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Asymmetric diffusion and charged particles in turbulence*

Andrey Beresnyak[1]

[1] *Plasma Physics Division, Naval Research Laboratory, Washington, DC 20375, USA*

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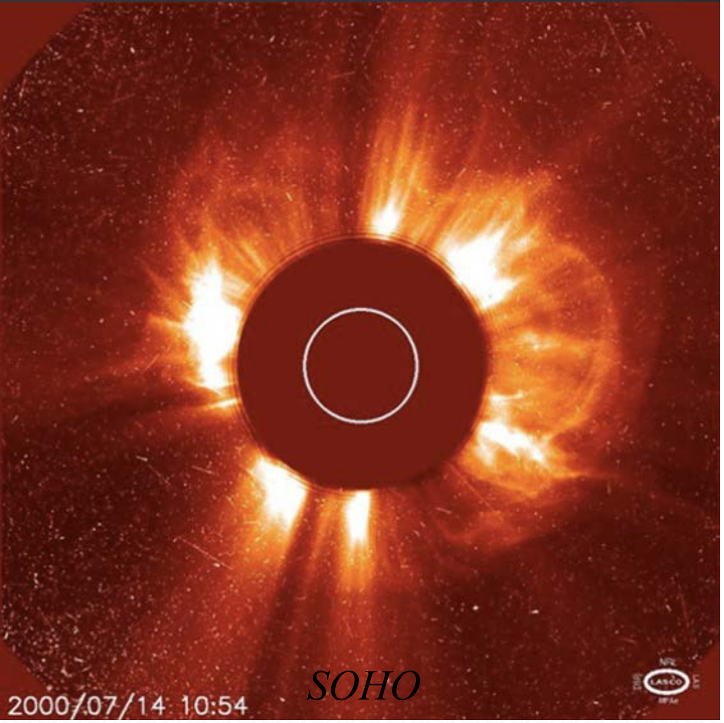
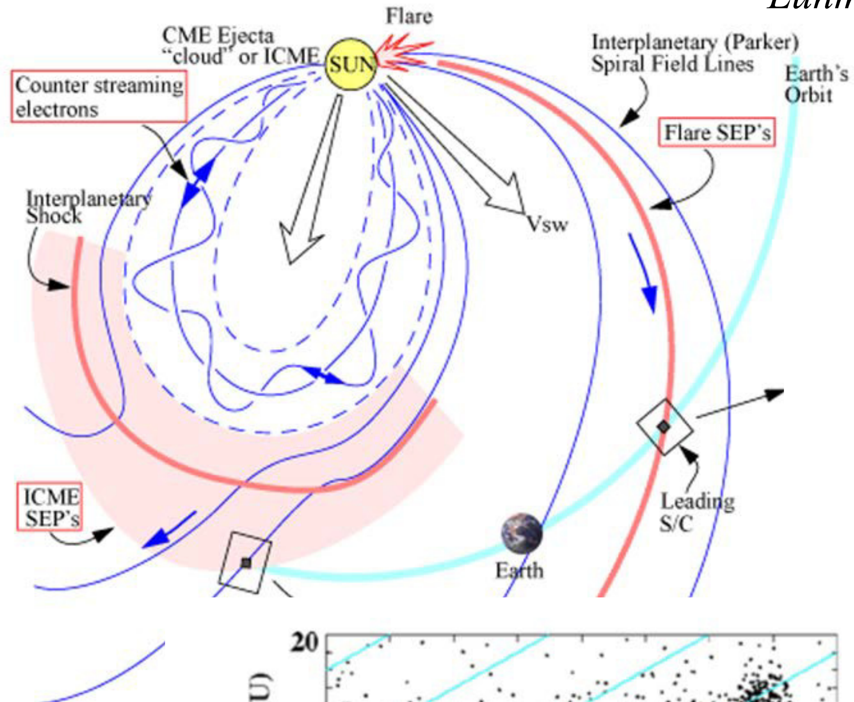
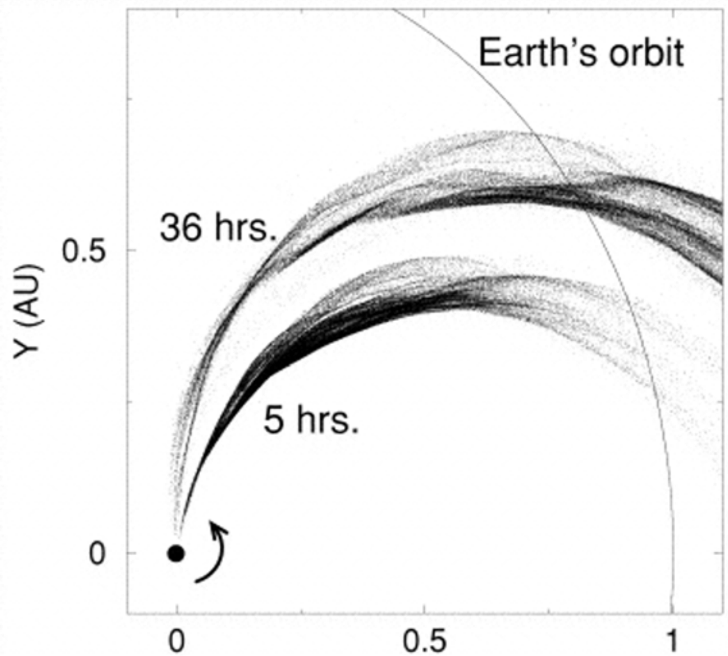
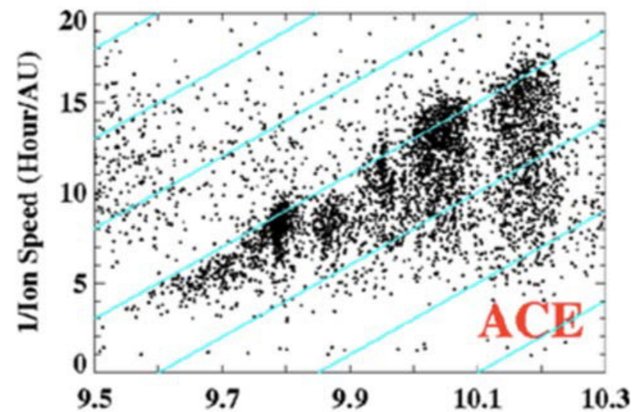
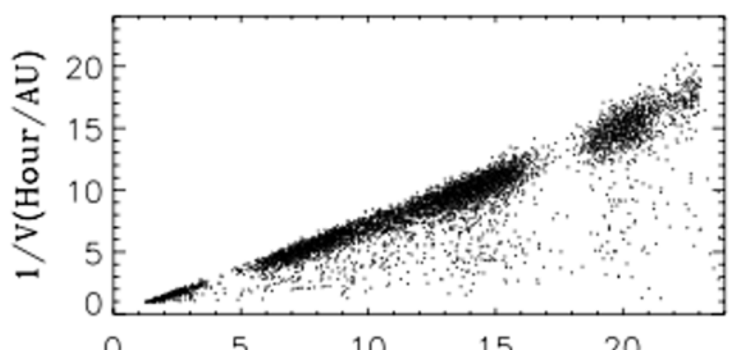
Luhman et al 2008







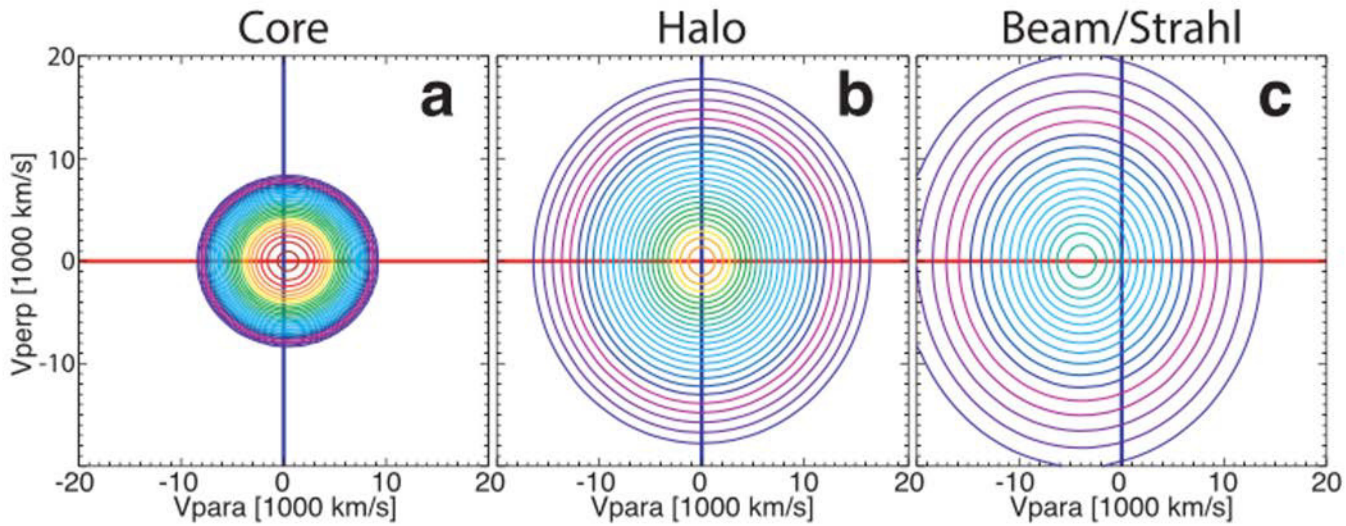




Solar particles from WIND data (3DP)

NRL PPD

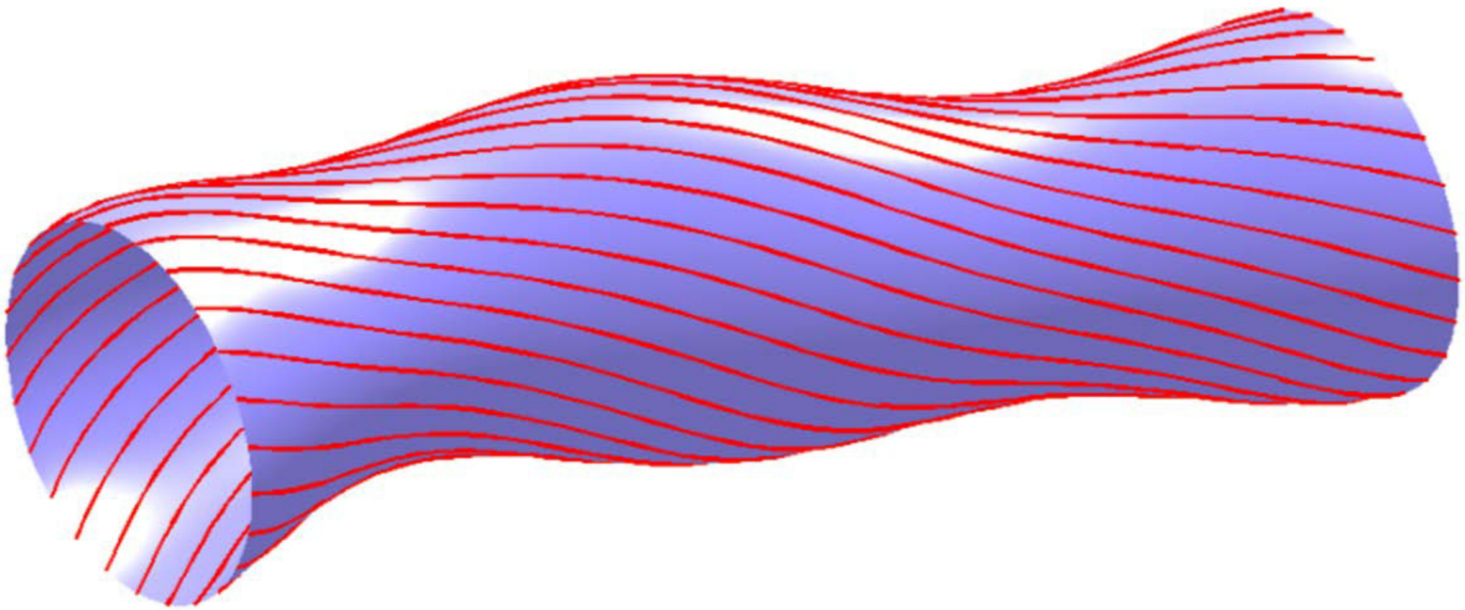
L. Wilson et al 2019



B

Magnetic flux surfaces (“flux ropes”, etc)

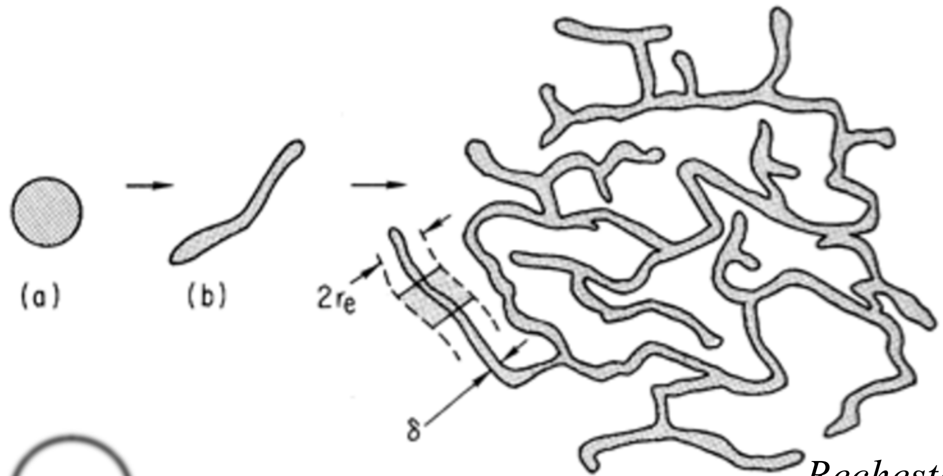
A concept born out of flux conservation $\nabla \cdot \mathbf{B} = 0$. These were expected to magn



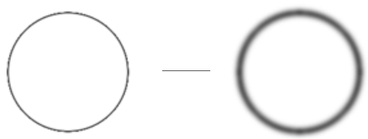
Theory of “classic transport”: relies on collisions, predicts that magnetic confinement is ex

Practice: transport is much larger than predicted.

Destroyed magnetic surfaces

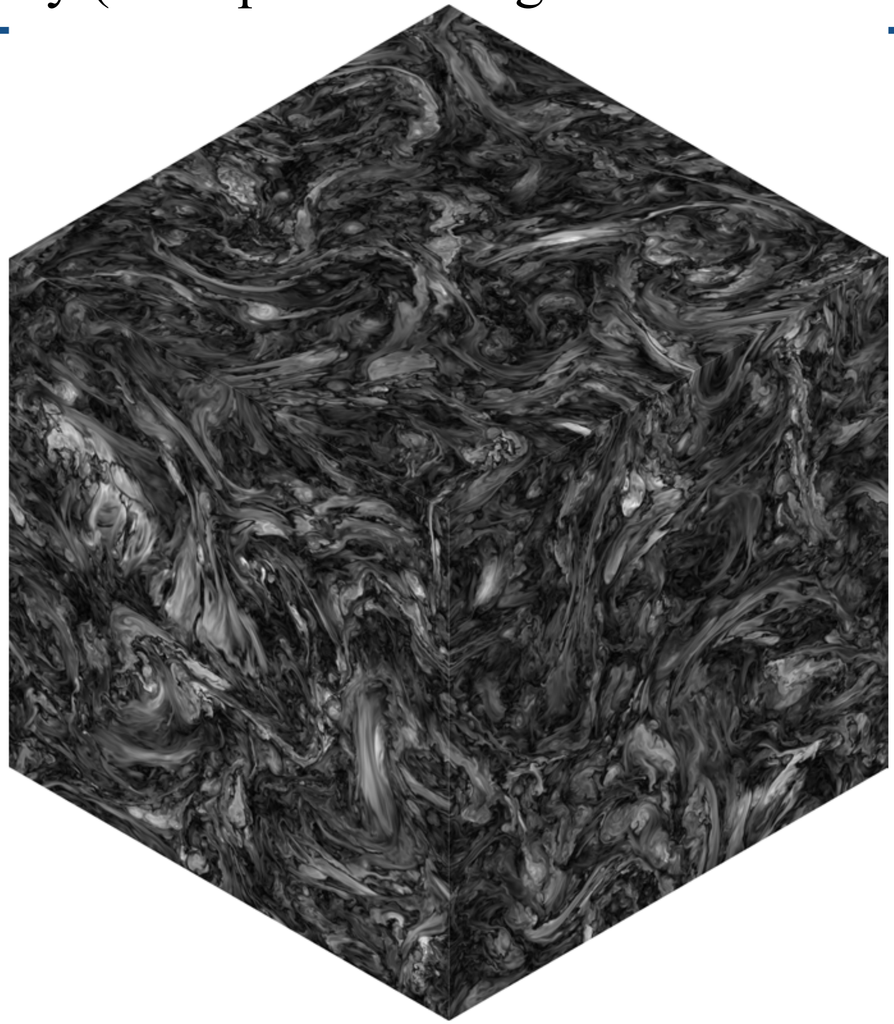


Rechester & Rosenbluth (1978)



1) analogy with classical mechanics: how conservative systems increase their en
2) separation of field lines is <u>not restricted</u> by flux conservation.
3) this separation causes mixing that <u>does not depend on collisions or gyro-radius</u>

DNS – direct numerical simulations. Solve dynamical equations with prescribed accuracy (no dependence e.g. on numerical scheme, etc)

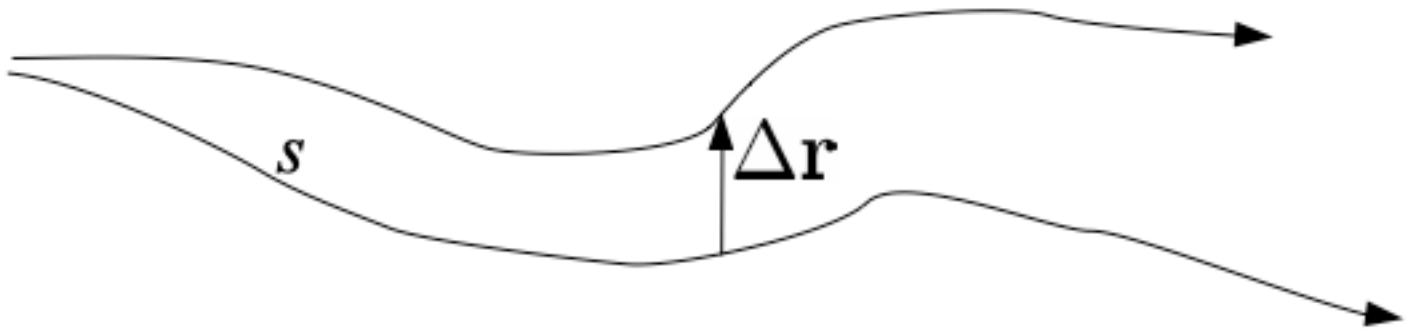


Datacubes up to 4096^3, spectra, SFs and Python scripts are public:
https://github.com/beresnyak/rmhd_large

Magnetic field line wandering

$$d\mathbf{r}_1/ds = \mathbf{B}_1/B_0$$

$$d\mathbf{r}_2/ds = \mathbf{B}_2/B_0$$



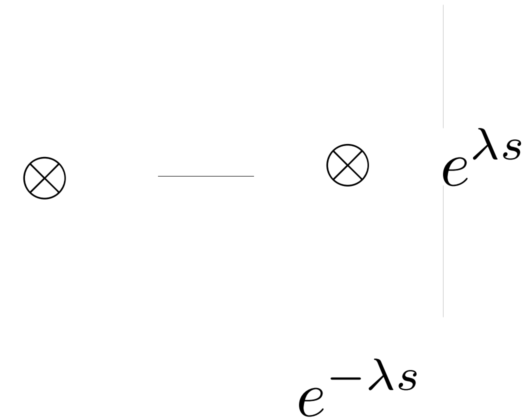
$$\langle(\Delta\mathbf{r})^2\rangle = f(s)$$

Case 1: small separations

$$d\mathbf{x}_{1,2}/ds = \mathbf{B}_{1,2}/B_0$$

$$d\Delta\mathbf{x}/ds = \Delta\mathbf{B}/B_0$$

Taylor-expand **B:** $\frac{1}{B_0}\frac{\partial B_i}{\partial x_j}$ $\begin{pmatrix} -\lambda & 0 \\ 0 & \lambda \end{pmatrix}$



$$\langle \Delta x^2 \rangle \sim \exp(2\lambda|s|)$$

$< \Delta\mathbf{x^2} >$ symmetric with respect to the sign of s

Result: **exponential** separation at small distances is **exactly symmetric**

Case 2: large separations

$$d\mathbf{r}_{1,2}/ds = \mathbf{B}_{1,2}/B_0$$

$\Delta\mathbf{r}$ experiences random walk, symmetric

$$\langle \Delta r^2 \rangle = \frac{B_{\mathrm{rms}}|s|}{LB_0}$$

Result: **diffusive** separation at large distances is **exactly symmetric**

Case 3: inertial range separations

$$d\mathbf{r}/ds = \mathbf{B}_r/B_0$$

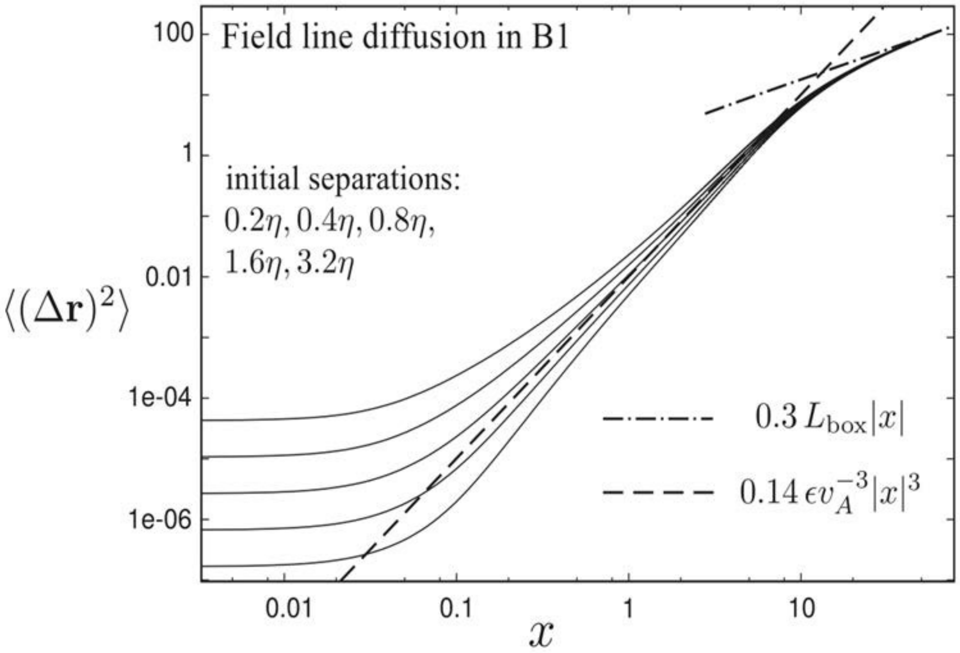
assume $B_{r\|} =< \hat{\mathbf{r}} \cdot (\mathbf{B}(\mathbf{x}-\mathbf{r}) - \mathbf{B}(\mathbf{x})) >_x = C_\| r^{1/3}$

Assumption: random Markovian process

$$dr^2/ds = 2rC_\| r^{1/3}/B_0$$

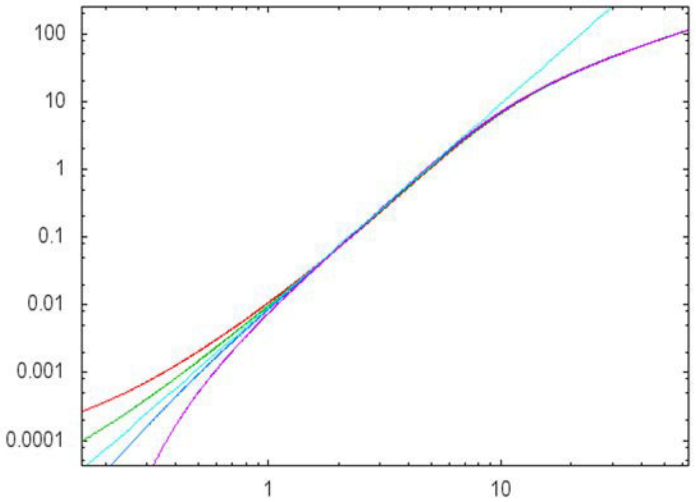
$$\langle r^2 \rangle = \frac{8}{27} C_\|^3 \frac{|s|^3}{B_0^3}$$

Simulation results



$$\langle \Delta r^2 \rangle =?\ \frac{8}{27} C_{\parallel}^3 \frac{|x|^3}{B_0^3}$$

Good collapse to x^3 law



Diffusion of particles in turbulence
Richardson's diffusion (1926)

Two-particle separation

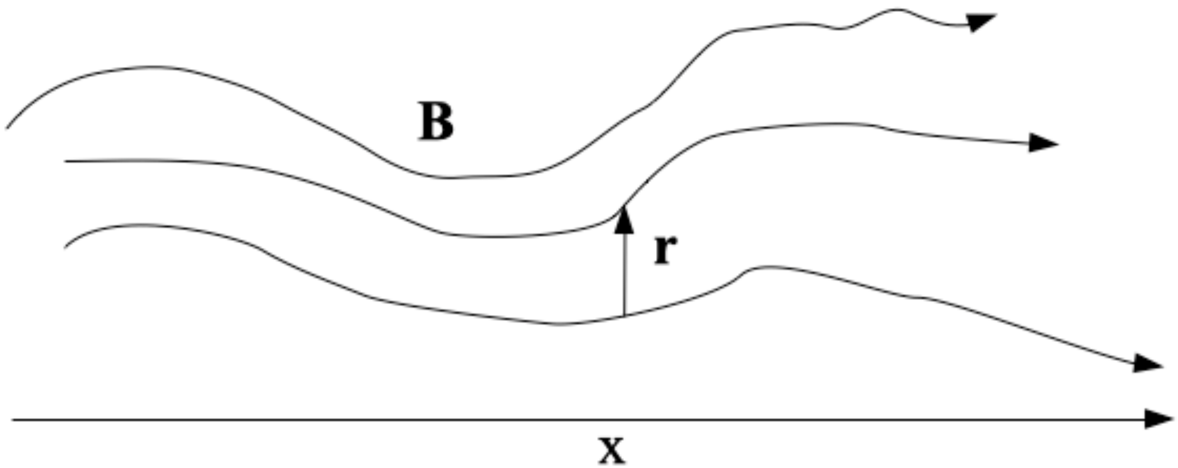
$$D \sim l^{4/3}$$

$$\langle (\Delta \mathbf{r})^2 \rangle = g_0 \epsilon t^3$$

ϵ – dissipation rate per unit mass – cm^2/s^3

g_0 – dimensionless number, "Richardson constant"

Turbulent separation of magnetic field lines

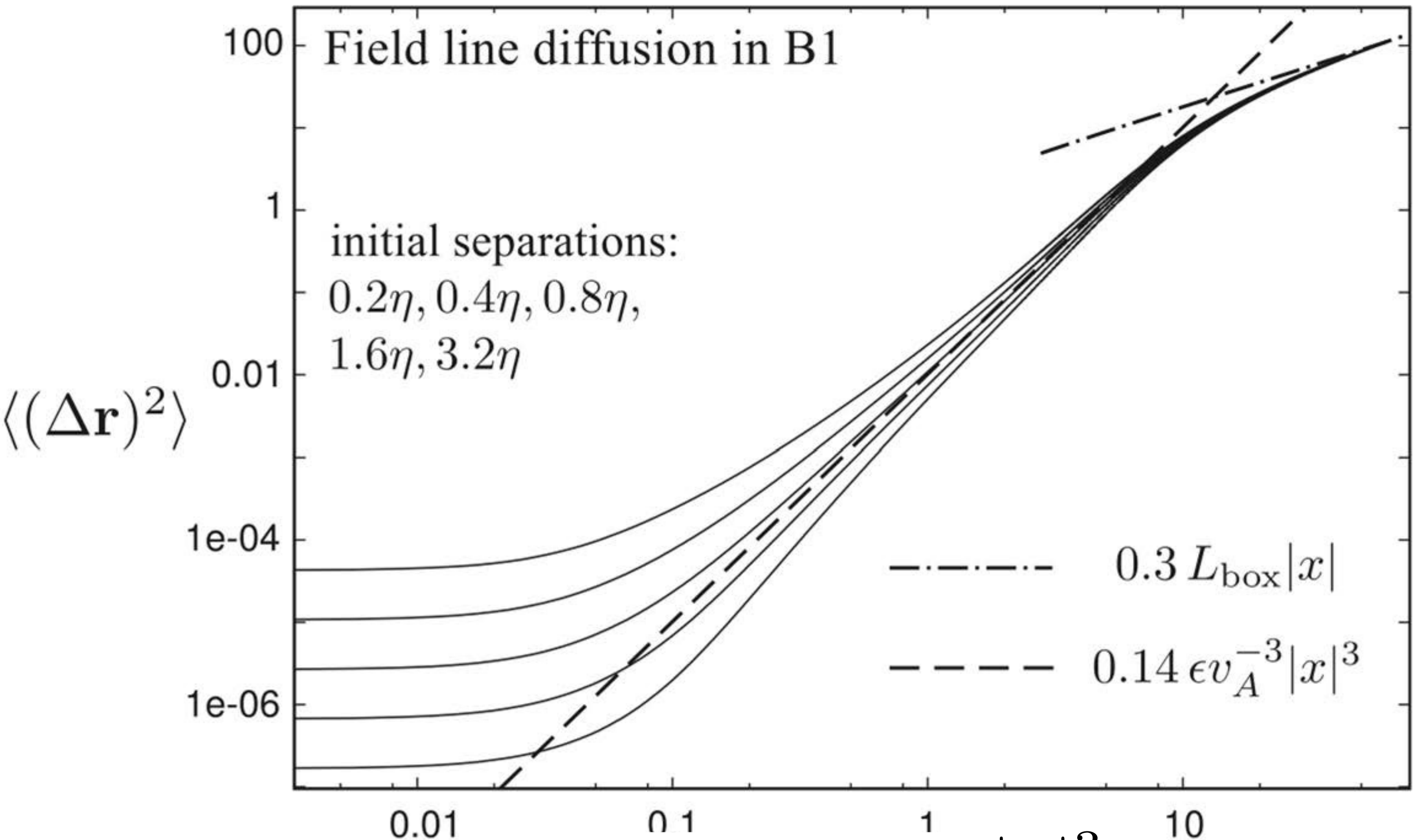


$$\langle(\Delta\mathbf{r})^2\rangle = g_m \epsilon v_A^{-3} |x|^3$$

"Richardson-Alfven" diffusion

(note: must satisfy **exact** Alfven symmetry, i.e. *x* only enters as x/v_A)

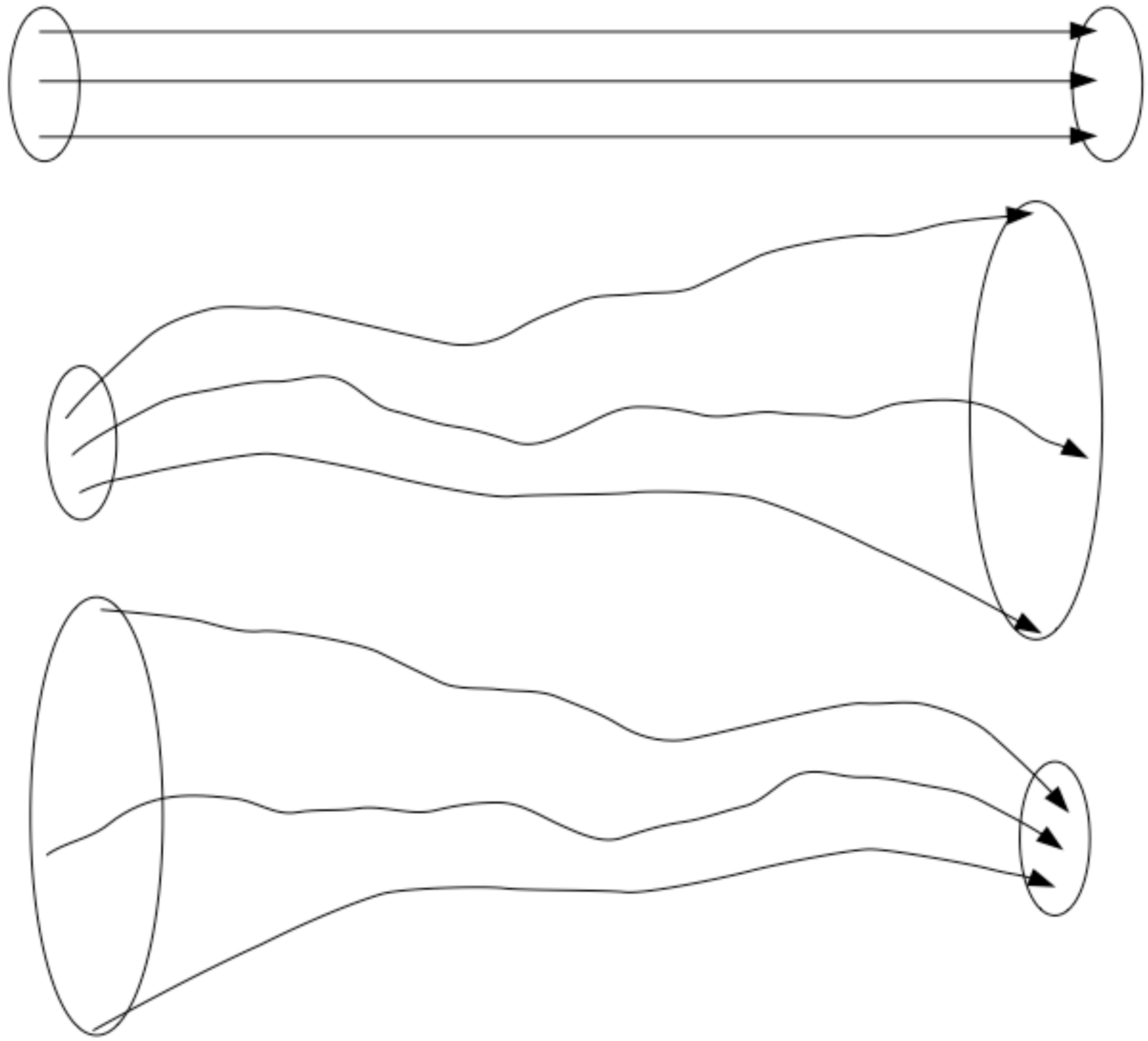
Diffusion in simulated turbulence



$$\langle \Delta r^2 \rangle =? \; \frac{8}{27} C_\parallel^3 \frac{|x|^3}{B_0^3}$$

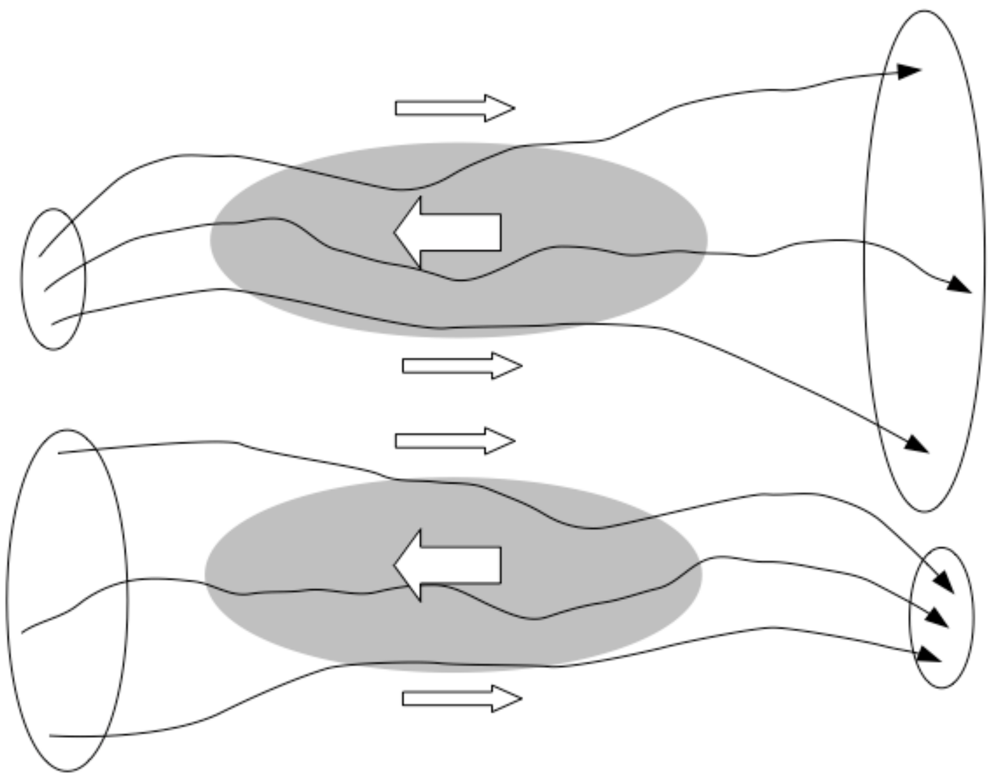
$$\langle (\Delta \mathbf{r})^2 \rangle = g_m \epsilon v_A^{-3} |x|^3$$

Diffusion along the **B** direction and backward



Why would you expect diffusion to be symmetric?

Asymmetric diffusion will turn Maxwellian distribution into non-Maxwellian.



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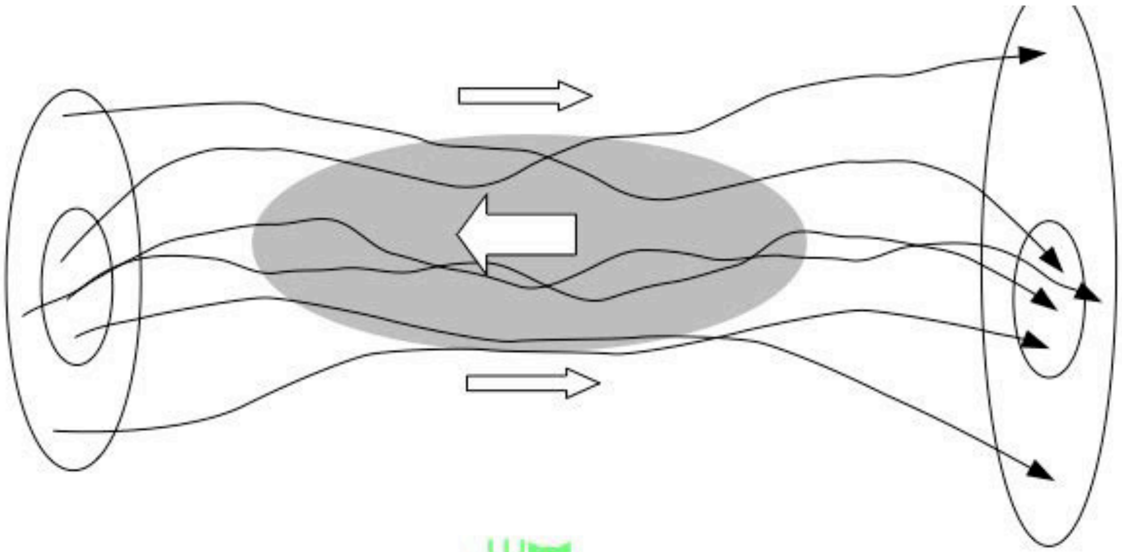
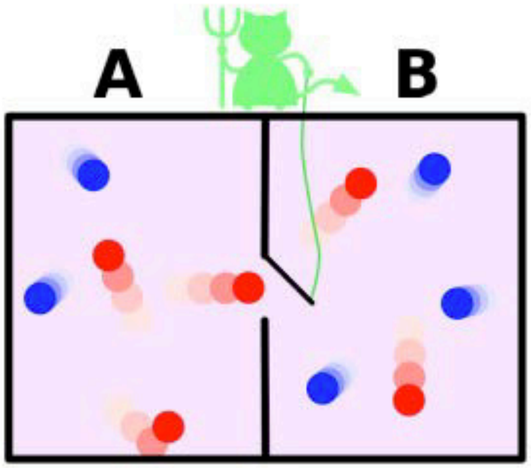
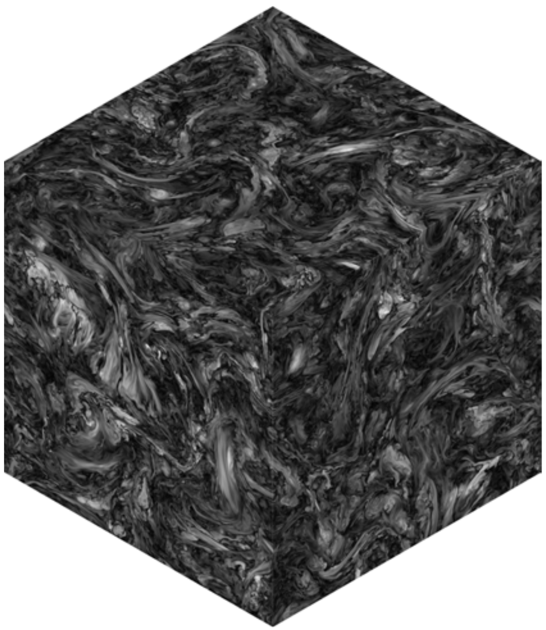
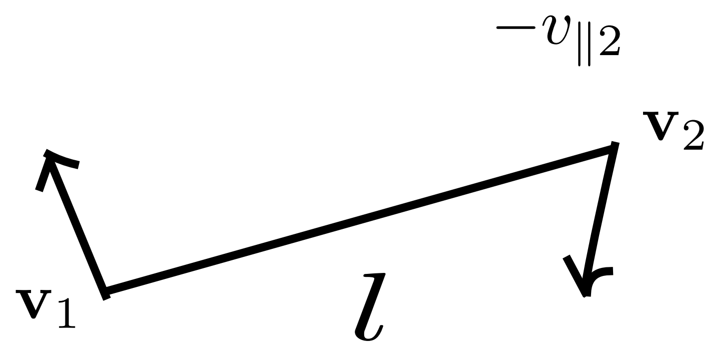




Turbulence is non-time reversible





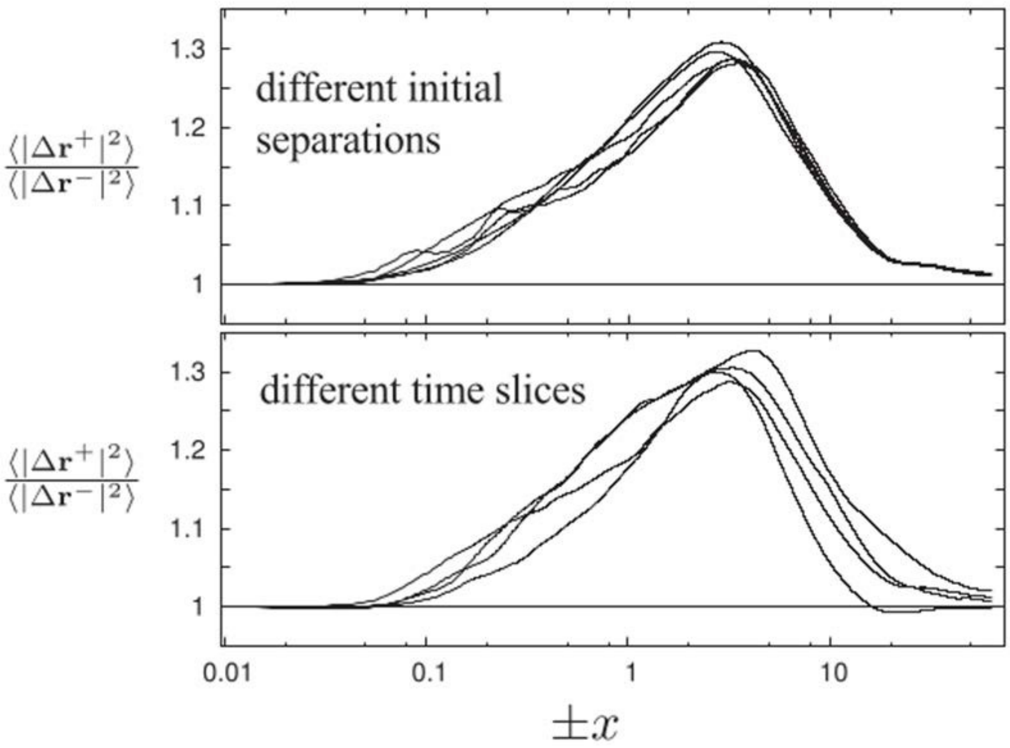
$$\delta v_{\parallel} = v_{\parallel 2} - v_{\parallel 1}$$

$$\langle \delta v_{\parallel l}^3 \rangle = -\frac{4}{5}\epsilon l$$ Kolmogorov -4/5 law

Backward in time: $$\langle \delta v_{\parallel l}^3 \rangle = +\frac{4}{5}\epsilon l$$

Empirical measurement of diffusion of magnetic field lines

Forward and backward:



Imbalanced simulation with ratio of fluxes

$$\epsilon^+/\epsilon^- = 2$$

NRL PPD

Forward and backward are different, why?

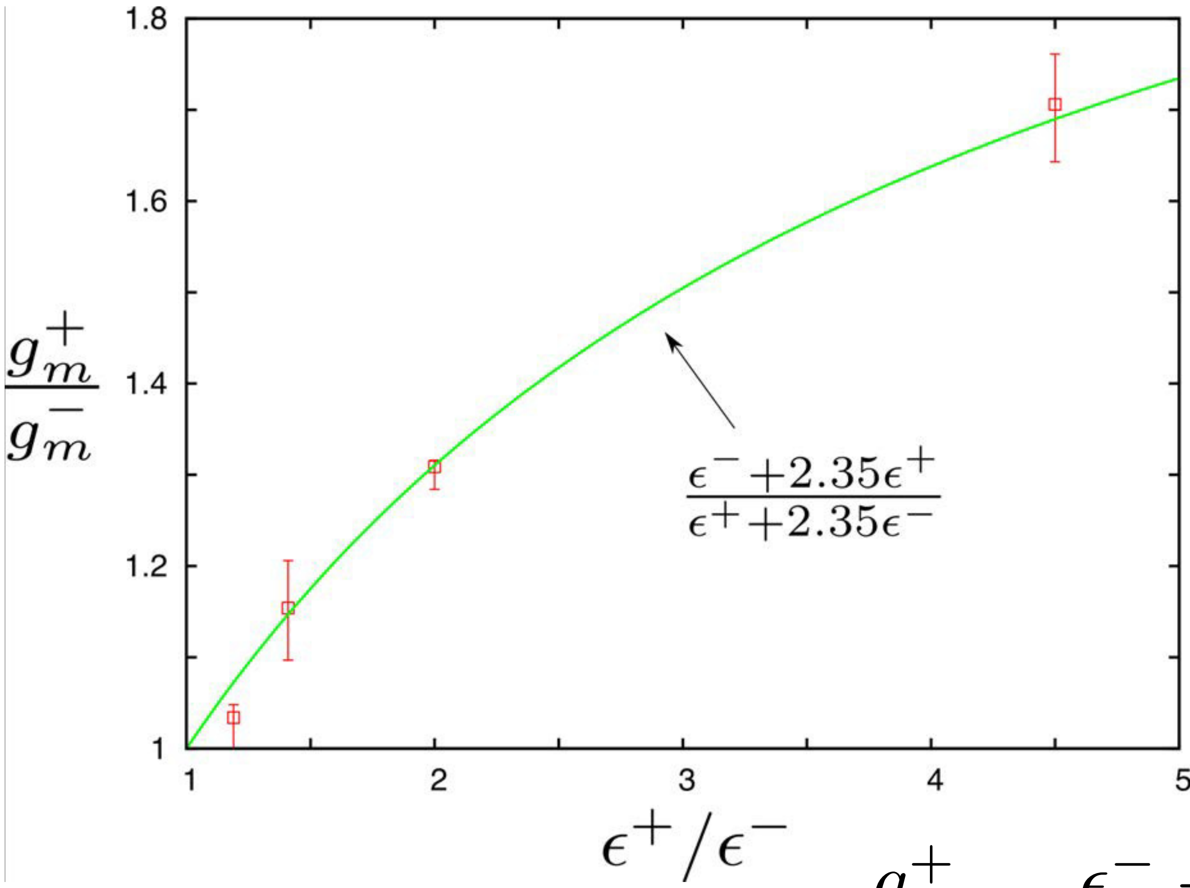
a) **B** consists of two components, $\mathbf{b}^+$ and $\mathbf{b}^-$

b) hydrodynamic Richardson's diffusion is known to be time-asymmetric.

c) b^- propagates along **B** and b^+ and against **B.** When we go along B, we follow evolution of b^- forward in time and b^+ backward in time.

$$\frac{g_m^+}{g_m^-} = \frac{\epsilon^- + a_m \epsilon^+}{a_m \epsilon^- + \epsilon^+}$$

Empirical measurement of diffusion of magnetic field lines



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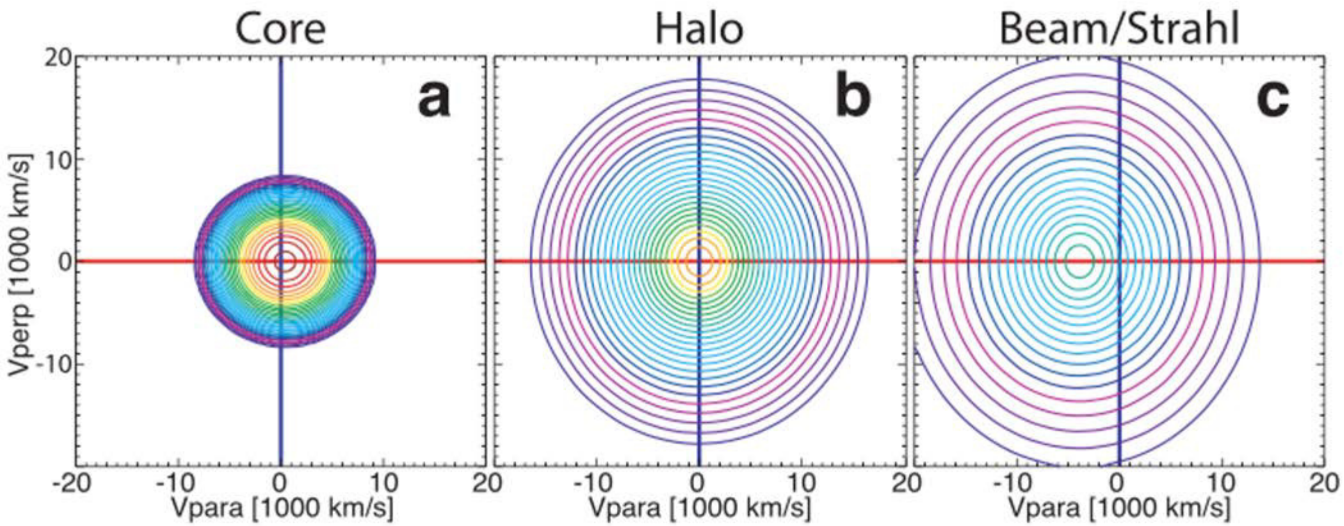
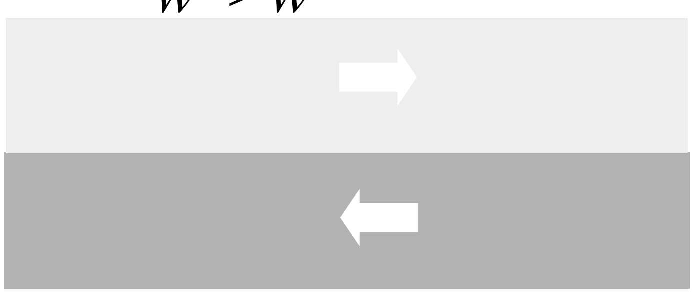


B

$w^+ > w^-$



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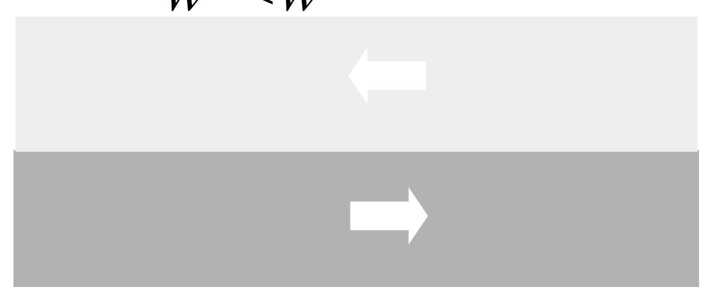

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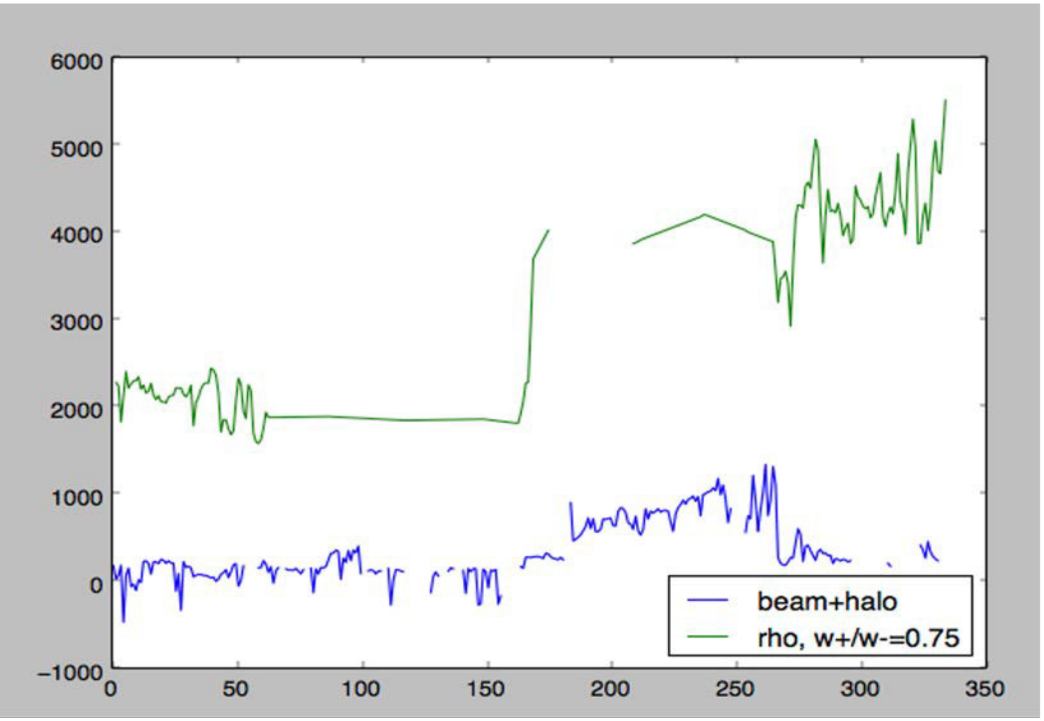

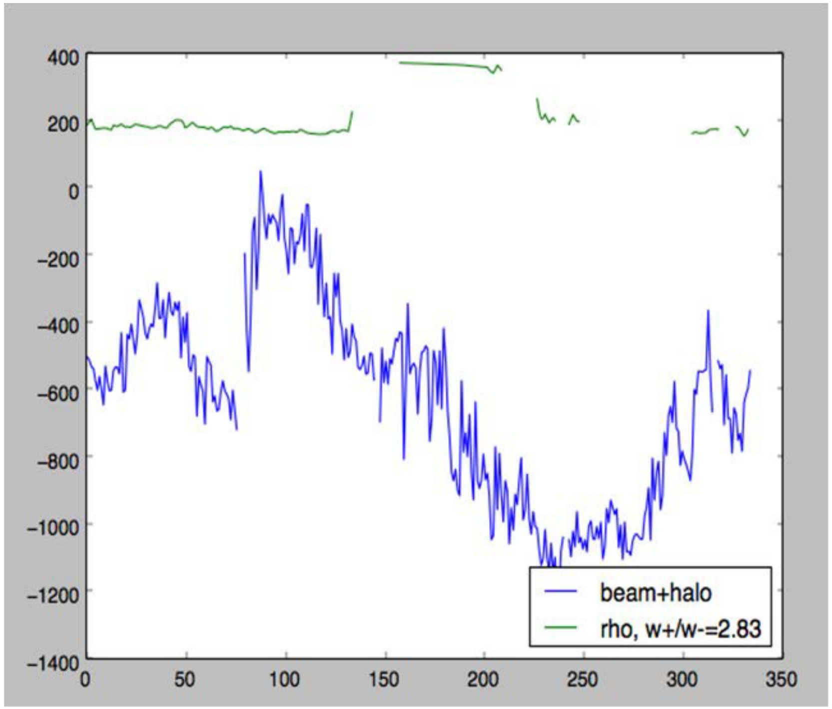

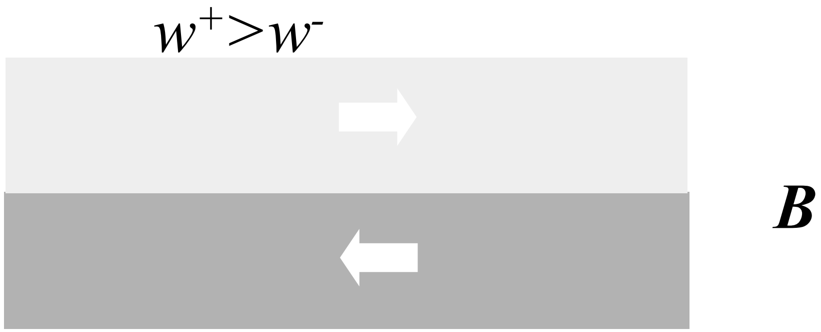

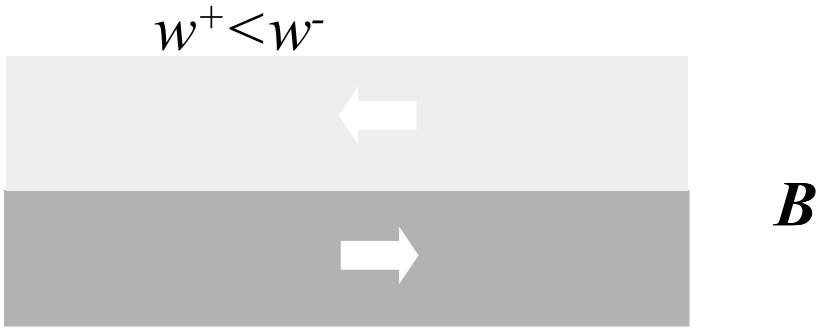

Conclusion

NRL PPD

- Asymmetric diffusion of magnetic field lines is an interesting geometrical effect present in imbalanced turbulent systems.

- It was observed in MHD fluid and plasma PIC simulations.

- This effect can create free energy in particles at all scales of turbulent subrange. This free energy is not “free” as it comes from essential non-reversibility of turbulence and a finite dissipation rate per unit mass (in synthetic time-symmetric random fields it is absent).

- The effect of back-reaction of particles on the field will be a subject of future work.

- I am studying asymmetric diffusion in the solar wind, which is often imbalanced and has high perpendicular density gradients.