

Collisional and collisionless dynamics in post-disruption plasma

Daniela Grasso

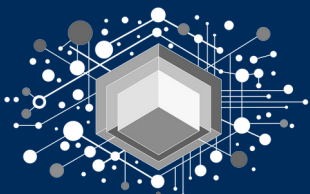
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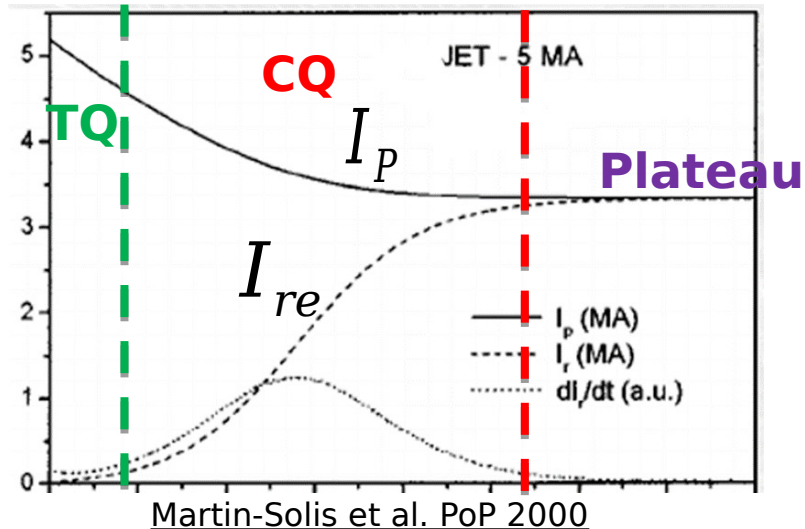
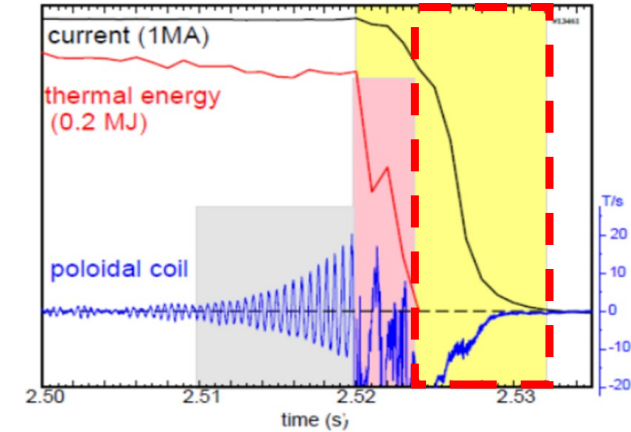


Motivation

Runaway electrons (REs) are high-energy particles that can accelerate to near-light speeds (10MeV)

REs are generated during a plasma disruption, a rapid and sudden loss of plasma magnetic confinement

Disruptions are triggered by the presence of impurities or a global MHD instability and causes a rapid loss of plasma thermal energy



Thermal quench (TQ): rapid drop of electron temperature ($\tau < 1\text{ms}$), increase of plasma resistivity and parallel electric field E_{\parallel} , that accelerates thermal electrons reaching the runaway regime. **The RE current begins to increase**

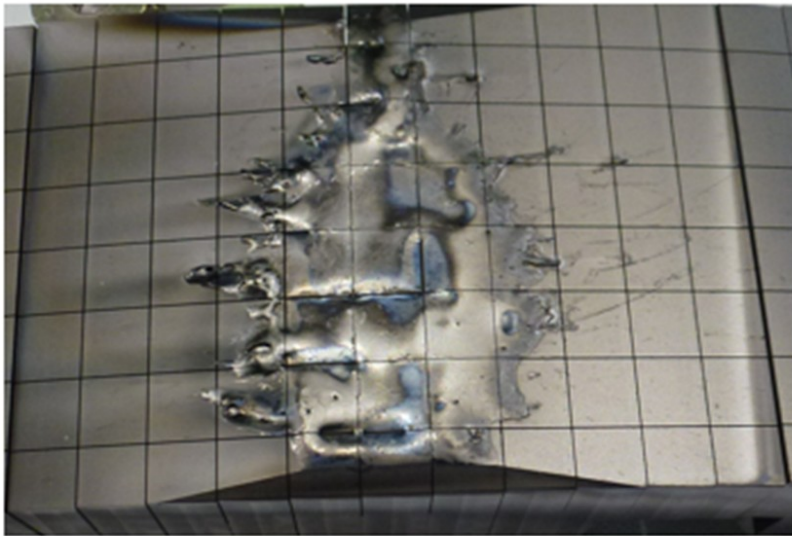
Current quench (CQ): decrease of plasma current and E_{\parallel} , due to the increase of resistivity ($\tau \sim 1\text{ms}$). **The RE population** undergoes an avalanche-like exponential growth, triggered by the Coulomb collisions between RE seeds and thermal electron, **increases significantly and partially compensates for the decrease in the thermal electron current.**

Plateau: at the end of CQ the RE current intensity remains almost constant for a certain period and REs can carry all the remaining plasma current.

Motivation

REs are a serious threat for large tokamak machines. In machines such as ITER these can acquire energies up to hundreds of MeV damaging the Plasma Facing Components (PFC). In extreme cases, the PFCs cooling system can be affected as well.

Damaged caused by the RE beam in JET #86801



L. Chen, 5th AAPPs, 26 Sep-1 Oct 2021

Disruption mitigation systems:

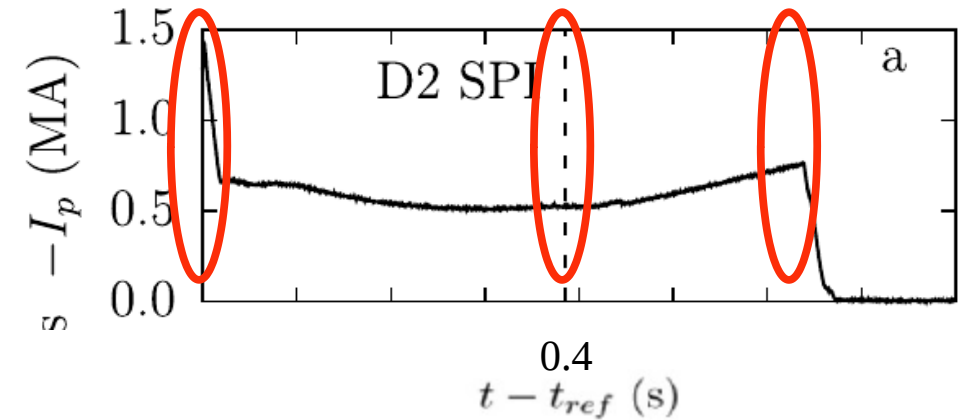
- **Shattered Pellet Injection** (SPI) (JET, ITER): impurity injection (D, Ne) for increasing electron density and collision radiation losses
- **Resonant magnetic perturbation coils** (REMC) (SPARC, DIII-D): increase the RE radial transport through the magnetic field lines stochastisation
- **Sacrificial limiters** (DEMO, DTT)

Robust RE avoidance/mitigation strategies require an improved understanding of plasma dynamics during the lifetime of the RE beam. The interplay between REs and plasma instabilities becomes important in this respect.

Motivation

An example of this interaction: Ex. JET #95135

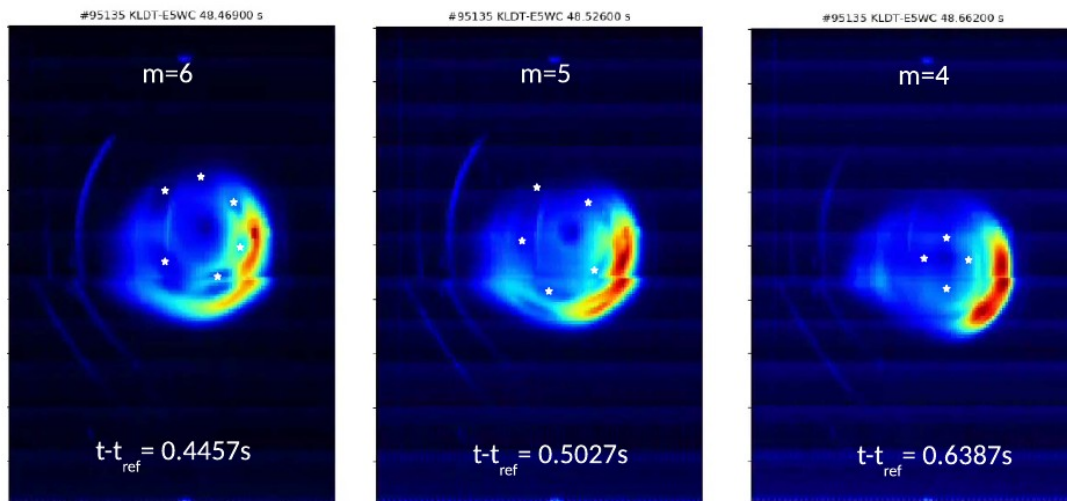
- Disruption triggered by massive injection of Argon which leads to the generation of RE
- At $t-t_{ref}=0.4$ second injection of D_2 to causes the plasma to recombine and the flush out of Argon from the plasma. This leads to a drop in the effective resistivity and an increase of the plasma current
- When the current reaches 0.75 MA a fast and near complete loss of RE takes place and leads to a benign termination, without damaging the wall
- The loss is due to the onset of a large MHD instability



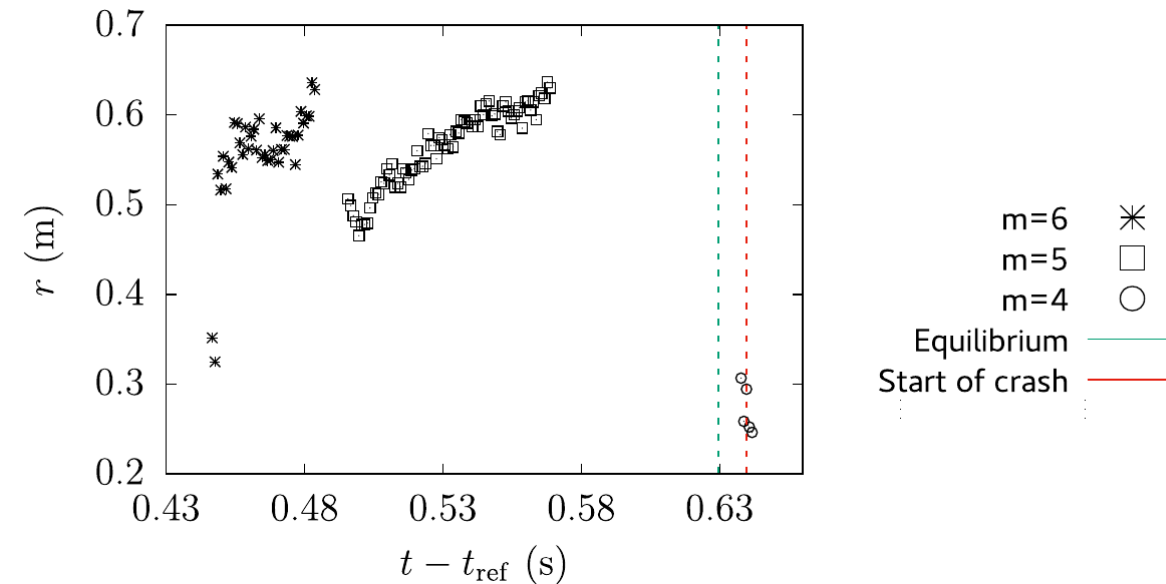
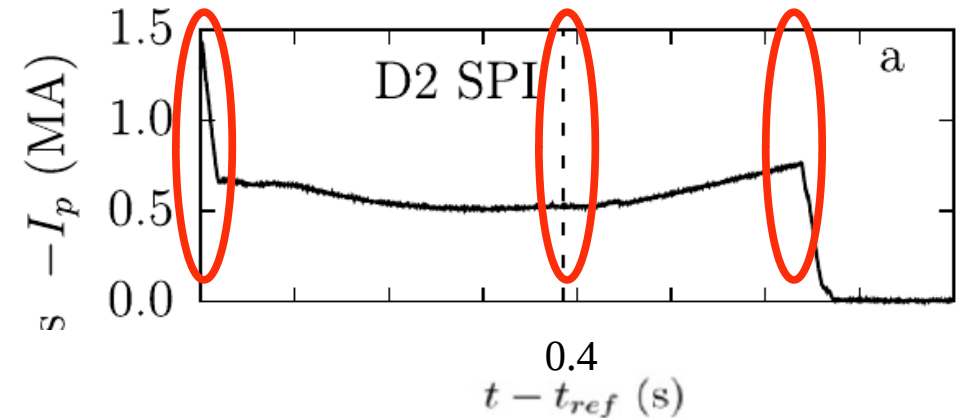
Motivation

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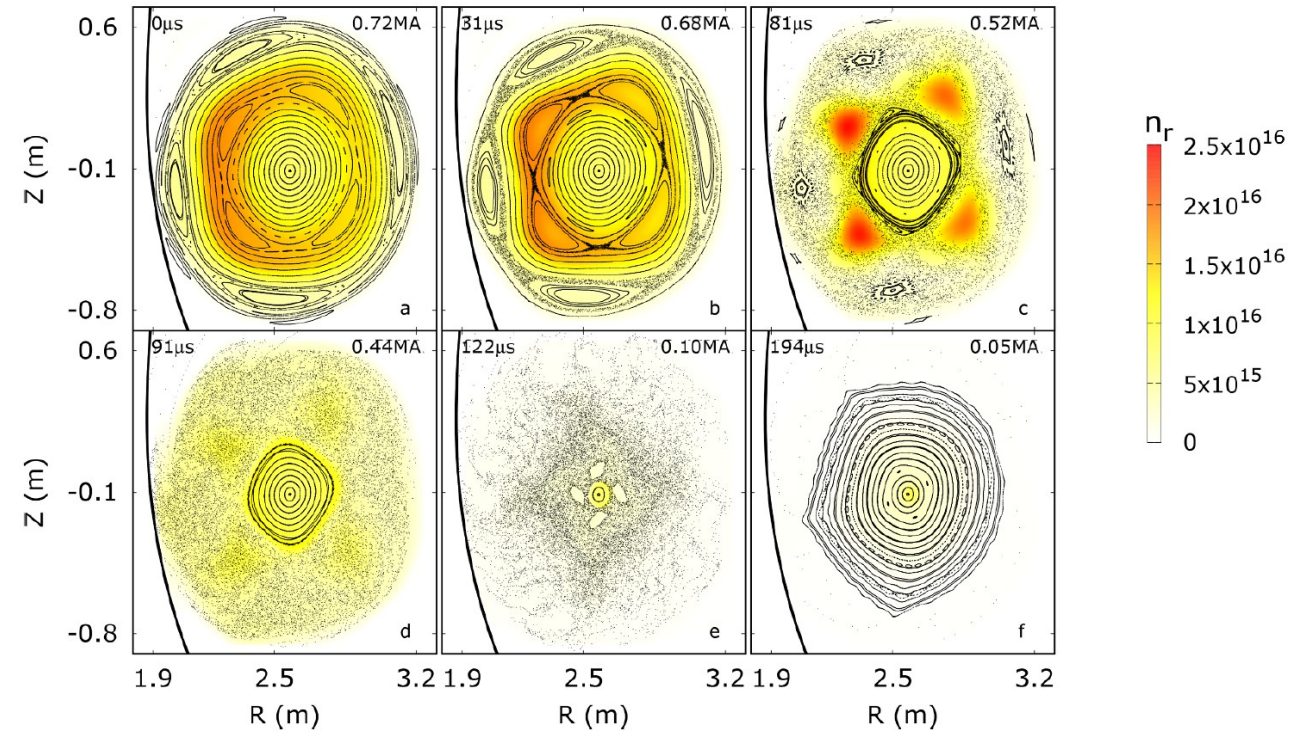


Infrared synchrotron data



Motivation

- **Bandaru et al. (PPCF 2021) carried out MHD numerical simulations with JOREK solver**, starting from the experimental magnetic configuration several ms before the crash
- Temporal evolution of magnetic field line topology (Poincaré plots on a poloidal section) and RE density distribution
- **Early stage of the evolution characterized by $m=4, n=1$**
- double chain of magnetic islands (*double tearing mode*)
- Magnetic field **stochastization in the nonlinear phase**
- **Large RE losses in presence of magnetic chaos occur within a duration of $20\sim\mu\text{sec}$**
- Final reformation of the magnetic flux surfaces and a nearly complete loss of RE
- **RE deposition shows significant toroidal and poloidal broadening that potentially reduces the REs load on the wall.**



The interaction of REs and magnetic reconnection is a fundamental process that needs to be understood.

How to model the post-disruption plasma?

The post disruption plasma is characterized by a rather cold resistive background plasma at $T_e \sim 10$ eV, where the equilibrium current is entirely carried by the runaway electrons

Collision frequency:

$$\nu_{ei} \sim \frac{n_e e^4 \ln \Lambda}{m_e^{1/2} T_e^{3/2}}$$

Mean free path:

$$\lambda_{mfp} \sim \frac{v_{th}}{\nu_{ei}} \propto \frac{T_e^2}{n_e \ln \Lambda}$$

At $T_e = 10$ eV, $n_e = 5 \times 10^{19} \text{ m}^{-3}$, $\ln \Lambda \sim 10$ \longrightarrow $\nu_{ei} \sim 10^8 \text{ s}^{-1}$; $\lambda_{mfp} \sim \text{few mm}$

For these parameters we have $\eta_{\parallel} = 1.96 n_e e^2 \tau_{ei} / m_e \approx 18 \mu\Omega\text{m}$ which is 10^3 times that of copper.

With a magnetic field $B \sim 3\text{T}$ the Alfvén velocity is substantially smaller than the speed of light:

$$v_A = B / (\mu_0 m_i n_e)^{1/2} \approx 0.02c$$

The normalized (to the minor radius $a \sim 1\text{m}$) resistive tearing mode layer width is $w \sim 1\text{cm}$ and $\gamma \sim 2 \times 10^3 \text{sec}$

Hence: $\lambda_{mfp} \ll w \ll a$ \longrightarrow

Local thermalization occurs before particles cross macroscopic scales: a fluid description is justified

Why is resistive MHD valid for the bulk plasma?

Can we use a resistive MHD description for the background plasma?

Three conditions need to be satisfied for a MHD description:

Quasi neutrality:

$$L \gg \lambda_D \approx 3.3 \mu\text{m}$$

Low frequency phenomena:

$$\gamma \sim 10^3 \ll v_{ei} \sim 10^8 \ll \omega_{ci} \sim 10^{11} \text{ s}^{-1}$$

FLR can be neglected:

$$\rho_i \sim 0.5 \text{ mm} \ll w \sim 10 \text{ mm}$$

Collision dominated phenomena:

$$\omega_A \sim 2 \times 10^6 \text{ s}^{-1} \ll v_{ei} \sim 10^8 \text{ s}^{-1}$$

$\beta \sim 0$: a reduced description is also allowed

We can use the Reduced Resistive MHD model

The RRMHD model for the bulk plasma

$$\frac{\partial \psi}{\partial t} + \nabla_{\parallel} \phi = \eta \nabla_{\perp}^2 \psi, \quad \leftarrow \text{Ohm's law for resistive thermal electrons} \quad \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

$$\frac{\partial \nabla_{\perp}^2 \phi}{\partial t} + [\phi, \nabla_{\perp}^2 \phi] + \nabla_{\parallel} \nabla_{\perp}^2 \psi = 0, \quad \leftarrow \text{Motion equation}$$

$$\mathbf{B} = B_0 \mathbf{e}_z + \nabla \psi_{eq} \times \mathbf{e}_z \quad \leftarrow \text{Magnetic field} \quad \mathbf{v} = \mathbf{e}_z \times \nabla \phi \quad \leftarrow \text{ExB velocity field}$$

$$[f, g] = \partial_x f \partial_y g - \partial_x g \partial_y f \quad \nabla_{\perp}^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \quad \nabla_{\parallel} f = \frac{\partial f}{\partial z} - [\psi, f]$$

**The cold resistive bulk is well described by resistive MHD
Runaway electrons require a separate collisionless treatment**

Why are runaway electrons collisionless?

The Coulomb drag decreases with velocity — above the Dreicer threshold, acceleration wins over collisions

Coulomb drag goes as $1/v^2$

$$v_{ei} \propto \frac{1}{v^3} \rightarrow 0 \quad \text{as} \quad v \gg v_{th}$$

Dreicer critical field:

$$E_D = \frac{n_e e^3 \ln \Lambda}{4\pi\epsilon_0 m_e v_{th}^2}$$

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} + q(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}) \cdot \frac{\partial f}{\partial \vec{v}} = 0 \quad \leftarrow$$

For $v \gg v_{th}$: collision frequency vanishes, mean free path diverges beyond machine size. The electron escapes the collisional regime.

For $E_{\parallel} > E_D$: runaway production begins

Vlasov equation: no collision term: particles stream freely along \mathbf{B} , conserving phase-space volume (Liouville theorem).

Once $v \gg v_{th}$, collisions are physically negligible
The collisionless treatment is exact, not an approximation

Collisionless plasmas in nature:

Fusion: runaway electrons, high-T core ($T_e \sim \text{keV}$)

Space: solar wind, Earth magnetosphere

Astro: accretion disks, galaxy clusters, pulsar magnetospheres

From Vlasov to a fluid equation for runaways current

Integrating the Vlasov equation over velocity space yields a closed fluid equation for the runaway current density

Step 1 — define the runaway current density

$$J_{RE} = e \int v_{\parallel} f d^3v$$



first moment of the distribution function over velocity space

Step 2 — Assume $v_{\parallel} = -c$ All REs move at $v \approx c$ along B

Step 3 — Integrating the Vlasov equation over velocity space you obtain (Helander et al., PoP 2007)

$$\frac{\partial J_{RE}}{\partial t} + [\varphi, J_{RE}] - \frac{c}{v_A} \nabla_{\parallel} J_{RE} = 0$$



parallel advection at $v = c$ (collisionless streaming) (v_A is due to the normalization)

**No resistive term: collisionless REs do not dissipate energy.
J_RE is frozen into the magnetic field (Alfven theorem).**

MR in a post-disruption plasma

The two regimes are coupled through J_{RE} in Ohm's law. REs are collisionless and hence subtracted from the collisional term. Equations are normalized on the Alfvén velocity and the magnetic field equilibrium scale length

$$\frac{\partial \psi}{\partial t} + \nabla_{\parallel} \phi = \eta (\nabla^2 \psi + J_{RE})$$

Ohm's law for resistive thermal electrons and collisionless REs

$$\frac{\partial \nabla^2 \phi}{\partial t} + [\phi, \nabla^2 \phi] + [\nabla_{\parallel} \nabla^2 \psi] = 0$$

Motion equation

$$\frac{\partial J_{RE}}{\partial t} + [\phi, J_{RE}] - \frac{c}{v_A} \nabla_{\parallel} J_{RE} = 0$$

REs current density equation

EQUILIBRIUM

$$\psi_0 = \psi(x)$$

Sheared magnetic field

$$\phi_0 = 0$$

No flow

$$J_0 = -\nabla_{\perp}^2 \psi_0$$

Current carried by REs

$$\mathbf{B} = B_0 \mathbf{e}_z + \nabla \psi_{eq} \times \mathbf{e}_z$$

Magnetic field

$$\mathbf{v} = \mathbf{e}_z \times \nabla \phi$$

Velocity field

$$v_A = B / (\mu_0 m_i n_e)^{1/2} \approx 0.02c$$

$$c/v_A \approx 50$$

$$[f, g] = \partial_x f \partial_y g - \partial_x g \partial_y f \quad \nabla_{\perp}^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \quad \nabla_{\parallel} f = \frac{\partial f}{\partial z} - [\psi, f]$$

MR driven by REs: **single helicity case**

$$\psi_1(x, y, z, t) = \sum_{k_y} \sum_{k_z} \hat{\psi}_{k_y k_z}(x, t) e^{i(k_y y + k_z z)} = \sum_{k_y} \hat{\psi}_{k_y k_z}(x, t) e^{ik_y(y + \alpha z)} \leftarrow$$

SH linearly unstable perturbation

$$\alpha = \frac{k_z}{k_y} \leftarrow \text{Helicity}$$

Simulations are carried out with our solver SCOPE3D

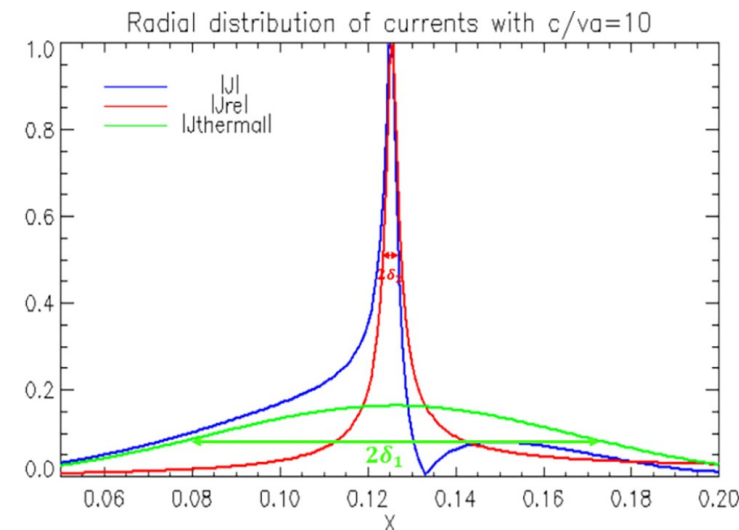
Helander et al. (**Phys Plasmas 2007**) and Liu et al. (**Phys Plasmas 2020**) studied both analytically and numerically the tearing mode instability induced by a runaway current in a weakly unstable plasma in the single helicity case

Linear dispersion relation

$$\frac{\gamma^{5/4}}{\eta^{3/4} k_y^{1/2}} \frac{2\pi\Gamma(3/4)}{\Gamma(1/4)} = \Delta' - i\pi \frac{k_y J'_{RE0}}{|k_y|}$$

Weak influence of the RE current on the growth rate which follows the FKR growth

Poloidal rotation frequency of the magnetic island proportional to the REs profile



Scale lengths

$$\delta_1 = \gamma^{1/4} \eta^{1/4} k_y^{-1/2}$$

$$\delta_2 = \frac{\gamma v_A}{k_y c}$$

MR driven by REs: **single helicity case**

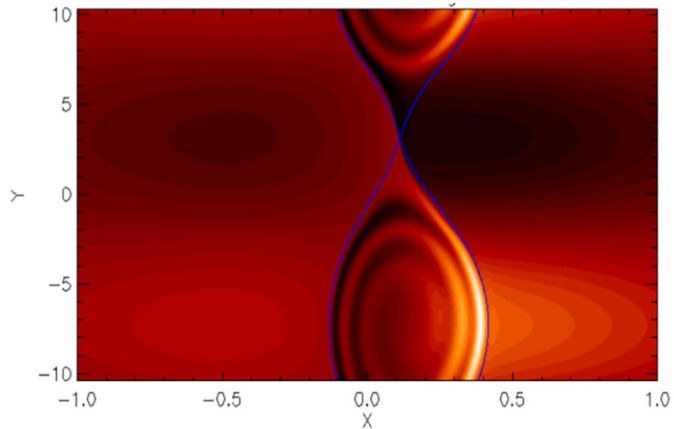
Nonlinear evolution

REs become nonlinear when $w \sim \delta_2$, while thermal electrons when $w \sim \delta_1$

REs enter the nonlinear regime earlier than the thermal electrons

In the early nonlinear phase RE current is distributed in a spiral structure within the magnetic island. Small scales tend to disappear as the saturated state approaches.

As soon as the system moves to the nonlinear phase, the magnetic island rotation slows down, until it finally stops



$$w = -\frac{j_0(0)}{j_0''(0)} \frac{\Delta'}{0.272}$$



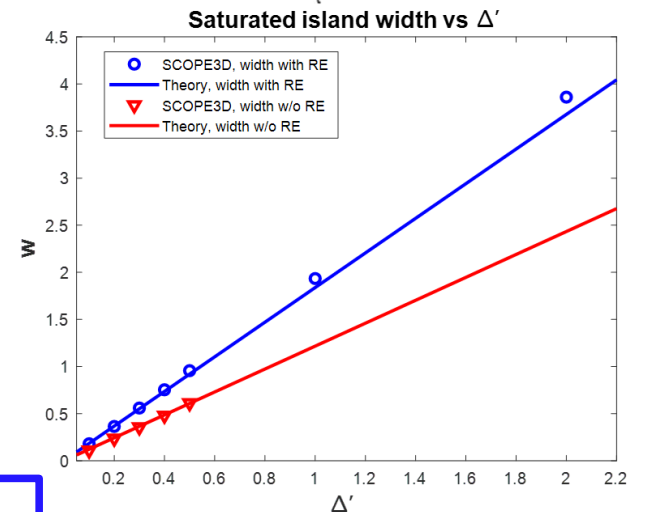
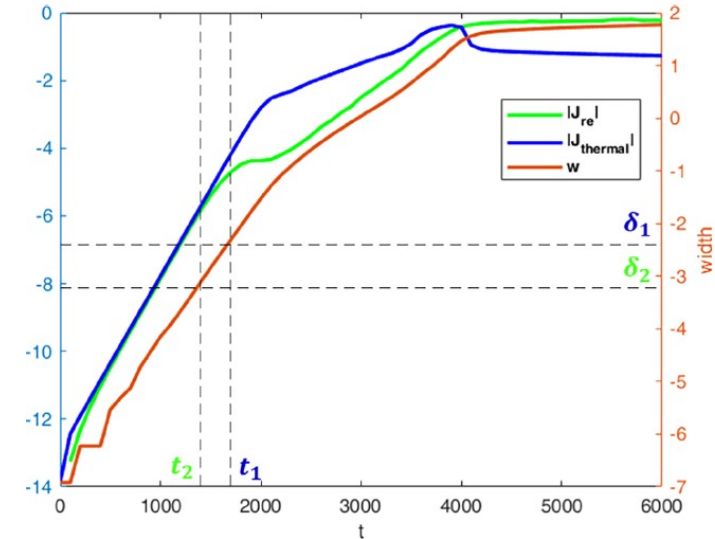
Saturation with REs

$$w = -\frac{j_0(0)}{j_0''(0)} \frac{\Delta'}{0.411}$$



Saturation w/o REs

REs lead to a 50% increase in the magnetic island width at saturation



MR driven by REs: multiple helicity

Is this scenario still valid in MH case?

We studied numerically the evolution of an initial magnetic perturbation with a dominant (k_{y*}, k_{z*}) mode component and a set of linearly unstable fluctuations of small amplitude, uncorrelated and equipartitioned, with multiple helicities

$$\psi_1(x, y, z) = \hat{\psi}_{k_{y*}, k_{z*}}(x) e^{i(k_{y*}y + k_{z*}z)} + \sum_{k_y} \sum_{k_z} \hat{\psi}_{k_y, k_z}(x, t) e^{i(k_y y + k_z z + \xi_{k_y, k_z})}$$

$$\begin{array}{ccc} \xi_{k_y, k_z} & \leftarrow & \text{Random phase} \\ & & \hat{\psi}_{k_y, k_z} = 10^{-3} \hat{\psi}_{k_{y*}, k_{z*}} \\ \left. \begin{array}{l} k_{y*} = \frac{2\pi m^*}{L_y} \\ k_{z*} = \frac{2\pi n^*}{L_z} \end{array} \right\} & \leftarrow & \text{Main wave vectors and wave numbers} \\ & \rightarrow & \left\{ \begin{array}{l} m^* = 1 \\ n^* = 1 \end{array} \right. \end{array}$$

MR driven by REs: multiple helicity

Poincarè maps of the magnetic field lines allow us to visualise the **topology of the magnetic field on $z=const$ sections**

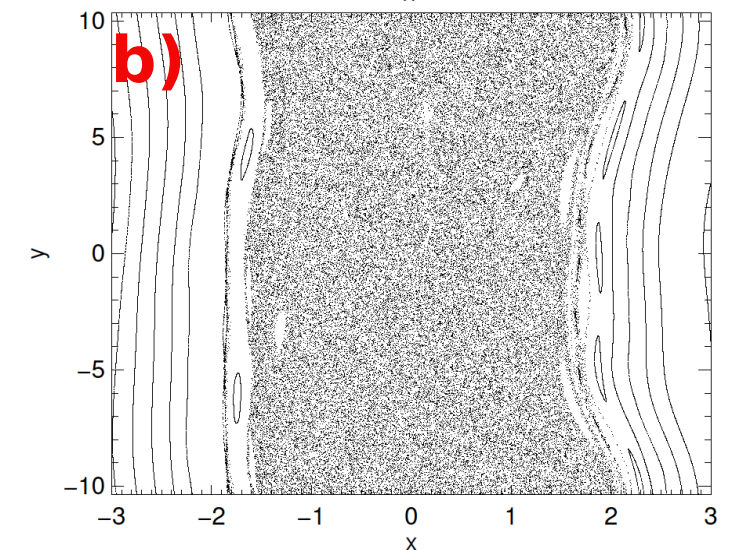
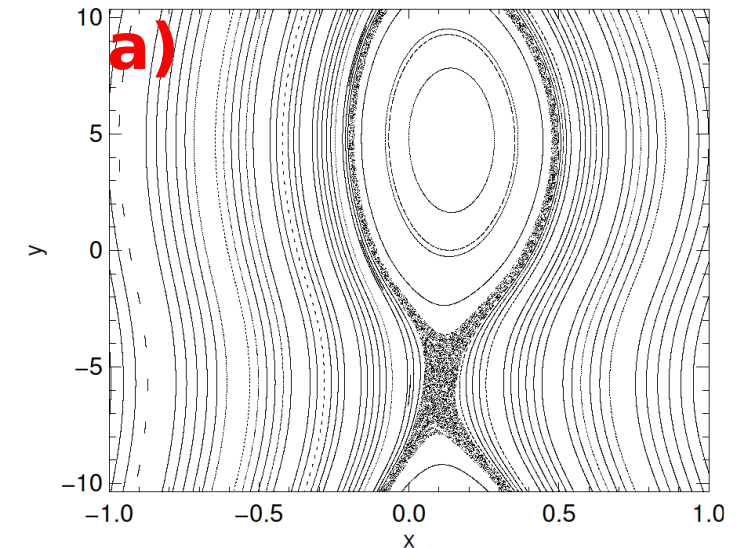
Magnetic field line equations



$$\left\{ \begin{array}{l} \frac{dx}{dz} = \frac{1}{B_0} \frac{\partial \psi}{\partial y} = \frac{B_x}{B_0} \\ \frac{dy}{dz} = -\frac{1}{B_0} \frac{\partial \psi}{\partial x} = \frac{B_y}{B_0} \end{array} \right.$$

a) The early phase of the process is characterised by the dominant component of the initial condition. Magnetic field lines **chaoticity** localized around the **separatrices** of the unperturbed island

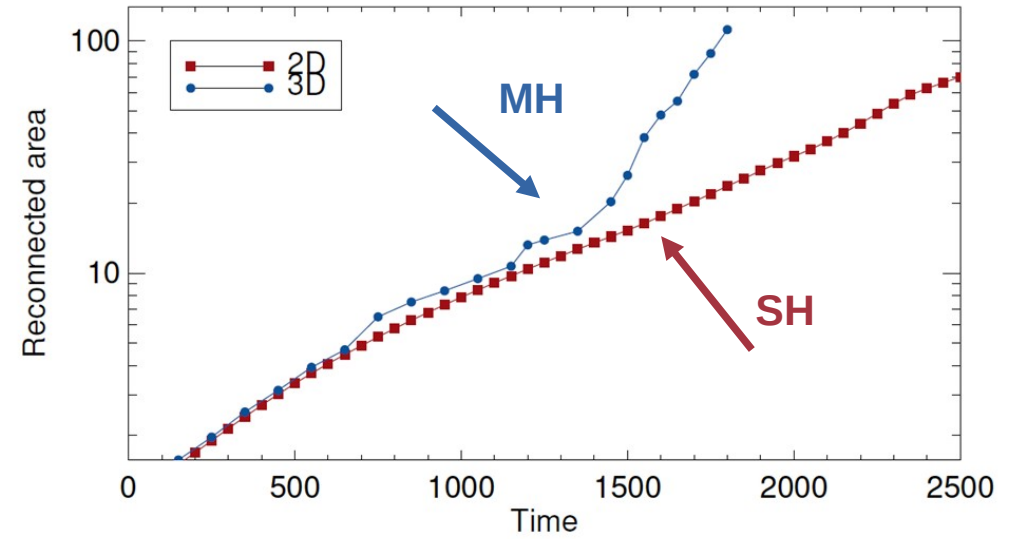
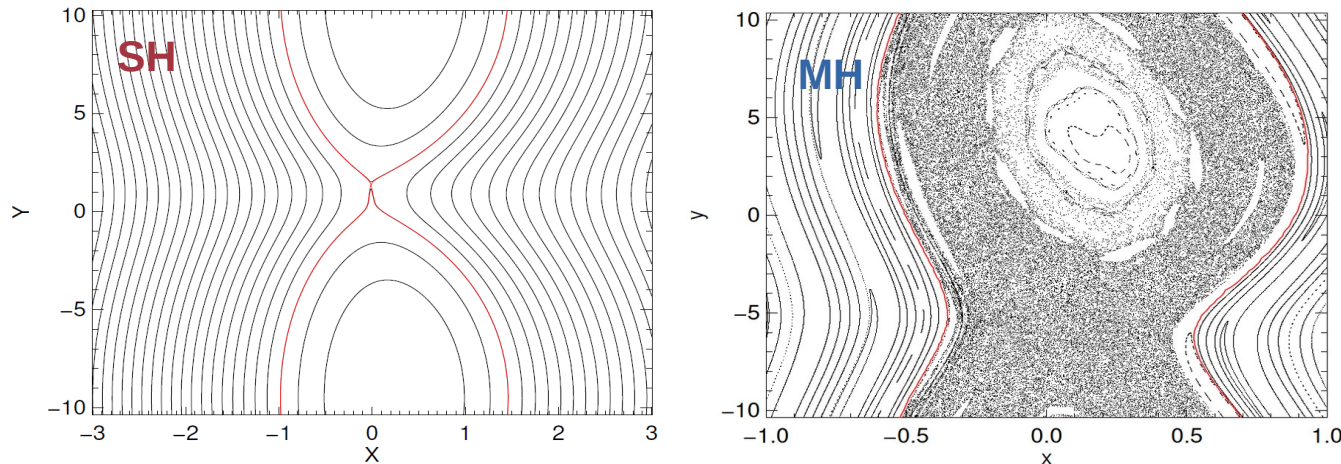
b) In the nonlinear phase of the evolution, multiple helicity modes with comparable amplitude coexist. Their interaction leads to the **global stochastization of the domain**



MR driven by REs: **multiple helicity**

According to Borgogno et al. PoP 2005 we measure the **reconnection rate by means of the area of the region where the topological modifications induced by MR are located**

This measure reduces to the magnetic island area in 2D



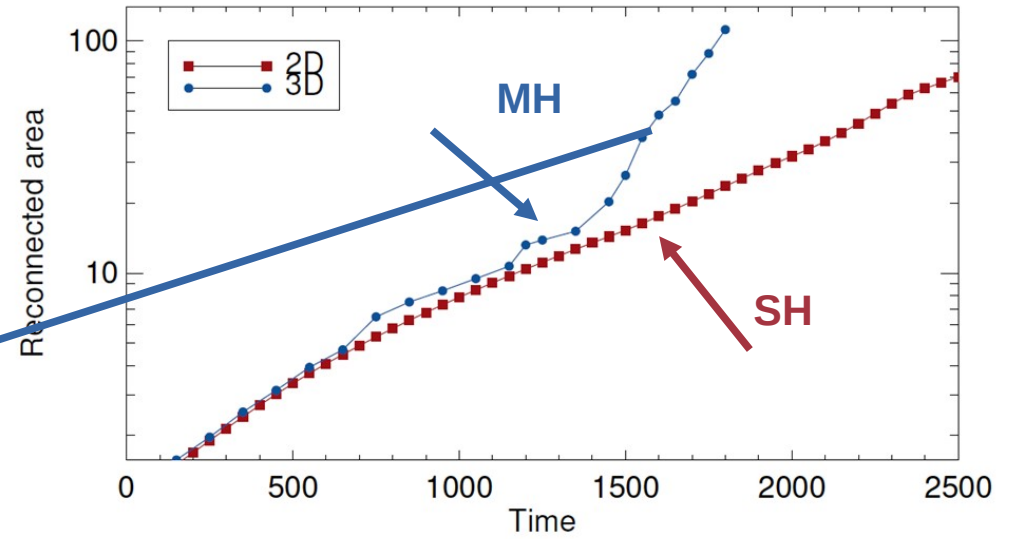
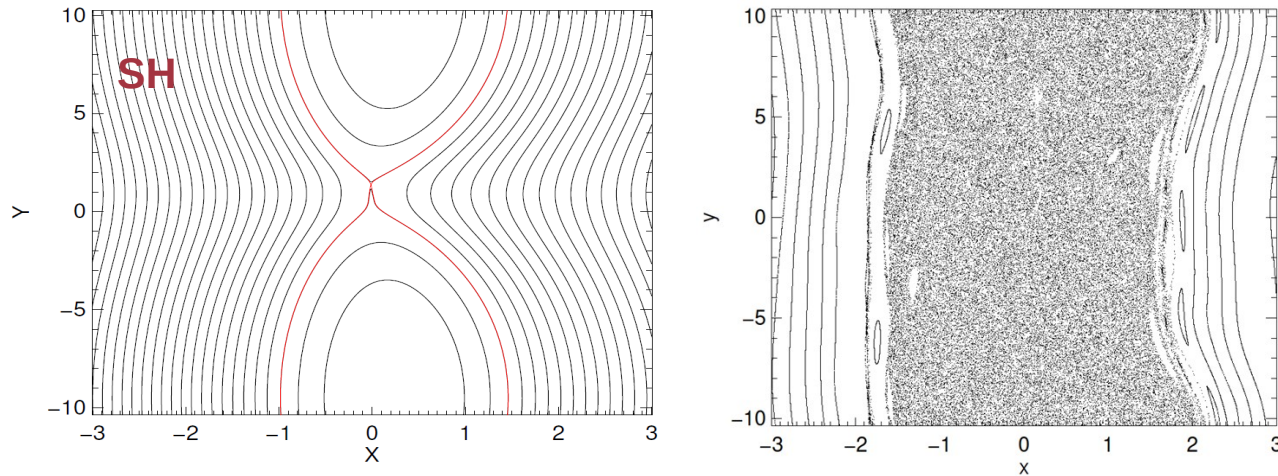
MR exhibits a **similar evolution in SH and MH configurations in the linear and early nonlinear phase**

Explosive growth of the reconnected area in 3D begins when the stochastic layer width is of the order of the equilibrium scale length

MR driven by REs: **multiple helicity**

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This measure reduces to the magnetic island area in 2D



Global stochastization of the domain before reaching the saturation

How do collisionless and collisional electrons behave?

As time progresses and the chaotic region expands, the RE current tends to homogenize and decrease over a significant portion of the reconnected region

$$\overline{J_{RE}}(x,t) = \frac{\iint J_{RE}(x,y,z,t) dy dz}{\iint dy dz}.$$

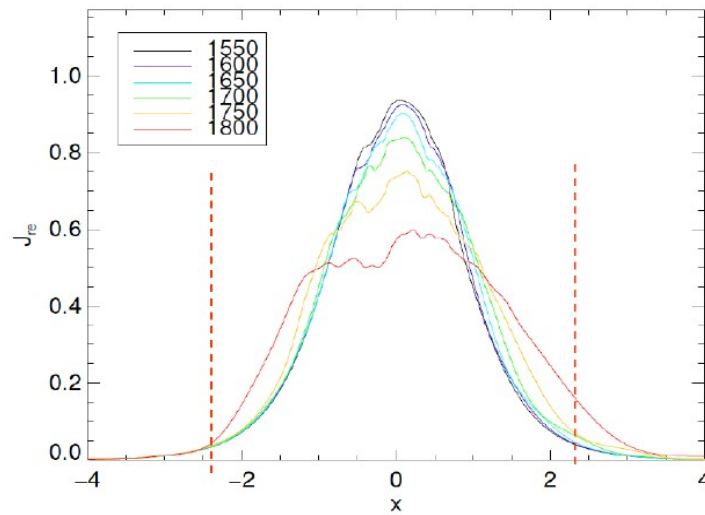


FIG. 5: Radial profile of the RE current averaged over y and z , according to the formula in eq. 9. The red dashed lines represent the averaged position of the boundaries of the chaotic region.

By contrast, the averaged Ohmic current remains strongly localized in small-scale structures.

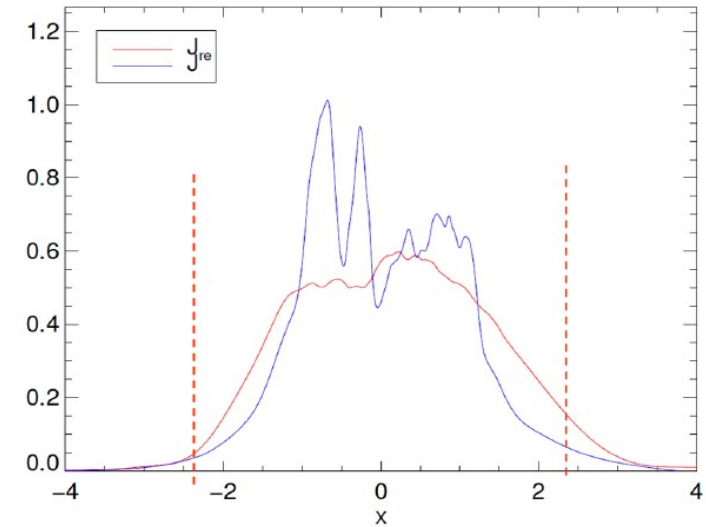
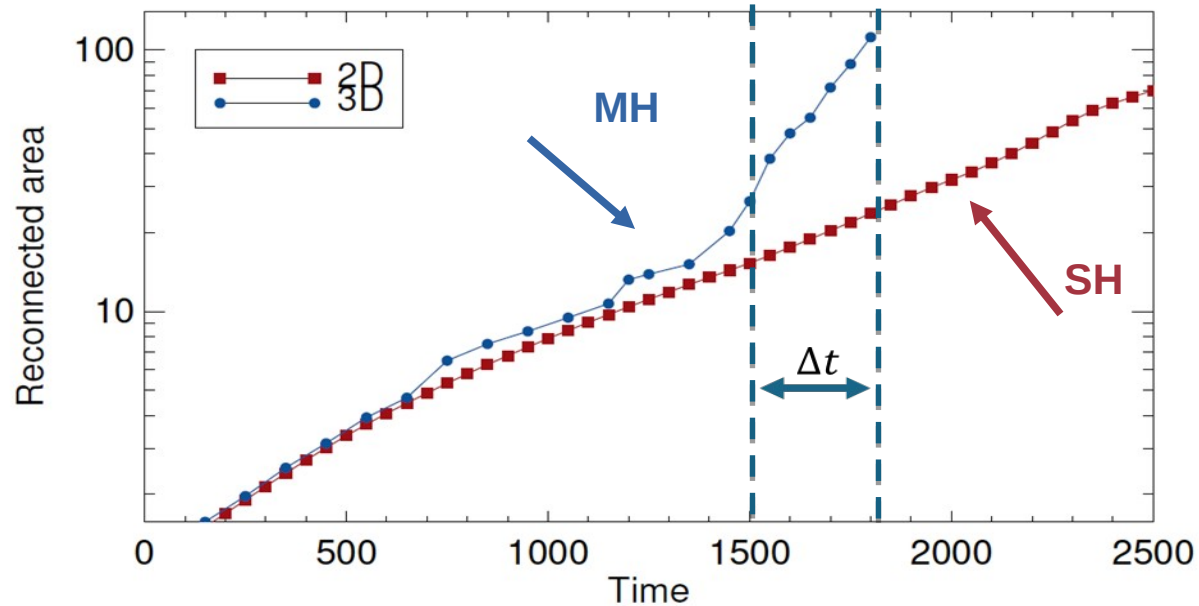


FIG. 6: Radial profile of the RE (red) and Ohmic (blue) current, averaged over y and z , at $t = 1800$. The red dashed lines represent the averaged position of the boundaries of the chaotic region.

This contrasting behaviour reflects the fundamentally different nature of the two populations: collisionless REs are transported along the field, while thermal electrons remain locally anchored by resistivity.

MR driven by REs: **multiple helicity**

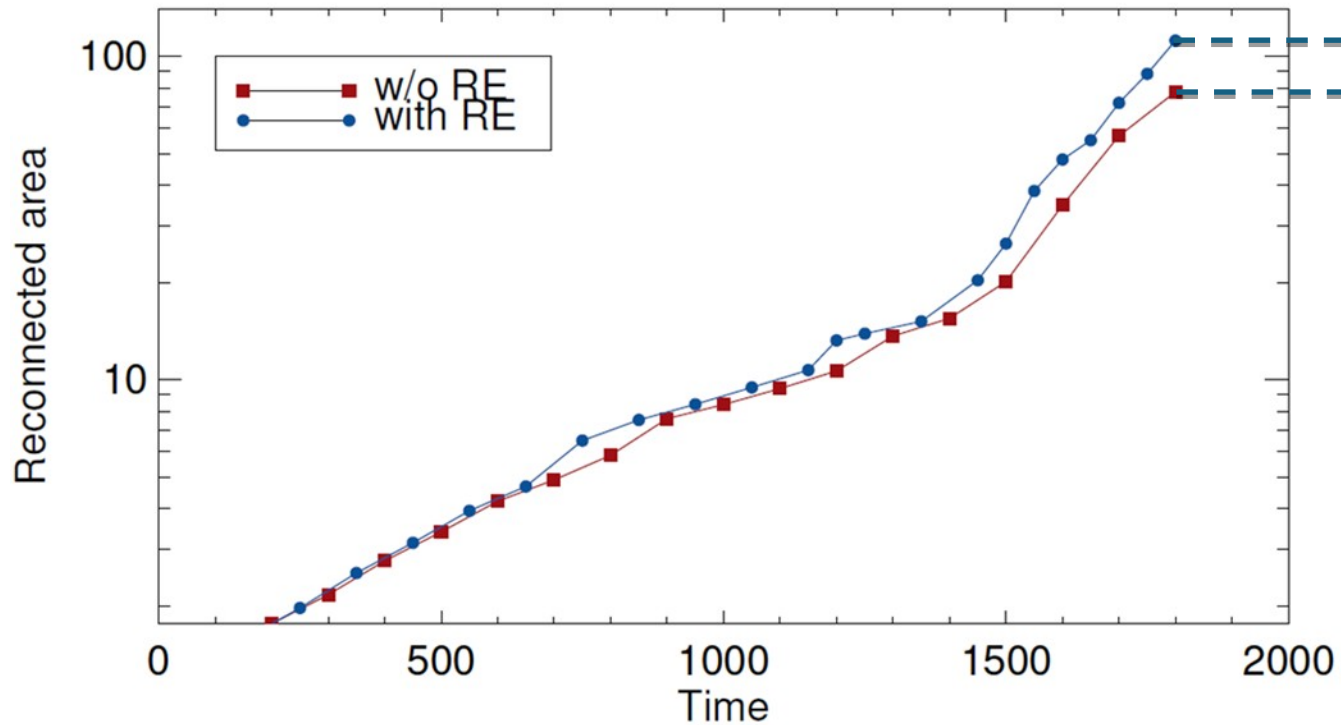
This supports the idea that REs are redistributed and potentially lost due to reconnection-enhanced chaotic transport. Runaway electrons, moving at nearly the speed of light along magnetic field lines, are strongly affected by the chaoticity of the field, which leads to their dispersion across the reconnection region



Transition from local to global stochasticity in $\Delta t \sim 300\tau_A \sim 10\mu\text{s}$ ($B_0=3.49\text{T}$, $n=10^{19}\text{m}^{-3}$, $L=1\text{m}$), in agreement with Bandaru et al., PPCF 2021.

MR driven by REs: **multiple helicity**

Comparison of MH case with and w/o REs



$\Delta A_{rec} 50\%$

Nonlinear explosive growth both with and w/o REs

In presence of REs the saturated amplitude is almost 50% larger than in the Ohmic case, in agreement with SH results

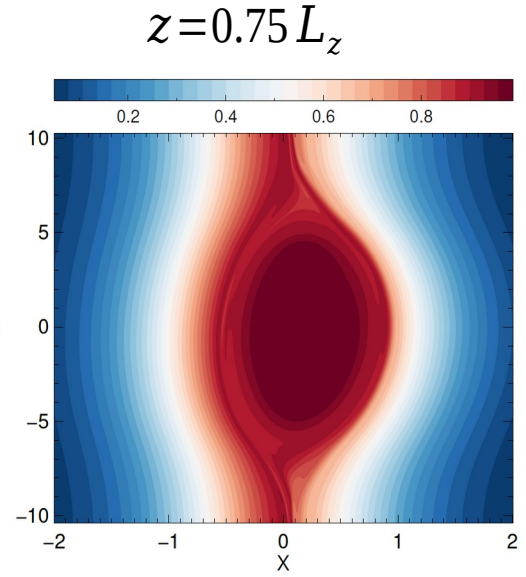
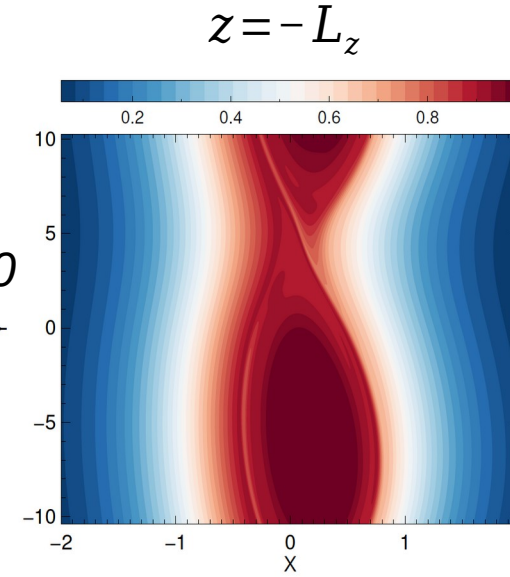
MR driven by REs: **multiple helicity**

Contour plot of the REs current on two different poloidal sections at two different nonlinear times

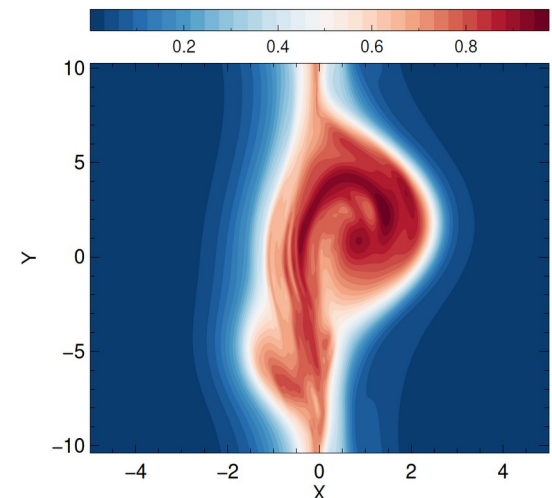
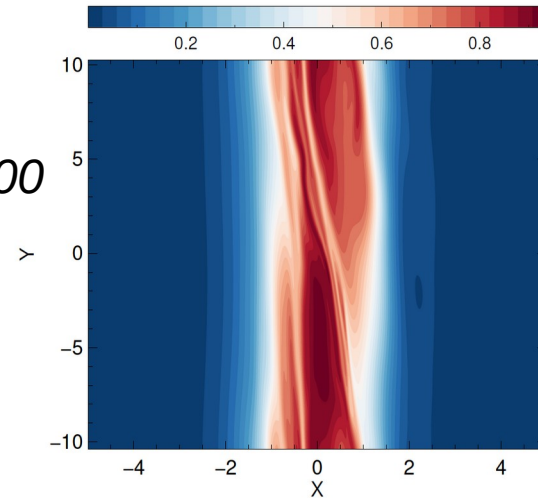
Generation of small scale filaments due to a mixing process



$t=1450$



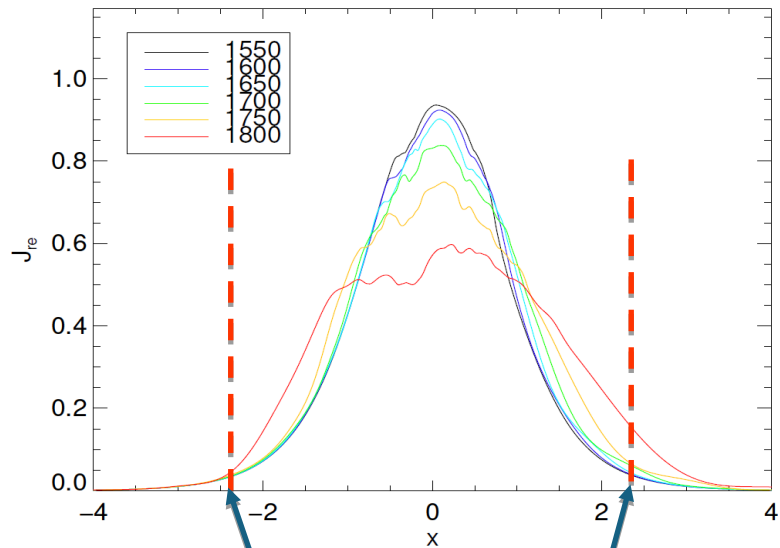
$t=1700$



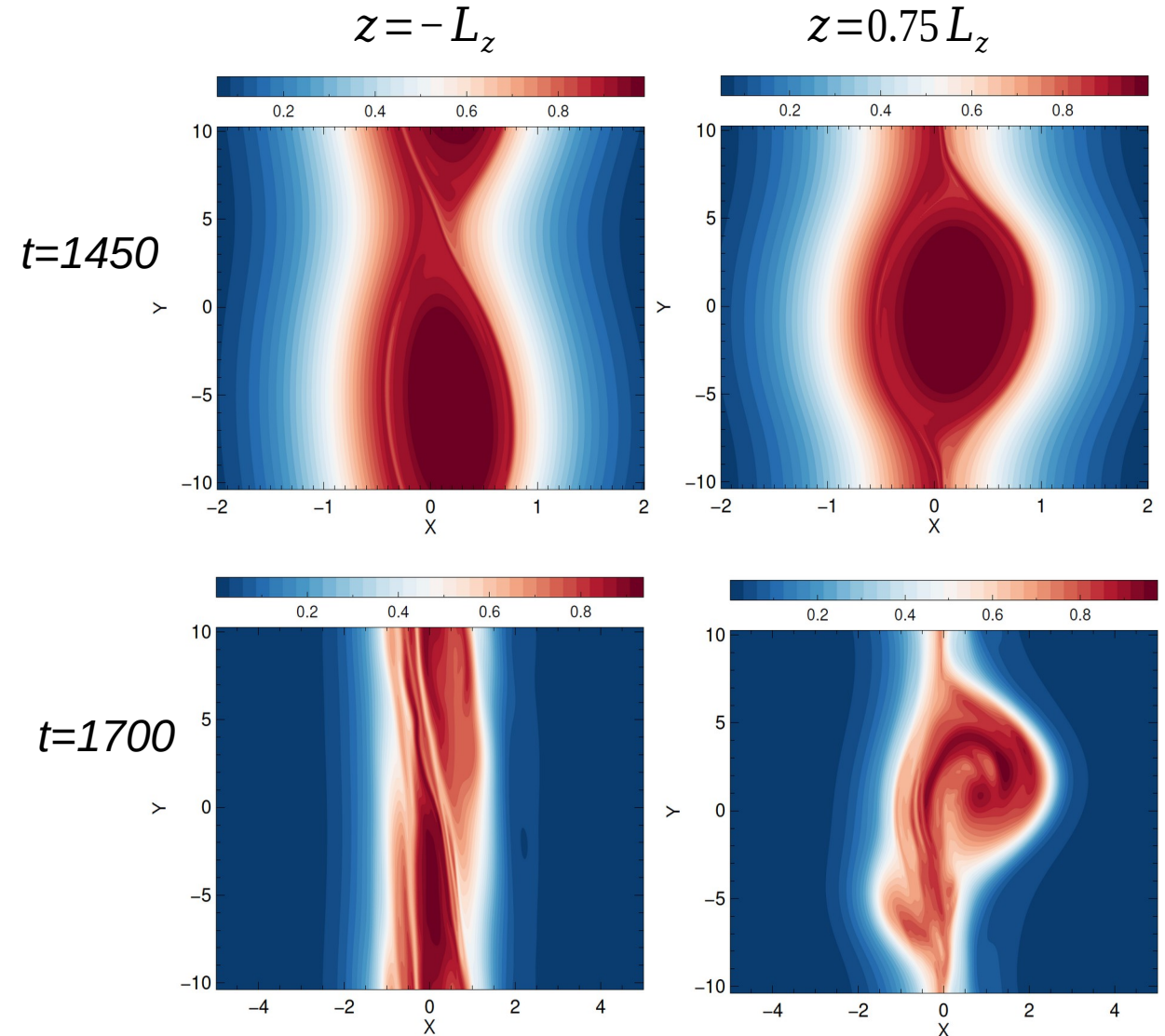
MR driven by REs: **multiple helicity**

Tendency of the runaway current to uniformly distribute in the chaotic region as the time increases

Radial profile J_{RE} averaged over y and z



Averaged position of the boundaries of the chaotic region



Conclusions

A detailed knowledge of the mutual interaction between runaway electrons and magnetic reconnection is crucial for designing effective RE mitigation systems

MR driven by REs current in a post disruption plasma develops peculiar behaviors compared to the standard thermal electron regimes

SH:

- i) magnetic island rotation**
- ii) development of small scale layers in the REs current, where kinetic contributions may become relevant**
- iii) larger magnetic island at saturation**

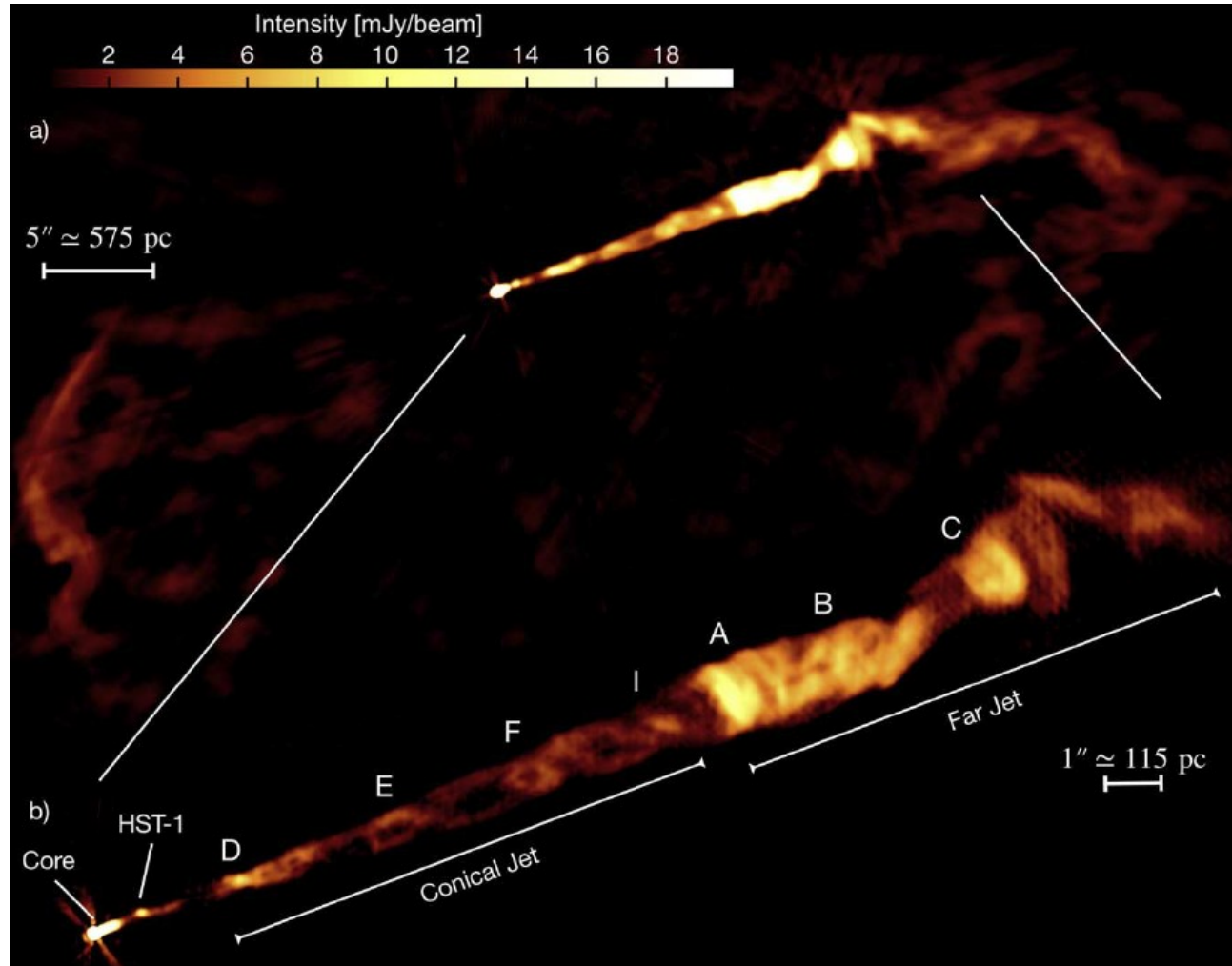
MH:

- i) explosive acceleration of the magnetic reconnection growth in 3D**
- ii) Fast decrease in runaway electron current and homogenisation on the chaotic regions**

Ongoing work:

- i) new campaign of 3D numerical simulations aimed at clarifying the role of resistivity**
- ii) investigation of the influence of the magnetic chaos on the runaway electron dynamics by means of test-particle simulations**

Imagining a future together....



Thank you!