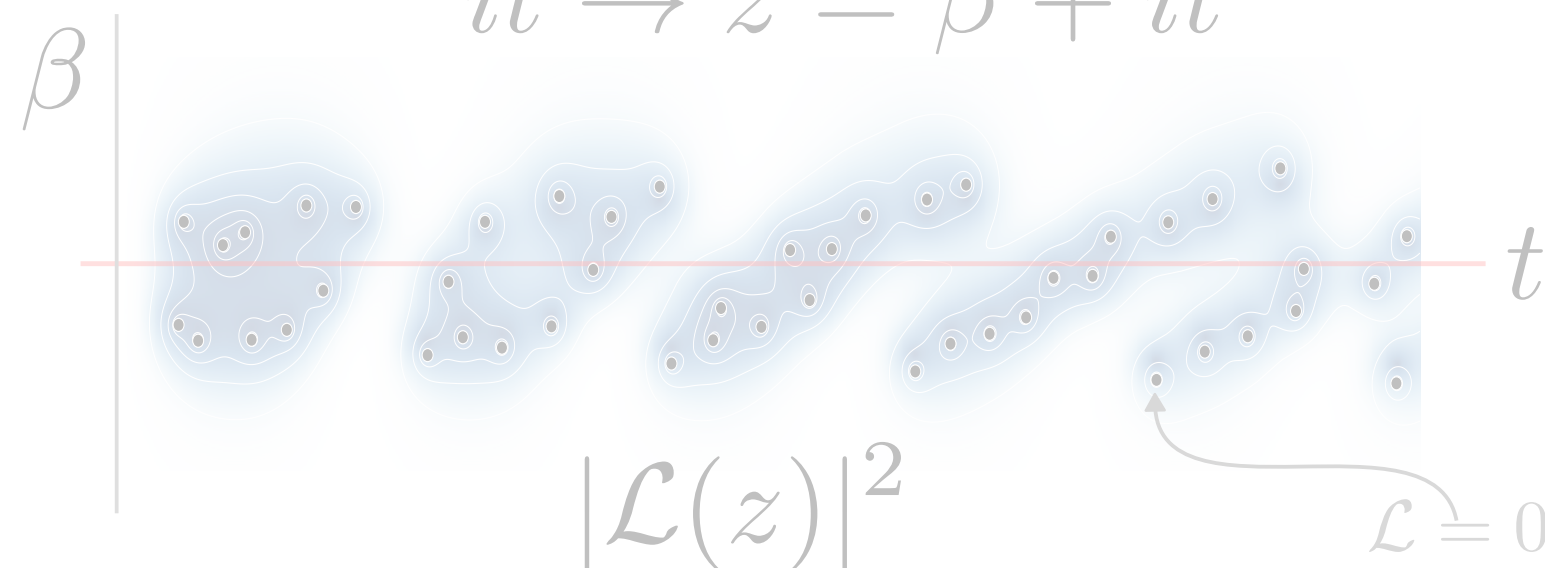


unique complex extension

$$it \rightarrow z = \beta + it$$

Log
→

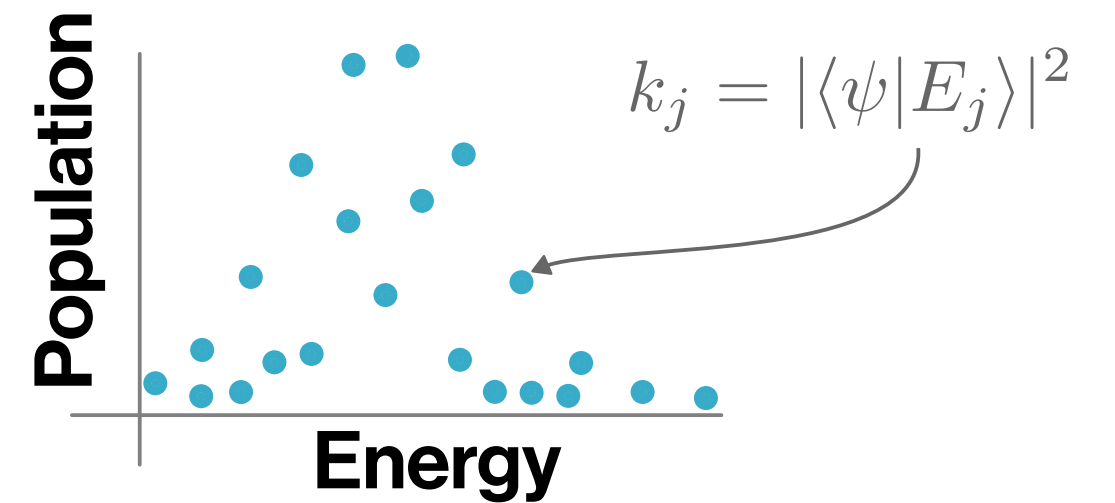
$$\Delta \log |\mathcal{L}(z)| = 2\pi \sum \delta(z - z_k)$$



Survival amplitude

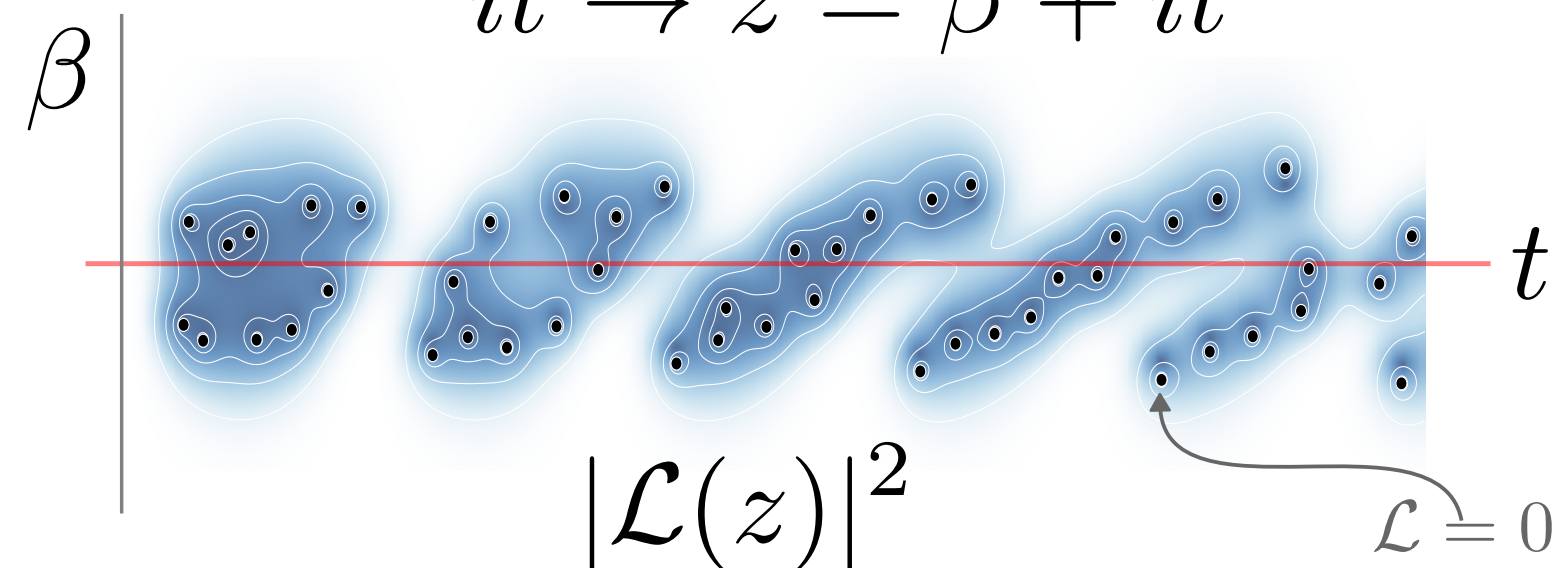
$$\mathcal{L}(t) = \langle \psi | \hat{U}(t) | \psi \rangle = \sum_j k_j e^{-iE_j t}$$

State
 $|\psi\rangle$



unique complex extension

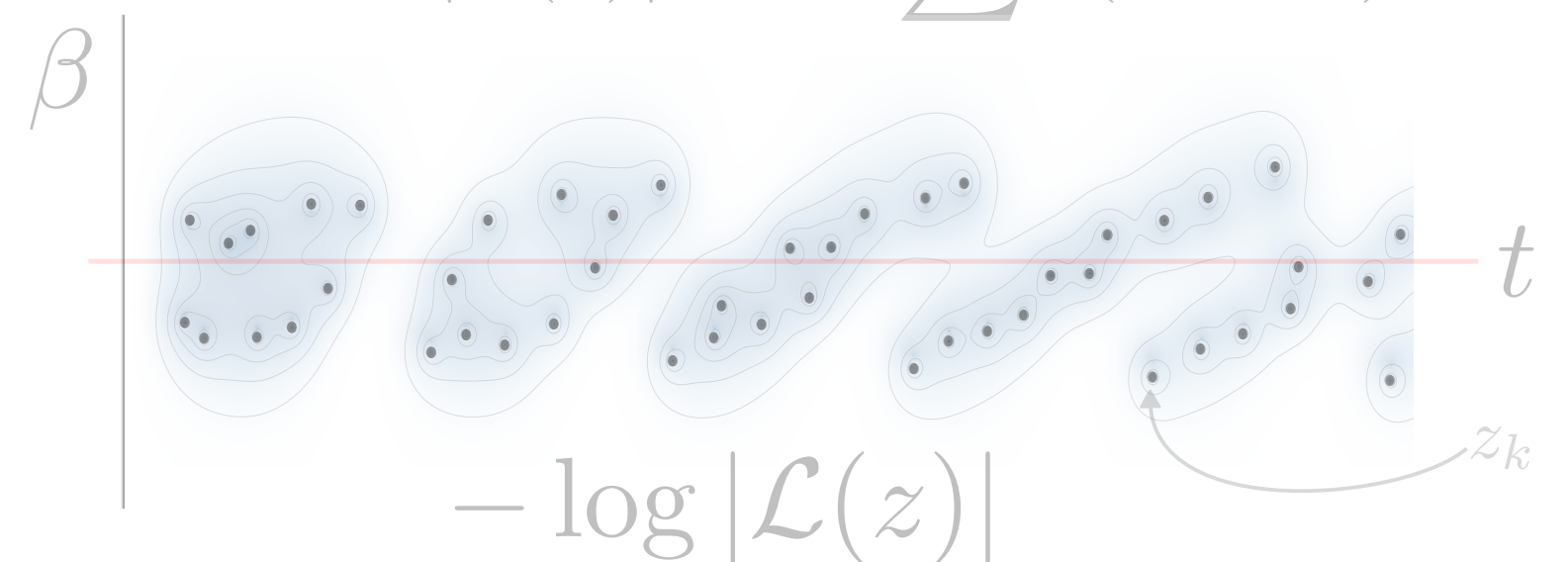
$$it \rightarrow z = \beta + it$$



Log
→



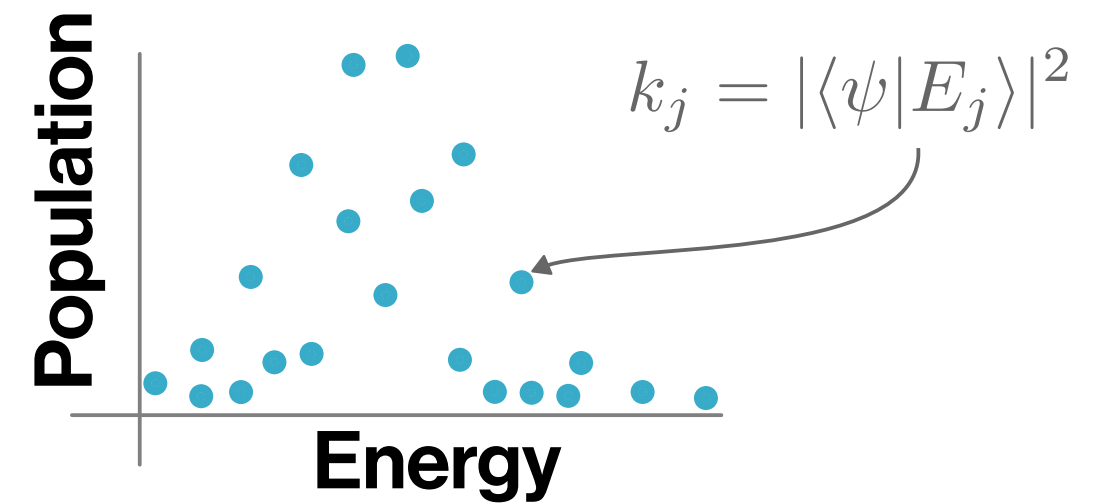
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Survival amplitude

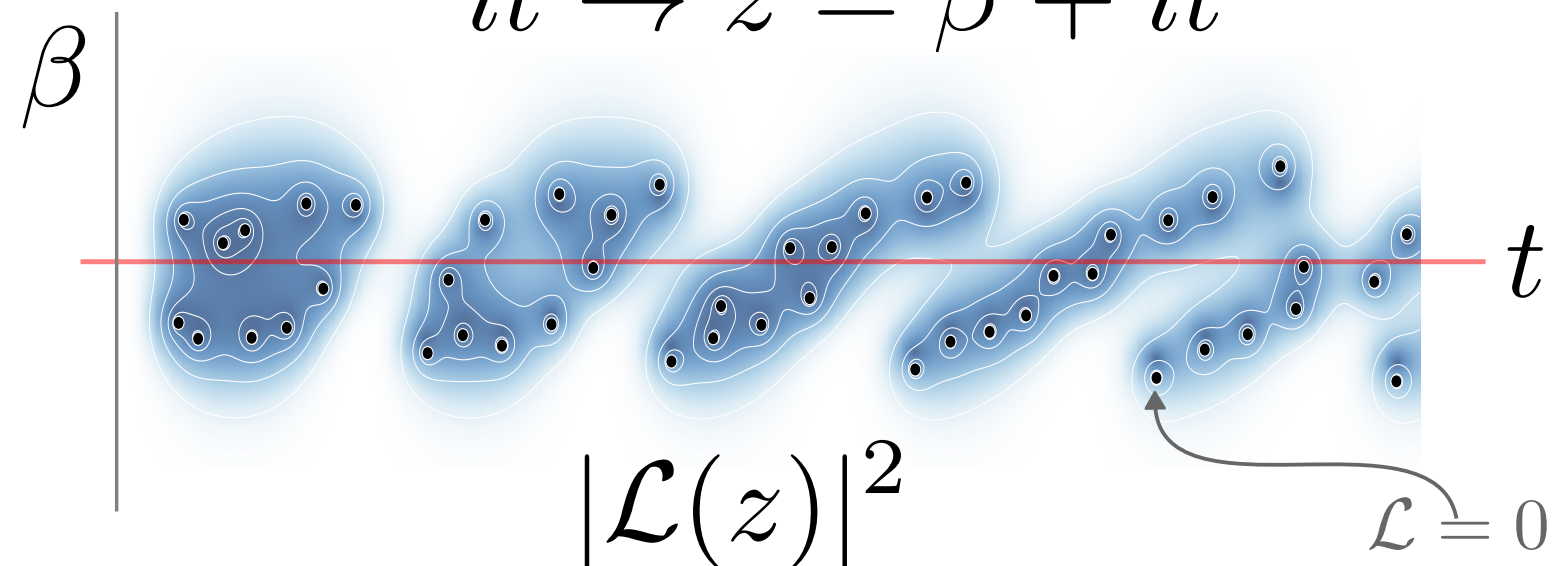
$$\mathcal{L}(t) = \langle \psi | \hat{U}(t) | \psi \rangle = \sum_j k_j e^{-iE_j t}$$

State
 $|\psi\rangle$

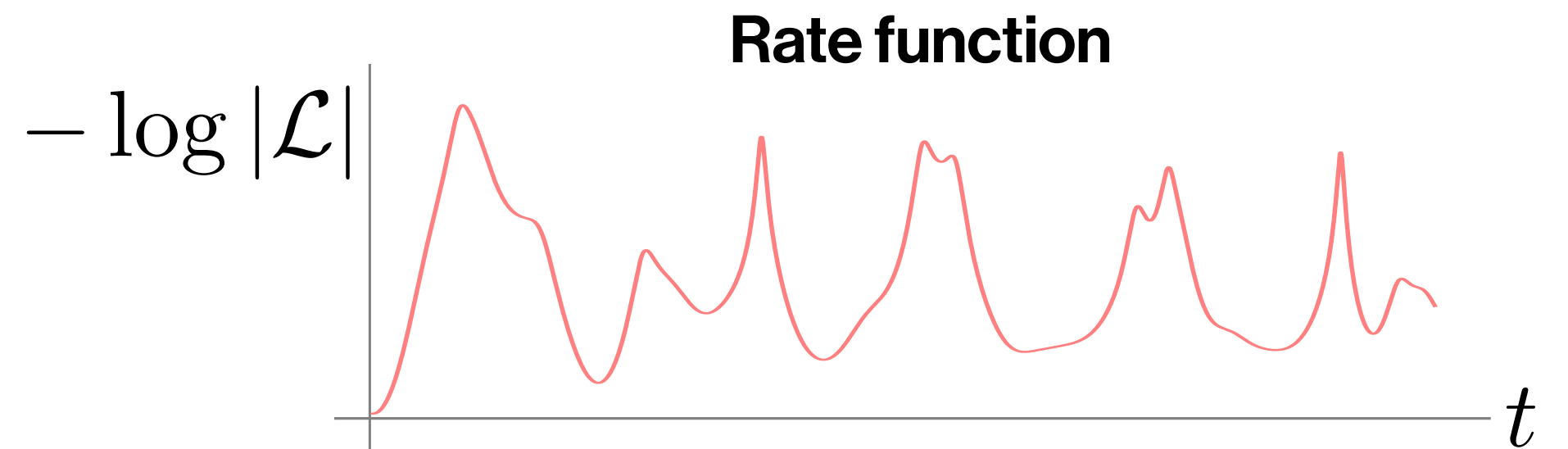


unique complex extension

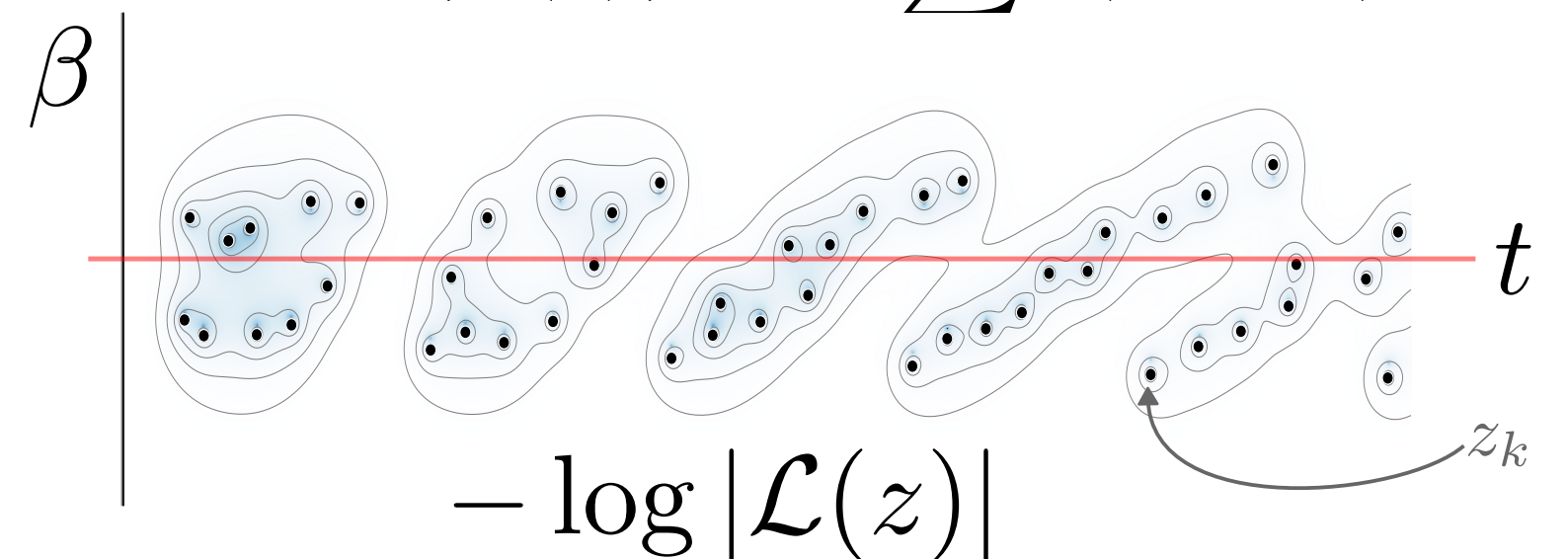
$$it \rightarrow z = \beta + it$$



Log
→



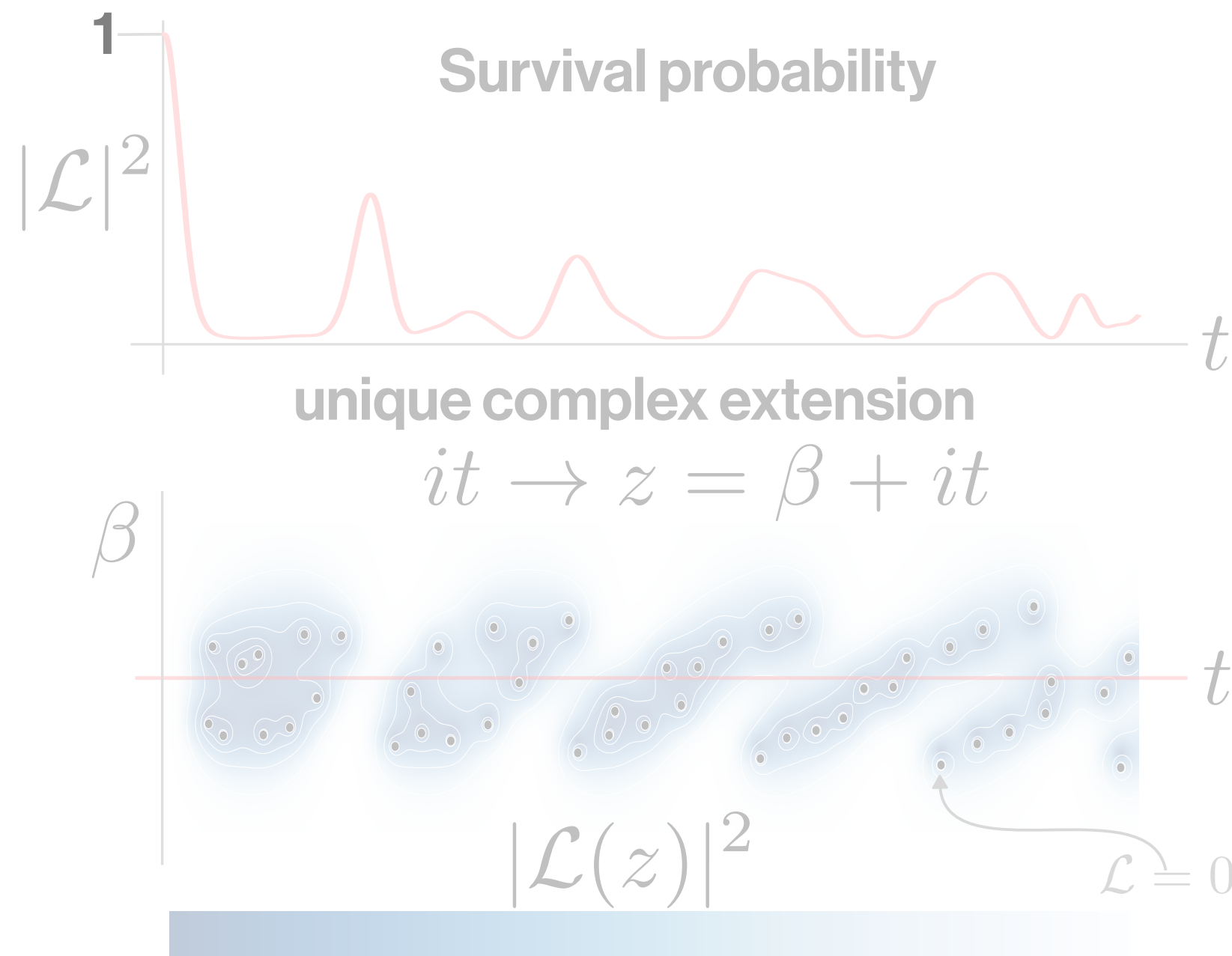
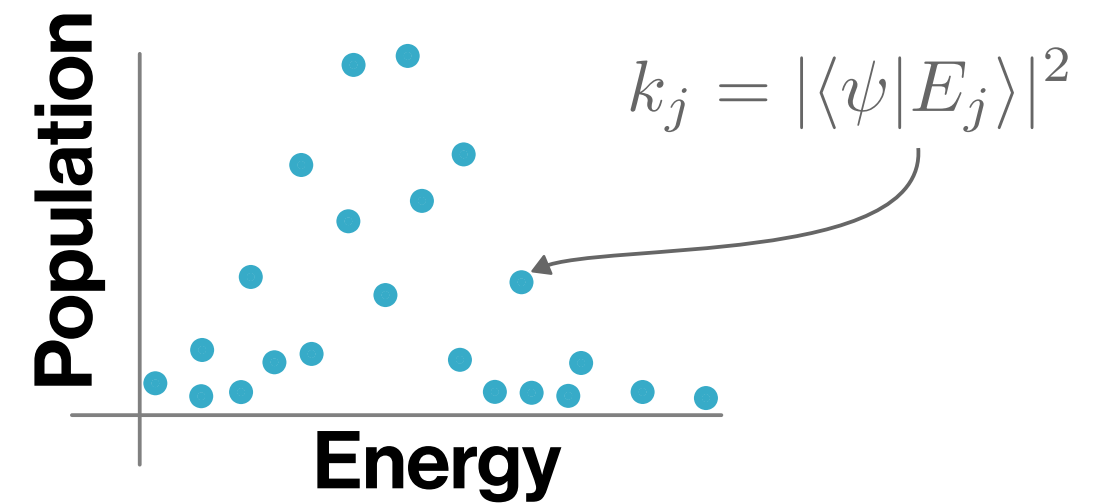
$$\Delta \log |\mathcal{L}(z)| = 2\pi \sum \delta(z - z_k)$$



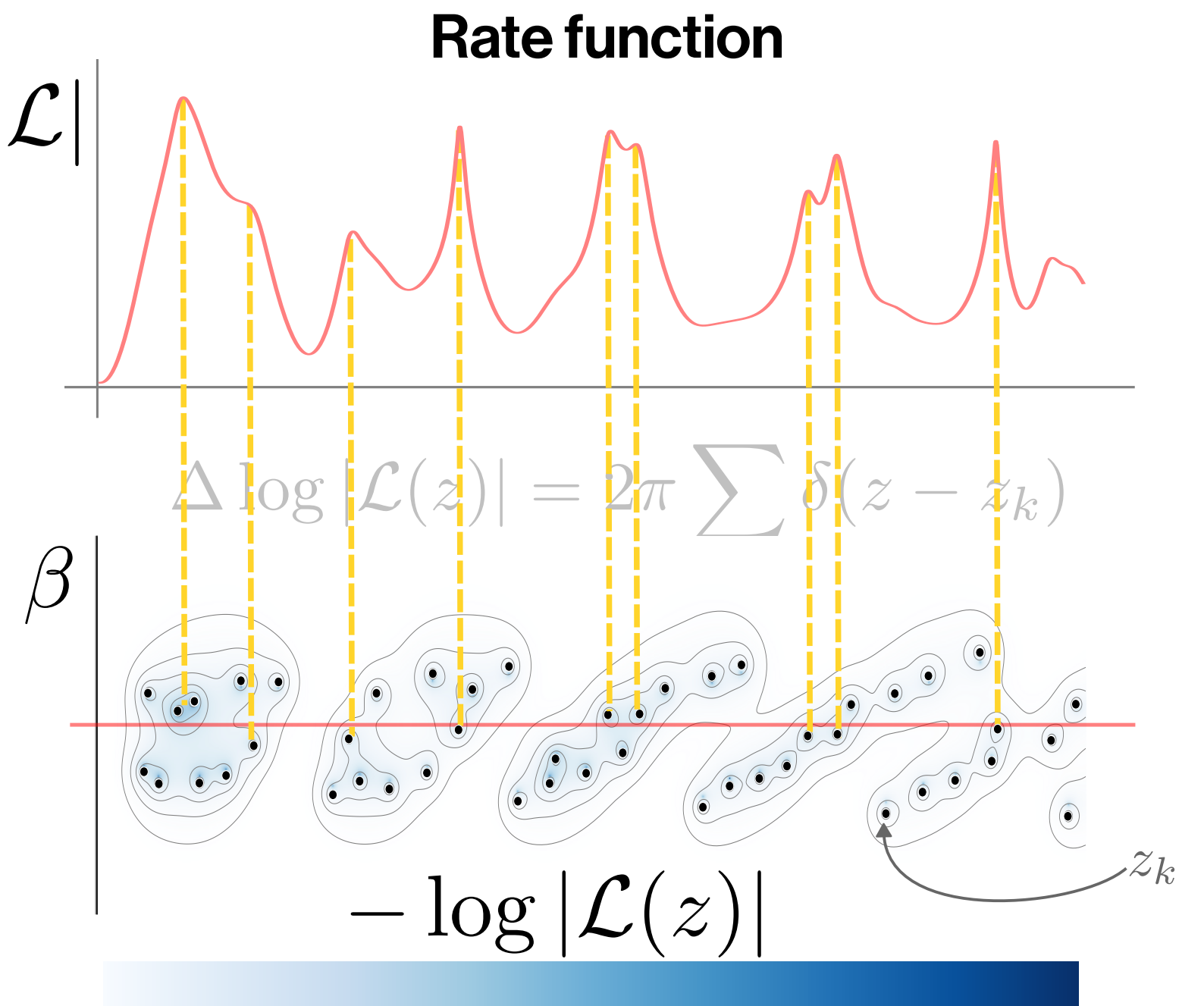
Survival amplitude

$$\mathcal{L}(t) = \langle \psi | \hat{U}(t) | \psi \rangle = \sum_j k_j e^{-iE_j t}$$

State
 $|\psi\rangle$

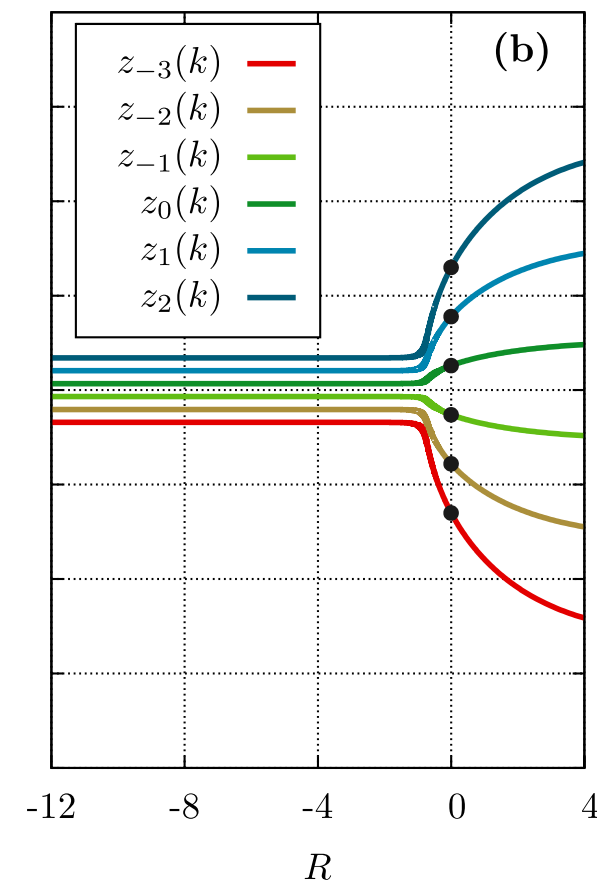
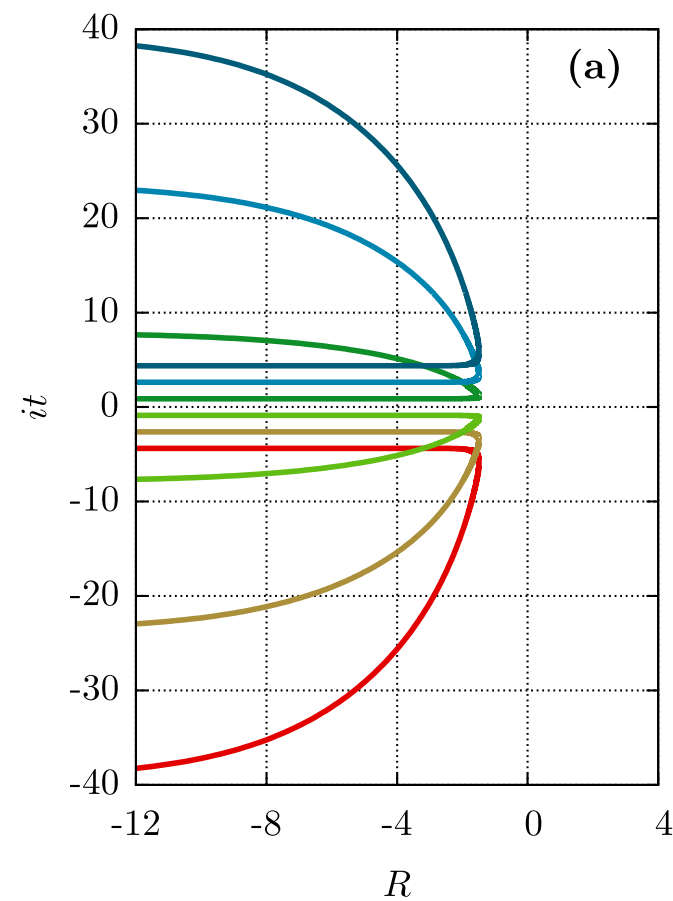
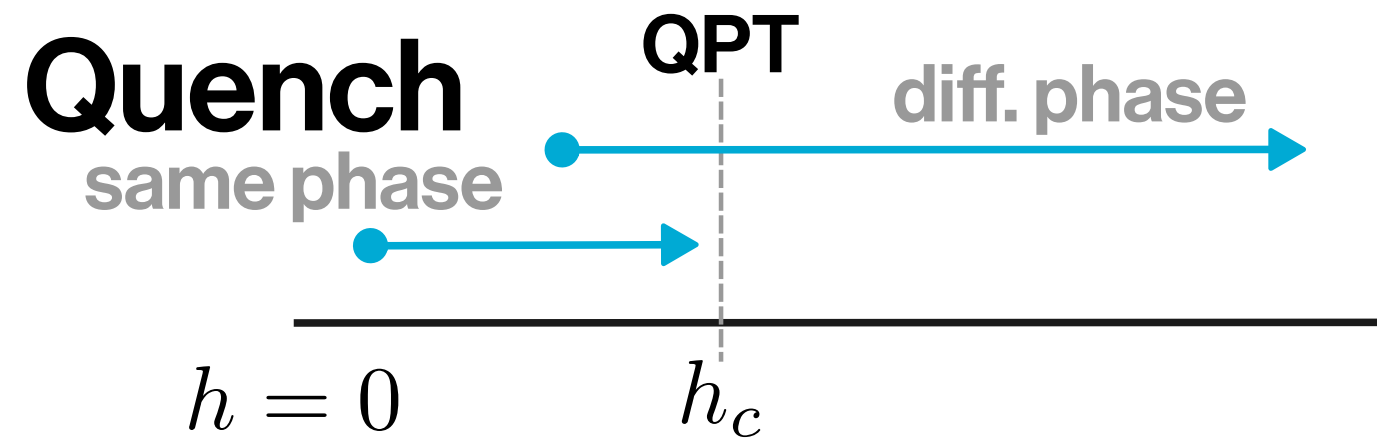


Log
→

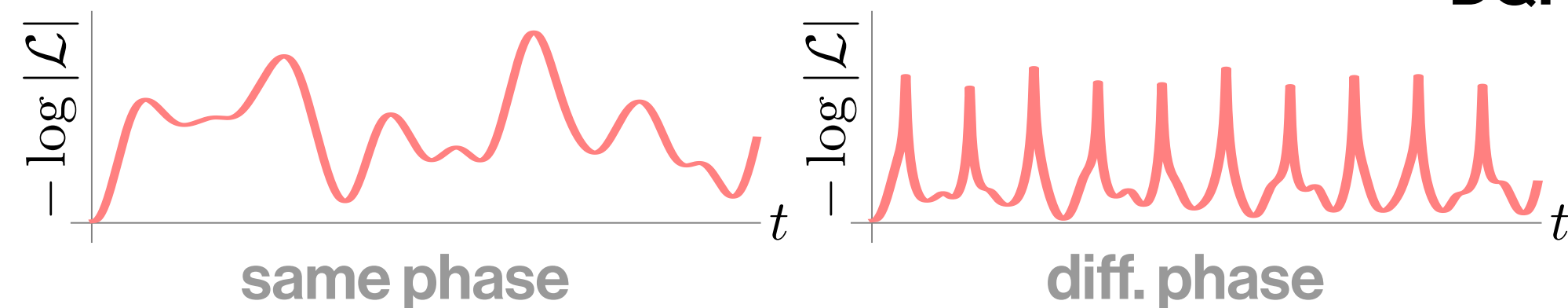


Heyl et al.
PRL 110, 135704 (2013)

$$\hat{H} = -J \sum_{i=1} \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + h \sum_{i=1} \hat{\sigma}_i^x$$



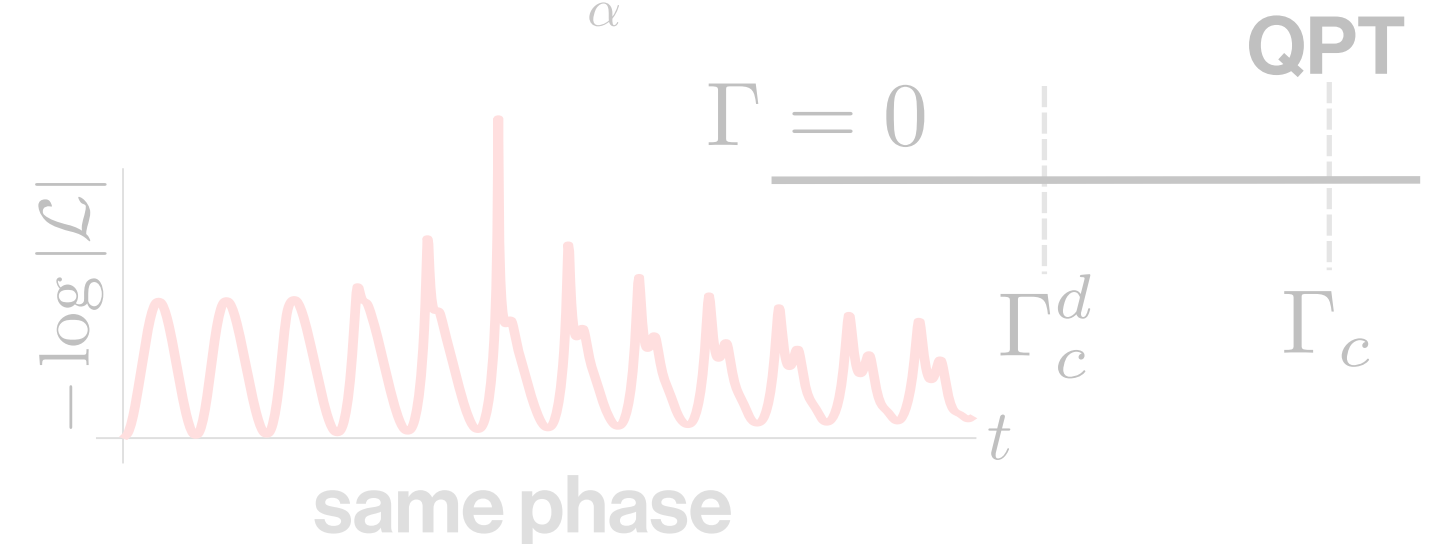
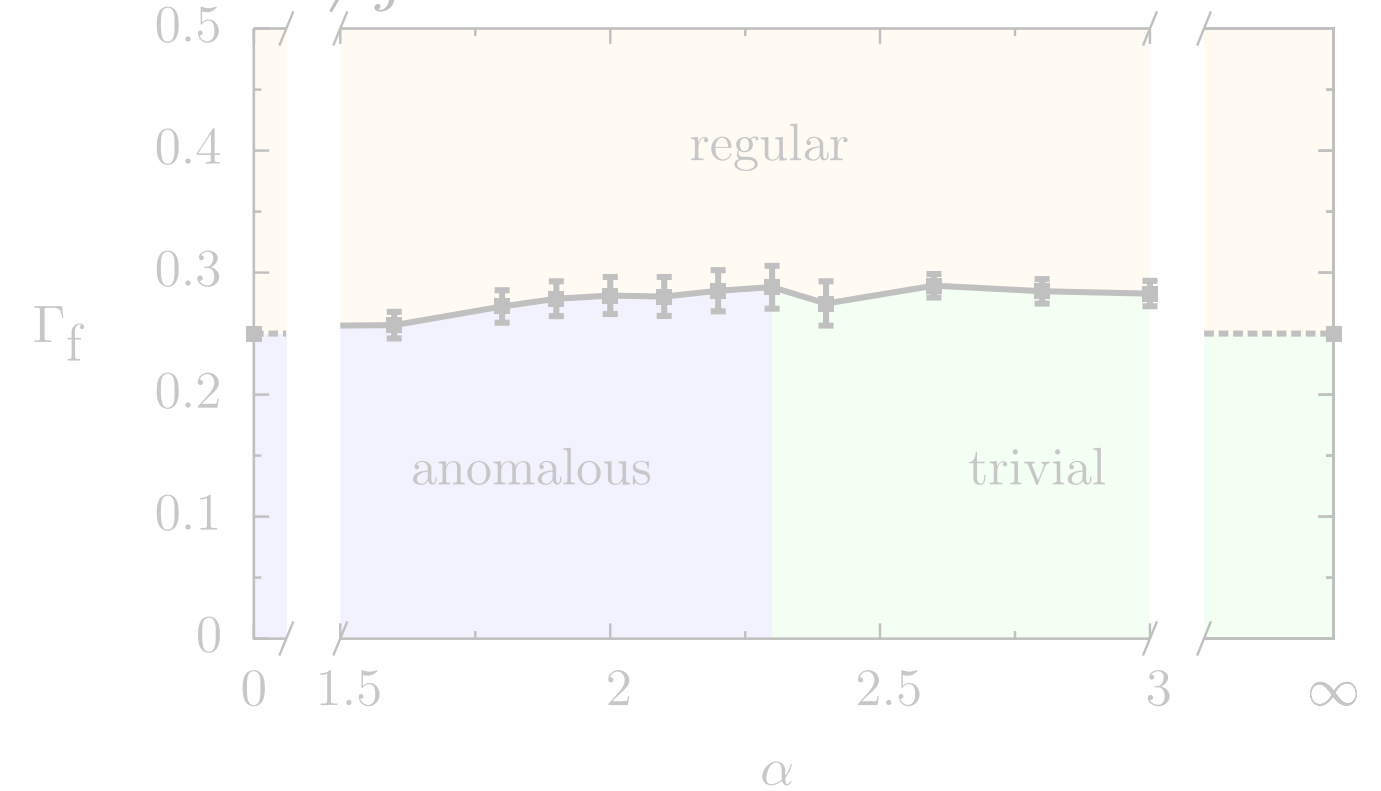
$N \rightarrow \infty$
DQPT (-II)



Why zeros?

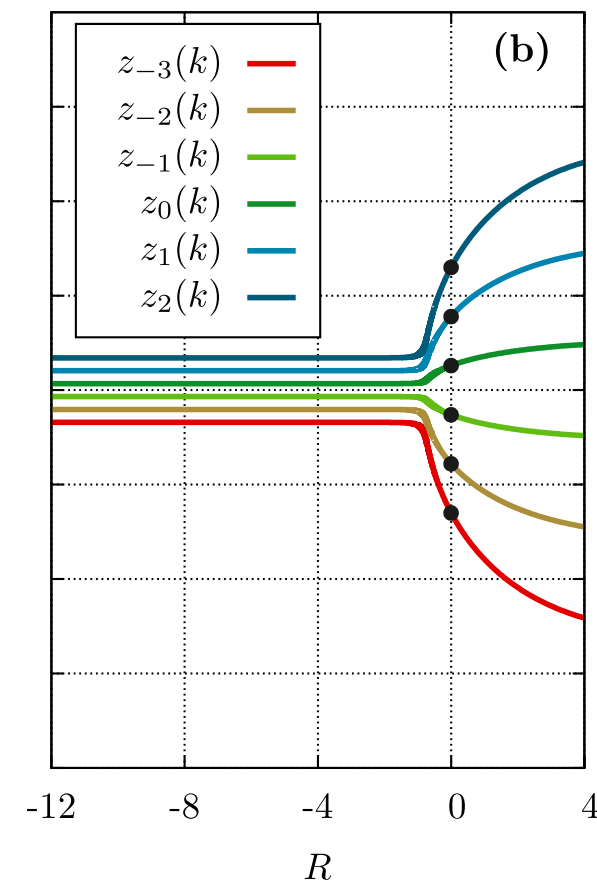
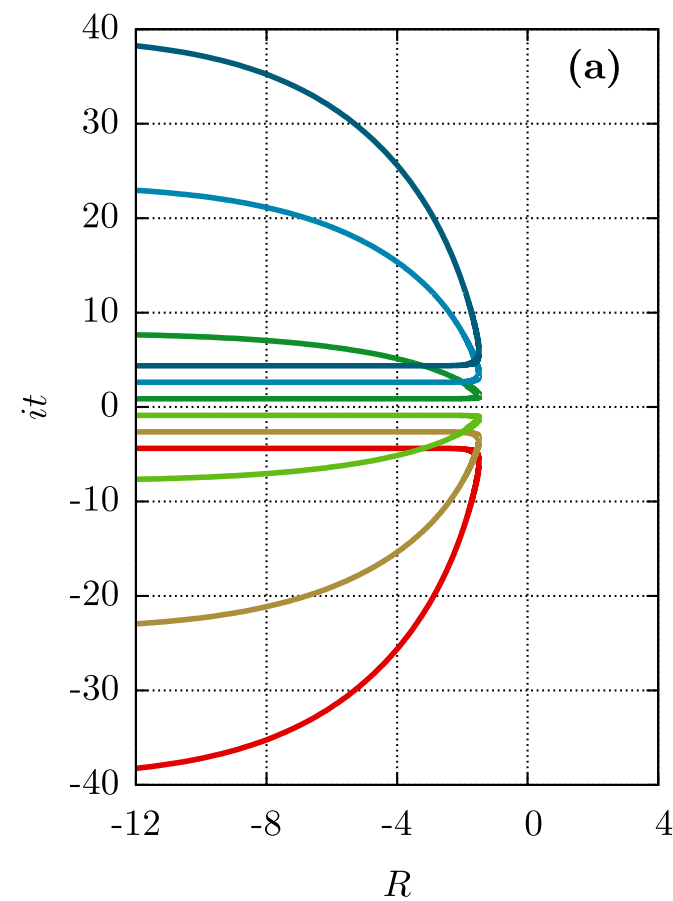
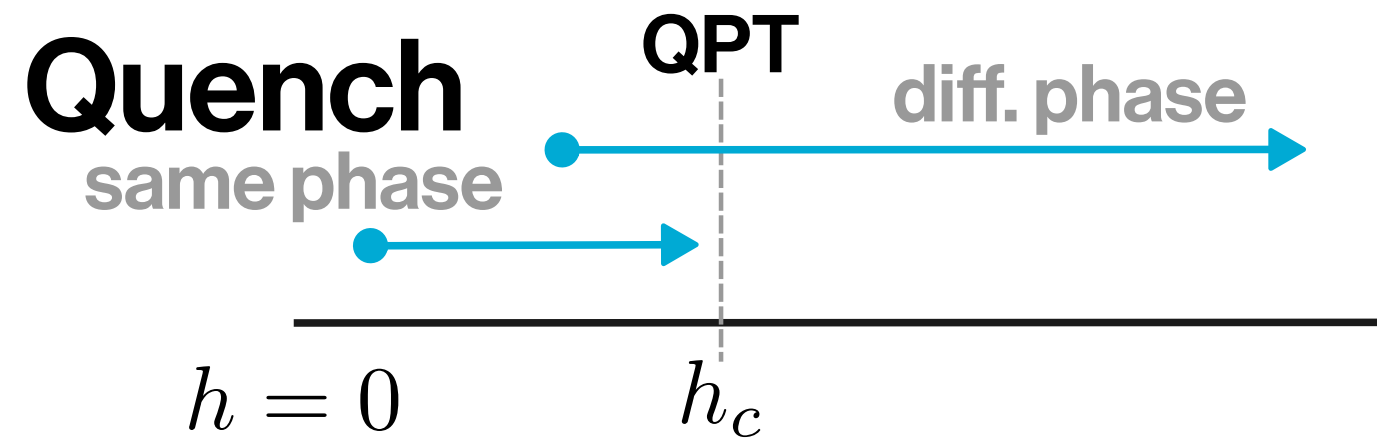
Homrighausen et al.
PRB 96, 104436 (2017)

$$\hat{H} = -\frac{J}{2N} \sum_{i \neq j} \frac{1}{|i - j|^\alpha} \hat{S}_i^z \hat{S}_{i+1}^z + \Gamma \sum_{i=1} \hat{S}_i^x$$

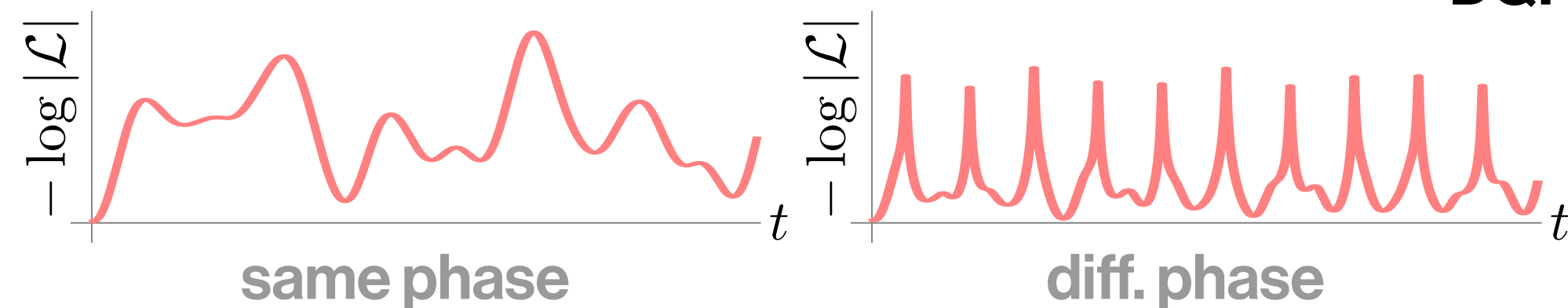


Heyl et al.
PRL 110, 135704 (2013)

$$\hat{H} = -J \sum_{i=1} \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + h \sum_{i=1} \hat{\sigma}_i^x$$



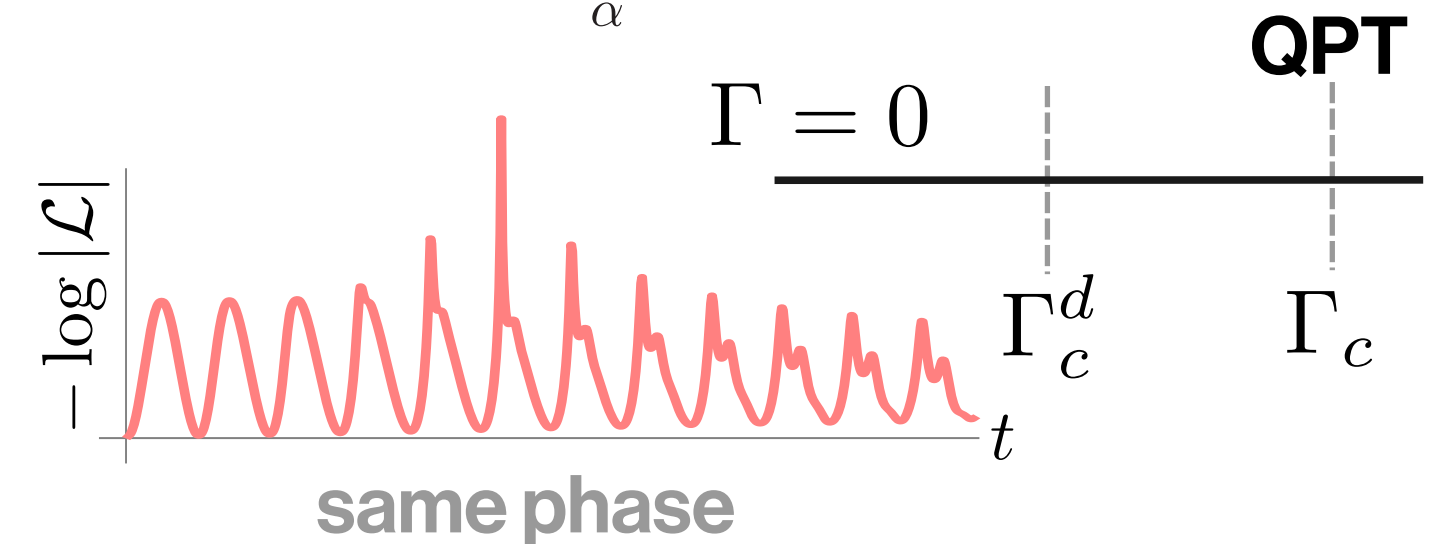
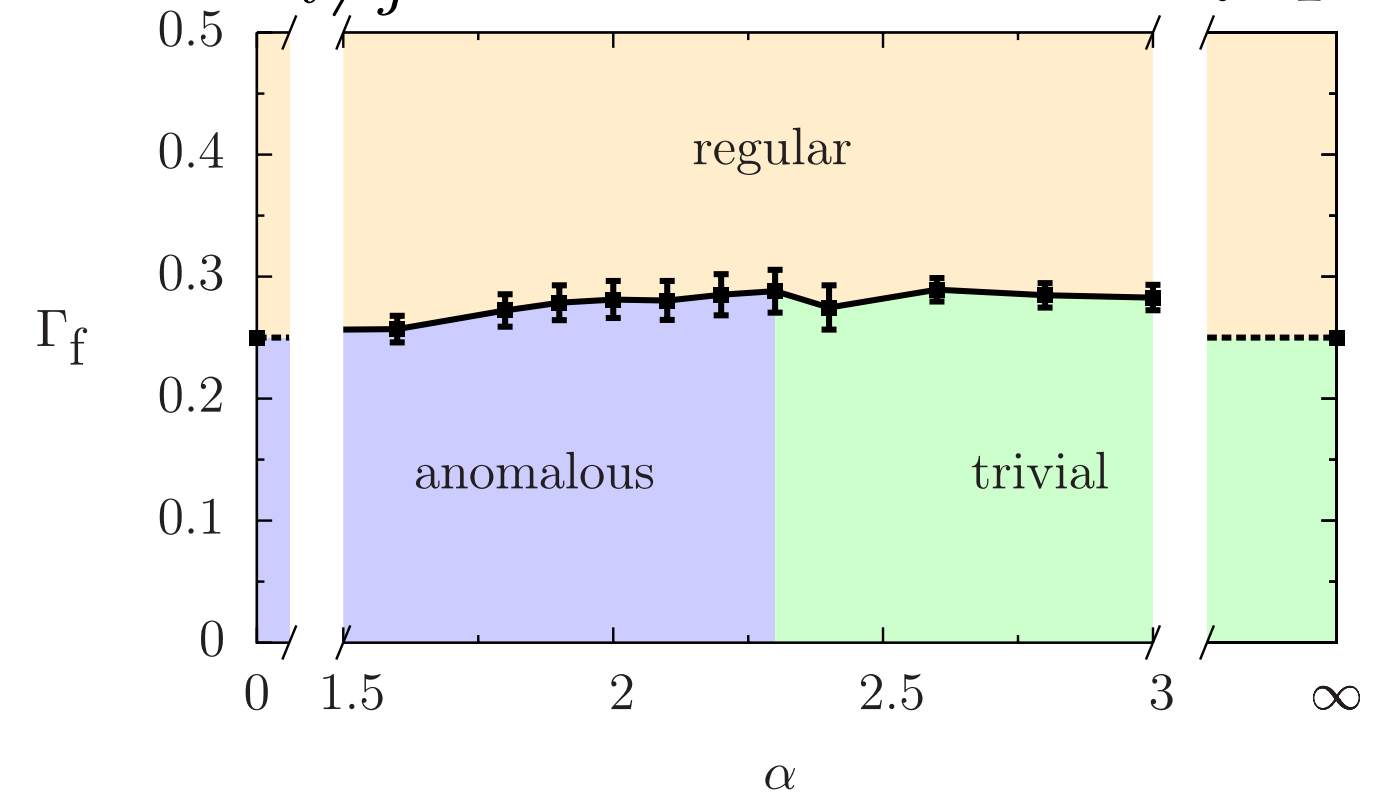
$N \rightarrow \infty$
DQPT (-II)

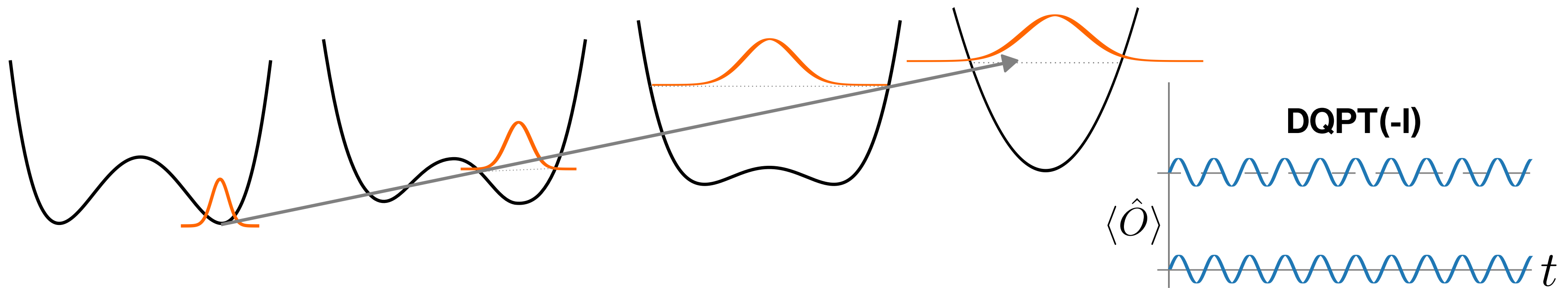


Why zeros?

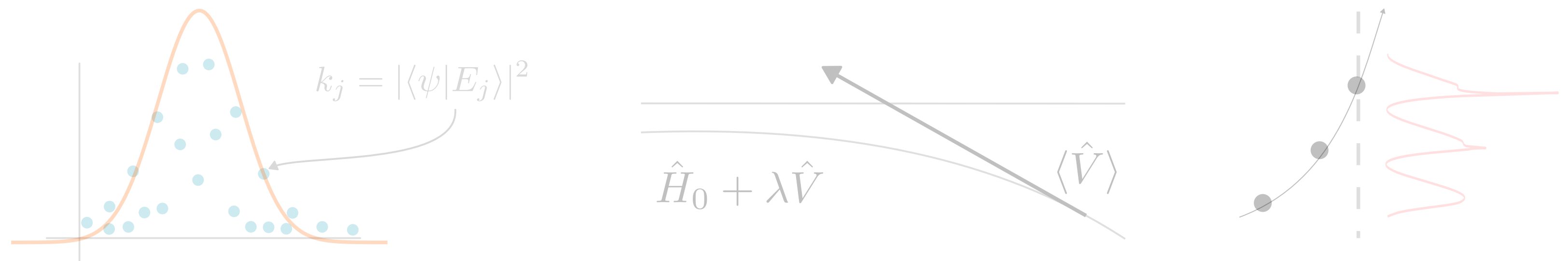
Homrighausen et al.
PRB 96, 104436 (2017)

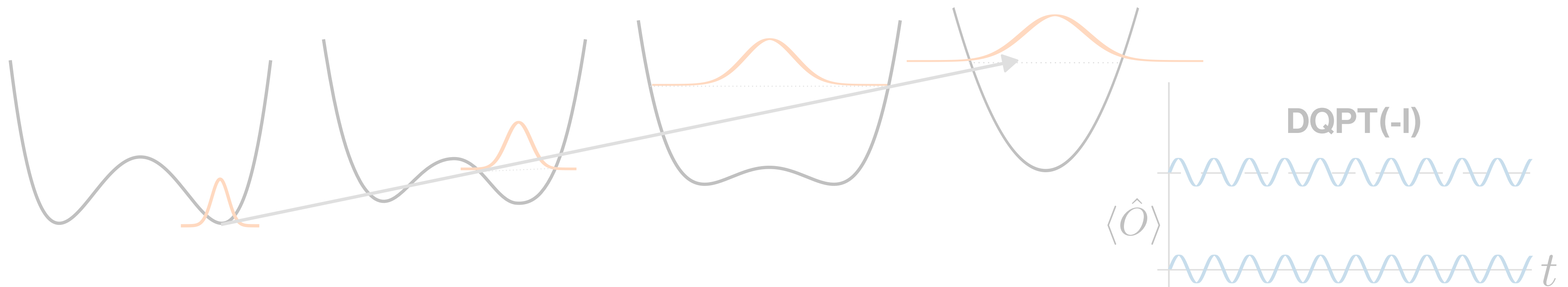
$$\hat{H} = -\frac{J}{2N} \sum_{i \neq j} \frac{1}{|i - j|^\alpha} \hat{S}_i^z \hat{S}_{i+1}^z + \Gamma \sum_{i=1} \hat{S}_i^x$$



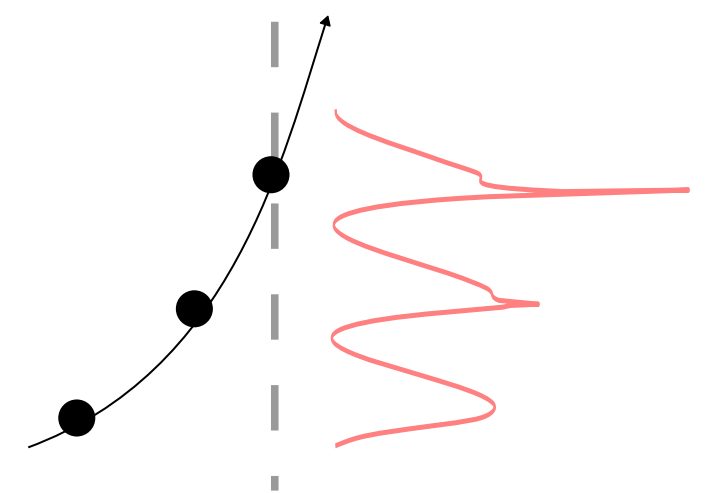
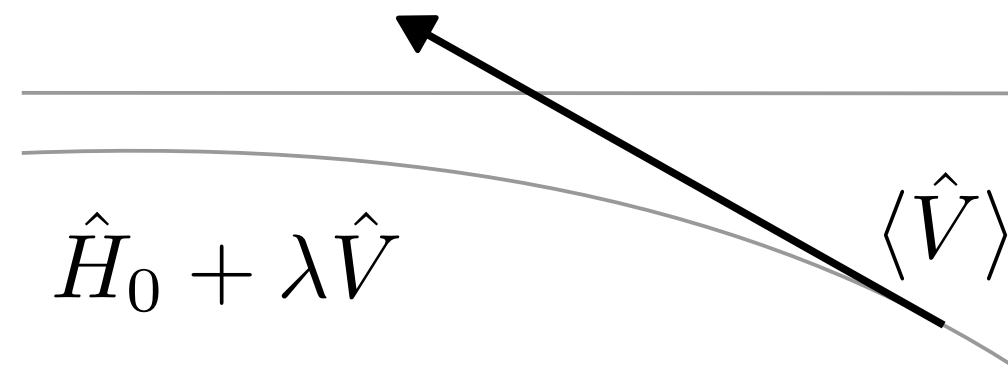
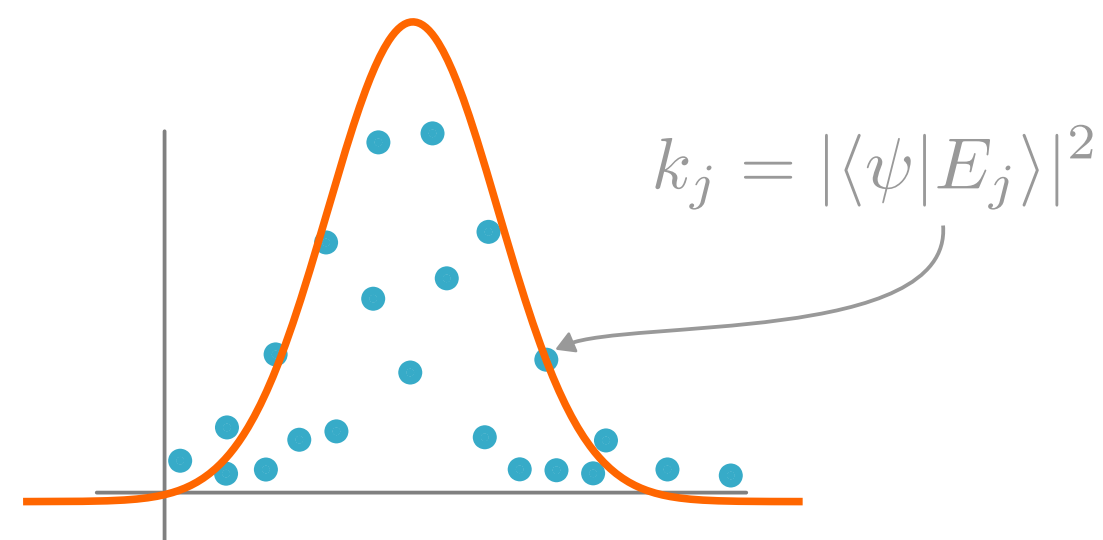


What determines the distribution of the zeros?





What determines the distribution of the zeros?

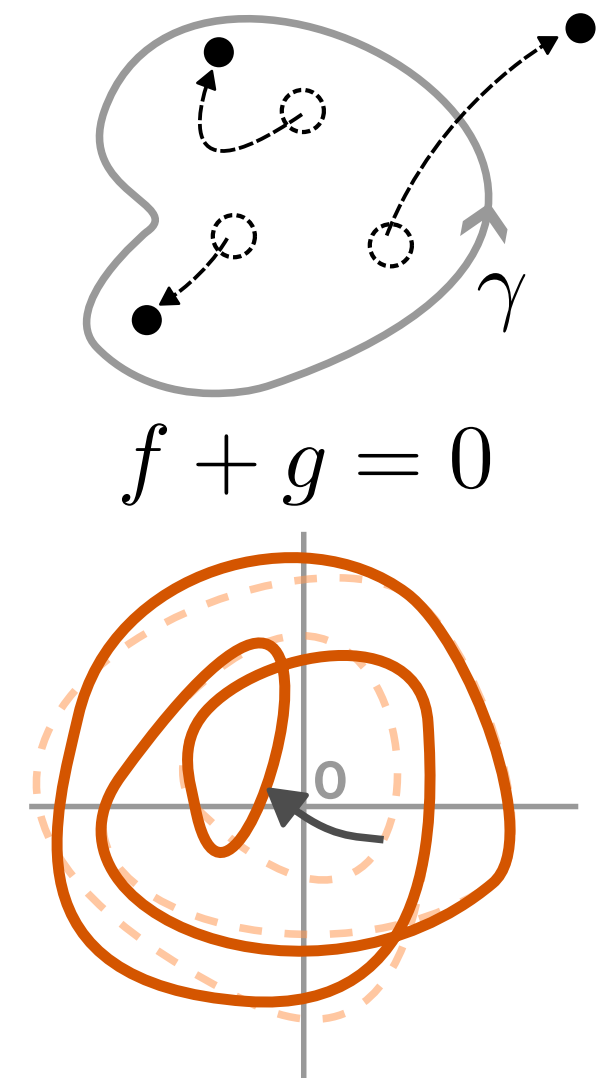


Zeros of holomorphic functions

Zeros cannot be created or annihilated by a hol. perturbation

$$\# \text{ zeros of } f(z) \text{ in } \gamma = \text{winding number} = W[f \circ \gamma] = W \left[\begin{array}{c} \text{Diagram of a curve } \gamma \text{ in the complex plane} \end{array} \right]$$

The diagram shows a curve γ in the complex plane, with the origin 0 marked. The curve is a solid orange line that winds around the origin. The axes are labeled t and β .



Num. of zeros is determined by the dominant part

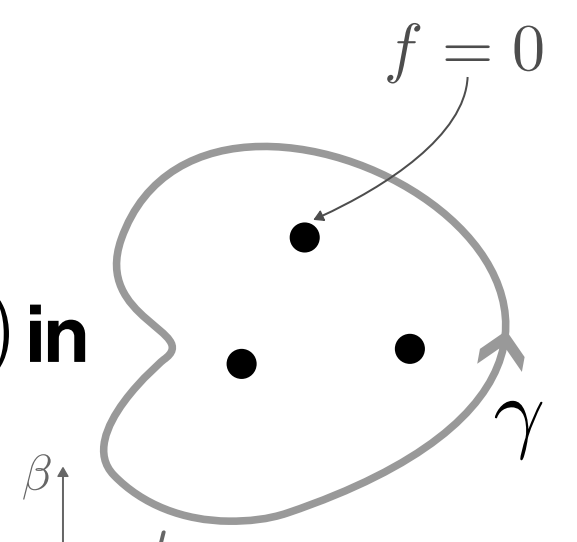
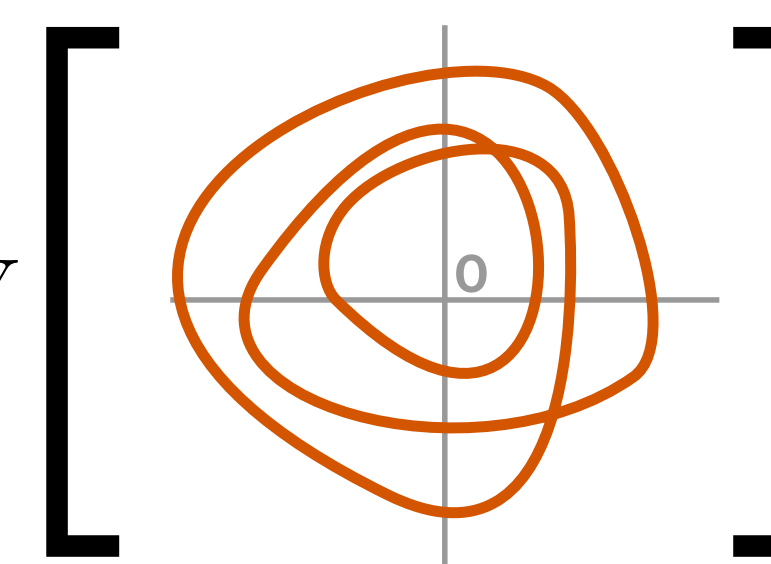
$$\# \text{ zeros in } \gamma \text{ for } f(z) = \# \text{ zeros in } \gamma \text{ for } f(z) + g(z) \quad \text{if } g < f \text{ on } \gamma$$

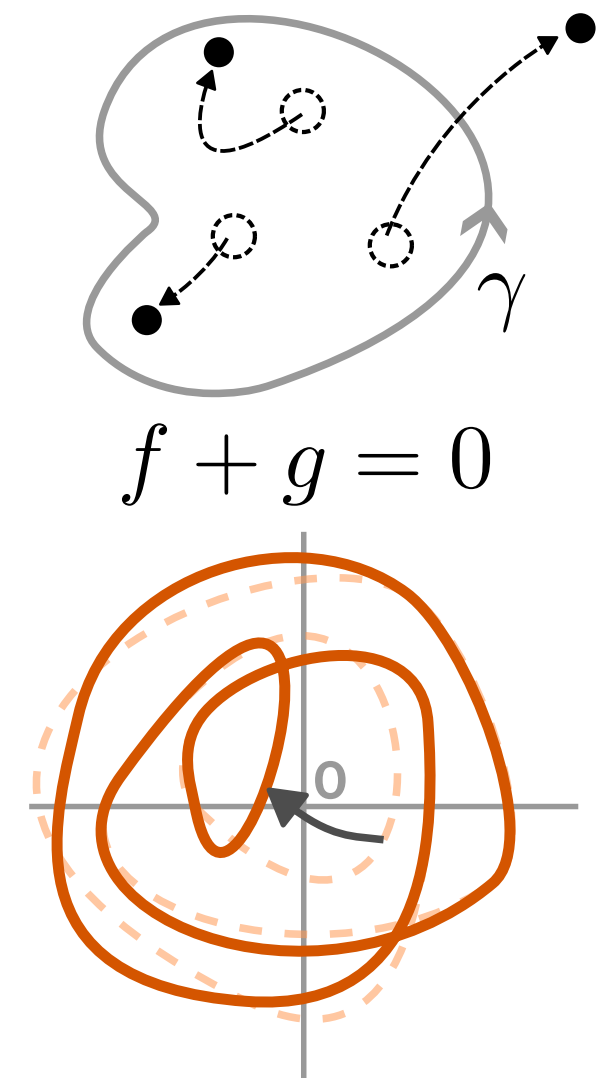
Rouché's theorem

The diagram shows two regions, each containing three zeros (black dots). The left region is labeled $f(z)$ and the right region is labeled $f(z) + g(z)$. Both regions are bounded by a curve γ .

Zeros of holomorphic functions

Zeros cannot be created or annihilated by a hol. perturbation

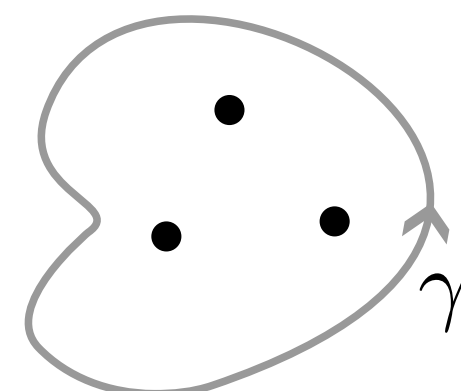
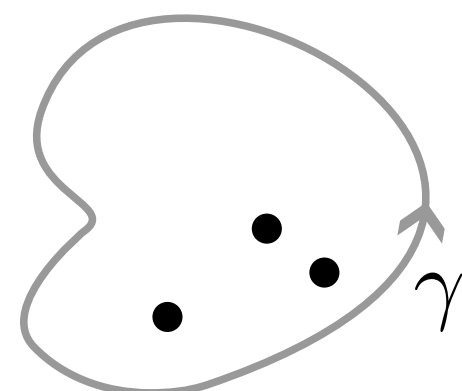
$$\# \text{ zeros of } f(z) \text{ in } \gamma = W[f \circ \gamma] = W \left[\begin{array}{c} \text{winding number} \\ \text{diagram} \end{array} \right]$$





Num. of zeros is determined by the dominant part

$$\# \text{ zeros in } \gamma \text{ for } f(z) = \# \text{ zeros in } \gamma \text{ for } f(z) + g(z) \quad \text{if } g < f \text{ on } \gamma$$

Rouché's theorem

Different time scales

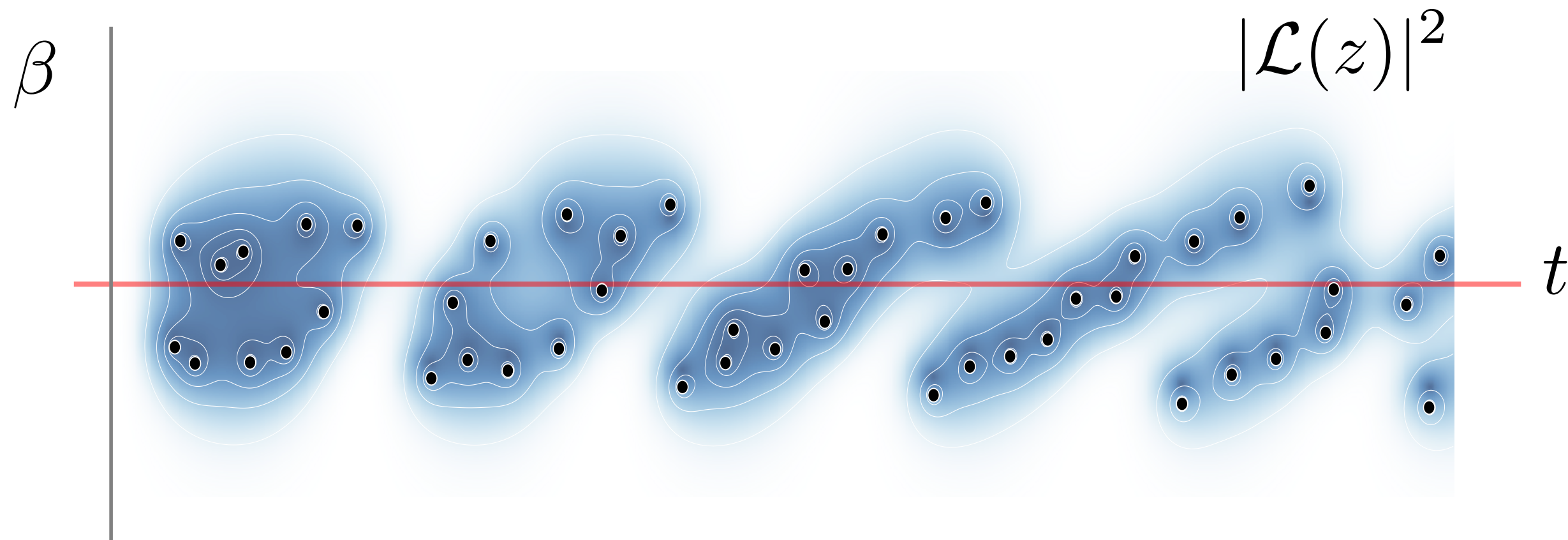
Level spacing

$$\Delta_i = \Delta + \varepsilon_i$$



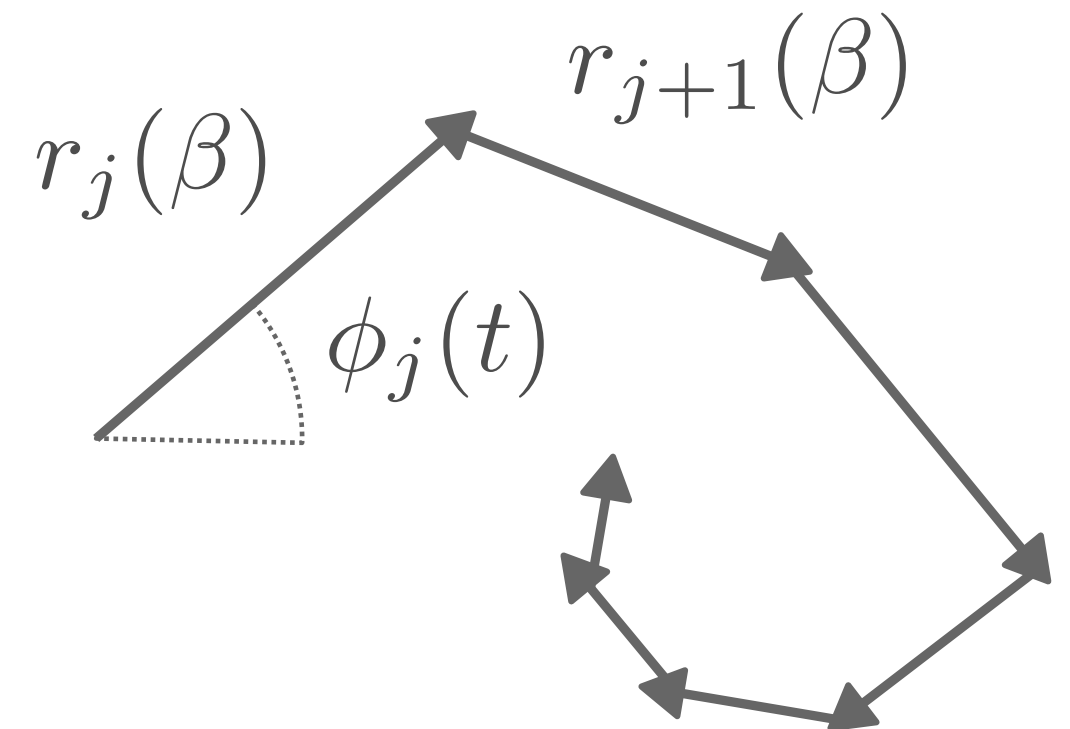
Dephasing

$$\mathcal{L}(\beta + it) = \sum_j k_j \underbrace{e^{-\beta E_j}}_{r_j(\beta)} e^{-iE_j t}$$



Short times

Long times

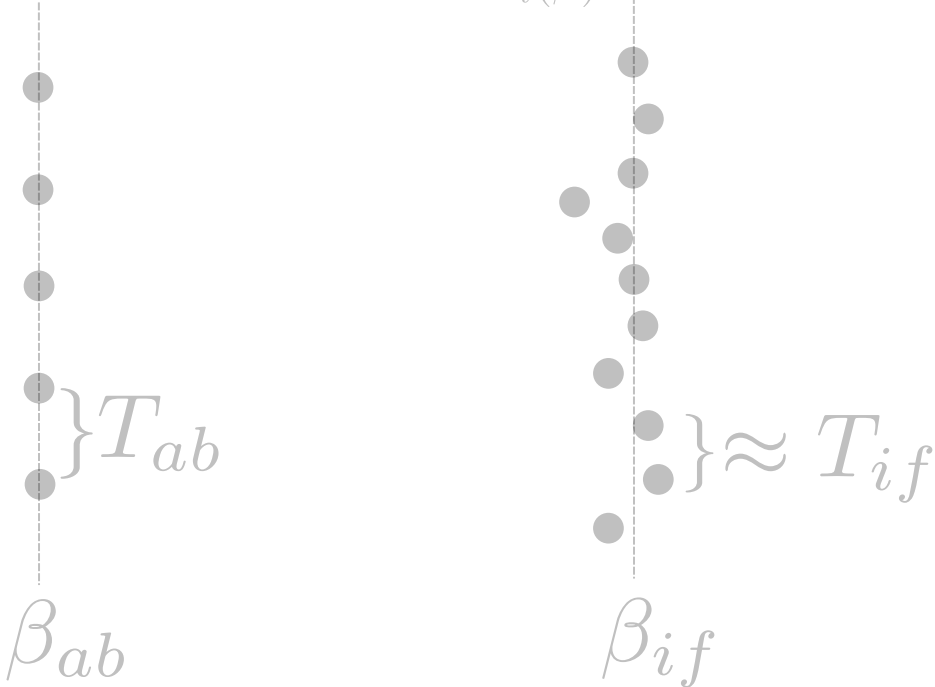
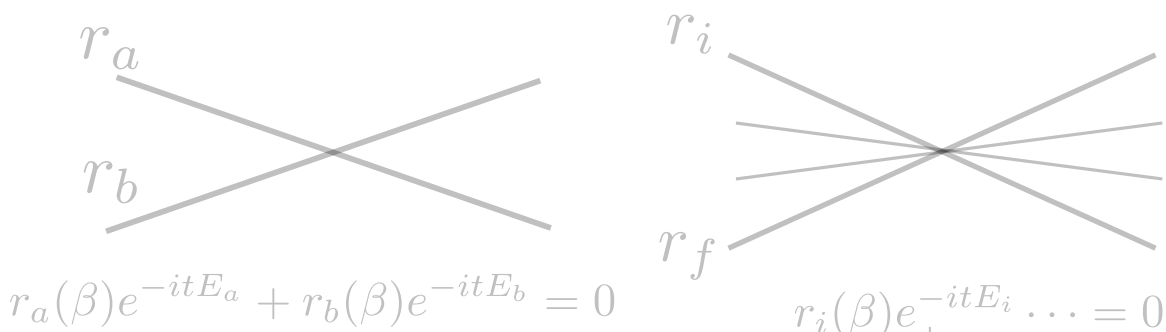
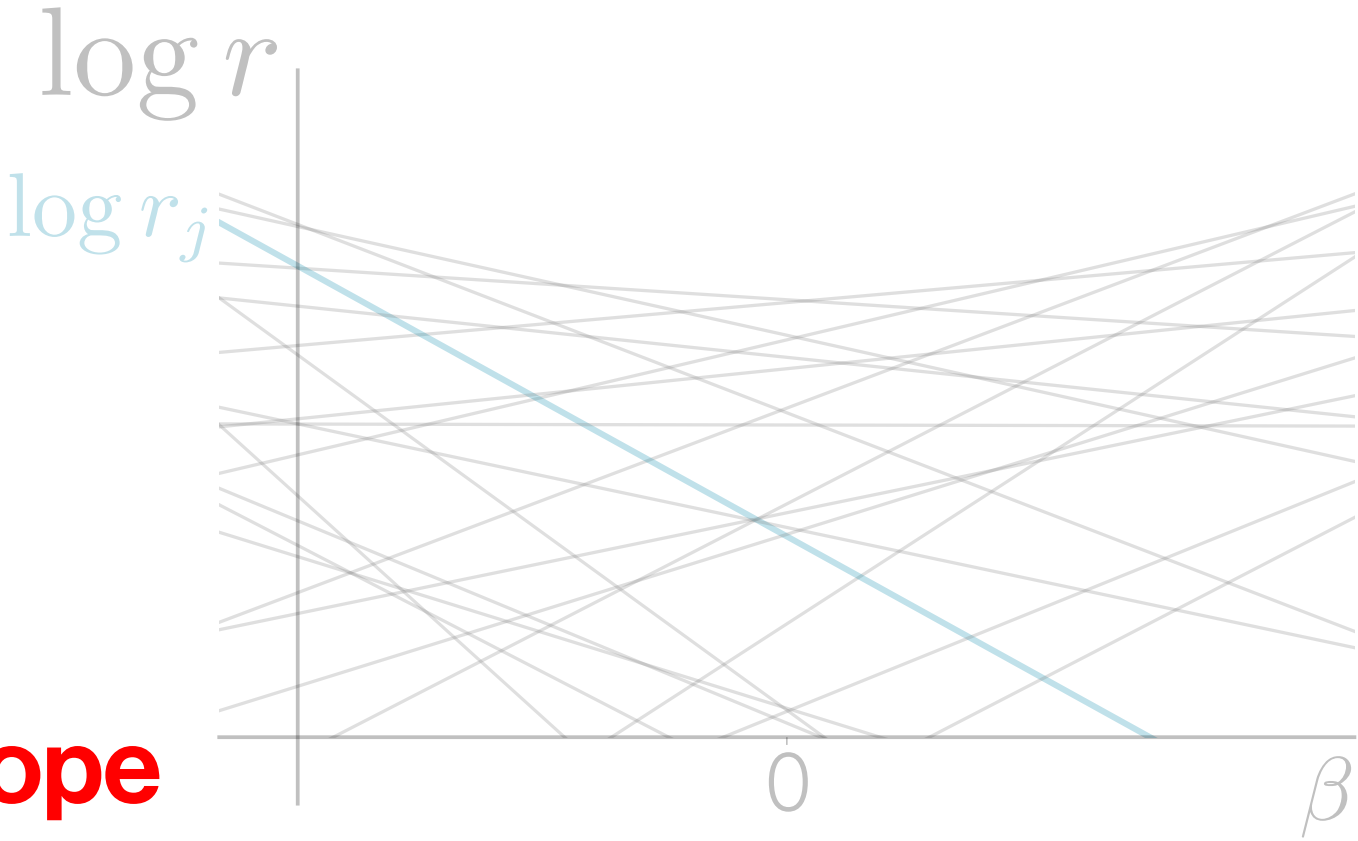
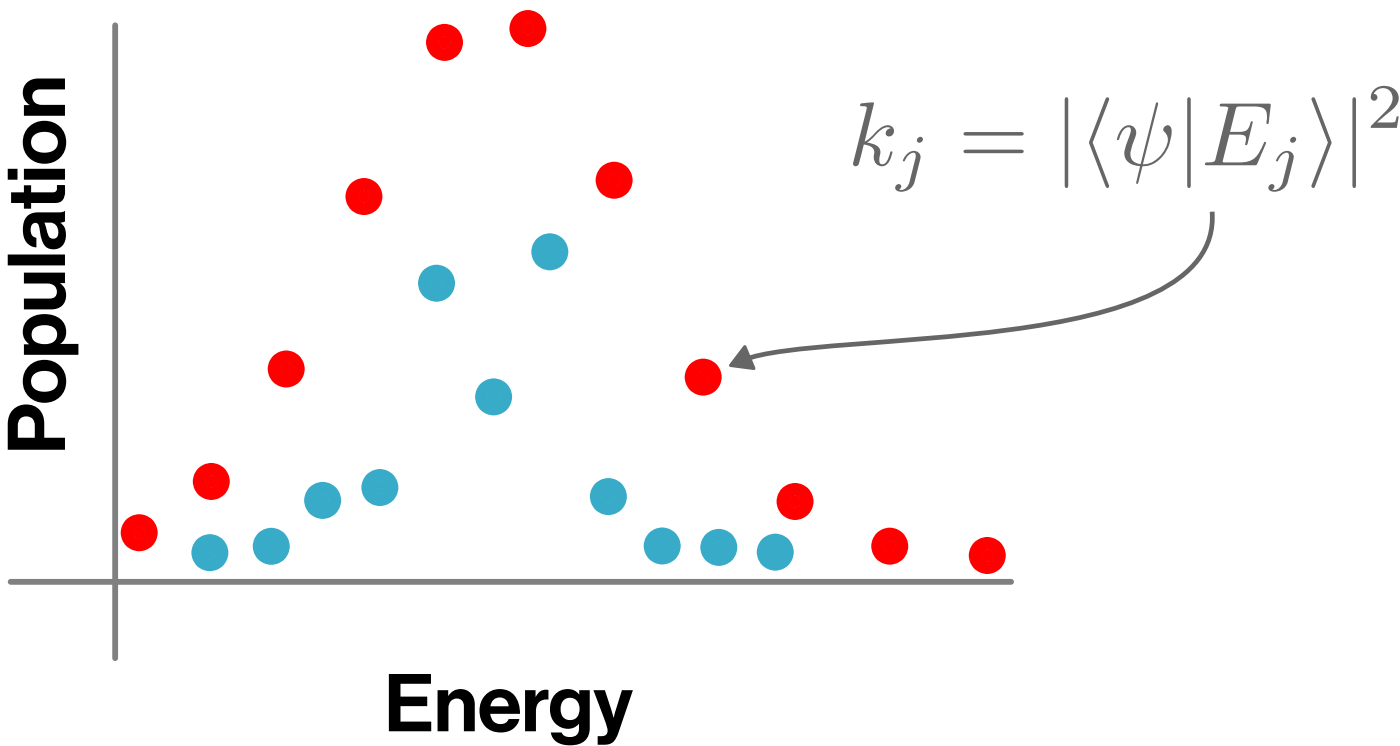


Long times

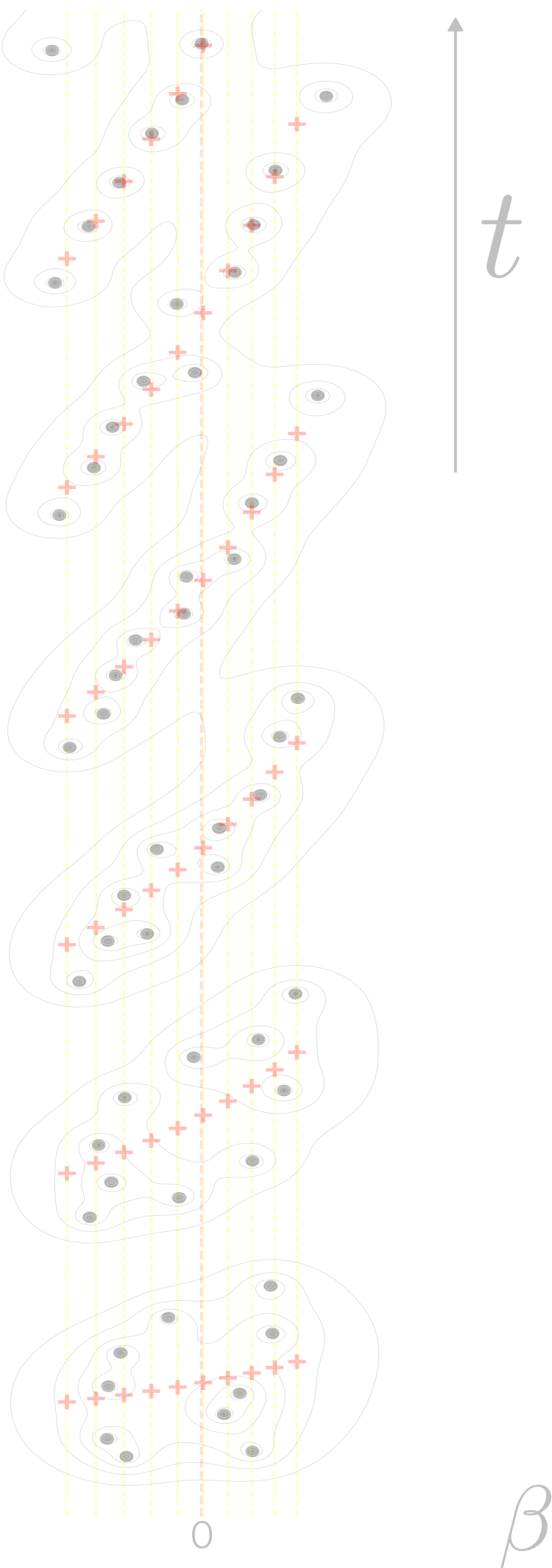
$$\mathcal{L}(\beta + it) = \sum_j r_j(\beta) e^{-iE_j t}$$

The dominant part = **The Envelope**

$$k_a \left(\frac{k_a}{k_b} \right)^{-\frac{\Delta_{ac}}{\Delta_{ab}}} \geq k_c, \quad \forall k_c, E_c$$



$$z_{ab}^*(n) = \frac{1}{\Delta_{ab}} \left(\log \left(\frac{k_a}{k_b} \right) + i\pi (2n + 1) \right)$$

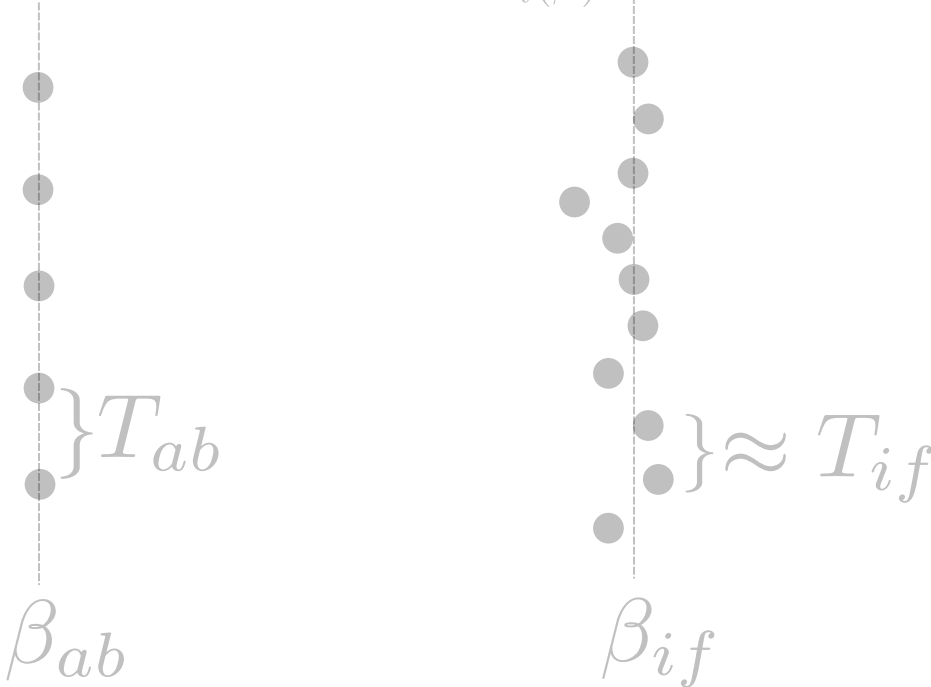
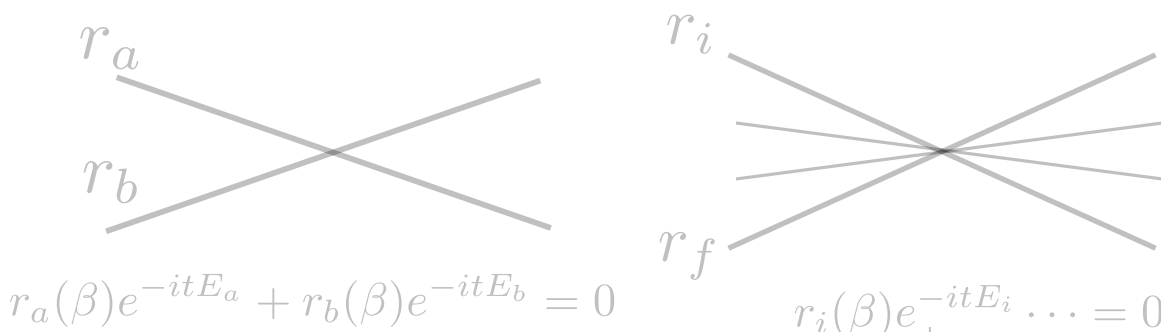
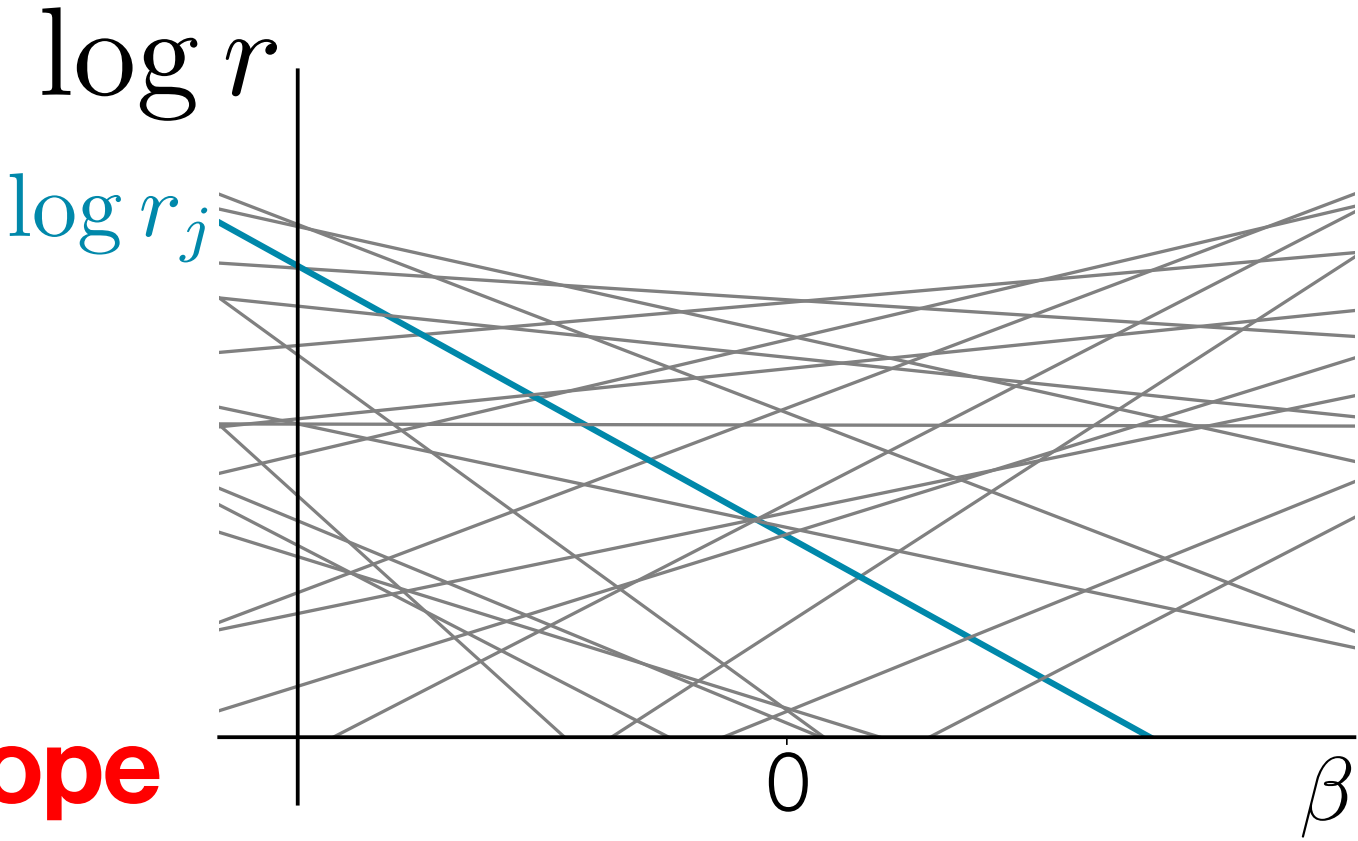
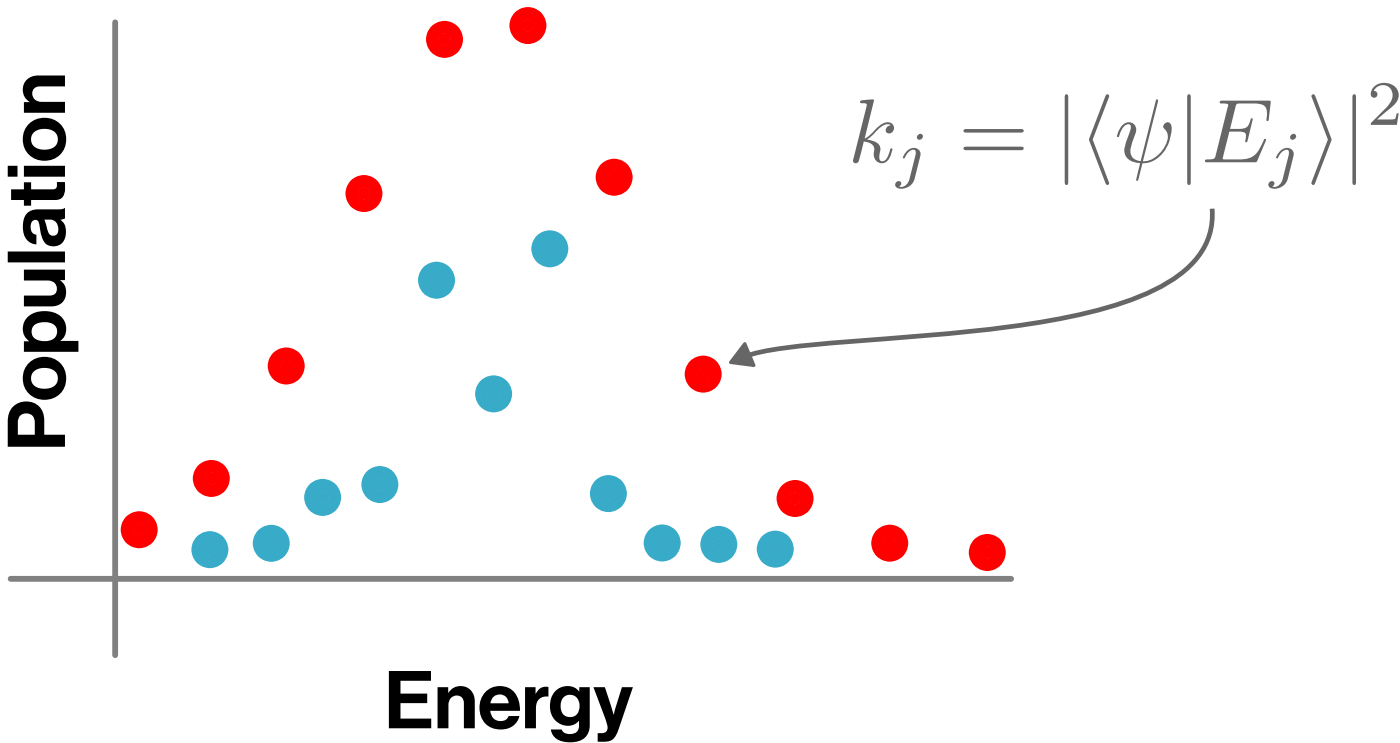


Long times

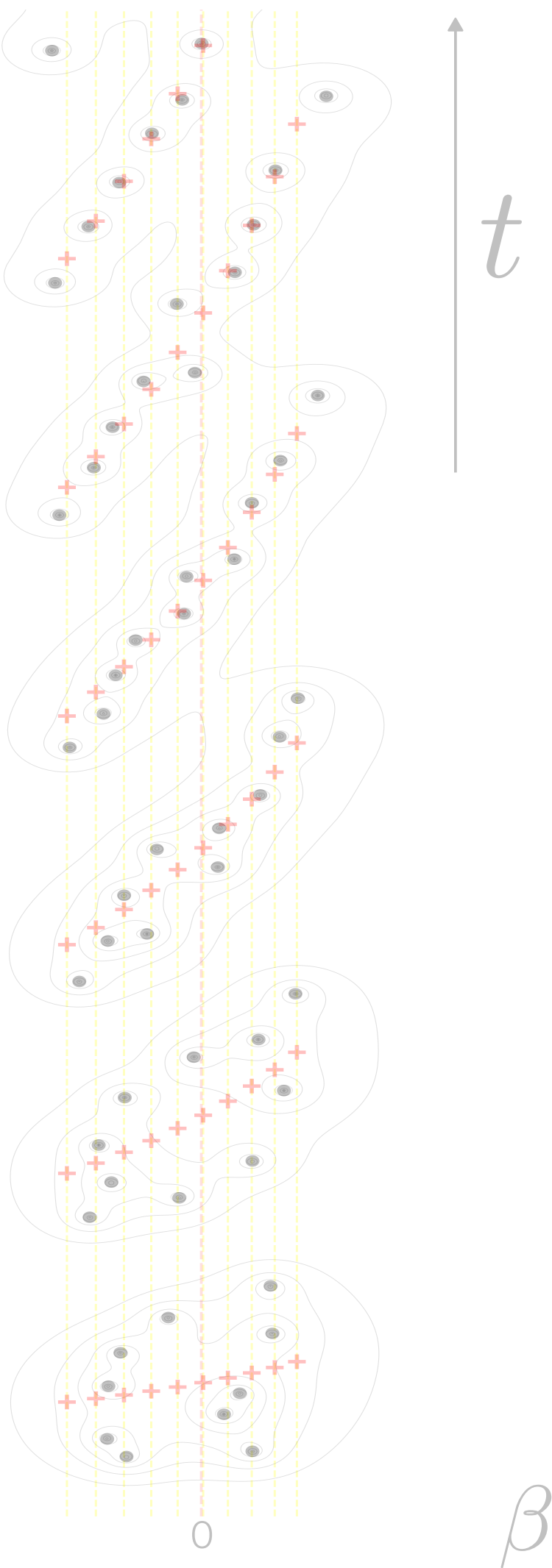
$$\mathcal{L}(\beta + it) = \sum_j r_j(\beta) e^{-iE_j t}$$

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$$z_{ab}^*(n) = \frac{1}{\Delta_{ab}} \left(\log \left(\frac{k_a}{k_b} \right) + i\pi (2n + 1) \right)$$

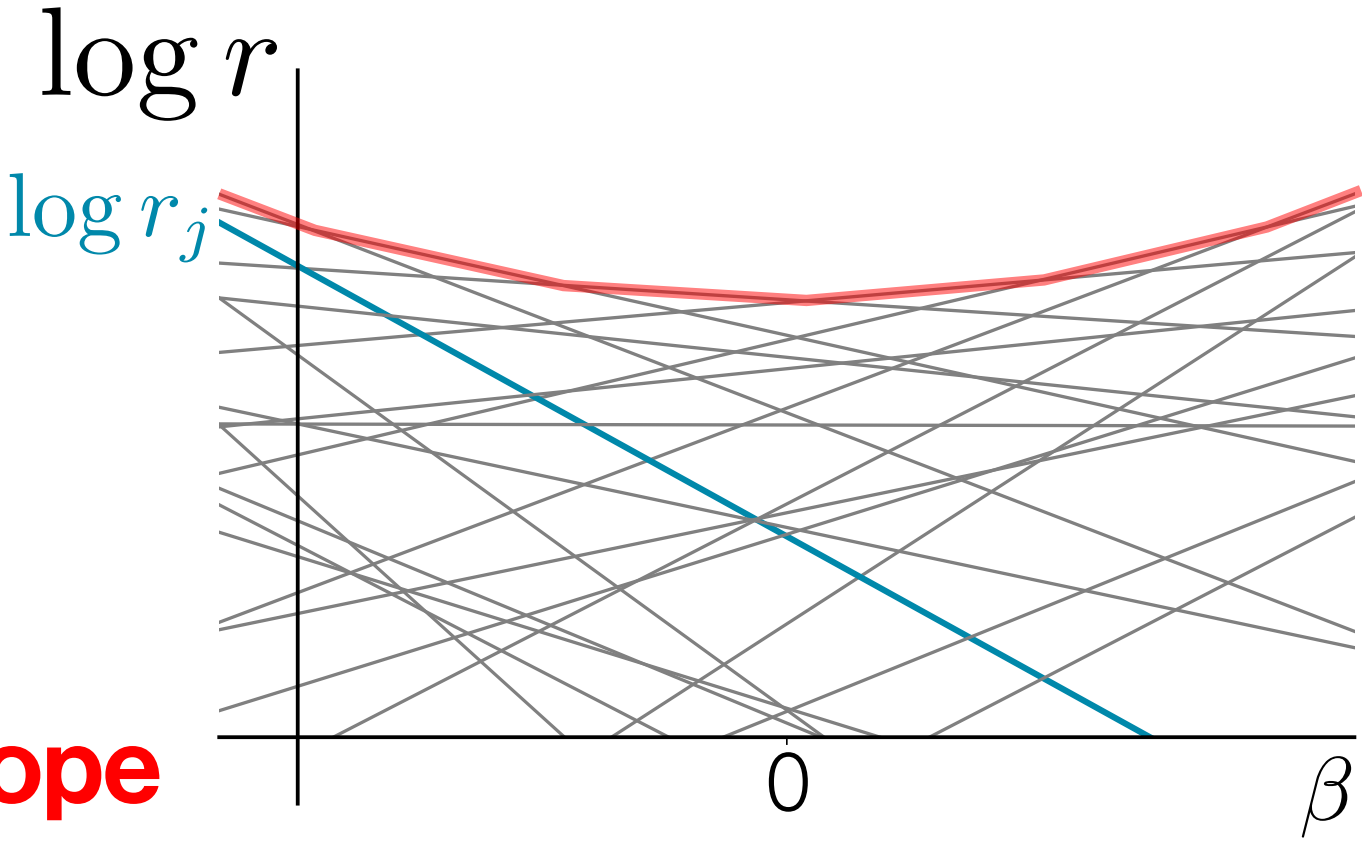
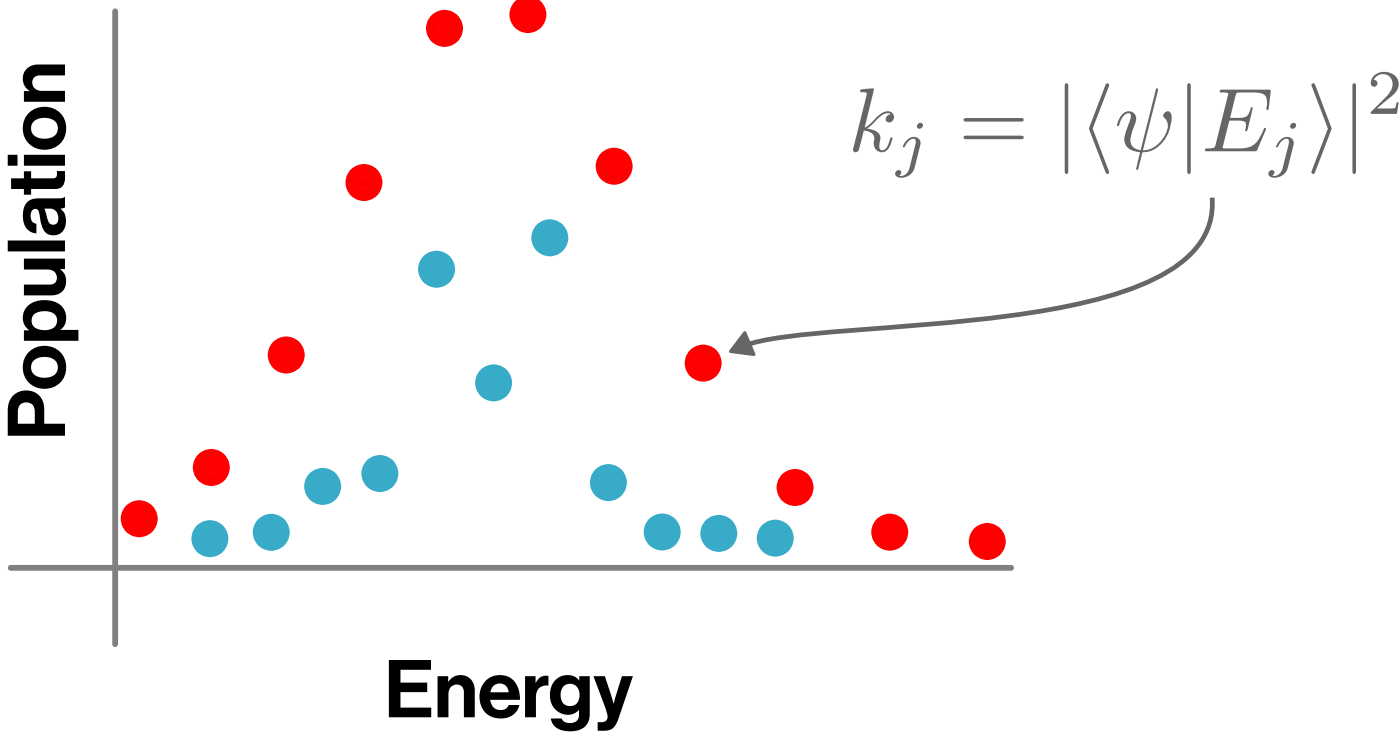


Long times

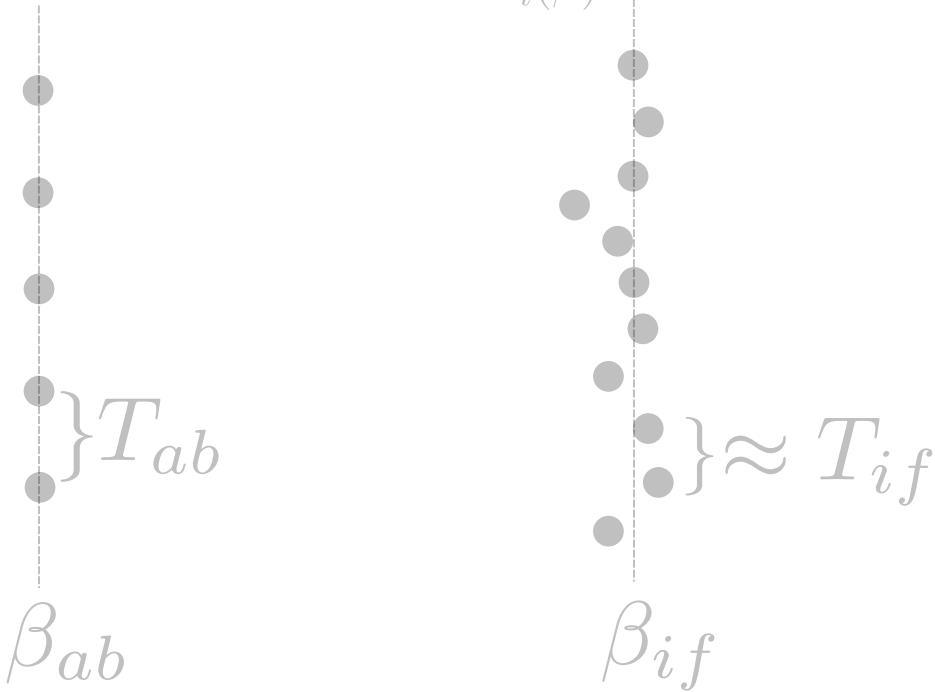
$$\mathcal{L}(\beta + it) = \sum_j r_j(\beta) e^{-iE_j t}$$

The dominant part = **The Envelope**

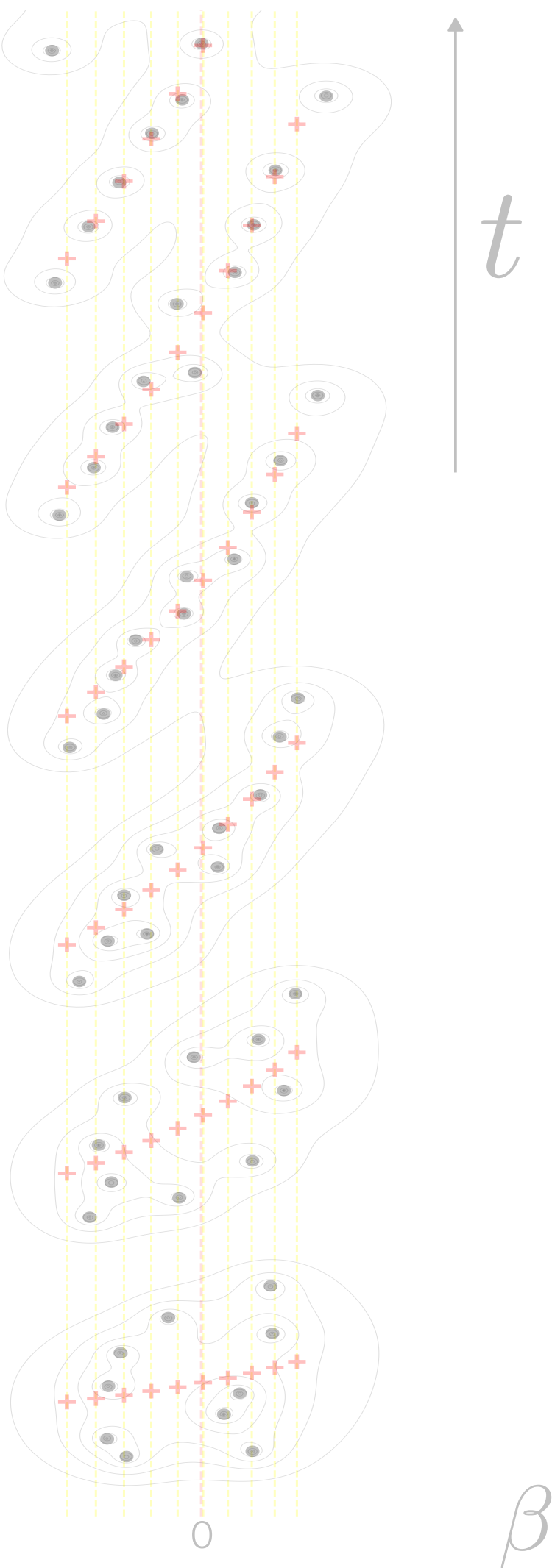
$$k_a \left(\frac{k_a}{k_b} \right)^{-\frac{\Delta_{ac}}{\Delta_{ab}}} \geq k_c, \quad \forall k_c, E_c$$



$$r_a(\beta) e^{-itE_a} + r_b(\beta) e^{-itE_b} = 0$$
$$r_i(\beta) e^{-itE_i} \dots = 0$$



$$z_{ab}^*(n) = \frac{1}{\Delta_{ab}} \left(\log \left(\frac{k_a}{k_b} \right) + i\pi (2n + 1) \right)$$

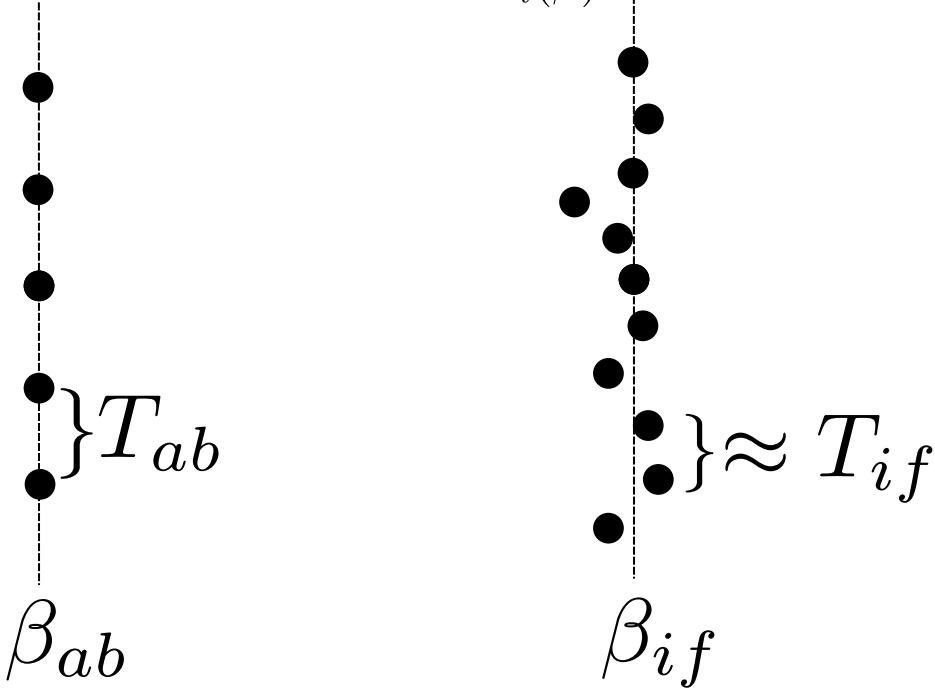
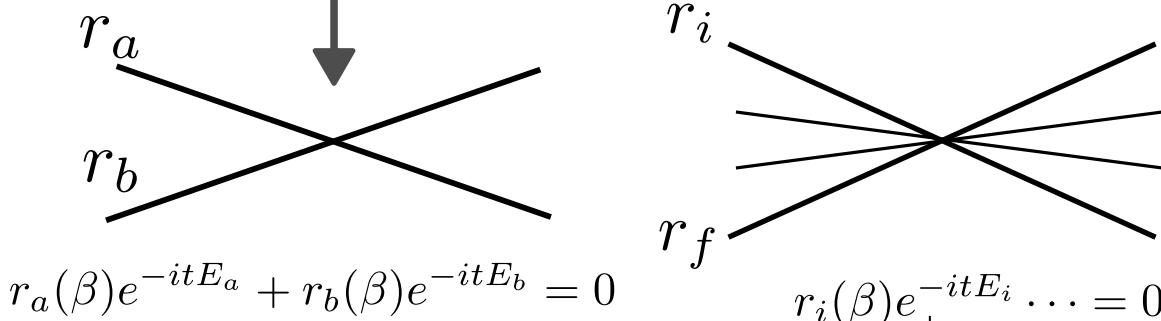
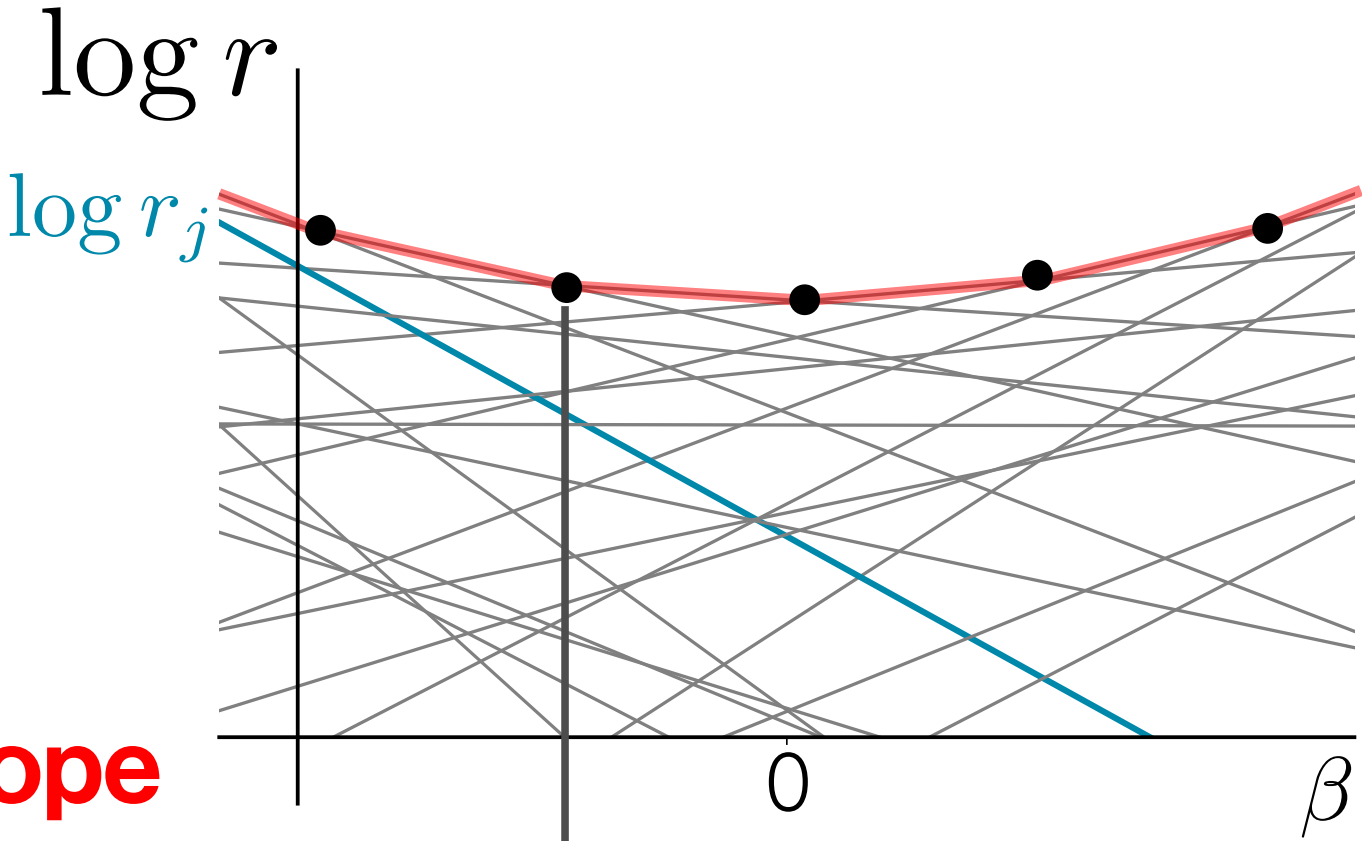
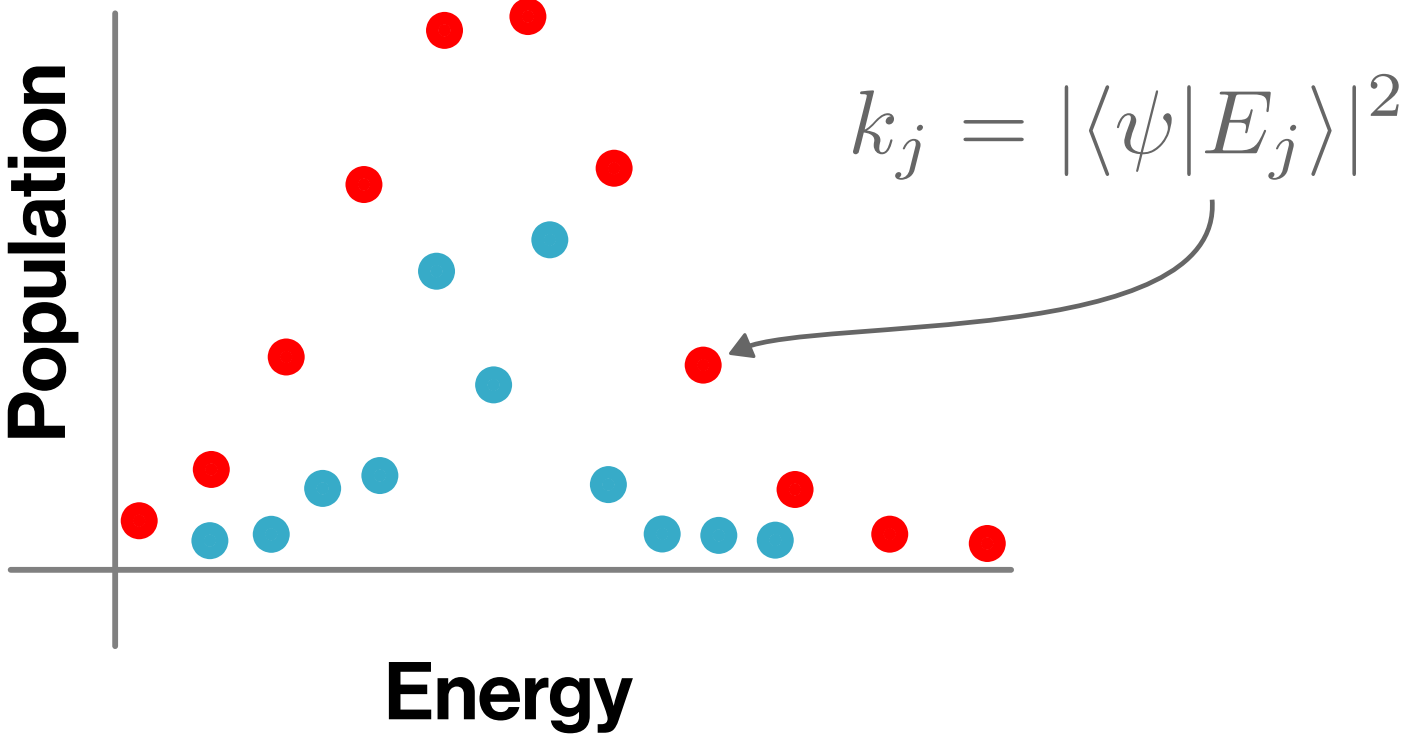


Long times

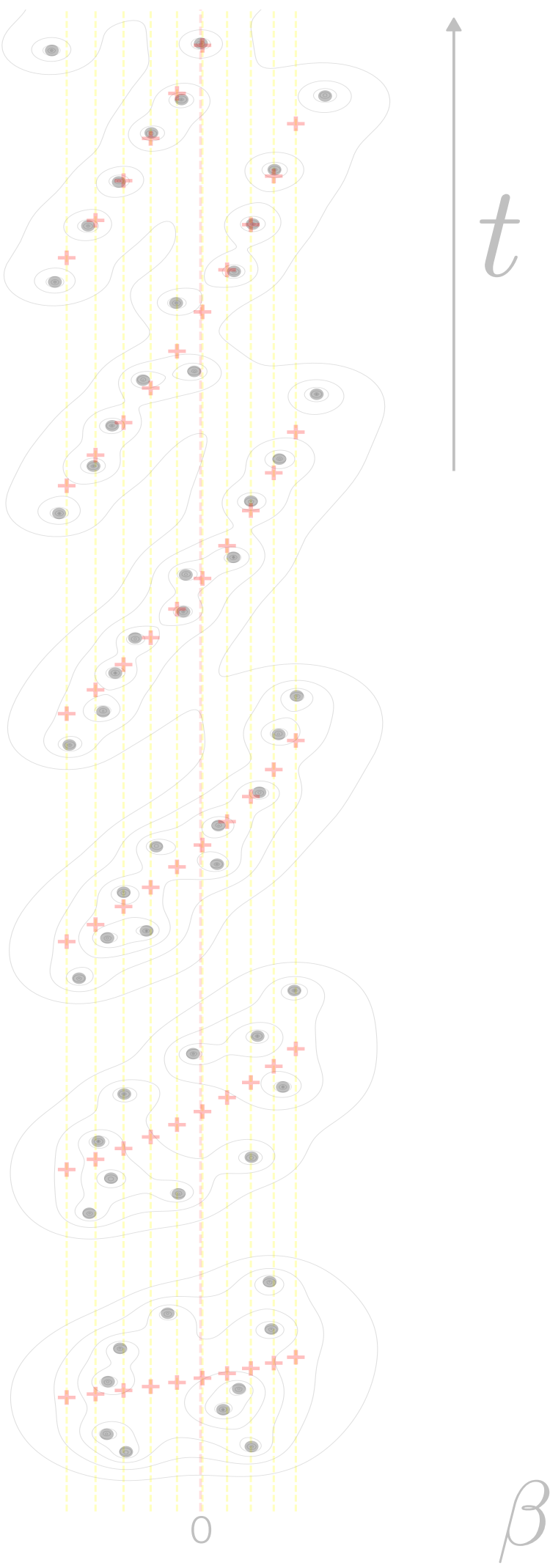
$$\mathcal{L}(\beta + it) = \sum_j r_j(\beta) e^{-iE_j t}$$

The dominant part = **The Envelope**

$$k_a \left(\frac{k_a}{k_b} \right)^{-\frac{\Delta_{ac}}{\Delta_{ab}}} \geq k_c, \quad \forall k_c, E_c$$



$$z_{ab}^*(n) = \frac{1}{\Delta_{ab}} \left(\log \left(\frac{k_a}{k_b} \right) + i\pi (2n + 1) \right)$$

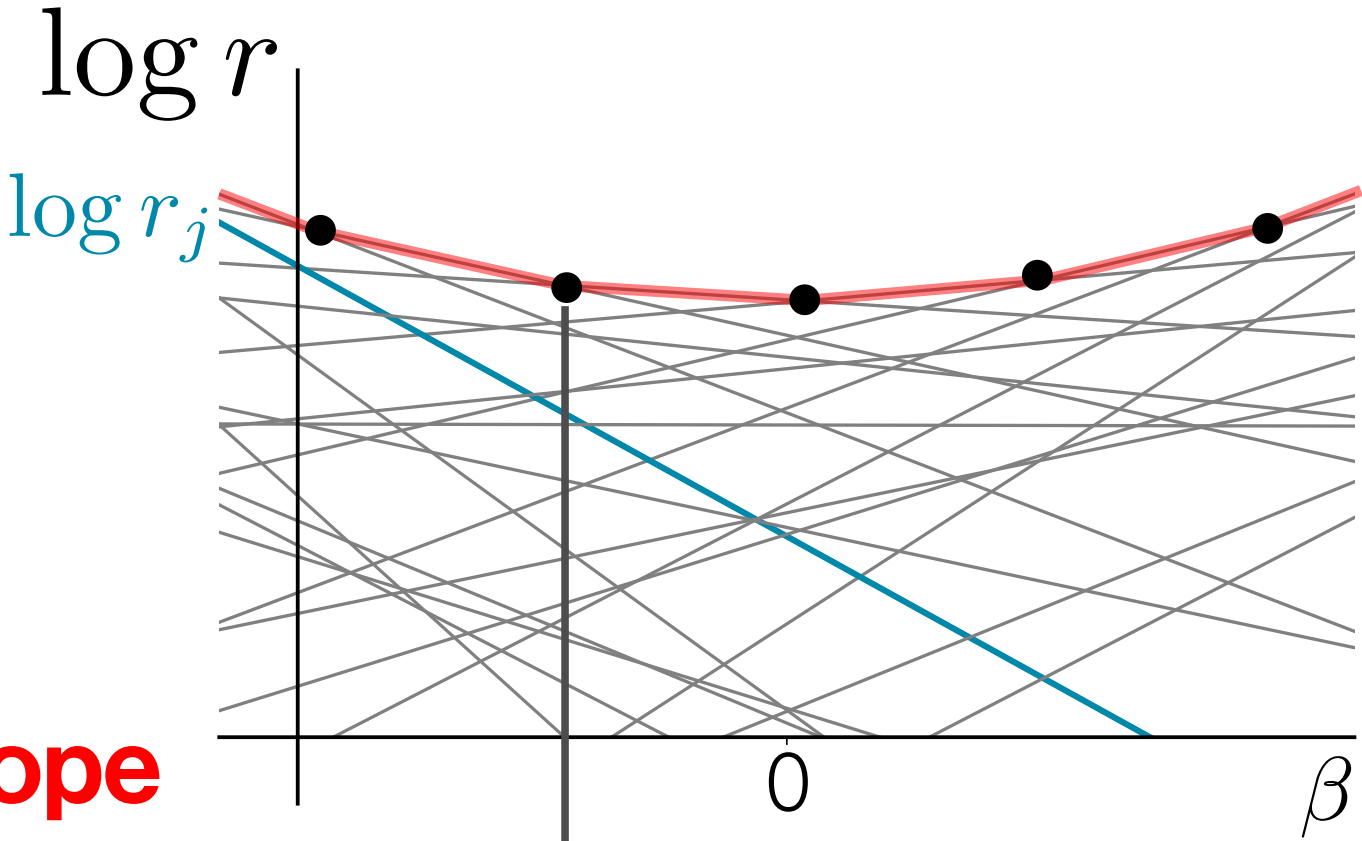
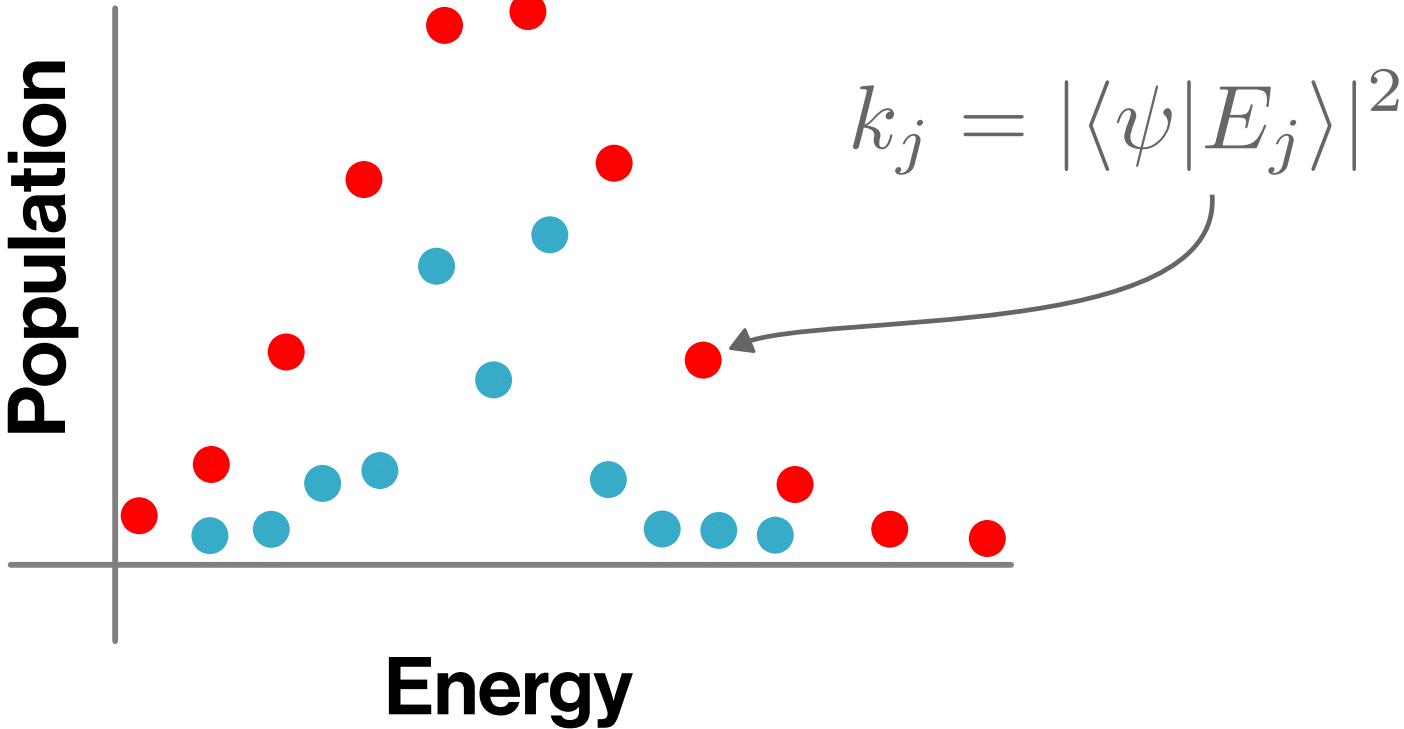


Long times

$$\mathcal{L}(\beta + it) = \sum_j r_j(\beta) e^{-iE_j t}$$

The dominant part = **The Envelope**

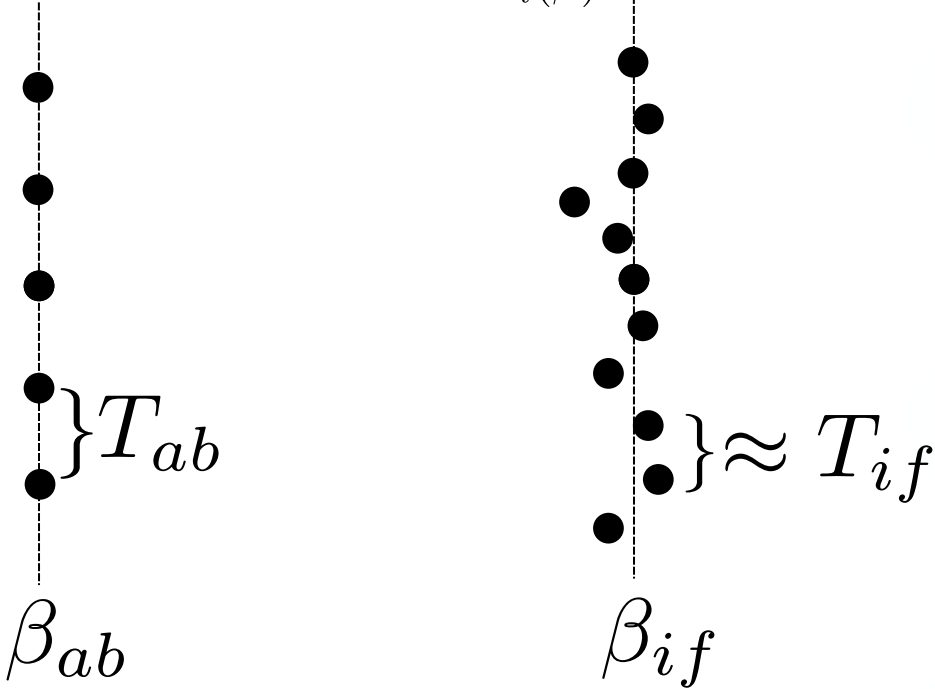
$$k_a \left(\frac{k_a}{k_b} \right)^{-\frac{\Delta_{ac}}{\Delta_{ab}}} \geq k_c, \quad \forall k_c, E_c$$



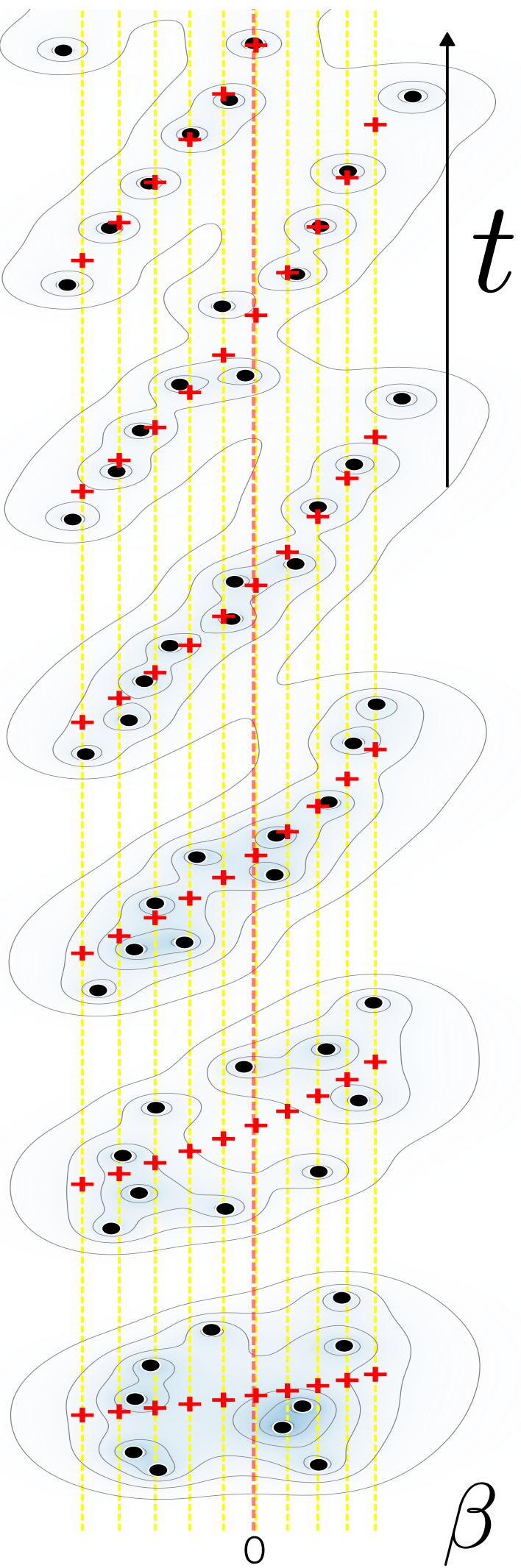
Diagrams illustrating the envelope formation:

Left: Two lines r_a and r_b intersecting. Below: $r_a(\beta)e^{-itE_a} + r_b(\beta)e^{-itE_b} = 0$

Right: Multiple lines r_i and r_f intersecting. Below: $r_i(\beta)e^{-itE_i} \dots = 0$



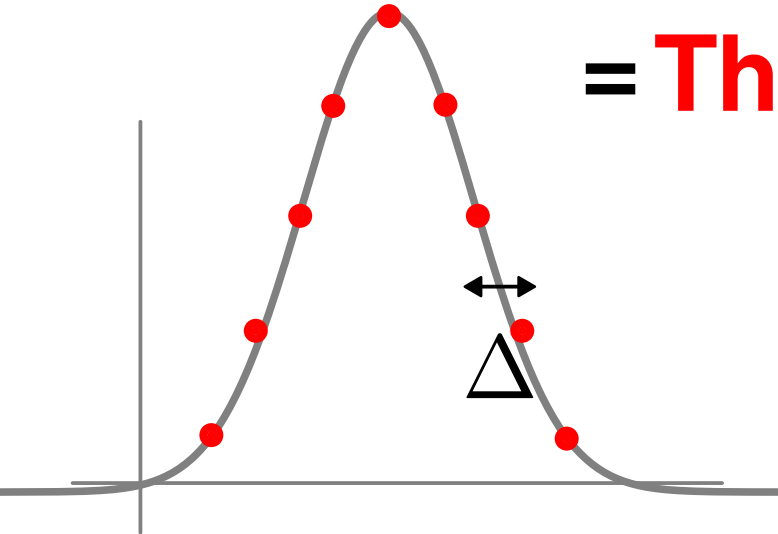
$$z_{ab}^*(n) = \frac{1}{\Delta_{ab}} \left(\log \left(\frac{k_a}{k_b} \right) + i\pi (2n + 1) \right)$$



Short times

The dominant part

= **The equid. model**



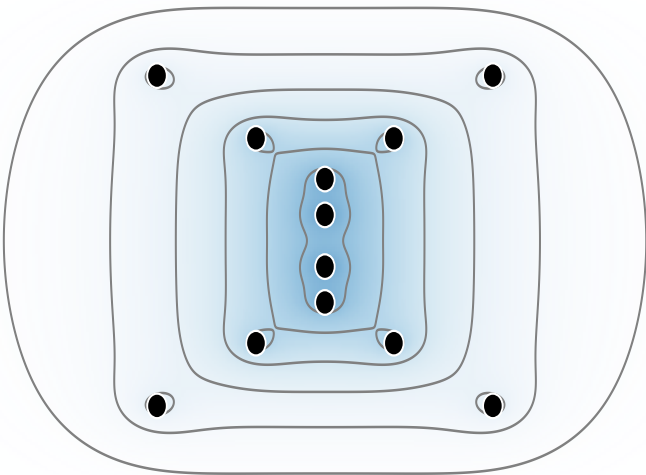
$$\Delta_i = \Delta$$

$$2\pi$$

$$\Delta$$

period

$$0$$



$$\phi_i = T\Delta + 3T\varepsilon_i$$

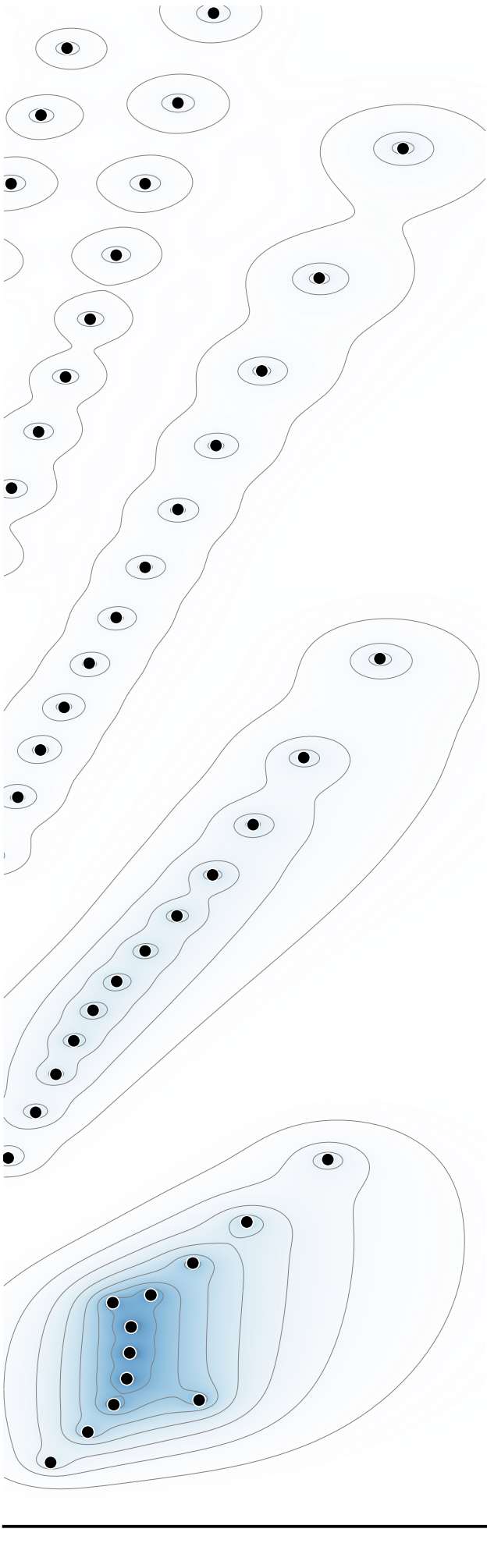
period

$$\phi_i = T\Delta + 2T\varepsilon_i$$

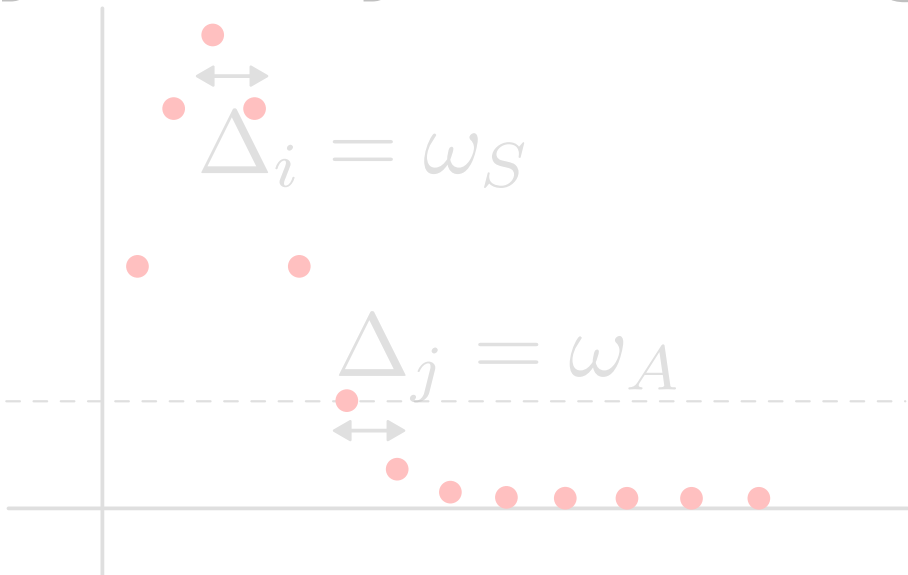
period

$$\phi_i = T\Delta + T\varepsilon_i$$

period



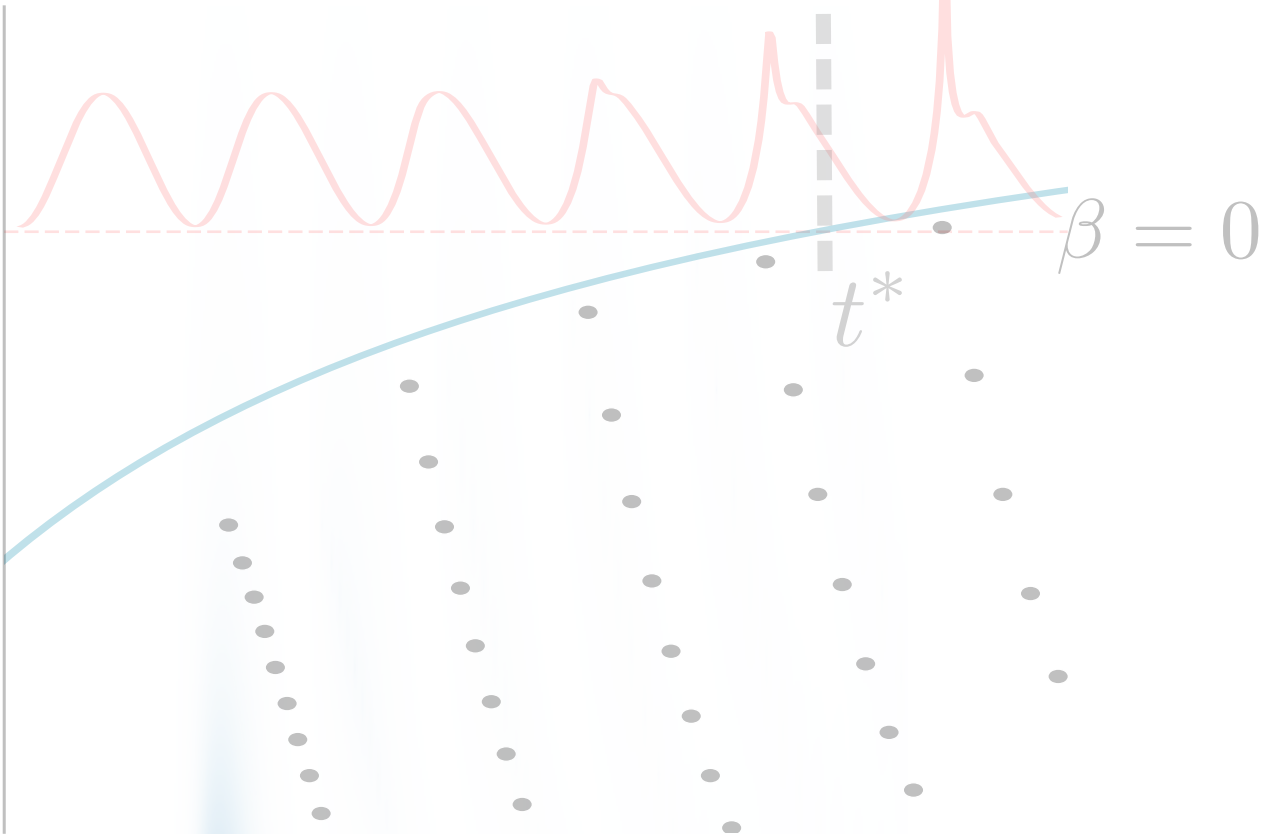
Asymmetry → running zeros



Arrival of the First zero

$$t^*(\beta) \approx t_0 + \frac{\beta - \beta_0}{|\omega_S(\beta) - \omega_A(\beta)|}$$

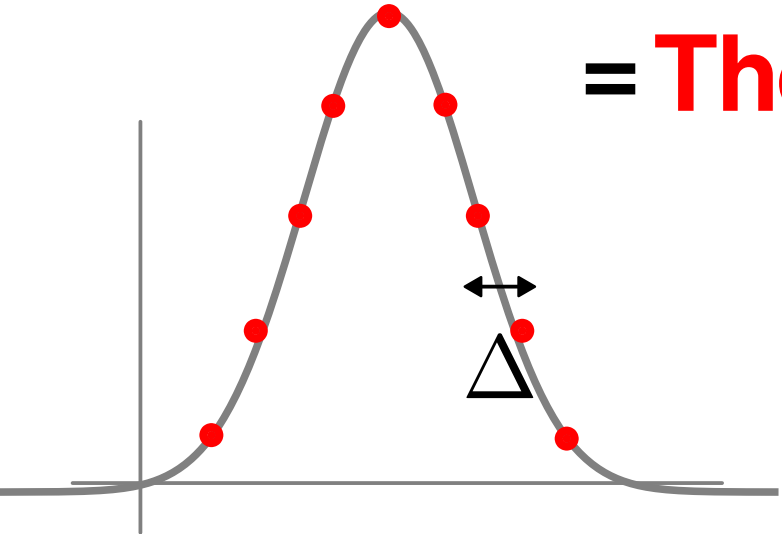
Lipkin model $\hat{H} = -\frac{J}{N} \hat{S}_x^2 + h \hat{S}_z$



Short times

The dominant part

= **The equid. model**



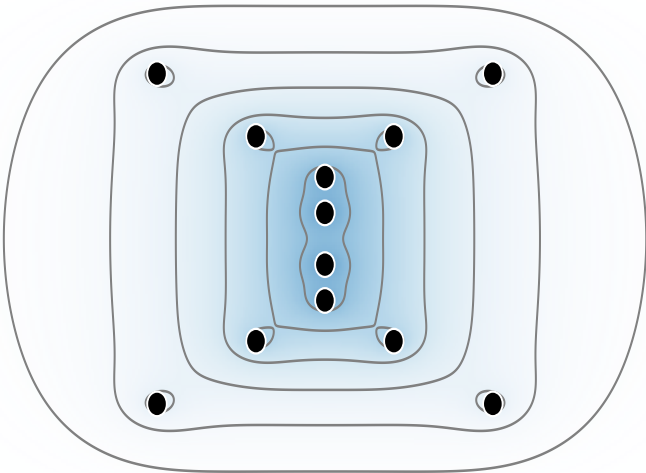
$$\Delta_i = \Delta$$

2π

Δ

period

0



$$\phi_i = T\Delta + 3T\varepsilon_i$$

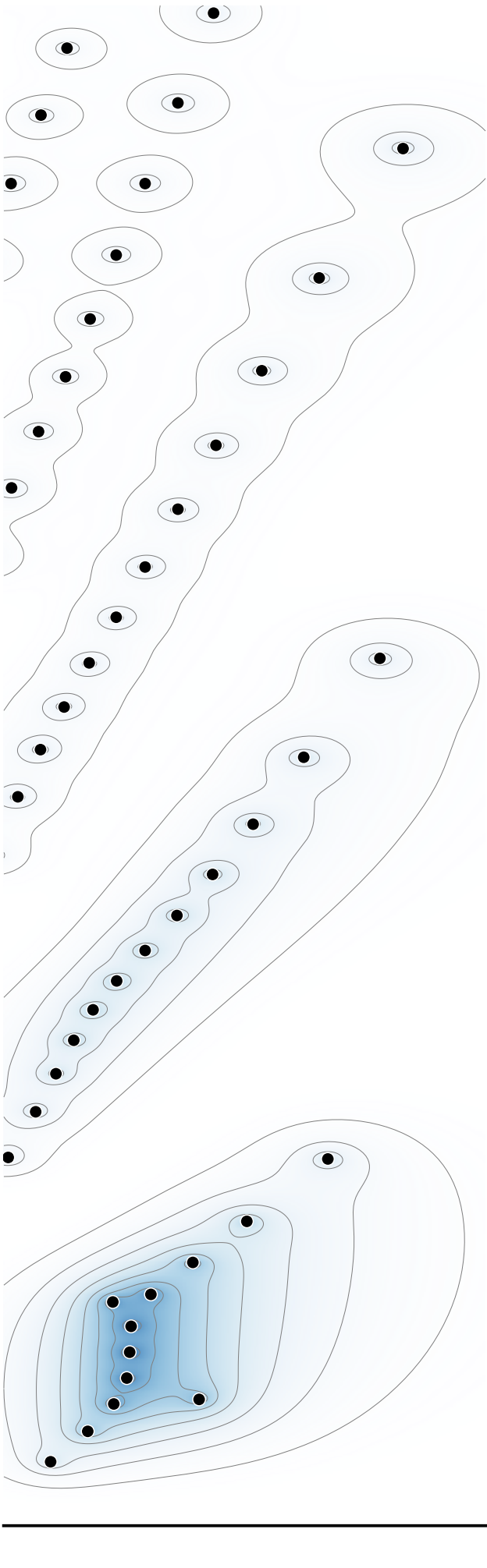
$$\phi_i = T\Delta + 2T\varepsilon_i$$

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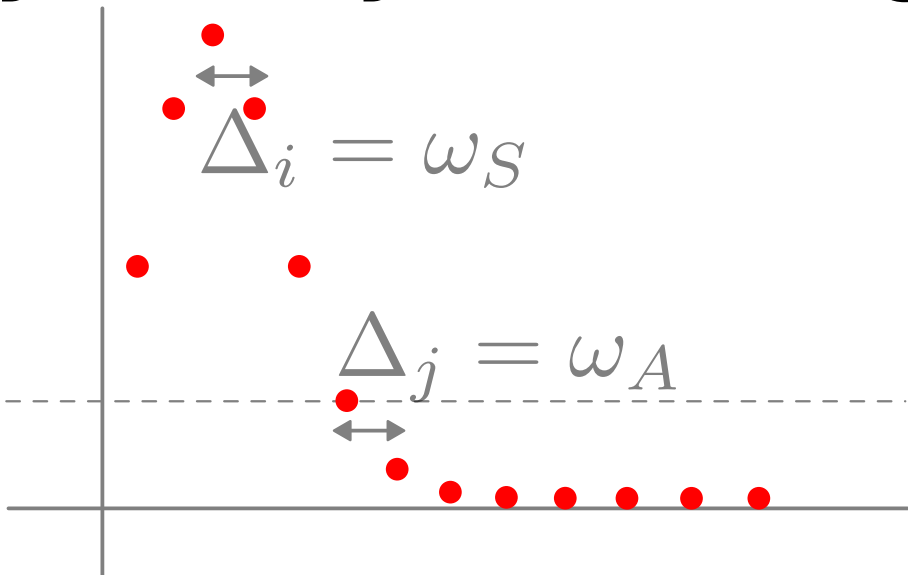
period

period

period



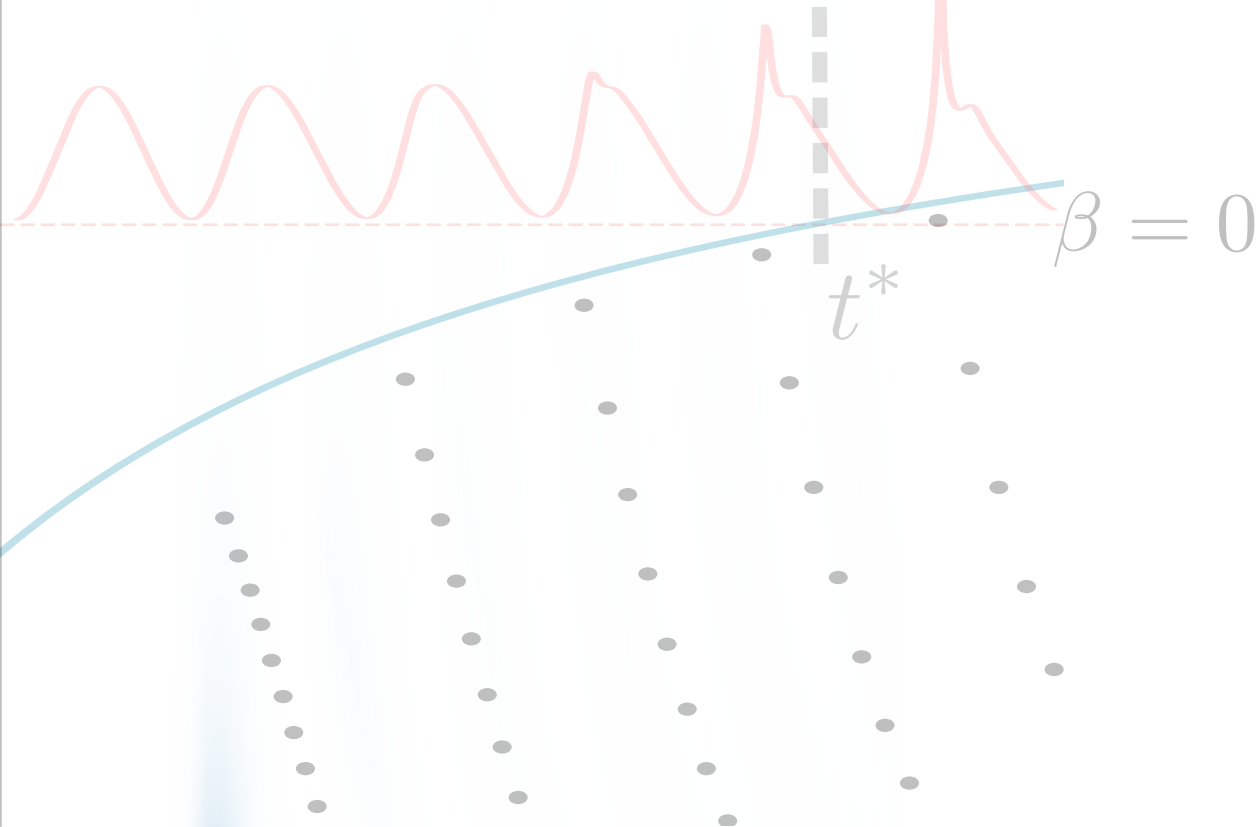
Asymmetry → running zeros



Arrival of the First zero

$$t^*(\beta) \approx t_0 + \frac{\beta - \beta_0}{|\omega_S(\beta) - \omega_A(\beta)|}$$

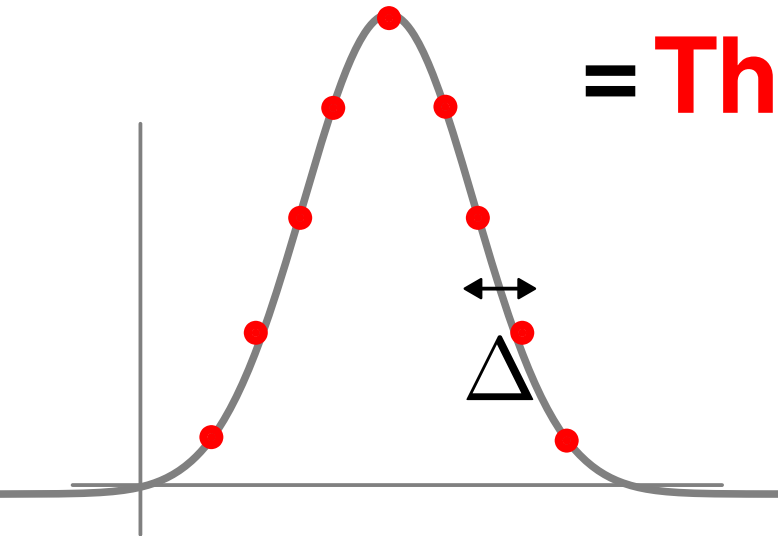
Lipkin model $\hat{H} = -\frac{J}{N} \hat{S}_x^2 + h \hat{S}_z$



Short times

The dominant part

= **The equid. model**



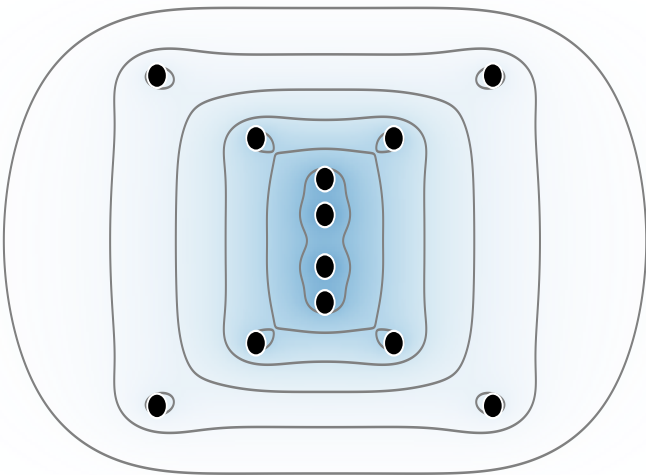
$$\Delta_i = \Delta$$

$$2\pi$$

$$\Delta$$

period

$$0$$



$$\phi_i = T\Delta + 3T\varepsilon_i$$

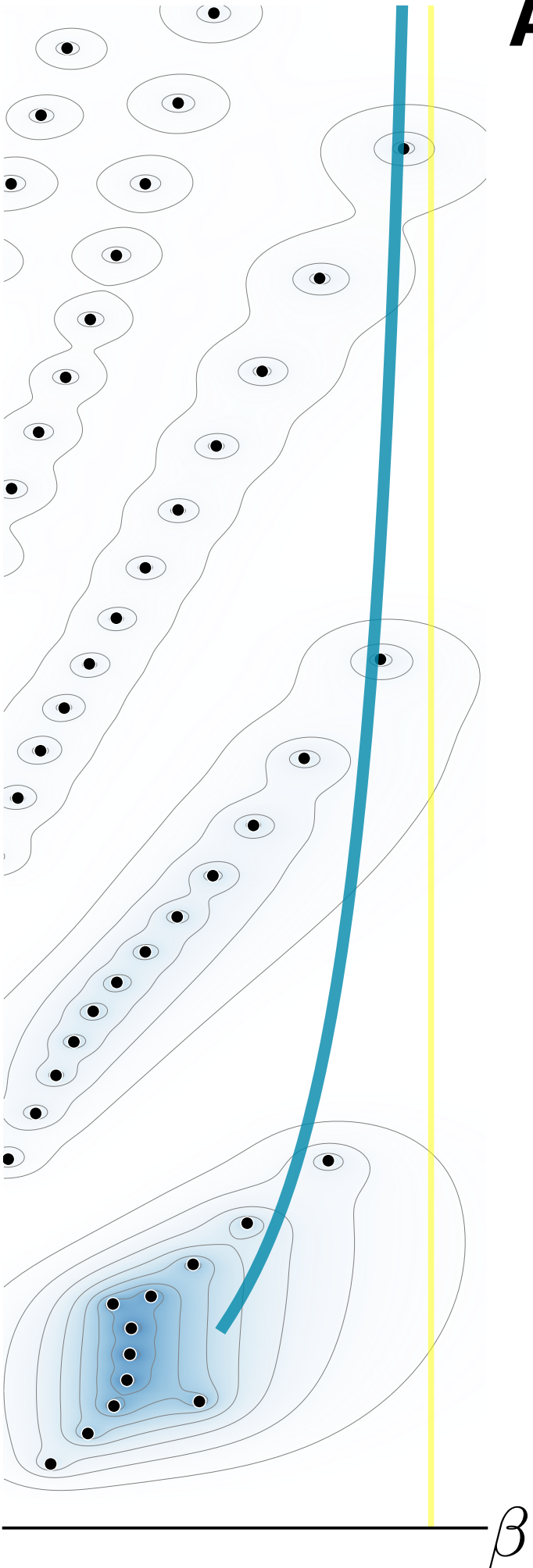
$$\phi_i = T\Delta + 2T\varepsilon_i$$

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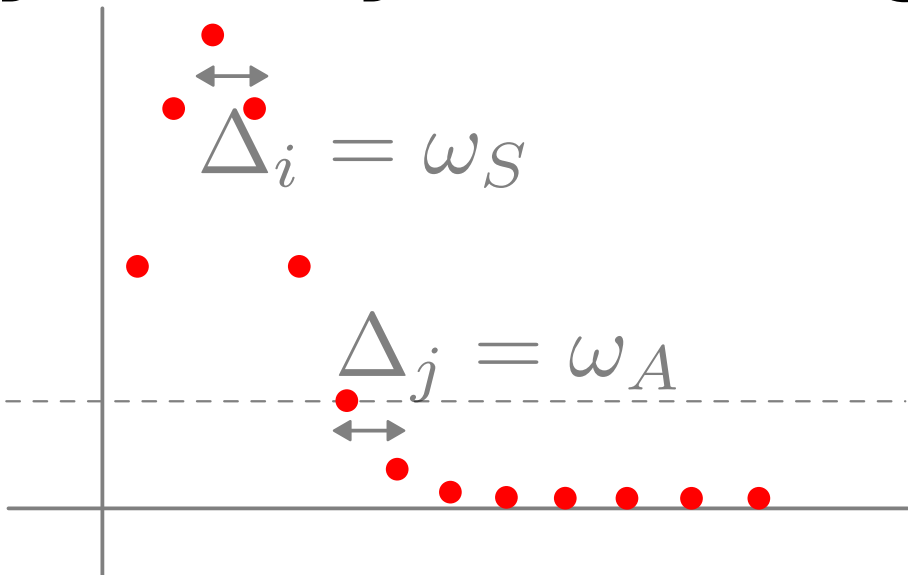
period

period

period



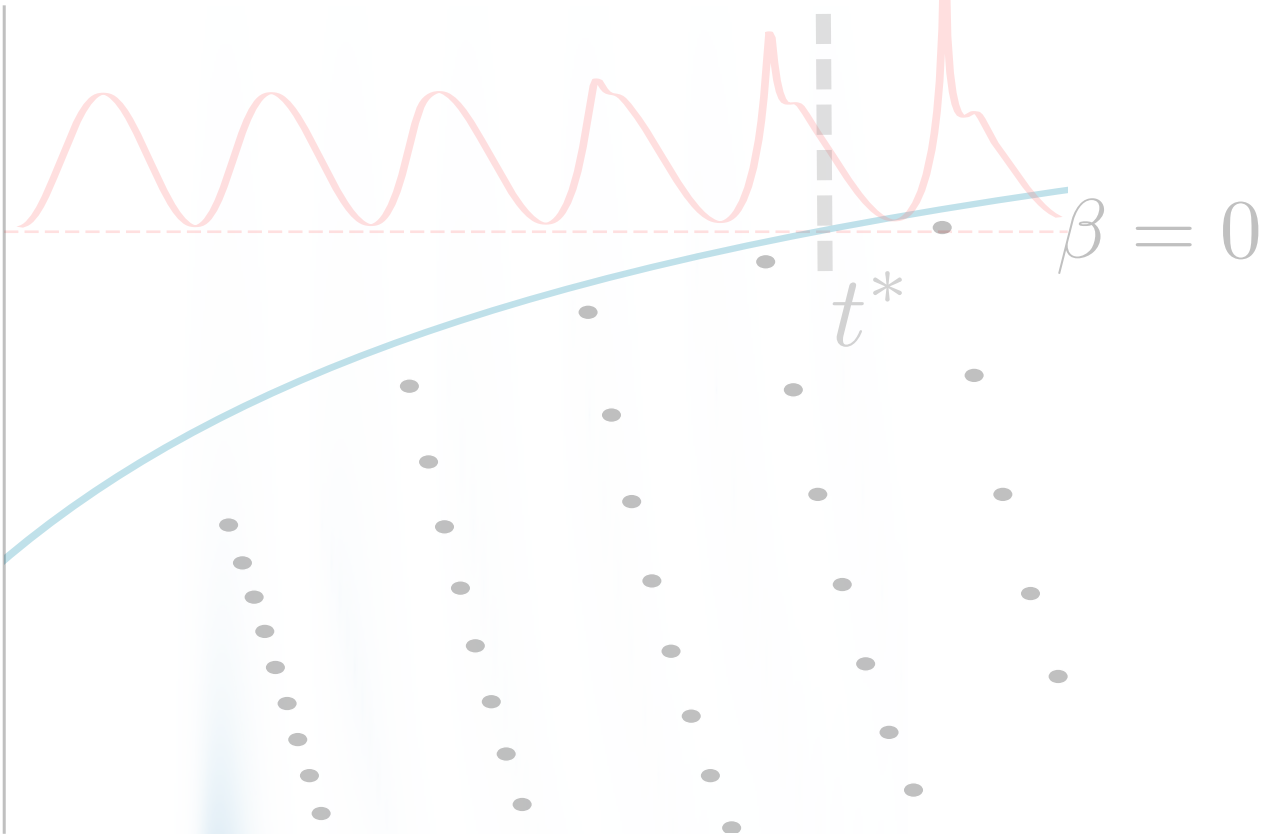
Asymmetry → running zeros



Arrival of the First zero

$$t^*(\beta) \approx t_0 + \frac{\beta - \beta_0}{|\omega_S(\beta) - \omega_A(\beta)|}$$

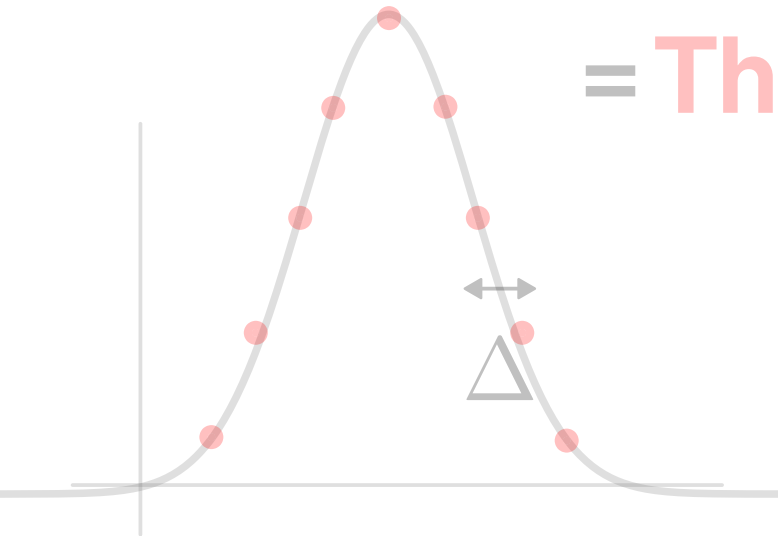
Lipkin model $\hat{H} = -\frac{J}{N} \hat{S}_x^2 + h \hat{S}_z$



Short times

The dominant part

= **The equid. model**



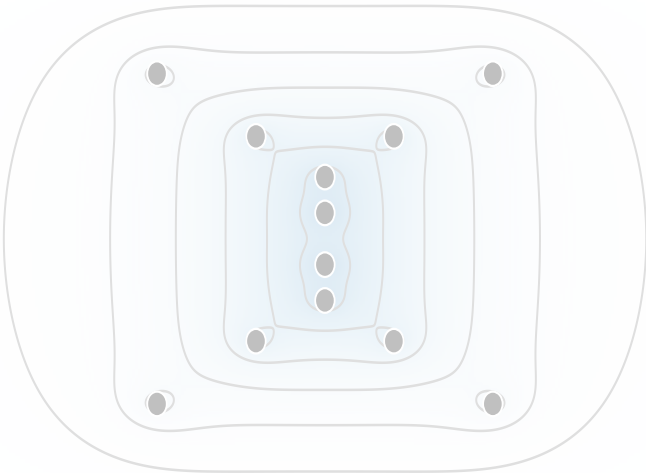
$$\Delta_i = \Delta$$

2π

Δ

period

0



$$\phi_i = T\Delta + 3T\epsilon_i$$

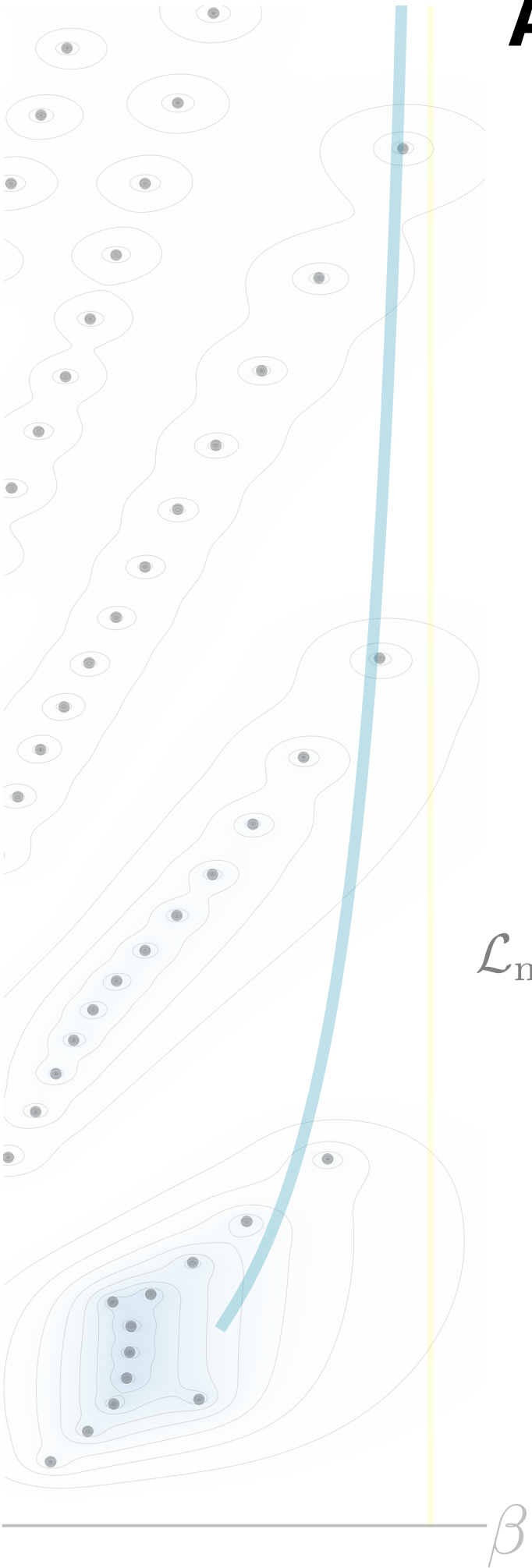
$$\phi_i = T\Delta + 2T\epsilon_i$$

$$\phi_i = T\Delta + T\epsilon_i$$

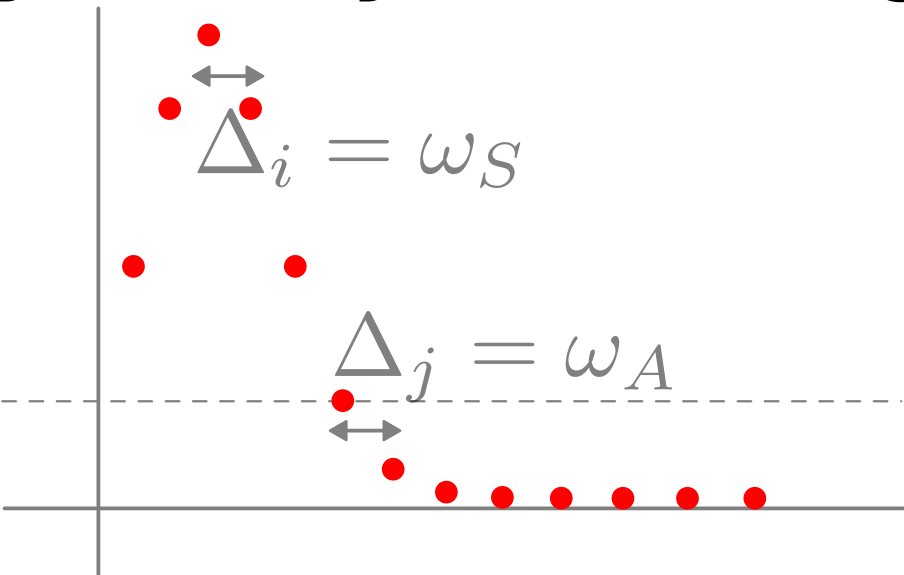
period

period

period



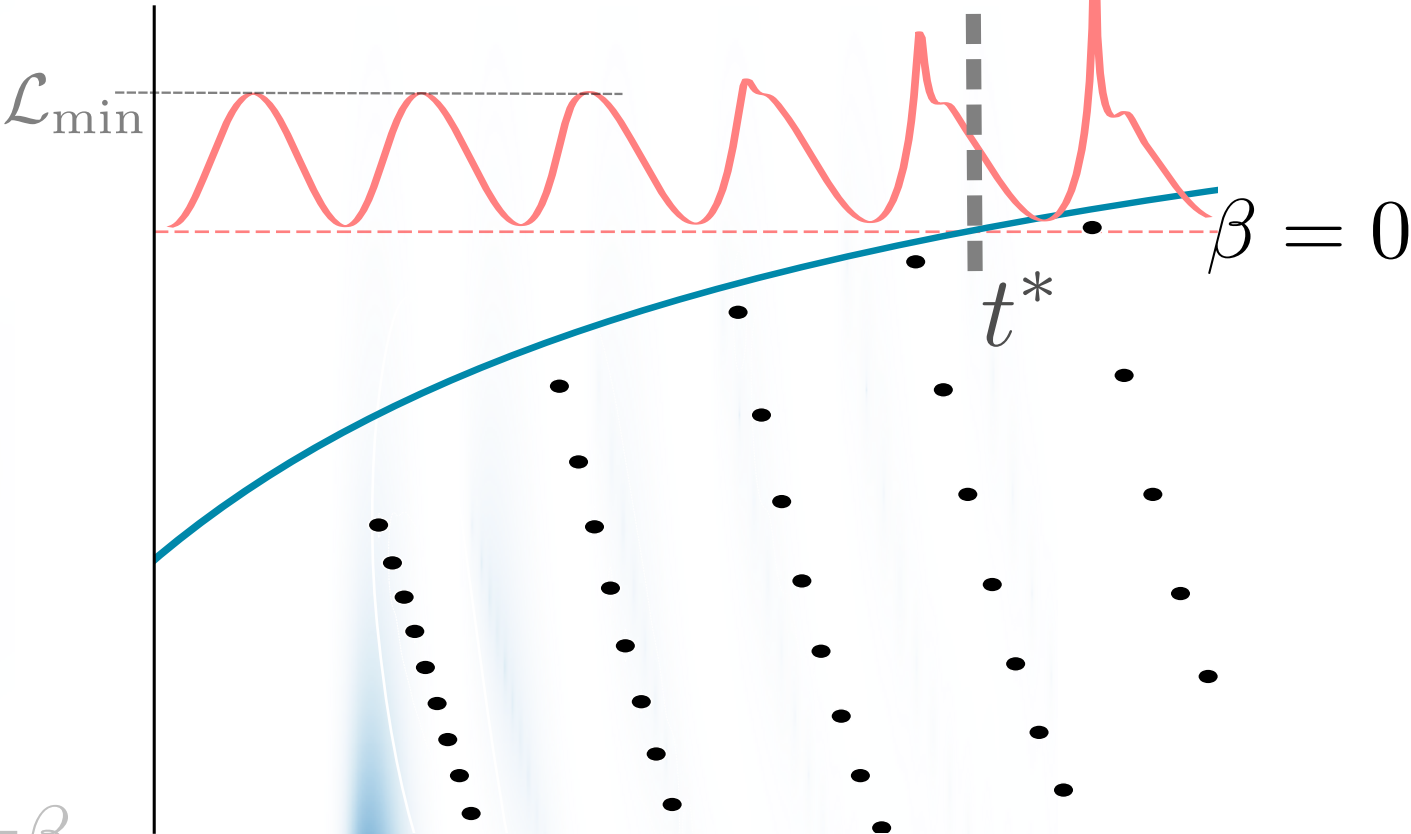
Asymmetry → running zeros



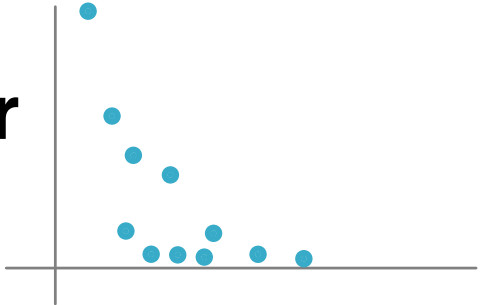
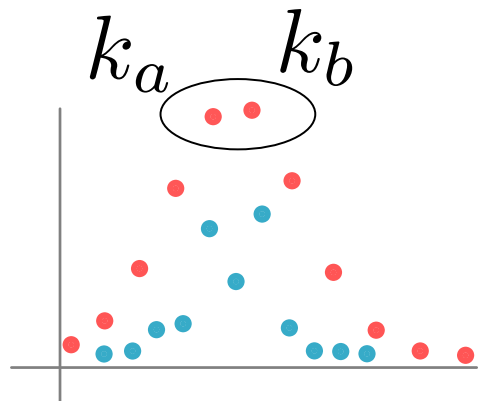
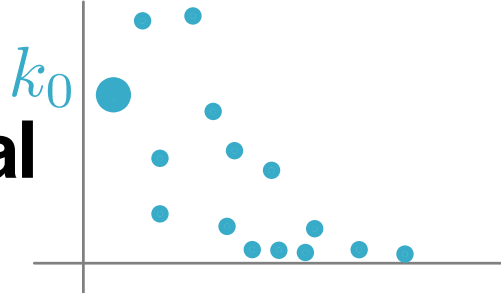
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Lipkin model $\hat{H} = -\frac{J}{N} \hat{S}_x^2 + h\hat{S}_z$

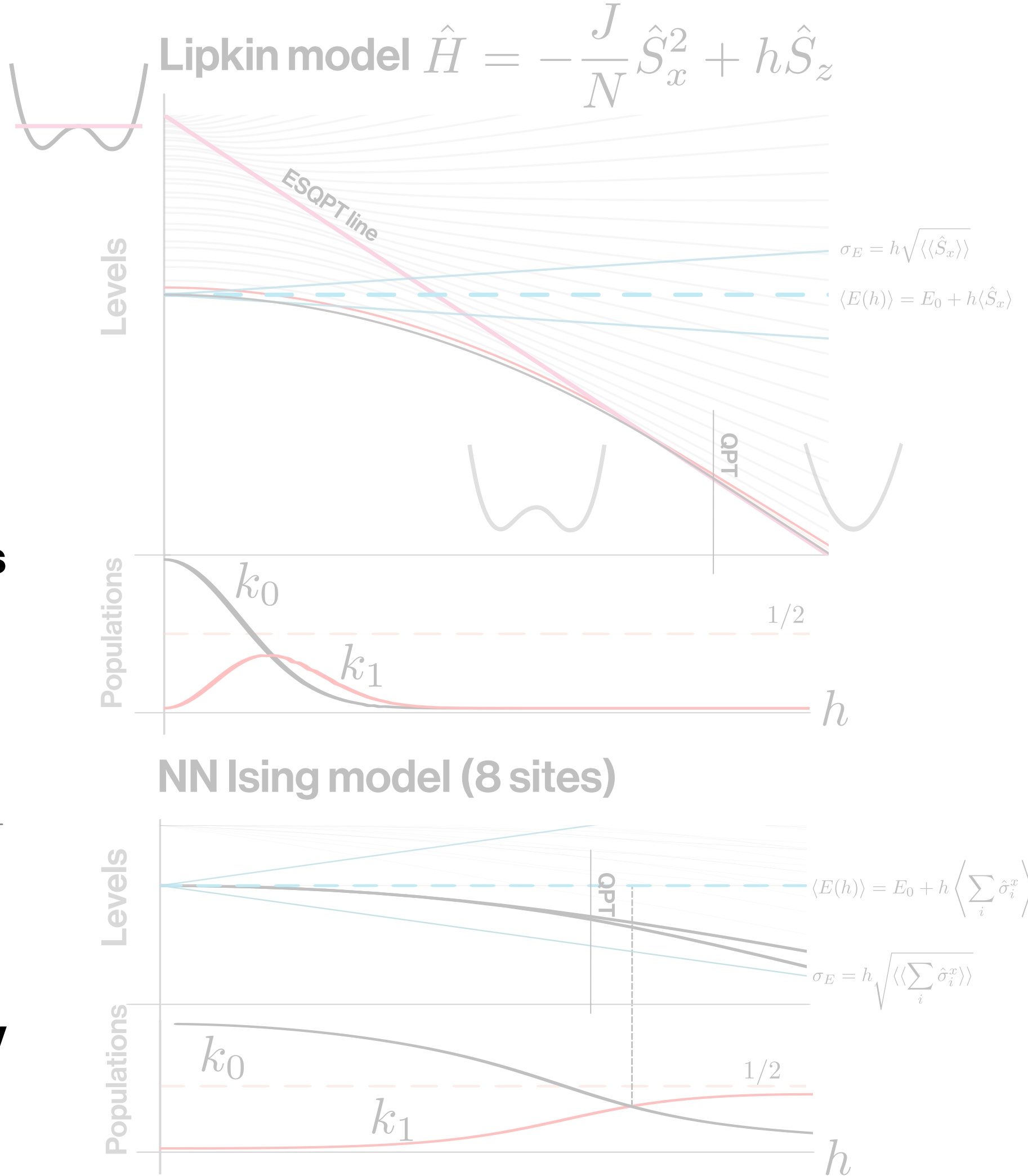


Quenches

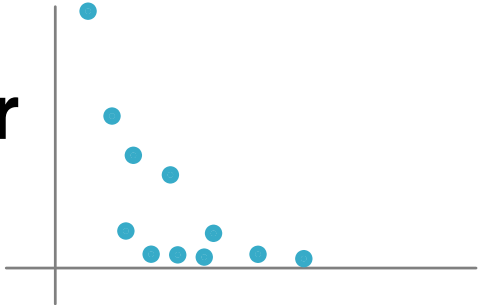
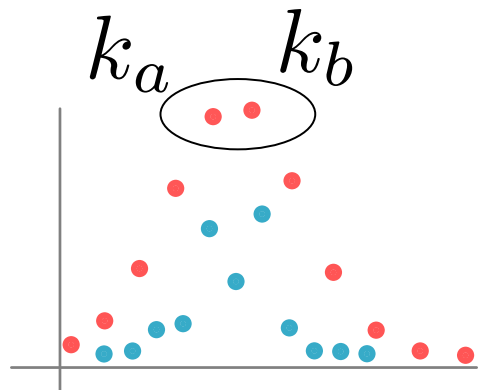
- $\bullet \quad \max\{k_j\} > \frac{1}{2}$
no zeros near the time axis

- $\bullet$

maximum determines the closest zeros to time axis
 $T_{ab} \quad \beta_{ab}$
- $\bullet \quad k_0 > \mathcal{L}_{\min}$
late time arrival


Ground state quench

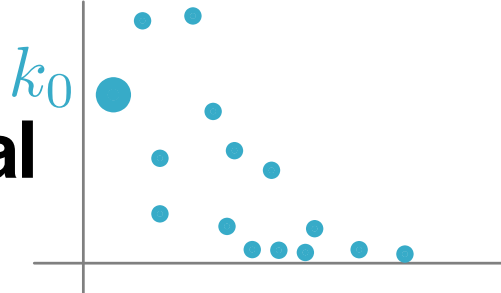
"zeros close to the time axis" $\longleftrightarrow$ "two (or more) highly populated states"



Quenches

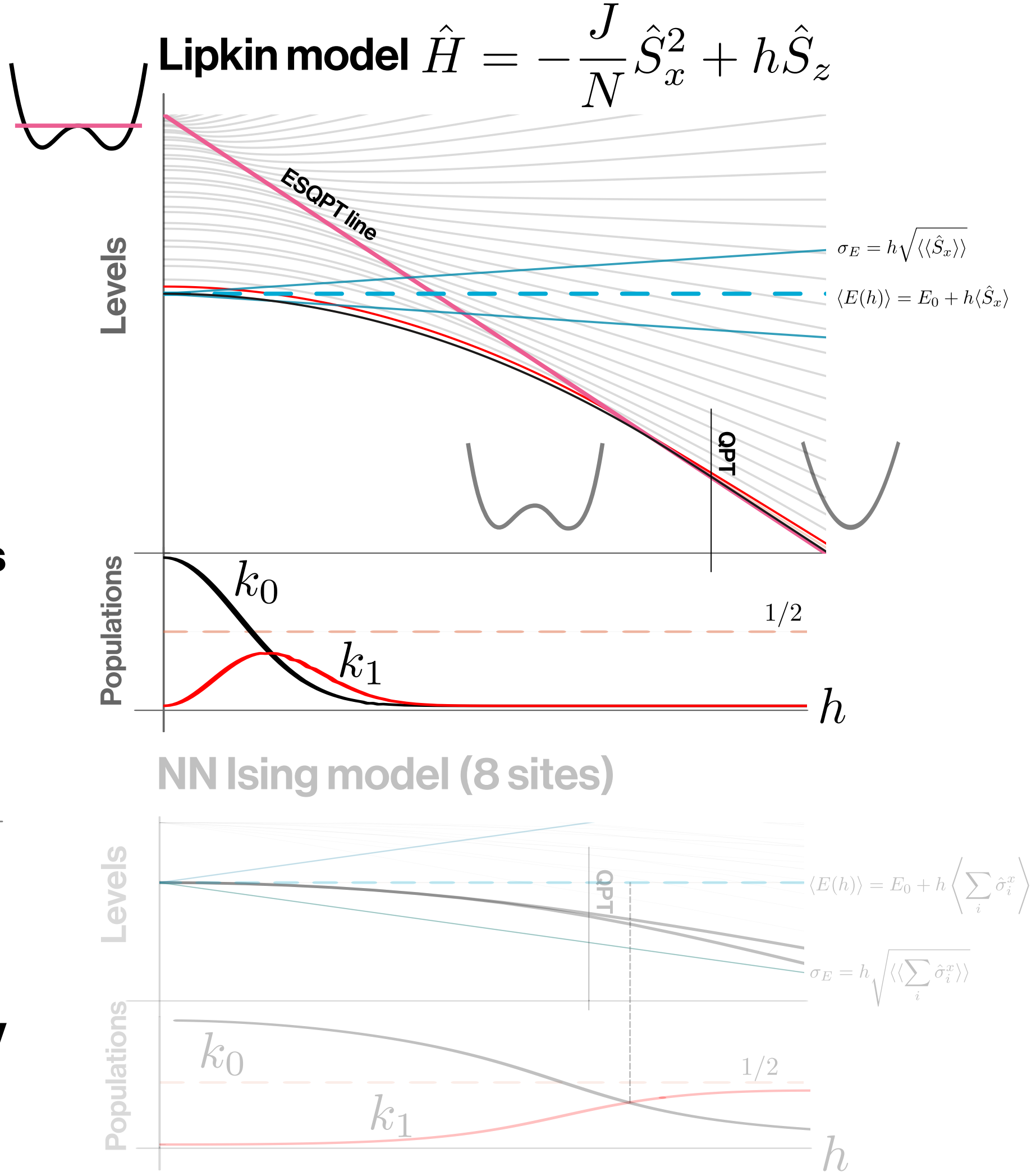
- $\bullet \max\{k_j\} > \frac{1}{2}$
no zeros near the time axis

- $\bullet$

maximum determines the closest zeros to time axis

T_{ab}
 β_{ab}


- $\bullet k_0 > \mathcal{L}_{\min}$
late time arrival

Ground state quench

"zeros close to the time axis" $\longleftrightarrow$ "two (or more) highly populated states"

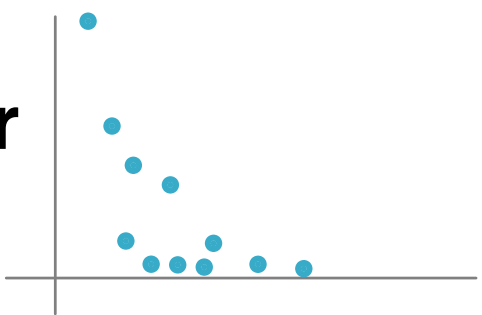


Quenches

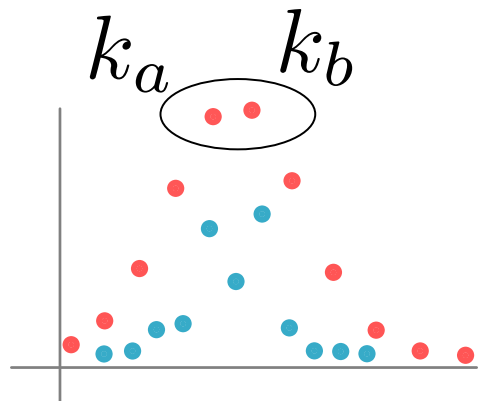
Cejnar et al.
JPA (2021)

Lipkin model $\hat{H} = -\frac{J}{N} \hat{S}_x^2 + h \hat{S}_z$

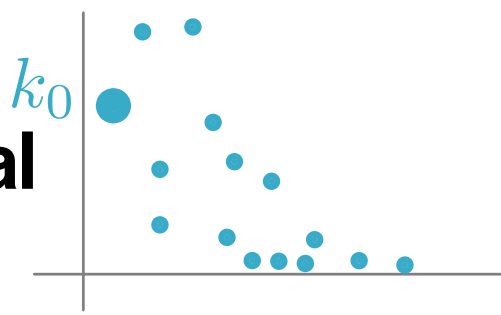
● $\max\{k_j\} > \frac{1}{2}$ no zeros near the time axis



● k_a, k_b maximum determines the closest zeros to time axis

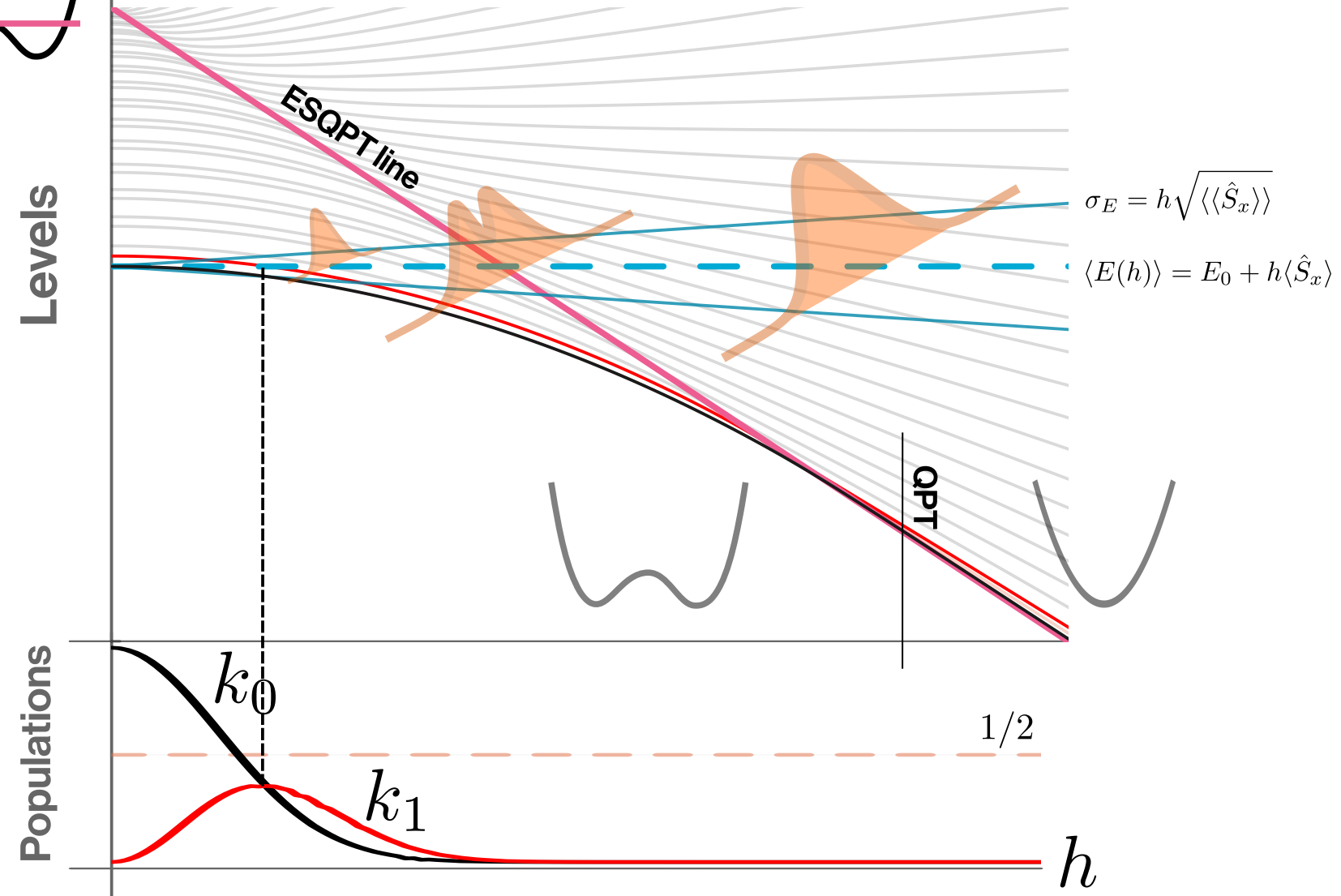


● $k_0 > \mathcal{L}_{\min}$ late time arrival

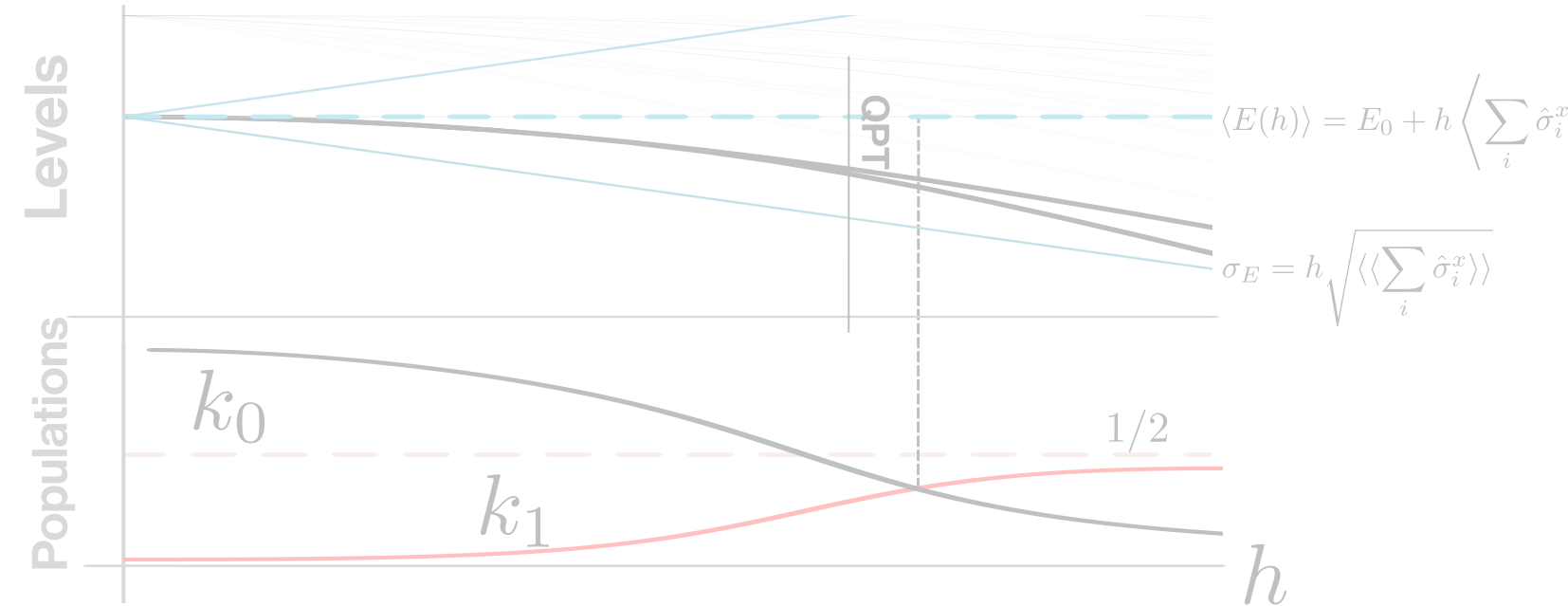


Ground state quench

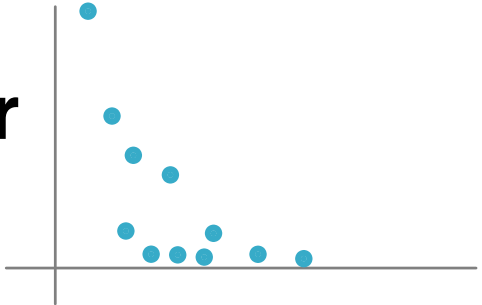
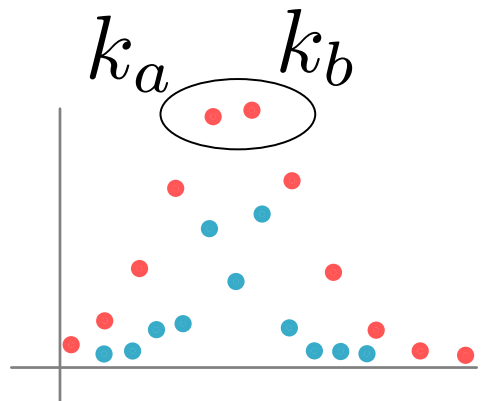
"zeros close to the time axis" ↔ "two (or more) highly populated states"



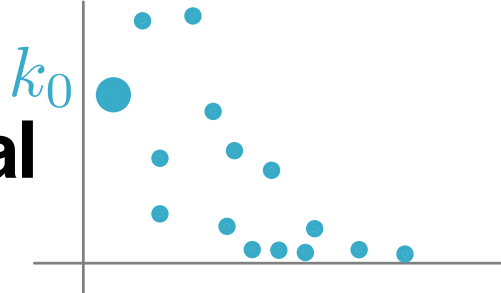
NN Ising model (8 sites)



Quenches

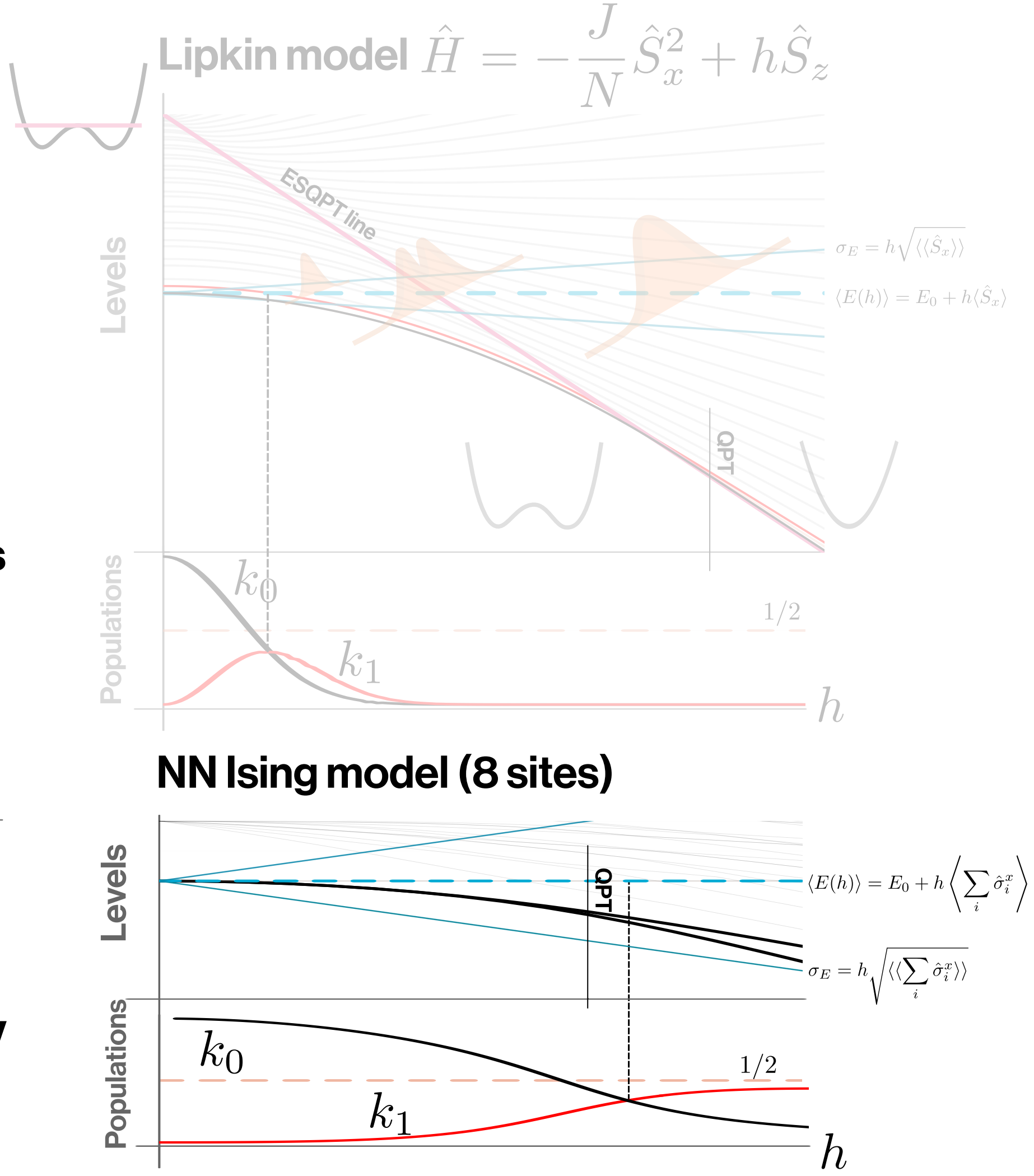
- $\bullet \quad \max\{k_j\} > \frac{1}{2} \quad \text{no zeros near the time axis}$

- $\bullet \quad$


maximum determines the closest zeros to time axis

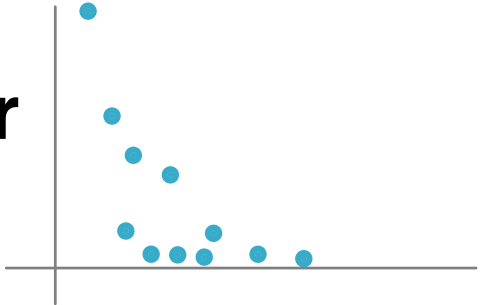
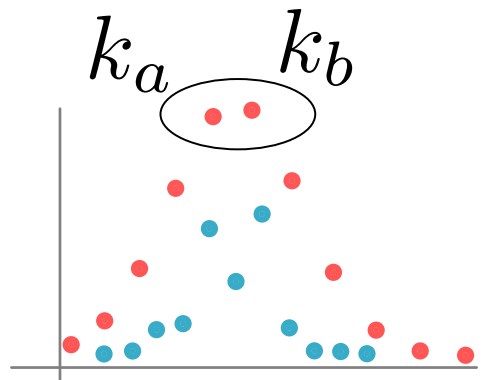
$T_{ab} \quad \beta_{ab}$
- $\bullet \quad k_0 > \mathcal{L}_{\min} \quad \text{late time arrival}$


Ground state quench

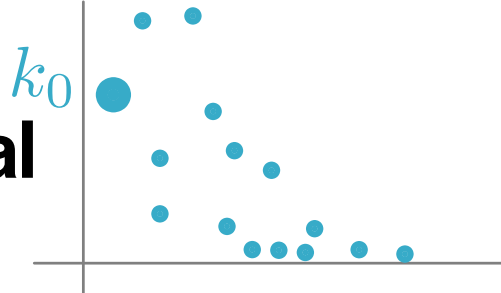
"zeros close to the time axis" $\longleftrightarrow$ "two (or more) highly populated states"



Quenches

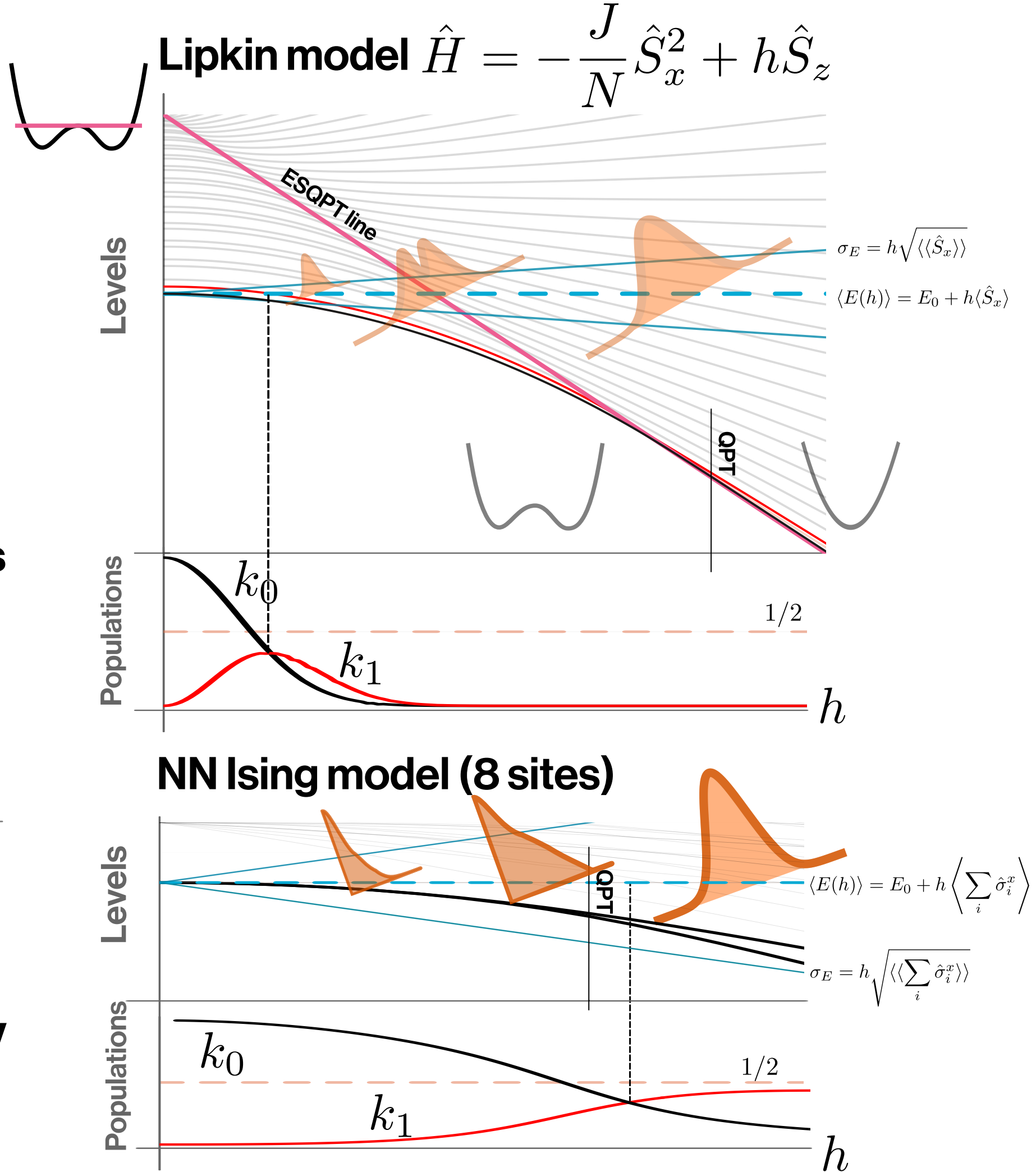
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Ground state quench

"zeros close to the time axis" $\longleftrightarrow$ "two (or more) highly populated states"



Conclusions

Zeros comes from the dominant parts of the initial state
- the envelope

Zeros form chain-like structures around β_{ab} with (quasi-) periods T_{ab}

Asymmetry of the initial state may delay the first zero

"zeros close to the time axis"  **"two (or more) highly populated states"**

Thank you for your attention

The work was supported by the The Charles University Grant Agency
(grant no. 215323)

