The Earth's Ionosphere and the Transition from Collisional to Collisionless Regimes

Stephan Buchert (scb@irfu.se)

Nordita 2026, Stockhom, May 11-15

Synergies Between Astrophysical, Space, Laboratory, and Fusion Plasma Physics

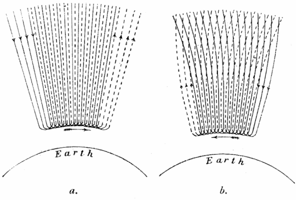
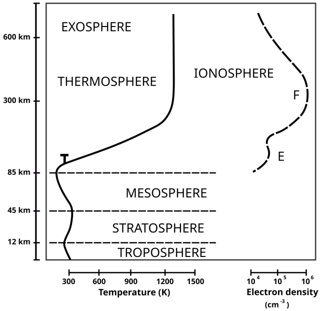


Fig. 50.

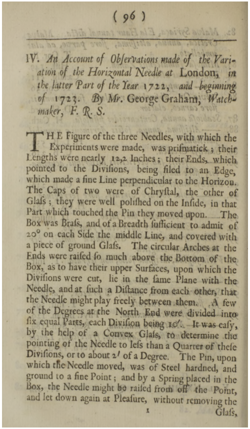
Outline



Wikipedia public domain

- the Earth's ionosphere, field-aligned currents and their closure;
- basic steady-state equations;
- including collisions (charged-neutral and charged-charged);
- single fluid MHD equations for the ionosphere-thermosphere;
- source of Joule/frictional heating and momentum transfer;
- coupling to atmospheric dynamics.

The First "In-situ" Observations of Effects of Space Plasma Physics



(96)

IV. *An Account of Observations made of the Variation of the Horizontal Needle at* London, *in the latter Part of the Year* 1722, *and beginning of* 1723. *By Mr.* George Graham, *Watchmaker,* F. R. S.

THE Figure of the three Needles, with which the Experiments were made, was prismatick; their Lengths were nearly 12,2 Inches; their Ends, which pointed to the Divisions, being filed to an Edge, which made a fine Line perpendicular to the Horizon. The Caps of two were of Chrystal, the other of Glass; they were well polished on the Inside, in that Part which touched the Pin they moved upon. The Box was Brass, and of a Breadth sufficient to admit of 20° on each Side the middle Line, and covered with a piece of ground Glass. The circular Arches at the Ends were raised so much above the Bottom of the Box, as to have their upper Surfaces, upon which the Divisions were cut, lie in the same Plane with the Needle, and at such a Distance from each other, that the Needle might play freely between them. A few of the Degrees at the North End were divided into six equal Parts, each Division being 10′. It was easy, by the help of a Convex Glass, to determine the pointing of the Needle to less than a Quarter of these Divisions, or to about 2′ of a Degree. The Pin, upon which the Needle moved, was of Steel hardned, and ground to a fine Point; and by a Spring placed in the Box, the Needle might be raised from off the Point, and let down again at Pleasure, without removing the Glass,

- George Graham (1674–1751)
- noticed, ≈300 years ago,
- daily variations in the direction of a magnetic needle.
- in Uppsala Hiorter and Celsius (1741),
- having one of Graham's needles,

Magnetic Needle of Hans Christian Ørsted

- recorded large variations when aurora was in the sky,
- and confirmed Graham's observation.

More History: Kristian Birkeland (1867–1917)

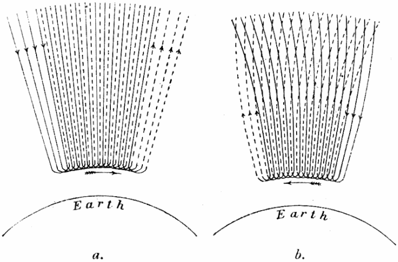


Fig. 50.

- Birkeland currents[a] flow (originally vertically)
- parallel to the magnetic field $\vec{B}$.
- $\nabla \cdot \vec{j} = 0 \rightarrow$ the closure of Birkeland currents is crucial
- for a transfer of energy and momentum between different regions/regimes!

[a] commonly they are also called FACs.

The "traditional" view in ionospheric physics:

- To close Birkeland currents, we need
  1. an electric field $\vec{E}$,
  2. (ion-neutral) collisions ν_{in},
- Neutral gas with velocity $\vec{u}$;
- To get the electric current $\vec{j} = e\,N_e\,(\vec{v}_i - \vec{v}_e)$,
- solve the steady-state momentum balance for ions

$$e\left[\vec{E} + \vec{v}_i \times \vec{B}\right] - m_i \nu_{in}\,(\vec{v}_i - \vec{u}) = m_i n_e \left[\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \cdot \nabla)\,\vec{v}_i\right] \ldots \approx 0 \tag{1}$$

The "traditional" view in ionospheric physics:

- To close Birkeland currents, we need
  1. an electric field $\vec{E}$,
  2. (ion-neutral) collisions ν_{in},
- Neutral gas with velocity $\vec{u}$;
- To get the electric current $\vec{j} = e\,N_e\,(\vec{v}_i - \vec{v}_e)$,
- solve the steady-state momentum balance for ions

$$e\left[\vec{E} + \vec{v}_i \times \vec{B}\right] - m_i\nu_{in}\left(\vec{v}_i - \vec{u}\right) \approx 0 \qquad (1)$$

- for $\vec{v}_i$

- With κ_i the ratio of the ion cyclotron to ion-neutral collision frequencies, $\kappa_i = eB/m_i\nu_{in}$.

$$\vec{v}_{i,\parallel} = \vec{u}_\parallel + \frac{e}{m_i\,\nu_{in}}\vec{E}_\parallel \tag{2}$$

$$\begin{aligned}\vec{v}_{i,\perp} &= \vec{u}_\perp + \frac{\kappa_i}{1+\kappa_i^2}\left(\vec{E}_\perp + \vec{u}\times\vec{B}\right)/B \\ &\quad + \frac{\kappa_i^2}{1+\kappa_i^2}\left(\vec{E}_\perp + \vec{u}\times\vec{B}\right)\times\vec{B}/B^2.\end{aligned} \tag{3}$$

- alternative writing of (3):

$$\vec{v}_{i,\perp} = \frac{\vec{u}_\perp + \kappa_i\left(\vec{E}_\perp + \vec{u}\times\vec{B}\right)/B + \kappa_i^2\vec{E}\times\vec{B}/B^2}{1+\kappa_i^2} \tag{4}$$

Limiting cases:

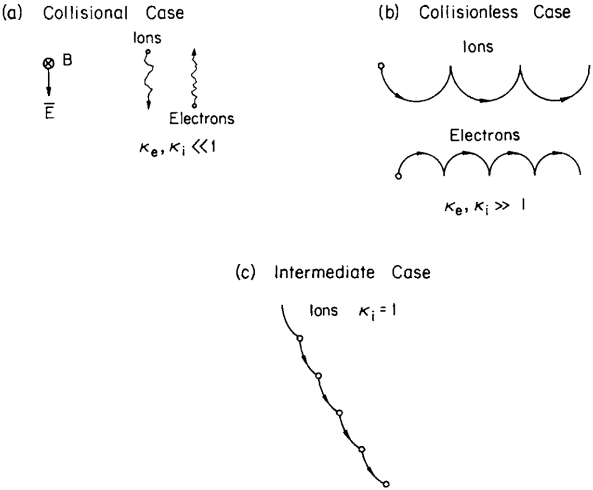
near-collisionless, $\kappa_i \gg 1$: $\vec{v}_{i,\perp} \approx \vec{E}\times\vec{B}/B^2$

very collisional, $\kappa_i \ll 1$: $\vec{v}_{i,\perp} \approx \vec{u}$

Limiting cases:

near-collisionless, $\kappa_i \gg 1$: $\vec{v}_{i,\perp} \approx \vec{E} \times \vec{B}/B^2$

very collisional, $\kappa_i \ll 1$: $\vec{v}_{i,\perp} \approx \vec{u}$

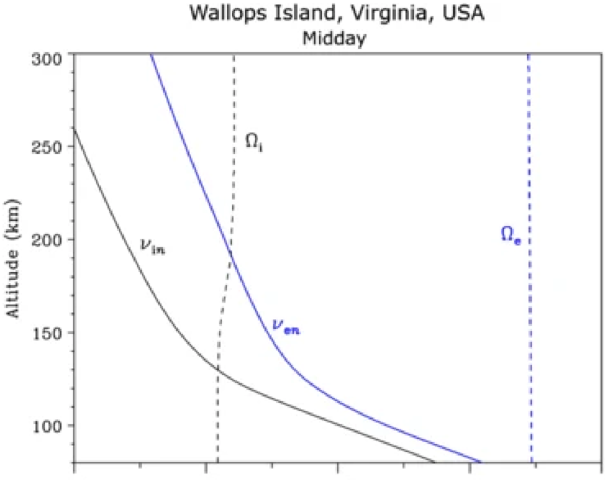


M. C. Kelley (1989), The Earth's Ionosphere

Electrons

- analogous to ions,
- but in the Earth's ionosphere $\kappa_e \gg 1$ (from about 80 km and upwards), therefore

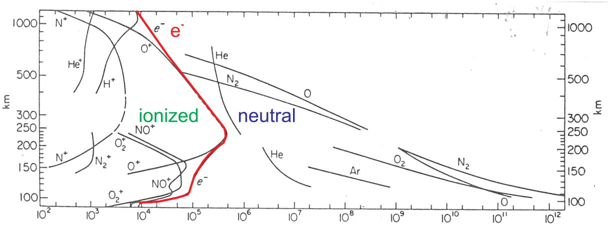
$$\vec{v}_{e,\perp} \approx \vec{E} \times \vec{B}/B^2? \tag{5}$$



Electrons

However, both

- electron-neutral, ν_{en}, and electron-ion (Coulomb), ν_{ei}, collisions need to be considered
- in the F region (above about 250 km altitude) $\nu_{ei} > \nu_{en}$



M. C. Kelley (1989), The Earth's Ionosphere

- To the take into account effects of Farley-Buneman turbulence in the lower E region,
- anomalous collisions (ν_{ei}^*) are assumed to represent those.

Analogous to ion-neutral, but $\vec{v}_i \rightarrow \vec{v}_e$ and $\vec{u} \rightarrow \vec{v}_i$ we get

Electron-ion collisions

$$\vec{v}_{e,\parallel} = \vec{v}_{i,\parallel} - \frac{e}{m_e\, \nu_{ei}} \vec{E}_\parallel \tag{6}$$

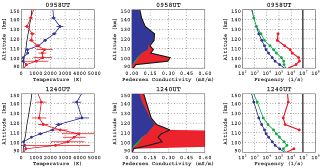
$$\vec{v}_{e,\perp} = \vec{v}_{i,\perp} - \frac{\kappa_{ei}}{1+\kappa_{ei}^2}\left(\vec{E}_\perp + \vec{v}_i \times \vec{B}\right)/B + \frac{\kappa_{ei}^2}{1+\kappa_{ei}^2}\left(\vec{E}_\perp + \vec{v}_i \times \vec{B}\right) \times \vec{B}/B^2. \tag{7}$$

$$\vec{v}_{e,\perp} = \frac{\vec{v}_{i,\perp} - \kappa_{ei}\left(\vec{E}_\perp + \vec{v}_{i,\perp} \times \vec{B}\right)/B + \kappa_{ei}^2 \vec{E} \times \vec{B}/B^2}{1+\kappa_{ei}^2} \tag{8}$$

- for near-collisionless ions electron-ion collisions do not contribute much to closure of Birkeland currents, $\vec{v}_{e,\perp} \approx 0$;
- for very collisional ions, $\vec{v}_{i,\perp} \approx \vec{u}$ electron-ion collisions behave like electron-neutral collisions and can very well close Birkeland currents.

According to Poynting's theorem a heating rate is given by

$$q_J = -\vec{j} \cdot \vec{E} \approx e\, N_e\, \vec{v}_{e,\perp} \cdot \vec{E} \qquad (9)$$



Observations with the EISCAT IS radar and estimate of ν_{ei}^*

- for $T_e > T_n$ the heating of e^-s is balanced by their cooling in inelastic collisions with neutrals;
- even the most intense heating by solar EUV raises T_e only a few 10 K over T_n;
- heating by Farley-Buneman waves/anomalous ν_{ei}^* can be enormously strong!
- relevant in the Sun's chromosphere?

Intermediate Sunummary

- two categories of collisions:
  1. ion-electron or Coulomb collisions ν_{ei}
  2. ion-neutral ν_{in} and electron-neutral collisions ν_{en} in a partially ionized plasma only;
- both are dissipative,
- ion-neutral collisions are required to close FACs
- and therewith are involved in the transfer of energy and momentum between different regions;
- Coulomb collisions can enable closure of FAC
- when ions are frozen into the neutral gas, $\kappa_i \ll 1$
- with possibly very large dissipation.

"Paradigms" $\vec{B}, \vec{v}$ vs $\vec{E}, \vec{j}$

- ✓ E. Parker
- ✓ Crafoord prize for Astronomy in 2020 (given to one person for achievements over life by the Royal Swedish Academy of Sciences/Crafoord Foundation)
- $>$ recommends to not think of $\vec{E}$ as a cause:
  - (JGR 1996): "…we suggest that the …task is ..effectively attacked with the B, v paradigm"

$$\alpha \equiv \frac{cB(M/\tau_i - m/\tau_e)}{4\pi ne(M/\tau_i + m/\tau_e)} \qquad (114)$$

representing the Hall resistive diffusion coefficient,

$$\beta \equiv \frac{B^2}{4\pi n\,(M/\tau_i + m/\tau_e)} \qquad (115)$$

as the Pedersen resistive diffusion coefficient, and

$$\eta \equiv \frac{c^2}{4\pi ne^2}\left[\frac{(M/\tau_i)(m/\tau_e)}{M/\tau_i + m/\tau_e} + \frac{m}{\tau}\right] \qquad (116)$$

- the paper is difficult to read: CGS units, diffusion coefficients α, β, η;
- but the ionospheric community uses conductivities σ_P, σ_H, and $\sigma_{\|}$

Ohm's law for the ionosphere

$$
\begin{aligned}
\vec{j} &= \vec{j}_{\|} + \vec{j}_{\perp} \\
&= \sigma_{\|}\vec{E}_{\|} + \sigma_P\left(\vec{E}_{\perp} + \vec{u}\times\vec{B}\right) - \sigma_H\left(\vec{E}_{\perp} + \vec{u}\times\vec{B}\right)\times\vec{B}/B \qquad (10)
\end{aligned}
$$

However, to eliminate $\vec{E}$ (Parker, 1996) we need a "generalized" Ohm's law of the form $\vec{E} = \ldots$!

The GOL for a PIP

$$\vec{E}_\perp = -\vec{u} \times \vec{B} + \frac{\sigma_P \vec{j}_\perp + \sigma_H \vec{j} \times \vec{B}/B}{\sigma_P^2 + \sigma_H^2} \tag{11}$$

- different versions of GOLs (generalized Ohm's laws) for FIPs (fully ionized ...),

> here $\vec{u}$ is the neutral velocity/wind!

The GOL for a PIP

$$\vec{E}_\perp = -\vec{u} \times \vec{B} + \frac{\sigma_P \vec{j}_\perp + \sigma_H \vec{j} \times \vec{B}/B}{\sigma_P^2 + \sigma_H^2} \tag{11}$$

- different versions of GOLs (generalized Ohm's laws) for FIPs (fully ionized ...),

$>$ here $\vec{u}$ is the neutral velocity/wind!

$$\mathbf{E} = \frac{B}{c}\{-\mathbf{v} \times \mathbf{b} - \beta[(\nabla \times \mathbf{b}) \times \mathbf{b}] \times \mathbf{b} + \alpha(\nabla \times \mathbf{b}) \times \mathbf{b} + \eta \nabla \times \mathbf{b}\} \tag{117}$$

---

appeared in Fujii et al., 1997
1^{st} in Leake et al., SSR, 2014

Joule Heating

with $\vec{E}$, $\vec{j}$ (and $\vec{E}_{\|} = 0$),

- according to Vasyliunas and Song (2005)

$$q_J = \vec{j} \cdot \left(\vec{E}_{\perp} + \vec{u} \times \vec{B}\right) \tag{12}$$

with $\vec{B}$, $\vec{v}$,

- eliminate $\vec{E}$ using the GOL:

$$q_J = \frac{\sigma_P}{\sigma_P^2 + \sigma_H^2} j_{\perp}^2 + j_{\|}/\sigma_{\|} \tag{13}$$

- also JH has the "Cowling" factor $\sigma_P / \left(\sigma_P^2 + \sigma_H^2\right)$;
- the Hall effect reduces JH,
- in the Earth's ionosphere by a factor ~ 5

Conservation of Momentum

with $\vec{E}$, $\vec{j}$ there is "ion drag"

$$m_n n_n \left(\frac{\partial}{\partial t} + (\vec{u} \cdot \nabla)\right) \vec{u} = m_n \vec{g} + \nabla \left(n_n k_B T_n\right) + m_n\, n_n\, \nu_{ni} \left(\vec{v}_i - \vec{u}\right) \tag{14}$$

with $\vec{B}$, $\vec{v}$:

$$\vec{v}_i - \vec{u} = \vec{v}_e + \vec{j}/N_e e - \vec{u} = \frac{\vec{E} \times \vec{B}}{B^2} + \vec{j}/N_e e - \vec{u} \tag{15}$$

eliminate $\vec{E}$ with the GOL:y

- For magnetized electrons the result is:

$$m_n n_n \left(\frac{\partial}{\partial t} + (\vec{u} \cdot \nabla)\right) \vec{u} = m_n \vec{g} + \nabla \left(n_n k_B T_n\right) + \vec{j} \times \vec{B} + \dots$$

> the Lorentz force $\vec{j} \times \vec{B}$ directly appplies to the neutral gas!

> (but still needs proof for demagnetized e^{-}s)

Self-contained dynamical system in $\vec{u}$ and $\vec{B}$

$$m_n n_n \frac{d\vec{u}}{dt} = m_n \vec{g} + \nabla \left(n_n k_B T_n \right) + \vec{j} \times \vec{B} \tag{16}$$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \left(-\vec{u} \times \vec{B} + \frac{\sigma_P \vec{j}_\perp + \sigma_H \vec{j} \times \vec{B}/B}{\sigma_P^2 + \sigma_H^2} \right) \tag{17}$$

$$\vec{j} = \mu_0 \nabla \times \vec{B} \tag{18}$$

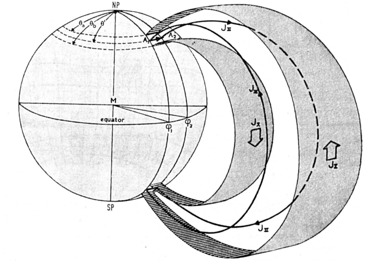
- $\partial \vec{B}/\partial t \neq 0$ would include Alfvénic dynamics and enforce very small time steps, rather use

$$\partial \vec{B}/\partial t \approx \vec{0} \tag{19}$$

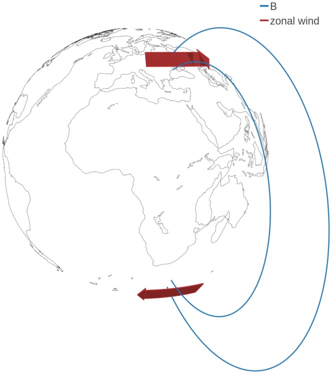
for practical codes and analysis.

Graham's daily magnetic variations (Sq)

- are not caused by a (tidal) neutral wind creating an "effective" $\vec{E}' = \vec{E} + \vec{u} \times \vec{B}$
- but rather by neutral wind differences $\Delta\vec{u}$ between northern and southern hemisphere at conjugate points:

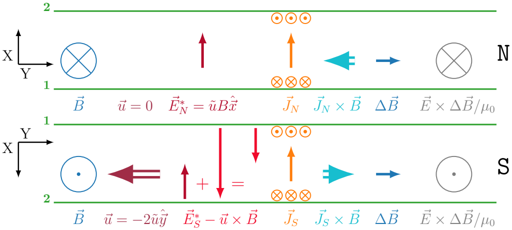


van Sabben (1966), JASTP



Buchert (2000), Ann. Geophys.

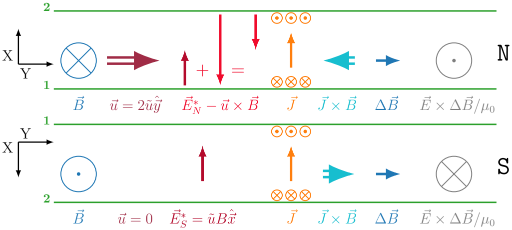
The View of the Northern Hemisphere



- Poynting flux into the Northern hemisphere
- and out of the Southern hemisphere

> the Southern hemisphere is the dynamo, Northern the load

The View of the Southern Hemisphere



- Poynting flux into the Southern hemisphere
- and out of the Northern hemisphere

$>$ the Northern hemisphere is the dynamo, Southern the load

- the frame-independent $\vec{j} \times \vec{B}$ acts to reduce $\Delta\vec{u}$

- a very high conductivity $\sigma_{\|}$ short-circuits electric potential differences along $\vec{B}$
- $\rightarrow \vec{u} \times \vec{B}$ not constant along $\vec{B}$ drives currents/magnetic tensions
- $\vec{j} \times \vec{B}$ forcing tries to relax to $\vec{u} \times \vec{B}$ constant along $\vec{B}$
- effectively transfering momentum between different regions

The real driver of Sq dissipation

with $\vec{B}$, $\vec{v}$ (sort of):

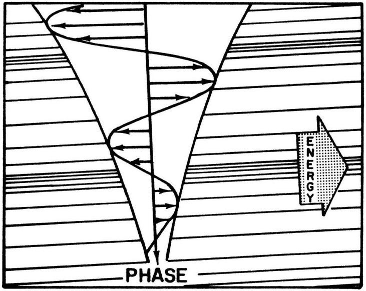
- Zonal kinetic energy flux, sum of N and S:

$$E_{kin} = \frac{nm}{2}\frac{\Delta x}{2}\left((u-\Delta u)^2 + (u+\Delta u)^2\right) \tag{20}$$
$$= \frac{nm}{2}\Delta x\left(u^2 + \Delta u^2\right) \tag{21}$$

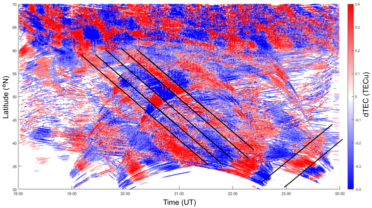
- assume that Sq succeeds in dynamically eliminating Δu
- conserving momentum;
- then E_{kin} has been reduced to

$$E_{kin} = \frac{nm}{2}\Delta x\, u^2 \tag{22}$$

Atmospheric Gravity waves in Partially Ionized Plasma



C. Hines, 1960



Traveling ionospheric Disturbances, Verhulst et al. (2022), JSWSC

$$m_n n_n \frac{d\vec{u}}{dt} = m_n \vec{g} + \nabla \left(n_n k_B T_n \right) + \vec{j} \times \vec{B}$$

- the basis for AGWs is the momentun equation for the neutrals gas with gravity and pressure term,
- coupling to the partially ionized ionosphere via $\vec{j} \times \vec{B}$;
- linearize and dispersion relations?
- not yet done, AFAIK, Nordita project?

Summary

- Including partially ionized plasmas there are two types of collisions in plasmas:
  - I charged-neutral and
  - II Coulomb collisions
- ion-neutral or both combined can effectively close field-aligned currents;
- closing FACs/releasing magnetic tension dissipates energy and transfers momentun between regions/layers potentially over large distances;
- atmospheric waves may collisionally couple to plasma and the magnetic field, and so to plasma dynamics;
- this would be relevant in the Earth's ionosphere/thermosphere, other planetary atmospheres, stellar and galactic atmospheres (?)
- the complete linear theory, dispersion relations etc remain to be solved.

References

- T. G. Cowling, The Electrical Conductivity of an Ionised Gas in the Presence of a Magnetic Field, Monthly Notices of the Royal Astronomical Society, Volume 93, Issue 1, November 1932, Pages 9097, https://doi.org/10.1093/mnras/93.1.90
- Parker, E. N. (1996), The alternative paradigm for magnetospheric physics, J. Geophys. Res., 101(A5), 1058710625, doi:10.1029/95JA02866.
- Pfaff, R.F. The Near-Earth Plasma Environment. Space Sci Rev 168, 23112 (2012). https://doi.org/10.1007/s11214-012-9872-6
- Leake, J.E., DeVore, C.R., Thayer, J.P. et al. Ionized Plasma and Neutral Gas Coupling in the Suns Chromosphere and Earths Ionosphere/Thermosphere. Space Sci Rev 184, 107172 (2014). https://doi.org/10.1007/s11214-014-0103-1
- Ballester, J.L., Alexeev, I., Collados, M. et al. Partially Ionized Plasmas in Astrophysics. Space Sci Rev 214, 58 (2018). https://doi.org/10.1007/s11214-018-0485-6