Third-Order Law in MHD Turbulence, Examining Different Dissipation Mechanisms

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Introduction

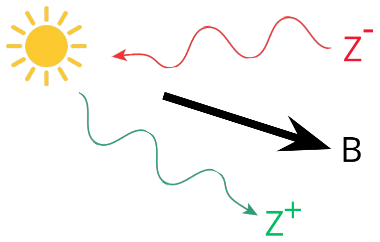
- We study the energy cascade in Alfvénic solar wind turbulence under non-ideal MHD conditions.

- Key point: viscosity (ν) and resistivity (η) differ, acting at separate scales.

- Using a phenomenological model ($\nu \neq \eta$), we analyze the impact on:
  - Energy balance for viscous-resistive MHD
  - Third-order Yaglom law in Elsässer variables

- Goal: improve interpretation of in-situ solar wind data.

Equation from Previous Work



Starting from the work in *[V. Carbone et al., 2009]*

$$\partial_t \langle |\Delta Z_i^{\pm}|^2 \rangle + \frac{\partial}{\partial r_\alpha} \langle \Delta Z_\alpha^{\mp} |\Delta Z_i^{\pm}|^2 \rangle = 2\nu \frac{\partial^2}{\partial r_\alpha} \langle |\Delta Z_i^{\pm}|^2 \rangle - \frac{4}{3} \frac{\partial}{\partial r_\alpha} (\epsilon_{ii}^{\pm} r_\alpha).$$

$$\epsilon_{ij}^{\pm} = \nu \langle (\partial_\alpha Z_i^{\pm})(\partial_\alpha Z_j^{\pm}) \rangle.$$

They have a quantity that depends only on $\boldsymbol{\nu}$, under the assumption that

$$\lambda^{\pm} = \lambda^{\mp} = \nu. \qquad \lambda^{\pm} = (\nu \pm \mu)/2$$

NEXTSTEP

Analitical Results

We rewrite the MHD equations express in terms of increments in space, and in average.

Under the assumptions of incompressibility and local homogeneity, we obtain the MHD von Kármán–Howarth equation

$$\frac{\partial}{\partial t} < |\Delta \vec{z}^{\pm}|^2 > + \frac{\partial}{\partial r} < \Delta \vec{z}^{\mp} |\Delta \vec{z}^{\pm}|^2 > = \frac{\partial^2}{\partial r^2} \left[(\nu + \eta) < |\Delta \vec{z}^{\pm}|^2 > + (\nu - \eta) < \Delta \vec{z}^{+} \cdot \Delta \vec{z}^{-} > \right] - 2\epsilon$$

- Trasport Term
- Cascade Term (Yaglom)
- Dissipation Term
- Energy Dissipation Rate

$$\epsilon = (\nu + \eta) < |\vec{\nabla} z^{\pm}|^2 > + (\nu - \eta) < (\vec{\nabla} \times z^{\mp}) \cdot (\vec{\nabla} \times z^{\pm}) >$$

Simulations

Geometry: 2D periodic box (x,y) → homogeneous

Domain: Lx=Ly=2π, grid 4096×4096

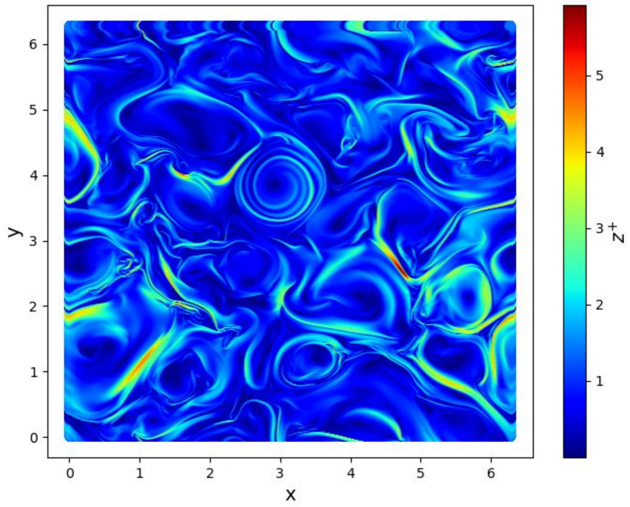
$$Pr_m = \frac{\nu}{\eta}$$

- **Pr_m = 1** (Balanced)**:** $\nu = \eta$
- **Pr_m > 1** (Viscous Domain): $\nu > \eta$
- **Pr_m < 1** (Resistive Domain): $\nu < \eta$

Run	ν	η	**Pr_m**
run1	0.0001	0.0001	1
run2	0.001	0.0001	10
run3	0.0001	0.001	0.1

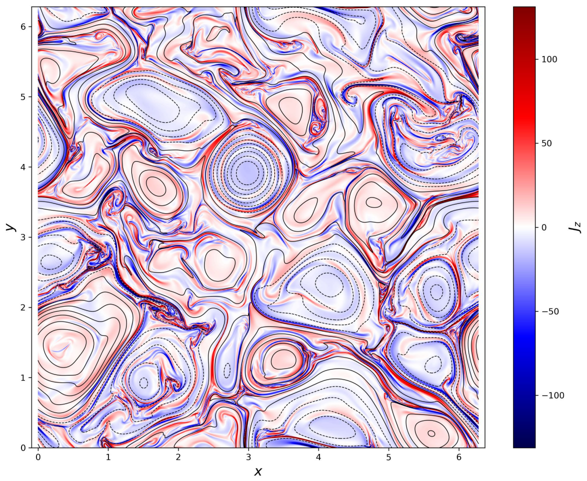
Table 1: Summary of the main simulation runs and their parameters

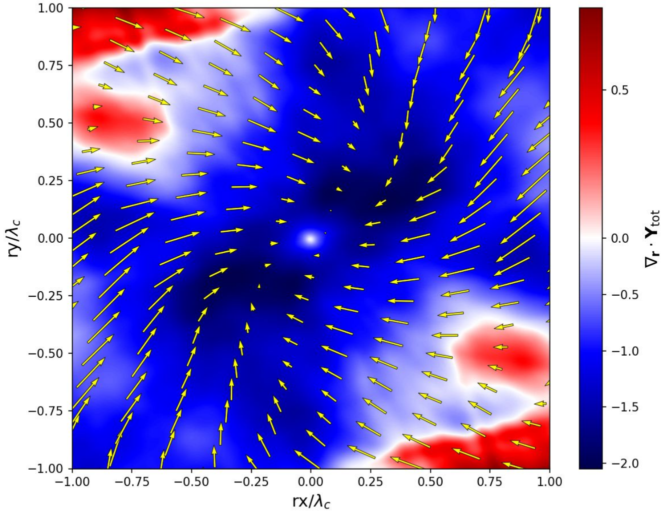
Structures and models



Spatial distribution of the Elsässer variable z, consistent with statistically homogeneous turbulence.

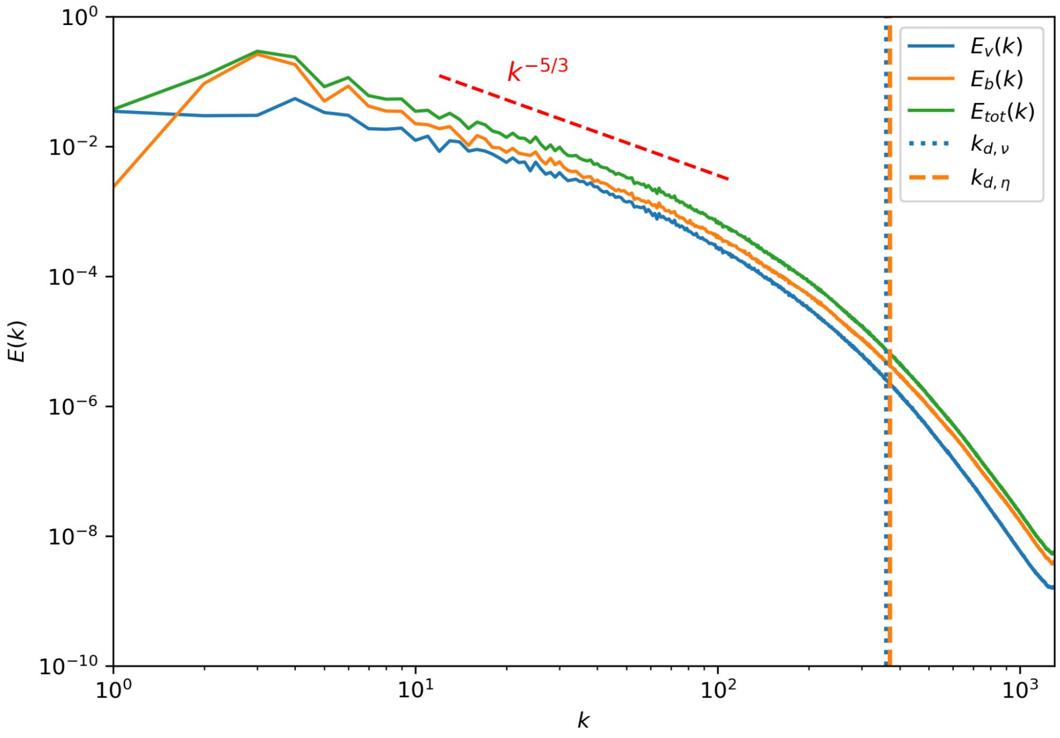
Spatial distribution of J_z with contours of A_z, representing in-plane magnetic field lines and highlighting coherent current structures in MHD turbulence.



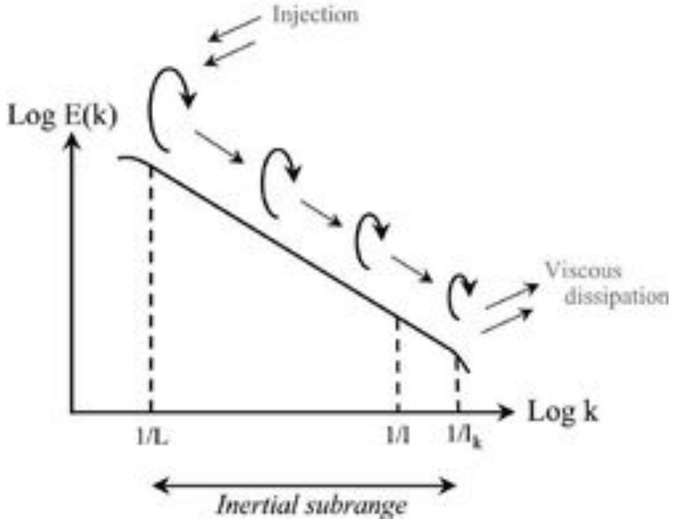


Divergence of the total Yaglom flux and the associated vector field, illustrating the intensity, direction, and anisotropy of energy transfer across scales in MHD turbulence.

Energy Spectrum Pr_m=1

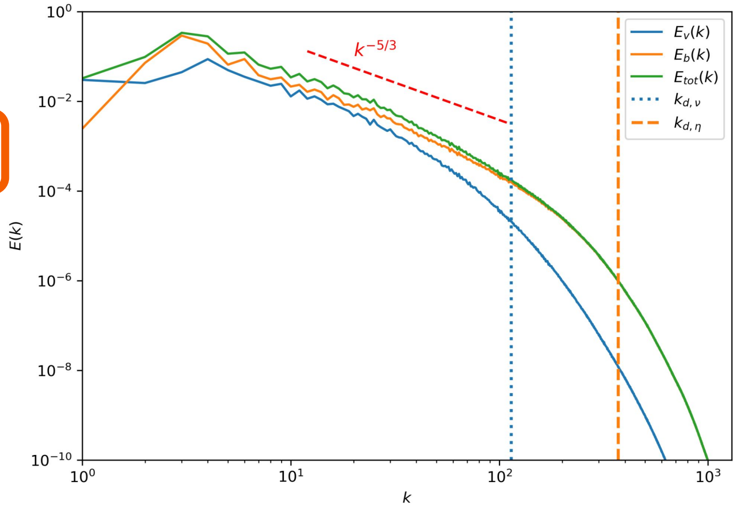


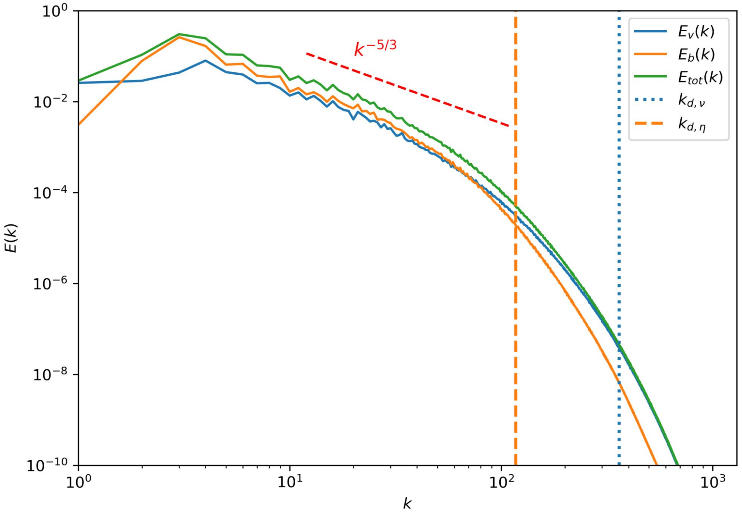
Energy spectrum E(k) as a function of wavenumber k, in logarithmic scale, showing a $k^{-5/3}$ scaling, typical of Kolmogorov inertial-range turbulence.



Energy Spectrum $Pr_m \neq 1$

Pr_m = 10





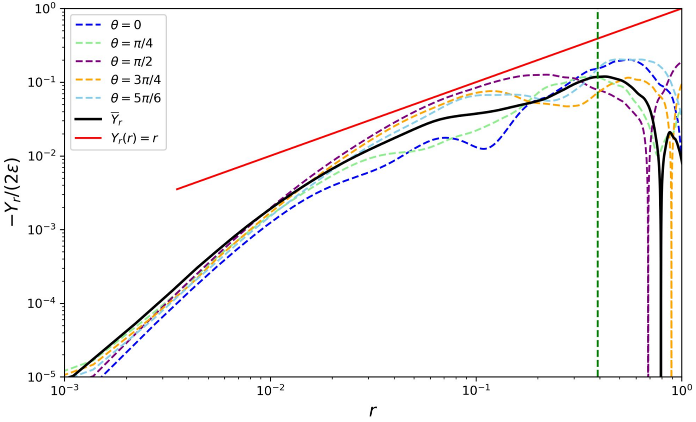
Pr_m = 0.1

- **(Active Dynamo):** Despite the premature viscous decay of velocity, low diffusivity allow the magnetic energy to self-sustain and dominate at the smallest scales.
- **(Quenched Dynamo):** High magnetic diffusivity suppresses the field at large scales. In MHD turbulence, this early magnetic quenching disrupts the wave interactions driving the cascade, prematurely dragging down the velocity field.

Yaglom law

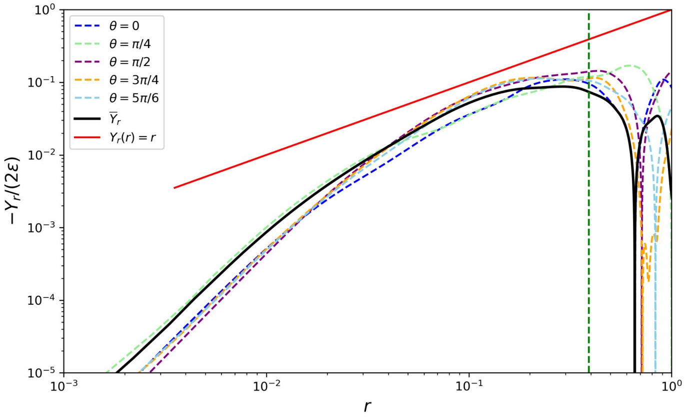
$$\frac{\partial}{\partial r} < \Delta\vec{z}^{\mp}|\Delta\vec{z}^{\pm}|^2 >= -2\epsilon$$

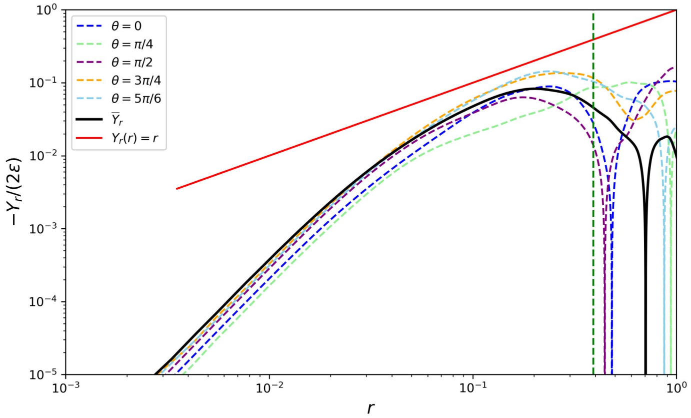
$$\overline{Y}_r(r) = -2\varepsilon r$$



Pr_m=1

Pr_m=10



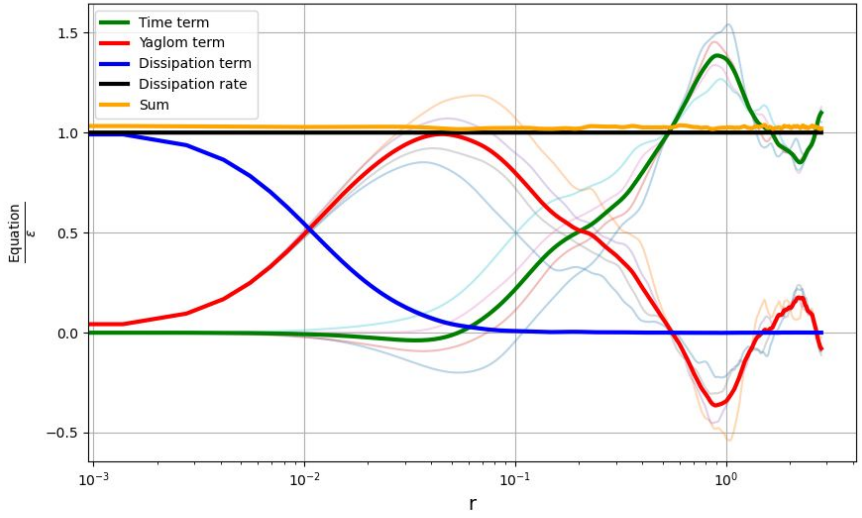


Pr_m=0.1

Radial component of the Yaglom flux Yr(r), normalized via −2ε, at several angles. The polar average (along θ) is reported with black line.

Energy Balance

Pr_m=1



Terms of the von Kármán–Howarth equation computed from the simulation.

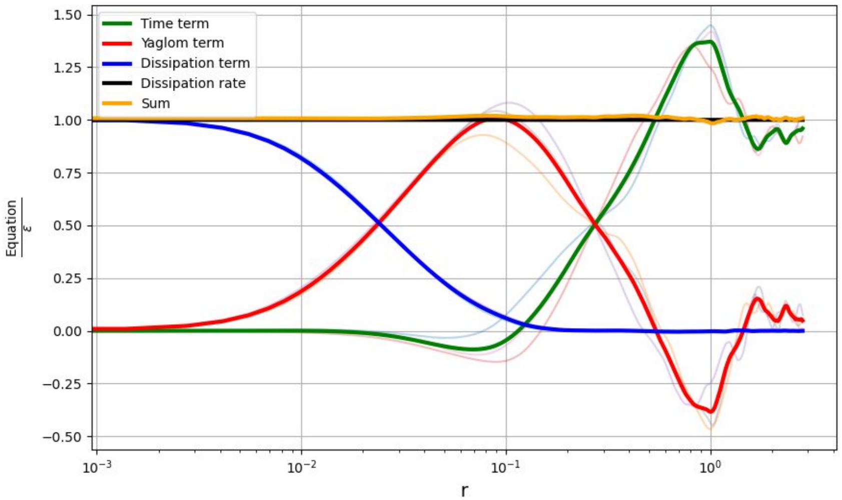
The colored curves represent: temporal variation (green), nonlinear transfer (red), and dissipation (blue), averaged over all directions of the separation r.

The sum (yellow line) matches the directly computed dissipation rate, confirming the energy balance.

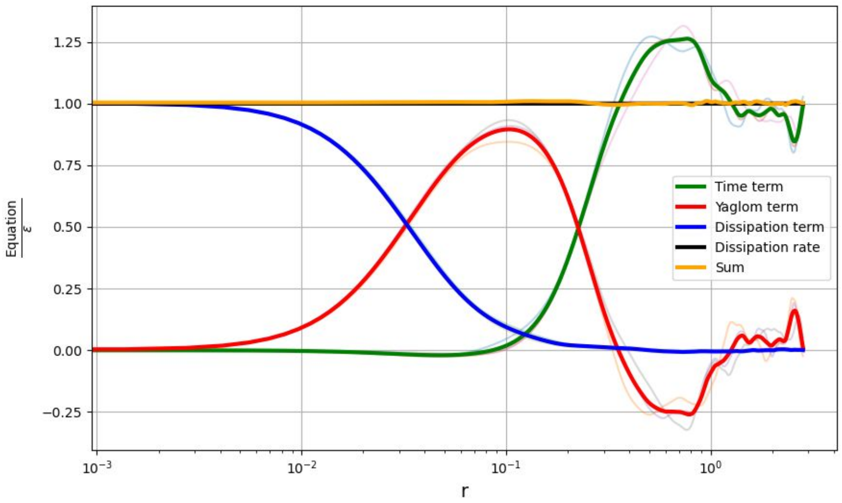
- **(Ideal Case):** The turbulent flux is fully developed (Yaglom term = 1) because both kinetic and magnetic channels dissipate simultaneously at the smallest scales.

Energy Balance

Pr_m=10



Pr_m=0.1



- Viscosity dampens the fluid at larger scales, but the dynamo redirects energy into the efficient magnetic channel. The Yaglom flux is preserved and still reaches the theoretical value of 1.

- High resistivity destroys the magnetic field. Without its magnetic partner, the cascade stops: the Yaglom term fails to reach the theoretical value of 1, and energy is lost to premature dissipation.

Conclusions

- The energy balance plot, showed the equilibrium between production, transfer, and dissipation, validating the analytical results and providing a consistent comparison.
- In summary, the results obtained allow for a comprehensive description of the energy cascade process on MHD turbulence.

Our results, supported by direct numerical simulations, could improve the interpretation of solar wind and magnetosheath observations.

SYNERGIES

Between Astrophysical, Space, Laboratory, and Fusion Plasma Physics

Thank you

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