MAGNETIC RAYLEIGH-TAYLOR INSTABILITY AND MIXING

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Synergies between Astrophysical, Space, Laboratory, and Fusion Plasma Physics, NORDITA

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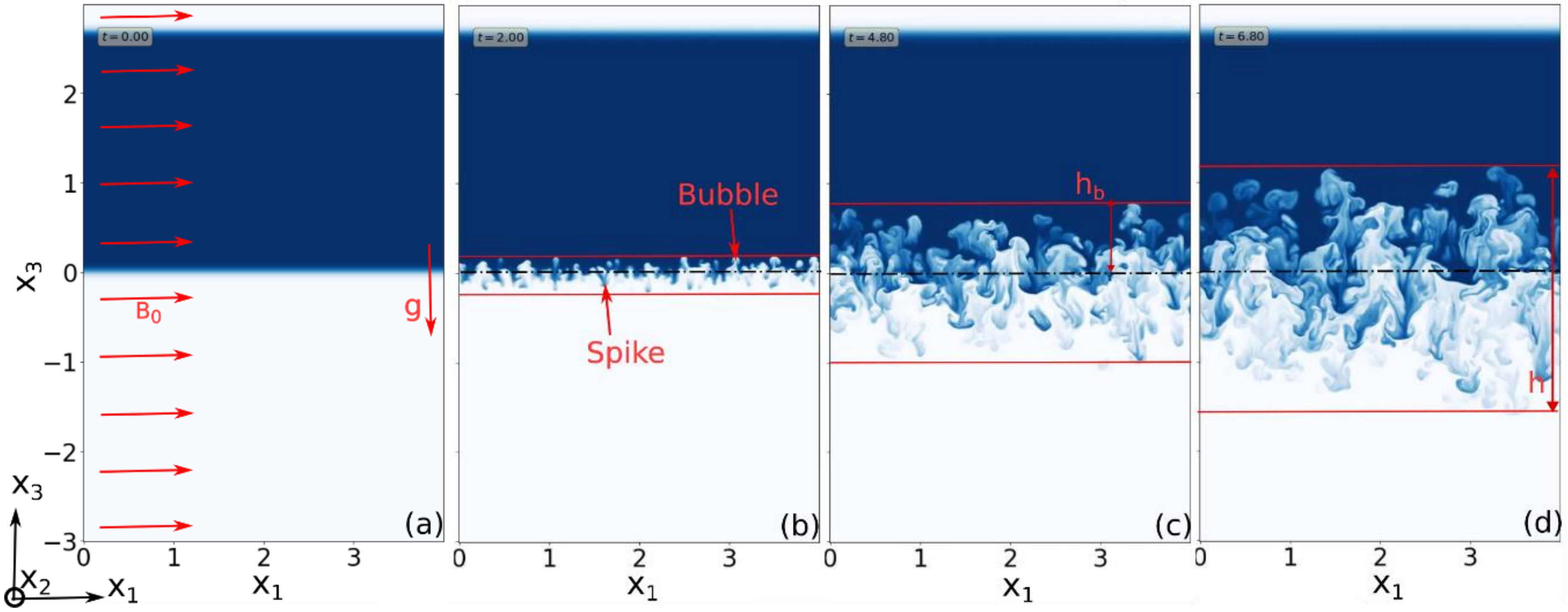
MAGNETIC RAYLEIGH -TAYLOR

- Configuration: High density plasma superimposed over low density plasma *(Inherently unstable)*
- *Destabilizing force:* Gravity; *Stabilizing force:* Magnetic tension
- In the regime where gravity dominates magnetic tension, MRTI develops
- Upon perturbation, instability grows over time through plasma penetration or mixing.



KEY QUESTIONS AND TAKE AWAYS:

Self-similarity

- Does the instability evolve/grow over time in a **self-similar** fashion?
  - We derived a proof proving MRTI evolution is conditionally, but eventually, self-similar
- What parameters contribute to the **(non-linear) instability growth**?
  - Energy partition across magnetic, kinetic, and thermal energies
- What is the effect of imposed magnetic **field strength**?
  - We derived scaling relations. TME/TKE α B_0^2 ; C_{diss} α $1 + C_1 B_0^2$ (in low B_0)
- What is the **role of reconnection** on the large-scale energy dynamics?
  - Necessary for long-term instability evolution
  - Important for energy conversion from magnetic to kinetic energy
  - Does not play a key role in magnetic energy dissipation

Reconnection and turbulence

- MRTI dynamics in **2D vs. 3D**?
  - Instability always evolves in 3D, but not always in 2D
  - 2D MRTI may not have self-similar evolution
  - 3D case exhibits more mixing, anisotropy, and energy dissipation

2D vs. 3D

MAGNETIC RTI: EVOLUTION

- Does the instability evolve/grow over time in a self-similar fashion?
  - Earlier studies assumed MRTI follows HD self-similar scaling, with ***no mathematical proof***.
  - ***Scaling law???*** Simple exercise shows it is a redundant problem:

$$\partial_t h \propto f(\mathcal{A}g, h, v_A) \qquad\qquad \partial_t h \propto (\mathcal{A}g)^p h^q v_A^r$$

$$\implies LT^{-1} = (LT^{-2})^p L^q (LT^{-1})^r \equiv L^{p+q+r} T^{-2p-r}$$

$$\implies p + q + r = 1; 2p + r = 1$$

  - ***So, what do we do???***

Proof by contradiction: Assume system follows HD self-similar scaling

Write the governing equations in terms of self-similar variable framed from HD scaling.

MAGNETIC RTI: SELF SIMILARITY

- Governing equations:

$$\partial_t \rho u_i + \partial_j(\rho u_j u_i) = -\partial_i p + \partial_j(B_{0j} b_i) + \partial_j(b_j b_i) - \delta\rho g \delta_{i3},$$
$$\partial_t b_i + \partial_j(u_j b_i) = \partial_j(B_{0j} u_i) + \partial_j(b_j u_i),$$
$$\partial_t \rho + \partial_j(u_j \rho) = 0,$$
$$\partial_i u_i = \partial_i b_i = 0.$$

- Non dimensionalization: cf. $h \,\alpha\, Agt^2 \Rightarrow u_c = d_t h \,\alpha\, Agt$

$$\rho = \rho_m \tilde{\rho}, \quad u = \mathcal{A}gt\tilde{u}, \quad b = \sqrt{\rho_m}\mathcal{A}gt\tilde{b}, \quad p = \rho_m(\mathcal{A}gt)^2\tilde{p}.$$

Characteristic density — *Characteristic velocity* — *Characteristic magnetic field* — *Characteristic pressure*

- Non dimensionalized equations

$$t\partial_t\langle\tilde{\rho}\tilde{u}_i\rangle + \langle\tilde{\rho}\tilde{u}_i\rangle + \mathcal{A}gt^2\partial_3\langle(\tilde{\rho}\tilde{u}_3\tilde{u}_i)\rangle + \mathcal{A}gt^2\partial_3\langle\tilde{p}\rangle - \mathcal{A}gt^2\partial_3\langle\tilde{b}_3\tilde{b}_i\rangle + \frac{1}{\mathcal{A}}\langle\delta\tilde{\rho}\delta_{i3}\rangle = 0,$$
$$\partial_3\langle\tilde{u}_3\tilde{b}_i\rangle - \partial_3\langle\tilde{b}_3\tilde{u}_i\rangle = 0,$$
$$\partial_t\langle\tilde{\rho}\rangle + \mathcal{A}gt\partial_3\langle\tilde{u}_3\tilde{\rho}\rangle = 0.$$

Equations are functions of x_3 and t

Note that, $\partial_1\langle\star\rangle=0$, $\partial_2\langle\star\rangle=0$, since x_1 and x_2 are directions of statistical homogeneity.

MAGNETIC RTI: SELF SIMILARITY

- Constructing self-similar variable: $\xi = \sqrt{\frac{x_3}{\mathcal{A}g}}\frac{1}{t}$ *(Note that, Xi is dimensionless)*

- Rewriting derivatives in terms of *Xi* : $\partial_t = \frac{-\xi}{t}\partial_\xi,\ \partial_3 = \frac{\xi}{2x_3}\partial_\xi$

- Rewriting the non-dimensional equations in terms of *Xi*

$$-\xi \mathrm{d}_\xi \langle \tilde{\rho}\tilde{u}_i \rangle + \xi \langle \tilde{\rho}\tilde{u}_i \rangle + \frac{1}{2}\mathrm{d}_\xi \langle \tilde{\rho}\tilde{u}_3\tilde{u}_i \rangle + \frac{1}{2}\mathrm{d}_\xi \langle \tilde{p} \rangle - \frac{1}{2}\mathrm{d}_\xi \langle \tilde{b}_3\tilde{b}_i \rangle + \xi \langle \frac{\delta\tilde{\rho}\delta_{i3}}{\mathcal{A}} \rangle = 0,$$

$$\mathrm{d}_\xi \langle \tilde{u}_3\tilde{b}_i \rangle - \mathrm{d}_\xi \langle \tilde{b}_3\tilde{u}_i \rangle = 0,$$

$$\xi^2 \mathrm{d}_\xi \langle \tilde{\rho} \rangle + \frac{1}{2}\mathrm{d}_\xi \langle \tilde{u}_3\tilde{\rho} \rangle = 0.$$

- Momentum, induction and density equations become ODEs i.e., follow self-similar scaling. But the system is not yet necessarily self-similar. Need to check higher order equations.

MAGNETIC RTI: SELF SIMILARITY

- Turbulent kinetic energy equation:

$$\partial_t \left\langle \rho \frac{u_i u_i}{2} \right\rangle + \partial_3 \left\langle u_3 \rho \frac{u_i u_i}{2} \right\rangle + \partial_3 \langle u_3 p \rangle - B_0 \langle u_i \partial_1 b_i \rangle - \langle u_i b_j \partial_j b_i \rangle + \langle u_i \delta\rho g \delta_{i3} \rangle = 0$$

- Rewriting the non-dimensional equations in terms of Xi

$$\xi \langle \tilde{\rho} \tilde{u}_i \tilde{u}_i \rangle - \xi^2 \partial_\xi \left\langle \tilde{\rho} \frac{\tilde{u}_i \tilde{u}_i}{2} \right\rangle + \frac{1}{2} \partial_\xi \left\langle \tilde{u}_3 \tilde{\rho} \frac{\tilde{u}_i \tilde{u}_i}{2} \right\rangle - \frac{1}{2} \partial_\xi \langle \tilde{u}_3 \tilde{p} \rangle - \frac{1}{2\sqrt{\rho_m} C_1 \mathcal{A} g t} B_0 \langle \tilde{u}_i \partial_\xi \tilde{b}_i \rangle - \frac{1}{2} \left\langle \tilde{u}_i \partial_\xi \left(\frac{\tilde{b}_1 \tilde{b}_i}{C_1} + \frac{\tilde{b}_2 \tilde{b}_i}{C_2} + \tilde{b}_3 \tilde{b}_i \right) \right\rangle$$

$$- \xi \frac{1}{\mathcal{A}} \langle \tilde{\rho} \tilde{u}_i \delta_{i3} \rangle = 0,$$

External magnetic field term

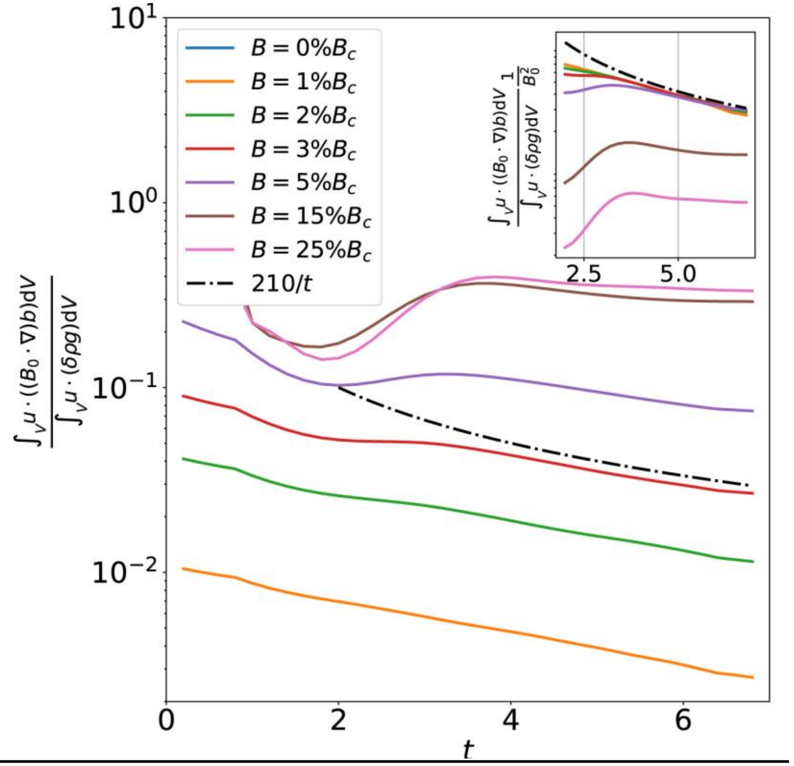
Gravitational potential energy term

- TKE does not reduce to equation of Xi alone, i.e., it does not follow HD self-similar scaling.
- Imposed magnetic field term follows a different scaling law.

MAGNETIC RTI: SELF SIMILARITY

Does MRTI never achieve self-similar state?

- Ratio of imposed magnetic field to other nonlinear terms is *1/t* i.e., with time other nonlinear terms dominate over imposed magnetic field term, tending the system toward self-similarity.
- Thus, over time the MRTI evolution tends towards self-similar state



GROWTH IN SELF SIMILAR STATE

Now, in the eventual self-similar state, ***how does the mixing layer grow?***

Starting with the ***energy conservation:***

$$\underbrace{-\int_V \delta\rho g x_3 \mathrm{d}V}_{\text{GPE released}} = \underbrace{\int_V \frac{1}{2}\rho u^2 \mathrm{d}V}_{\text{TKE}} + \underbrace{\int_V \frac{1}{2} b^2 \mathrm{d}V}_{\text{TME}} + \underbrace{D_E}_{\text{Total energy dissipated}}.$$

$$-\int_V \delta\rho g x_3 \mathrm{d}V = (1+C_{diss})\left(\int_V \frac{1}{2}\rho u^2 \mathrm{d}V + \int_V \frac{1}{2} b^2 \mathrm{d}V\right)$$

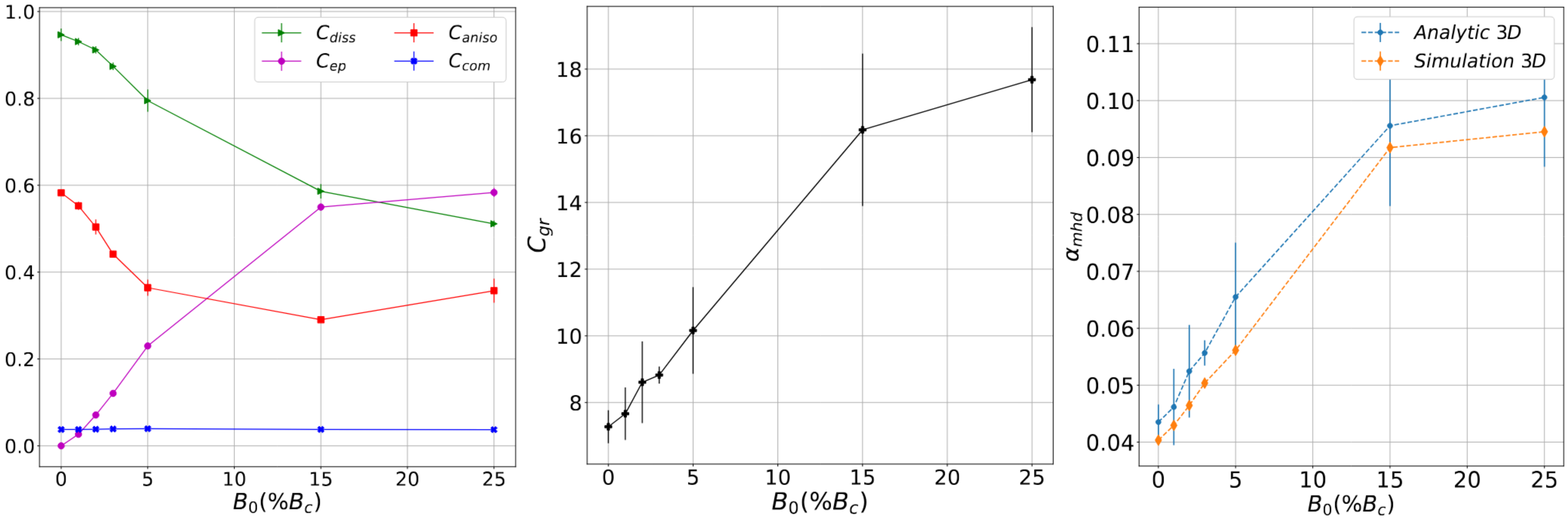
$$-\int_V \delta\rho g x_3 \mathrm{d}V = (1+C_{diss})(1+C_{ep})\left(\int_V \frac{1}{2}\rho u^2 \mathrm{d}V\right)$$

$$-\int_V \delta\rho g x_3 \mathrm{d}V = (1+C_{diss})(1+C_{ep})(1+C_{aniso})\left(\int_V \frac{1}{2}\rho u_3^2 \mathrm{d}V\right)$$

GROWTH IN SELF SIMILAR STATE

$$C_{com} g L_x L_y \Delta\rho h^2 = \frac{(1+C_{diss})(1+C_{ep})(1+C_{aniso})}{C_{gr}} h(\partial_t h)^2 L_x L_y \bar{\rho}$$

$$h = \alpha_{mhd} \mathcal{A} g t^2 + 2\sqrt{\alpha_{mhd} \mathcal{A} g h_0}\, t + h_0, \text{ where } \alpha_{mhd}(\mathcal{A}, B_0) = \frac{C_{com} C_{gr}}{2(1+C_{diss})(1+C_{ep})(1+C_{aniso})}$$



SCALING LAWS WITH B_0

$$D_E \propto \left(\eta \int_0^t \int_V \frac{b_{rms}^2}{l_j^2} \mathrm{d}V \mathrm{d}t + \nu \int_0^t \int_V \frac{\rho u_{rms}^2}{l_\omega^2} \mathrm{d}V \mathrm{d}t \right)$$

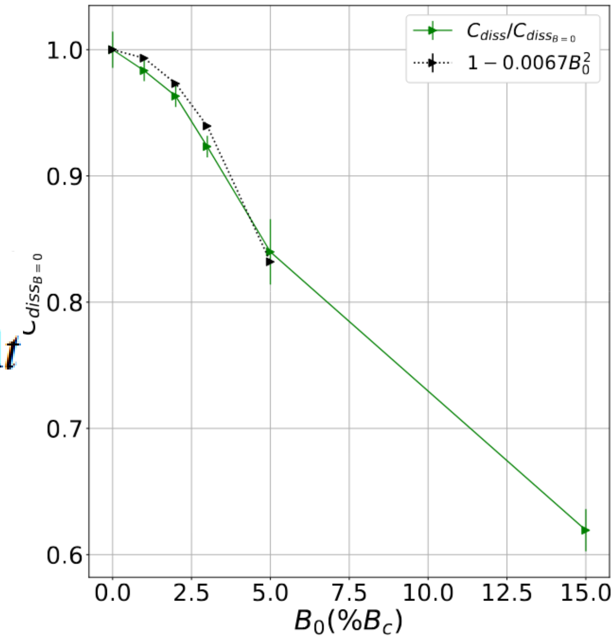
$$D_E \propto \left(\frac{l_\omega^2}{l_j^2} \frac{1}{Pr_m} C_{ep} + 1 \right) \int_0^t \int_V \nu \frac{\rho u_{rms}^2}{l_\omega^2} \mathrm{d}V \mathrm{d}t, \text{ where } C_{ep} = \frac{\int_V b_{rms}^2 \mathrm{d}V}{\int_V \rho u_{rms}^2 \mathrm{d}V}, Pr_m = \frac{\nu}{\eta}$$

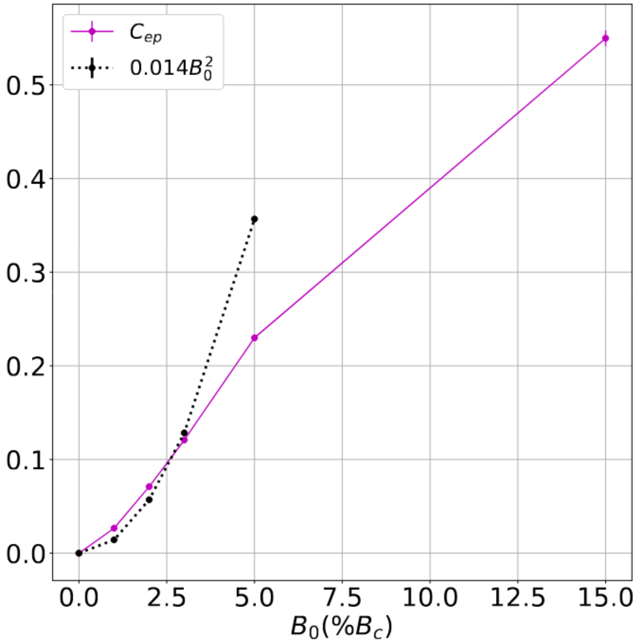
$$C_{ep} \equiv \frac{\int_V b_{rms}^2 \mathrm{d}V}{\int_V \rho u_{rms}^2 \mathrm{d}V} \propto B_0^2$$

(Lazarian et al. 2024)

$$D_E \propto \left(\frac{l_\omega^2}{l_j^2} \frac{1}{Pr_m} a B_0^2 + 1 \right) \int_0^t \int_V \nu \frac{\rho u_{rms}^2}{l_j^2} \mathrm{d}V \mathrm{d}t$$

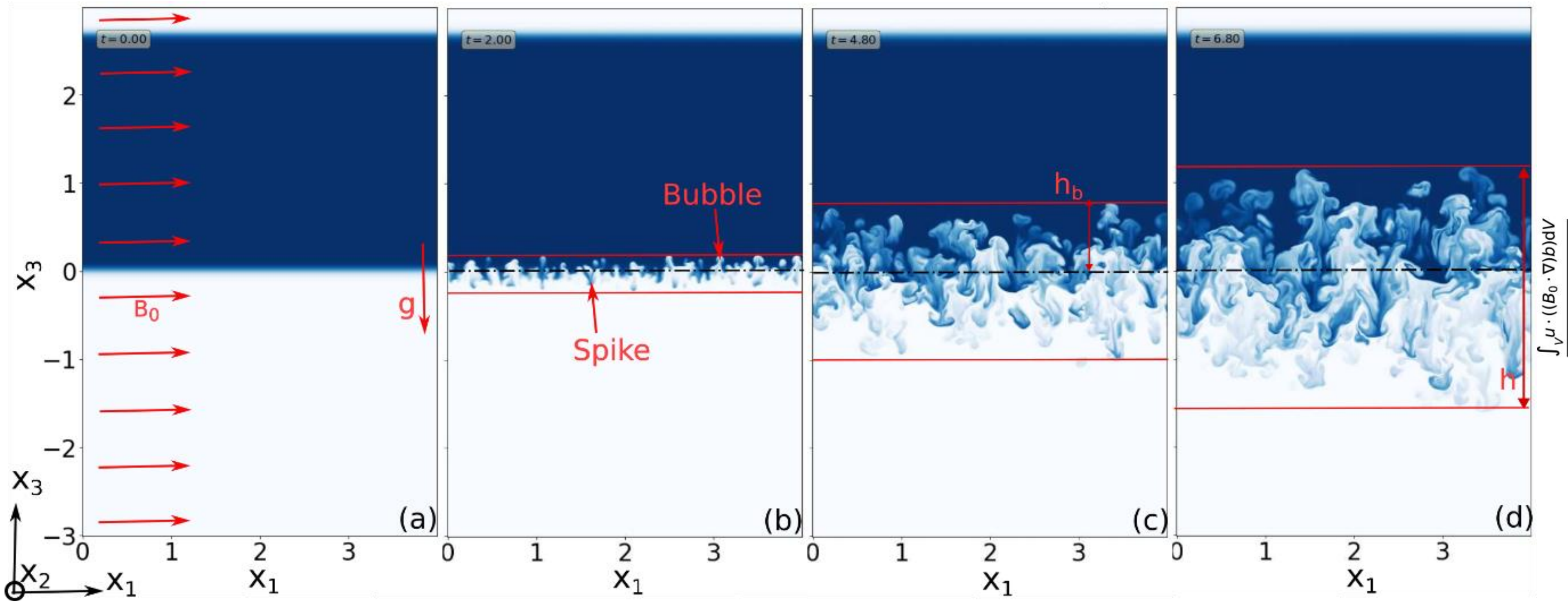
$$C_{diss} \propto \left(\frac{l_\omega^2}{l_j^2} \frac{1}{Pr_m} a B_0^2 + 1 \right) C_{diss_{B=0}}.$$

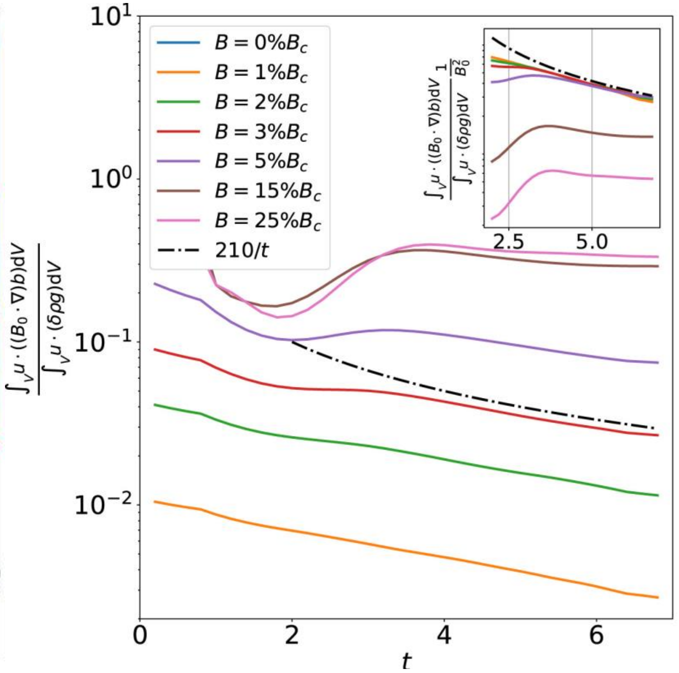




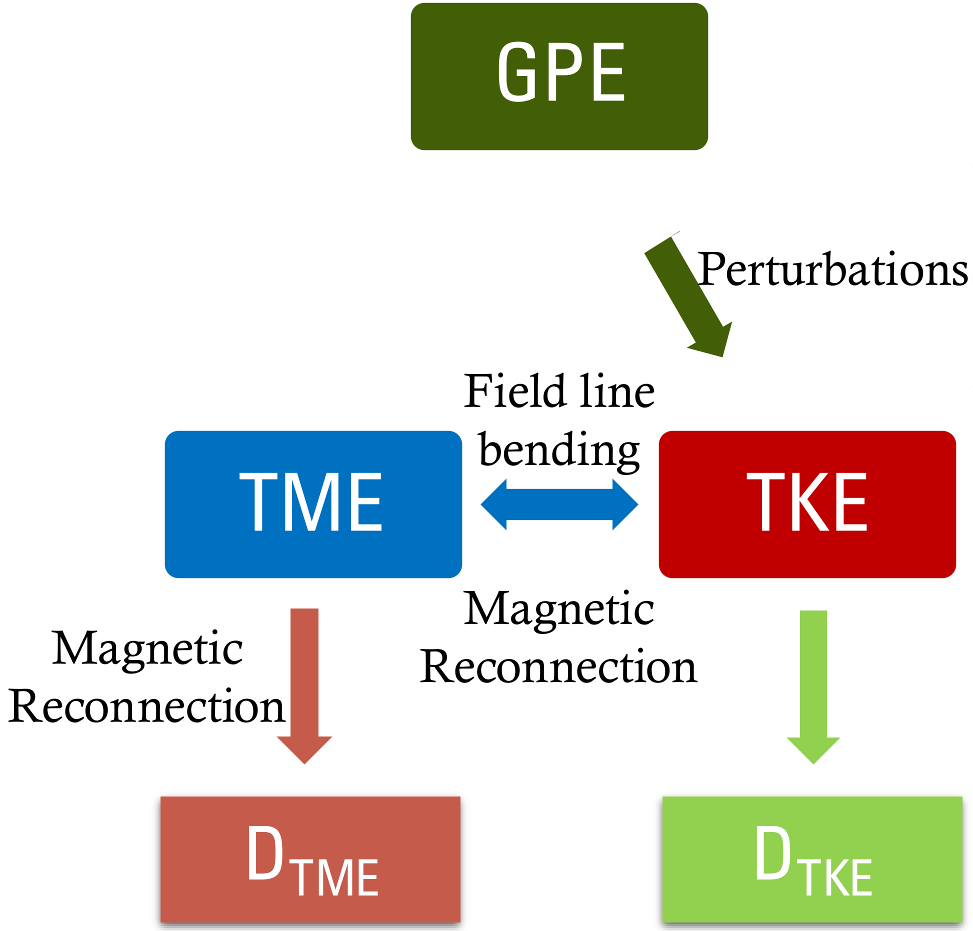
SUMMARY: ONSET AND EVOLUTION

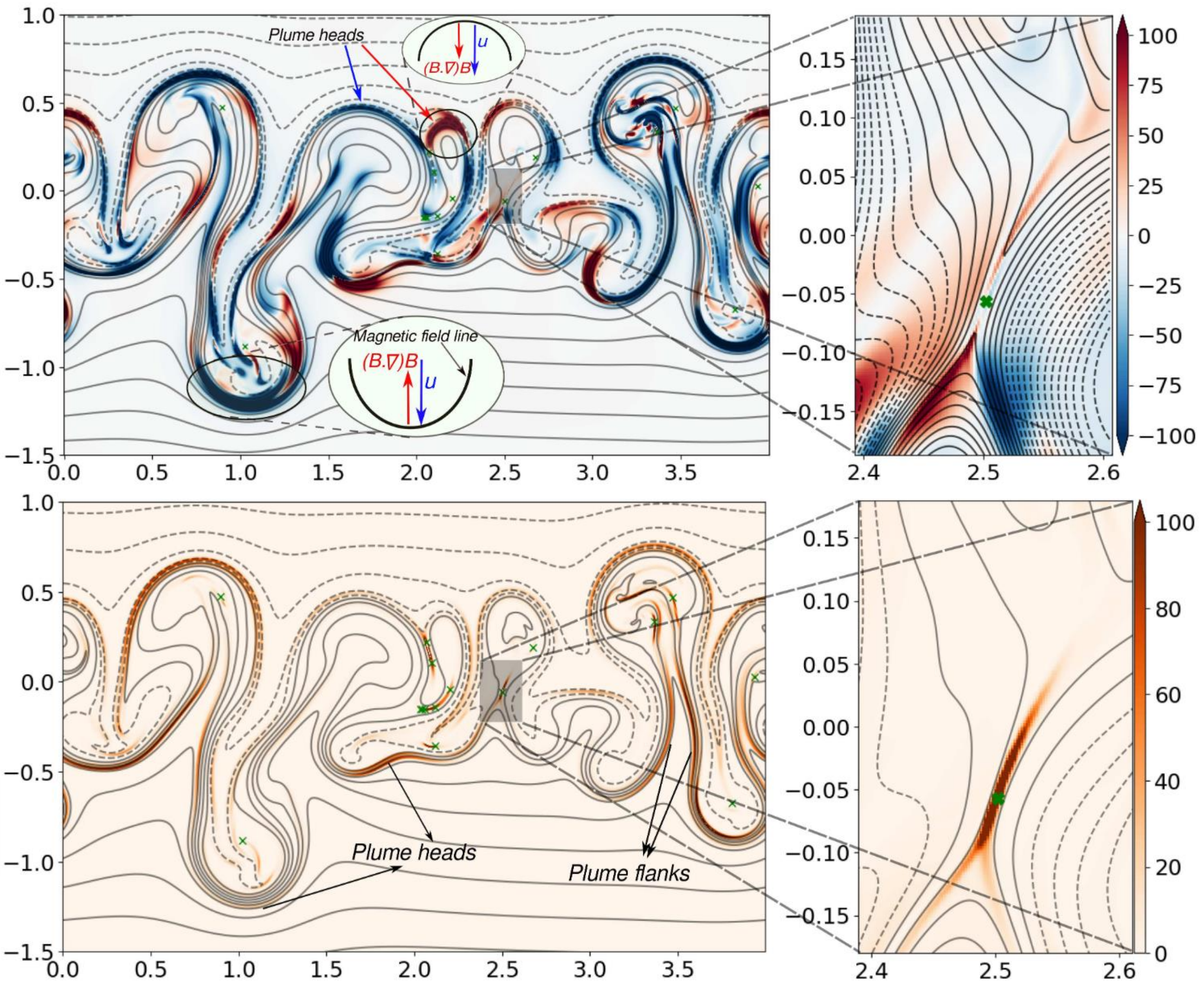
- Investigated **temporal evolution of MRTI**: self-similarity, non-linear growth constant ($\alpha = h/t^2$)
- Proved mathematically that the self-similar growth of MRTI is conditional. (t >> 1)
- Derived a formula for α highlighting processes controlling instability growth.





ENERGY FLOW

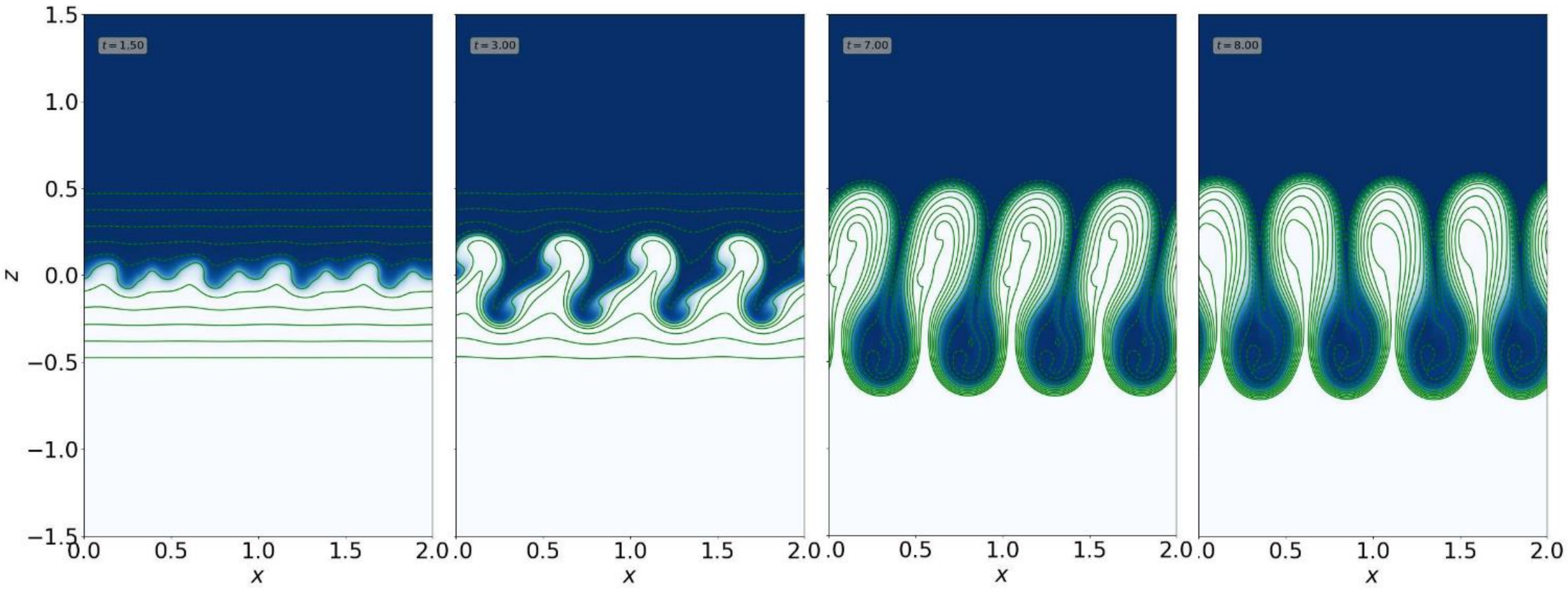


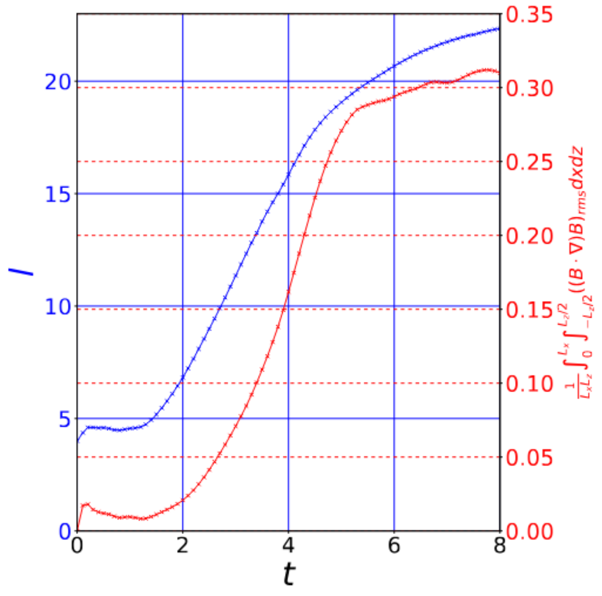


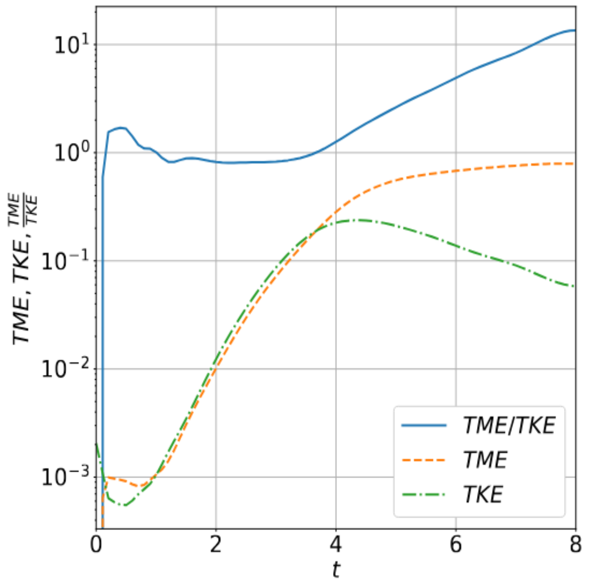
Contours of: *(top)* $u.(B.\nabla)B$; *(bottom)* j^2

ROLE OF RECONNECTION IN INSTABILITIES EVOLUTION

- Let us first see the instability evolution without reconnection

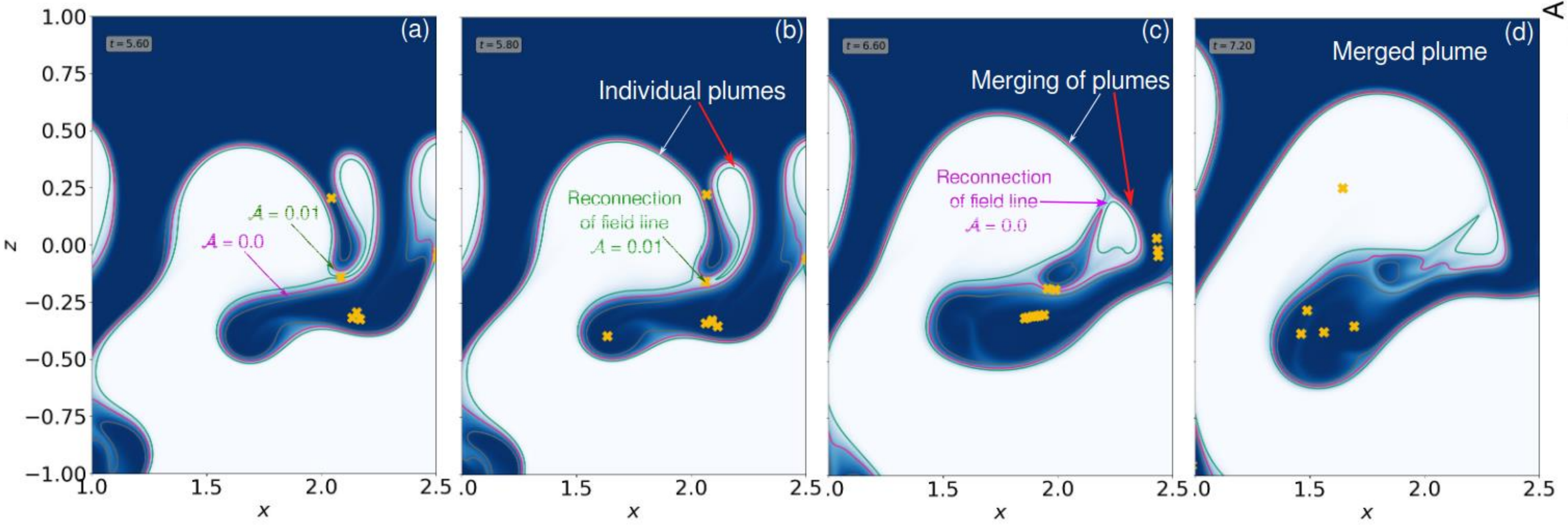


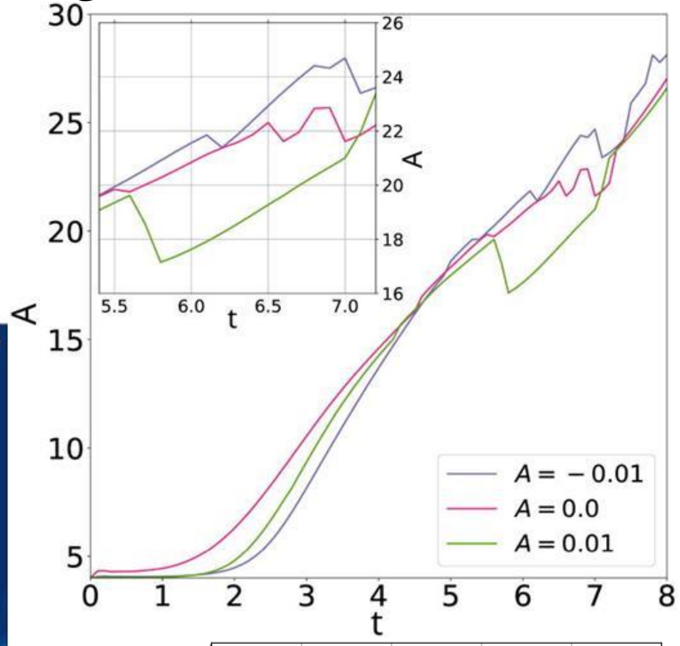


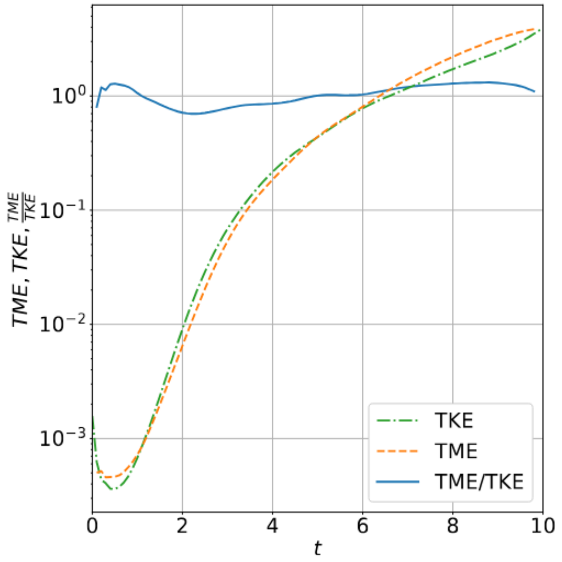


ROLE OF RECONNECTION IN INSTABILITIES EVOLUTION

- Reconnection sustains instability evolution over long time by relieving magnetic tension.
- If not for reconnection, the MRTI evolution would saturate.

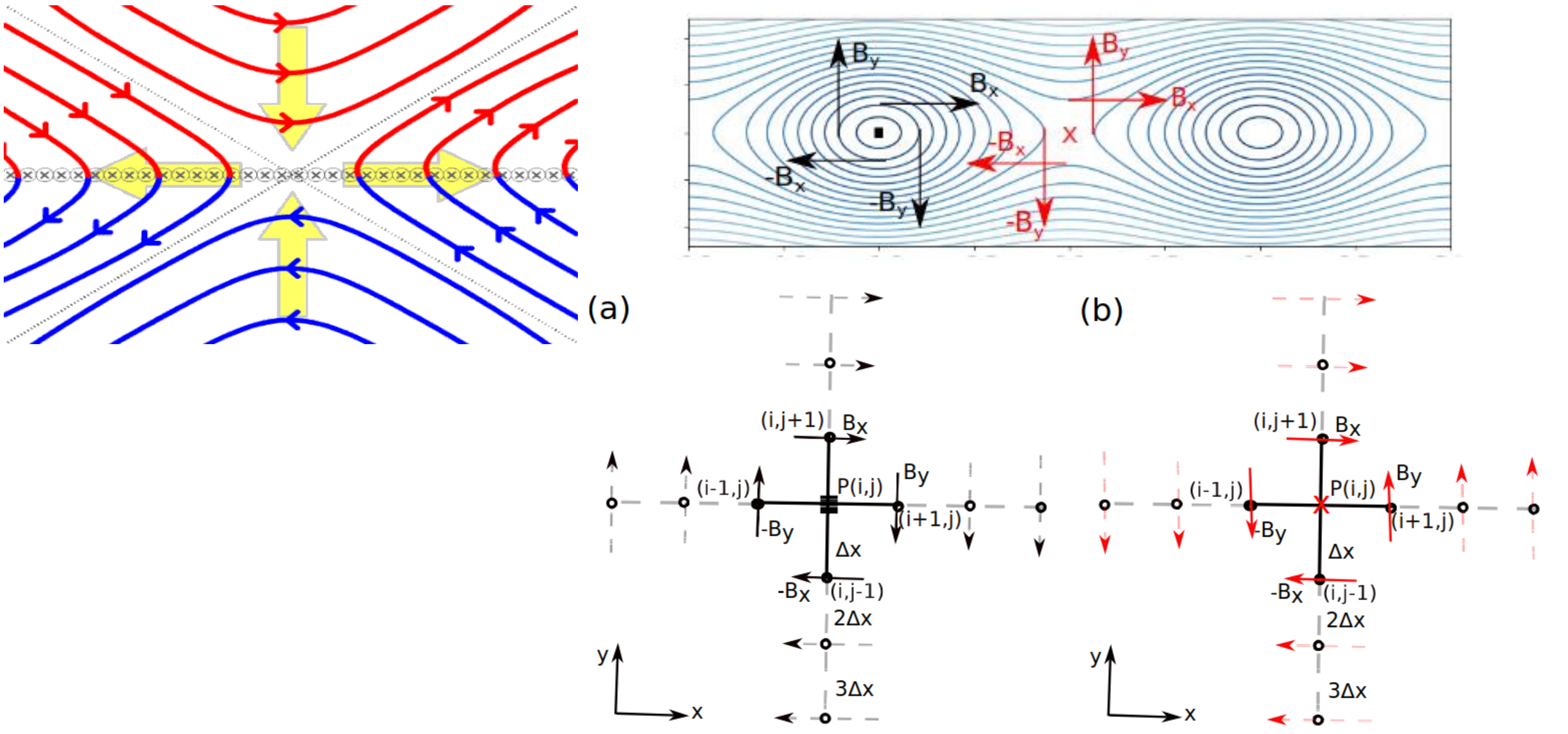






RECONNECTION DETECTION

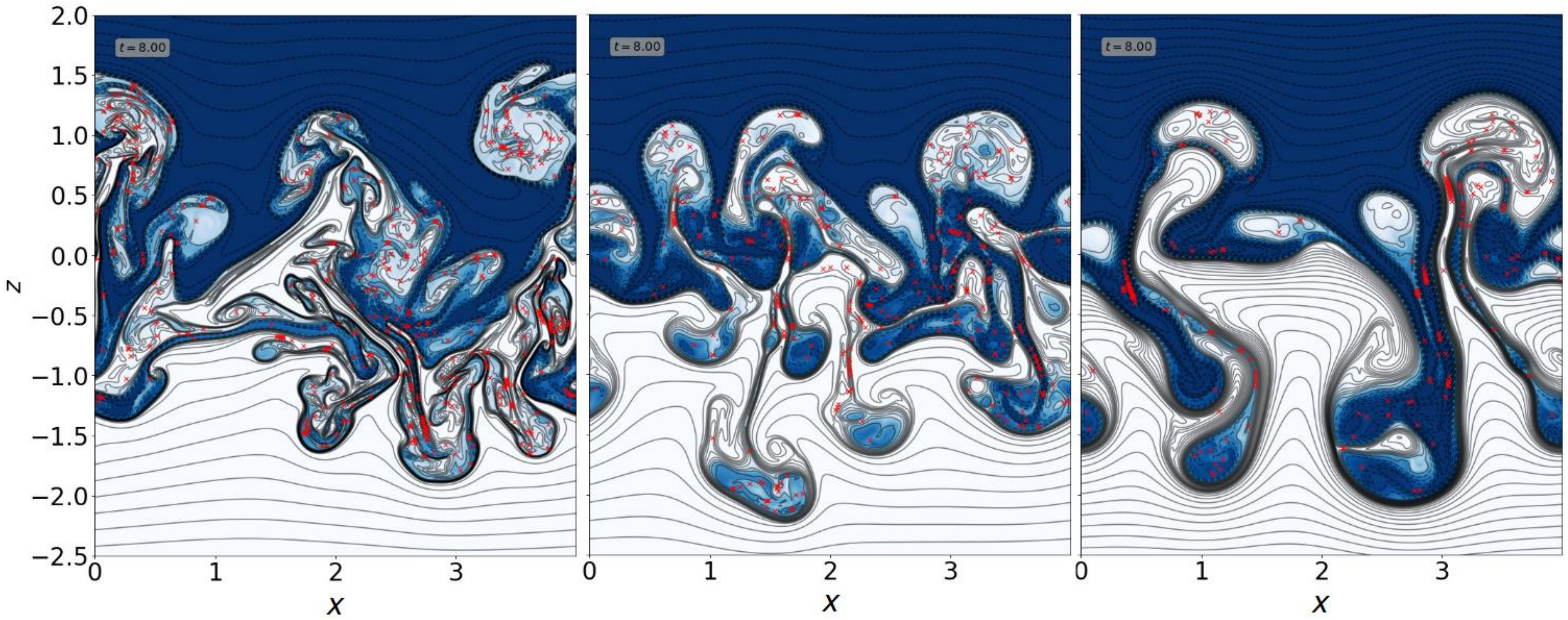
Topology based reconnection detection algorithm



EFFECT OF MAGNETIC FIELD STRENGTH

Why n_x reduces with magnetic field strength?

Magnetic field suppresses small scales => Suppresses turbulence => reconnection events decreases



2D VS. 3D

2D system:

Has either undular or interchange modes.

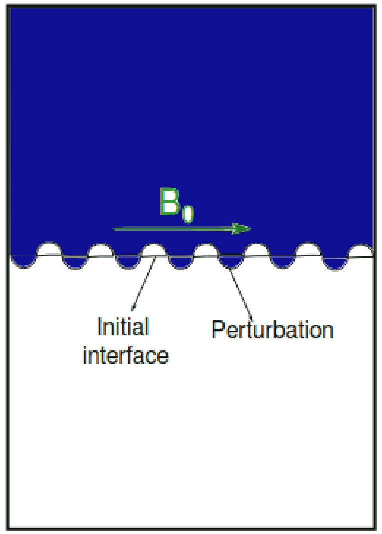
3D system:

Undular ($\theta = 0$, $k_x \neq 0$, $k_y = 0$) +

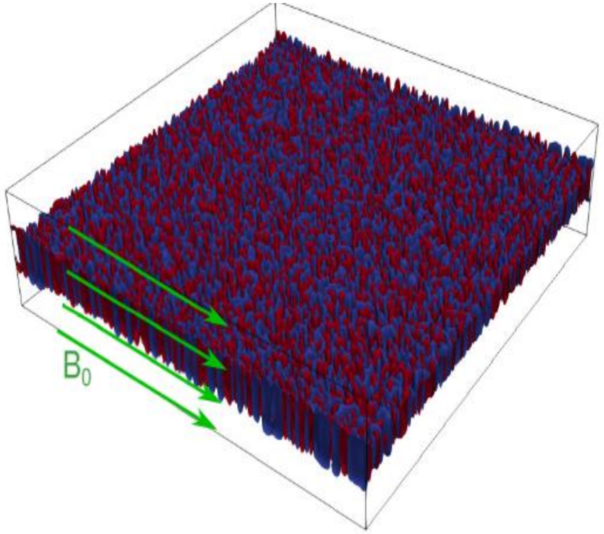
Interchange ($\theta = \pi/2$, $k_x = 0$, $k_y \neq 0$) +

Mixed modes ($0 < \theta < \pi/2$, $k_x \neq 0$, $k_y \neq 0$).

Mixed modes has a lot more energy than undular/interchange modes



2D



3D

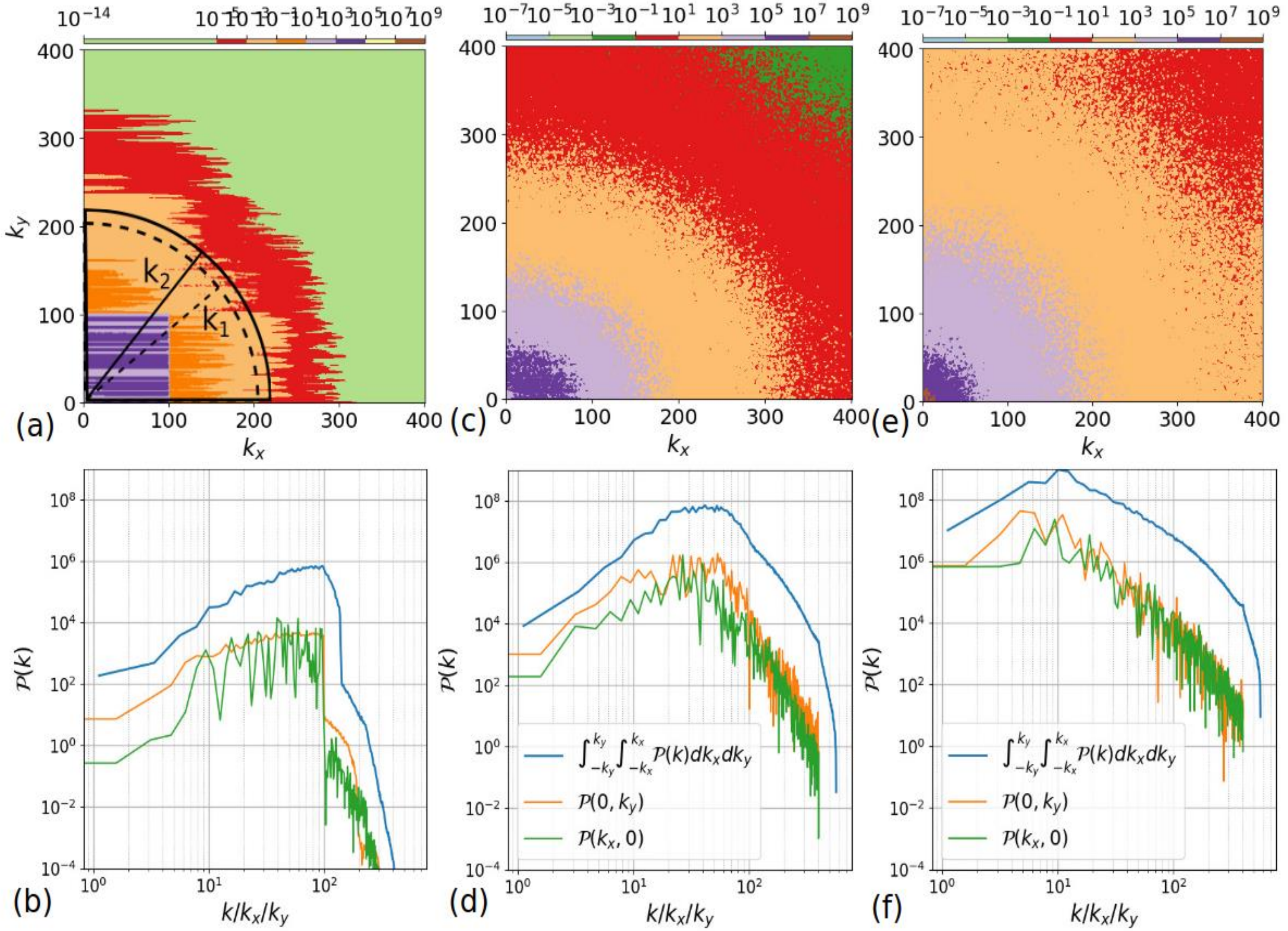
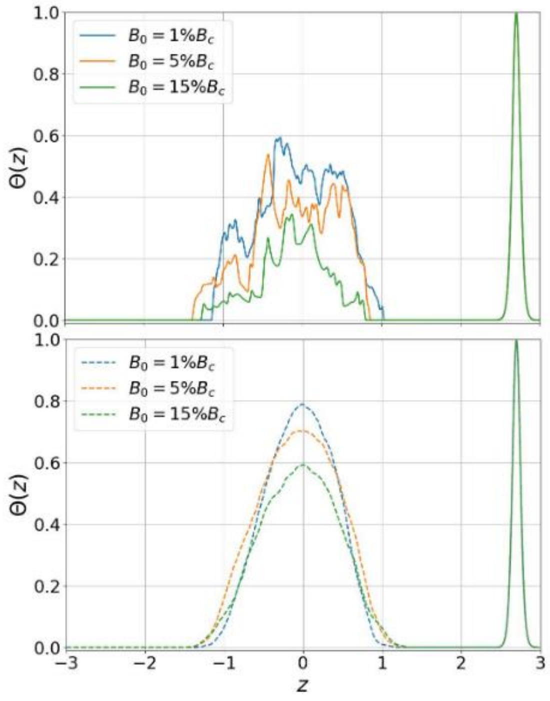


Figure 4. Figure showing: *(top)* first quadrant of 2D contour of power spectrum and *(bottom)* the energy spectrum plot of all wave modes, interchange modes and undular modes at different time instants: $t = 0.2$ *(left)*; $t = 2$ *(centre)*; $t = 6$ *(right)*. The total energy spectrum is obtained by binning the k into 250 bins and summing the energy in across all kx and ky in each bin. The data used to plot the energy spectrum is vertical velocity at the mid z plane.

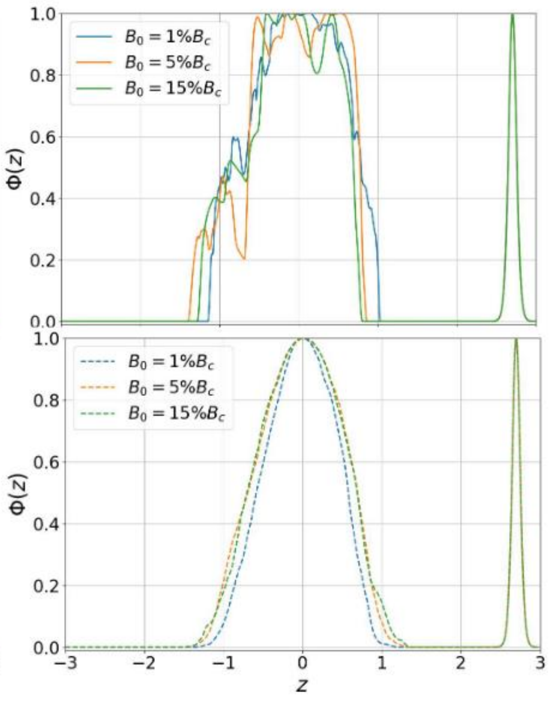
MIXING LAYER IN 2D AND 3D

- ***2D:*** Higher dispersion, Lower mixing, Large scale structures
- ***3D:*** Lower dispersion, Higher mixing, Small scale structures

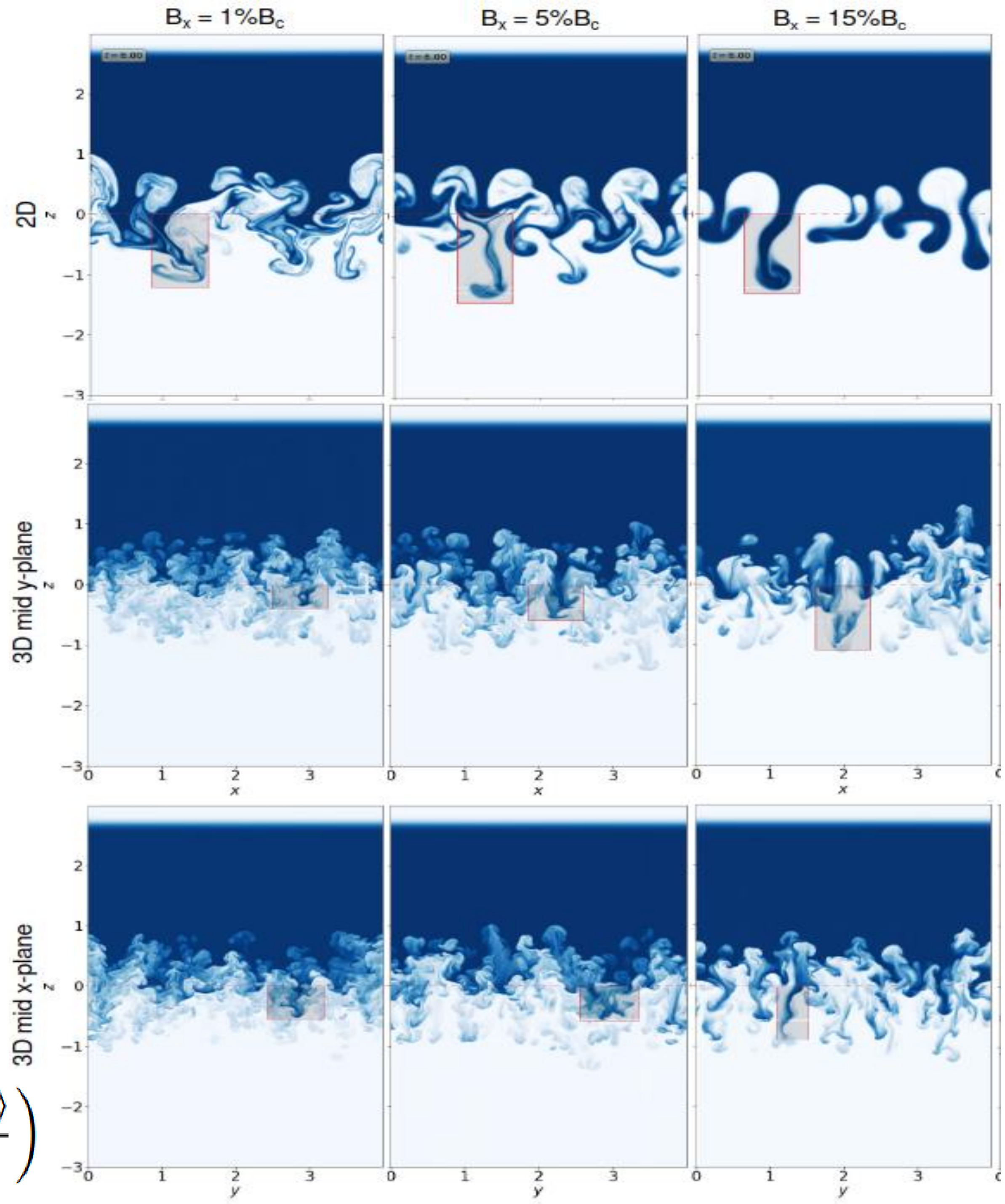
Mixing



Dispersion

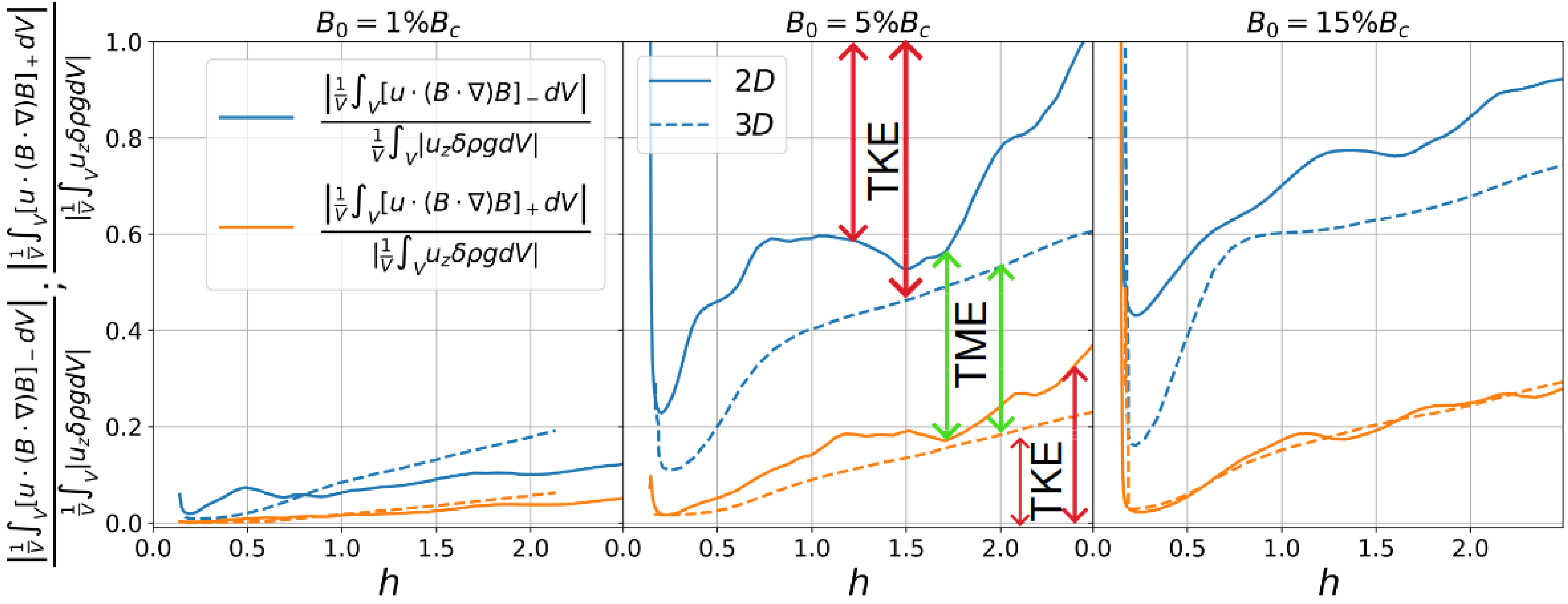


$$\Theta = 4\left\langle \left(\frac{\rho - \rho_l}{\rho_h - \rho_l}\right)\left(\frac{\rho_h - \rho}{\rho_h - \rho_l}\right)\right\rangle \quad \Phi = 4\left(\frac{\langle\rho\rangle - \rho_l}{\rho_h - \rho_l}\right)\left(\frac{\rho_h - \langle\rho\rangle}{\rho_h - \rho_l}\right)$$



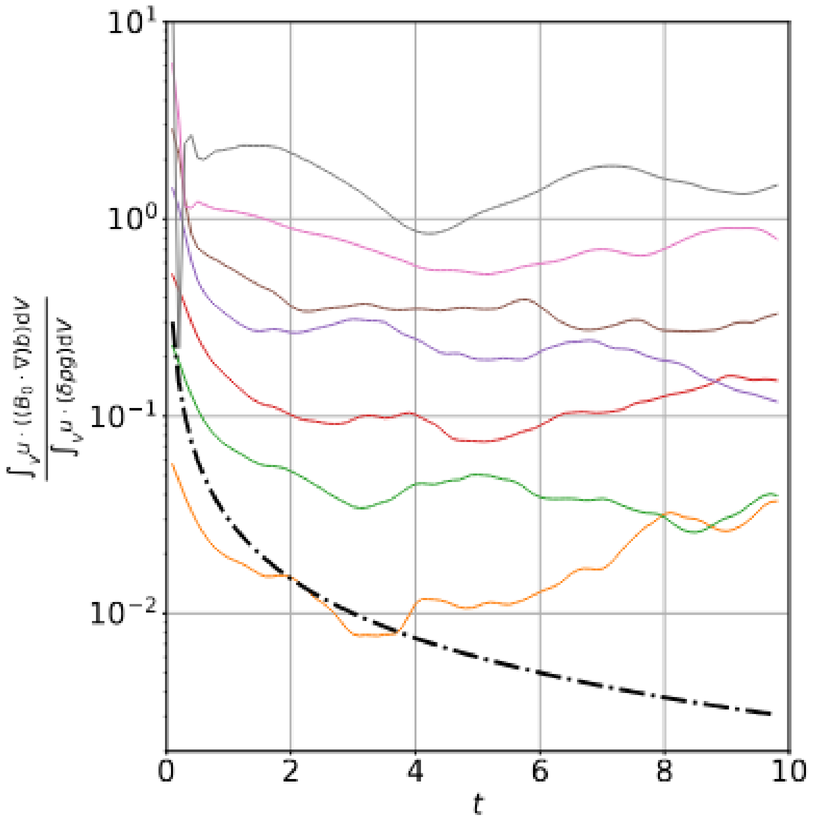
ENERGY DYNAMICS BETWEEN 2D AND 3D

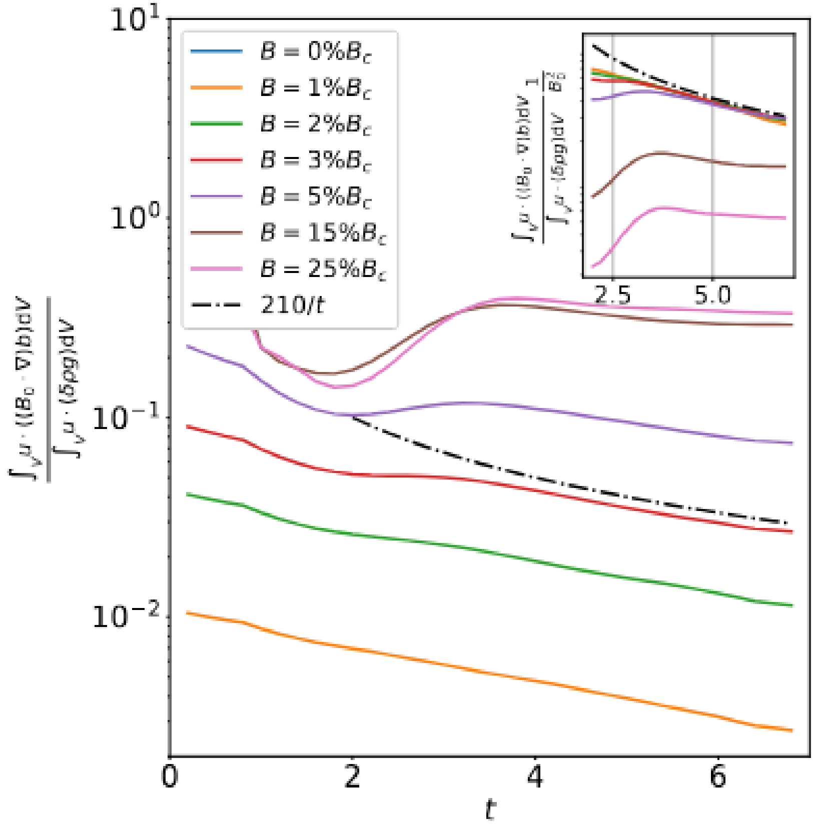
- Energy required to bend magnetic field lines (blue line) is higher in 2D. In 2D magnetic field lines ***"must"*** be bent. In 3D, magnetic field lines ***"can be displaced"***.
- Energy transfer from magnetic to kinetic energy is similar in 2D and 3D.



SELF SIMILARITY: 2D VS. 3D

- In 3D system, magnetic field strength is dominated by other non-linear dynamics over time and the system approaches towards self-similarity
- 2D system, magnetic field always plays a dominant role, self-similar **may** never be achieved

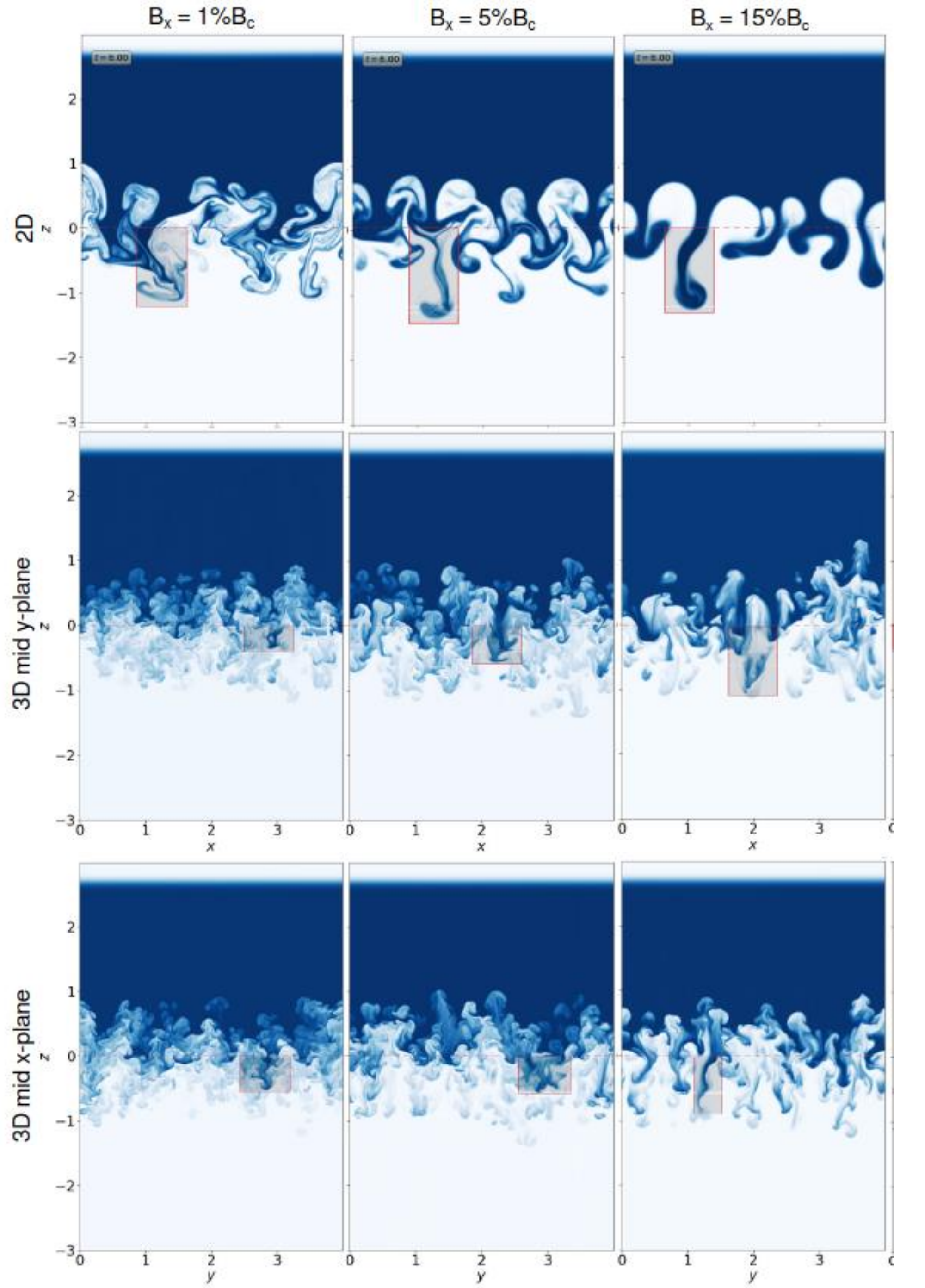




2D VS. 3D

	3D	2D
Dispersion		More
Mixing	More	
Stirring	More	
Released GPE		More
Energy to bend B lines		More
ME to KE	Equal	Equal
Energy dissipation	More	
TKE, TME		More
Energy anisotropy	More	

- 2D MRTI differs significantly from the 3D
- 2D simulations might be able to reflect phenomena like reconnection which help energy transfer from ME to KE.



THANK YOU

LET'S DISCUSS YOUR QUIERIES AND COMMENTS

Feel free to connect, ask queries or collaborate:

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