Runaway electrons beams, Hall-MRI instabilities, and Reconnection.

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About me

- PhD at the University of Wisconsin.
  - Thesis work on runaway electron modeling for tokamaks using reduced models in extended-MHD.
- NSF funded postdoc project at Princeton.
  - Global shear flow-driven Hall-MHD instabilites in accretion flows.
  - Extended MHD simulations of MRX and FLARE (ongoing)

RE-MHD

A. P. Sainterme and C. R. Sovinec, "Resistive hose modes in tokamak runaway electron beams", Physics of Plasmas **31**, 10.1063/5.0183530 (2024)

A. P. Sainterme and C. R. Sovinec, "Resistive hose modes in tokamak runaway electron beams ii", Physics of Plasmas **32**, 10.1063/5.0273401 (2025)

Runaway electron generation may be understood via Coulomb collision physics.

The presence of a strong $\boldsymbol{E}$ field can accelerate electrons beyond the point where they can be slowed by collisions.

$$e|\boldsymbol{E}| \gtrsim mv\nu(v) = \frac{e^4 n \ln \lambda}{4\pi\epsilon_0^2 m v^2}$$

Classically, for a given electric field strength, any electrons with $v^2 \gtrsim \frac{e^3 n \ln \lambda}{4\pi\epsilon_0 m_e}|\boldsymbol{E}|$ would be expected to runaway. Considering relativistic effects sets a lower limit[1].

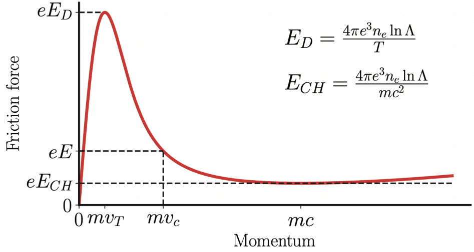

Figure 1. Friction force resulting from collisions as a function of electron momentum[2].

[1] J. Connor and R. Hastie, "Relativistic limitations on runaway electrons", Nuclear Fusion **15**, 415–424 (1975)

[2] B. N. Breizman et al., "Physics of runaway electrons in tokamaks", Nuclear Fusion **59**, 10.1088/1741-4326/ab1822 (2019)

REs are produced during a tokamak thermal quench.

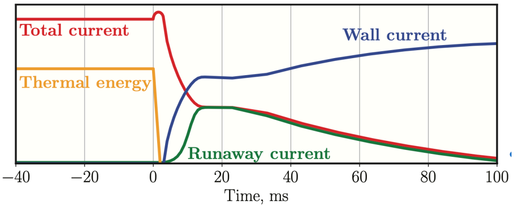

Figure 9. Anticipated evolution of the plasma thermal energy (orange), total plasma current (red), RE current (green) and the wall current (blue) in ITER. At 20 ms the density of the background plasma increases to facilitate mitigation of the RE current.

Rapid cooling of the plasma increases resistivity $\implies E_{\|} = \eta J_{\|}$ is large.
Electrons are accelerated into the runaway region, RE current plateau is reached.
RE current is eventually terminated - possibly damaging PFCs. This figure from Breizman illustrates the process schematically[a].

[a] B. N. Breizman et al., "Physics of runaway electrons in tokamaks", Nuclear Fusion **59**, 10.1088/1741-4326/ab1822 (2019).

RE+MHD model starts with resistive MHD.

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} \tag{1}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\boldsymbol{v}) = 0 \tag{2}$$

$$mn\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}\right) = \boldsymbol{J} \times \boldsymbol{B} - \nabla p - \nabla \cdot \boldsymbol{\Pi} \tag{3}$$

$$\frac{n}{\gamma - 1}\left(\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T\right) = -\frac{p}{2}\nabla \cdot \boldsymbol{v} - \nabla \cdot \boldsymbol{q} \tag{4}$$

With a modified Ohm's law:

$$\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} + \eta\left(\boldsymbol{J} - \boldsymbol{J}_r\right) \tag{5}$$

The runaway electrons MHD model is defined by a modified resistive Ohm's law.

$$\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} + \eta \left(\boldsymbol{J} - \boldsymbol{J}_r \right) \tag{6}$$

Here, $\boldsymbol{J}_r \equiv -e n_r \boldsymbol{u}_r$ is the current density associated with a beam of REs of density n_r moving at at $\boldsymbol{u}_r$. This Ohm's law is valid for

$$\frac{n_r}{n_e} \ll 1, \qquad m_e n_r \frac{\mathrm{d}\gamma_r \boldsymbol{u}_r}{\mathrm{d}t} \approx 0$$

The fluid model of REs evolves a beam-like population with density n_r - including volumetric sources.

Continuity equation for runaway electron population:

$$\frac{\partial n_r}{\partial t} + \nabla \cdot (n_r \boldsymbol{u_r}) = S_D(\mathcal{E}_{||}) + S_A(E_{||}), \tag{7}$$

S_D, S_A are source terms

$$\mathcal{E}_{||} \equiv \frac{E_{||}}{E_D}, \quad \boldsymbol{u_r} = \mathrm{sgn}(E_{||}) c_r \hat{\boldsymbol{b}} + \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2}, \quad c_r = \text{const.} \sim c$$

$$E_D = \frac{n_e e^3 \ln \Lambda}{4\pi\epsilon_0 T_e}.$$

The source terms model the Dreicer and avalanche mechanisms.

The model for the Dreicer source S_D is that presented by Connor and Hastie[3]

$$S_D = n_e^2 \frac{e^4 \ln \Lambda}{4\pi\epsilon_0^2 m_e^2 v_{th}^3} \mathcal{E}_{||}^{-\frac{3}{16}(1+Z_{eff})} \exp\left\{ -\frac{1}{4\mathcal{E}_{||}} - \sqrt{\frac{1+Z_{eff}}{\mathcal{E}_{||}}} \right\}$$
$$\times \exp\left\{ -\frac{T_e}{mc^2} \left(\frac{1}{8\mathcal{E}_{||}^2} + \frac{2}{3}\sqrt{\frac{1+Z_{eff}}{\mathcal{E}_{||}^3}} \right) \right\}$$

The avalanche source S_A is given by Rosenbluth and Putvinski[4]:

$$S_A = \frac{n_r}{\tau}\sqrt{\frac{\pi a}{3(Z+5)}} \left(\frac{E_{||}}{E_c} - 1 \right) \left(1 - \frac{E_c}{E_{||}} + \frac{4\pi(Z+1)^2}{3a(Z+5)(E_{||}^2/E_c^2 + 4/a^2 - 1)} \right)^{-1/2}$$

$$E_c = \frac{n_e e^3 \ln \Lambda}{4\pi\epsilon_0^2 mc^2}, \quad a(\varepsilon) = (1 + 1.46\sqrt{\varepsilon} + 1.72\varepsilon)^{-1}, \quad \tau = \frac{mc \ln \Lambda}{eE_c}$$

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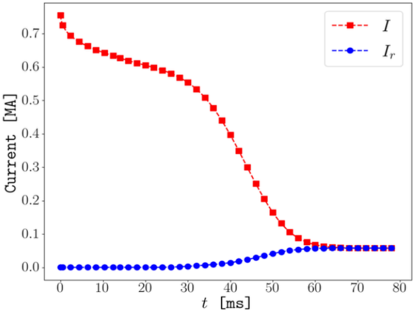
[3] J. Connor and R. Hastie, "Relativistic limitations on runaway electrons", Nuclear Fusion **15**, 415–424 (1975)

[4] M. N. Rosenbluth and S. V. Putvinski, "Theory for avalanche of runaway electrons in tokamaks", Nuclear Fusion **37**, 1355–1362 (1997)

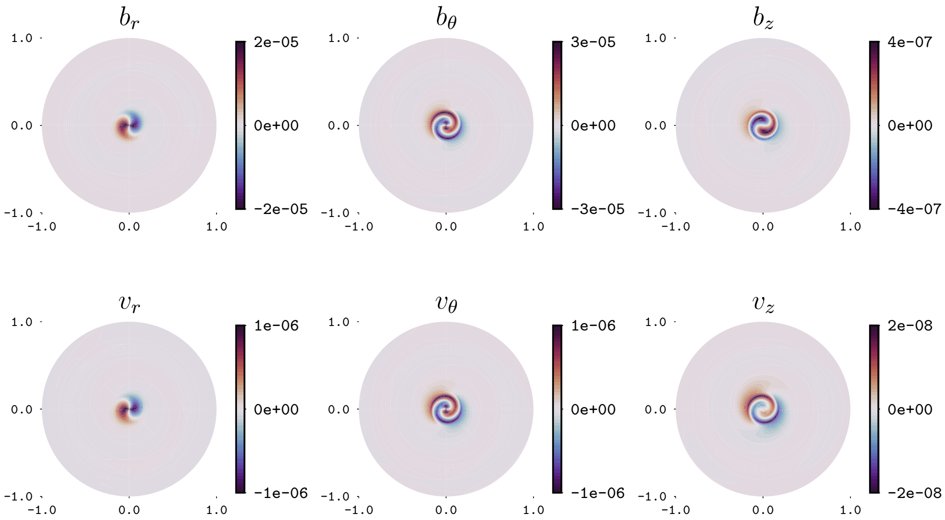
Source terms are effective at capturing the current plateau phase of a diruption.



- The density of runaways is zero at $t = 0$.
- At $t = 13.3$ ms, the perpendicular thermal conductivity, $\kappa_\perp$, is increased by a factor of 100.
- As the temperature in the simulation drops, the resistivity increases $\implies$ increase in $E_\parallel$ $\implies$ REs are generated via source terms

[4] C. Sovinec et al., "Nonlinear magnetohydrodynamics simulation using high-order finite elements", Journal of Computational Physics **195**, 355–386 (2004)

Resistive hose mode[6] at $S = 10^{-3}$.



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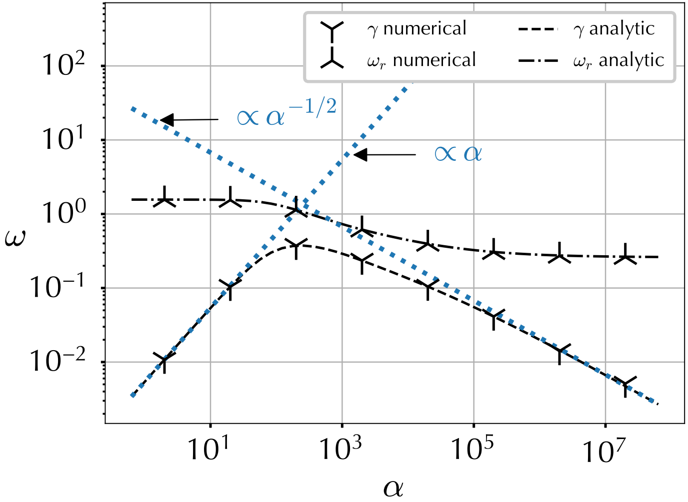
[5] A. P. Sainterme and C. R. Sovinec, "Resistive hose modes in tokamak runaway electron beams", Physics of

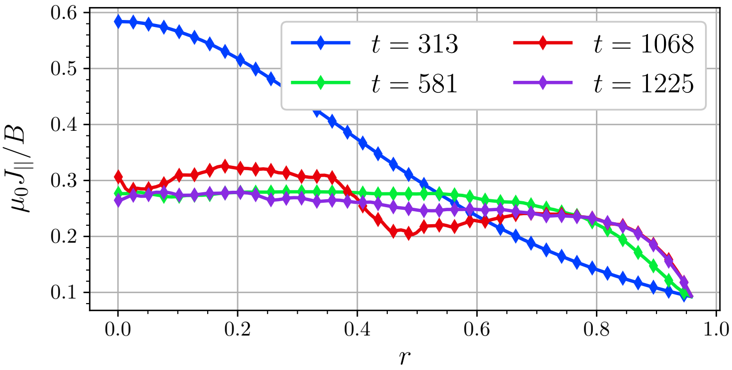
Resistive hose mode growth rate depends on $\alpha \equiv ac_r/\eta = Sc_r/V_A$.



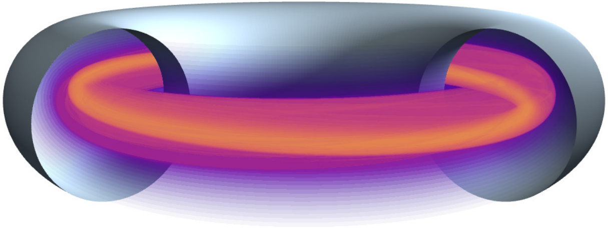
[7]

[7] A. P. Sainterme and C. R. Sovinec, "Resistive hose modes in tokamak runaway electron beams", Physics of Plasmas **31**, 10.1063/5.0183530 (2024).

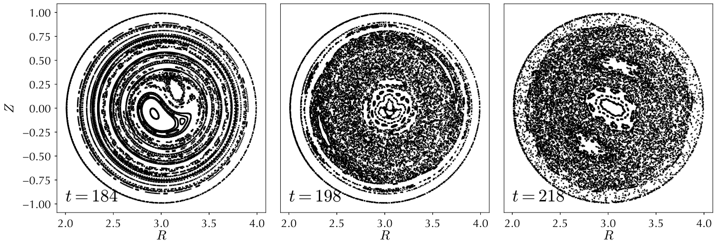
Flattening of parallel current profile does not extend to the wall in toroidal case.

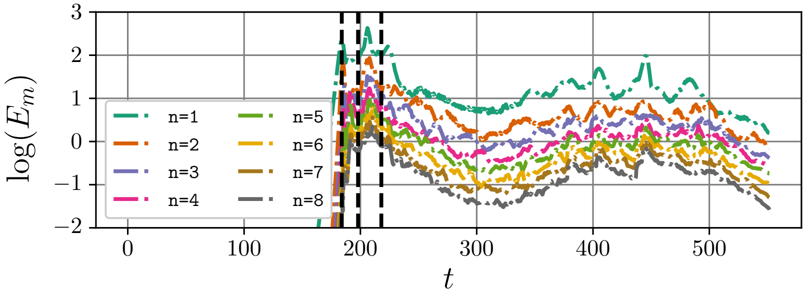


Parallel current isosurfaces show kinked helical structure



Nonlinear consequences of hose modes on field line topology.





Hall-MHD in Rotating Flows

A. Sainterme and F. Ebrahimi, "Global non-axisymmetric hall instabilities in a rotating plasma", The Astrophysical Journal **1001**, 160 (2026)

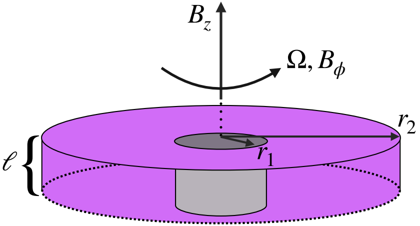
Hall-MHD includes the Hall term in the Ohm's law

$$\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B} + \boldsymbol{J} \times \boldsymbol{B}/en_e \tag{8}$$

- Generates an EMF perpendicular to $\boldsymbol{J}$ and $\boldsymbol{B}$.
- $\boldsymbol{E}$ depends on $\mu_0 \boldsymbol{J} = \nabla \times \boldsymbol{B}$, but the effect is non-dissipative. i.e. electromagnetic energy is conserved.

Here we use simple viscous stress and adiabatic heat flux closures:
$\boldsymbol{\Pi} = \rho\nu(\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^T - \frac{2}{3}(\nabla \cdot \boldsymbol{v}\boldsymbol{I}))$, and $\nabla \cdot \boldsymbol{q} = 0$

Model



- Plasma Couette flow rotating with Keplerian frequency $\Omega(r) \propto r^{-3/2}$ around the z-axis.
- Rigid, conducting boundaries at $r = r_1$ and $r = r_2$. Periodic boundary conditions at $z = \pm\ell/2$,. Define the aspect ratio $A \equiv \ell/(r_2 - r_1)$. Here, $\ell = 8r_1$
- Either vertical or azimuthal magnetic field: $\boldsymbol{B} = B_z\hat{\boldsymbol{z}}$ or $\boldsymbol{B} = B_\phi\hat{\boldsymbol{\phi}}$.
- Uniform B_z, and current free $B_\phi \implies B_\phi \propto 1/r$

Consider of the linearized, incompressible Hall-MHD equations.

The incompressible Hall-MHD equations for a fully-ionized plasma linearized about a steady, differentially rotating azimuthal flow are given, in cylindrical coordinates, by

$$\rho\partial_t \boldsymbol{v} + \Omega\partial_\phi \rho \boldsymbol{v} + \left(\frac{\kappa^2}{2\Omega}\hat{\boldsymbol{\phi}}\hat{\boldsymbol{r}} - 2\Omega\hat{\boldsymbol{r}}\hat{\boldsymbol{\phi}}\right)\cdot\rho\boldsymbol{v} = -\nabla\tilde{P} + \frac{1}{\mu_0}\left(\boldsymbol{B}\cdot\nabla\boldsymbol{b} + \boldsymbol{b}\cdot\nabla\boldsymbol{B}\right),$$

$$\partial_t \boldsymbol{b} + \Omega\partial_\phi \boldsymbol{b} - r\Omega'\hat{\boldsymbol{\phi}}b_r = (\boldsymbol{B}\cdot\nabla)\left(\boldsymbol{v} - \frac{d_i}{\sqrt{\mu_0\rho}}\nabla\times\boldsymbol{b}\right) - \left(\boldsymbol{v} - \frac{d_i}{\sqrt{\mu_0\rho}}\nabla\times\boldsymbol{b}\right)\cdot\nabla\boldsymbol{B}$$

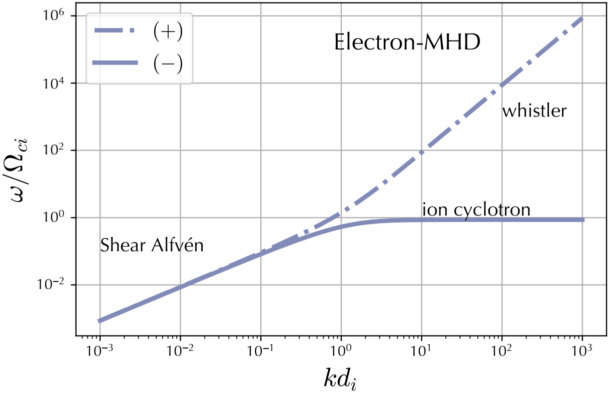
$$\nabla\cdot\boldsymbol{v} = \nabla\cdot\boldsymbol{b} = 0.$$

- Ω is the azimuthal rotation frequency
- $\kappa^2 \equiv 4\Omega^2 + 2\Omega r\Omega'$ is the square of the epicyclic frequency ($=\Omega^2$ for Keplerian rotation)
- $d_i = c/\omega_{pi}$ is the ion skin depth
- solns. of the form $\boldsymbol{v}, \boldsymbol{b} \propto \exp(ikz + im\phi - i\omega t)$

Incompressible Hall-MHD supports two waves.

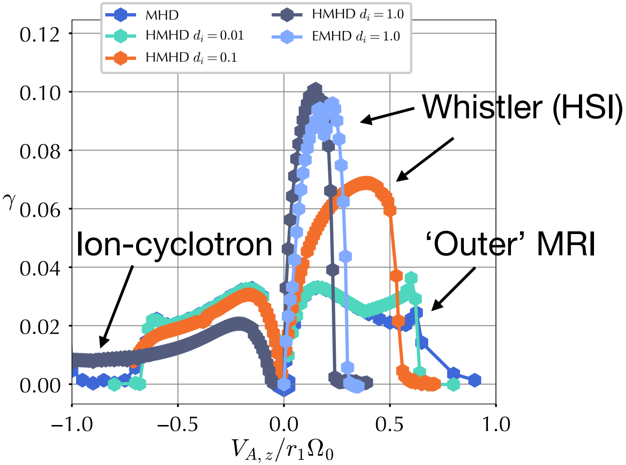
Uniform $\boldsymbol{B}$, single ion species: q, m_i, n_i. $\omega_A \equiv (\boldsymbol{k} \cdot \boldsymbol{B})/\sqrt{\mu_0 \rho}$, $\Omega_{ci} \equiv qB/m_i$.

$$(\omega^2 - \omega_A^2) = \pm k d_i \omega_A \omega \tag{9}$$



- $kd_i \ll 1$ - MHD limit. Magnetic field and fluid motions are well coupled.
- $kd_i \gg 1$ - Hall regime. Magnetic field decouples from ion motion – moves with electron drift.
- Low frequency limit of the R and L cold plasma waves.

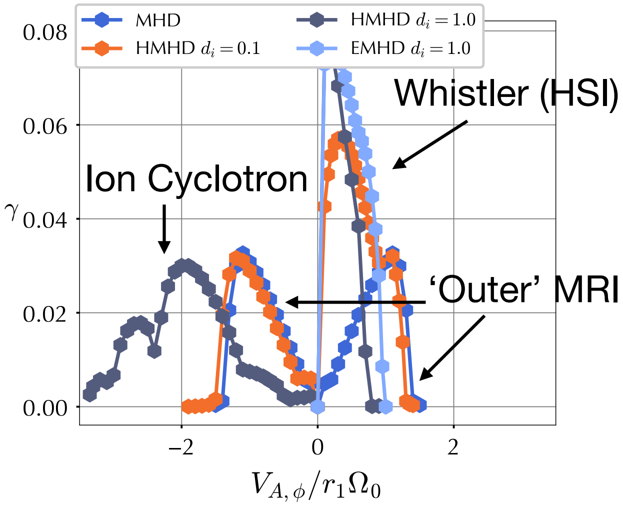
Hall-MHD have both whistler and Alfvénic/ion cyclotron instabilities.



- $m = 1, k = \pi/4$ mode growth rate as a function of vertical field strength
- $V_{A,z} > 0 \implies \boldsymbol{B} \parallel$ to the vorticity of the steady flow.
- The MHD limit recovers the 'outer' MRI modes with resonances near the boundary[1]

[1] [9]

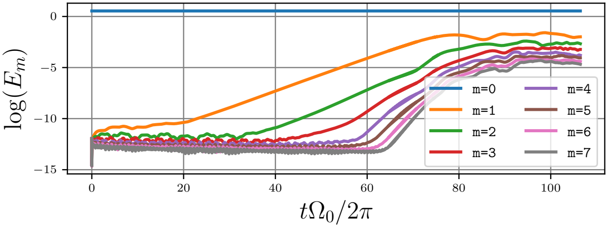
The asymmetry is exaggerated in the azimuthal field case



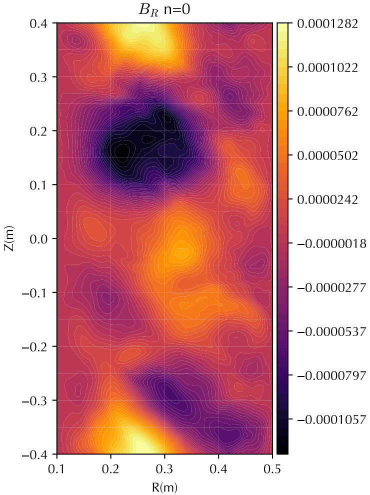
- $\boldsymbol{B} = B_\phi \hat{\boldsymbol{\phi}}$
- $V_{A,\phi} > 0 \implies \boldsymbol{B} \parallel$ to the flow direction.
- $m = 1,\ k = \pi/4$ mode growth rates as a function of azimuthal field strength at the inner boundary.

[1] [10]

Nonlinear NIMROD calculations show a combination of turbulent and laminar structures.



- Contours of $m = 0$ component of solution at $t\Omega_0/2\pi \sim 90$



Turbulence changes magnetic field line topology