Collisional Versus Collective (Instability) Processes for Space Plasmas

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Synergies Between Astrophysical, Space, Laboratory, and Fusion Plasma Physics, May 11-29, 2026, Nordita, Stockholm, Sweden

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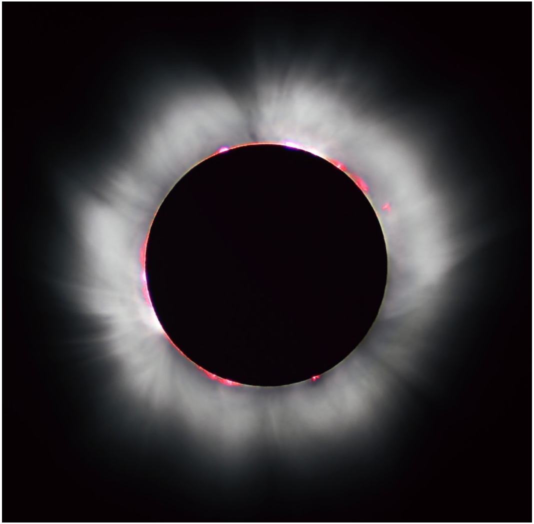
Motivation: **Coronal Heating** and **Solar Wind Acceleration** – An Unsolved Problem in Astronomy and Astrophysics



- The temperature of the Sun's surface is at about 5800 kelvin.
- The corona is at about 1 to 3 MK (parts of the corona can even reach 10 MK).[1]
- Near the surface of the sun the solar atmosphere shows no sign of organized motion but in the transition region, solar atmosphere begins acceleration outward – this is the solar wind
- The coronal heating and solar wind acceleration is a twin problem still not completely solved.

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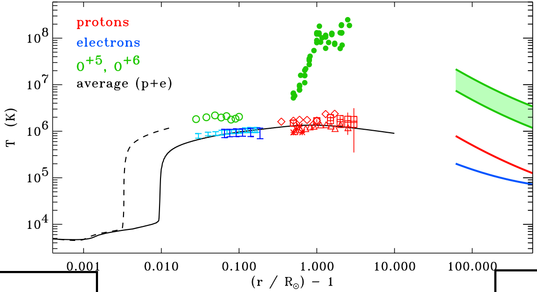
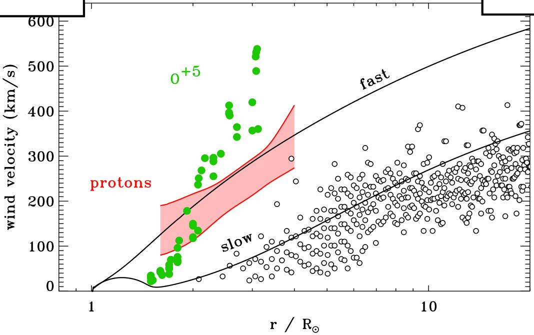
[1] c.f., fusion plasmas can reach 100 MK or more (fusion plasma is denser, 10^{13}–10^{15} pcc, versus coronal density of 10^{6} – 10^{10} pcc)



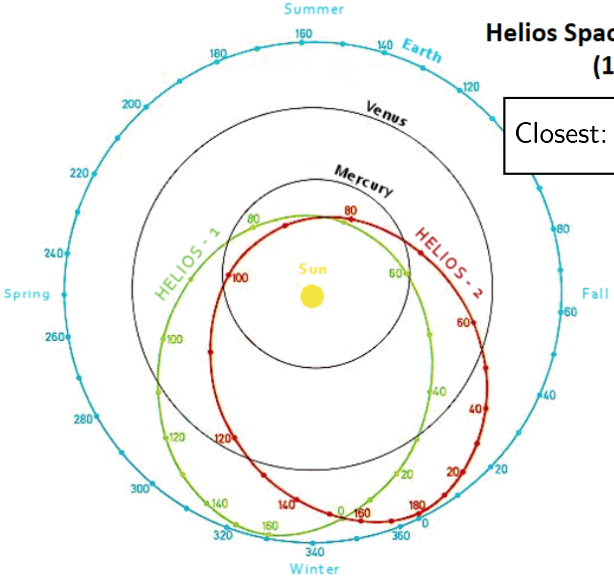
[Credit] Cranmer, 2009

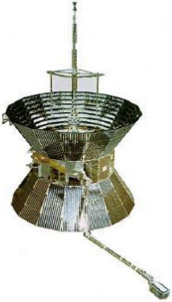
$R_\odot \sim 7 \times 10^5$ km

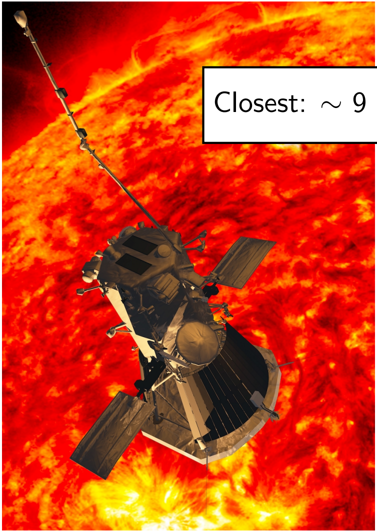


Helios Spacecraft Mission (1970s)

Closest: 0.3 AU $\sim$ 64.5 $R_\odot$

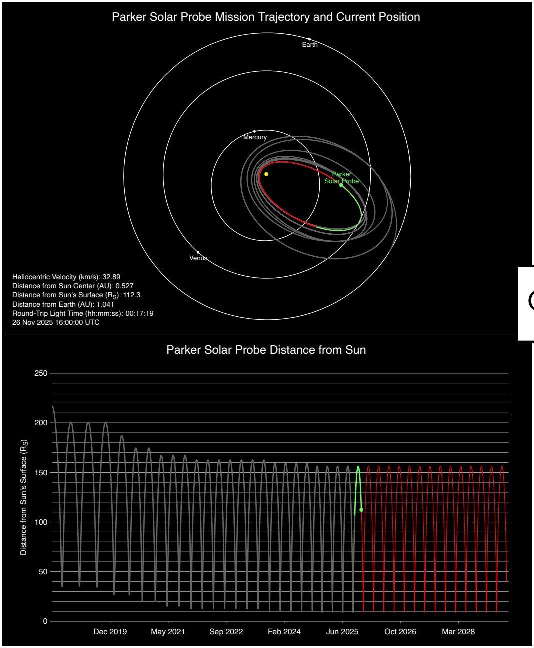






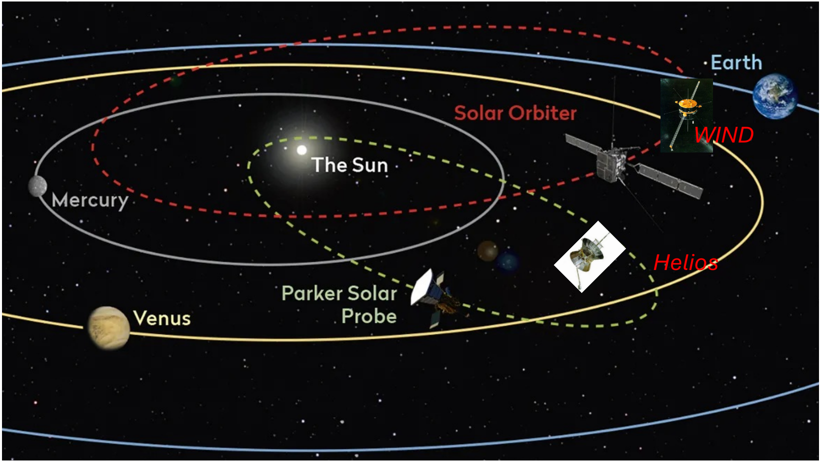
Closest: $\sim 9\ R_\odot$ or 0.04 AU

Parker Solar Probe (2018 –)



Heliocentric Velocity (km/s): 32.89
Distance from Sun Center (AU): 0.527
Distance from Sun's Surface (R_S): 112.3
Distance from Earth (AU): 1.041
Round-Trip Light Time (hh:mm:ss): 00:17:19
26 Nov 2025 16:00:00 UTC

Closest: 9 $R_\odot$ or 0.04 AU



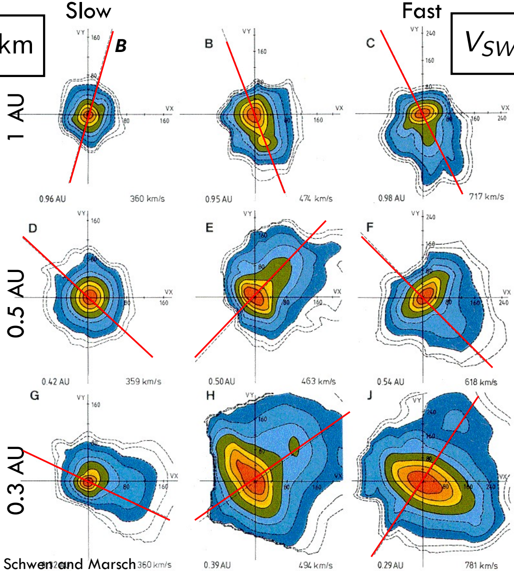
- PSP: 0.04 AU (9 $R_\odot$)
- Helios & Solar Orbiter (the new Helios): 0.3 AU (64.5 $R_\odot$)
- WIND: 1 AU (215 $R_\odot$)

- Despite many missions, past and present, and a plethora of theories, the twin problem of coronal heating and solar wind acceleration has not been solved yet.
- From the perspective of *in-situ* observation, in spite of the impressive achievement by the Parker Solar Probe (PSP), its closest approach of $\sim 9R_\odot$ is not enough, since the transition region where the abrupt coronal heating and wind acceleration take place is less than $0.1R_\odot$ (in fact, close to $0.01R_\odot$).
- Nevertheless, the historic Helios mission, contemporary PSP and Solar Orbiter (SolO), and WIND space probes have led to a rich understanding of the plasma physical processes taking place in the interplanetary environment.
- Among them is the interplay of **collisions** and plasma **instabilities**.

Collisions Versus Instabilities: Solar Wind Protons – Helios Observation

$V_{SW} < 300\text{km}$

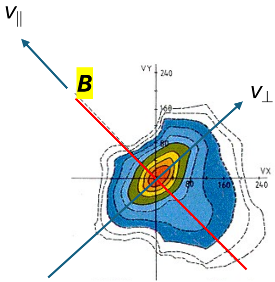
$V_{SW} > 600$ - 700km



Schwen and Marsch

Bi-Maxwellian fitting:

$$f(v_\perp, v_\parallel) \propto \exp\left(-\frac{mv_\perp^2}{2T_\perp} - \frac{mv_\parallel^2}{2T_\parallel}\right)$$



Organize data as a function of

$$\frac{T_\perp}{T_\parallel} \qquad \text{and} \qquad \beta_\parallel = \frac{8\pi n T_\parallel}{B^2}$$

Solar wind density (n) and magnetic field (B),

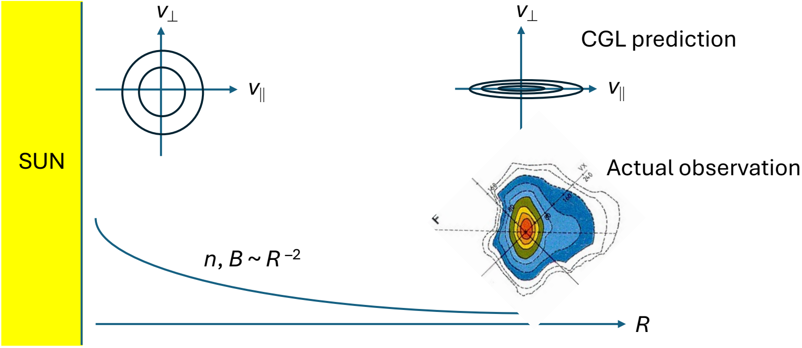
$$n,\ B \propto \frac{1}{R^2}$$

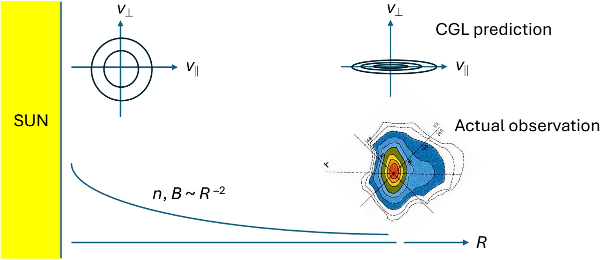
Helios (and 1AU WIND, PSP, SolO) observations show kinetic processes (**instabilities** and **collisions**) are necessary. According to CGL (two-fluid) theory,

$$\frac{nT_\perp}{B^2} = \mathrm{const} \qquad \frac{nT_\parallel}{B} = \mathrm{const}$$

For $n,\ B \propto R^{-2}$,

$$T_\perp \sim R^{-2}, \qquad T_\parallel \sim \mathrm{const}$$



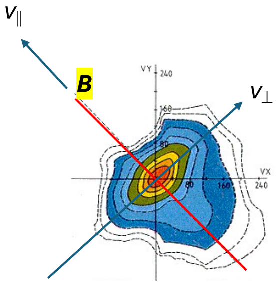


– **Perpendicular heating** of the protons is taking place (**will discuss this issue later if we have time**).
– Given the observed temperature anisotropies, $T_\perp / T_\parallel$, there seems to be certains limits on how high or low this value can get.
– This **"temperature anisotropy regulation"** can be explained by combined collisions and instabilities.

Part I. Temperature Anisotropy Regulation by Kinetic Instabilities (Collective Processes)

Recall the bi-Maxwellian fitting:

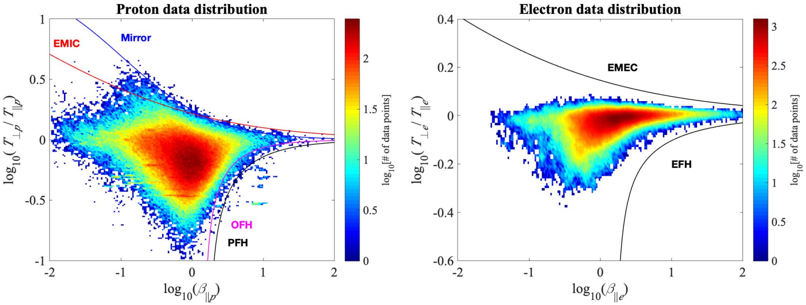
$$f(v_\perp, v_\parallel) \propto \exp\left(-\frac{mv_\perp^2}{2T_\perp} - \frac{mv_\parallel^2}{2T_\parallel}\right)$$



Organize data as a function of

$$\frac{T_\perp}{T_\parallel} \qquad \text{and} \qquad \beta_\parallel = \frac{8\pi n T_\parallel}{B^2}$$

1 AU proton data distribution in $(\beta_\parallel, T_\perp / T_\parallel)$ space



[Data credit] C. Salem – see also, Kasper, Bale, Hellinger, ... Štverák, ...

The high-beta (or right-hand side) boundaries are readily associated with various temperature anisotropy instabilities.

Proton Instabilities

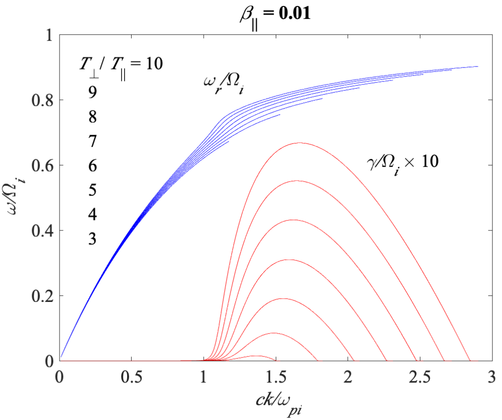
- EMIC: Parallel Electromagnetic ion (proton) cyclotron instability.
- PFH: Parallel proton fire-hose instability.
- Mirror: Obliquely propagating proton mirror instability.
- OFH: Obliquely propagating proton fire-hose instability.

Electron Instabilities

- EMEC: Parallel Electromagnetic electron cyclotron instability.
- EFH: Parallel electron fire-hose instability.

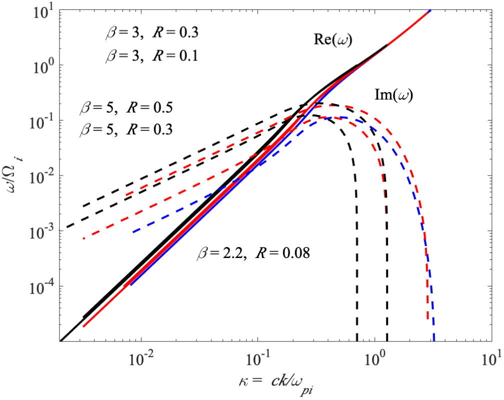
Electromagnetic Ion Cyclotron Instability (EMIC)

$$0 = \frac{c^2k^2}{\omega_{pi}^2} + \frac{\omega}{\Omega_i} - \left(\frac{T_{\perp i}}{T_{\parallel i}} - 1\right) - \left[\frac{T_{\perp i}}{T_{\parallel i}}\,\omega - \left(\frac{T_{\perp i}}{T_{\parallel i}} - 1\right)\Omega_i\right]\frac{1}{k\alpha_{\parallel i}}\,Z\left(\frac{\omega - \Omega_i}{k\alpha_{\parallel i}}\right).$$



Parallel Proton Firehose Instability (PFH)

$$0 = \frac{c^2k^2}{\omega_{pi}^2} - \frac{\omega}{\Omega_i} + 1 - \frac{T_{\perp i}}{T_{\parallel i}} - \left[\frac{T_{\perp i}}{T_{\parallel i}}\,\omega - \left(1 - \frac{T_{\perp i}}{T_{\parallel i}}\right)\Omega_i\right]\frac{1}{k\alpha_{\parallel i}}\,Z\!\left(\frac{\omega+\Omega_i}{k\alpha_{\parallel i}}\right).$$



VOL. 81, NO. 7 JOURNAL OF GEOPHYSICAL RESEARCH MARCH 1, 1976

Proton Temperature Anisotropy Instabilities in the Solar Wind

S. PETER GARY,[1] M. D. MONTGOMERY,[2] W. C. FELDMAN, AND D. W. FORSLUND

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The linear dispersion properties of proton temperature anisotropy instabilities in a homogeneous infinite Vlasov plasma are studied by using a configuration appropriate to the solar wind at 1 AU. The proton distribution is taken to consist of two components, a cooler $T_\perp > T_\parallel$ 'core' and a hotter $T_\parallel > T_\perp$ 'halo.' For the parameters considered the $\mathbf{k} \| \mathbf{B}_0$ fire hose and ion cyclotron instabilities are the most important modes. Resonant proton effects enhance both instabilities, and the presence of the cooler component can substantially reduce the threshold anisotropy of the halo-driven fire hose.

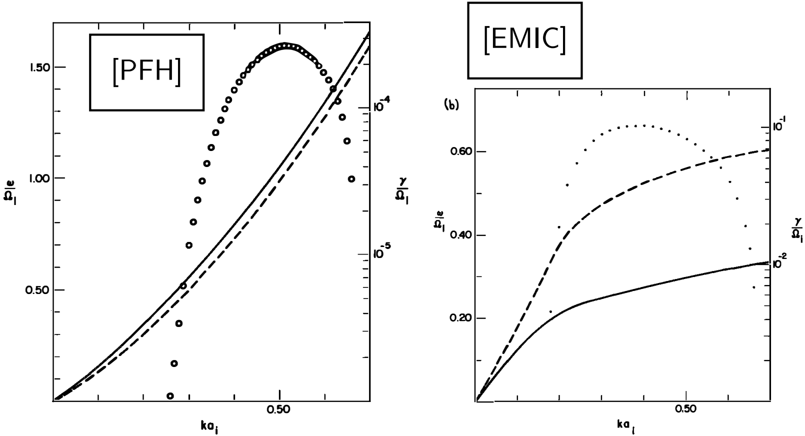
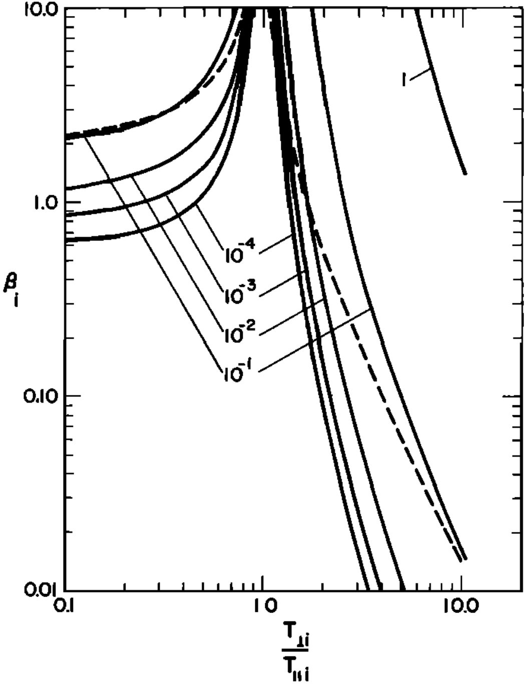
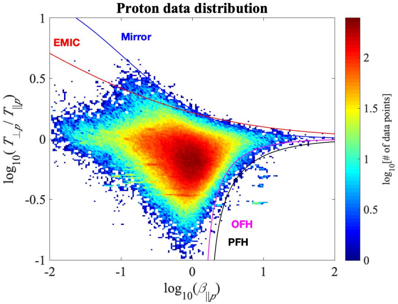
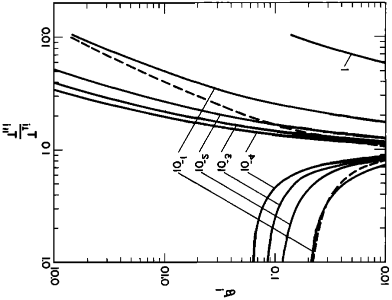


Fig. 1. Frequencies and growth rates as functions of wave number for $\mathbf{k} \parallel \mathbf{B}_0$. A single bi-Maxwellian ion component is used here; parameters are the same as those given in the tabulation of values of dimensionless parameters except that $n_i = n_e$. The solid line is ω for isotropic ions and is stable. (*a*) Right-hand mode (fire hose instability). The dashed line is ω for $T_{\perp i} = 0.4 T_{\parallel i}$, and the associated γ is a line of open circles. (*b*) Left-hand mode (ion cyclotron instability). The dashed line is ω for $T_{\perp i} = 2.5\, T_{\parallel i}$, and the associated γ is a dotted line.

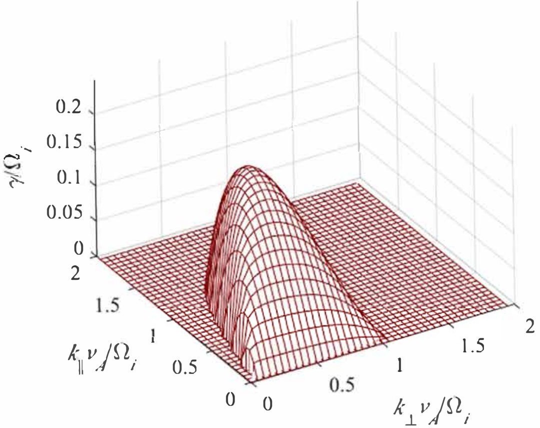

10.0
1.0
0.10
0.01
β_i
0.1
1.0
10.0
$\frac{T_{\perp i}}{T_{\parallel i}}$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1


Proton data distribution
EMIC
Mirror
OFH
PFH
$\log_{10}(T_{\perp p}/T_{\parallel p})$
$\log_{10}(\beta_{\parallel p})$
$\log_{10}$[# of data points]
1
0.5
0
-0.5
-1
-2
-1
0
1
2
2
1.5
1
0.5
0


$\frac{T_\perp}{T_\parallel}$
$\beta_\parallel$
10.0
1.0
0.10
0.01
10.0
1.0
0.1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1

Proton Mirror Instability

$$
\begin{aligned}
0 &= \frac{c^2k^2}{\omega_{pi}^2} - 2\lambda\left[I_0(\lambda) - I_1(\lambda)\right]e^{-\lambda}\left(\frac{T_{\perp i}}{T_{\| i}} - 1 + \frac{T_{\perp i}}{T_{\| i}}\,\xi\,Z(\xi)\right), \\
\lambda &= \frac{k_\perp^2\alpha_{\perp i}^2}{2\Omega_i^2}, \qquad \xi = \frac{\omega}{k_\| \alpha_{\| i}}.
\end{aligned}
$$

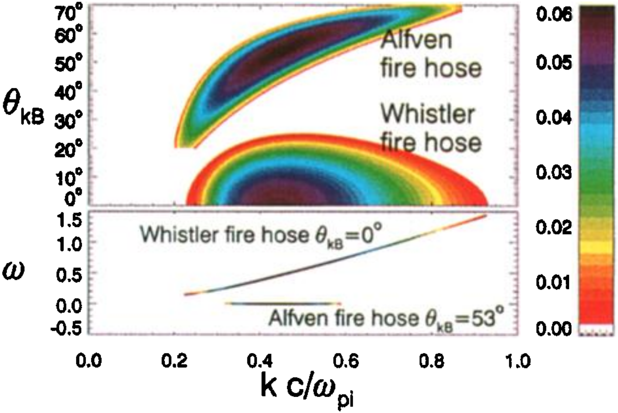


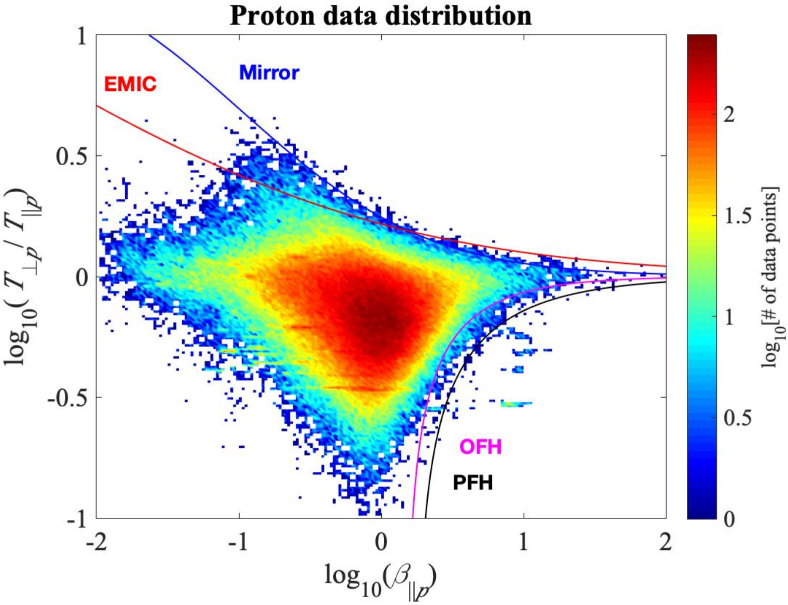
JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 105, NO. A5, PAGES 10,519–10,526, MAY 1, 2000

New kinetic instability: Oblique Alfvén fire hose

P. Hellinger and H. Matsumoto

Radio Atmospheric Science Center, Kyoto University, Uji, Japan




Proton data distribution
EMIC
Mirror
OFH
PFH
$\log_{10}(T_{\perp p} / T_{\parallel p})$
$\log_{10}(\beta_{\parallel p})$
$\log_{10}$[# of data points]
1
0.5
0
-0.5
-1
-2
-1
0
1
2
0
0.5
1
1.5
2

- Marginal instability conditions, or threshold conditions, can explain the partial (high-beta side) boundaries, but they cannot explain the left-hand (or low-beta) boundaries. These are explained by collisions, but before we discuss that, we discuss the dynamic theory of instabilities.
- The simplest dynamic theory is quasilinear theory. Dynamics is important because growth rate calculation alone is insufficient to understand the time scale of instability saturation. It is not sufficient to determine the saturated wave level either.

Quasilinear Theory of EMIC and PFH Instabilities

Quasilinear particle kinetic equation

$$\begin{aligned}\frac{\partial f}{\partial t} &= \frac{ie_a^2}{4m_a^2}\int dk_\parallel \int d\omega \sum_{+,-} \frac{1}{v_\perp}\mathscr{L} v_\perp \frac{<|E_x \mp iE_y|^2>_{k_\parallel,\omega}}{\omega - k_\parallel v_\parallel \pm \Omega_i}\mathscr{L} f,\\ \mathscr{L} &= \left(1 - \frac{k_\parallel v_\parallel}{\omega}\right)\frac{\partial}{\partial v_\perp} + \frac{k_\parallel v_\perp}{\omega}\frac{\partial}{\partial v_\parallel}.\end{aligned}$$

By assuming that f is given by a bi-Maxwellian form,

$$f = \frac{m_i^{3/2}}{(2\pi)^{3/2} T_{\perp i} T_{\parallel i}^{1/2}} \exp\left(-\frac{m_i v_\perp^2}{2T_{\perp i}} - \frac{m_i v_\parallel^2}{2T_{\parallel i}}\right),$$

We solve for time evolution of $T_{\perp i}$ and $T_{\parallel i}$.

$$\begin{aligned}
\frac{dT_\perp}{dt} &= -\frac{e^2}{m_i}\int_{-\infty}^{\infty}\frac{dk_\parallel}{c^2k_\parallel^2}\,\omega_i(k_\parallel)\left(\frac{c^2k_\parallel^2}{\omega_{pi}^2}\pm\frac{\omega_r(k_\parallel)}{\Omega_i}\right)\delta B^2(k_\parallel),\\
\frac{dT_\parallel}{dt} &= \frac{e^2}{m_i}\int_{-\infty}^{\infty}\frac{dk_\parallel}{c^2k_\parallel^2}\,\omega_i(k_\parallel)\left(\frac{c^2k_\parallel^2}{\omega_{pi}^2}\pm\frac{2\omega_r(k_\parallel)}{\Omega_i}\right)\delta B^2(k_\parallel).
\end{aligned}$$

$$\frac{\partial\delta B^2(k_\parallel)}{\partial t} = 2\omega_i(k_\parallel)\,\delta B^2(k_\parallel).$$

Upper/lower signs are for EMIC and PFH instabilities.

Electromagnetic ion cyclotron instability driven by ion energy anisotropy in high-beta plasmas

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Joan M. Ogden

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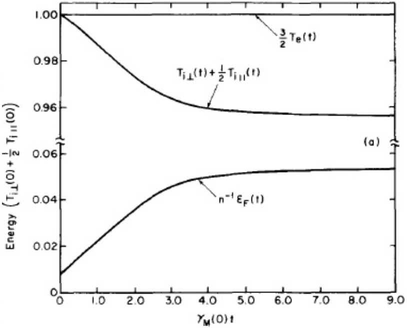
(Received 13 March 1975)

$$\frac{\partial K_{j\perp}}{\partial t} = \frac{\omega_{pj}^2}{2}\sum_{+,-}\int_{-\infty}^{\infty} dk_z \frac{\mathcal{E}_{k_z}}{c^2k_z^2}\left[2\gamma - (2\gamma \mp i\omega_{cj}) \times \int d^3\mathbf{v}\, \frac{(k_z v_\perp^2 \partial F_j/\partial v_z \pm 2\omega_{cj}F_j)}{\omega - k_z v_z \pm \omega_{cj} + i\gamma}\right], \tag{8}$$

$$\frac{\partial K_{j\parallel}}{\partial t} = \frac{\omega_{pj}^2}{2}\sum_{+,-}\int_{-\infty}^{\infty} dk_z \frac{\mathcal{E}_{k_z}}{c^2k_z^2}\left[(-i\omega + \gamma \mp i\omega_{cj}) \times \int d^3\mathbf{v}\, \frac{(k_z v_\perp^2 \partial F_j/\partial v_z \pm 2\omega_{cj}F_j)}{\omega - k_z v_z \pm \omega_{cj} + i\gamma}\right], \tag{9}$$

where $\sum_{+,-}$ denotes summation over right- and left-hand polarizations as defined in Ref. 27, and $\mathcal{E}_{k_z}(t) = |\delta\mathbf{B}_{k_z}(t)|^2/8\pi$, the spectral energy density associated with the magnetic field fluctuations, evolves according to

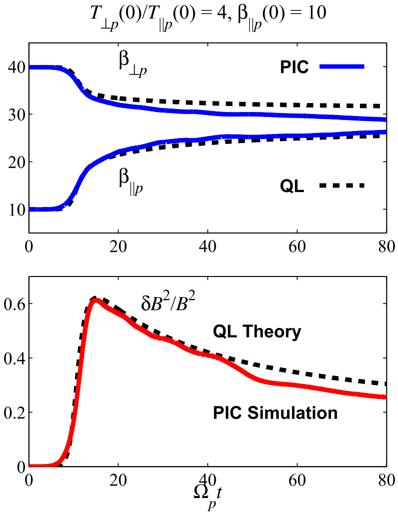
$$\frac{\partial}{\partial t}\mathcal{E}_{k_z} = 2\gamma\mathcal{E}_{k_z}. \tag{10}$$

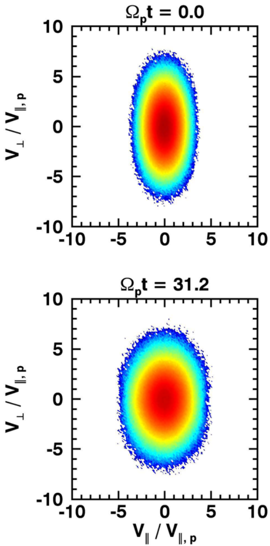


PHYSICS OF PLASMAS **21**, 062118 (2014)

Quasilinear theory and particle-in-cell simulation of proton cyclotron instability

Jungjoon Seough,[1,a)] Peter H. Yoon,[2,3,a)] and Junga Hwang[1,4,a)]

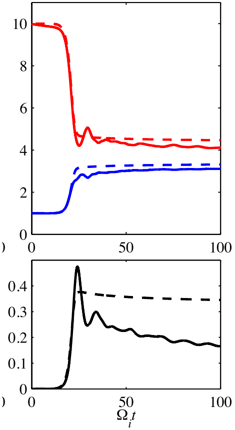


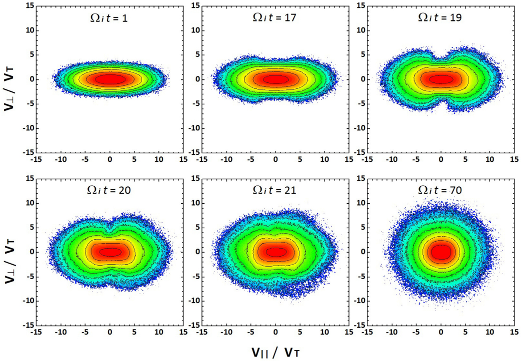


PHYSICS OF PLASMAS **22**, 012303 (2015)

Simulation and quasilinear theory of proton firehose instability

Jungjoon Seough,[1,2] Peter H. Yoon,[3,4] and Junga Hwang[1,5]





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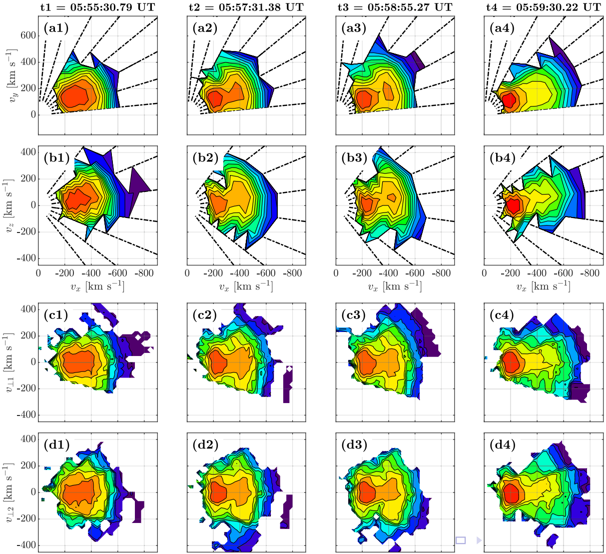
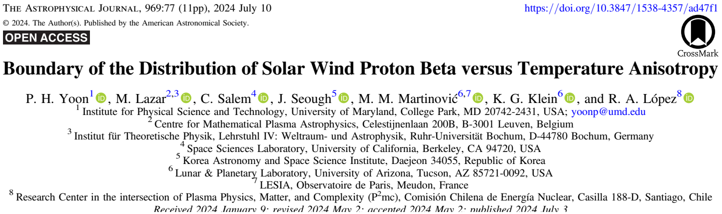
Coexistence of Antisunward and Sunward Ion Cyclotron Waves in the Near-Sun Solar Wind: Excitation by the Proton Cyclotron Instability

Chen Shi[1], Jinsong Zhao[2], Si Liu[1], Fuliang Xiao[1], Yifan Wu[3], Trevor A. Bowen[4], Roberto Livi[4], and S. D. Bale[4]





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Boundary of the Distribution of Solar Wind Proton Beta versus Temperature Anisotropy

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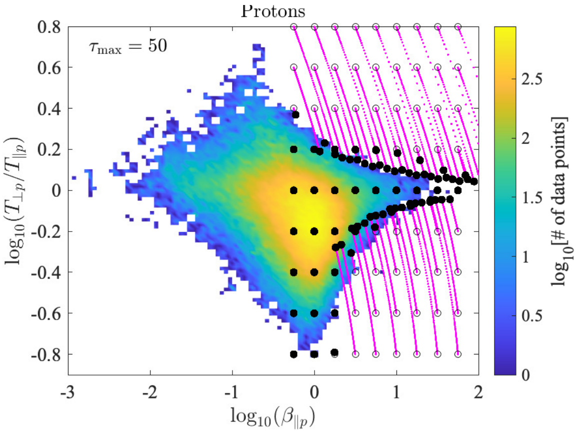
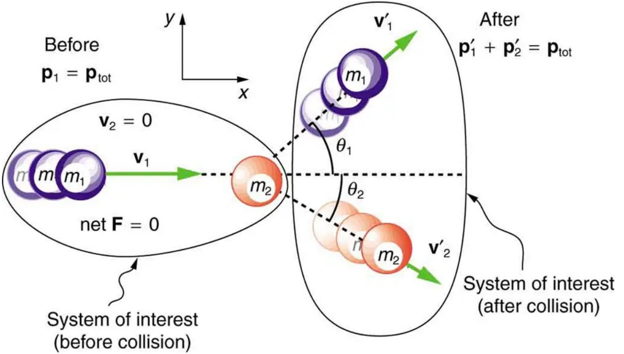


Proton_movie_inst.mp4

Part II. Temperature Anisotropy Regulation by Collisions

To many textbooks in plasma physics discuss the collisional processes in plasma by resorting to the outdated Rutherford scattering picture, like the one shown below.



However, plasma scattering can be discussed systematically from Klimontovich kinetic theory rather easily, so I will spend a few slides talking about the fundamentals.

Fundamentals

Klimontovich equation,

$$\boxed{\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} + \frac{e_a}{m_a}\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{v}}\right) N_a(\mathbf{r},\mathbf{v},t) = 0,}$$

$$\boxed{\nabla\cdot\mathbf{E}(\mathbf{r},t) = \sum_{a=e,i} 4\pi e_a \int d\mathbf{v} N_a(\mathbf{r},\mathbf{v},t),}$$

Klimontovich equation is mathematically identical to Vlasov equation except that N_a is exact N-body phase space distribution,

$$\boxed{\begin{array}{l} N_a(\mathbf{r},\mathbf{v},t) = \displaystyle\sum_{i=1}^{N} \delta[\mathbf{r}-\mathbf{r}_i^a(t)]\delta[\mathbf{v}-\mathbf{v}_i^a(t)], \\ \displaystyle\frac{d\mathbf{r}_i^a(t)}{dt} = \mathbf{v}_i^a(t), \qquad \frac{d\mathbf{v}_i^a(t)}{dt} = \frac{e_a}{m_a}\mathbf{E}[\mathbf{r}_i^a(t),t]. \end{array}}$$

Klimontovich equation for free-particles,

$$\boxed{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right)N_a^0(\mathbf{r},\mathbf{v},t)=0,}$$

$$\boxed{N_a^0(\mathbf{r},\mathbf{v},t)=\sum_{i=1}^{N}\delta(\mathbf{r}-\mathbf{r}_i^a-\mathbf{v}_i^a t)\delta(\mathbf{v}-\mathbf{v}_i^a).}$$

Subtract the two equations,

$$\boxed{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right)[N_a(\mathbf{r},\mathbf{v},t)-N_a^0(\mathbf{r},\mathbf{v},t)]+\frac{e_a}{m_a}\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{v}}N_a(\mathbf{r},\mathbf{v},t)=0.}$$

This equation describes **collective** processes where purely single particle dynamics are taken out of the picture.

Split total microscopic quantities into averages and fluctuations,

$$\boxed{N_a(\mathbf{r},\mathbf{v},t) = \langle N_a(\mathbf{r},\mathbf{v},t)\rangle + \delta N_a(\mathbf{r},\mathbf{v},t) \equiv f_a(\mathbf{r},\mathbf{v},t) + \delta N_a(\mathbf{r},\mathbf{v},t),}$$

$$\boxed{\mathbf{E}(\mathbf{r},t) = \delta\mathbf{E}(\mathbf{r},t),}$$

where ensemble averages $\langle\cdots\rangle$ of the fluctuations are zero, and $f_a(\mathbf{r},\mathbf{v},t)$ is the smoothed one particle distribution function. Then Klimontovich equation becomes

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}} + \frac{e_a}{m_a}\,\delta\mathbf{E}\cdot\frac{\partial}{\partial\mathbf{v}}\right)(f_a + \delta N_a) = 0.$$

Taking ensemble average: formal particle kinetic equation,

$$\boxed{\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}}\right) f_a = -\frac{e_a}{m_a}\frac{\partial}{\partial\mathbf{v}}\cdot\langle\delta\mathbf{E}\,\delta N_a\rangle\,.}$$

Equation for the perturbed distribution becomes

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}}\right)(\delta N_a - \delta N_a^0) + \frac{e_a}{m_a}\,\delta \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{v}}(f_a + \cancel{\delta N_a}) = 0,$$

Upon ignoring nonlinear terms,

$$\boxed{\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}}\right)(\delta N_a - \delta N_a^0) + \frac{e_a}{m_a}\,\delta \mathbf{E} \cdot \frac{\partial f_a}{\partial \mathbf{v}} = 0.}$$

Poisson equation

$$\boxed{\nabla \cdot \delta \mathbf{E} = \sum_a 4\pi e_a \int d\mathbf{v}\, \delta N_a}$$

closes the system.

Recap:

$$\begin{aligned}
&\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} + \frac{e_a}{m_a}\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{v}}\right) N_a = 0, \\
&\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right) N_a^0 = 0, \\
&\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right) [N_a - N_a^0] + \frac{e_a}{m_a}\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{v}} N_a = 0.
\end{aligned}$$

$\Rightarrow$

$$\begin{aligned}
&\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right) f_a = -\frac{e_a}{m_a}\frac{\partial}{\partial \mathbf{v}}\cdot\left\langle \delta\mathbf{E}\,\delta N_a\right\rangle, \\
&\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right) (\delta N_a - \delta N_a^0) + \frac{e_a}{m_a}\,\delta\mathbf{E}\cdot\frac{\partial f_a}{\partial \mathbf{v}} = 0.
\end{aligned}$$

In spectral form,

$$\frac{\partial f_a}{\partial t} = -\frac{e_a}{m_a} \int d\mathbf{k} \int d\omega \, \frac{\mathbf{k}}{k} \cdot \frac{\partial}{\partial \mathbf{v}} \left\langle \delta E^*_{\mathbf{k},\omega} \, \delta N^a_{\mathbf{k},\omega} \right\rangle ,$$

$$\delta N^a_{\mathbf{k},\omega} = \underbrace{\delta N^{a0}_{\mathbf{k},\omega}}_{\text{source term}} - \frac{ie_a}{m_a} \frac{1}{\omega - \mathbf{k}\cdot\mathbf{v}} \delta E_{\mathbf{k},\omega} \frac{\mathbf{k}}{k} \cdot \frac{\partial f_a}{\partial \mathbf{v}} ,$$

$$\delta E_{\mathbf{k},\omega} = -i \sum_a \frac{4\pi e_a}{k} \int d\mathbf{v} \, \delta N^a_{\mathbf{k},\omega} .$$

$\Rightarrow$

$$\frac{\partial f_a}{\partial t} = \frac{e_a}{m_a} \int d\mathbf{k} \int d\omega \, \frac{\mathbf{k}}{k} \cdot \frac{\partial}{\partial \mathbf{v}}$$

$$\times \left[\mathrm{Im} \sum_b \frac{4\pi e_b}{k \epsilon^*(\mathbf{k},\omega)} \int d\mathbf{v} \; \underbrace{\left\langle \delta N^0_a \delta N^{0*}_b \right\rangle_{\mathbf{k},\omega}}_{\text{source fluctuation}} \right.$$

$$\left. + \frac{\pi e_a}{m_a} \delta(\omega - \mathbf{k}\cdot\mathbf{v}) \left\langle \delta E^2 \right\rangle_{\mathbf{k},\omega} \frac{\mathbf{k}}{k} \cdot \frac{\partial f_a}{\partial \mathbf{v}} \right],$$

$$\epsilon(\mathbf{k},\omega) = 1 + \sum_a \frac{4\pi e_a^2}{m_a} \int d\mathbf{v} \frac{1}{\omega - \mathbf{k}\cdot\mathbf{v}} \frac{\mathbf{k}}{k^2} \cdot \frac{\partial f_b}{\partial \mathbf{v}} .$$

Source Fluctuation $\langle \delta N_a^0 \delta N_b^{0*} \rangle_{\mathbf{k},\omega}$

From the definition

$$N_a^0(\mathbf{r}, \mathbf{v}, t) = \sum_{i=1}^{N} \delta(\mathbf{r} - \mathbf{r}_i^a - \mathbf{v}_i^a t)\delta(\mathbf{v} - \mathbf{v}_i^a),$$

we obtain

$$\begin{aligned} &\left\langle \delta N_a^0(\mathbf{r}, \mathbf{v}, t)\, \delta N_b^0(\mathbf{r}', \mathbf{v}', t') \right\rangle \\ &= \delta_{ab}\delta[\mathbf{r} - \mathbf{r}' - \mathbf{v}(t - t')]\delta(\mathbf{v} - \mathbf{v}') f_a(\mathbf{r}, \mathbf{v}, t), \end{aligned}$$

or in spectral form

$$\boxed{\left\langle \delta N_a^0(\mathbf{v})\, \delta N_b^0(\mathbf{v}') \right\rangle_{\mathbf{k},\omega} = (2\pi)^{-3}\delta_{ab}\delta(\mathbf{v} - \mathbf{v}')\delta(\omega - \mathbf{k}\cdot\mathbf{v}) f_a.}$$

In the end we have

$$\boxed{\begin{aligned}\frac{\partial f_a}{\partial t} &= \frac{\pi e_a^2}{m_a^2}\int d\mathbf{k}\int d\omega\left(\frac{\mathbf{k}}{k}\cdot\frac{\partial}{\partial\mathbf{v}}\right)\delta(\omega-\mathbf{k}\cdot\mathbf{v})\\ &\quad\times\left[\mathrm{Im}\,\frac{m_a}{2\pi^3 k\,\epsilon^*(\mathbf{k},\omega)}\,f_a+\langle\delta E^2\rangle_{\mathbf{k},\omega}\left(\frac{\mathbf{k}}{k}\cdot\frac{\partial f_a}{\partial\mathbf{v}}\right)\right],\\ \left\langle\delta E^2\right\rangle_{\mathbf{k},\omega} &= \frac{2}{\pi}\frac{1}{k^2|\epsilon(\mathbf{k},\omega)|^2}\sum_a e_a^2\int d\mathbf{v}\delta(\omega-\mathbf{k}\cdot\mathbf{v})f_a.\end{aligned}}$$

Kinetic Equation: Collision versus Instability

Collisional kinetic equation and quasilinear kinetic equation follow from the same formal particle kinetic equation.

$$\boxed{\begin{aligned}\frac{\partial f_a}{\partial t} = \frac{\pi e_a^2}{m_a^2} \int d\mathbf{k} \int d\omega \left(\frac{\mathbf{k}}{k} \cdot \frac{\partial}{\partial \mathbf{v}} \right) \delta(\omega - \mathbf{k} \cdot \mathbf{v}) \\ \times \left[\mathrm{Im}\, \frac{m_a}{2\pi^3 k\, \epsilon^*(\mathbf{k},\omega)}\, f_a + \langle \delta E^2 \rangle_{\mathbf{k},\omega} \left(\frac{\mathbf{k}}{k} \cdot \frac{\partial f_a}{\partial \mathbf{v}} \right) \right].\end{aligned}}$$

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If we treat the electric field fluctuations as solely determined by the spontaneous emission, then we have the collision integral:

$$\frac{\partial f_a}{\partial t} = \frac{\pi e_a^2}{m_a^2} \int d\mathbf{k} \int d\omega \left(\frac{\mathbf{k}}{k} \cdot \frac{\partial}{\partial \mathbf{v}} \right) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$

$$\times \left[\mathrm{Im}\, \frac{m_a}{2\pi^3 k\, \epsilon^*(\mathbf{k}, \omega)}\, f_a + \underbrace{\langle \delta E^2 \rangle_{\mathbf{k},\omega}}_{\uparrow} \left(\frac{\mathbf{k}}{k} \cdot \frac{\partial f_a}{\partial \mathbf{v}} \right) \right].$$

$$\boxed{\left\langle \delta E^2 \right\rangle_{\mathbf{k},\omega} = \frac{2}{\pi} \frac{1}{k^2 |\epsilon(\mathbf{k}, \omega)|^2} \sum_a e_a^2 \int d\mathbf{v} \delta(\omega - \mathbf{k} \cdot \mathbf{v}) f_a}$$

Spontaneously emitted thermal fluctuation

Balescu-Lenard Collision Integral

$$\boxed{\begin{aligned}\frac{\partial f_a}{\partial t} \quad = \quad & \frac{2e_a^2}{m_a^2}\sum_b e_b^2 \int d\mathbf{k} \int d\mathbf{v}' \frac{\mathbf{k}}{k} \cdot \frac{\partial}{\partial \mathbf{v}} \frac{\delta(\mathbf{k}\cdot\mathbf{v} - \mathbf{k}'\cdot\mathbf{v}')}{k^2|\epsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v})|^2} \\ & \times \left[\frac{\mathbf{k}}{k}\cdot\frac{\partial f_a(\mathbf{v})}{\partial \mathbf{v}} f_b(\mathbf{v}') - \frac{m_a}{m_b}\frac{\mathbf{k}}{k}\cdot\frac{\partial f_b(\mathbf{v}')}{\partial \mathbf{v}'} f_a(\mathbf{v})\right].\end{aligned}}$$

If $\epsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v}) \to 1$, the we have **Landau collision integral** with appropriate lower- and upper k interal cutoffs, namely, the Coulomb logarithm,

$$\int_0^\infty \frac{dk}{k} \to \int_{k_{\min}}^{k_{\max}} \frac{dk}{k} = \ln \Lambda.$$

If we consider the electric field fluctuation as being solely determined by the collective processes (i.e., waves and instabilities)

$$\left\langle \delta E^2 \right\rangle_{\mathbf{k},\omega} = \frac{2}{\pi}\frac{1}{k^2|\epsilon(\mathbf{k},\omega)|^2}\sum_a e_a^2 \int d\mathbf{v}\,\delta(\omega-\mathbf{k}\cdot\mathbf{v}) f_a,$$

$$\rightarrow \qquad \epsilon(\mathbf{k},\omega)\left\langle \delta E^2 \right\rangle_{\mathbf{k},\omega} = \frac{2}{\pi}\frac{1}{k^2\epsilon^*(\mathbf{k},\omega)}\sum_a e_a^2 \int d\mathbf{v}\,\delta(\omega-\mathbf{k}\cdot\mathbf{v}) f_a,$$

$$\uparrow \qquad \omega = \omega_{\mathbf{k}}, \qquad \omega \rightarrow \omega + i\,\frac{\partial}{\partial t}, \qquad \left\langle \delta E^2 \right\rangle_{\mathbf{k},\omega} = I_{\mathbf{k}}\,\delta(\omega-\omega_{\mathbf{k}})$$

$$\begin{aligned}\rightarrow \qquad & \frac{i}{2}\frac{\partial \mathrm{Re}\epsilon(\mathbf{k},\omega_{\mathbf{k}})}{\partial \omega_{\mathbf{k}}}\frac{\partial I_{\mathbf{k}}}{\partial t}\,\delta(\omega-\omega_{\mathbf{k}}) + i\mathrm{Im}\epsilon(\mathbf{k},\omega_{\mathbf{k}}) I_{\mathbf{k}}\,\delta(\omega-\omega_{\mathbf{k}}) \\ & = \underbrace{\frac{2}{\pi}\frac{i\pi\delta(\omega-\omega_{\mathbf{k}})}{k^2\partial \mathrm{Re}\epsilon(\mathbf{k},\omega_{\mathbf{k}})/\partial\omega_{\mathbf{k}}}\sum_a e_a^2 \int d\mathbf{v}\,\delta(\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{v}) f_a,}_{\text{ignore}}\end{aligned}$$

$$\rightarrow \qquad \frac{\partial I_{\mathbf{k}}}{\partial t} = 2\gamma_{\mathbf{k}} I_{\mathbf{k}}, \qquad \gamma_{\mathbf{k}} = -\frac{\mathrm{Im}\epsilon(\mathbf{k},\omega_{\mathbf{k}})}{\partial \mathrm{Re}\epsilon(\mathbf{k},\omega_{\mathbf{k}})\omega_{\mathbf{k}}}.$$

Then we have the quasilinear theory:

$$\frac{\partial f_a}{\partial t} = \frac{\pi e_a^2}{m_a^2} \int d\mathbf{k} \int d\omega \left(\frac{\mathbf{k}}{k} \cdot \frac{\partial}{\partial \mathbf{v}} \right) \delta(\omega - \mathbf{k} \cdot \mathbf{v})$$
$$\times \left[\underbrace{\mathrm{Im} \, \frac{m_a}{2\pi^3 k \, \epsilon^*(\mathbf{k}, \omega)} \, f_a}_{\downarrow} + \underbrace{\langle \delta E^2 \rangle_{\mathbf{k},\omega}}_{\uparrow} \left(\frac{\mathbf{k}}{k} \cdot \frac{\partial f_a}{\partial \mathbf{v}} \right) \right].$$

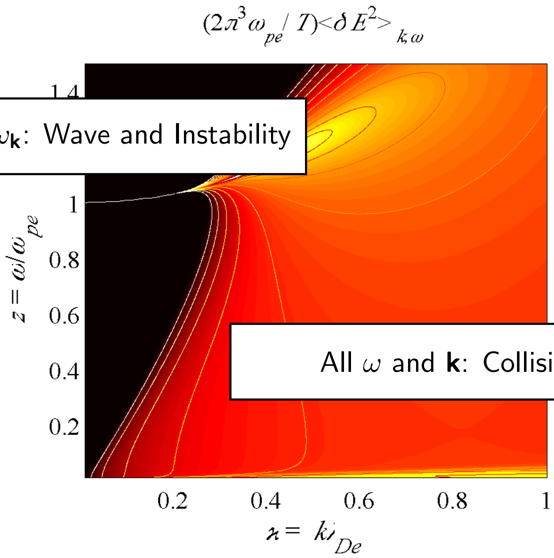
$$\boxed{\frac{m_a \delta(\omega - \omega_{\mathbf{k}})}{2\pi^2 \partial \mathrm{Re}\epsilon(\mathbf{k}, \omega_{\mathbf{k}}) / \partial \omega_{\mathbf{k}}}} \qquad \boxed{\left\langle E^2 \right\rangle_{\mathbf{k},\omega} = I_{\mathbf{k}} \delta(\omega - \omega_{\mathbf{k}})}$$

Plasma eigenmodes

Quasilinear Kinetic Equation

$$
\boxed{
\begin{aligned}
\frac{\partial f_a}{\partial t} &= \frac{\partial}{\partial v_i}\left(A_i f_a + D_{ij}\frac{\partial f_a}{\partial v_j}\right). \\
A_i &= \frac{e^2}{4\pi m_e}\int d\mathbf{k}\,\frac{k_i}{k^2}\,\omega_{\mathbf{k}}\,\delta(\omega_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v}), \\
D_{ij} &= \frac{\pi e^2}{m_e^2}\int d\mathbf{k}\,\frac{k_i k_j}{k^2}\,\delta(\omega_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v}) I_{\mathbf{k}}, \\
\frac{\partial I_{\mathbf{k}}}{\partial t} &= 2\gamma_{\mathbf{k}} I_{\mathbf{k}}.
\end{aligned}
}
$$

This equation describes *collective* processes, i.e., instabilities.
(Note that the velocity friction term A_i can be ignored.)



Only $\omega = \omega_{\mathbf{k}}$: Wave and Instability

All ω and $\mathbf{k}$: Collisions

Collisional Temperature Relaxation

General collision integral for unmagnetized plasmas is Balescu-Lenard equation,

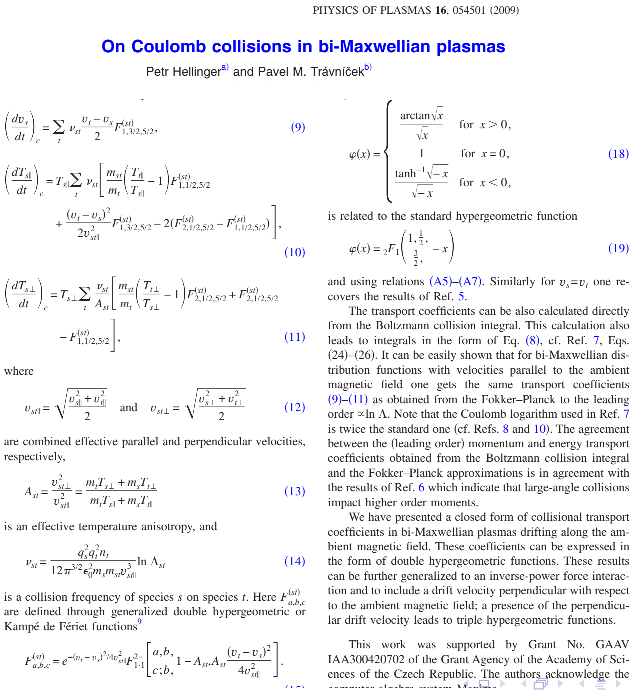
$$\begin{aligned}
\frac{\partial f_a(\mathbf{v})}{\partial t} &= -\frac{e_a^2}{m_a^2}\int d\mathbf{k}\int d\omega\left(\frac{\mathbf{k}}{k}\cdot\frac{\partial}{\partial\mathbf{v}}\right)\mathrm{Im}\,\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\\
&\quad\times\left[\frac{m_a\,\mathrm{Im}\,\epsilon(\mathbf{k},\omega)}{2\pi^3k|\epsilon(\mathbf{k},\omega)|^2}\,f_a(\mathbf{v})+\langle\delta E^2\rangle_{\mathbf{k},\omega}\left(\frac{\mathbf{k}}{k}\cdot\frac{\partial f_a(\mathbf{v})}{\partial\mathbf{v}}\right)\right],\\
\langle\delta E^2\rangle_{\mathbf{k},\omega} &= -\frac{2}{\pi^2k^2|\epsilon(\mathbf{k},\omega)|^2}\sum_a e_a^2\int d\mathbf{v}\,\mathrm{Im}\,\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\,f_a(\mathbf{v}),\\
\epsilon(\mathbf{k},\omega) &= 1+\sum_a\frac{4\pi e_a^2}{m_ak^2}\int d\mathbf{v}\,\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}\right)f_a(\mathbf{v}).
\end{aligned}$$

We take the velocity moments by assuming bi-Maxwellian distribution.

After lengthy mathematical manipulations, we arrive at

$$\begin{aligned}
\frac{dT_\perp}{dt} &= \frac{2\pi^{1/2} n e^4 \ln\Lambda}{m^{1/2} T_\perp^{3/2}} F\,(T_\parallel - T_\perp), \\
\frac{dT_\parallel}{dt} &= \frac{4\pi^{1/2} n e^4 \ln\Lambda}{m^{1/2} T_\perp^{3/2}} F\,(T_\perp - T_\parallel), \\
F &= \frac{(A+1)^{3/2}}{A^2}\left((A+3)\,\frac{\mathrm{arctan}\sqrt{A}}{\sqrt{A}} - 3\right), \qquad (A>0), \\
&= \frac{(A+1)^{3/2}}{A^2}\left((A+3)\,\frac{\mathrm{arctanh}\sqrt{-A}}{\sqrt{-A}} - 3\right), \qquad (A<0), \\
&= \frac{4}{15}, \qquad (A=0), \qquad A = \frac{T_\perp}{T_\parallel} - 1,
\end{aligned}$$

This is the same as that found in the NRL Plasma Formulary.



PHYSICS OF PLASMAS **16**, 054501 (2009)

On Coulomb collisions in bi-Maxwellian plasmas

Petr Hellinger[a)] and Pavel M. Trávníček[b)]

$$\left(\frac{dv_s}{dt}\right)_c = \sum_t \nu_{st}\frac{v_t - v_s}{2}F^{(st)}_{1,3/2,5/2}, \tag{9}$$

$$\left(\frac{dT_{s\|}}{dt}\right)_c = T_{s\|}\sum_t \nu_{st}\left[\frac{m_{st}}{m_t}\left(\frac{T_{t\|}}{T_{s\|}} - 1\right)F^{(st)}_{1,1/2,5/2} + \frac{(v_t - v_s)^2}{2v^2_{st\|}}F^{(st)}_{1,3/2,5/2} - 2(F^{(st)}_{2,1/2,5/2} - F^{(st)}_{1,1/2,5/2})\right], \tag{10}$$

$$\left(\frac{dT_{s\perp}}{dt}\right)_c = T_{s\perp}\sum_t \frac{\nu_{st}}{A_{st}}\left[\frac{m_{st}}{m_t}\left(\frac{T_{t\perp}}{T_{s\perp}} - 1\right)F^{(st)}_{2,1/2,5/2} + F^{(st)}_{2,1/2,5/2} - F^{(st)}_{1,1/2,5/2}\right], \tag{11}$$

where

$$v_{st\|} = \sqrt{\frac{v^2_{s\|} + v^2_{t\|}}{2}} \quad \text{and} \quad v_{st\perp} = \sqrt{\frac{v^2_{s\perp} + v^2_{t\perp}}{2}} \tag{12}$$

are combined effective parallel and perpendicular velocities, respectively,

$$A_{st} = \frac{v^2_{st\perp}}{v^2_{st\|}} = \frac{m_t T_{s\perp} + m_s T_{t\perp}}{m_t T_{s\|} + m_s T_{t\|}} \tag{13}$$

is an effective temperature anisotropy, and

$$\nu_{st} = \frac{q_s^2 q_t^2 n_t}{12\pi^{3/2}\epsilon_0^2 m_s m_{st} v^3_{st\|}}\ln \Lambda_{st} \tag{14}$$

is a collision frequency of species s on species t. Here $F^{(st)}_{a,b,c}$ are defined through generalized double hypergeometric or Kampé de Fériet functions[9]

$$F^{(st)}_{a,b,c} = e^{-(v_t - v_s)^2/4v^2_{st\|}}F^{2\cdot\cdot}_{1\cdot 1}\left[\begin{matrix} a,b, \\ c;b, \end{matrix} 1 - A_{st}, A_{st}\frac{(v_t - v_s)^2}{4v^2_{st\|}}\right].$$

$$\varphi(x) = \begin{cases} \dfrac{\arctan\sqrt{x}}{\sqrt{x}} & \text{for } x > 0, \\ 1 & \text{for } x = 0, \\ \dfrac{\tanh^{-1}\sqrt{-x}}{\sqrt{-x}} & \text{for } x < 0, \end{cases} \tag{18}$$

is related to the standard hypergeometric function

$$\varphi(x) = {}_2F_1\left(\begin{matrix} 1, \frac{1}{2}, \\ \frac{3}{2}, \end{matrix} - x\right) \tag{19}$$

and using relations (A5)–(A7). Similarly for $v_s = v_t$ one recovers the results of Ref. 5.

The transport coefficients can be also calculated directly from the Boltzmann collision integral. This calculation also leads to integrals in the form of Eq. (8), cf. Ref. 7, Eqs. (24)–(26). It can be easily shown that for bi-Maxwellian distribution functions with velocities parallel to the ambient magnetic field one gets the same transport coefficients (9)–(11) as obtained from the Fokker–Planck to the leading order $\propto \ln \Lambda$. Note that the Coulomb logarithm used in Ref. 7 is twice the standard one (cf. Refs. 8 and 10). The agreement between the (leading order) momentum and energy transport coefficients obtained from the Boltzmann collision integral and the Fokker–Planck approximations is in agreement with the results of Ref. 6 which indicate that large-angle collisions impact higher order moments.

We have presented a closed form of collisional transport coefficients in bi-Maxwellian plasmas drifting along the ambient magnetic field. These coefficients can be expressed in the form of double hypergeometric functions. These results can be further generalized to an inverse-power force interaction and to include a drift velocity perpendicular with respect to the ambient magnetic field; a presence of the perpendicular drift velocity leads to triple hypergeometric functions.

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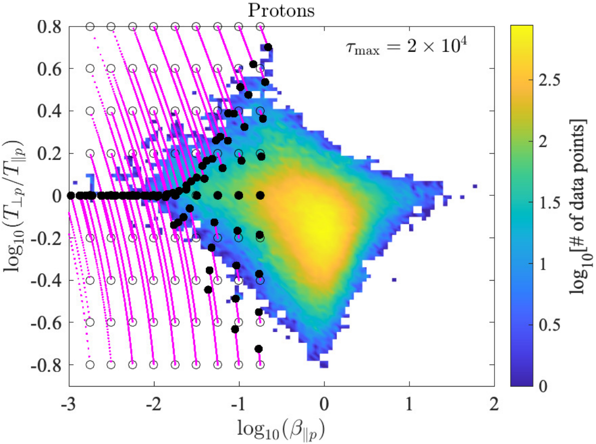
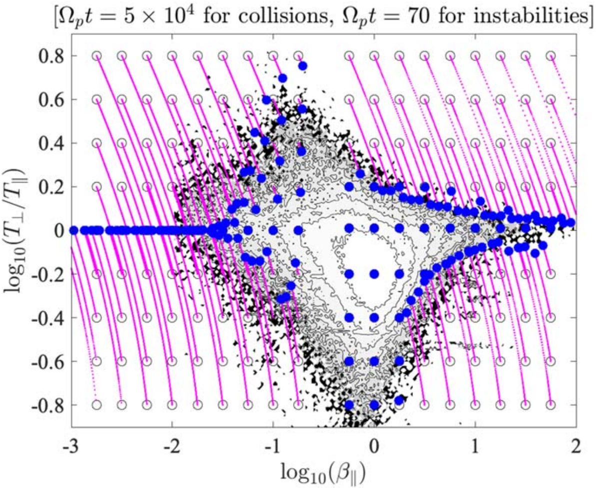
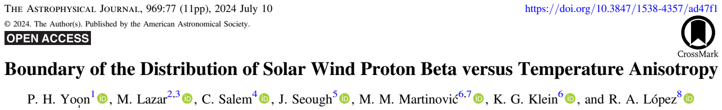

Protons
$\tau_{\max} = 2 \times 10^4$
$\log_{10}(T_{\perp p}/T_{\parallel p})$
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-3
-2
-1
0
1
2
$\log_{10}(\beta_{\parallel p})$
$\log_{10}$[# of data points]
2.5
2
1.5
1
0.5
0

Proton_movie_coll.mp4



$\Omega_p t = 5 \times 10^4$ is typical transit time between the solar source to 1 AU.



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Boundary of the Distribution of Solar Wind Proton Beta versus Temperature Anisotropy

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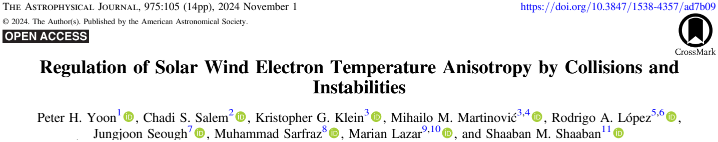

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Regulation of Solar Wind Electron Temperature Anisotropy by Collisions and Instabilities

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Regulation of temperature anisotropy for solar wind protons and alpha particles by collisions and instabilities

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Parker Solar Probe movie: combined_v2.mp4

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Perpendicular Heating

Back to the quasilinear temperature equation in dimensionless form,

$$\partial \overbrace{\beta_\perp}^{8\pi n T_\perp/B^2} / \partial \overbrace{T}^{\Omega_p t} = -4 \int d \overbrace{\kappa}^{k v_A/\Omega} \, z_i^C \, \overbrace{W_C(\kappa)}^{\delta B^2(k)/B^2} \left(1 + \frac{z_r^C}{\kappa^2}\right),$$

$$\partial \overbrace{\beta_\parallel}^{8\pi n T_\parallel/B^2} / \partial T = 4 \int d\kappa \, z_i^C \, W_C(\kappa) \left(1 + \frac{2 z_r^C}{\kappa^2}\right),$$

$$\partial W_C(\kappa)/\partial T = 2 z_i^C W_C(\kappa),$$

$$0 = \kappa^2 + \overbrace{z_C}^{\omega/\Omega} - A - \frac{(A+1) z_C - A}{\kappa\sqrt{\beta_\parallel}} Z\left(\frac{z_C - 1}{\kappa\sqrt{\beta_\parallel}}\right),$$

$$A = \beta_\perp/\beta_\parallel - 1,$$

if the initial wave intensity is finite (because of the background solar wind turbulence),

$$W_C(\kappa, 0) = W_0/(1 + \kappa^2),$$

and if we include the effect of radial expansion (this is a topic that deserves a separate discussion but ...),

$$\frac{\partial \beta_\perp}{\partial T} = \overbrace{-\frac{2Tf\beta_\perp}{T_*^2}}^{\text{expansion}} - 4\int d\kappa\, z_i^C\, W_C(\kappa)\left(1+\frac{z_r^C}{\kappa^2}\right),$$

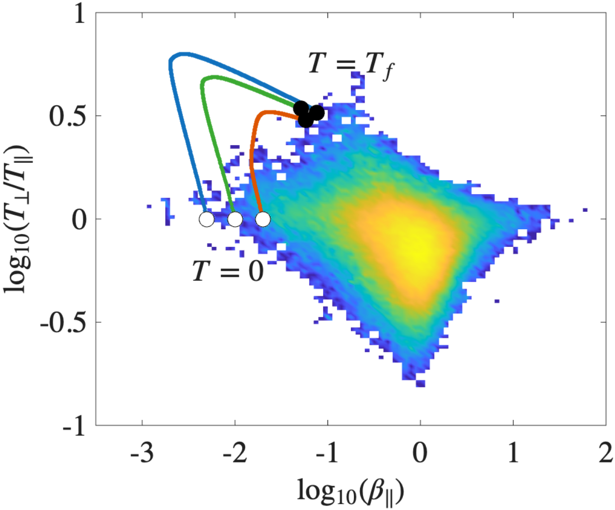
$$\frac{\partial \beta_\parallel}{\partial T} = 4\int d\kappa\, z_i^C\, W_C(\kappa)\left(1+\frac{2z_r^C}{\kappa^2}\right),$$

$$\partial W_C(\kappa)/\partial T = 2z_i^C W_C(\kappa),$$

$$0 = \frac{\kappa^2}{f} + \frac{z_C}{f} - A - \frac{(A+1)z_C - A}{\kappa\sqrt{\beta_\parallel}}\, Z\left(\frac{z_C - f}{\kappa\sqrt{\beta_\parallel}}\right),$$

$$A = \beta_\perp/\beta_\parallel - 1, \qquad \underbrace{f = \frac{1}{1+(T/T_*)^2},}_{\text{expansion factor}}$$

then we can explain the perpendicular heating.


$T = T_f$
$T = 0$
1
0.5
0
-0.5
-1
-3
-2
-1
0
1
2
$\log_{10}(T_\perp/T_\parallel)$
$\log_{10}(\beta_\parallel)$