Nordita PhD School Tutorial Problems

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Problem 1: Worldline EFT for Classical Electromagnetism

Classical electromagnetic dynamics for massive point particles can also be described in a worldline effective field theory formalism with interactions mediated by a massless vector field A_μ. Consider a massive particle with mass m and charge q.

(a) Using all symmetry constraints, write down the minimal (leading order) point-particle action. *[Hint: you should only use the dimensionless charge q as your coupling of the worldline to the gauge field A_μ.]*

(b) Build the action to higher orders by adding tidal interactions.

(c) Build the action to higher orders by adding multipole moments.

(d) Write down the equations of motion. *[You need to impose a gauge condition on the gauge field A_μ to have an invertible EOM. A common choice is the Lorentz gauge, $\partial_\mu A^\mu = 0$.]*

Problem 2: Massive Two-Body Scattering in EM

Consider two particles with charge q_1, q_2 and masses m_1, m_2 that are described by the leading order point-particle action from Problem 1(a).

(a) Calculate the gauge field A_μ produced by a single particle. Show that it reproduces the Coulomb field. *[How does this problem differ from the gravitational field generated by a massive particle?]*

(b) Calculate the leading order deflection and the waveform generated by the two particle interaction.

(c) Calculate the next-to-leading order deflection.

Problem 3: Compton Scattering in a Coulomb Background

As with the gravitational case, we can choose to study two-body electromagnetic scattering as an expansion about the background Coulomb field of one particle. We can also study the scattering of an incoming electromagnetic plane wave against this single source with charge q and mass m.

(a) By inserting the expansions $A_{\mu\nu} = \bar{A}_{\mu\nu} + \delta A_{\mu\nu}$ and $x^\mu = \bar{x}^\mu + \delta x^\mu$ in the action from Problem 1(a), derive the leading non-zero corrections to the background action for $\bar{A}_{\mu\nu}$ and $\bar{x}^\mu$. Integrate out the explicit dependence on δx^μ using its equation of motion.

(b) Derive the Feynman rules for the recoil operator.

(c) Calculate the leading order Compton amplitude for a photon scattering against a scalar charged particle.